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Comparison of the Gottfried and Adler sum rules within the large-N_c expansion

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Abstract

The Adler sum rule for deep inelastic neutrino scattering measures the isospin of the nucleon and is hence exact. By contrast, the corresponding Gottfried sum rule for charged lepton scattering was based merely on a valence picture and is modified both by perturbative and by non-perturbative effects. Noting that the known perturbative corrections to two-loop order are suppressed by a factor $1/N_c^2$, relative to those for higher moments, we propose that this suppression persists at higher orders and also applies to higher-twist effects. Moreover, we propose that the *differences* between the corresponding radiative corrections to higher non-singlet moments in charged-lepton and neutrino deep inelastic scattering are suppressed by $1/N_c^2$, in all orders of perturbation theory. For the first moment, in the Gottfried sum rule, the substantial discrepancy between the measured value and the valence-model expectation may be attributed to an intrinsic isospin asymmetry in the nucleon sea, as is indeed the case in a chiral-soliton model, where the discrepancy *persists* in the limit $N_c \to \infty$.

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1. Introduction

Alone among the various sum rules of deep inelastic scattering (DIS) the isospin Adler sum rule [1] has the special feature that its quark–parton model expression

$$I_A \equiv \int_0^1 \frac{\mathrm{d}x}{x}\big[F_2^{\nu p}(x,Q^2) - F_2^{\nu n}(x,Q^2)\big] = 2\int_0^1 \mathrm{d}x\,\big(u(x) - d(x) - \bar{u}(x) + \bar{d}(x)\big) = 4I_3 = 2 \tag{1}$$

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coincides with its QCD extension and receives neither perturbative nor non-perturbative corrections (for a discussion, see Ref. [2]). Moreover, this sum rule is supported by the existing neutrino–nucleon DIS data, which show no significant Q^2 variation in the range $2\ \mathrm{GeV}^2 \leqslant Q^2 \leqslant 30\ \mathrm{GeV}^2$ and give [3]

$$I_A^{\exp} = 2.02 \pm 0.40. \tag{2}$$

Though the error-bars are quite large, the precision could in principle be improved by future νN DIS experiments at neutrino factories (for discussion of such a program, see Ref. [4]).

Within the quark–parton model, the corresponding isospin sum rule in the case of charged-lepton–nucleon DIS has the form

$$\begin{aligned} I_G(Q^2) &\equiv \int_0^1 \frac{\mathrm{d}x}{x}\left[F_2^{\ell p}(x, Q^2) - F_2^{\ell n}(x, Q^2)\right] \\ &= \frac{1}{3}\int_0^1 \mathrm{d}x\,\big(u(x) - d(x) + \bar{u}(x) - \bar{d}(x)\big) = \frac{1}{3} - \frac{2}{3}\int_0^1 \mathrm{d}x\,\big(\bar{d}(x) - \bar{u}(x)\big). \end{aligned} \tag{3}$$

If the nucleon sea were flavour symmetric, with $\bar{u}(x) = \bar{d}(x)$, we would obtain the original Gottfried sum rule [5], $I_G = 1/3$, in strong disagreement with the most detailed analysis of muon–nucleon DIS data, by the NMC Collaboration, which gave the following result [6]:

$$I_G(Q^2 = 4\ \mathrm{GeV}^2) = 0.235 \pm 0.026. \tag{4}$$

In contrast to the Adler sum rule, the original quark–parton model expression for the Gottfried sum rule is modified by perturbative QCD contributions, analyzed numerically at the α_s^2-level in Ref. [7]. These corrections turn out to be small and cannot be responsible for the significant discrepancy between I_G and the naive expectation of $1/3$. This discrepancy can be associated with the existence of non-perturbative effects in the nucleon sea, which generate light-quark flavour asymmetry, and lead to the inequality $\bar{u}(x, Q^2) < \bar{d}(x, Q^2)$ over significant ranges of the Bjorken variable x (for reviews, see Refs. [8–10]).

In this Letter we examine the QCD corrections to the moments of parton-model densities, for non-singlet neutrino and charged-lepton DIS, with the $N = 1$ moments corresponding to the Adler and Gottfried sum rules, and comment upon a striking feature which they exhibit in the large-N_c limit [11] at the two-loop level.

2. Radiative corrections at large N_c

First we present an analytical result for the two-loop radiative correction that was evaluated numerically in Ref. [7] and then comment on its structure as $N_c \to \infty$.

2.1. Analytical two-loop correction to the Gottfried sum rule

Following Ref. [7], we write the radiative corrections to the $N = 1$ non-singlet charged-lepton moment of Eq. (3), in the case of light-quark flavour symmetry, as

$$I_G = A(\alpha_s)C^{(\ell)}(\alpha_s), \tag{5}$$

with an anomalous-dimension term

$$A(\alpha_s) = 1 + \frac{1}{8}\frac{\gamma_1^{N=1}}{\beta_0}\left(\frac{\alpha_s}{\pi}\right) + \frac{1}{64}\left(\frac{1}{2}\frac{(\gamma_1^{N=1})^2}{\beta_0} - \frac{\gamma_1^{N=1}\beta_1}{\beta_0^2} + \frac{\gamma_2^{N=1}}{\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3), \tag{6}$$

where β_0 and β_1 are the first two scheme-independent coefficients of the QCD β-function, namely,

$$\beta_0 = \left(\frac{11}{3}C_A - \frac{2}{3}N_F\right), \tag{7}$$

$$\beta_1 = \left(\frac{34}{3}C_A^2 - 2C_F N_F - \frac{10}{3}C_A N_F\right), \tag{8}$$

with N_F active flavours and Casimir operators $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$, in the fundamental and adjoint representations of $\mathrm{SU}(N_c)$.

The one-loop anomalous dimension vanishes and the leading correction from the two-loop result of Ref. [12], confirmed in Ref. [13], has the form

$$\gamma_1^{N=1} = -4\left(C_F^2 - \frac{C_F C_A}{2}\right)\left[13 + 8\zeta(3) - 12\zeta(2)\right] \approx 2.557552, \tag{9}$$

with two conspicuous features:

- the appearance of $\zeta(2) = \pi^2/6$, which is absent from even non-singlet moments of the charged-lepton–nucleon structure function F_2, and from odd moments of the corresponding neutrino–nucleon structure function, but occurs at odd moments for charged-lepton scattering, and at even moments for neutrino scattering, by analytic continuation in N of results from QCD Feynman diagrams [13];
- the distinctive non-planar colour-factor, $(C_F^2 - C_F C_A/2) = O(N_c^0)$, which exhibits an $O(1/N_c^2)$ suppression at large-N_c, in comparison with the individual weights C_F^2 and $C_F C_A$, which are associated with planar two-loop diagrams that do not show this large-N_c cancellation at two loops [13] for moments $N > 1$. Nor is there any sign of such large-N_c cancellation in the three-loop results of [14], obtained for even moments.

The second factor in Eq. (5) comes from radiative corrections to the coefficient function, of the form

$$C^{(\ell)}(\alpha_s) = \frac{1}{3}\left[1 + C_1^{(\ell)N=1}\left(\frac{\alpha_s}{\pi}\right) + C_2^{(\ell)N=1}\left(\frac{\alpha_s}{\pi}\right)^2 + O\left(\alpha_s^3\right)\right] \tag{10}$$

with a vanishing one-loop term, $C_1^{(\ell)N=1} = 0$ [15]. The scheme-independent two-loop coefficient $C_2^{(\ell)N=1}$ can be defined through the general non-singlet Mellin moment of charged-lepton–nucleon (ℓ) DIS scattering

$$C_2^{(\ell)N} = 3\int_0^1 \mathrm{d}x\left[C^{(2),(+)}(x,1) + C^{(2),(-)}(x,1)\right]x^{N-1} \tag{11}$$

taken at $N = 1$, where the expressions for the functions $C^{(2),(-)}(x,1)$ and $C^{(2),(+)}(x,1)$ were given in Ref. [16] and confirmed later with the help of another technique in Ref. [17]. The "1" in the argument of these functions denotes the choice of renormalization scale $\mu^2 = Q^2$, where μ^2 is associated to the $\overline{\mathrm{MS}}$-scheme and the coupling α_s is evaluated at Q^2.

Explicit numerical integration of the $N = 1$ moment of Eq. (11) gave the result [7]

$$C_2^{(\ell)N=1} = 3.695C_F^2 - 1.847C_F C_A, \tag{12}$$

with a contribution from the colour factor $C_F N_F$ which was consistent with zero, to the accuracy of that numerical work. At the time, the approximate emergence in Eq. (12) of the same non-planar structure $(C_F^2 - C_F C_A/2)$, already observed in the two-loop $N = 1$ anomalous dimension coefficient of Eq. (9), went unremarked. Now we are able to derive an exact result, by comparing the charged-lepton moments (11) with the corresponding non-singlet moments of the F_2 structure function for neutrino–nucleon (ν) DIS, which can also be expressed in terms of the functions $C^{(2),(-)}(x,1)$ and $C^{(2),(+)}(x,1)$, but now in the combination

$$C_2^{(\nu)N} = \frac{1}{2}\int_0^1 \mathrm{d}x\,\bigl[C^{(2),(+)}(x,1) - C^{(2),(-)}(x,1)\bigr]x^{N-1}. \tag{13}$$

To obtain an analytic expression for the correction (11) to the Gottfried sum rule we remark that the $N=1$ case of the moment (13) corresponds to the Adler sum rule, which is free of radiative corrections. Hence, $C_2^{(\nu)N=1} = 0$ and by elimination of

$$\int_0^1 \mathrm{d}x\, C^{(2),(+)}(x,1) = \int_0^1 \mathrm{d}x\, C^{(2),(-)}(x,1) \tag{14}$$

we obtain

$$C_2^{(\ell)N=1} = 2\cdot 3\int_0^1 \mathrm{d}x\, C^{(2),(-)}(x,1). \tag{15}$$

Noting that the $C^{(2),(-)}(x,1)$ term in Ref. [16] is explicitly proportional to $C_F(C_F - C_A/2)$, we are left with a single integration over x, multiplied by this distinctive non-planar colour structure. Unlike the contributions from $C^{(2),(+)}(x,1)$, this integral is free of singularities as $x \to 1$, and hence requires no regularization. The integrand involves trilogarithms, but elementary integration by parts reduces it to a regular integral whose integrand involves nothing more complicated than the product of dilogarithms and logarithms. Maple then provided a speedy evaluation of the numerical coefficient of $C_F(C_F - C_A/2)$ to 20 significant figures, for which we readily found a simple fit with a rational linear combination of the expected structures $\{1, \zeta(2), \zeta(3), \zeta(4)\}$. Increasing the accuracy of integration to 30 significant figures we confirmed, with overwhelming confidence, the analytical form

$$\begin{aligned} C_2^{(\ell)N=1} &= \left[-\frac{141}{32} + \frac{21}{4}\zeta(2) - \frac{45}{4}\zeta(3) + 12\zeta(4)\right]C_F(C_F - C_A/2) \\ &\approx 3.69439249494141137892516966638 C_F(C_F - C_A/2), \end{aligned} \tag{16}$$

which validates the first 3 significant figures of the approximate terms of Eq. (12), obtained in Ref. [7] by the far more difficult procedure of evaluating an integral in Eq. (11) that has three apparently distinct colour factors and requires delicate regularization at the singular endpoint, $x = 1$, of the $C^{(2),(+)}(x,1)$ function, interpreted as a distribution.

We now interpret the vanishing of the one-loop corrections to the anomalous dimension and coefficient function of the $N = 1$ non-singlet moment of charged-lepton–nucleon DIS structure functions as a simple consequence of the vanishing of all radiative corrections to the Adler sum rule and the absence of a non-planar one-loop diagram that distinguishes charged-lepton scattering from neutrino scattering. As already remarked, this makes the two-loop anomalous dimension coefficient $\gamma_1^{N=1}$ and the two-loop correction $C_2^{(\ell)N=1}$ scheme-independent. The first place that scheme-dependence may appear is in the three-loop anomalous dimension coefficient $\gamma_2^{N=1}$, which appears in Eq. (6) at order α_s^2, albeit divided by β_0. This contribution is in the process of calculation (see, for example, Ref. [18]). We expect its contribution to be small in the $\overline{\text{MS}}$-scheme, for reasons discussed in Ref. [7], based on experience of next-to-next-to-leading order fits [19] to the data on xF_3 in νN DIS from the CCFR Collaboration.

Moreover, we offer our first conjecture, which is that the 6 possible colour structures in the three-loop term $\gamma_2^{N=1}$ will occur only in those 3 combinations suppressed in the large-N_c limit, namely, $C_F^2(C_F - C_A/2)$, $C_F C_A(C_F - C_A/2)$ and $C_F(C_F - C_A/2)N_F$. If this guess turns out to be wrong, then much of our subsequent discussion will become questionable. It should be noted that this conjecture applies exclusively to the $N = 1$ moment of the non-singlet charged-lepton structure function F_2. We derive it from the wider hypothesis that the *differences* between non-singlet moments of F_2 in charged-lepton scattering and neutrino scattering will continue to exhibit non-planar suppressions, beyond the two-loop order at which we have observed them. Then the suppression of $\gamma_2^{N=1}$ in charged-lepton scattering at large N_c becomes a special consequence of the complete vanishing of radiative corrections to the Adler sum rule.

We also note how quickly the two-loop corrections change their colour structure when one considers moments with $N > 1$. For example, the ratio

$$R_2^N \equiv \frac{C_2^{(\ell)N} - 6C_2^{(\nu)N}}{C_2^{(\ell)N} + 6C_2^{(\nu)N}} = \frac{\int_0^1 \mathrm{d}x\, C^{(2),(-)}(x,1)x^{N-1}}{\int_0^1 \mathrm{d}x\, C^{(2),(+)}(x,1)x^{N-1}} \tag{17}$$

is forced to take the value $R_2^{N=1} = 1$ at $N = 1$, by virtue of the vanishing of radiative corrections to the Adler sum rule. But for $N = 2$, we obtained from Ref. [17] the ratio

$$R_2^{N=2} = -\frac{0.505931104}{5.4183241N_c^2 - 4N_cN_F - 8.4480127} \tag{18}$$

which is negative and small in magnitude at large N_c, and also at $N_c = 3$ with $N_F = 3$ active flavours, where it takes the value $R_2^{N=2} = -0.117197668$. Moreover, the magnitude of R_2^N continues to decrease very rapidly with the moment-number, N, because the integral in the numerator of Eq. (17) has an integrand that is strongly suppressed as $x \to 1$. Similarly, we expect the currently known results for the charged-lepton anomalous dimension γ_2^N, at several even values of N, to give little guidance as to the eventual value at $N = 1$, which must be obtained by analytic continuation of a complete set of even-N results.

2.2. *Planar approximation, renormalons and $1/Q^2$ corrections*

The limit $N_c \to \infty$ and the $1/N_c$-expansion [11] are known to be rather useful for analyzing the non-perturbative structure of QCD. Here we will use this framework to characterize our conjecture about the perturbative corrections and then seek a non-perturbative consequence.

To do this, we use the planar approximation formulated in Ref. [20]. In this approximation one retains, after extracting an overall factor of C_F, only those terms at order $(\alpha_s/\pi)^n$ that contain the leading N_c behaviour for each possible power of N_F. In the case of the order $(\alpha_s/\pi)^n$ contribution to the coefficient function of Eq. (10) this prescription then amounts to selecting

$$C_n^{(\ell)N=1}\big|_{\text{planar}} = C_F \sum_{i=0}^{n-1} \mathcal{C}_{n,i}^{(\ell)N=1} N_F^{n-1-i} N_c^i, \tag{19}$$

where the $\mathcal{C}_{n,i}^{(\ell)N=1}$ are pure numbers. By definition, the planar approximation differs from reality by (at most) terms of order $1/N_c^2$. So far we have seen that $\mathcal{C}_{1,0}^{(\ell)N=1} = 0$, since there is no one-loop correction to the coefficient function, and that $\mathcal{C}_{2,1}^{(\ell)N=1} = \mathcal{C}_{2,0}^{(\ell)N=1} = 0$, since only the colour structure $C_F(C_F - C_A/2) = -\frac{1}{2}C_F N_c^{-1}$ survives at two-loop order in this moment, because of the vanishing of all radiative corrections to the Adler sum rule and the appearance of a non-planar factor in the difference between charged-lepton and neutrino structure functions at two loops. Now let us analyze the consequences of the rather strong conjecture that the planar approximation (19) also vanishes at all orders $n > 2$.

In general, when it is non-vanishing, a planar approximation provides us with information in two distinct limits, namely, in the large-N_c limit and also in the large-N_F limit. The intriguing link that it provides between these limits is underwritten by the way the large-order behaviour of perturbation theory is built by renormalon singularities, as was considered in QCD in the pioneering work of Ref. [21] and reviewed in detail in Ref. [22]. This leads one to expect that the asymptotic behaviour of terms in perturbation theory in nth order is of the form $C_n \sim K\beta_0^n n^\delta n!$ (where β_0 is the first coefficient of the QCD β-function) and so it is natural to develop perturbative coefficients as an expansion in powers of β_0. The planar approximation is indeed polynomial in β_0 and hence can be rewritten as

$$C_n^{(\ell)N=1}\big|_{\text{planar}} = C_F \sum_{i=0}^{n-1} \tilde{\mathcal{C}}_{n,i}^{(\ell)N=1} \beta_0^{n-1-i} N_c^i, \tag{20}$$

where again the $\tilde{\mathcal{C}}_{n,i}^{(\ell)N=1}$ are pure numbers. This expansion is closely related to the procedure of naive nonabelianization (NNA) or large-β_0 approximation proposed in Refs. [23,24] in which one replaces N_F by $(11N_c - 3\beta_0)/2$ (for recent applications see Refs. [25,26]). The expansion of Eq. (20) in N_c/β_0 can be regarded as involving different numbers of effective renormalon bubble chains involving powers of β_0 [22], inserted in planar diagrams [20]. There is a related expansion in N_F/β_0 which is obtained by replacing N_c by $(3\beta_0 + 2N_F)/11$ [24,27]

$$C_n^{(\ell)N=1}\big|_{\text{planar}} = C_F \sum_{i=0}^{n-1} \hat{\mathcal{C}}_{n,i}^{(\ell)N=1} \beta_0^{n-1-i} N_F^i, \tag{21}$$

and here again the $\hat{\mathcal{C}}_{n,i}^{(\ell)N=1}$ are pure numbers. This expansion, which has been termed the “dual NNA”, has no direct Feynman diagrammatic interpretation, but turns out to be rather useful in making estimates of perturbative corrections to various physical quantities (see, for example, Ref. [25]).

We now consider how the planar approximation is related to renormalon singularities. Following the work of Parisi [28] one expects that there will be singularities in the Borel transforms of QCD observables. We stress that we are focusing here on a coefficient function, say C, and ignoring any anomalous dimension part, since the latter will not contain renormalon effects [29]. C will have a Borel representation

$$C(a) = \int_0^\infty \mathrm{d}z\, e^{-z/a} B[C](z). \tag{22}$$

Here $a \equiv \alpha_s/\pi$ and $B[C](z)$ is the Borel transform. The work of Parisi implies that one expects branch point singularities in z along the real axis at positions $z = \pm z_n$ where $z_n \equiv 4n/\beta_0$, $n = 1, 2, 3, 4, \ldots$. Those on the positive real axis are referred to as infrared renormalons (IR_n), and those on the negative real axis as ultraviolet renormalons (UV_n). Near each of these singularities one expects the structure

$$B[C](z) = \sum_i \frac{K_i + O(1 \pm z/z_n)}{(1 \pm z/z_n)^{\delta_i}}, \tag{23}$$

where the sum is over the contributions of various operators, and the δ_i exponents depend on their anomalous dimensions. The large-order asymptotic behaviour of the perturbation theory will be determined by the dominant renormalon singularity nearest the origin, and its corresponding operator with largest δ_i. The analysis has been carried out for the Adler e^+e^--annihilation function, and for moments of the DIS structure functions F_1, F_2 and F_3, in Ref. [30]. UV_1 gives the dominant contribution for the Adler e^+e^--annihilation function, and contributes, together with IR_1, to the moments of DIS structure functions. The same dimension-six operator gives the dominant contribution to UV_1 in all the cases considered. In the planar approximation one finds the exponent [20]

$$\delta_+ = 2 - \frac{\beta_1}{\beta_0^2} + \frac{2N_F}{3\beta_0} + \frac{\sqrt{16{N_F}^2/9 + 9N_c^2}}{2\beta_0} - \frac{3N_c}{2\beta_0}, \tag{24}$$

and one obtains the asymptotic large-order behaviour for the coefficient function of the Nth non-singlet moment of F_2

$$C_n^N \approx K_N \left(-\frac{\beta_0}{4}\right)^n n^{\delta_+ - 1} n!. \tag{25}$$

In the large-N_c limit one finds the asymptotic behaviour,

$$C_n^N \approx K_N \left(-\frac{11}{12}\right)^n N_c^n n^{19/121} n!. \tag{26}$$

Only the overall constant K_N depends on the moment taken; the remaining n-dependence is universal [30]. Notice that in fact the same n-dependence also applies to the moments of F_1 and F_3 [30].

Our present conjecture is that the non-singlet moments of F_2 in charged-lepton DIS and in neutrino DIS have essentially the *same* planar approximation, as a consequence of some generalization of the Cutkosky rules that were investigated to two-loop order in Ref. [16]. One obvious consequence is that $K_1 = 0$ for the Gottfried sum rule, since clearly there are no corrections to the Adler sum rule. For higher moments the K_N $(N > 1)$ will be nonzero, but very simply related. At $n = 2$ loops, one sees from Eqs. (11) and (13) that both the ℓ and ν non-singlet F_2 moments are dominated by $C^{(2),(+)}(x,1)$, at large N_c. If it remains true beyond two-loop order that only the $(+)$ component receives a contribution from planar diagrams, then one would expect that $6C_n^{(\nu)N}|_{\text{planar}} = C_n^{(\ell)N}|_{\text{planar}}$ with the factor of 6 simply resulting from the normalization of the Adler and Gottfried sums rules in the most naive quark–parton model. Not only would we expect $6C_n^{(\nu)N} - C_n^{(\ell)N}$ to be suppressed by a factor of $1/N_c^2$, but also to decrease rapidly with the moment number, N, as is the case at two-loop order.

So far we have considered only the leading UV renormalon contribution. One may anticipate that there is an equally important IR_1 contribution, but to compute the corresponding δ one would need the anomalous dimensions of twist-four operators contributing to the operator product expansion (OPE) for the non-singlet moments of F_2, which are not known explicitly. The expectation would, however, be that the corresponding constant K_N^{IR} would vanish for $N = 1$, and for $N > 1$ should differ by a factor of 6 for the ν and ℓ DIS moments.

Since the leading $1/Q^2$ OPE corrections to the moments of DIS structure functions are connected with the leading IR_1 renormalon (for a review, see Ref. [22]), we thus expect higher-twist contributions to the Gottfried sum rule to be suppressed by a factor of

$$\frac{\alpha_s}{\pi N_c} \sim \frac{1}{N_c^2 \log(Q^2/\Lambda^2)}$$

as $N_c \to \infty$, relative to comparable effects in the Bjorken sum rules [26,31], because in the Gottfried sum rule a renormalon chain starts to develop only in a non-planar three-loop diagram, while in the case of the Bjorken sum rules it starts to develop in a two-loop planar diagram.

3. The nucleon sea at large N_c

The previous discussion leads us to believe that the naive quark–parton model expression for the Gottfried sum rule, namely, $I_G = 1/3$, is not modified by perturbative effects, or by their resummations as renormalon chains generating higher-twist effects, in the large-N_c limit. But in the real world, at $N_c = 3$, the experimental data of the NMC Collaboration (see Eq. (4)) show a very significant discrepancy from the naive expectation of 1/3.

There are several ways out of this puzzle. One is to say that $1/N_c^2 = 1/9$ is not small enough for our considerations to be relevant. Another is to say that the $1/N_c^2$ suppression to two-loop order was an accident that will not be repeated at higher loops. To our minds, the most interesting response is to allow that 1/9 may be a small enough factor to take seriously, and that such a suppression of radiative corrections may persist beyond two loops and hence be reflected in a suppression of higher-twist corrections, associated with IR renormalons. Then that leaves the failure of the naive Gottfried sum rule to be explained by an intrinsically non-perturbative flavour asymmetry of the nucleon sea that is inaccessible to renormalon analysis but should still be apparent in the $N_c \to \infty$ limit, to which we have appealed in our perturbative conjectures and their resummations.

It was interesting to learn from the authors of Ref. [32] that this is indeed the distinctive feature of a chiral-soliton model based on the work of Ref. [33]. Briefly, their large-N_c picture, at a very low normalization point, around 0.6 GeV, is as follows. Isosinglet unpolarized distribution functions are large, since they give rise to sum rules that are proportional to N_c; isovector unpolarized distribution functions appear only at next-to-leading order in $1/N_c$, with the Adler sum rule satisfied in the form

$$\frac{1}{2}I_A = 1 = \int_{-1}^{1} \mathrm{d}x\,\bigl(u(x) - d(x)\bigr), \tag{27}$$

where the integrand at $x > 0$ corresponds to a "constituent" quark contribution and at $x < 0$ to an antiquark contribution coming from $u(x) - d(x) = -(\bar{u}(-x) - \bar{d}(-x))$. The failure of the Gottfried sum rule at large N_c is attributed to the integral

$$\frac{1}{2}(3I_G - 1) = -\int_{-1}^{0} \mathrm{d}x\,\bigl(u(x) - d(x)\bigr) = \int_{0}^{1} \mathrm{d}x\,\bigl(\bar{u}(x) - \bar{d}(x)\bigr) = O\bigl(N_c^0\bigr), \tag{28}$$

which measures the flavour asymmetry of the nucleon sea at this very low normalization point. Values of I_G between 0.219 and 0.178 were obtained for a range of constituent quark masses between 350 and 420 MeV, in fair agreement with $I_G^{\mathrm{exp}} = 0.235 \pm 0.026$ at $Q^2 = 4\ \mathrm{GeV}^2$. Note, however, that the NMC data are at a substantially higher momentum scale than can be accessed directly by the chiral-soliton model. For that reason, the authors also compared their predictions for $\bar{u}(x) - \bar{d}(x)$ with the parton distributions of Ref. [34], which were initialized at a comparably low scale. Here too, they claim fair agreement.

There are, of course, several other approaches to the problem of estimating the light-quark flavour asymmetry of the nucleon sea, based on meson-cloud models, instanton models and other considerations (see the reviews of Refs. [8,9] and the recent work in Ref. [35]). We have highlighted the results of the chiral-soliton model because it is based on the large-N_c expansion, used throughout this work.

4. Conclusions

Within the large-N_c expansion we have made the following conjectures, based on rather limited two-loop input:

1. Within the framework of light-flavour symmetry, the radiative corrections to the Gottfried sum rule are suppressed by a factor $1/N_c^2$, relative to the typical expectation

   $$O\bigl((N_c\alpha_s/\pi)^n\bigr) \sim \frac{1}{(\log(Q^2/\Lambda^2))^n}$$

   at n loops. We base this on the facts that they vanish at the one-loop level and are merely of order

   $$(\alpha_s/\pi)^2 \sim \frac{1}{(N_c \log(Q^2/\Lambda^2))^2}$$

   at $n = 2$ loops.
2. We expect the unknown three-loop anomalous-dimension coefficient $\gamma_2^{N=1}$ to be restricted to only 3 of 6 possible colour structures, namely, $C_F^2(C_F - C_A/2)$, $C_F C_A(C_F - C_A/2)$ and $C_F(C_F - C_A/2)N_F$.
3. We expect the ratio of the non-singlet moments, with $N > 1$, for the charged-lepton–nucleon and neutrino–nucleon F_2 structure functions, to maintain the naive ratio 6 : 1, at large N_c, within the framework of light-quark symmetric perturbative QCD, after one discounts quark-loop terms involving $N_F d^{abc} d_{abc}/N_C$, which will contribute to the neutrino–nucleon moments. We have exposed the behaviour

   $$\frac{6C_n^{(\nu)N}}{C_n^{(\ell)N}} = 1 + O\bigl(1/N_c^2\bigr)$$

for all $N > 1$ at $n = 2$ loops and expect it to persist at higher loop orders in the quenched approximation, $N_F \to 0$.
4. Moreover, even at finite N_c, we expect this ratio to tend to unity at high moment-number N, as is the case at two loops.
5. We expect higher-twist corrections, of order $1/Q^2$, to follow the same patterns and hence to be negligible in the Gottfried sum rule at large N_c.
6. In attempting to reconcile this large-N_c perturbative picture with the significant discrepancy between the measured value for the Gottfried sum rule and the naive expectation of 1/3, we note with interest the low-energy picture of the nucleon as a chiral soliton in the large-N_c limit, which leads to an intrinsically non-perturbative flavour asymmetry of the nucleon sea [32]. We believe that current phenomenological analyses which incorporate a flavour-asymmetric sea as non-perturbative input, as, for example, in the most recent parton distributions of Refs. [36–39], capture the essence of this situation, in a manner that cannot be achieved by radiative corrections, or by their resummations in the form of higher-twist effects.

Note added in proof

Shortly after we submitted our paper, an impressive determination of three-loop non-singlet splitting functions appeared in Ref. [40]. Using that work, we are now able to determine the three-loop anomalous-dimension coefficient $\gamma_2^{N=1} \equiv -2\int_0^1 \mathrm{d}x\, P_{\mathrm{ns}}^{(2)+}(x)$, with $P_{\mathrm{ns}}^{(2)+}(x)$ given by Eq. (4.9) of Ref. [40]. To evaluate it, we note that the corresponding integral of the splitting function $P_{\mathrm{ns}}^{(2)-}(x)$ of Eq. (4.10) of Ref. [40] vanishes and hence that $\gamma_2^{N=1} = 2\int_0^1 \mathrm{d}x\,[P_{\mathrm{ns}}^{(2)-}(x) - P_{\mathrm{ns}}^{(2)+}(x)]$ indeed has the colour structure that we anticipated. Performing the integral analytically, we obtained

$$\begin{aligned}\gamma_2^{N=1} &= \left(C_F^2 - C_A C_F/2\right)\Big\{C_F\big[290 - 248\zeta(2) + 656\zeta(3) - 1488\zeta(4) + 832\zeta(5) + 192\zeta(2)\zeta(3)\big]\\ &\quad + C_A\left[\frac{1081}{9} + \frac{980}{3}\zeta(2) - \frac{12856}{9}\zeta(3) + \frac{4232}{3}\zeta(4) - 448\zeta(5) - 192\zeta(2)\zeta(3)\right]\\ &\quad + N_F\left[-\frac{304}{9} - \frac{176}{3}\zeta(2) + \frac{1792}{9}\zeta(3) - \frac{272}{3}\zeta(4)\right]\Big\}\\ &\approx 161.713785 - 2.429260 N_F\end{aligned}$$

by systematic reduction of integrals of harmonic polylogarithms to Euler sums [41] with weights up to 5. This result was checked, to 30 significant figures, by numerical integration of an integrand involving products of dilogarithms, obtained after integration by parts. Within the framework of light-flavour symmetry, it leads to radiative corrections

$$3I_G \approx \begin{cases} 1 + 0.035521\alpha_s/\pi - 0.58382\alpha_s^2/\pi^2, & \text{for } N_F = 3,\\ 1 + 0.038363\alpha_s/\pi - 0.56479\alpha_s^2/\pi^2, & \text{for } N_F = 4,\end{cases}$$

that are even smaller than those estimated in Ref. [7], since the anomalous dimension terms of order α_s^2 cancel about 30% of the order α_s^2 contribution from the coefficient function.

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