An isotropic zero Poisson’s ratio metamaterial based on the aperiodic ‘hat’ monotile

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ABSTRACT
Metamaterials are synthetic materials, engineered to have desirable mechanical properties. This paper is concerned with a class of cellular metamaterials that minimise the Poisson’s ratio, a metric that characterises the deformation behaviour of materials according to their orthogonal response to applied force. This secondary deformation is usually visually apparent as a sideways “bulge”, and can limit the longevity of multi-material components, from aircraft parts to medical implants, because of the interference shear stresses that arise between different materials due to deformations in different directions by different amounts. As a result, low Poisson’s ratio can be a desirable mechanical property and the challenge of producing cellular metamaterials that have reduced Poisson’s ratio has received significant research attention. However, solutions that have been proposed tend to be limited in their applicability, either because they are anisotropic, so behaviour varies greatly according to direction of the applied force, or because they are very low in relative density, so have low stiffness. Here we introduce a new cellular metamaterial, a honeycomb based on the recently discovered aperiodic ‘hat’ monotile, which offers nominally zero Poisson’s ratio. Results from compression testing and computational modelling show that the behaviour of this metamaterial is isotropic, and is consistent across a range of relative densities, leading to the exciting conclusion that zero Poisson’s ratio is possible for different stiffnesses. We envisage that this metamaterial will facilitate otherwise unfeasible technologies, such as morphing wing structures and medical devices that better mimic the behaviour of tissues such as cartilage.

1. Introduction
The Poisson’s ratio of a material provides a metric for its elastic deformation behaviour. Specifically, it is defined as the ratio of the change in the width per unit width, to the change in length per unit length, as a result of applied load. In modern engineering, materials with zero Poisson’s ratio are desirable because they permit the change of shape in one direction without altering the shape in another. Aeronautical devices, such as morphing wings [1], that change shape to alter aerodynamic properties but without changing length, are an ideal application. Another application is in biomedical devices that need to absorb shocks and impacts but without transferring lateral displacement into the surrounding tissues [2].

Typically, solid materials exhibit a Poisson’s ratio between 0.2 and 0.5. This means that application of force in an axial-direction can result in orthogonal deformation of 20-50% of the axial deformation. For some synthetic materials, such as concrete and other composites, the Poisson’s ratio can be as low as 0.1, while a few exotic materials have a negative Poisson’s ratio, which results in orthogonal deformation that is opposite to normal expectations [3]. For example, when such material is stretched, it becomes thicker in the orthogonal direction, and when it is loaded in compression it becomes thinner. Materials with a Poisson’s ratio of zero are extremely rare, and the few examples that have been identified with Poisson’s ratio below 0.2 are biomaterials, such as cork [4], tendon [5] and cartilage [6]. However, research into metamaterials offers the possibility to develop synthetic materials with zero Poisson’s ratio, which could be exploited in a broad range of engineering applications, including aerospace and medical engineering.

This research is concerned with honeycombs, a class of cellular metamaterials that offer significant benefits, compared to solid materials, in terms of specific energy absorption [7–9] and heat transfer [10], as well as attractive mechanical properties at reduced weight [11]. Honeycombs consist of two-dimensional patterns that are extruded out of the plane, and they are used in a broad range of applications, from packaging and protective equipment to aerospace sandwich panels [12]. Traditional methods of manufacturing have historically restricted the
range of honeycombs that can be efficiently produced, in terms of geometry and materials [11]. However, developments in additive manufacturing (3D printing) offer much greater flexibility, enabling production of cellular metamaterials with wide ranging, and otherwise unimaginable, topological designs, and resulting in new opportunities to control mechanical properties and introduce material behaviours that were previously impossible [13].

For example, in the absence of synthetic materials that exhibit zero Poisson’s ratio, an area of research that is particularly active is concerned with auxetic materials, e.g. [14]. These are cellular metamaterials that are designed to exhibit negative Poisson’s ratio, so compressive loading results in a lateral deformation that decreases the volume dramatically, thus increasing effective density. As a result, applications are found in areas such as impact protection and body armour [15]. Investigations into the auxetic behaviour of cellular metamaterials have found that such behaviour tends to be sensitive with respect to the relative density of the metamaterial and the orientation of the applied force [16]. For example, investigations of re-entrant [17] and “wheel” type designs [18] show a significant dependence of the Poisson’s ratio on the relative density of auxetic honeycombs. This should be expected because Poisson’s ratio is commonly linked to other mechanical properties, such as the effective elastic modulus, which in turn are directly impacted by relative density [14].

Negative Poisson’s ratio is also often accompanied by anisotropy, so the deformation behaviour is directionally dependent [14,16]. A potential source of this anisotropy is the translational and rotational symmetries of the underlying patterns, due to their periodic order [19]. Isotropic Poisson’s ratio has been reported for chiral type structures [20] but only at densities low enough to allow hinging to occur [14]. At such low densities, these honeycombs are inappropriate for most engineering applications due to their stiffness being three or four orders of magnitude lower than their base materials, and because the required resolution is beyond what is possible with most manufacturing methods. A honeycomb that minimises Poisson’s ratio, whilst being isotropic in the plane and having a relative density high enough to offer useful stiffness, has previously not been introduced.

Aperiodic patterns, such as the famous Penrose tilings [21], offer an emerging area of investigation, as potential patterns for producing isotropic honeycombs. Aperiodic patterns exhibit “forbidden” rotational symmetry, beyond the 2-, 3-, 4- and 6-fold symmetries of classical crystallography, and no translational symmetry [22]. In material science, they model quasicrystals which were first observed in 1982 in rapidly cooled aluminium manganese alloys [23], an unexpected and fundamental discovery that was recognised by the award of the 2011 Nobel Prize in Chemistry to Dan Shechtman. Quasicrystals are typically intermetallic alloys characterised by aperiodic point-like diffraction patterns which, due to their microstructure, offer unusual isotropic behaviours [24]. Similarly, honeycombs based on aperiodic structures, but at a meso rather than an atomic scale, are also mechanically isotropic, and at a range of relative densities [25,26]. A large number of aperiodic patterns exist, and those that have been used as the basis of aperiodic honeycombs have been shown to give rise to an exceptional variety of mechanical properties with a broad range of stiffness, as measured by a normalised elastic (Young’s) modulus between 0.1 and 0.5, and a broad range of deformation behaviours, as measured by a Poisson’s ratio between 0.2 and 0.4, all without loss of isotropy [25].

Recent research offers a new aperiodic pattern to explore. In March 2023, the discovery of an entirely new aperiodic pattern was announced by Smith et al. [27], based on the ‘hat’ tile shown in Fig. 1. It is the first identified example of an aperiodic monotile, a single tile that can only tile the plane aperiodically. This surprisingly simple shape has remained elusive, despite over 50 years of searching by professional and recreational mathematicians, and its discovery has opened new opportunities for mathematical and mechanical investigation.

In the context of cellular metamaterials, the introduction of the ‘hat’ monotile offers an opportunity to investigate the mechanical properties of honeycombs based on a new aperiodic pattern. The ‘hat’ is defined according to a grid of hexagons, as shown in Fig. 1a. Each hexagon is divided into six polykites by connecting the midpoints of the edges with the centre, and the ‘hat’ is defined on the resulting grid. It has fourteen edges of two different lengths that are defined by half the side-length of the hexagon grid (given by \( l \) in Fig. 1), and the distance between the centre of a hexagon and the centre of one of its sides (given by \( \sqrt{3} l \)). The resulting tile cannot tile the plane periodically, but does give rise to an aperiodic tiling as shown in Fig. 1b.

Unlike previously explored periodic and aperiodic tilings, e.g. [16, 25], the ‘hat’ shape has multiple concave corners of differing size and position. Concavity is a typical feature of auxetic honeycombs [17] and can be exploited to achieve zero Poisson’s ratio if only in a single orientation direction, for example by balancing features of re-entrant shapes with hexagons [1]. Also, the ‘hat’ tilings are aperiodic, and have the potential to present isotropy as reported with other aperiodic tilings [25]. Therefore, from visual inspection alone, it is reasonable to suggest honeycombs based on the ‘hat’ tiling will exhibit isotropic behaviour and low Poisson’s ratio.

2. Experimental

The methods used in this manuscript are nominally identical to those in [16], however in this case applied to the ‘hat’ monotile rather than periodic honeycombs. To evaluate the mechanical properties of honeycombs based on the ‘hat’ monotile a range of samples were selected, tested and simulated, as illustrated in Fig. 2. Fig. 2a shows how the isotropy of the mechanical properties was assessed by extracting 50 \( \times \) 50 mm patches of pattern at different orientations; the examples shown are for 0° and 45°. Similarly, the influence of relative density on the mechanical properties was assessed by taking different sized sub patches and scaling the pattern to give an effective change in relative density, as shown in Fig. 2b, for samples with higher and lower relative density.

Nominally identical physical samples were additively manufactured with dimensions 50 \( \times \) 50 \( \times \) 50 mm in accordance with ASTM D1621 [28] out of Polylactic acid (PLA) using fused deposition modelling. A constant wall thickness of 0.5 mm was used across all samples which ensured that each wall was fabricated from a pair of tool paths, as shown in Fig. 3, with a layer height of 0.2 mm.

The mechanical properties were characterised experimentally using

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**Fig. 1.** The aperiodic ‘hat’ monotile.
compression testing, as illustrated in Fig. 2c, on an Instron 6800 mechanical testing machine with a 50 kN load cell. All tests were conducted at a constant strain rate of 0.5 mm/min to a maximum strain of 25% and were controlled using crosshead displacement measurement. Strain was measured using digital image correlation (DIC) [29] for greater accuracy and to negate the need to account for machine compliance. For quantitative data analysis, LaVision DaVis 10.2.1.81611 software\(^1\) was used to apply live virtual strain gauges where subsets of 31 pixels at each end of the strain gauge were tracked in real time. The vertical gauge was placed between the bottom and top surfaces to measure the normal strain and two horizontal gauges were placed between the vertices closest to each side so that Poisson’s ratio could be calculated as the average of the two strain gauges over the elastic region. Spatially resolved data was calculated via post processing, again with LaVision DaVis 10.2.1.81611. Holes within the honeycomb were masked using an algorithmic mask, and displacements were calculated using a 15-pixel window and data extracted after applying a rigid body motion subtraction. This ensured that the centre point of each colour map in Fig. 5 exhibited approximately zero displacement.

Fabrication and physical testing is too slow a process to conduct the full range of investigation necessary to understand the isotropic behaviour of honeycombs, so the physical results are supplemented by computational models that supported simulation using a rapid modelling technique, as illustrated in Fig. 2d. Simulation of the elastic properties was performed using a bespoke tool based on Asymptotic Expansion Homogenisation (AEH) developed by the authors using Fenics\(^2\), as reported and validated by Imediegwu et al. [19]. AEH [30] is an established method that is used to determine the properties of materials constructed of periodic arrangements of micro-architecture. By leveraging the FEA method AEH can be used to approximate the effective properties of lattices with a reduction of the problem size, resulting in significant reduction in computational overhead, when compared to other FEA methods. Using AEH, full rotational analysis across parametric studies of honeycombs becomes viable. Numerical simulations of the honeycombs were carried out using element-based assignment of material properties, as illustrated in Fig. 2d. Here the structures are treated as composites consisting of material and void, the void being assigned as weak isotropic material, in this case void was assigned a Young’s modulus of 100 Pa. Digital representations of the manufactured samples were created with identical size and parameters. One key difference in the method of constructing the honeycombs for modelling is that cells are not scaled down to define the boundaries of the structure as with the method for producing CAD models for manufactured samples. Instead, a mesh is created that covers the entire sample area and the structure is created mathematically by assigning properties based on the elements’ proximity to the cell wall. The element nodes are first classified as inside or outside the structure, this can be seen in Fig. 2d. A node fraction is then assigned to each element based on the fraction of nodes that lie inside the structure. Intermediate properties are assigned to elements that do not fully lie within the cell wall based on their node fraction. The effective properties were approximated by using an

\(^1\) https://www.lavision.de/en/products/davis-software/.

adapted method of AEH [19]. The honeycombs were simplified to two-dimensional lattices with displacements obtained using Hooke’s law for plain strain conditions. Local strain for each node was calculated, and an example is shown in the stress distribution map in Fig. 2d. For this study the overall compliance matrix was of most use. The compliance matrix from each simulation was transformed for each degree of rotation and the Young’s modulus and Poisson’s ratio extracted. By using this method, repeated meshing of the sample is not required for each rotation and the rotational analysis can be automated and carried out with no input from the analyst. This approach has been validated against known mechanical properties of well-understood honeycombs [19], and against results from compression testing [16].

FDM can produce cost effective models quickly, but the method often introduces isotropy into models [31]. And, any printed specimen may exhibit different mechanical properties depending on toolpath and print parameters such as layer height [32]. Datasheet values for mechanical properties do not always yield equivalent results when used for FEA models and when compared with experimental results [33]. For this reason, when normalising results, the same correction factor has been applied to all experimental data to compensate for deviations from datasheet values, accuracy of manufacturing processes and added stiffness caused by the addition of top and bottom plates.

To contextualise the findings, comparison will be made with honeycombs based on hexagons, which are commonly used in engineering applications, and the properties of which are well understood [16,25].

3. Results and Discussion

Fig. 4a shows an overall summary of the results from computational modelling of the mechanical properties of honeycombs based on the ‘hat’ tiling at a range of relative densities (given by $\rho^s/\rho_b$ where $\rho^s$ is the density of the honeycomb, and $\rho_b$ is the density of the solid base material). It can be seen that the normalised elastic modulus (given by $E^*/E_o$, where $E^*$ is the apparent Young’s modulus of the structure, and $E_o$ is the Young’s modulus of the solid base material) increases with increasing relative density; this is a phenomena that is apparent in most honeycombs [16]. The isotropic nature of the behaviour is shown by the width of the band, which represents $\pm$1 standard deviation of the computational modelling at different orientations. Poisson’s ratio (given by $\nu = \varepsilon_{\text{trans}}/\varepsilon_{\text{axial}}$, where $\varepsilon_{\text{trans}}$ is the transverse strain, and $\varepsilon_{\text{axial}}$ is the axial strain) fluctuates slightly, just above zero, with values between 0.010 and 0.048 for relative densities above 0.225 and between 0.018 and 0.075 at relative densities of 0.225 and below. The increased spread in Poisson’s ratio at low relative density is potentially due to increased cell size and fixed sample geometry, resulting in a smaller number of cells being considered.

The isotropic behaviours shown in Fig. 4a are further confirmed in Fig. 4b and 4c, where the Poisson’s ratio and the normalised elastic modulus are calculated with respect to all in plane orientations (see Fig. 2a). In these rotational plots, results from compression testing of the honeycombs based on the ‘hat’ tiling are indicated by markers, while curves show results from computational modelling. In all the plots, the strong agreement between results from testing and modelling is apparent. Although very isotropic, the results in Fig. 4b and 4c are not perfectly circular, indicating a deviation from perfect isotropy. This is believed to be an artefact of the modelling technique and is discussed in detail in [16].

In Fig. 4b, a comparison is made between honeycombs based on the ‘hat’ tiling and honeycombs based on a hexagon tiling, both at a relative density of 0.30. Data for the hexagon-based honeycomb is from Clarke et al. [16] and was derived using the same methods as described here. It might be expected that, since the ‘hat’ tiling is based on a hexagonal grid, the two types of honeycomb should have comparable behaviour, but this is not what was found. In Fig. 4b-i, the near circular shape of the results for the hexagon-based honeycomb shows it has an isotropic Poisson’s ratio of approximately 0.6. In comparison, the Poisson’s ratio for the ‘hat’-based honeycomb has a Poisson’s ratio that fluctuates slightly above 0.0, as shown in the thickness of the plot in Fig. 4a. Fig. 4b-ii, shows that the hexagon-based honeycomb has an isotropic normalised elastic modulus of approximately 0.03, compared to an isotropic value of approximately 0.01 for the ‘hat’-based honeycomb.

Similarly, Fig. 4c shows a comparison of the mechanical properties of ‘hat’-based honeycombs of different relative densities (see Fig. 2b). Fig. 4c-i shows how the Poisson’s ratio fluctuates slightly above 0.0 for all relative densities, while Fig. 4c-ii shows how the isotropic elastic modulus increases with relative density, and confirms the trends seen in Fig. 4a.

Fig. 5 shows digital image correlation (DIC) analysis of the honeycombs under compression testing at an applied strain of 1.5%. Arrows represent deformation vectors, and the colour map represents the spatially resolved lateral displacement. As a result of the analysis steps taken, the centre point of each map is close to zero displacement. The combination of arrows and colour plots provide an effective visualisation of the elastic Poisson effect and provide a comparison between ‘hat’-based and hexagon-based honeycombs.

Fig. 5a shows analysis for the ‘hat’ at 0° rotation and 0.30 relative
density; the displacement vectors are approximately vertical, and the colour map shows low lateral displacement, thereby qualitatively confirming a near-zero Poisson’s ratio. Similar results are apparent for the ‘hat’ at 90° rotation and 0.30 relative density in Fig. 5b, and at 0° rotation and 0.35 relative density in Fig. 5c. Fig. 5d shows analysis for the hexagon at 0° rotation and 0.35 relative density and the contrast with the ‘hat’-based honeycomb is visually apparent. The displacement vectors clearly show a “bulge” towards the central edges of the sample, indicating the secondary orthogonal displacement as measured by Poisson’s ratio. The lateral displacement is also visually apparent in the colour map. The hexagon-based honeycomb exhibits lateral displacements exceeding ±200 um while the lateral displacement exhibited by the ‘hat’-based honeycombs are all under 50 um.

Fig. 6 further corroborates the existence of zero or near zero Poisson’s ratio in the ‘hat’ based structure, presenting the Poisson ratio throughout the elastic portion of loading. The values obtained for the ‘hat’ based samples are consistently low throughout this portion of loading, ranging from -0.05 to 0.05 for different densities, and showing negligible change as strain increases. These can be compared to the values obtained for the hexagon based structure which shows an approximate linear increase in Poisson’s ratio from approximately 0.5 at a strain of 0.004 to approximately 0.55 at a strain of 0.011.

Fig. 7 presents a compilation of findings from the literature that shows how the ‘hat’-based honeycombs compare to other honeycomb structures, with respect to their Poisson’s ratio and normalised elastic modulus. This graph does not include all honeycomb structures that have been reported in the literature, because studies that report both Poisson’s ratio and normalised elastic modulus are not common. This is perhaps due to the complexity of quantifying Poisson’s ratio experimentally and the necessity of bulk mechanical properties to calculate normalised elastic modulus.

In Fig. 7, different clusters of honeycombs are apparent and emphasised by shaded areas. Types of honeycomb are identified by coloured markers and are further classified according to isotropic and anisotropic behaviour, as distinguished by filled and unfilled markers respectively. Traditional periodic honeycombs are represented by red markers, and typically have high Poisson’s ratio [16]. Green markers represent aperiodic honeycombs which have smaller Poisson’s ratio [25]. Auxetic honeycombs, with reported negative Poisson’s ratio [14], are represented by black markers. Pink markers represent the ‘hat’-based honeycombs, with a near-zero Poisson’s ratio. And blue markers represent honeycombs with reported zero Poisson’s ratio.

Fig. 5. 2D plots of displacement vectors for a range of ‘hat’ specimens and a hexagon for comparison.

Fig. 6. Poisson’s ratio for hat and hexagon honeycombs through the elastic portion of loading.
The discovery of metamaterials with low Poisson’s ratio will be of benefit in many engineering sectors. Presently, the design of morphing wings is a strong motivation [1], because of the requirement for materials capable of deforming in one direction, without disturbing the aerodynamics in other directions. For other applications, the complex computations that are required to find appropriate metamaterials, e.g. [37], can be cost prohibitive. However, the simplicity and accessibility of the ‘hat’ monolite offers zero Poisson’s ratio with minimal computational effort, so it can be used in a broad range of lower-cost applications. For example, to produce shock absorbing layers with the absence of shear stresses forming at the interface to stiffer structures; seals and bungs analogous to the wine bottle cork; and body contour conforming surfaces like seats and benches which maintain overall geometry when deforming. There are also many potential biomedical applications, such as cartilage replacements and orthopaedic implants which would benefit from minimised strain transfer to bone. These structures are essentially shock absorbers where zero Poisson’s ratio prevents lateral strains interfering with the component-bone bond. The non-proprietary nature of the unique geometry of the ‘hat’ monolite opens opportunities for zero Poisson’s metamaterials to be used in many new and exciting applications.

CRediT authorship contribution statement

Daniel John Clarke: Conceptualization, Methodology, Software, Investigation, Writing – review & editing. Francesca Carter: Conceptualization, Writing – review & editing. Iestyn Jowers: Conceptualization, Writing – original draft, Writing – review & editing, Supervision. Richard James Moat: Conceptualization, Writing – original draft, Writing – review & editing, Supervision, Formal analysis.

Declaration of Competing Interest

The authors have no conflicting interests in the work presented in this manuscript

Data availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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