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Engagement and online mathematics enrichment for secondary students

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Online may not be the ideal format for a mathematics enrichment event, but in some circumstances, it may be the only option available. This article considers a mathematics enrichment programme consisting of a series of masterclasses which were held live online for secondary students in the UK during the Covid-19 pandemic. The series of masterclasses were part of the Royal Institution of Great Britain's Mathematics Masterclass Programme which runs annually across the UK. In this study, we investigate how and to what extent students were able to engage with this series of online masterclasses. Learner engagement is researched through in-session observations, student work, attendance data, participant feedback and interviews. While the online masterclass series lost some of its traditional in person features, such as hands-on live social interaction and a university environment, it appeared that the participants perceived the online sessions as interactive enabling them to both enjoy the sessions and enjoy learning mathematics in the sessions. The evidence found suggests that the participating students could engage behaviourally, emotionally and cognitively in online mathematics enrichment. However, constructing mathematical knowledge in online sessions can be difficult for some students and social interaction may need to rely on existing social groups among school friends.

1. Introduction

Mathematics enrichment programmes offer students opportunities to develop their mathematical skills and confidence through a broad range of activities beyond the classroom setting. The Royal Institution of Great Britain (Ri) Mathematics Masterclass programme for secondary school students is UK-wide mathematics enrichment programme which has been running since 1981. The programme is organized as separate local series of masterclasses each typically consisting of six sessions held on Saturday mornings at a local university. Schools are invited to nominate students to take part in their local masterclass series. Each session is run as an interactive workshop lead by an expert in their field. The Ri's vision for these masterclasses is for them to offer an extended and in-depth exploration of mathematics beyond the classroom to show real-life applications of mathematics, inspire further engagement with mathematics and allow participants to grow in confidence and make new friends (The Royal Institution, 2018a). While the notion of mathematics enrichment has been long debated in literature (Barbe, 1960; Worcester, 1979;

Jones & Simons, 2000; Piggott, 2007; Nemirovsky *et al.*, 2017), the Ri's Masterclass vision reflects the aspirations of mathematics enrichment outlined in a variety of studies (see, e.g., Feng, 2010; Santos & Barmby, 2010; Richardson & Mishra, 2018; Wright, 2021).

Before the COVID-19 pandemic, the Ri Masterclass programme annually ran over 900 events and attracted over 6200 students (The Royal Institution, 2018b). However, during the pandemic, many series could not take place, were disrupted, or had to move online. The present study considers an online Ri Masterclass series held in Wales in spring 2021. The sessions took place at a time when schools in Wales were open but operating under strict social distancing measures and whole classes or year groups were repeatedly required to study from home for prolonged periods of time due to the need to self-isolate.

Studies have found that during these pandemic school closures, student engagement was low and unequal (see, e.g. Montacute, 2020; Vegas, 2020). The preferences of teachers at this time were reported to be for setting basic routine tasks which were perceived to help schools and teachers adapt to the new teaching environment while reducing the feelings of anxiety in children and preventing a widening of the attainment gap between more and less able students (Hodgen *et al.,* 2020). Yet, such tasks also limited student opportunities for mathematical discussion, metacognitive activities and problem solving, leading to a reduced level of challenge for high-attaining students.

In this context, there was perhaps a greater need for mathematical enrichment programmes, but it could also be questioned whether students would be willing or able to engage in yet another online activity, in particular, one that was not directly relevant to their school study (Lyakhova *et al.,* 2021). This served as motivation for the authors of the present article to examine how successful the online Ri Masterclass series in Wales was at engaging students and what lessons could be drawn for such events in future. In our approach, we use the notion of learner engagement which can be defined as a 'students' psychological investment in and effort directed towards learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote' (Newmann *et al.,* 1992, p. 12) and take the view that engagement with mathematics is a key outcome of mathematics enrichment (Santos & Barmby, 2010).

2. Background

2.1. Mathematics enrichment

There have been several attempts to define what is meant by 'enrichment' in mathematics in terms of descriptions of the types of events held or the desired outcomes and perceived benefits of such events (see e.g. Feng, 2005). However, 'enrichment' is often used without strict definition to simply refer to a wide range of activities taking place beyond the school curriculum such as summer schools, in-school clubs, workshops or series of masterclasses (Feng, 2005; Nemirovsky *et al.*, 2017; Gavin & Renzulli, 2021). While research on enrichment in mathematics is sparse, authors are consistent in their remarks about enrichment remaining under-researched, under-conceptualized and under-theorized (Feng, 2010; Nemirovsky *et al.*, 2017).

The outcomes and perceived benefits of mathematics enrichment for learners reported in the literature include their mathematical, personal and social development (see e.g., Young *et al.*, 2011), support for their mathematics learning at school (Feng, 2005, 2010) and exposure to new environments such as higher education (see, e.g. Ní Shuilleabhain *et al.*, 2020). Given that mathematical tasks can often be experienced as difficult and demanding, the desired outcome of a mathematics enrichment activity can be the sustained engagement of participants, an increased sense of confidence, a perception of being a more rounded mathematician or an appreciation that doing mathematics is not confined to exercises in a textbook (Piggott, 2007; Griffiths, 2010; Mun & Hertzog, 2018).

Mathematics enrichment is often valued for giving opportunities and motivation for participants to explore complex ideas and discover unexpected connections between apparently distinct areas of mathematics (Koshy *et al.*, 2009; Griffiths, 2010). Hands-on activities and topics that are of interest to students or that are presented in an engaging manner with a strong collaborative element have been argued to benefit a range of students. Through such collaborative activities, participants can develop an appreciation for the need to share ideas and engage in debate when doing mathematics helping to promote both independence of thought and creativity (Koshy *et al.*, 2009; Feng, 2010; Griffiths, 2010). Griffiths (2010) argued that enrichment activities were perceived as more successful when they were able to reveal some underlying beauty to the mathematical structures which appear to create opportunities for participants to encounter the aesthetic aspects of mathematics and see the discipline from a more holistic point of view.

Similar outcomes have been found when considering broader STEM (science, technology, engineering and mathematics) activities including more awareness and appreciation of STEM in real life, an increased motivation to choose STEM subjects in future, as well as gaining practical skills in working together, learning to apply knowledge in real-life situations and an increased perseverance in tackling unfamiliar problems (see, e.g. Van-Meter *et al.*, 2014; Kitchen *et al.*, 2018; Vennix *et al.*, 2018; Stringer *et al.*, 2020).

2.2. Engagement and online mathematics enrichment

Engagement is commonly defined as a multifaceted construct with three overlapping components: the behavioural, emotional and cognitive engagements (Jimerson *et al.*, 2003; Kong *et al.*, 2003; Fredricks *et al.*, 2004). The behavioural dimension draws on the idea of participation and includes learner behaviours to become involved in tasks. The emotional dimension refers to the affective reactions of learners to tasks, and the cognitive dimension builds on the idea of psychological investment in learning and incorporates thoughtfulness and willingness to exert effort to comprehend complex ideas and master advanced tasks. While engagement may begin with liking and participating, it can result in commitment and investment to enhance learning and so the notions of commitment, effort and investment are seen as central to engagement.

Behavioural engagement can be captured not only through attendance and participation data but also through observing participatory behaviours such as completing tasks, complying with class rules or being persistent when confronted with difficult problems. In an online setting, this can also be captured automatically through measures such as time on task, number of posts to a discussion board, as well as log in details (Henrie *et al.*, 2015). However, caution is needed as such observations can give little information on the quality of the effort, participation or thinking (Fredricks *et al.*, 2004). In contrast, in both online and in person settings, emotional and cognitive engagement are often captured through self-report measures such as surveys, by making inferences from observations, or through conducting interviews with students (Kong *et al.*, 2003; Fredricks *et al.*, 2004; Fredricks & McColskey, 2012; Henrie *et al.*, 2015). For example, emotional engagement can be captured through survey items on emotions related to class, classwork and people in class whilst cognitive engagement can be captured through survey items on metacognition, effort and volition and the use of cognitive strategies. Goal theory and self-regulation theory have been used in some research to capture the cognitive component (Fredricks *et al.*, 2004). Within these, being committed to understanding is contrasted with wanting to get a good grade, and deep-level learning (such as elaboration or organization) is contrasted with surface level work (such as memorization). In the online setting, cognitive engagement can also be explored through inference from a student's written messages by considering their behaviours of seeking, interpreting, analysing and summarizing information, critiquing and reasoning and making decisions in online discussions (Zhu, 2006; Shukor *et al.*, 2014).

Santos & Barmby (2010) used the concept of engagement to define enrichment in mathematics. They argued that the outputs of the engagement activities that they considered in a face-to-face environment (such as the enjoyment of doing mathematics, understanding new mathematics, a raised awareness of mathematical careers) corresponded to the three components of engagement. From this, they defined mathematics enrichment as those activities which successfully bring about engagement in mathematics. In this, they stressed that all three aspects of engagement were important to successful enrichment as it is important that students are involved in mathematical thinking and developing their knowledge and understanding as well as taking part in and enjoying the activities. In the face-to-face environment, the factors in planning an enrichment event that were reported to encourage engagement with mathematics included the involvement of hands-on activities, activities that were different from those completed in school and activities which demonstrated the usefulness and importance of mathematics.

Yang *et al.* (2018) in their review of research on online learner engagement found that most factors that were effective in encouraging engagement in face-to-face settings also apply to online learning environments. However, they emphasized that online learning is often characterized by the need for fewer resources but a need for greater concentration and time commitment from students. In their definition of online learning engagement, they proposed that the sense of commitment should prevail over any learning outcomes, so that positive behaviour and commitment may not result in high achievement but instead in a sense of well-being.

Indeed, there is evidence that encouraging engagement in an online learning environment can be more challenging than in an in-person setting. There can be an increased cognitive load, an increased feeling of isolation and higher drop-out rates which are suggestive of disengagement (Park & Choi, 2009; Onah *et al.*, 2014). Studies often highlight the role of cognitive engagement, including effort regulation, meta-cognitive regulation when performing online tasks in environments where the presence of a teacher and opportunities for social interaction are diminished (see, e.g. Wysocki, 2007; Shukor *et al.*, 2014; Lee *et al.*, 2015; Kim *et al.*, 2015).

While synchronous forms of interactions are known to stimulate learner engagement in online learning, safeguarding reasons often led during the pandemic to a preference for asynchronous forms of online engagement or synchronous engagement with limited live interaction (see, e.g. Hodgen *et al.*, 2020; Hodges *et al.*, 2020; Lyakhova, 2020). In one recent study of a mathematics enrichment programme which offered asynchronous video materials to post-16 students in spring/summer 2020 (Lyakhova *et al.*, 2021), participants were found to engage behaviourally, emotionally and cognitively, despite no-real time interaction, no controlled channels for peer support, no real-time live feedback and no external rewards, such as a grade or certificate. In part, the authors attributed this to mature approaches to learning in these students, who had self-selected to take part in the enrichment programme. Along with the materials that students found interesting, and the asynchronous format that they found convenient, Covid-19 played a role and engaging in the enrichment programme was perceived to compensate for feelings of anxiety, abandonment or boredom during the pandemic.

3. Planning and design of the online masterclass series

Before the pandemic, the organizers of the online Ri Masterclass series would annually arrange two programmes of in-person Ri Masterclasses: one for any student aged 13–14 and one for female students aged 14–15. Schools from across the South West Wales region (the local authority areas of Swansea, Neath Port Talbot, Carmarthenshire and Pembrokeshire) were invited to select two students from each age group (four students in total) to attend the programmes with a maximum of 60–80 participants accepted onto each programme. The programmes would run simultaneously across six Saturday mornings between January and April. Each class would last for 2 h and would take place in Swansea in a university lecture theatre. These in-person masterclasses were designed to be interactive with students working through problems and interacting with hands on practical activities.

Following the start of the pandemic, it was not possible to run in-person events and so it was decided to instead run a single series of four live online sessions over four Saturdays in April and May 2021. In compliance with the online safeguarding policy in Wales, it was decided to use the Microsoft Teams platform to run the sessions and that the interactions between students and presenters would be controlled. Each session would be led by a presenter assisted by a moderator and several administrators. In sessions, students would be able to use the chat box to ask and answer questions posed by the presenters, but no student video or audio would be allowed. Breakout rooms would not be used, and the sessions would not be recorded. No unsupervised or anonymous chat would be allowed between the students as all the participants would be required to be under supervision and to always be identifiable. The moderator would be responsible for monitoring the chat and would view student comments and pass these anonymously to the presenters. Administrators would be responsible for handling technical difficulties during the session with the students advised to email a designated administrator if they had a problem and did not want to, or could not, use the chat.

As limitations on both geography and space in a venue would not apply to the online sessions, schools from across Wales were invited to nominate as many pupils aged 13–15 as they wanted to attend the online series. In line with the Ri practice, schools were invited to subscribe students whom they considered mathematically able, curious and well-motivated, who would not otherwise have access to such enrichment activities and advise students that whilst the organizers aim to establish an enjoyable working atmosphere, the pupils who would derive the greatest benefit would be those who are most attentive and hard working.

Following the initial invitation sent to schools (185 schools in total), 249 students were registered to participate from 66 schools. There was a wide geographic spread with schools participating from 20 out of 22 local authority areas in Wales. Taking the number of students in each school in receipt of free school meals as a measure of deprivation, the profile of the registered students and their schools was similar to the overall national picture but with proportionally more students registered from schools with both the highest and lowest levels of deprivation (see Fig. 1). The median number of registered students per school was 3, with 20 schools registering more than the previous restriction of 4 pupils (see Fig. 2). More than half of registered students (52%) were female. Twelve of the registered participants had attended the in-person masterclasses held the previous year.

The number of applications considerably exceeded the organizers' expectations and so, for practical reasons, it was decided to split the cohort into two groups with students either attending sessions in the morning or early afternoon.

The pedagogical approach to designing the sessions was influenced by the experience of online teaching that the organizers had acquired through running the Further Mathematics Support Programme Wales (FMSPW) which provides online mathematics tuition for students aged 16–18 across Wales. A previous study of learner experience of FMSPW online courses found that learners that make the most of the course display characteristics of self-regulated learners (Lyakhova & Joubert, 2022), including initiating and taking notes, organizing their learning materials, revising and watching lesson recordings after lessons and asking for help from their peers, teachers and family. As the masterclass students were 2 or 3 years younger than the FMSPW learners, and so less likely to be self-regulated learners, it was felt necessary to include tips on how to make the best of the online classes in the first session (see Appendix C).

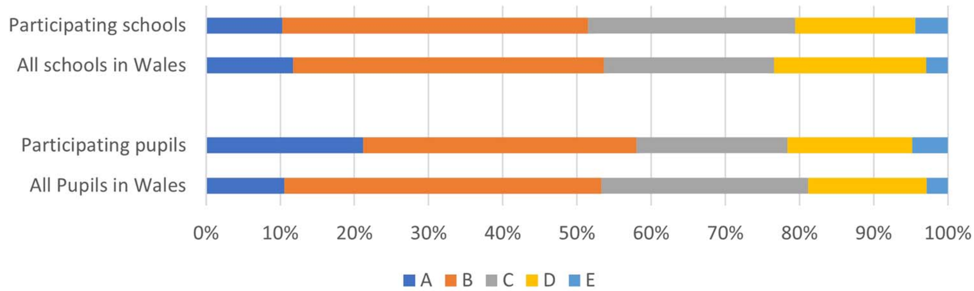

FIG. 1. Participating school and pupil data with comparison to all Wales data. Categories A–E correspond to the percentage of students at the school in receipt of free school meals (FSM) as a measure of deprivation. (A < 8%, 8% < B < 16%, 16% < C < 24%, 24% < D < 32%, 32% < E). School data details percentage of schools in categories A–E. Pupil data details percentage of pupils attending a school in categories A–E.

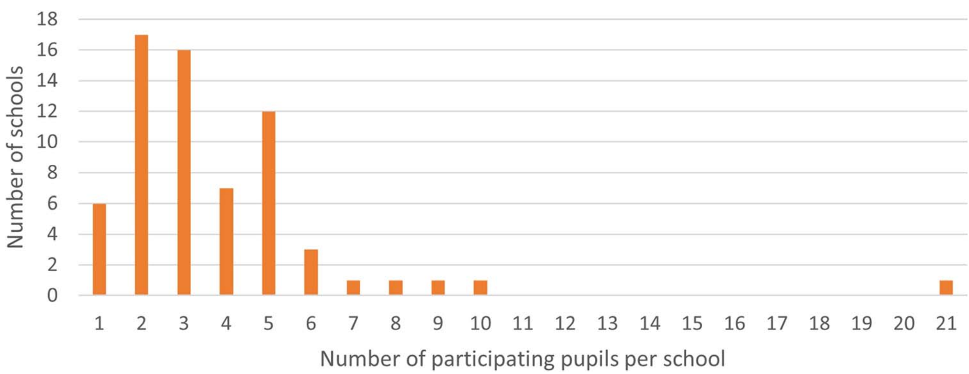

FIG. 2. Frequency plot of participating pupils per school.

As motivation for students to actively participate in the sessions, a presentation about a distinguished female mathematician, Maryam Mirzakhani, was included in the first session. This included Maryam's tips on doing mathematics to emphasize that succeeding in mathematics lies in effort, in asking questions, in allowing yourself time to work on interesting challenging problems outside the classroom and in working with other people (Mirzakhani, 2008).

Traditionally masterclasses would be structured with the presenter explaining new material to the whole class followed by extended periods of students working individually or in groups. The presenter would circulate around the class to monitor the work, ask questions and answer students' questions. This would normally be followed up by a plenary for the whole class. As it was not possible to reproduce this format online, it was decided to employ a whole class interactive lesson format instead. As online learning is known to be more cognitively demanding, the length of sessions was reduced to 1.5 h and presenters were instructed to build short regular breaks for students to work on questions and tasks and to encourage students to use the chat box to explain their solutions or ask questions. The breaks would also allow the speakers to reflect on the learners' comments from the chat. After such periods of individual work, the presenter would explain the solution to the problem relating their explanation to students' comments from the chat and also taking into account that there may be students who encountered greater difficulty while not disclosing this in the chat. Between half and two thirds of each session was devoted to individual work. Whenever PowerPoint slides were used, speakers were advised to ensure that the slides had some content prewritten and enough space for them to write mathematics live on screen to force a teacher presence and expose their mathematical thinking.

The presenters for the programme were chosen from a pool of experienced masterclass speakers who could offer sessions on a variety of topics to help the organizers to plan a programme that would include opportunities to do mathematics creatively (less traditional calculations and algebra) and could include hands-on activities. The topics had been presented in-person previously and so the organizers felt confident that they would interest students, however adjustments needed to be made to present the material online. The team of organizers met with each presenter in advance to explain the online platform and allowed interaction to decide on a general plan for each session.

The first class was on Combinatorics and was planned as an exercise in problem solving requiring no prior knowledge apart from the ability to count. It focussed solely on problems whose solution would be numerical. Two types of problems were considered in class, one set in a real-life context (the western calendar) and another about the properties of integer numbers. The second class was on selected topics from topology, requiring students to look at familiar shapes from a new perspective with new classes of equivalent shapes, different from Euclidean geometry, introduced. Both of these first two classes were based around PowerPoint presentations. The third class was also on topology but focussed on the Mobius strip. This class had a practical and hands-on emphasis where students were cutting and glueing shapes such as Mobius strips themselves. In this session, students watched the presenter's hands making shapes from paper and they were expected to make these along with the presenter. The fourth and final class was on Codes and Cyphers and involved an 'escape room' activity which was created using a Google document containing puzzles to do during and after the session.

It was decided that in addition to the online sessions, a collection of follow-up materials would be sent to students after each session to encourage them to look back and fill gaps in their understanding. These contained a pre-recorded video of the presentation or a link to a video available on internet and covering a similar topic and a list of problems and solutions in English and Welsh. After the first session, students were also invited to participate in a competition to invent a new problem that could be solved by methods similar to those considered in class and submit their solution via email to the organizers. This aimed to give the organizers an extra insight in how well students understood the material and if they were willing to spend extra time on doing mathematics after sessions.

4. Methods

We adopt the view of Santos & Barmby (2010) that engagement is the key output of a mathematics enrichment activity. Following Newmann *et al.* (1992), we define engagement as a 'students' psychological investment in and effort directed towards learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote', and consider engagement to be constructed from the three components: behavioural, emotional and cognitive. To address how and if students engaged with the online mathematics masterclass series, several types of data were collected in two separate phases and then mapped onto these three components (see Table 1).

In the first phase of the research, data were collected during and immediately after each masterclass session. This was done through the collection of attendance data, recording the observations of the administrators, moderators and presenters, an online feedback survey of students completed at the end of each session and an analysis of the competition entries submitted by students after the first session.

TABLE 1. *Summary of data sources and mapping to the components of engagement*

Phase	Data source		Components of engagement		
			Behavioural	Emotional	Cognitive
1: Collected during/after each session	Attendance data		✓		
	Observations of administrators, moderators and presenters		✓		
	Online feedback survey	Perceptions of the class (five-point scale)		✓	
		Free text responses on what students enjoyed most and least	✓	✓	✓
	Competition entries (first session only)		✓		✓
2: Collected at end of programme	Participant interviews		✓	✓	✓

The attendance data was collected at every session and consisted of the number of registered students who logged in at the start of the session. This was used in the analysis of the behavioural component of engagement.

The observations of the presenters, administrators and moderators were collected through informal conversations at the end of each session and recorded as notes on their perceptions of how the session went, the behaviour of students in the session and how students interacted using the chat. These aspects were also used to inform the analysis of the behavioural component of engagement.

All participants were asked to complete an online feedback survey at the end of each session (see Appendix 1). As this survey was to be repeated after each session, a short survey was used covering the students' overall perceptions of the class, what they enjoyed most and least, and if they experienced any problems with the administrative or technological aspects of the class. Student perceptions of aspects of the class were rated on a five-point scale, and these data were used in the analysis of the emotional component. The other questions of the survey used free-text responses to collect rich data which could potentially be used in the analysis of all three components. Within these free-text responses, comments about student's developing interest, motivation and appreciation of mathematics were considered in relation to the emotional dimension whilst comments on their own learning that revealed aspects of their metacognitive strategies and approaches and comments about mathematical content of the sessions and motivational goals were considered for the cognitive dimension. Responses indicating learners involvement or non-involvement in tasks were mapped to the behavioural dimension. The free text responses to the questions on aspects that the students enjoyed the most and least were also analyzed thematically. The themes used in this analysis were identified from the survey responses, informed by the common outcomes and benefits of engagement events identified in the literature.

In total across the four sessions, there were 311 responses to the survey consisting of 128 responses after the first session (56% response rate), 60 after the second (28%), 62 after the third (30%) and 61 after the fourth (28%). To encourage participation, the survey was completely anonymous, and consequently, it is not possible to say if the same or different students completed the feedback after each session.

Competition entries were collected after the first session via email and were marked by the presenter. The presenter made a note of entries being valid, that is, clearly written or typed, relevant and mathematically sensible and these aspects were considered against the cognitive component whilst participation in the competition was considered in the behavioural component.

The second phase of the research was conducted after the masterclass programme had completed. Students who had participated in the masterclasses were invited to give a telephone interview and some 26 responded. From these volunteers, six were selected for interview. The selection was made to ensure participants came from a range of schools including Welsh and English speaking schools, urban and rural locations, schools with high and low measures of deprivation and schools with higher proportions of ethnic minority students. One of the interviewed students had attended the masterclasses in-person the year before.

The students were interviewed remotely either by telephone or via an online platform. The interviewer was not involved in the delivery of the masterclass programme and followed a pre-prepared list of open ended questions (see Appendix 2). Some of these questions followed up on the thematic analysis of the phase one data collection, focussing on aspects of the interaction in the classes, social aspects of the classes and their preferences between online and face-to-face classes. The interviews also covered two themes where there were limited findings from the phase one data: how students coped when they found the mathematical content challenging and how they used the follow-up materials that were provided after each session. Transcripts from these interviews were then analyzed with responses categorized using the three dimensions of engagement not only for the purposes of identifying patterns but also for understanding the themes identified in the analysis of the survey. Again, comments on their developing interest, motivation and appreciation of mathematics, were considered in relation to the emotional dimension, those relating to participating in activities were considered for the behavioural dimension, and those related to their metacognitive strategies and approaches, the mathematical content or their motivational goals were considered for the cognitive dimension.

5. Findings

In the following sections, we consider the data collected from both phases of the study against the three components of student engagement (see Table 1).

5.1. Students participating in the masterclass (the behavioural dimension)

Attendance across the series was sustained with 85% of participants attending all sessions, and only two students missing more than one session (see Fig. 3).

From the observations of organizers and presenters, students were perceived to display positive behaviours during the sessions as they were seen to follow instructions and participate in tasks with no examples of misbehaviour recorded. The students' comments in the survey also commonly mentioned on-task behaviours highlighting learning, doing mathematics, solving problems and investigating as well as interacting with the class (see Fig. 4). This was also reflected in the interviews with all the interviewed students reporting that they made notes during the sessions. Students in the interviews also indicated that they were working out questions on paper whilst the presenters were working on the screen, e.g., 'when they asked questions, I was working it while they did it [on screen]'.

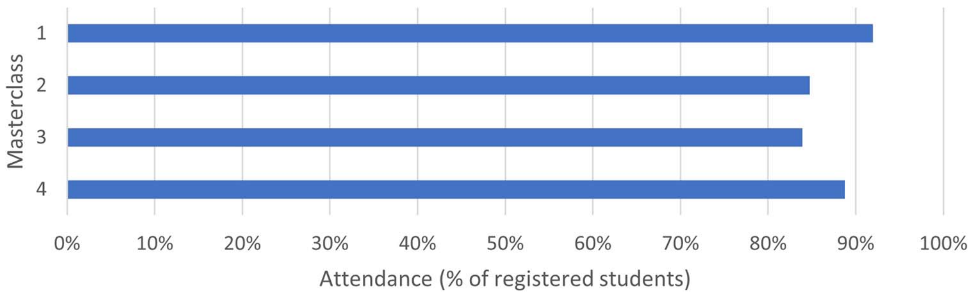

FIG. 3. Attendance at each Masterclass as a % of overall registrations.

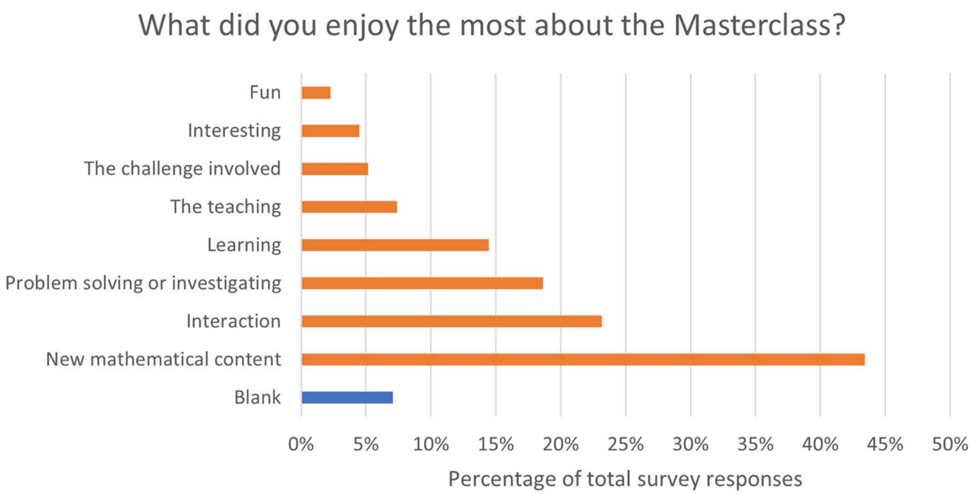

FIG. 4. Themes identified in free text responses to 'What did you enjoy the most about the masterclass?' aggregated across all masterclasses. Some responses categorized in more than one theme.

Indeed, in all sessions, it was noted by the chat moderators that students enthusiastically interacted through the chat to answer questions posed by the presenter with around 40+ chat entries registered to most questions. One interviewed student described how he used the chat:

'. . . like every time they had a question that they would tell us to write it in the chat and then they would read out some of the answers, which was really good.'

In some instances, students posted comments that they found a mistake in their answers posted earlier. Students also corrected other student's mistakes in the chat, e.g. saying that they also found the same incorrect answer first and then they noticed what they did wrong. Some also used the chat to ask for technical help from the administrator and, as the sessions progressed, to seek and give peer help to sort out more subtle technical difficulties.

As well as note taking and interacting with the chat, the interviewed students also reported doing other activities during the sessions for example, in the third session which involved practical activities, one of the interviewed students noted that:

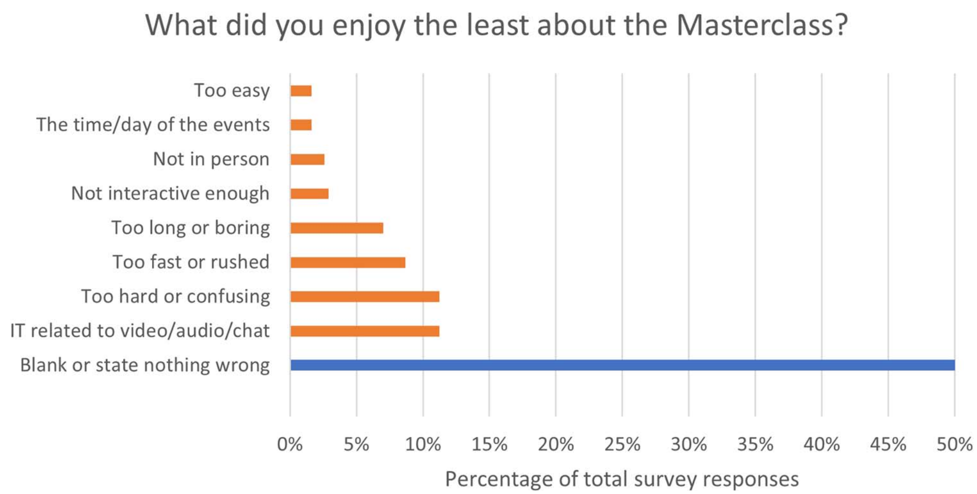

FIG. 5. Themes identified in free text responses to 'What did you enjoy the least about the masterclass?' aggregated across all masterclasses. Some responses categorized in more than one theme.

'there was a lesson in which we stuck things with paper like the Mobius strip. So I found that really nice, so I was basically doing everything that the host of the teacher was doing and yeah it was fun.'

This session was also highlighted in the survey comments as making students feel particularly involved because the session gave participants 'something to go away and try'. As one student put it, they were 'physically taking part, rather than writing on paper'. Common terms used in such comments included making or creating shapes, 'making experiments with paper', 'creating models using paper', 'doing practical things' or simply 'physically interacting with the material'. One of the interviewed students reported cutting out the shapes with her family after the workshop which she enjoyed.

While many of the survey comments indicate active participation, a small number of survey respondents felt there was not much for them to do, describing themselves as 'being idle for the entire session', or that it was difficult for them to actively partake in the sessions because of the technology (see Fig. 5). For example, some students commented in the survey that:

"At times, there were a few technical problems. I was unable to view the whiteboard at the beginning."

"The [MS Teams] notifications from other people blocked me from typing [in the chat]."

"The video was glitchy, so I could not watch it."

"I could not use the chat in the beginning."

Those who could not access the chat in the first session due to technical problems were active in seeking help about this, with more than 70 emails exchanged between students or parents and the administrator during the first session.

The interaction between students in the sessions was limited by their lack of cameras and microphones. However, there is evidence that students were interacting through pre-existing social groups. The interviewed students who joined the classes with school friends described reaching out to them both during the classes and after sessions:

”. . . when there was a question we spoke together and answered so it was quite fun.”

“. . . every time after the masterclass, we would go back to school and then talk about how it went and was it easy or not, stuff like that.”

However, this was not the case for everyone. Another interviewed student explained that although they knew there were other people from their school in the sessions, they did not know them personally and so did not engage with them.

After the sessions, some of the interviewed students reported looking back at their notes as was encouraged in the classes:

“it’s very useful, so afterwards I could look bac*k* on if there’s anything I didn’t understand and go over it again”.

The follow up materials provided after each session were also accessed by all the students in our interview sample, but none of the students completed all the activities for example:

“I watched some of the videos and I did about half of them.”

“I kind of rushed and but still I enjoyed doing it.”

“I gave some of them a go . . . .yeah they were enjoyable yeah.”

One interviewed student shared his reason for not engaging with the follow-up materials:

“It’s just because we’ve been having lots of tests recently and I don’t really want to [do extra work].”

Only 36 students (16% of those attending the first session) entered the competition which included 22 entries from girls. All but three entries were valid. Most solutions appeared to be well-presented with some entries type written.

5.2. Interest, motivation and appreciation of mathematics and the masterclasses (the emotional dimension)

The feedback from both the survey and the interviews demonstrated that most students felt positive about the masterclass series. In the survey, there were many more comments identifying what students enjoyed the most rather than what they enjoyed the least (see Figs. 4 and 5), and all students who left comments in the least enjoyable category, also left comments in the most enjoyable category. Across all survey responses, 94% said that they would recommend the sessions to others, whilst most students rated the content of the sessions and the quality of delivery excellent or good (see Fig. 6).

Within the survey there was a range of attitudes displayed in the student comments which can be broadly categorized into four groupings; those with a completely positive attitude (group A), those who were rather positive but with minor or qualified negatives (group B), those with both clear positive and negative statements (group C) and those who displayed a generally negative attitude to the experience with no substantive positive (group D). Illustrative examples of comments from students in each of these categories are shown in Table 2. The distribution of survey responses between these four groups shows a positive attitude overall with 68% of responses in categories A and B (see Fig. 7).

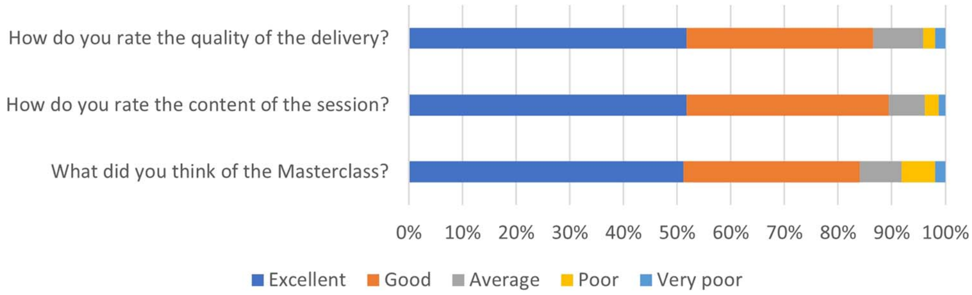

FIG. 6. Overall survey responses on what students felt about the masterclasses.

TABLE 2. *Selected student opinions*

Group	'The most enjoyable' comment	'The least enjoyable' comment
A	*'The lesson was at a good pace to follow and overall was very interesting.'*	*'I didn't dislike anything about the masterclass.'*
B	*'I enjoyed discovering methods, rather than just thinking about answers.'*	*'At times, there were a few technical problems. I was unable to view the whiteboard at the beginning.'*
C	*'I enjoyed learning about the various ways in which mathematics can be implemented.'*	*'I believe that the masterclasses became tedious after a while, as one and a half hour long sessions can become monotonous.'*
D	*'Something else to do.'*	*'Lengthy and didn't really captivate my interest.'*

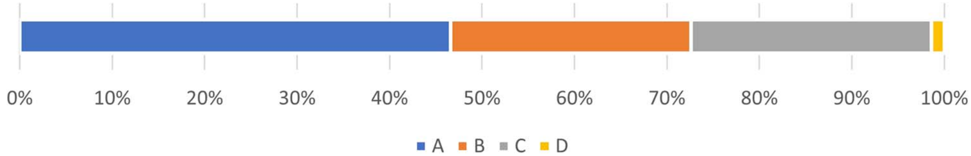

FIG. 7. Distribution of survey comment responses between groups A–D (see Table 2).

From the survey (see Fig. 4), we can see that across the sessions participants most commonly enjoyed the new mathematical content, interacting in the sessions, engaging in problem solving and investigating and the opportunity to learn. Notably, for the third session in the series, the most common theme for the most enjoyable aspect was its interactivity (56% of responses compared to 23% overall) with a corresponding reduction in comments on its mathematical content (27% of responses compared to 43% overall) and problem solving (0% of responses compared to 19% overall).

In these comments, the mathematical content in the sessions was appreciated because it was new ('lots of different things like topology'), contextualized, different from school ('not normally in GCSE curriculum or we learn in class'; 'not just questions and equations') or more advanced. There were comments on making surprising discoveries about mathematics, e.g. 'the link between normal everyday things to mathematics advanced questions', 'a simple thing such as a calendar could be used to make a variety of interesting questions', 'maths could be used for mostly anything' and that cutting and shaping paper that was done 'as part of the maths'. One of the interviewed students said that attending masterclasses 'made it seem more, like a lot more variety' in terms of mathematical topics. Another explained that learning new mathematics led them to feel that they were ‘more innovative’ in mathematics in general.

Both an expectation of doing mathematics or learning new mathematics and learning new things about mathematics as well as their previous positive experience of doing mathematics were motivating factors for students to join the masterclasses according to the interviews:

> “Learning new maths is good, because you can learn lots of different things that you usually wouldn’t and that’s why I like coming to the masterclasses.”

> “I find maths enjoyable so I thought this would be really fun.”

It follows from the remarks of the interviewed students that enjoying the first session was a good enough reason to continue attending the classes.

In terms of online interaction, both survey participants and interviewed students enjoyed that there were a lot of questions to answer, that they could be active in the chat, that they ‘didn’t just have to listen to 1.5 hours’, that they had chances to ask questions and felt being ‘listened to by the teachers’. From the interviews, it appeared that interviewed students enjoyed interacting and the teaching in the online masterclasses especially when comparing it with their experience of online school classes:

> “It was a lot more challenging [in the masterclasses] and it was better because you could hear them explaining it to you, instead of just putting worksheets on like Google classroom.”

> “I think the masterclasses were a lot better [then school] because it’s easier to follow this, more explaining things.”

> “I preferred [masterclasses] to [school] online learning because I felt it was more interactive.”

> “In school it was okay [but] it wasn’t very interactive, and it was more just drilling the knowledge in your mind”.

Among the responses in the survey categories C and D, the least enjoyable experience was often associated with the format of the sessions. There were students who were frustrated about problems arising from technical glitches and streaming problems, and some who described the classes as lengthy, ‘monotonous’, ‘tedious’, or boring and not engaging. While the survey did not show much preference for face-to-face classes over online classes, some interviewed students expressed a clear preference for face-to-face masterclasses because they were missing the social interaction. For example, the student who attended the masterclasses series in person the year before explained:

> “I enjoyed the in person one way more because I felt like it was more interactive, and I did make friends like throughout it but with this one it’s hard to make friends when everyone’s watching the same video.”

Finally, we would like to remark that some of the survey participants reflected aesthetically about the mathematics they experienced in class, noticing ‘astounding properties’, ‘clear and elegant solutions’ and connections between topics, and referring to creativity in doing mathematics.

5.3. Learning habits and participants’ remarks on effort, doing and comprehending mathematics in sessions (cognitive dimension)

Learning mathematics, that is ‘actually doing maths’, solving problems or investigating, was a common thread among the themes identified in the survey comments about what students enjoyed the most (see Fig. 4). However, both among the comments on the most and the least enjoyable aspect of the masterclasses comprehending mathematics were associated with a challenge.

In the survey, students raised different aspects of the nature of this challenge. Some highlighted that the mathematics was different or novel whilst others highlighted that they had to work independently. For example, one of the survey comments said that ‘the problems didn’t have an immediate apparent solution’ which for this student ‘led to solving them [being] more rewarding and enjoyable’. It was noted that emphasis on the method, rather than getting to the answer, was seen as something that made some students think, so they worked to ‘discover methods rather than just thinking about the answer’. In contrast another student noted that they had ‘to think and to solve problems ourselves’ and some reflected on how they needed to be persistent, e.g., ‘I found a few things a bit fiddly, but I understand that that was part of working it out myself.’ Similarly, in interview one student described that when faced with something they did not understand they would continue trying: ‘To understand was a bit tricky, but when you are, you know, looking at properly I got it in the end, but it just took me a few tries.’

Technical issues were mentioned by some students in the survey as a reason for having difficulty in understanding the material, e.g. ‘the video would lag and be hard to understand’ and a feeling that when their connection dropped they ‘likely missed some information’ and after that could not understand what to do.

A theme of delayed understanding (‘took me a while to understand’) and, in particular, relying on teacher explanation was notable both among the survey comments and interviews:

> “There were a few things that were difficult to understand but they were all explained after.” (Survey participant)

> “A certain question I didn’t understand and I only started to understand as we went through the method.” (Survey participant)

> “a few things I found a little bit tricky, I got them after they’ve been explained properly . . . . I waited for them to explain.” (Interview participant)

Whilst some of the interviewed students explained that they would ask questions straight away in the chat if they did not understand, some others decided to wait for someone else to do this because, as one student put it ‘usually somebody else would have asked a question anyway.’ Another interviewed student used a combination of strategies including reaching out to their peers:

> “I did wait for the teacher to explain and then, when I went to school on Monday, I discussed it with my friends and they explained it to me, to the best of their ability and yeah I kind of got it then.”

Evidently, there were students who left sessions while still not understanding something (11% of the survey participants, see Fig. 5). As one survey participant put it ‘The thing I enjoyed least about this masterclass was that I didn’t completely understand all of the work.’ We observe that many of these were able to identify particular elements that they could not understand:

> “I didn’t understand the question which the method at the end was showing me how to do.”

> “I was confused at times by the definitions of some terms.”

> “There was something called rh2 that I didn’t understand and wasn’t explained.”

> “I didn’t fully understand the way the enigma machine works.”

> “I did not fully understand the given explanation of a Klein bottle.”

In relation to persevering with difficult mathematics, for some students who felt positive about their learning, the challenge and interest of the more advanced questions served as a motivator to 'push and improve in maths'. Learning about mathematicians (both presenters and historical figures) was also mentioned in this respect. Some other factors that helped participants to learn appeared to reflect the design of the classes. Survey respondents appreciated that they 'could interact with the presenters anonymously' while not feeling 'pressured to answer quickly'. This seemed to be supported by the pace of the sessions: '[the sessions] were quite interactive allowing people to attempt to figure out the answer before it got explained'. More generally, students reflected on the high quality of the teaching emphasizing the explanations given by the teachers, 'the level the work was set' and the building in of pauses to work on the questions.

Survey comments of the least enjoyable aspect of the masterclasses give a glimpse on cognitive strategies that students wished they could employ. Some wished they could pause the video to go through it again, had more time for explanations or more time to complete the tasks and problems. There were remarks about presenters adopting a faster pace as they approached the end of the session, which made it harder for participants to follow the material. Others had wanted clearer instruction on 'what notes to take' and some found visualizing verbal explanations or reading diagrams online difficult and there were those who wished for being given more than one way to solve problems.

Enthusiasm for continuing to work on novel mathematics was reflected in the range of problems students presented as competition entries after the first session. From the 36 competition entries received, most demonstrated a good understanding of the mathematics learnt in class. Some seven submissions contained a problem and an answer (usually correct) but no working or a statement that the solution method would be the one discussed in class. Five submissions were only a slight variation on the questions presented in class and nine submissions considered questions that were already solved in class. For example, one problem solved in class initially considered cases of a certain parameter $n = 2$ and $n = 3$ and then the method was generalized for values of n between 2 and 10. A corresponding new problem presented as part of the competition considered the same problem for one value of the parameter, say $n = 8$, that was already covered by the general case considered in class. Finally, eight submissions were more original including questions applied to entirely new situations or contexts or attempting to combine several types of problems considered in class in one new question. Although solutions to these appeared to be incomplete, they were mathematically interesting.

6. Discussion

Our primary aim in this study was to assess how and to what extent students were able to engage in a series of live online mathematics masterclasses during the Covid-19 pandemic. To achieve this we considered evidence using the three dimensions of engagement: behavioural, emotional and cognitive.

The evidence for emotional engagement was perhaps the clearest and strongest dimension in this study. Our results show that students in the masterclasses displayed positive feelings towards both mathematics and the masterclass series. There is evidence that the surveyed participants felt positively about the teaching and learning in the sessions and how interactive the sessions were. This was despite some students having technical difficulties, some not being able to understand all the mathematical content and the need for students to work independently. The ability to interact with the mathematics by working along at home and with the presenter and the class through the chat function were clearly appreciated by the students. Through the programme the perceptions of interaction from the students varied from class to class with many different aspects of interaction highlighted including solving problems, responding to questions in the chat and engaging in hands-on activities.

The factors that students reported that they enjoyed about the masterclasses such as the mathematical content, the problem solving and the challenge involved can all be seen as typical for mathematics enrichment and would coincide with the expectations from a traditional face-to-face event (Feng, 2005). The percentages associated to the themes identified in Fig. 4 are lower than one may expect for an enrichment event but this could be a reflection on the wording of the question as it asked students to identify the single aspect they enjoyed the most and least and so most students comments fell only into one of these themes. There are also some responses that would typically be expected in such a survey that are missing with no comments about students making friends or working with likeminded students. Indeed, this aspect of a face-to-face enrichment event was missing by design in this series due to the restrictions placed on student camera and microphone use in the safeguarding rules. Such rules allow online events to take place but do so at the expense of social interaction. Instead, the interviews revealed that social interaction between students occurred in this series only through their existing social groups and social media channels both during and after the events.

However, it cannot be overlooked that the context of the Covid-19 pandemic played a role in the emotional engagement of students. The repeated lockdowns and school closures coupled with 'emergency remote teaching' (Hodges *et al.,* 2020) that students had experienced over the preceding year could have influenced their perception of the interaction and experience provided by the masterclasses. In both the interviews and comments, students compared the level of interaction in the masterclasses positively with their experiences of emergency remote teaching from school. Indeed, very few students highlighted that the lack of in-person classes was the worst aspect of the masterclass programme, but this may have been because students had accepted that such events were not possible.

In the behavioural dimension, there is also evidence that students were participating actively within the masterclasses, but one may argue that this evidence is limited. In the online environment, we have seen evidence of behavioural engagement through attendance, student participation in the chat and through the lack of negative behaviours in the sessions. However, as has been noted with behavioural engagement online in general (Yang *et al.,* 2018), the evidence of non-participation is not perceptible. For example, attendance data tells us that students logged on to follow sessions but not whether they then watched and followed the content. Similarly, our evidence indicates that many students were using the chat actively and enthusiastically, but for those who did not use the chat it is not possible to draw conclusions on whether they were engaged or not. For example, the evidence in the interviews suggests that some students who were not writing in the chat were actively following the messages, writing notes and working on paper. The lack of evidence on every participant, together with the evidence of students not understanding some mathematics in the sessions, may encourage one to ask if there were students 'silently' disengaging, which this study has not registered. However, given that the masterclasses were voluntary, and participation was not extrinsically rewarded, the data on attendance suggests that many students did find a way to engage and benefit from the classes as attendance across the classes was high with little variation between sessions.

The wide geographic spread of schools suggests that moving to an online series had broadened access to areas and schools which would not previously have had the opportunity to engage in the Ri Masterclass programme. The profile of schools who participated is similar in terms of deprivation to the national profile of schools. However, the profile of students who participated in the programme is skewed towards those from schools from the least deprived areas which suggest that schools from such areas are more likely to successfully encourage greater participation in such events from their pupils. Statistics on the number of students nominated by each school illustrates that some schools did nominate more students than was previously possible whilst others only registered 1 or 2 students. It would be interesting to understand how teachers make decisions about such nominations and whether coming from a larger group encourages greater engagement.

The findings on cognitive engagement show that constructing meanings may be difficult for students in an online masterclass. Comments that the least enjoyable aspect of the masterclasses was not being able to understand the content are not typical for face-to-face enrichment. Such comments are supported by studies on teachers teaching online during the pandemic (Crisan *et al.,* 2021) who felt that they were not able to judge student understanding in online classes. This represents a challenge for enrichment where the content is deliberately challenging, the instructor does not typically have experience with the students and the sessions are one off events. However, the fact that students were able to identify bits of mathematics that they did not understand, does imply that students were cognitively engaged. Indeed, from our findings it could be hypothesized that students needed to constantly check their understanding against what was happening on screen in order to make judgements about their understanding.

More generally our findings suggest that online masterclasses are cognitively demanding, and we would propose that this needs to be taken into account when planning the content and delivery of online enrichment. The strategies that the interviewed students employed when they got stuck or did not understand something (working together, revisiting questions, making decisions to wait for teacher explanations) were self-initiated and could be interpreted as self-regulation strategies (see, e.g., Lyakhova & Joubert, 2022). The emphasis placed on waiting for explanations from teachers in some students' quotes could be perceived as students relying too heavily on teachers instead of actively working through problems themselves. However, it is important to note that the quotes also highlight that students sought explanations specifically for the parts they found challenging, indicating that they had already put in some effort to identify areas of difficulty. Moreover, the quotes suggest that students perceived their understanding to have improved as a result of the explanations provided. This observation may be attributed to the intentional design of the series, which requested presenters to place particular emphasis on providing clear explanations.

Our findings also suggest that the chat function is a powerful tool for interaction, which is in line with other studies on online engagement (Hidayah *et al.*, 2021). For example, a study by Shukor *et al.* (2014) found that posting written messages in a virtual learning environment was significant for student cognitive engagement. However, our study would have benefited from a more elaborate analysis of the messages posted in the chat.

While only a small proportion of students took part in the competition, the quality of the entries implies that those who did had engaged with the questions in class and were happy to spend extra time doing mathematics and took the trouble to write their solution, to take a picture and send an email. This may be interpreted as engagement resulting in achievement in this small group of students. However, it seems that for the majority, the result of the engaging in the maths masterclasses is well-being (feeling positively) rather than achievement (cf., Yang *et al.,* 2018).

There were also comments in the survey data to indicate some students were making aesthetic judgements about the mathematics in the masterclasses which stood out to the authors. Following convention, we placed these comments with the emotional dimension; however, we agree with Sinclair (2004) that aesthetics play an important role in motivating and sustaining inquiry and is intimately connected with cognitive growth. Sinclair further argues that the emphasis school mathematics places on the mechanics of solving problems might discourage students from recognizing the value of aesthetical judgements and operating aesthetically when doing mathematics. From this point of view, the masterclass environment should encourage the aesthetic aspect of mathematical inquiry, and it is positive to see students engaging in aesthetic judgement about the content of the classes.

Prior attainment is one of the factors that can influence engagement, and, in this respect, the findings of our study are limited, as no data on the background of the participants is available and, as noted above, we have no knowledge of how participants were actually selected by the schools. We can only speculate that if schools followed the advice on selecting their students given by the organizations (as described in section 3), high achieving students would have been selected. More data may need to be collected to understand if positive engagement in our study could be attributed to participants' prior experience of success in mathematics and/or online learning. Further limitations of the study include that the feedback was collected anonymously thus not permitting to compare the experience of students coming from schools in low and high areas of deprivations. We are also unable to compare the answers of students who previously attended the masterclasses in person with those who only experienced it online.

In conclusion, our findings highlight that students can engage in online mathematics enrichment programmes but engaging in online enrichment is different to engaging in traditional in-person enrichment. Online enrichment classes are more cognitively demanding and offer less benefits in terms of social interaction. However, where access to in person enrichment is limited, such online programmes can play an important role in engaging students in mathematics learning. Focusing on mathematics while not having access to peer support and teacher help is an important feature of online classes and may result in some students perceiving that they do not understand all of the mathematics in the classes. This needs to be taken into consideration when designing classes. Yang *et al.* (2018) highlighted that the design and implementation of online learning needs to be connected to the engagement framework.

In this direction, many studies of online course design have used the Community of Inquiry (COI) framework (Garrison & Arbaugh, 2007) which suggests that three elements—teaching presence, social presence and cognitive presence work together to facilitate effective learning process in online environments. Recently, the COI framework was studied in relationship with the engagement framework among online graduate students (Kucuk & Richardson, 2019), where it was found that the three presences could be predictors of four forms of engagement: emotional, cognitive, behavioural and agentic (students' constructive contribution to the flow of the instruction they receive). For example, an increased cognitive presence was found to be a predictor of cognitive engagement of students, and an increase in both cognitive and teaching presence was found to be a predictor for emotional engagement. Kucuk & Richardson (2019) suggested that the COI framework could be considered for design of online courses to improve engagement. We agree that this could apply to the masterclasses series considered in this study. While the organizers guided by safeguarding policies may have limited freedom in orchestrating the social presence, the teaching presence can be modelled with more freedom and is found to influence cognitive engagement as well as being a predictor of other forms of engagement and learner satisfaction. Indeed, one may hypothesize that overall satisfaction of the masterclasses noted in the present study, may have been due to the teaching presence. Overall, the importance of the design of masterclass sessions and the online environment used should be considered in future in light of the COI framework.

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