Evaluation and optimisation of the Small Ring Test and its amalgamation with the Small Punch Test

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Abstract

This thesis explores the limitations of conventional mechanical testing methods and assesses two miniaturised specimen testing techniques: the small ring test (SRT) and the small punch test (SPT).

Given the nascency of the SRT, especially as a tensile test, this thesis rigorously assesses the SRT for 48 pin displacement rates on Stainless Steel (Grade 316L), employing machine learning and inter-test comparison to evaluate rate dependency. The gathered data facilitates new conversion relationships for translating SRT data to equivalent stress-strain data. Material properties are obtained through inverse finite element analysis and optimisation routines, with test standardisation recommendations also proposed.

Next, Digital image correlation (DIC) is applied to various SRT ring specimen locations, suggesting the design of an extensometer for the SRT. A 30° claw-like extensometer, gripping the ring at 30° points on either side of the horizontal axis, is found to be optimal.

This thesis also studies the concurrent use of SRT and SPT, where SPT specimens are machined from the SRT’s blank space. Consequently, two discs are extracted from each SRT ring’s blank space and their results are compared with those from discs extracted normally. This combined testing method is applied to SS-316L, Nimonic-75 creep tests, and a plate with weld deposits. The SRT reveals encouraging results, showing a good match between the rings extracted normally and those with discs taken from their blank space. Despite rig compliance issues inhibiting proper result conversion for the SPT, the approach exhibits potential, as the results for discs from the ring’s blank space align well with normally extracted discs.

The combination shows encouraging results, suggesting future research on different materials. It offers substantial material savings by theoretically reducing the volumetric material required by 97.89% compared to conventional uniaxial testing.
Thesis declaration

The work reported in this thesis was carried out by Aniket Joshi in the School of Engineering and Innovation, STEM, The Open University, between October 2019 and May 2023, under the supervision of Dr. Salih Güngör, Dr. Alex Forsey, Dr. Richard Moat for the Doctorate of Philosophy degree at The Open University.

No part of this report has been submitted for a degree at this or any other university. Where the work of others has been used, this has been acknowledged in the text.

Signed:

Aniket Joshi
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Glad to be done with this.

Wonder who reads these. Email/message me if you do. I should be easy to find.
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List of Abbreviations

AGR  Advanced Gas-cooled Reactor.

CDR  Constant Displacement Rate.

CEGM  Constitutive Equation Gap Method.

CEN  European Committee for Standardization.

DE  Differential Evolution (optimisation).

DIC  Digital Image Correlation.

EDM  Electrical Discharge Machining.

EGL  Equivalent Gauge Length.

EGM  Equation Gap Method.

FBD  Free Body Diagram.

FEA  Finite Element Analysis.

FEMU  Finite Element Method - Updating.


LSQNONLIN  Least Squares Non-Linear.

LVDT  Linear Variable Differential Transformer.

MAE  Mean Absolute Error.

MDBT  Miniature Disk Bend Test.

ML  Machine Learning.

MSE  Mean Squared Error.

N75  Nimonic-75.

NM  Nelder Mead (optimisation).

NPP  Nuclear Power Plant.

R2  R-squared or Coefficient of Determination.
**List of Abbreviations**

**RFR** Random Forest Regression.

**RHTT** Ring Hoop Tension Test.

**RMSE** Root Mean Squared Error.

**S2BT** Small 2 Bar Test.

**SPCT** Small Punch Creep Test.

**SPT** Small Punch Test.

**SPTT** Small Punch Tensile Test.

**SRT** Small Ring Test.

**SRTT** Small Ring Creep Test.

**SRTT** Small Ring Tensile Test.

**SS3104LN** Stainless Steel (304, Low Nitrogen).

**SS316L** Stainless Steel (316L).

**UTS** Ultimate Tensile Strength.

**VFM** Virtual Fields Method.

**XGB** XGBoost.
Chapter 1

Introduction
1.1 Background

Nuclear Power Plants are thermal power stations that convert the energy released via nuclear fission, which is usually obtained via the splitting of uranium or plutonium nucleus, into thermal energy which is then converted to electricity. A nuclear power plant is made up of numerous components such as the containment structure, reactor pressure vessel, turbines, and heat exchangers to name a few. A schematic of a typical nuclear power plant is shown in fig. 1.1 [1]. The schematic is of an Advanced Gas-cooled Reacted (AGR).

Figure 1.1: Schematic of an NPP: Advanced Gas-cooled Reactor (AGR) [1]

The gas used in an AGR is carbon dioxide, which is responsible for carrying the heat from the fuel and enters the boiler (steam generator) at around 620°C [2]. The carbon dioxide exits approximately 300°C cooler once it turns the water to steam. This steam is circulated through a turbine, which in turn produces electricity. The flow-rate of the carbon dioxide can be as high as 4 tonnes per second [2].

A nuclear power plant usually operates for 40 years or more [3]. The material properties of all the components in the reactor system may change dramatically over time as a result of extended exposure to stress and high temperatures. A particular time and temperature-dependent phenomenon, called ‘creep’, becomes potentially hazardous in this system as time progresses.

Creep is the cold flow of the metals at elevated temperatures of around 0.35 times the melting point of the material [4]. The components are regularly exposed to high temperatures in many areas of the reactor systems, as mentioned previously for AGRs,
for instance. Over the lifespan of a power plant, the high temperature and loads will lead to creep, which can ultimately lead to failure of the component. This issue is especially exacerbated in areas where the temperatures are extremely high and under areas of complex loading, either chemical, mechanical, or a mixture of both.

In order to assist life extension cases and component evaluation, numerous methodologies exist, such as non-destructive evaluation, hardness testing and so on. Small sample scooping is one of the techniques as well [5], which involves scooping out material from the component for testing. Given the limited amount of material available in the scoop, it is of paramount importance to make use of the material as efficiently as possible [5] [6] and miniaturized specimens are preferred.

The focal point of this thesis is the tensile tests performed on miniaturized specimens via the small ring test (SRT) and the small punch test (SPT). Miniature-specimen tensile test methods are included since they are relatively more researched (than creep) and provide a good stepping stone to creep testing, which has also been addressed with the small ring creep test. The testing methods will be coupled with digital image correlation (DIC) techniques to explore how these tests behave. This is in addition to inverse finite element modelling techniques that are coupled with optimisation algorithms to better understand the test outputs from these miniature-specimen testing methods. This inverse modelling technique will provide useful insight into constitutive parameters obtained via these testing methodologies.
1.2 Tensile and Creep testing in metals

Tensile testing is a fundamental mechanical testing method used to determine the strength and ductility of materials under uniaxial loading conditions. It involves subjecting a specimen to a gradually increasing load until it fractures, and the data obtained is used to calculate various mechanical properties, such as tensile strength and yield strength \[ \text{Tensile strength} \] and \[ \text{Yield strength} \]. The testing is typically performed at room temperature, but it can be carried out at elevated or reduced temperatures to simulate specific operating conditions \[ \text{Elevated or reduced temperatures} \].

The stress-strain curve is a graphical representation of the relationship between the applied stress and the resulting strain in the material during tensile testing. This curve can be divided into several distinct regions, which reveal various mechanical properties and deformation mechanisms \[ \text{Regions of stress-strain curve} \]. An example is shown in fig. 1.2, which showcases how the stress-strain behaviour evolves throughout the test for a typical brittle and a ductile material. The curve can be plotted in two ways: engineering stress-strain and true stress-strain. The former is more commonly used, while the latter provides a more accurate representation of the material’s behaviour \[ \text{Accuracy of curve representation} \]. True stress-strain are derived from the engineering stress-strain values, with the true strain obtained from the natural logarithm of the summation of 1 and engineering strain \[ \text{Derivation of true strain} \], while true stress is derived by multiplying the engineering stress with the exponential function value of the true strain. This is done to account for the decrease in the section area as the test progresses \[ \text{Correction for section area decrease} \].

Figure 1.2: A typical engineering stress-strain curve for a ductile and brittle material. Adapted from \[ \text{Adapted from} \].
The elastic region of the stress-strain curve corresponds to the initial linear portion, where the material deforms elastically, and the deformation is reversible. The slope of the elastic region is known as the elastic modulus or Young’s modulus (denoted by ‘E’ usually) \[4\].

The yield point, or the point at which the material transitions from elastic to plastic deformation, is characterized by a sudden increase in strain without a significant increase in stress. The stress at the yield point is referred to as the yield strength (usually denoted by \(\sigma_y\)) or the 0.2% proof strength (usually denoted by \(R_{p0.2}\)) \[4\]. It is known as the 0.2% proof strength, since it usually corresponds to the strength of the material at 0.2% of strain value.

The plastic region of the stress-strain curve corresponds to the non-linear portion, where the material undergoes irreversible plastic deformation. This region is characterized by strain hardening, where the material’s resistance to deformation increases with increasing strain \[4\]. The tensile strength (usually denoted by \(\sigma_{uts}\)) is the maximum stress the material can withstand before fracture, and it occurs at the peak of the stress-strain curve. Beyond this point, the material undergoes necking and then fractures \[4\].

Tensile testing is widely used across industries to evaluate material performance, for quality control, and for research applications. However, at elevated temperatures and over long periods of time (in years), a time and temperature-dependent phenomenon known as ‘Creep’ needs to be accounted for as well. Creep is the plastic flow of metals occurring above \(0.35T_m\) (melting temperature) and when it is subjected to a value of stress above the critical limit in the creep deformation maps \[9\]. The creep mechanism experienced by a specimen is dependent on the applied stress and temperature, and varies for each material. The Ashby deformation map is a commonly used tool to identify which deformation mechanism may be prominent for given operating conditions \[10\]. An example deformation mechanism map for SS316 is given in fig. 1.3.
Figure 1.3: Ashby deformation map for SS316. Adapted from [11, 12].

The operating parameters are normalized as a function of the Young’s Modulus (for operating stress) and the melting temperature (for operating temperature). These normalized parameters are depicted in fig. 1.3. Numerous studies have been performed for varying operating parameters, with stresses varying from 50 MPa to 350 MPa and temperature varying from 500°C to 750°C for SS316 [12]. This map (fig. 1.3) also includes the varying value of shear stress at a fixed temperature.

The application area for the varying stress and temperature can also be used to identify the physics-driven creep material model which governs the creep mechanism. This is shown in fig. 1.4. The map (fig. 1.4) also depicts the creep deformation rate – a parameter whose value increases with temperature and applied stress [12].
1.2. TENSILE AND CREEP TESTING IN METALS

Figure 1.4: Application area based on Ashby deformation map depicting the mathematical creep material model controlling the deformation mechanisms. Adapted from [10, 12].

A typical profile of the strain variation with respect to time obtained from creep testing techniques is depicted in fig. 1.5. The initial strain in the creep curve is due to the loading of the specimen [13]. For completeness of the discussion, creep deformation mechanisms are discussed next (1.2.1).

Figure 1.5: A typical creep response curve indicating the three different regions of creep. Based on the schematic given in [13].
1.2.1 Commentary on Mechanisms

The deformation of materials can occur under various conditions, such as during tensile testing or creep. Both involve the movement of dislocations and strain hardening, but they differ in the stages and specific mechanisms involved.

For the uniaxial creep tests [15], the variation of strain with respect to time was shown in fig. 1.5. This is a good representation of a typical creep curve and can be separated into three distinct regions, both in terms of the shape of the curve and the underlying mechanisms occurring. Meanwhile, in tensile testing, the stress-strain curve exhibits elastic and plastic deformation regions, followed by necking and fracture.

The primary stage is characterised by a rapid increase in strain followed by a gradual stabilisation in strain rate. Strain hardening is a common phenomenon that occurs during deformation, such as in tensile testing and the primary stage of creep. This is not solely due to the production of dislocations, but also due to the movement of these dislocations under the applied stress. This movement of these dislocations causes them to become entangled and resist further deformation, thus causing the hardening of the material [16, 17]. This resistance to deformation, combined with the applied load, contributes to the material’s strength and can lead to distinct stages in the tensile stress-strain curve or creep curve. However, like the elastic portion of the tensile curve, the creep curve is not reversible due to the plastic and permanent nature of the deformations. It is a quasi-steady state which is characterised by a constant strain rate. This strain rate is referred also known as Minimum Creep/Strain Rate (MCR/MSR) [13, 14, 17]. The secondary stage of creep is predominantly where most components are utilised throughout their lifetime. Thus, this stage is the subject of most creep related investigations. However, it must be noted that some materials do not observe this secondary stage at all [6]. For instance, the research on the creep of A508/533 RPV steel by Wright [18] observed no significant steady-state creep behaviour in that material.

The tertiary stage is characterised by a rapid increase in strain in a relatively short time. The dislocations formed in the secondary stage can be the pathways through which a crack propagates and ruptures the specimen [17]. Understanding the crack propagation helps design the components in a way that this is avoided [17]. The time to rupture is given as $t_f$ and the strain at which it fails is known as “uniaxial creep ductility” or “failure strain” ($\varepsilon_f$) [17].

The microstructural changes that occur during deformation, such as dislocation movement, grain boundary sliding, and the formation of new phases, depend on the material, load, and temperature. Fig. 1.6 illustrates the major creep deformation mechanisms associated with the secondary stage.
Figure 1.6: Creep deformation mechanisms [13, 14].
1.3 Conventional testing methods

The first national standard for creep testing (in UK) was published in 1948 [19] and it was primarily used for aircraft material testing. Uniaxial creep testing was established as the standard test to generate creep damage models. It was not until 1986 that Manahan et al. [20] devised a completely different form of testing technique, that a true alternative was available to uniaxial creep testing.

Uniaxial tests became standardised quite early on and have remained the default standard for mechanical testing worldwide. Thoroughly validated and repeated testing has given rise to standards for uniaxial tests, such as the testing standard for uniaxial tensile tests [8] and the testing standard for creep in uniaxial tensile form [21, 22].

A uniaxial test is usually performed on a flat or cylindrical specimen with one end fixed and a load (tensile) applied on the other end. A basic schematic is shown in fig. 1.7. The right side is clamped while the left side will be pulled.

![Figure 1.7: Schematic of loading in a uniaxial test. The right end is fixed, and the left end has the stretching force applied.](image)

The effect of miniaturisation can lead to ‘size effects’. This refers to the phenomenon where the miniaturised specimen is unable to represent the properties of the parent material due to the insufficient amount of grains present. For many techniques, a critical ratio must be established between the grain size and specimen thickness. Certain studies recommend 6-8 grains across to be sufficient enough to make the specimen independent of flow-stress effect [23]. The study by Song et al. [24] evaluates various grain sizes and specimen thicknesses and recommends a thickness ($t$) to grain size ($d_g$) ratio ($t/d_g$) of 19 and 25 for estimation of yield strength and ultimate tensile strength respectively in small punch testing. The thickness to grain-size ratio was found to have a significant effect on their results, thus prompting these findings. Beyond these critical values that they present for SS316L, the SPT is not representative of the bulk material results. Thus, this size effect should be borne in mind when miniaturization of specimens is done for testing.

A renaissance in small specimen testing techniques has emerged in the past 20 years, especially in the nuclear industry [23]. The need for the miniature-specimen testing techniques is discussed in subsequent sections.
1.4 Introduction and need for miniature testing

The need for small specimen testing techniques is necessitated by 5 reasons, primarily:\n
1. To make effective use of limited material, especially from reactors, and use the smallest possible amount of irradiated material.
2. Extract the least amount of material possible to not detrimentally affect the structural integrity.
3. Aid in design of new alloys for application in fusion and fission power industry.
4. Make the best possible use of active specimen and the possibility of requiring cheaper equipment for a relatively shorter timescale.
5. More tests can be performed where material is scarce, such as in weld regions and heat affected zones.

As mentioned previously, the application of miniature-specimen testing techniques can prove to be extremely advantageous for weld regions and heat affected zones. Weld regions, typically, do not have much material to spare and structurally interfering with the heat affected zone can affect the outcome of a mechanical test. Miniature-specimen tests provide a means of performing mechanical tests using less material than is needed for conventional uniaxial testing. Studies have been performed on welded regions, and they show promising results\cite{25,27}. Samples for evaluation can be scooped from an in-service component and specimens can then be manufactured, as depicted in fig. 1.8. The contrast between the size of a sub-size tensile specimen and a miniature-sized tensile specimen is also depicted in fig. 1.9.

Figure 1.8: Test specimen layouts from sample scooping as proposed by Kumar et al.\cite{28}.
CHAPTER 1. INTRODUCTION

Figure 1.9: Conventional tensile and miniature-sized tensile test specimen side by side to visually highlight the size difference by Kumar et al. [28].

This project aims to analyse the miniaturization of specimen for tensile as well as creep tests, since the former builds towards the latter.

Numerous techniques for miniature-specimen test techniques have been evaluated worldwide [29]. The upcoming section (1.5) informs about the miniaturization of tensile and creep testing techniques in detail.
1.5 Miniaturized testing approaches

1.5.1 Small Punch Test

Tensile tests

In recent years, the advancement in small specimen testing techniques has propelled a plethora of relevant studies. One significant advancement in this field is the European Code of Practice for Small Punch Testing presented in 2007 and updated in 2018. This Code of Practice is a significant step in pushing forth a worldwide standard for the small punch test (SPT) technique.

The small punch technique was first envisioned by Manahan et al. It was initially envisioned as the “Miniature Disk Bend Test” (MDBT) and this forms the basis for the small punch creep test as well as other types of small punch tests. A schematic of this testing rig is shown in fig. 1.10. It can be seen that the receiving die, denoted by ‘3’, has a chamfer denoted by ‘1 x 45 deg’. The CEN standards recommend this value of chamfer to be 0.2 mm at 45°. The recommended punch tip radius \( r \) is 1.25 mm, and the recommended disc height \( h \) is 0.5 mm and a disc diameter \( d_1 \) of 8 mm. The exposed diameter \( d_2 \) of the disc is 4 mm \( d_2 \). The deflection measurement rod is responsible for measuring the disc deformation \( u_2 \), while the punch travel is given by \( u_1 \).

Figure 1.10: Small Punch Testing Apparatus schematic. 1– Specimen; 2– Punch; 3– Receiving Die; 4– Clamping Die; 5– Deflection Measurement Rod. Image adapted from [30].
The theory of the small punch technique for tensile tests (SPTT) is also developed on the basis of the work by Manahan et al. [32], along with the work of Mao and Hideki [33]. Building off of the work by Kumar et al. [34], Altstadt et al. [35] provide a critical overview of using the SPTT for this purpose. The SPTT curve can be visualized as shown in Fig. 1.11.

![Figure 1.11: Typical force-deflection diagram from a CDR SPT [36].](image)

SPT techniques are primarily divided into three modes: a) constant force (CF) mode b) constant deflection rate (CDR) mode c) constant deflection (CD) mode [37]. The CF mode is analogous to performing a conventional creep test, the CDR mode is analogous to performing a tensile test, and the CD mode is equivalent to stress relaxation testing at elevated temperatures [37]. The curve obtained from a typical CDR SPT was shown in Fig. 1.11.

As shown in the load/force-deflection diagram, the typical test can be divided into 6 distinct regions and the $E_{SP}$ represents the energy taken to break the specimen. The 6 distinct regions represent [38]:

I. Elastic region
II. Transition stage
III. Strain hardening of specimen
IV. Geometrical softening and damage of material
V. Macro-crack initiation and growth
VI. Rupture of specimen

One of the biggest challenges for SPT tests is the translation of the SPT test data to conventional uniaxial test data. The SPT test works by pushing the punch into the specimen until rupture, while the uniaxial test works by pulling one end of the specimen to rupture. Thus, by its nature, the SPT has a multiaxial state of stress compared to the largely uniaxial state of stress it needs to be translated to for estimating mechanical parameters [6].
The CDR’s load-displacement curve informs about the maximum load applied to the specimen for the test. However, the estimation of the ultimate tensile strength of the specimen is an issue. One of the initial approaches developed by Mao and Takahashi is given as,

\[ \sigma_{UTS} = 130 \frac{P_{\text{max}}}{t_0^2} - 320 \]  

(1.1)

where \( P_{\text{max}} \) is the maximum applied load (N) and \( t_0 \) is the initial specimen thickness (mm). However, this method has been criticised by Garcia et al. \[40\]. The authors suggest that the maximum load is not indicative of the onset of the necking, and this alone should not be used as a calculation parameter. They recommend using the displacement at maximum load (\( d_{\text{max}} \)) as a calculation factor for calculating UTS. Their formula is given as \[34, 40\],

\[ \sigma_{UTS} = 0.277 \frac{P_{\text{max}}}{d_{\text{max}}t_0} \]  

(1.2)

Published in 2014, the study by Garcia et al. \[40\] is one of the more recent studies evaluating this condition. This argument, however, is challenged by Kumar et al. \[34\] who argue that the scatter in the data makes it difficult to obtain an appropriate value of \( d_{\text{max}} \). They conducted around 70 tests on different specimens and concluded that readings corresponding to 0.48 mm of displacement are the closest to necking zone and these should be used instead of the values taken at \( P_{\text{max}} \). They perform these 70 tests on 3 different materials (20MnMoNi55, CrMoV steel, and SS304LN) at 25°C, 100°C, 200°C, 250°C, and 300°C \[34\]. Pend et al. \[41\] largely agree with this study and use the Holloman law to describe elasto-plastic constitutive relationship of ductile metals and obtain the total strain energy required to break the specimen. Their study recommends taking the parameters from the load-displacement curve when displacement = initial thickness, which is 0.5 mm in the CEN standards \[30\]. This value is very close to the 0.48 mm displacement as proposed by Kumar et al. \[34\]. A snippet of the SPTT results obtained by Kumar et al. is shown in fig. 1.12.
The results they obtain by using their correlation give an error of 8.54% as compared to a traditional sub-size specimen tensile test. Using Mao and Takahashi’s correlation meanwhile yields an error of 12.24%.

Altstadt et al. critically evaluated the results by Kumar et al. and simulated around 100 different SPTT tests and argue that this is still not perfect. They perform these simulations on various hypothetical materials by varying the ultimate tensile strength (UTS) and elongation capabilities of these hypothetical materials. They recommend that the parameters obtained at 1.29 times the specimen thickness should be used, since this point marks the onset of plastic instability. The authors validate their results on P91 and use FEA modelling as well to further bolster their results. Studies like this put forth a case of using SPTT to evaluate irradiated specimen as well. Other studies have also been performed in this field to evaluate various mechanical properties. Numerous relationships for $P_{max}$ and UTS have been proposed and consolidated by Moreno and this work goes on to show the amount of work being conducted in this field.

With all these studies considered and the disagreement in different studies, it would be prudent to say that Bruchhausen et al.’s revision of the CEN standard should be followed when starting out on SPT. The methodology proposed by them is a slight modification of the $P_{(y/10)}$ method and makes use of a trilinear fit on the function below:

$$f(v) = \begin{cases} 0 & \text{for } 0 \leq v < v_0 \\ (v - v_0)\frac{f_A}{v_A - v_0} & \text{for } v_0 \leq v < v_A \\ f_A + (v - v_0)\frac{f_B - f_A}{v_B - v_A} & \text{for } v_A \leq v \leq v_B \end{cases}$$

With the minimization error metric defined as:
1.5. MINIATUREIZED TESTING APPROACHES

\[
\text{err} = \int_0^{v_B} [F(v) - f(v)]^2 \, dv
\]  

(1.3)

The proof stress is determined by the formula:

\[
R_{p0.2} = \beta R_{p0.2} \frac{F_e}{h_0^2}
\]  

(1.4)

where \(F_e\) is the force \(f_A\) from the trilinear function fit, while \(h_0\) is the disc height. The constant \(\beta\) depends on the geometry of the test rig. For a punch with tip radius of 1.25 mm (as set by the standard), the value of \(\beta\) is given as 0.479.

There is a bilinear function fit possible according to this standard, but that can be used only if specimen deflection is obtained. If punch displacement was the parameter, a trilinear function fit is recommended.

Similar evaluation procedure is available for the estimation of the ultimate tensile strength as well. This is given by 2 approaches listed below:

\[
R_m = \beta R_m \frac{F_m}{h_0 u_m}
\]  

(1.5)

\[
R_m = \beta R_m \frac{F_i}{h_0^2}
\]  

(1.6)

The first equation makes use of the maximum force reached in the test \(F_m\) and \(u_m\) is the corresponding displacement. The second equation makes use of the force at a specific location, \(i\), which is geometry dependent. For the tests performed in this study, the value of \(F_i\) must be taken at punch displacement 0.645 mm with a \(\beta\) value of 0.179 which is valid for both the equations. All these values are taken from the new standard laid out by Bruchhausen et al. [31].

The displacement measurement system (for disc deformation) could be augmented by novel strain measurement systems as well. A 3D DIC setup would allow capturing the full strain field and would therefore also allow visualisation of strain evolution at various locations, and not just to rely on the displacement of the punch. This has been attempted by Vijayananad et al. [51] on SS316L. They achieve a 3D DIC setup with the help of mirrors placed at the bottom of the specimen and capture the reflected images. The mirror reflects light towards the camera at 90 degrees. They also attempt to perform inverse analysis and use the inverse finite element modelling (FEMU) approach (described in chapter 1.7). Given the 3D DIC approach adopted by the authors, this opens the possibility of using detailed inverse analysis methodologies, such as the virtual fields method (VFM). Additionally, it also enables proper specimen deflection tracking.

Creep tests

Small Punch Creep Tests (SPCT) were codified in the European Code of Practice in 2007 [30]. The SPCT should preferably be performed in an inert atmosphere of argon.
to prevent oxidation at higher temperatures [55]. Just like the SPTT, the evaluation of parameters is a subject of debate [6]. The CEN standards base the SPCT’s data conversion relationship on the Chakrabarty membrane theory [30]. The Chakrabarty membrane theory allows the stretched specimen to be considered as a membrane spreading over the punch. A schematic of this is presented in fig. 1.13.

![Figure 1.13: Chakrabarty membrane theory—schematic. Adapted from [6, 56].](image)

where, $\theta_0$ is the contact angle between punch and specimen, $R_s$ is the punch radius, $\theta$ is the angle between the surface normal and the axis of rotation, $\Delta$ is the displacement, and $a_p$ is the half-length of the specimen. It should be noted that the angle $\theta$ should be on the other side of the normal (on the right side) in the diagram, but for visual purposes, is shown on the left side.

The generalised expression to determine the load to be applied for the required testing stress, as per the CEN CoP, is [30, 31],

$$F = 3.33k_{SP}R_{s}^{-0.2}r^{1.2}h_0$$

(1.7)

where $k_{SP}$ is the SPCT correlation factor, $h_0$ is the initial specimen thickness, $R$ is the die hole radius, and $r$ is the punch radius. A typical SPCT displacement-time curve is shown in fig. 1.14.
1.5. MINIATURIZED TESTING APPROACHES

The deformation-time curve appears similar to the traditionally obtained creep curve from uniaxial specimens (fig. 1.15). However, the conventional creep curve is a strain-time curve. Thus, the two need to be related in some manner. The curve can be broken down into three distinct regions [6, 55, 57]:

1. Primary stage: Involves only bending deformation and is elastic in nature. This is due to a highly localised loading region.

2. Secondary stage: Related to membrane stretching. The specimen starts changing shape from disc to cone. Deformation mode changes from bending to membrane stretching. The region is also related to the formation of circumferential (for a ductile material) or central cracks (for a brittle material).

3. Tertiary stage: This region is attributed to the development of macro-cracks and eventually the rupture of the specimen.

Analytically, the Chakrabarty membrane approach is preferable to model the membrane stretching, which occurs primarily in the secondary stage. However, this has faced heavy criticism, particularly directed towards it not including the punch angle into consideration [58] and also its validity being limited only to exponential hardening law [55]. Alternative approaches have been proposed to this methodology, such as the ‘Reference stress approach’ [55]. This involves obtaining a multiplier (reference stress multiplier) with the help of FE analyses that helps in the conversion from the observed deformation rate in the small punch creep test to the equivalent uniaxial strain rate. However, given the deformation in the disc is quite high, this may not be the most suitable approach [55]. The use of any methodology needs to be careful in application since some materials do not observe the secondary stage at all, or it is insignificant [54].

To model the continuum damage mechanics, FEA has also been done for SPCT to analyse the creep crack growth and creep crack initiation in the specimen using numerous models, one of them being the Liu-Murakami model [55]. The multiaxial form of this...
damage-coupled creep evolution constitutive model is given as \[60–62\],

\[
\dot{\varepsilon}_{ij}^c = \frac{3}{2} C n_2 \sigma_{eq}^2 S_{ij}^{eq} \exp\left( \frac{2(n_2 + 1)}{\pi \sqrt{1 + 3/n_2}} \left( \frac{\sigma_1}{\sigma_{eq}} \right)^2 \cdot \omega^{1.5} \right)
\]  

(1.8)

where \( C \) and \( n_2 \) are material constants. \( \dot{\varepsilon}_{ij}^c \) is the equivalent strain, \( \sigma_{eq} \) is the equivalent stress, \( \sigma_1 \) is the maximum principal stress, and \( S_{ij} \) is the deviatoric stress tensor. The deviatoric stress is the stress tensor with the hydrostatic stress subtracted from it. The damage variable \( \omega \) is given as \[60\],

\[
\dot{\omega} = D \frac{(1 - e^{-q_2})}{q_2} \sigma_{eq} e^{q_2 \omega}
\]  

(1.9)

where \( D, q_2, \) and \( p \) are material constants. The rupture stress \( \sigma_r \) is defined as \[60\],

\[
\sigma_r = \alpha \sigma_1 + (1 - \alpha) \sigma_{eq}
\]  

(1.10)

However, these models are not always deemed adequate. For instance, the Liu-Murakami damage model fails at zero-stress areas and thus needs to be modified to predict failure properly \[58\] and also take into account initial plasticity \[63\]. The latter aspect is especially important since initial plastic deformation can have a significant impact on the SPCT curves generated \[63\]. The modification of Liu-Murakami damage model as done by Cortellino et al. \[63\] by modifying the Young’s modulus based on the damage to the material. This modification is better at reducing the error in the prediction of the minimum displacement rate (MDR) but is still not satisfactory in predicting the failure time \( (t_f) \). Their results are shown in table 1.1. MDR corresponds to minimum displacement rate and \( t_f \) corresponds to time to failure (in hours). The modification that they propose is also given below, with \( E \) representing the Young’s modulus of the damaged material, \( E_0 \) representing the Young’s modulus of the undamaged material, and \( \omega \) representing the damage parameter.

\[
E(\omega) = \begin{cases} 
E_0(1 - \omega) & 0 \leq \omega < 0.9901 \\
0.01\text{MPa} & \omega \geq 0.9901
\end{cases}
\]  

(1.11)

<table>
<thead>
<tr>
<th>Load (kg)</th>
<th>Method</th>
<th>Error % of MDR</th>
<th>Error % of ( t_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>Experimental data</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Liu-Murakami</td>
<td>208.07</td>
<td>84.00</td>
</tr>
<tr>
<td>-</td>
<td>Modified Liu-Murakami</td>
<td>8.74</td>
<td>73.20</td>
</tr>
<tr>
<td>28</td>
<td>Experimental data</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Liu-Murakami</td>
<td>1109.30</td>
<td>82.24</td>
</tr>
<tr>
<td>-</td>
<td>Modified Liu-Murakami</td>
<td>0.78</td>
<td>78.77</td>
</tr>
<tr>
<td>30</td>
<td>Experimental data</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>Liu-Murakami</td>
<td>1002.80</td>
<td>83.33</td>
</tr>
<tr>
<td>-</td>
<td>Modified Liu-Murakami</td>
<td>24.30</td>
<td>34.87</td>
</tr>
</tbody>
</table>

Table 1.1: Summary of the results by Cortellino et al. \[63\].

For the estimation of creep residual life, Dymacek illustrates the residual creep life measurement for P92 steel with SPCT \[37\]. The Monkman-Grant relationship is used, which for SPCT is,
where \( m_{SP} \) and \( C_{SP} \) are constants, \( t_f \) is time to failure, and \( \dot{u}_{\text{min}} \) is the minimum deflection rate \[37\]. A typical deflection rate variation graph is shown in fig. 1.15.

![Deflection Rate Variation Graph](image)

**Figure 1.15:** A deflection rate variation graph of the SPCT. Adapted from \[64\]

The approach taken by Dymacek is promising, but is limited in terms of number of experiments performed. However, it raises the question of the role of friction coefficient in estimating creep residual life via SPCT \[37\]. The research by Dobes and Milicka \[65\] summarises the various residual creep life approaches applicable for SPCT. They conclude that the Sud-Aviation model fits the best, however this is still a topic of debate since it doesn’t take into account contact angle and coefficient of friction. The residual creep life methods and their corresponding parameters are listed in table 1.2.

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larson-Miller</td>
<td>( T(C + \log(t_f)) )</td>
</tr>
<tr>
<td>Fisher-Dorn</td>
<td>( \log(t_f) - \frac{C}{T} )</td>
</tr>
<tr>
<td>Goldhoff-Sherby</td>
<td>( \frac{\log(t_f) + C}{T + T_a} )</td>
</tr>
<tr>
<td>Manson-Haferd</td>
<td>( \frac{T + T_a}{\log(t_f) + C} )</td>
</tr>
<tr>
<td>Manson-Succop</td>
<td>( \log(t_f) + CT )</td>
</tr>
<tr>
<td>Suv-Aviation</td>
<td>( \log(t_f) + \log(T) )</td>
</tr>
</tbody>
</table>

Table 1.2: Summary of all methods and parameters evaluated by Dobes and Milicka \[65\] for evaluating creep residual life.

Mathew et al. \[64\] use SPCT to test a new alloy. Numerous modelling studies have been
performed to delineate alternatives to the Chakrabarty membrane theory, to understand the creep damage mechanics, or to establish life estimation methods \[5, 18, 31, 66\].

The SPCT’s strength of giving results in a shorter time-frame is also its weakness, since it does not allow for the observation of microstructure evolution \[49\]. Thus, there is a strong impetus to conduct interrupted SPCT and validate the conversion relationships, such as the ‘reference stress approach’. Rasche et al. \[67\] expand upon this need for microstructure understanding as they analyse various materials. It can be seen from their results, which are shown again in fig. 1.16, that the failure in materials differs extensively. An interrupted test would allow for the monitoring of this microstructure evolution and specimen profile to better understand how the specimens behave in the small punch test \[49, 67\]. This testing methodology could then be used to validate one of the many conversion relationships proposed that translate the test data from the punch displacement to strain \[49\], which is needed as the test standards keep evolving as new studies are performed \[30, 31\].

![Figure 1.16: Schematic of the load-displacement curve of specimens, showing different stages of deformation at the moment of fracture. The materials range from purely brittle (technical alumina ceramics) to fully ductile (ferritic pressure vessel steel). Adapted from \[67\]](image)

Additionally, there is potential in adapting the SPCT to 3D DIC as was done for the SPTT by Vijayanaud et al. \[54\], since this would provide further information on disc deformation profiles. Cacciapuoti et al. \[58\] use a curve-fitting technique for their specimen profile analysis. The profile captured by the interferometer was fit with the help of a 5-term Fourier series. A 3D DIC measurement could theoretically help in the selection of regions and better fit the specimen deformation profiles.
1.5.2 Miniaturised uniaxial test

Tensile tests

The size effects make it difficult for the extrapolation of relevant properties from these tests \cite{ref68}. In some recent works, Kumar et al. \cite{ref69} illustrate their method of extracting specimens via the boat sampling technique and establish their testing procedure. Strong emphasis is placed on polishing and the number of grains included in the specimen. This is to fulfil the Hall-Petch relationship for grain size \cite{ref68}. However, it must be mentioned that they do not use an LVDT strain measurement tool, but use the cross-head displacement as a measure. The guidelines they establish are,

\begin{align}
\text{Gauge length} & \geq 5.65\sqrt{\text{Area of gauge section}} \tag{1.13} \\
\text{Thickness} & \geq 10 \times \text{Grain size} \tag{1.14} \\
\text{Width} & \geq 8 \times \text{Thickness} \tag{1.15}
\end{align}

The study by Kartal et al. \cite{ref26} for miniaturised-specimen testing on weld regions shows promising results, but the number of samples tested is too small to draw strong conclusions from. The specimen used by Kartal et al. \cite{ref26} is shown in fig. 1.17. They use a miniaturized specimen of thickness 0.7 mm, width 0.7 mm, and gauge length 3.75 mm. They find a considerable variation across the gauge length of their specimen and in the direction of extraction as well, highlighting further how grain size affects the test results for miniaturized specimens.

![Miniature tensile test specimen as used by Kartal et al.](image)

In their study, Kumar et al. \cite{ref28} delineate the effect of specimen preparation on results. They also succinctly elaborate on the problems faced by this testing method \cite{ref28}:

1. Representation of parent material: It is said that the test specimen may not represent the bulk properties of the parent material, since the effects of specimen preparation may introduce residual stresses.

2. Strain measurement: Conventional extensometers cannot be used, and other methods need to be relied on.
3. Validity of results: Variation in results is observed due to change in ductility, which is attributed to the change in shape and size of the test specimens. They advocate for standardising the $L/D$ ratio for cylindrical specimens and $L/\sqrt{A}$ ratio for flat specimens.

In an effort to further this research, they try to optimise the thickness of the miniature specimens [27] by conducting tests on 3 very different materials with thickness varying from 0.15 mm to 0.4 mm (variation step of 0.05 mm) and a specimen width of 1 mm. They find, based on all three materials, that a minimum thickness of $\approx 0.25$ mm is appropriate to give satisfactory results [27].

Creep tests

Miniaturised uniaxial creep tests have the distinct advantage of not needing any data conversion. Cao et al. [70] propose miniature-specimen creep testing on uniaxial specimens on nickel-based superalloys by the use of mild notches. Their specimen is shown in fig. 1.18.

![Specimen used by Cao et al.](image)

Figure 1.18: Specimen used by Cao et al. [70]. (a) Desired specimen dimensions (measurements in mm) (b) Manufactured specimen.

Their use of mild notches, however, will alter the stress state of the specimen [70]. While they do find that no notch root cracking occurred (cracking which occurs at the notch’s starting point), their FEA results and experiments do show that the displacements found in the notched specimen are larger than the displacements in the sub-size uniaxial creep specimens [70].

The geometry used by Dymacek et al. [71] is shown in fig. 1.19.
1.5. MINIATURIZED TESTING APPROACHES

They perform tensile and creep tests on both the specimens. Their fractographic analysis shows no significant differences between the sub-size and miniature specimen (creep test) but they fracture differently as compared to their tensile counterparts. The tests performed also indicated a 30-40% quicker time to failure \( t_f \) for the miniature specimen as compared to the uniaxial specimens. Based on the results, some suggestions they recommend are [71]:

1. Specimen dimensions should be optimised and standardised.
2. More testing of more materials, similarly as done for SPCT is needed.
3. Define size effect of microstructure in miniature specimens.

Hyde and Sun [72] cite another problem with this geometry that at small gauge lengths (<10mm), the strain sensitivity is reduced and it is difficult to manufacture and handle such specimens. The problem of standardisation needs rigorous experimental programs to address it. Similar to its uniaxial counterpart, the minimum dimensions need to be rigorously established while taking into account the size effects. The effect of notches altering the stress state should be investigated alongside the size effect and recommend a standardised notch radius.

1.5.3 Small Ring Test

Tensile tests

When the Small Ring Test (SRT) was developed by Hyde and Sun in 2009 [72], it was aimed primarily at creep testing. Kazakevičiūtė et al. [73, 74] modified the original elliptical geometry to analyse a purely circular one. This is shown in fig. 1.20. The loading mechanism is the same for the tensile and creep testing modes. The ring is loaded with the help of pins at diametrically opposite ends. These pins then pull the ring apart due to tensile forces at a constant force (for creep test) or at a constant displacement rate (for tensile test).
Their specimen has the dimensions of outer diameter ($\phi_o$) of 11 mm, inner diameter ($\phi_i$) of 9 mm, and a thickness ($t$) of 2 mm. The displacement rates chosen by them are based on preliminary FEA which used the Ramberg-Osgood plasticity model \[73\]. All studies also strongly indicate that the loading pin behaves as a rigid body throughout their test, with frictionless contact being adequate for modelling \[73\]–\[75\]. This is attributed to the minimal contact area between the loading pin and the ring. This evolution in loading sets the SRT apart from the Ring Hoop tests (covered in the next section \[1.5.4\]), since the latter makes use of thick discs to load the specimen.

They also provide detailed analytical solutions and demonstrate that SRTT is insensitive to pin misalignment, due to its self-aligning ability. After establishing equivalent gauge section and gauge length, they demonstrate promising results for this test methodology in obtaining the mechanical parameters, but the estimation of yield strength is not perfect \[73\]. Given the nascency of this testing method, there is a lack of literature. The promising results in the preliminary studies indicate that this testing methodology needs further exploration.

**Creep tests**

The SRT test for creep testing was devised in 2009 \[72\]. This was originally envisioned as an elliptical ring, as shown in fig. \[1.21\].
1.5. MINIATURIZED TESTING APPROACHES

Figure 1.21: Original SRT specimen from Hyde and Sun [72].

The test output from this SRCT is the deformation (Δ) vs time curve [6]. The analytical solution originally developed for the secondary creep stage neglects shear stresses and assumes the material to follow Norton’s creep law under bending stresses only [6,72]. This form of testing does not require the loading pins to be made of a material with superior creep strength, since these have negligible effects on the result. This is a significant advantage over some of the other creep testing methods, such as the Impression Creep Test [72].

The conversion relationship to uniaxial parameters was originally given as [29, 72],

\[
\sigma_{ref} = \eta \frac{P a}{b_0 d^2} \tag{1.16}
\]

\[
\dot{\varepsilon}^c(\sigma_{ref}) = \eta \frac{d}{4 a b \beta} \dot{\Delta} \tag{1.17}
\]

where \( \sigma_{ref} \) is the equivalent uniaxial reference stress, \( \eta \) and \( \beta \) are test reference parameter, \( P \) is the applied load, \( a \) and \( b \) are ring dimensions, \( \dot{\varepsilon}^c \) is the creep strain rate, \( d \) is the thickness, \( b_0 \) is the specimen depth, and finally \( \dot{\Delta} \) is the ‘load line deformation rate’ [29]. This analytical solution was derived from a reference stress approach [76] and promising results have been obtained for nickel-based superalloys [76]. This is noteworthy since nickel-based superalloys are notoriously creep resistant and testing them is quite difficult [77].

In their recent study, Hyde et al. [78] perform the SRCT on Inconel-718. The paper concludes with the authors mentioning the development of an in-house software that will take into account the change in reference stress which occurs due to the changes in the specimen geometry. This change in references stress causes a small curvature, which reduces the overall strain rate [78]. A shortcoming of all these results, however, is that this testing method cannot provide with the data for the tertiary stage of creep [79]. Thus, bearing this in mind, there is a reasonably strong impetus to further develop this testing technique.

1.5.4 Ring Hoop Test

The Ring Hoop test is largely similar to the small ring test, but with key differences in the loading mechanism.
The ASTM E8 standard has provisions for different specimen geometries, including flat, cylindrical, and even tubular and pipe-shaped specimens. However, it must be noted that even the tubular and pipe-shaped specimens must be longitudinally loaded, that is, the samples are stretched instead of being expanded in diameter.

This restriction comes in addition to the size restrictions that are generally imposed by the test standard.

To overcome this restriction, the Ring Hoop Tensile Test has been proposed. A schematic of this is shown in fig 1.22.

![Schematic of the RHTT setup. Adapted from [80].](image)

Developed by Arsene and Bai [81, 82], this test has been pivotal in the nuclear industry and the similarities in the basic geometry with the small ring test (SRT) cannot be ignored. The sample is loaded with split discs that are attached to the machine’s crosshead. These split discs are in contact with the inner surface of the sample as the sample is stretched due to tensile forces, with the split discs moving apart during a test.

However, this test is primarily used for the determination of hoop parameters. While studies have also reported the unfortunate nature of friction coefficient dependence since all the dies are in contact with the ring [83], this test has enabled the ability to obtain material parameters not conventionally available.

With conventionally cited parameters being mostly in the longitudinal direction, this test provides insight into obtaining material parameters where hoop stress is likely to be dominant.

Of particular relevance here is the study by Calaf et al. [84], which adapts the RHTT to the benefits of the SRT. That is to say that they are able to combine the efficacy of garnering insight into material properties via the RHTT and use the loading pins
approach of the SRT to alleviate any friction dependency. Consequently, this study has been referred to extensively in the testing chapters.

1.5.5 Small Two Bar test

The Small Two Bar Creep Test (S2BT) was first proposed by Hyde et al. \[85\] in 2012. A schematic of this is shown in fig. 1.23 \[86\]. The sample is loaded via circular pins on each end of the cavity in the sample, with the sample being subjected to a tensile form of testing at either a constant force or a constant displacement rate.

![Schematic of the S2BT specimen. Adapted from \[86\].](image)

where \(d\) is the thickness, \(b\) is the width, \(D_i\) is the diameter of loading pins, \(L_0\) is the loading bar length, and \(k\) is the length of the loading pin’s end \[86\]. A conversion relationship has been proposed along with an equivalent gauge length (EGL) formula as,

\[
\sigma_{\text{ref}} = \eta \sigma_{\text{nom}}
\]

\[
\dot{\varepsilon}^c \approx \frac{\Delta_{ss}}{EGL}
\]

\[
EGL = L_0 \beta
\]

where, \(\sigma_{\text{ref}}\) is the reference stress, \(\sigma_{\text{nom}}\) is the nominal stress, \(\eta\) is a reference stress parameter, \(\Delta_{ss}\) is the minimum deformation rate, \(\dot{\varepsilon}^c\) is the minimum creep strain rate, and \(L_0\) is the reference gauge length parameter. \(\eta\) and \(\beta\) are dependent on the geometry and not the material and this is a huge advantage for this testing methodology \[86\]. These parameters (\(\eta\) and \(\beta\)) can be obtained with finite element modelling of the test using appropriate damage models.

Ali et al. \[86\] analyse the effect of the variation of specimen size on these reference parameters in detail and recommend a range of dimension ratios. This is shown in Table 1.3. The reference stress method used for obtaining the two geometric parameters is delineated in the work by Cacciapuoti \[6\].
Table 1.3: Dimension ratios for S2BT as recommended by Ali et al. [86].

<table>
<thead>
<tr>
<th>Dimension Ratio</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_0/D_i$</td>
<td>$\geq 2 - 5$</td>
</tr>
<tr>
<td>$k/D_i$</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>$b/D_i$</td>
<td>$0.2 - 0.47$</td>
</tr>
<tr>
<td>$b/d$</td>
<td>1</td>
</tr>
</tbody>
</table>

The output of this test is close to a uniaxial creep test due to both its bars undergoing a largely uniaxial state of stress [6]. This geometry has the advantage, like the SRCT, of being insensitive to misalignment of loading since they can be self-aligned (up to a certain degree) [87]. Currently, there has not been much research on this specimen geometry and further testing is needed before this can be standardised.
1.6 Summary of testing methods

This chapter summarized the following experimental testing techniques:

1. Small Punch Test
   (a) Tensile testing (SPTT)
   (b) Creep testing (SPCT)

2. Miniaturized uniaxial test
   (a) Tensile testing
   (b) Creep testing

3. Small Ring Test
   (a) Tensile testing (SRTT)
   (b) Creep testing (SRCT)

4. Ring Hoop Tension Test (RHTT)

5. Small Two-Bar Test (S2BT)

Exhaustive efforts have been made to comprehensively analyse these tests, yet more work is needed in this field. Some consolidated literature can be found in the works of Cacciapuoti [6], Hyde et al. (2007) [23], Mathew et al. [88], and Kumar et al. [52]. Numerous creep damage models are available to assist in finite element modelling, and a consolidation of these models is found in the work by Meng and Wang [62].
1.7 About inverse modelling

While a lot of studies focus on establishing empirical correlations between miniature and conventional tests, there is a strong need for standardising an inverse modelling procedure based on experiments which can provide with the material properties [89, 90].

In a mathematical notation, the relationship between parameters $g$ and the measurements $f$ can be written as [91],

$$Q(g) = f$$

where $Q$ is the $g$-dependent function which allows for the mapping. Essentially, the process of fitting the simulation model to the experimental obtained is inverse modelling.

Inverse modelling has various sub-branches. The first organic division to delineate would be between static and dynamic models. Many materials tests (such as the tensile test or the creep test) can be qualified as a quasi-static test as well, such as those performed in this study. Dynamic models would refer to systems that are often used in structural monitoring, such as monitoring footbridges, etc [92]. These are especially useful for vibrating systems, since these are described by distributed parameters that require adjustment of the model based on damping and stiffness [93–95].

Another subdivision to think of can refer to parametric optimisation, or it can refer to model optimisation. There is a nuanced difference between the two. Model optimisation can refer to modification of the FE model to adjust for model inaccuracies to match the ground truth [96] and is usually performed via topological optimisation [97, 98]. In this section, novel technologies such as physics-informed inverse mapping are gaining traction as well with their promising results [99].

However, if there is sufficient confidence that the model is working well, one may not need to update the model. This can be fixed, and the unknown parameters can then be extracted via inverse simulations [100].

Lastly, another way of classification of inverse simulations is via direct and indirect approaches. The direct approaches are one of the oldest techniques used. These update the stiffness matrices (global or local) directly [100, 104].

However, this method is rather unwieldy and requires extensive knowledge of the system. Moreover, it is reliant on the user making the correct choices. Indirect systems, meanwhile, tweak the parameters that affect the resultant model and make the best possible guesses based on user metrics such as minimisation of R-squared values, mean squared error, and so on [100].

Ereiz et al. [100] and Avril et al. [104] provide comprehensive overviews of these methodologies.

Although, it should be noted that there are severe inconsistencies in the literature regarding the terminologies [100]. This has been observed consistently while perusing the relevant literature as well.
For instance, there seems to be a no common ground regarding the terminologies about inverse Finite Element Method Updating (FEMU). This can refer to iterative simulations that update parameters in some literatures, or it can even refer to the model updating techniques.

Based on the literature for structural health monitoring (SHM), a field where inverse methods are used extensively, some available methods for inverse modelling via the finite element route are [95, 100, 105, 106],

1. Bayesian stochastic updating
2. Population Based methods
3. Gradient Free methods
4. Sensitivity-based updating
5. Fuzzy updating
6. Regularization

While this is a good starting point, this is not an exhaustive list. It must also be noted that these FEMU methods may benefit from proper optimisation of the goal/objective function as well. For robustness, many analyses prefer a multi-objective optimisation technique [107]. However, for simpler problems, a single-objective optimisation may be just as useful while saving on computational costs [100].

What has been consistently observed is that a larger set of measurements is always helpful. These are usually full-field measurements, and they have their own dedicated techniques that extend beyond finite element. Some of these methods, in addition to FEMU, are:

1. CEGM: Constitutive Equation Gap Method
2. VFM: Virtual Fields Method
3. EGM: Equilibrium Gap Method

These methods are often used in mechanical testing given the ability to obtain full-field measurements from the tests via techniques such as alternate current potential difference (ACPD), digital image correlation (DIC), and so on. The availability of full-field measurements allows for more precise and accurate model and material calibration. Some of these are elaborated upon below.

1.7.1 Finite Element Method - Updating (FEMU)

The FEMU is approach is perhaps the most widely studied approach out of all the full-field inverse analysis approaches. For the context of this study, model updating techniques are not discussed in the context of FEMU and only parametric updating is considered.

This is because the model involved in this study is relatively simple and does not require updating, as will be seen in section 1.7.5.

Although, the techniques used in the global updating of the model can very well be used in the local updating to find the unknown material parameters [100]. Two of the most common techniques belong to the family of gradient-free optimisation and
population-based optimisation. This is touched in detail in section 3.4 since these are used extensively in this study.

The FEMU approach was adopted by Ktari et al. [96] for the RHTT, which is a test method that is somewhat similar to the SRT. Usually, model updating is preferred when there is uncertainty about the initial (FE) model [96]. This has been the preferred approach to calibrate models and was performed for the RHTT by Ktari et al. [96]. However, their study takes a unique approach by making the use of feed-forward ANNs (Artificial Neural Networks) to calibrate their model, consequently performing shape optimisation on their initial FE model. They use back-propagation in their training phase with the Levenberg-Marquardt (LM) algorithm to update their ANN model. This algorithm aims to minimise the cost function (the parameter that governs the error) by following the path of steepest descent and optimising the trial values based on the gradient of this descent. To prevent overfitting while being cognizant of their computational resources, the early-stopping method was used instead of Bayesian regularisation. As the name suggests, the method involves stopping the training early.

However, equal importance should be given to the methods that have not been used for mechanical testing relevant to the SRT or SPT.

Other optimisation methods for inverse FE (or FEMU) include the fireworks algorithm [108], the ant-swarm optimisation [109], or even the genetic algorithm [110, 111]. These are in addition to the plethora of studies that use particle swarm optimisation techniques for inverse FEMU [100]. Most of these techniques are population-based optimisation techniques that make use of the full sampling space and seed this space with candidate solutions. The fireworks algorithm, for instance, seeds the sample space with a population of fireworks patterns and creates new fireworks based on the best possible solutions in the previous firework. The ant-swarm optimisation is inspired by the behaviour of ant colonies, the differential evolution algorithm is inspired by the genetic algorithms in biology, and the particle-swarm optimisation is inspired by the behaviour of bird flocks [100, 112].

Although, it must be noted here that the best results are obtained when multiple methodologies are combined. For instance, Fu et al. [113] combine the genetic algorithm (as the training function) with neural networks to bolster their analysis.

It would be apt to conclude that the selection of the FEMU methodology is dependent on the user and good engineering judgement is required to select the appropriate methodology based on multiple factors such as the model’s complexity, number of unknowns, resource availability, and the objective function to name a few.

1.7.2 Constitutive Equation Gap Method (CEGM)

CEGM is based on the measurement of stress fields. It tries to measure the distance (or gap) between the current stress field \( \tau \) at a given node with another stress field computed through the constitutive model. This data is derived from the displacement field \( u \) [104].

Principally, CEGM is suited for experiments where over-determined data is available [104]. CEGM is again divided into two sub-methods: the first variant and the second invariant methods.

The first variant method is one which exactly enforces the kinematic measurements to the system [104]. This is somewhat analogous to an implicit analysis in finite element modelling.
The second invariant method on the other hand works on the principle of a penalty function. This is analogous to the explicit analysis done in FEA. This method does not exactly enforce the kinematic measurements, and the penalty term is specified by the user.

While CEGM started out as being applicable only to elastic field measurements, recent efforts poise it well to tackle non-linear constitutive models \[104, 114\].

### 1.7.3 Virtual Fields Method (VFM)

Virtual Fields method is applicable to situations where the full strain-field is known. This method is extremely well-suited when measurement techniques such as 3D DIC are applied, since this method gives the full strain tensor.

VFM, just like the previous methods, also needs a constitutive model from which it can extrapolate \[104\]. If the parameter $\theta$ is the one to be identified, the constitutive model chosen should allow the Cauchy stress tensor $\sigma$ to be related to this parameter and the measured strain-field \[104\].

Since this technique would rely on surface measurements, it would be prudent to have a relationship between the strain fields inside the solid and the strain fields on the surface.

A chosen virtual field then introduces a displacement governed field. There are numerous ways to construct a virtual field, with the simplest one being constructing an analytical one with the help of a polynomial function over the whole specimen. However, on the other spectrum of VFM, lies the ability to create a piece-wise virtual field over the whole specimen. The latter method provides better estimation of parameters, since a polynomial over the whole field has the assumption of homogeneity built into it \[104\].

Virtual fields also suffer from the issue of non-linearity when the virtual displacement field reduces to 0 (as it may happen in the case of creep cracks) in very localised fields. However, appropriate assumptions can be used to circumvent this peculiarity \[104\].

### 1.7.4 Equation Gap Method (EGM)

EGM is used for cases where elastic heterogeneity, as it might happen when there is stress concentration (for instance in notches), takes the form of a scalar field \[104\].

This method involves the construction of a finite element model mesh whose integration points coincide with the measurement points of the experiment. The stiffness matrix is subsequently established based on the contrast with adjacent elements. Based on this stiffness matrix, equilibrium is established at one specific node and the summation of similar nodes satisfies the equilibrium equation \[104\].

The definition of this method in the form of such elements makes it equivalent to VFM, with the fields being constructed piece-wise \[104\]. The EGM is somewhat similar to the CEGM method as well, but is less computationally expensive. The main difference between the two methods is that the CEGM uses the constitutive equations to define the gap between the measured and predicted responses, while the EGM uses the equilibrium equations. For the CEGM, the objective function (that is, the cost function to minimise) is based on the difference between an updated statically admissible stress field and the stress field reconstructed using the experimental strain field and the constitutive model, while for the EGM, local equilibrium of the material is assessed using the continuity of the stress vector at the interfaces. For inverse modelling, local equilibrium can be
calculated using finite differences between the elements interfaces, and their gap must be minimized \cite{104,114}.

Regardless of the technique used, it would be astute to say that selecting the correct model and the parameters to optimise requires good engineering judgement \cite{100}.

1.7.5 Applicability to creep and tensile tests

Given the nascency of the Small Ring Test (creep and tensile), few relevant studies have been performed on this testing methodology that incorporate inverse modelling. Perhaps one of the most relevant studies to this section for the SRT is the study done by Rouse et al. \cite{75}, which builds on the study by Kazakeviceite et al. \cite{73,74}. Rouse et al. \cite{75} derive an analytical expression for their small rings. Subsequently, they define an elastic-plastic material model (via Armstrong-Frederick cyclic plasticity) to describe their material, which is incorporated in ABAQUS. However, this is only used for initial estimates for the SRT optimisation. The authors do go on to say that they used the “well-known Ramberg-Osgood equation as a smoothing function in this procedure to limit the effect of noise in the experimental data” \cite{75}. This is an important insight since there are very limited number of studies for the SRT. Additionally, their inverse FE modelling via these material models relies on the basic ‘LSQNONLIN’ (Least Squares Non-Linear) procedure in MATLAB. It acts to minimise the sum of squares objective function with respect to the experimental dataset and the simulation’s values.

More and more studies have also come out in recent years that make use of the inverse finite element analysis in related fields. Zinonven et al. \cite{115} describe Nelder-Mead (NM) method, a gradient-free method, as the best approach suited for them for parametric optimisation alongside their inverse FE methodology. While this is applied only to conventional uniaxial specimen to determine the parameters of tungsten at elevated temperatures for use in the ITER project, the use of NM method deserves further exploration due to its promising results.

The NM method was originally introduced in 1965 \cite{116} and has been extensively used since then. Its implementation has been extremely streamlined in SciPy \cite{117} and is quite intuitive and user-friendly. The NM method is a part of the gradient-free methods for optimisation. Optimisation problems that don’t require gradient calculations are the most suited for this method. This is because the minimisation criterion for the FE analysis would be a comparison metric (root mean squared error, mean absolute error, and so on) and not a differentiable function. This value is obviously not a differentiable quantity. Neither is the function that is used to calculate this value. Thus, only gradient-free methodologies can be used for this optimisation for inverse FEA.

The NM method, or the downhill simplex method, is a gradient-free direct search algorithm. It works by iteratively searching the neighbourhood of the simplex. A simplex is a name given to the entity that describes the available solution space. Since this method requires searching in its neighbourhood, it is extremely sensitive to the initial guess, also known as the initial simplex.

However, the NM method sometimes runs the risk of getting stuck in local minima. This can be overcome by using good initial seeding points, but this requires some knowledge about the material. Thus, it is not recommended for a completely unknown material, which is why studies have proposed a combination of methodologies to overcome this shortcoming \cite{118,120}.
1.7. ABOUT INVERSE MODELLING

The other facet of the combination of methodologies is the Differential Evolution (DE) algorithm, which is a population-based optimisation method. The DE algorithm is not prone to the pitfalls of the NM method, since it samples the whole population space for possible solutions. The same error metrics as the NM method (RMSE, MAE, etc) can be used to calibrate this optimisation as well.

However, since DE samples the whole solution space, this method is much more computationally expensive and takes significantly longer to converge. Thus, for the combination of the methods, it is usually recommended that the DE method be used on a wide set of bounds and then be stopped once a semblance of convergence is found. These values can subsequently be used to set the bounds for the NM method [117, 118].

For the SPT, meanwhile, there are many more studies that make use of inverse modelling. The study by Wen et al. [121] focuses on inverse modelling in creep tests for the SPT. They use a non-linear least square optimisation routine for their inverse finite element modelling of the Small Punch Creep Test (SPCT) and minimise the errors between their experimental data and simulation data. However, their optimisation algorithm is informed by their uniaxial creep tests, which does not make this approach well-poised for unknown materials. However, this can be easily overcome with the help of empirical formulas described in the CEN standards [30, 31, 121].

The study by Peng et al. [41] describes a model based on equivalent energy principle. The Small Punch tensile Test (SPTT) is simulated, and the authors essentially split the curve obtained into elastic and plastic regions and use the total strain energy for their calculations. This is again an FEA-driven method, with its underlying FE approach largely similar to that adopted by Wen et al. [121]. However, their approach is largely empirical and is based only on FEA, with no optimisation. They use numerous experiments and simulation of fictional materials via FEA to analyse the ductility of metals in the SPT. The approach to use numerous materials to analyse a test is a sound approach that should be adopted for recently developed tests, especially the SRT.

Abendroth [38] opts for a different route and uses neural networks for SPTT. The load-displacement curve is divided into its 6 distinct regions, which were showcased in fig. [1.11]. The author then proposes a feed-forward three-layer neural network, All the layers are fully connected, and the algorithm is trained on the experiments performed by the author to obtain excellent results [38]. With enough diverse data on materials, as the author recommends, this remains a promising avenue to explore not just for the SPT, but for all miniature-specimen testing techniques.

The work by Yang et al. [89] uses the least squares method as its base. This is modified to yield the Least Squares Support Vector Machine (LS-SVM) model. The authors use FEM combined with a gold section search algorithm on the experimental data. The core function generated by them maps to a high dimensional feature space and uses a regularised cost function to accommodate the noise in the data. Essentially, the authors use FEMU to match with the experimental curve and train their LS-SVM based on that calibrated FEM model. The model is then trained to approximate and find the least possible error. However, the neural network they used (back-propagation and generalised regression neural network) is deemed as computationally expensive and time-consuming by themselves [89]. While, the authors say a lot of further work is still needed, this remains a promising approach to eventually obtain a generalised model for small-scale specimen testing, especially for translating the data to conventional stress-strain data.
Liu et al. \cite{122} use a FEMU-based approach as well for the SPT. In many ways, their approach is similar to VFM and CEGM, but they do not use a constitutive model. Instead, they perform a comparison between FEM and DIC results. They observe good results for low values of strain, but deviation starts to arise as the ultimate tensile strength point is approached. Additionally, these tests are performed only at room temperature. This could be a promising avenue to explore, but the nascency of the SRT means that the onset of the UTS is not fully known.

Cornaggia et al. \cite{123} perform FE simulations (after sensitivity analyses) and formulate an error function which needs to be minimised. They use experiments (SPTT) to assist this and an extremely quick and mathematically robust methodology (Trust Region Algorithm + Proper Orthogonal Decomposition + Radial Basis Function Interpolation) makes their approach unique in terms of quick computational output while also incorporating machine learning. Again, these experiments are done at room temperature and different constitutive models need to be tried for different parameters. However, this study does show promise that a Reduced Order Model can eventually be realised with the help of inverse modelling techniques. This is a unique study in its approach and the results are highly commendable. Their approach to use dimensionality reduction for the inverse FEA highlights how time-consuming and expensive numerical simulations can be. The authors, however, do observe that uncertainty quantification is needed with a lot more emphasis on stochastic modelling for such studies. Should a reduced order model eventually be desired, this would be a good approach to follow for small-scale specimen testing.

The study by Kamaya et al. \cite{124} is noteworthy as well. They obtain constitutive parameters from irradiated SS316L by using DIC coupled with FEMU. It is difficult to criticise the results, since there is a lot of scatter in irradiation testing data. The constitutive model for FEA is guided by the help of DIC data, which is then used to predict, with the help of a curve-estimation technique (k-fit), the constitutive parameters. This is a relevant study in the realm of coupling DIC with inverse FEA. While the optimisation techniques used are basic, they work well due to the robustness of the experimental data, thus showcasing the importance of a well-designed experiment.

Lastly, the thesis by Brown \cite{125} is an important study for the SPT. They use the Nelder-Mead method on the Small Punch Test. This is an interesting choice given the simplicity of the model, but they obtain excellent results for their study, and it shows that the NM can potentially work on its own as well for miniaturised mechanical testing when informed by proper initial seeding points, as mentioned previously.

Thus, it can be seen that a plethora of avenues exist for the inverse modelling of mechanical testing. These exist for creep testing as well as tensile testing. However, as mentioned previously, most of these are focused on the SPT. With the revision of the SPT standard by Bruchhausen et al. \cite{31}, there is more a more standardised approach to direct conversion of the SPT data directly from the SPT test output. However, the same cannot be said for the SRT and consequently, inverse modelling approaches need to be applied here for better understanding of this testing methodology.

Based on the perused literature, two algorithms are promising and could potentially be used in this study for inverse FE optimisation for the SRT. The gradient-free method could potentially make use of the Nelder-Mead (NM) method (also known as the downhill-simplex method) and Differential Evolution (DE) could potentially be the population-based method. Clearly, inverse numerical simulations are imperative, especially for the SRT. While this is a time-consuming process, it is necessary given the nascency of the SRT.
The procedure to perform inverse FEA simply depends on the availability of time and resources. The perused literature also highlights the complexities that arise with data conversion. The longer the data conversion pipeline is, the higher the uncertainty and resource utilisation becomes. Thus, this is an informed choice that needs to be made based on the scope of a testing programme. For this study, this would entail starting with the data analysis of the SRT performed, and then evaluating the test matrix before performing inverse FEA.
1.8 Research question and thesis outline

The literature perused demonstrates a strong drive for the development of testing techniques oriented towards small-scale specimens.

The testing techniques listed in section [1.5] showcase only some testing techniques available for miniaturized specimens. None of the techniques listed are perfect, and all of them have shortcomings.

For instance, the Small Punch test, has extensive conversion issues pertaining to its test outputs. The Small Ring Test, for instance, is widely unexplored, especially for tensile tests. The Ring Hoop Test requires extensive machining and precision and is not able to service creep tests either. The miniaturised uniaxial tests, on the other hand, suffer from the issue of stress localisation as well. The Small 2 Bar test, on the other hand, is relatively larger than all the tests but produces outputs from both its arms.

More research is needed in some testing methods (such as the small ring and small 2 bar tests) while some are more widely studied and mature (such as the small punch and miniaturised uniaxial tests). Thus, it would be logical to focus either on the improvement of a promising testing methodology or advance a relatively mature testing methodology.

However, the ideal methodology would be to focus on both these aspects in the most efficient manner.

To evaluate this idealistic approach, there seems to be a largely unexplored avenue of combining different testing methodologies. Given the small size of the small punch disc, it might be prudent to analyse whether this can be combined with other tests. There is a significant research gap here to be filled, since studies don’t take into account the possibility of combining these tests.

Given the geometries usually considered for these tests (based on the literature perused), it seems logical to evaluate whether a combination of the small ring test and the small punch test is possible. This is the overarching research question that this thesis intends to answer.

However, given the infancy of the small ring test, it needs more thorough evaluation before it can be combined with the small punch test. Thus, to ensure robustness of this nascent technique, the questions below need to be answered:

1. Is the Small Ring Test reliable and repeatable enough to establish better conversion procedures of the force-displacement curve to the stress-strain curve?

2. If the SRTT is reliable enough, there is no pre-defined way to measure the test outputs. Is it possible to propose an extensometer or a DIC methodology for this?

3. If the SRTT passes all these checks and proves its robustness, can it be successfully used on known (non work hardened) materials?

4. Can this knowledge be extended to unknown materials, such as welds?
The last two questions are largely inter-linked, but hold their own merit as separate questions as well.

The next chapter (chapter 2) delineates the process of constructing the experimental rigs and the strain measurement techniques. This chapter also details the parent sub-size uniaxial specimen tests that have been performed. Additionally, it also delineates the analytical solution for the small ring tensile test.

The third chapter (chapter 3) serves to answer the first research objective. A matrix of small ring tensile tests have been performed, analysed, and thoroughly evaluated in this chapter.

The fourth chapter (chapter 4) serves to extend the applicability and consistency of the small ring test. This is done by evaluating the test with DIC and the possibility of a potential extensometer based on the displacements obtained at various points in the ring to ensure test standardization.

The fifth chapter (chapter 5) combines the SRT and SPT for tensile tests on SS316L. Additionally, it also analyses the capability of creep on the SRT and tensile tests on SPT with the help of a standard material: Nimonic-75.

The sixth chapter (chapter 6) presents a discussion of all the results presented in the thesis and how they address the research question and objectives overall.

The conclusions and proposed future research directions are summarized in chapter 7.

A visual aid for this outline is also presented in the form of fig. [24].
Can the Small Ring Test (SRT) and Small Punch test (SPT) be combined in any way?

Chapter 1

Review of current research field for miniature-specimen test techniques

Chapter 2

Construction of relevant rigs, strain measurement systems, Uniaxial testing, and discussion of analytical solution

Chapter 3

A matrix of small ring tests at different rates to analyse:
- Filtering out test noise
- Inverse Finite Element Optimization
- Rate dependency
- Conversion to stress-strain curves

Chapter 4

Using DIC on the matrix of small ring tests: can it be helpful in any way to determine the feasibility of a novel extensometer?

Chapter 5

Combining SRT and SPT for:
- Comparative tests on the same material as chapter 4
- Creep test on small ring and tensile tests on small punch
- Used ex-service material with a significantly welded portion

Chapter 6

Conclusion and future research

Figure 1.24: Thesis outline for all chapters.
Chapter 2

Methodologies and tools used, Uniaxial testing, and Analytical Solution
2.1 Testing Apparatus Construction: SRT

Performing the SRT requires a new rig since it differs in its loading mode as compared to a conventional uniaxial test. The rig needs to have the capability to load pins into it while also maximising the viewing area for the SRT. The viewing area should at least allow viewing 45° angles of the ring in all quadrants (see fig. 2.2 in next section for quadrant explanation) so that digital image correlation can be performed. Beyond this, there may be a risk of the new SRT rig failing. This rig should also be able to handle small ring tensile test and small ring creep test as well. This required that the pins do not bend significantly while the test was being run.

The authors of the original small ring creep test\cite{72} demonstrate how the ring creep tests is one of the few tests that does not require a stronger creep-resistant material for creep testing. This merit for the small ring creep test bolsters confidence in the fixtures as well, since only the loading pin material is the point of concern. No standardized recommendations are available for this test and its rig design, but this information is useful in selecting the pin material, since the authors say that the material for the pin can be the same as that of the test sample\cite{72}.

The rig schematic and constructed rigs are shown next.

2.1.1 Implementation

The overall rig was manufactured with SS316L, while the pins have been made out of Hastelloy X. The schematic is shown in fig. 2.1. The rig is designed to accommodate rings of thickness 2 mm, with some clearance left (0.20 mm) for loading. It can be seen from the selected rig geometry that a strong incline (around 75°) with respect to the horizontal edge of the rig has been given to the fixtures. This allows for sufficient visualization window since the areas of interest (at least 45 degree of the ring sample) are clearly visible without any direct effect of obstruction. The rationale for having at least 45° regions of the ring visible is to propose a potential extensometer design, eventually with the help of digital image correlation (DIC). This is covered in chapter 4. The quadrants, meanwhile, can be visualised with the help of fig. 2.2. This image showcases the division of the ring into 4 quadrants, with the 45-degree regions highlighted. The quadrants are heavily utilised for their symmetry in the DIC chapter (chapter 4), where the extensometer design is proposed.
2.1. TESTING APPARATUS CONSTRUCTION: SRT

Figure 2.1: SRT grip sketch with front view and side view. All dimensions are in mm.
Figure 2.2: Division of the ring for the SRT into quadrants, with the 45-degree lines annotated.
The rig is shown in fig. 2.3 as not yet mounted, while an experiment in progress shown in fig. 2.4.

Figure 2.3: SRT grip: assembled but not loaded on the machine.

Figure 2.4: SRT grip: experiment in progress.
2.2 Testing Apparatus Construction: SPT

2.2.1 Requirements

The biggest challenge for the small punch testing facility was the lack of any available testing rigs capable of compression testing that would be able to, potentially, do a creep test in an inert atmosphere as well. Additionally, it was also required that the SPT rig be capable of performing 3D DIC at elevated temperatures to better visualise the disc deformation.

The only available rig capable to do this was the ZwickRoell Kappa 250 DS, which has been chosen for this test. The rig has a vacuum chamber (90 mm diameter tube) and the capability to perform tests in vacuum or in an inert atmosphere. However, due to manoeuvrability issues imposed by the narrow access within the vacuum chamber, the eventually designed SPT rig needs to be loaded in a single piece. Additionally, the rig is capable of only tensile testing and not compression testing, which necessitates that the SPT rig adapt to this constraint as well.

Thus, the requirements can be summarised as:

1. Ability to function like a compression test with the testing rig in tension test mode.
2. Capability for 3D DIC.
3. Ability to load the whole rig as a single piece in the narrow vacuum chamber.

2.2.2 Implementation

The viewing area for the rig is restricted to 38 mm once the machine is fully loaded. An opened view of the machine can be seen in fig. 2.5, where the viewing window is the piece of circular glass on the outer window (neon green line) and the glass chamber is the vacuum chamber (also labelled). The vacuum chamber does not have an easy access point, which is why, as mentioned previously, the SPT rig needs to be loaded in one piece.
Figure 2.5: An opened view of the ZwickRoell rig used for SPT. The viewing window and vacuum chamber are also marked.
To address the problem of the rig being capable of only tensile experiments, it was decided that instead of the punch pressing down on the disc (akin to a compression test), the rig was inverted by 180°. This would put the punch beneath the disc while still being connected to the top (moving) screw-head. Consequently, the punch, in the tensile mode, would still be penetrating the disc as it is pulled upwards to pierce the disc. A first design of this idea is shown in fig. 2.6. The purple cylinder with a taper at the end is the punch, which is connected to the top (moving) screw-head. It can be seen that the dies are clamped in a cage that is connected to the bottom rig. It should be noted that this is only the preliminary design image and is intended to only showcase the inversion loading mechanism.

Figure 2.6: Isometric view of preliminary SPT rig design. The pink cylinder is the punch, and the yellow and brown cylinders are the dies that hold the disc for the SPT.

Fig. 2.7 showcases the section view of the previous figure. The top die in this inversion has a flare which allows for light to enter. This lead-in allows for the light to enter the cavity and illuminate the specimen to allow for 3D DIC. The cameras will eventually capture the specimen images with the help of a tilted 45° Magnesium Oxide mirror which will be placed on top of this flared die. Also, to note here, is the brown die that receives the purple punch to pierce the disc.
Figure 2.7: SPT grip: section view to visualize the lead-in on the top die, which receives the deformed disc.
A manufactured version’s top view is also shown in fig. 2.8 to show the flare of the top (yellow) die. Meanwhile, fig. 2.9 showcases the mirror holder affixed on the top die, which will aid visualisation of the specimen during the test. The mirror is not shown in this image and is kept covered at all times until the rig is loaded due to its high propensity for contamination.

Figure 2.8: SPT grip: top view of the bottom fixture to illustrate the flare of the top die that allows for light to illuminate the disc.
2.2. TESTING APPARATUS CONSTRUCTION: SPT

Additionally, it can also be seen in fig. 2.8 that the closed cage structure has been removed. This is to allow for the top screw head parts to slide in the cavity. These are then loaded on to shoulder mounts of the bottom rig. This allows for the loading of the whole rig is a singular structure. This is best visualised with the help of the image in fig. 2.10. The shoulder mounts are highlighted on the bottom rig. The top rig enters via the cavity in the cage structure, rotates, fixes below the mounts, and helps lift the whole SPT rig into the machine. It should be noted that the dies (and thus the specimen) are placed in the rig after the top rig is holding on to the mounts of the bottom rig. Otherwise, the top rig cannot enter the structure since the dies will be blocking the way. For clarity and visualisation, the punch was not assembled for this picture to visualize only the mounts and how they enable loading.

A fully assembled but not loaded into the machine view can be found in fig. 2.11. The dies are labelled, and the punch is loaded and labelled as well. The top SPT rig slots into the driving screw-head that pulls the punch upward, thereby penetrating the disc. The mounts are also labelled for more visualisation of the mechanism. The Magnesium Dioxide mirror is also shown, but is covered until the last moment to not contaminate the mirror.
Figure 2.10: SPT grip: top view of the bottom fixture to illustrate the flare of the top die that allows for light to illuminate the disc.
Figure 2.11: SPT grip: Assembled but not loaded in the machine. Note the presence of the punch beneath the dies, indicating the inversion of the rig.
Next, fig. 2.12 shows the full SPT rig before it is loaded into the machine. It can be seen the top rig is supporting the whole structure with the help of the mounts.

Figure 2.12: SPT grip: Loading into the machine. It can be seen that the whole structure is supported by the mounts at the bottom rig.

Lastly, fig. 2.13 showcases the rig completely loaded (chamber not closed for visualisation). The top rig is freed from the mounts by applying a small rotation on the bottom rig. Since both parts are loaded as a single unit, this small rotation of the bottom rig about the vertical direction is important, since this frees the top rig and essentially disconnects the two parts. Thus, the punch is now free to pierce the disc while the bottom rig is independent and fixed.
2.2. TESTING APPARATUS CONSTRUCTION: SPT

Figure 2.13: SPT grip: Loading into the machine. It can be seen that the whole structure is supported by the mounts at the bottom rig.
Some modifications were made to the top die for the 3D DIC, but these are detailed in the DIC sections next (see section 2.3.2).
2.3 Displacement measurement: DIC

2.3.1 2D DIC

Background

Digital Image Correlation (DIC) is an optical measurement technique that compares images taken at different times to determine any displacement on the surface imaged between the images taken [126]. In 2D DIC, the displacement tracking process can be fine-tuned by considering various key aspects, such as subset size, step size, pyramid levels, epsilon, and adaptive subset approaches [126]. A typical speckle pattern, which is a randomized pattern of blacks and whites on the specimen, is shown in fig. 2.14. This speckle pattern provides the DIC analysis with reference points to calculate displacement vectors. Additionally, also consider fig. 2.15 for all the DIC-related explanations to follow since this annotated image is used to explain the concepts of step size, subset size, and the equations relating to DIC.

Figure 2.14: Typical DIC Speckle pattern. Adapted from [127].
Consider light intensity functions $F(x, y)$ and $G(x', y')$ to be at $P(x, y)$ and $Q(x', y')$, respectively, as well.

Figure 2.15: Annotated image to understand the concept of DIC. The image on the left is the reference image, while the image on the right is the deformed image.
A raw image captured by the camera cannot be processed by a computer. The image must first be converted to a greyscale format (8-bit) which allows the computer to read it based on the greyscale intensity $I(x, y)$. Sakanashi points out that the subset displacement is a function of luminous intensity, which permits the use of correlation functions. This is given as,

\[
C_1 = |F(x, y) - G(x', y')|
\]

\[
C_2 = \frac{\sum(F(x, y) \times G(x', y'))}{\sqrt{\sum(F(x, y)^2)\sum(G(x', y')^2)}}
\]

where, $F(x, y)$ is the light intensity (greyscale) at coordinates $(x, y)$ while $G(x', y')$ is the value at $(x', y')$. Consider these points in fig. 2.15. The algorithm tries to find a minimum for the parameter $C_1$ and $C_2$. The first correlation function is responsible for calculating the image intensity variation, while the second one is used to express the similarity between images.

A subsequent problem is the determination of the new sub-grid (also known as a ‘subset’) and subsequently the particle which was being tracked. The displacement field must be updated at every time-step measurement (that is, for every subsequent image). With the initial image fixed and a reference point defined, it is computationally inexpensive to perform further analysis and subsequently determine the best position to capture deformation (based on the speckle pattern primarily).

However, the above methodology poses the problem of sub-pixel grid interpolation. Sakanashi acknowledges this and elucidates the interpolation methods used. For efficiency, the input signal can be modified by the use of Fourier transform to provide for quicker throughput in the software.

While the equations above describe the correlation parameters for intensity variation and similarity between images, a generalized correlation function $C(x, y, u, v)$ can be written as with fig. 2.15 acting as a visual aid,

\[
C(x, y, u, v) = \sum_{i,j=-\frac{n}{2}}^{\frac{n}{2}} (I(x + i, y + j) - \Gamma(x + u + i, y + v + j))^2
\]

where, $x$ and $y$ are the pixel coordinates in the reference image. It can be seen in the figure that these points are contained in the ‘subset’ enclosed by the green boundary, with the size of the subset varying from $-n/2$ to $n/2$, which is shown in the equation as well. $n$ is the number of points in this subset, with $i$ and $j$ being the local coordinates within this subset. Once the specimen is displacement by displacement $u$ and $v$ (horizontal and vertical directions, respectively), the magnitude of the new coordinates of at point Q is $x + u$ and $y + v$. Now, considering these coordinates in the reference of the subset (that is, by using $i$ and $j$), we get the associated local values, which are given as $I(x + i, y + j)$ (reference image) and $\Gamma(x + u + i, y + v + j)$. A lower value of $C$ is desirable.

These can be converted into $x'$ and $y'$ notations now in the context of the local $i$ and $j$ coordinates. Consider the same point $P$ and the tracked point as $Q$ in the deformed
The first-order function based mapping for this point’s coordinates, in a generic form, is given as [128],

\[ x_i' = x_i + u_x \Delta x + u_y \Delta y \] \hspace{1cm} (2.4)
\[ y_i' = y_i + v_x \Delta x + v_y \Delta y \] \hspace{1cm} (2.5)

where, \((x'_i, y'_i)\) is the mapped position \(Q\), \((x_i, y_i)\) is the position of \(P\), \((u_x, u_y, v_x, v_y)\) are the first-order displacement gradients of the reference subset, \(\Delta x\) is the \(x\) distance between \(Q\) and \(P\) points while \(\Delta y\) is the \(y\) distance between \(Q\) and \(P\) points [128, 129].

These equations are now helpful to understand the concept of step size and subset size in the context of LaVision’s DaVis 10 software [130]:

- **Subset Size**: In the context of the image shown previously in fig. 2.15, subset is the name given to the blue and green squares. These squares are essentially the calculation windows for the DIC analysis as it moves spatially over an image. The size of the subset is crucial for the accuracy of the correlation. A smaller subset size may lead to higher spatial resolution, but might compromise the measurement quality due to noise. On the other hand, a larger subset size can improve the signal-to-noise ratio but may not capture smaller deformations. A balance must be struck based on the specific application and experiment setup.

- **Step Size**: Again, in the context of the image shown previously in fig. 2.15, step size can simply be understood as the amount of shift between two subsequent subsets. A smaller step size results in a higher density of data points, which can provide more detailed information on the deformation field. However, it also increases the computational cost of the analysis. A larger step size reduces the computational effort, but may lead to a lower resolution in the deformation field.

DIC has been used successfully in numerous studies and has proven its effectiveness [124, 126, 129, 131–135]. While a lot of the studies focus on 2D DIC, 3D DIC has the capability to obtain full-field measurements and is briefly discussed in section 2.3.2.

**Implementation**

It must be noted that it is user-dependent to identify what technique works best for a given testing scenario. For the Small Ring Test, the 2D DIC would be sufficient, since a 3D visualization would not meaningfully augment the dataset. This is because the primary deformation for the Small Ring Test is the in-plane deformation (via ring expansion) [75].

While the implementation of this 2D DIC is discussed in detail in chapter 4, it should be mentioned here that Nikon D810 cameras were used for DIC and all the rings tested in this study were painted with a randomised speckle pattern.

2D DIC needs a flat surface on a specimen which has its optical axis perpendicular to the specimen. It is quite sensitive to out-of-plane deformation as well [135], which is why it cannot be used when the deformation is towards or away from the camera. 3D DIC is able to overcome these issues and is discussed in the next section since this is relevant for the SPT.
2.3.2 3D DIC

Background

3D Digital Image Correlation (3D DIC), also known as stereo DIC, is a powerful and versatile optical measurement technique that has emerged as a significant advancement over its 2D counterpart. The key difference between the two methods lies in their ability to capture spatial information: while 2D DIC provides surface displacement measurements within a single plane, 3D DIC can obtain full-field, three-dimensional measurements of surface displacements and deformations [136].

The 3D DIC system employs a pair of cameras positioned at an angle with respect to the specimen, with their optical axes intersecting on the specimen surface. The typical angle between the cameras ranges from 10 to 30 degrees, depending on factors such as the specimen size, camera resolution, and desired measurement accuracy [126]. This configuration enables the simultaneous capture of images from two different perspectives, which are then processed using stereo-triangulation algorithms to compute the three-dimensional coordinates of points on the specimen surface.

An essential aspect of both 2D and 3D DIC is the application of a random speckle pattern on the specimen surface. In the case of 3D DIC, the pattern must be discernible from the perspectives of both cameras, ensuring sufficient overlap in their fields of view. The correlation functions employed in 2D DIC, as outlined in the previous section (2.3.1), can be extended and adapted to incorporate the additional camera perspective in 3D DIC [137].

One of the primary advantages of 3D DIC is its resilience to out-of-plane deformations, which can pose challenges for 2D DIC measurements. The ability to accurately measure complex, three-dimensional geometries and deformations makes 3D DIC particularly well-suited for studying material behaviour under diverse loading conditions, characterising material properties, and understanding failure mechanisms [126]. Furthermore, the technique can provide valuable information on surface topography, which may be crucial for applications in fields such as biomechanics, geophysics, and civil engineering [104].

Despite its numerous advantages, 3D DIC also presents some challenges, including increased complexity and cost associated with the need for more sophisticated hardware and software. Additionally, the use of two cameras can introduce potential sources of error, such as misalignment or calibration inaccuracies [136]. It is essential to carefully weigh these factors when determining the suitability of 3D DIC for a specific application.

In some cases, such as the Small Ring Test discussed in the previous section (2.3), 2D DIC may be sufficient, as the added complexity of 3D visualisation does not significantly enhance the dataset. However, for tests involving more intricate geometries or deformations, such as the SPT, 3D DIC can offer valuable insights that are unattainable through 2D DIC alone [54]. In certain experimental setups, such as the one proposed in this study, space constraints may make it challenging to achieve the desired angle between the cameras. To overcome this limitation, a partial mirror with a 50-50 reflect-transmit ratio can be used to reflect the image from one camera to another camera positioned approximately 90 degrees to the first camera. This approach should, theoretically, allow for the effective use of 3D DIC in situations where the conventional camera arrangement is not feasible.
Implementation

Vijayanand et al. [54] designed a novel stereo DIC system for the small punch test. It worked by reflecting the image of the disc from a mirror positioned on top of the die, which was in turn reflected to the stereo DIC system.

Unfortunately, their system could not be replicated for the current setup proposed in this study, since the reflected image from within the die did not have a wide enough area to position 2 cameras for a stereo vision system. This was due to the restrictions posed by the 38 mm diameter window of the furnace.

Despite best efforts, two cameras could not be positioned to visualize the tests going on within the chamber. Additionally, the calibration of a stereo system requires the placing of a well-defined grid that is moved by fixed distances to calibrate the stereo system. Given the lack of access to the disc and the dies in general throughout the experiment, a workaround needed to be developed. This hurdle for calibration was in addition to the already existing hurdle of lack of camera access to the testing setup.

A potential methodology was developed to address this issue:

1. Mount 2 cameras at approximately right angle to each other on a stage. Note: this angle was later modified to test various angles as well.
2. Place a partial mirror in the line of sight of the first camera such that the first camera can see the specimen straight into the furnace, and the image is also reflected onto the other camera (Cam 2, perpendicular camera). Placing the mirror at approximately 45 degrees with respect to the first camera was found to be a good starting point.
3. Calibrate the stereo system with the help of a calibration pattern etched on the die. This is done by clicking a picture at the most focussed point of the calibration pattern and then moving the whole stage, in sequence of 1 mm, by 5 mm on both sides of this focussed point.
4. Once both the cameras have captured the required images, the whole stage is moved so that both cameras are now focussed on the small punch disc. Note: The physical focussing of the lens remains untouched.
5. Capture a reference image of the disc from both the cameras before starting the test.
6. Set a fixed interval for both the cameras and start the cameras at the same time as the experiment (both are controlled via a central relay switch).

A 50-50 partial mirror was used to achieve the imagining in both the cameras. The image seen by the second camera (Cam 2, perpendicular) would be a mirror image of the one seen by the first camera.

It is also important to note here that an exactly perpendicular angle would not be feasible, since the images in both cameras would be the same. Thus, a slight angle of 5° was found to be optimum after repeated testing.

Fig. 2.16 shows the die with the regular grid pattern (0.25 mm blank space and 0.75 mm squares) that was used for camera calibration. Fig. 2.17 shows the full camera setup used, with fig. 2.18 showing a zoomed-in version of the same.
2.3. DISPLACEMENT MEASUREMENT: DIC

Figure 2.16: Etching on the upper die (disc with lead-in for specimen visualisation) of SPT for stereo DIC calibration.

Figure 2.17: Full 3D DIC camera setup mounted on the stage, along with the partial mirror between the cameras to enable imaging in both the cameras. The furnace with the SPT rig is kept open for visualisation purposes.
Figure 2.18: Zoomed in version of fig. 2.17 to showcase the partial mirror near the lenses.
The next few images, from fig 2.19 to 2.26, show the progression of a typical experiment from the calibration image to the end of the test, with the punch visibly piercing the disc. While full-size images are being used to appreciate the images, it must be noted that some image compression is inevitable when viewing them on a document such as this. The images shown make use of the magnesium oxide mirror that allows for disc visualisation.
Figure 2.19: Camera 1: Calibration step. Focused on the calibration pattern.
Figure 2.20: Camera 2: Calibration step. Focused on the calibration pattern.
Figure 2.21: Camera 1: Start of experiment. Focus moved to disc.
Figure 2.22: Camera 2: Start of experiment. Focus moved to disc.
Figure 2.23: Camera 1: Midway during the experiment.
2.3. DISPLACEMENT MEASUREMENT: DIC

Figure 2.24: Camera 2: Midway during the experiment.
Figure 2.25: Camera 1: End of test.
2.3. DISPLACEMENT MEASUREMENT: DIC

A couple of observations can be made from the set of images showcased in fig. 2.21 to fig. 2.26, which are also indicative of some reasons for the failure of this methodology.

Firstly, as the camera setup is moved and the experiment is started, there is a movement of the disc from the original place in the calibration images to the disc position at the
start of the experiment. However, this should not be an issue since the whole image space is constant and the calibration is applied to the whole image space and not just the calibration pattern area.

The DIC analysis always calibrated well, however, the DIC analysis always failed because it could not find correspondence in the secondary camera (Cam 2) in DaVis 10. This could likely be because the disc was not centrally aligned. To address this, more experiments were performed to alleviate this by having the discs be in the centre of the image in both the cameras after they were focussed on. Unfortunately, the problem persisted and the DIC analysis always failed.

Additionally, as the experiment progresses, there seems to be movement in the rigs which affects the DIC greatly, thereby affecting the focus as well.

Thus, these 2 points (rig movement and disc position during DIC) should be borne in mind when future researchers want to attempt 3D DIC in restricted spaces. Given the failure of this aspect of the project, the crosshead displacement was relied upon while performing SPT, which will be shown in chapter 5. While it is not conventional to detail failures of a technique in such detail, this information is useful to disseminate given the use of partial mirror in restricted spaces for 3D DIC application in SPT.
2.4 Uniaxial Tensile Testing

2.4.1 Background and Material Information

The uniaxial tensile test applies a unidirectional load to a specimen in which the load is aligned with the longitudinal axis of the specimen. The specimen is subjected to a force along its lengthwise direction, resulting in an extension of the specimen in that same direction. This loading configuration allows for the measurement of material properties such as strength, ductility, and stiffness under uniaxial stress conditions and has been codified as well [8]. Uniaxial tensile tests were needed for this study, since these serve as the reference values for the SRT and the SPT performed.

The primary material for this study is stainless steel (grade 316L). This as-received material has a manufacturer given 0.2% proof strength of 480 MPa with a hardness of 202 HBw (Brinell hardness), which corresponds to Hv 0.3 (vickers hardness) of approximately 225. The chemical composition is given in table 2.1. The material was heat treated at 1060° for 4 hours and was cooled with standard water quenching. The material was also hyper-tempered and cold-drawn into cylinders of 20 mm diameter. The grain size of the material was found to be 54µm (±6µm) and an example micrograph is also shown in fig. 2.27.

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
<th>Cu</th>
<th>N</th>
<th>Co</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.019</td>
<td>0.60</td>
<td>1.59</td>
<td>0.035</td>
<td>0.029</td>
<td>17.18</td>
<td>2.03</td>
<td>11.20</td>
<td>0.46</td>
<td>0.019</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2.1: Chemical composition of the primary material tested in this study (SS316L). All values represent weight %.

Figure 2.27: Example micrograph of the material used.
2.4.2 Implementation and Results

The crosshead displacement rates chosen for the uniaxial reference tests were 0.5 mm/min and 0.8 mm/min. Reference tests were performed on a standard uniaxial specimen with a gauge length of 40 mm, thickness 2 mm, and width of 2 mm on the Instron 6800 Universal testing machine. A remark that can be made here is on the choice of these displacement rates for testing. An ideal scenario would be a spectrum of uniaxial tests, ranging from the lowest to the highest rates used for the small-scale specimens. However, due to an extensometer and material availability issue, this could not be fulfilled.

Nonetheless, the mean of both the test results is taken. At slower displacement rates, the strain rate effect is much lower, as evidenced by Jin et al. [138]. While their study is for SS304, this can be extended to SS316L given the same category of metals. Their study extensively analyses the strain rate dependency from $10^{-4}/s$ to $10^{3}/s$. They conduct a plethora of tests and find that the trends for the material parameters are largely constant below $10/s$ strain rate. The rates tested here are much lower than that.

At slower strain rates, the motion of dislocations isn’t very rapid, and there’s ample time for dislocation interactions, annihilation, or for dislocations to move to grain boundaries. This results in a more or less stable observed material properties. Thus, while there is a deviation expected in these tests, it is not expected to exhibit a strong strain rate dependency. Nevertheless, it must be conceded that the methodology adopted in this study could be enhanced by performing uniaxial tests at various rates, and this remains a strong point of improvement for future studies.

Additionally, it should also be noted here that the strain rates used in this study (for SRT and SPT as well) are considerably lower than those encountered in manufacturing processes. This is because the primary focus of this work is not to simulate high-rate processes, but to gain insights, by evaluating the feasibility of the SRT and SPT in conjunction, into the mechanical behaviour of materials in the context of maintenance of structural components and development of new alloys. The strain rates encountered in this domain are often much lower than those in manufacturing processes, hence the choice of these relatively low displacement rates.

A schematic of the test sample is shown in fig. 1.7. The results are shown in fig. 2.29 and 2.30 individually and combined in fig. 2.31. The ‘yield line’ is used to determine the proof strength at the 0.2% strain point via the intersection method. The values are given in the next subsection (2.4.3).
2.4. UNIAXIAL TENSILE TESTING

Figure 2.28: Schematic of the test specimen used for uniaxial tensile testing. All dimensions are in mm.

Figure 2.29: Uniaxial tensile test data at 0.5 mm/min displacement rate. ‘Yield Line’ represents the line at 0.2% strain that is parallel to the elastic portion of the curve.
Figure 2.30: Uniaxial tensile test data at 0.8 mm/min displacement rate. ‘Yield Line’ represents the line at 0.2% strain that is parallel to the elastic portion of the curve.

Figure 2.31: Results of both uniaxial tensile tests.
2.4.3 Discussion of Results

The elastic modulus from the 0.5 mm/min test was found to be 209.710 GPa and 206.905 GPa from the 0.8 mm/min test. The proof strength (at 0.2% strain) was found to be 526.08 MPa from the slower test and 501.7 MPa from the faster one. The mean values, thus, are 208.21 GPa for the elastic modulus and approximately 513.89 MPa for the proof strength. These will later be used in subsequent chapters to compare the SRT and SPT against uniaxial tensile test data.
2.5 Analytical solution for Small Ring Tensile Test

2.5.1 Background

Given the lack of literature about the small ring tensile test, there are not many references for the analytical solution for the SRTT.

The most recent study on it by Rouse et al. [75] attempts to provide a detailed explanation of the SRTT. However, it is obfuscated by the omission of some mathematical steps and the full implementation of the analytical solution is not presented in their study. In this section, this derivation is properly explained for ease of future researchers by elaborating on the crucial integration and substitution steps while also providing with the analytical solution in a python script (see Section 2.5.2 and Appendix A.1).

As in the original study by Rouse et al. [75], a quarter ring model is considered with a P vertical load and a Q dummy load. The same free-body-diagram (FBD) as the original study is considered here.

Rouse et al. [75] use an assumption that considers the loading pins as a point load in their 2D model, and their experiments have demonstrated this assumption is largely true.

The following FBD (fig. 2.32) from the original paper as used as a reference again in this section. Sub-figure a) showcases the quarter ring geometry, while sub-figure b) showcases the full free-body diagram used in the original study.

The effective radius for the elliptical section is defined as R'. The chord length between
the vertical symmetry point and the end of the section is defined as $\gamma$. The angle that this segment makes with the centre of the ring is defined as $\theta$. Meanwhile, $\phi$ is the angle made by the tangent with respect to the horizontal axis at the segment-end. Lastly, $\zeta$ is the angle made by the chord length $\gamma$ to the horizontal axis.

Most of these parameters can be derived right away. However, it is important to note that $R \sin(\theta)$ is the distance of horizontal component and its cosine component is the vertical distance.

The terms are described as,

$$ R' = ab \sqrt{\frac{a^2}{2} + \frac{b^2}{2} + \frac{(a^2 - b^2) \cos(2\theta)}{2}} \quad (2.6) $$

$$ \gamma = \sqrt{b^2 - 2bR \cos(\theta)} + R'^2 \quad (2.7) $$

$$ \zeta = \tan^{-1} \left( \cosec(\theta) \left( \frac{b}{R'} - \cos(\theta) \right) \right) \quad (2.8) $$

$$ \phi = \tan^{-1} \left( \frac{-b^2}{a^2} \tan(\theta) \right) \quad (2.9) $$

These 4 equations are derived by exploiting the geometrical relationships in the FBD. For instance, $\gamma$ is derived from the pythagorean theorem applied to the triangle made by sides $[R \sin(\theta)]$ and $[b - R' \cos(\theta)]$.

After the definition of these terms, the authors go on to describe the bending moment caused by the load $P$ and $Q$.

The load $P$ and $Q$ cause a bending moment $M_0$, which in turn causes the reaction moment $M_\theta$ at the end of this segment. This bending moment has been defined as:

$$ M_\theta = M_0 + \gamma [P \cos(\zeta) - Q \sin(\zeta)] \quad (2.10) $$

This is derived by simply balancing the forces. $\gamma (P \cos(\zeta))$ is the horizontal component of the force $P$ with respect to the angle $\zeta$. The multiplication with $\gamma$ is to make use of the trigonometric identities. Similar treatment has been given for the dummy load $Q$.

Once this moment has been delineated, the bending strain energy ($U_B^*$) has been calculated. For a given length of beam $L$, This relationship is given by:

$$ U_B^* = \int_0^L \frac{M_\theta^2}{2EI} \, dx \quad (2.11) $$

However, for this simplified assumption of a curved beam, this can be converted to in terms of polar coordinates by using the arc length relationship, where $x = R' \theta$. This changes the equation to:

$$ U_B^* = \int_0^{\pi / 2} \frac{M_\theta^2}{2EI} R' \, d\theta \quad (2.12) $$
The study by Rouse et al. [75] has a typo in the above equation while changing the limits, but the study by Kazakeviciute et al. [73] does not have this.

After the changing of this limit, the authors equate this complimentary strain energy due to bending as 0. This is done by assuming that there is no rotation at the vertical line of symmetry, i.e., the point where the load $P$ is acting. This is a reasonable assumption that is in line with the experimental observations. Thus, this gives,

$$0 = \frac{1}{2EI} \int_{0}^{\frac{\pi}{2}} M_0^2 R' \, d\theta \quad (2.13)$$

The $M_0^2$ term can be expanded to its original form now. This yields,

$$0 = \int_{0}^{\frac{\pi}{2}} (M_0 + \gamma[P\cos(\zeta) - Q\sin(\zeta)])^2 R' \, d\theta \quad (2.14)$$

Which then results in,

$$0 = \int_{0}^{\frac{\pi}{2}} (M_0^2 + 2M_0\gamma P\cos(\zeta) - 2M_0\gamma Q\sin(\zeta) - 2\gamma^2 PQ\cos(\zeta)\sin(\zeta)$$

$$+ \gamma^2(P^2\cos^2(\zeta) + Q^2\sin^2(\zeta))) R' \, d\theta \quad (2.15)$$

Getting $R'$ involved in these terms and splitting the integrals gives,

$$0 = \int_{0}^{\frac{\pi}{2}} R' M_0^2 \, d\theta + \int_{0}^{\frac{\pi}{2}} R'2M_0\gamma P\cos(\zeta) \, d\theta - \int_{0}^{\frac{\pi}{2}} R'2M_0\gamma Q\sin(\zeta) \, d\theta$$

$$- \int_{0}^{\frac{\pi}{2}} R'2\gamma^2 PQ\cos(\zeta)\sin(\zeta) \, d\theta + \int_{0}^{\frac{\pi}{2}} R'\gamma^2 P^2\cos^2(\zeta) \, d\theta$$

$$+ \int_{0}^{\frac{\pi}{2}} R'\gamma^2 Q^2\sin^2(\zeta) \, d\theta \quad (2.16)$$

Out of all these 6 integration terms, $M_0$, $P$, and $Q$ are not dependent on $\theta$. These can thus be considered as constants of integration. The remaining terms can be re-arranged now,

$$0 = M_0^2 \int_{0}^{\frac{\pi}{2}} R' \, d\theta + P^2 \int_{0}^{\frac{\pi}{2}} R'\gamma^2 \cos^2(\zeta) \, d\theta + Q^2 \int_{0}^{\frac{\pi}{2}} R'\gamma^2 \sin^2(\zeta) \, d\theta$$

$$- PQ \int_{0}^{\frac{\pi}{2}} 2R'\gamma^2 \cos(\zeta)\sin(\zeta) \, d\theta + M_0 P \int_{0}^{\frac{\pi}{2}} 2R'\gamma \cos(\zeta) \, d\theta$$

$$- M_0 Q \int_{0}^{\frac{\pi}{2}} 2R'\gamma \sin(\zeta) \, d\theta \quad (2.17)$$
The remaining integration terms are what Rouse et al. define as $A_1, A_2, ..., A_6$. These terms are thus defined as,

\[
A_1 = \int_0^\frac{\pi}{2} R' \, d\theta 
\]

\[
A_2 = \int_0^\frac{\pi}{2} R' \gamma^2 \cos^2(\zeta) \, d\theta 
\]

\[
A_3 = \int_0^\frac{\pi}{2} R' \gamma^2 \sin^2(\zeta) \, d\theta 
\]

\[
A_4 = \int_0^\frac{\pi}{2} 2R' \gamma^2 \sin(\zeta) \cos(\zeta) \, d\theta = \int_0^\frac{\pi}{2} R' \gamma^2 \sin(2\zeta) \, d\theta 
\]

\[
A_5 = \int_0^\frac{\pi}{2} 2R' \gamma \cos(\zeta) \, d\theta 
\]

\[
A_6 = \int_0^\frac{\pi}{2} 2R' \gamma \sin(\zeta) \, d\theta 
\]

Using the above terms in equation 2.17 gives,

\[
0 = M_0^2 A_1 + P^2 A_2 + Q^2 A_3 - PQA_4 + PM_0A_5 - QM_0A_6 
\]

This is now differentiated with respect to $M_0$ which gives,

\[
M_0 = \frac{QA_6 - PA_5}{2A_1} 
\]

The ring section also has corresponding tensile ($T$) and shear ($S$) forces. These can be expressed in the forms loads $P$ and $Q$ while considering their horizontal and vertical contributions as applicable. These are defined in equation 2.27 and 2.28. Consequently, just like the complimentary strain energy for bending equation, it is possible to use the equations for complimentary strain energy due to tensile forces ($U^{*}_T$) and due to shear forces ($U^{*}_S$). For the latter, however, the equation demands the answer in terms of the shear modulus $G$, which is given in equation 2.29.

\[
T = P \sin(\phi) + Q \cos(\phi) 
\]

\[
S = P \cos(\phi) - Q \sin(\phi) 
\]

\[
G = \frac{E}{2(1 + \nu)} 
\]

Rouse et al. then delineate the corresponding strain energy equations. These are given as,
\[ U_T^* = \int_0^L \frac{T^2}{2EA} \, dx = \int_0^{\frac{\pi}{2}} \frac{T^2}{2EA} R' \, d\theta \]  
\[ U_S^* = \int_0^L \frac{S^2}{2GA} \, dx = \int_0^{\frac{\pi}{2}} \frac{S^2(1 + \nu)}{EA} R' \, d\theta \]  
\[ (2.30) \]
\[ (2.31) \]

Similar treatment as given to the complimentary strain energy due to bending (equations (2.12) to (2.25)) yields the following expressions:

\[ U_T^* = \frac{1}{2AE} \left( P^2 B_1 + Q^2 B_2 + PQB_3 \right) \]  
\[ U_T^* = \frac{1 + \nu}{AE} \left( P^2 B_2 + Q^2 B_1 - PQB_3 \right) \]  
\[ (2.32) \]
\[ (2.33) \]

where,

\[ B_1 = \int_0^{\frac{\pi}{2}} R' \sin^2(\phi) \, d\theta \]  
\[ (2.34) \]
\[ B_2 = \int_0^{\frac{\pi}{2}} R' \cos^2(\phi) \, d\theta \]  
\[ (2.35) \]
\[ B_3 = \int_0^{\frac{\pi}{2}} 2R' \sin(\phi)\cos(\phi) \, d\theta = \int_0^{\frac{\pi}{2}} R' \sin(2\phi) \, d\theta \]  
\[ (2.36) \]
\[ (2.37) \]

It was important to delineate this information for future researchers should the need to thoroughly critique this analytical solution arise in the future.

Rouse et al. [75] use the summation of these terms to find the total strain energy. After perusing the literature, it can be understood that they have chosen to omit the strain energy due to torsion, since this is a plane stress assumption. This is an important assumption to reiterate and note.

The summation of all energies results in,

\[ \sum U = M_0^2 A_1 + P^2 A_2 + Q^2 A_3 - PQA_4 + PM_0 A_5 - QM_0 A_6 \]
\[ + \frac{1}{2AE} \left( P^2 B_1 + Q^2 B_2 + PQB_3 \right) + \frac{1 + \nu}{AE} \left( P^2 B_2 + Q^2 B_1 - PQB_3 \right) \]  
\[ (2.38) \]

Differentiating this with respect to P and Q yields the vertical and horizontal displacements, respectively. The partial differentiation of the strain energy yields the deflection for a linearly elastic material. Thus, for a vertical load P it will give the vertical displacement and for the horizontal load Q it will give the horizontal displacement. This is given as,
2.5. ANALYTICAL SOLUTION FOR SMALL RING TENSILE TEST

\[ u_v = \frac{1}{2EI} \left( 2PA_2 + M_0A_5 - QA_4 \right) + \frac{1}{2AE} \left( 2PB_1 + QB_3 \right) + \frac{1 + \nu}{AE} \left( 2PB_2 - QB_3 \right) \]

\[ u_h = \frac{1}{2EI} \left( 2QA_3 - PA_4 - M_0A_6 \right) + \frac{1}{2AE} \left( 2QB_2 + PB_3 \right) + \frac{1 + \nu}{AE} \left( 2QB_1 - PB_3 \right) \]  

(2.39)

\[ (2.40) \]

Q needs to be set to 0 since it is a dummy load and the value of \( M_0 \) must be substituted in from equation 2.26. This results in,

\[ u_v = P \left( \frac{A_2}{EI} - \frac{A_2^2}{4EI A_1} + \frac{B_1(3 + 2\nu)}{AE} \right) \]

\[ u_h = P \left( \frac{A_5A_6}{4EI A_1} - \frac{A_4}{2EI} - \frac{B_3(1 + 2\nu)}{2AE} \right) \]  

(2.41)

(2.42)

These equations are solved to calculate the horizontal and vertical displacements for each load iteration by Rouse et al. [75] and subsequently update the \( a \) and \( b \) values based on the change in displacements. The horizontal displacement values are subtracted from the \( a \) values (ring shrinkage in horizontal direction) for each successive iteration, and the vertical displacement values are added to the \( b \) values (ring elongation due to pin pulling in vertical direction) for each successive iteration.

2.5.2 Implementation

The implementation of this analytical solution in Python requires the creation of a couple of functions. These are the calculation of the second moment of area and the calculation of all the integration terms involved in the analytical solution. The second moment of area for the ring is given by:

\[ I = \pi \left( \frac{b_1a_1^3}{4} - \frac{b_0a_0^3}{4} \right) \]  

(2.43)

where \( I \) is the second moment of area, \( b_1 \) and \( a_1 \) are the outer ring dimensions and \( b_0 \) and \( a_0 \) are the inner ring dimensions.

In the script used, this routine is updated at every step of the loop as the dimensions change. The integrals \( A_1 \) through \( B_3 \) also need to be calculated and updated at every step. These terms were derived in the previous section (see eqn 2.18 to eqn 2.34).

It is important to note that symbolic integration does not converge, and numerical computation has to be used. Additionally, an important force correction parameter \( (f_p) \) is needed. This factor is multiplied with the force array to account for 3D effects of the ring, since the analytical solution assumes a 2D ring.

While it is quite unorthodox to have pages of code in a written body of work like this, this has been included for completeness (see Appendix A.1 for full code) because the
values obtained from this study for force correction parameter differ from those obtained by Rouse et al. \[75\]. The definition of all the relevant functions (moment of inertia and integration terms) can also be found in the appendix.

The script uses a linearly spaced array of force to determine the displacement values. The force value is used in the equations given in equation 2.41 and 2.42, as seen in the loop that iterates over the force array. The calculated displacement values are then used to update the ring dimensions, with the vertical displacement being added and the horizontal displacement being subtracted from the ring’s minor (vertical) and major (horizontal) axis, respectively. Once the dimensions are updated, the force correction factor is calculated. The function for the estimation of the correction factor was proposed by Rouse et al. \[75\] as,

\[
fp = a_1 \cdot \exp(a_2 \cdot u_v) + a_3 \cdot \exp(a_4 \cdot u_v)
\]  

(2.44)

where, \(a_1\) to \(a_4\) are the unknown coefficients to be determined and \(u_v\) is the analytical vertical displacement. Thus, this gives the correction factor for the analytically observed force a dependency on the vertical displacement.

The original force value is then multiplied with this correction factor. One way to interpret this multiplied force is to view it as the force in 3D that would be required to cause the displacement in the ring. This process repeats for every new value of force in the loop.

For preliminary evaluation purposes and testing the values given by Rouse et al. for their \(fp\), an elastic modulus of 200 GPa is assumed (for now). However, using the elastic modulus in its MPa form does not resolve in any convergence and the numerical solution breaks down when the parameters used by Rouse et al. are used. The solution only converges when used in its GPa form for their parameters. This is an important factor that must be noted, since it is quite unusual to have an analytical solution converge only via this form. For proper unit balancing in the equations, the MPa form is desirable, since this equates to \(N/mm^2\) units directly. Bearing this observation in mind, elastic modulus in GPa only have been considered while testing their force correction factor.

The values of the parameters obtained from this optimisation by Rouse et al. were:

1. \(a_1 = 0.986\)
2. \(a_2 = 0.103\)
3. \(a_3 = 3.567e-6\)
4. \(a_4 = 4.995\)

The result from this script, with and without the correction factor with the parameters defined by Rouse et al. \[75\] for their range of 1.6 mm vertical displacement, is shown in fig. 2.33. It can be seen that the modified analytical solution with the correction factor has almost no difference without the correction factor. Fig. 2.34 shows the variation of their correction factor (\(fp\)) with respect to the analytical displacement observed. A linearly rising trend is observed, which is consistent with the findings obtained by Rouse et al. \[75\] as well.
Lastly, fig. 2.35 showcases the variation of the ring dimensions and the horizontal displacement \( (u_h) \) variation as well with respect to the analytically observed vertical displacement \( (u_v) \). A consistently rising trend is obtained for the ring’s vertical dimensions, indicating that the ring is being stretched. This is corroborated by the horizontal dimensions shrinking as well (blue and green lines). It should be noted that the experimental data is a combination of elastic and plastic regimes, while the analytical solution considers only elastic behaviour. Thus, a straight line is expected for this initial part of the experiment, with a divergence expected as the test progresses and plasticity is introduced.

![SRTT: Analytical (with and without correction factor)](image)

Figure 2.33: Analytical solution of the small ring tensile test with and without the correction factor defined by Rouse et al. [75].
CHAPTER 2. METHODOLOGIES AND TOOLS USED

Figure 2.34: Variation of correction factor with respect to displacement using correction factor parameters defined by Rouse et al. [75].

Figure 2.35: Variation of ring geometry using correction factor defined by Rouse et al. [75].
To obtain the force correction factor’s parameters for this study, an optimisation procedure was run with the ‘L-BFGS-B’ method via SciPy [139]. This optimisation procedure allowed the placement of bounds on all the 4 values to be optimised. These bounds were set to be practically unlimited, with the constraints ranging from 0 to infinity. Additionally, the elastic modulus was also kept at 200 GPa, but was multiplied by 1000 to convert it to the MPa (or \(N/mm^2\)) form.

Given that Kazakeviciute et al. [73, 74] demonstrate that the analytical solution should be a straight line, the vertical displacement cut-off was set to account only for the linear part (up to 0.5 mm) for optimisation purposes. Any additional values would cause the function to wrongly bias away from the region of interest, which is pivotal since the region of interest is the elastic region only.

A heavily frontal-biased function was used on the RMSE (Root Mean Squared Error) which was computed between the modified force (modified via \(f_p\)) and the experimentally observed force. The weighting bias was introduced via the following function:

```python
def weighted_rmse(y_true, y_pred, weight_func):
    # y_true is the experimentally observed force
    # y_pred is the analytically predicted force
    n = len(y_true)
    weights = weight_func(np.linspace(0, 1, n))  # distribute
    rmse = np.sqrt(np.mean(((y_true - y_pred) * weights) ** 2))
    return rmse

weight_func = lambda x: 1 - x ** 12
```

The parameters of \(f_p\) were calibrated on a Small Ring Tensile Test on Stainless Steel (Grade 316L) at 0.9 mm/min. For now, this is only a representative value of the \(f_p\) parameters to indicate the difference between the original parameters and the correction parameters found in this study. Analytical solution is run on a multitude of tests in the next chapter. These parameters were:

1. \(a1 = 10.22\)
2. \(a2 = 0\)
3. \(a3 = 14.06\)
4. \(a4 = 0\)

Essentially, it is a constant value of 24.28 for this experiment. The results from this correction factor are now shown in fig. 2.36—2.38. It can be seen that the analytical solution now matches very well with the experimental data in fig. 2.36. The next image (fig. 2.38) showcases the variation of the force with respect to the normalised displacements. The first sub-figure uses the normalised displacement of vertical over horizontal displacement. The second sub-figure uses the normalised displacement of horizontal over vertical displacement, and the force has also been normalised with respect to the maximum force observed in the experiment (around 2400 N). Lastly, the final image (fig. 2.38) showcases the variation of the ring geometry with respect to the vertical displacement. The ring expansion follows a gradually rising trend, while the horizontal
displacement exhibits a gradually declining trend, therefore indicating the ring is expanding in the vertical direction and contracting in the horizontal direction.

Figure 2.36: Analytical solution of the small ring tensile test using the correction factor defined in this study.

Figure 2.37: Variation of force (non-normalised and normalised) against the normalised horizontal displacement
2.5.3 Discussion

A thorough evaluation of the analytical solution for the SRTT was showcased here. For a preliminary demonstration of how the correction parameter differs in this study with that derived by Rouse et al. [75], one of the tests was showcased here. The analytical solution is tested on all the tests performed in the next chapter, and the values showcased here should only be regarded as a demonstration to indicate how much these values can differ with those derived in the original study.

While largely similar to the values obtained from optimisation on all experiments in the next chapter, it should be noted that these values presented in this section are only for 1 experiment at 200 GPa. These might change for other experiments, and they do change in the next chapter. Thus, these images should be taken as an indicator of the analytical solution in this section and the procedure needed to solve the associated equations.

Additionally, it should again be noted that for illustrative purposes, the experimental data used here for comparison is from one of the experiments performed in chapter 3. This script is used extensively at the end of that chapter to critically analyse the SRTT and its elastic properties.

It now makes sense to have a look at the matrix of SRTTs performed. This is done next, in chapter 3, after a brief chapter summary in the next section.
2.6 Chapter Summary

This chapter delineated the tools and methodologies adopted in this study that will help answer the research questions.

The first section delineated the rigs for SRT and SPT and how they were constructed. This was especially difficult for the SPT rig given the narrow dimensions of the machine involved, in addition to the complete inversion of the rig required.

The second section delineated the background about 2D and 3D DIC, along with their implementation in this study. The 2D DIC implementation was briefly mentioned because chapter 4 addresses this in detail and how DIC is leveraged for the SRTT. Unfortunately, the hypothesis developed for 3D DIC for the SPT did not work despite repeated attempts.

The third section informed about the uniaxial test performed as a reference value for the comparison of the SRT and the SPT. The comparison values for these tests were 208.21 GPa for the elastic modulus and 513.89 MPa for the yield strength. The material background for SS316L was also provided in this section.

The last section laid the foundation for the analytical solution for the small ring tensile test. This section also discussed how the SRTT analytical solution was evaluated and compared against a single experiment. This foundation is helpful to understand its implementation in the next chapter where the analytical solution is used on all tests performed to discover new relationships within the data.
Chapter 3

A matrix of Small Ring Tensile Tests
3.1 Test impetus and information

This chapter aims to address the following research objective: is the Small Ring Tensile Test (SRTT) reliable and repeatable enough over a multitude of displacement rates to be widely used, or does it work well only for a select few displacement rates? Additionally, are the force-displacement output from the test consistent enough over a range of displacement rates to reliably obtain the stress-strain values via conversion factors for comparison with uniaxial test data?

To address these questions, the following steps have been adopted in this chapter:

1. Perform experiments on a multitude of displacement rates.
2. Clean data as required.
3. Analyse any rate dependency introduced by the test.
4. Perform Inverse Finite Element Analysis (FEA) to obtain required stress-strain values.
5. Compare obtained values from above with existing conversion relationships.
6. Propose new conversion relationships between force-displacement and stress-strain values, if applicable.

Given the nascency of this testing methodology, there is a lack of literature about the SRTT, especially about its performance on different materials. Stainless Steel (316L grade) has been used in this study. The material is the same SS316L as was described for uniaxial tests in the previous chapter (see 2.4.1). The authors of the original studies strongly recommend testing the SRTT with different materials, and this material has not been tested in those studies [73–75]. Materials tested in those studies include Aluminium alloy 7175-T7153 [73, 74] and additively manufactured alloy Ti-6Al-4V [75].

The tests were performed on the Instron 8982 Universal Testing machine. Table 3.1 showcases the displacement rates that have been tested. It should be noted here that there is no overwhelming significance attributed to any specific row and the table is shown as such only for better visualisation.

<table>
<thead>
<tr>
<th>Regime (mm/min)</th>
<th>0.0 to 0.09</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 to 0.5</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3333</td>
<td>0.35</td>
<td>0.4</td>
<td>0.45</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>0.55 to 0.85</td>
<td>0.55</td>
<td>0.6</td>
<td>0.65</td>
<td>0.6667</td>
<td>0.7</td>
<td>0.75</td>
<td>0.8</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>0.9 to 1.2</td>
<td>0.9</td>
<td>0.95</td>
<td>1.0</td>
<td>1.05</td>
<td>1.1</td>
<td>1.1111</td>
<td>1.15</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>1.25 to 1.7</td>
<td>1.25</td>
<td>1.3</td>
<td>1.35</td>
<td>1.4</td>
<td>1.45</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>1.8 to 4.0</td>
<td>1.8</td>
<td>1.9</td>
<td>2.0</td>
<td>2.3333</td>
<td>2.6667</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: All displacement rates tested for SRTT.

Covering this range of displacement rates for SS-316L has the added benefit of analysing the SRTT for a material previously not tested this way, while also addressing the research objective.
The next section delineates the way these tests were performed and how the data from the tests was handled and cleaned for data analysis.
3.2 Performing Tests and Data Cleaning

The dimensions of the SRT rig have been presented in the previous chapter (see fig. 2.1). All the rings tested have approximately an inner diameter of 10 mm, an outer diameter of 12 mm, and a thickness of 2 mm. Full details of the ring dimensions, with their corresponding pin displacement test rates, can be found in [140].

As mentioned previously, all tests were performed on the Instron 8862 Universal Testing machine. However, this machine intermittently gave a data logger overflow error, resulting in the affected experiments stopping midway. This logging error was not resolved despite extensive analysis by all personnel within the University or the affected machine provider’s engineers. Thus, the affected experiments had to be restarted manually. However, this did not have an effect on the resulting force-displacement values, with the test resuming at approximately the same load and displacement values. All the data obtained from this testing has been made open-source at [140], along with this affected data as well.

Additionally, for the evaluation of a potential extensometer design (covered in chapter 4) with the help of digital image correlation (DIC), the rings were painted with a randomized speckle pattern of a white background with black spots. A small plate was also affixed to the side of the bottom pin’s fixture for tracking the y-displacement. All the DIC images obtained have also been made open-source at [140]. For preliminary visualisation of how the test looks and progresses, fig. 3.1 to 3.3 are shown. Fig. 3.1 showcases the ring in its non-deformed state before the test has started. The next image (fig. 3.2) showcases the image at, approximately, the middle of the test’s overall pin displacement. The ring has been observed to deform from its original round shape to a more elliptical shape. Lastly, at the end of the test (fig. 3.3), the ring can be observed to be in an almost rectangular shape with 2 long sides. It should be noted that these are just representative images to inform how a typical SRT progresses.
3.2. PERFORMING TESTS AND DATA CLEANING

Figure 3.1: SRTT setup and specimen image at the start of the test. Pin displacement rate 0.75 mm/min. The specimen is not loaded, and the ring is in its non-deformed round shape.
Figure 3.2: SRTT setup and specimen image at, approximately, the middle of the test’s duration. Pin displacement rate 0.75 mm/min. The ring has been deformed to an elliptical shape.
Figure 3.3: SRTT setup and specimen image at the end of the test. Pin displacement rate 0.75 mm/min. The ring has deformed to an almost rectangular shape with 2 long sides.
A typical unprocessed graph obtained from the tests is shown in fig. 3.4. The ring is not loaded until 1 mm of displacement. The loading displacement value varies from test to test. This loading methodology has been used for all tests to ensure that the first image for DIC is captured right at the start of the test, and also to ensure that the ring is allowed to be self-centered as much as possible before a test begins.

![Graph showcasing raw output from the Instron 8862 UTM for SRTT. The test does not load until approximately 1 mm of pin displacement.](image)

Figure 3.4: Graph showcasing raw output from the Instron 8862 UTM for SRTT. The test does not load until approximately 1 mm of pin displacement.

It was found that the sampling rates of the tests resulted in a noisy test output of the force-displacement values. This poses an issue while properly evaluating the rate independency of the test, since the noise can be as large as 20 N. This was found to be especially prevalent in the test rates slower than 1.0 mm/min. Additionally, this can also bias the inverse FEA, since a smooth curve is required for proper material calibration.

A few data cleaning techniques have been presented and discussed in the next subsection to address this issue, and the relevant cleaned data has also been presented.

### 3.2.1 Data Cleaning and filtering

The experimental file (.csv format) is loaded onto a Pandas dataframe for data manipulation. Pandas allows for robust data manipulation and handling in Python. All data has been made open-source and is available at.

For quicker processing, only data up to 150 N is considered for initial processing. This is because it needs to be seen if the filter that is being evaluated is appropriate for SRTT. Applying the whole filter to the whole dataset does not make sense, since it is computationally expensive and harder to visualize the filter’s effects. Testing it on a
small piece of the dataset provides a better idea of the processing, while consuming less resources.

The ‘tsmoothie’ library \(^{143}\), which excels at filtering time-series data, was utilised for data cleaning. tsmoothie provides with various filters and smoothing techniques, and a ‘Convolution Smoother’ was chosen for this study. In this technique, a sliding window of a fixed size is moved across the data, and at each position, a weighted average of the data points within the window is calculated. The weights are determined by a kernel function that assigns a larger weight to the data points closer to the centre of the window and a smaller weight to those farther away.

This methodology is much simpler in contrast to the Butterworth filter, with only the kernel to be determined. tsmoothie provides with numerous weighing function methodologies, such as ‘ones’, ‘hanning’, ‘blackman’, and so on. More information about all types of available kernels can be found on their documentation page \(^{143}\). The ‘ones’ function corresponds to a simple moving average, since it assigns a weight of 1 to every data point in the window. The ‘hanning’ function meanwhile similarly provides a weighting that follows the Hanning function (see \(^{144}\)) over the window space. The weighting follows the Hanning function (similar to a bell-curve) in assigning the highest weight to the values at the centre of the window, with a gradual taper towards the end. This minimizes the loss in data that occurs due to windowing. An example of this is shown in fig. 3.5, which showcases the cleaned data superimposed on the raw data. A window length of 50 with the windowing type set to Hanning is shown in the image, with the smoothed data lying within the bounds of the raw data.

![tsmoothie filter](image)

Figure 3.5: Convolution Smoother, via tsmoothie, applied to test data from the SRTT experiments. Cleaned value (blue) is superimposed onto the raw value (orange). Window type Hanning is used.

While both the filters perform satisfactorily in smoothing the data, the ease of application via the Convolution Smoothing is preferred since it does not require the frequency calculation for each experiment. Regardless, this does not negate the use of the Butterworth filter, but is merely a commentary on the ease of its use. Thus, this
Convolution Smoothing (with Hanning windowing) was used in this study to smooth all the experimental data.

Fig. 3.6 showcases a selection of the cleaned data overlaid on top of one another, while fig.3.7 showcases most of the experiments compared with each other in separate subplots.

---

**Figure 3.6:** Visualizing some processed test outputs on a single overlay. Legend box indicates the rate (in mm/min).
Figure 3.7: Processed test outputs contrasted with each other. Legend box indicates the rate (in mm/min).
It can be seen there are differences in the graphs (fig. 3.6 and 3.7) as different pin displacement rates are tested. The differences are especially profound as the test progresses, thus suggesting that these differences arise in the plasticity region.

Whether the SRTT induces any artefacts of rate dependency throughout the whole set of experiments is the primary motivation behind performing so many of these tests on this material. One way to answer this question of rate dependency is to correlate the tests with another. Another overarching approach to this methodology would be to fit a machine learning model to these tests and use it as a tool to determine any rate dependency. This question is evaluated thoroughly in the next section (3.3).
3.3 SRTT rate dependency evaluation

To analyse how the SRTT behaves for all the tests that have been performed, two approaches have been explored in this section to determine the differences within each test. The first approach is comparing the tests against each other (section) and the second approach is using machine learning as a tool to evaluate dependency.

3.3.1 Inter-test comparison

Valuable insight can be found within comparative heatmaps generated for all the tests performed. A heatmap can be generated by comparing all the experiments with each other and using error metrics to quantify the difference between them. Metrics such as the RMSE (Root Mean Squared Error), MSE (Mean Squared Error), R2 score, and the MAE (Mean Absolute Error) are some metrics that be used for this evaluation. The first two metrics (RMSE and MSE) tend to penalize the larger outliers, which is why MAE was chosen since it provides with average magnitude of all errors. The R2 score, meanwhile, helps measure the similarity between the experimental dataframes.

For the generation of the heatmap, the larger dataframe in the comparison was resampled to the size of the smaller dataframe and the error metrics were computed for the force values. Heatmaps of the R2 score and MAE were generated for all the 48 experiments, contrasted against each other in ascending order of displacement rate.

This is shown in fig. 3.8 shows the R2 score heatmap with the colour bar ranging from the minimum observed value of R2 score to its maximum. The minimum R2 score was observed to be 0.64539 and while the maximum was observed to be 0.99989. The mean R2 score for all these scores, taken from either the upper or the lower triangle and excluding the diagonal, was observed to be 0.95632. It should be noted that the axis labels represent the test displacement rates in mm/min.
Figure 3.8: Heatmap of all SRTT experiments’ force-displacement values with the colour bar scaled from min to max of R2 score. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
3.3. SRTT RATE DEPENDENCY EVALUATION

Fig. 3.9 shows a similar heatmap for the Mean Absolute Error (MAE) as a percentage value, scaled from the minimum to the maximum. The minimum MAE is observed to be 0.39% and the maximum is observed to be 14.8%, with a mean MAE deviation of 4.95%. The tick labels showcase the test displacement rates in mm/min.

![Heatmap of SRTT experiments' force-displacement values](image)

Figure 3.9: Heatmap of all SRTT experiments’ force-displacement values with the colour bar scaled from min to max of MAE % value. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.

Further insight and better visualisation for the data can be found in fig. 3.10 and 3.11. The former highlights experimental comparison that results in an R2 score of less than 0.9 while the latter highlights the experimental comparison that results in an MAE % value higher than 5%. Both diagrams have the experimental displacement rate (in mm/min) in the tick labels.

It can be observed from both the partially shaded figures presented that some tests do not compare that well with one another. This is found to be especially true for the faster testing rates (between 1.15 mm/min to 1.8 mm/min) when contrasted with the slower testing rates (below 0.45 mm/min). It should be mentioned here that the interrupted tests (0.05, 0.06, 0.2, 0.6, and 1.2 mm/min) do not show this deviation significantly.
However, this could also be an artefact of testing, given how this trend is not true for all the faster displacement rates when contrasted against the slower ones. For instance, consider the 1.35 mm/min test, one of the faster displacement rates, compared to the 0.3 mm/min test, one of the slower displacement rates. The R2 score for this comparison is 0.98453 while the MAE is 2.10374%.

Figure 3.10: Heatmap of all SRTT experiments’ force-displacement values with the colour bar active only for experimental comparisons that yield an R2 score of less than 0.9. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
Figure 3.11: Heatmap of all SRTT experiments’ force-displacement values with the colour bar active only for experimental comparisons that yield an MAE % score of less than 5%. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
While some comparison metrics are high, there seems to be no clear pattern emerging in the variation between the tests. This would suggest that the difference in metrics could possibly be attributed as a testing anomaly when the experiments were performed. A potential reason could be the misalignment of the ring in the z-direction (direction towards the viewer during testing) before the experiment was started. Unfortunately, this cannot be identified with the DIC images that have been captured, since 2D DIC is unable to resolve for the z-direction.

The heatmaps are an important data visualisation tool since they showcase the performance of each test with respect to the other. Since no trend in deviation is observed that would indicate any dependency on test rates, a valid interpretation of these results would be to suggest a testing anomaly while these tests were performed and the tests could potentially be rate-independent. Another check on these results is shown in the forthcoming sections, which involves leveraging this data on machine learning tools.

3.3.2 Background on regression models

A secondary way to analyse the performance of these tests, as mentioned previously, is to fit regression models trained on all the tests and analyse the resulting models. This modelling that will help analyse the relationship between one or more independent variables and a dependent variable. The trained model will estimate how changes in the independent variable (displacement, for instance) are associated with changes in the dependent variable (force, for instance).

To evaluate any models trained on this matrix of tests further, 3 tests were separated out of the 48 tests, leaving 45 tests for the model training. The tests that were separated were of pin displacement rates (in mm/min) 0.3333, 0.6667, and 1.1111. These test rates were separated out to further evaluate the regression models that have been trained on the other 45 tests, thus allowing for more robust testing of the models, since this would be new data for the model to make predictions on.

Training a general-purpose regression model, at least for this material, is a logical step because it serves a two-fold purpose:

1. It allows for the examination of the difference between each test conducted. The variance and standard deviation can be computed for the trained model.

2. It allows for the model to be used on the SS316L samples in chapter 5, which involves combining the SRTT with the Small Punch Test (SPT), therefore providing a commentary on whether the hypothesis is feasible or not.

It was assumed appropriate to use regression models, since these are very well-suited for time-series problems. As mentioned previously, while the force-displacement curve is not exactly a time-series dataset, it can be assumed analogous to one due to the periodicity of displacements at fixed sampling intervals. The two models used here are Random Forest Regressor and XGBoost. While these two may originate from the same family of ensemble methods, there are pronounced differences between the two libraries.

Random Forests were conceptually introduced by Breiman. A detailed implementation of this can be found in the SciKit library. The algorithm works by splitting the input and output data into separate decision trees in a randomized manner.
3.3. SRTT RATE DEPENDENCY EVALUATION

Given training dataset vectors ‘X’ and ‘Y’, with the former being the input features and the latter being the target features, these values are randomly split into the number of decision trees contained within the forest as specified by the user.

To understand the workings of the RFR, it is prudent to first understand the underlying logic of its constituents: the decision trees. The decision trees are a set of supervised learning models that get allocated with a fixed amount of training data (input vectors X) and output data (output vectors Y). The split of the data is governed by a multitude of factors, but essentially comes down to the best split of data that is appropriate for a tree. This appropriateness is determined by a number of factors, but in SciKit-learn, the split is governed by the mean squared error (MSE) for the split. The quality of the split is for this analysis is predetermined in SciKit-learn via data impurity.

To summarise it succinctly, the split is determined in the tree node if “the split induces a decrease of the impurity greater than or equal” to 0.0 (default value). Impurity here referring to the homogeneity in a split within a tree. A high impurity indicates that the tree has a lot of target values that differ from each other greatly. Thus, if a potential split does not decrease the impurity by this amount (0.0), the split is not made. The impurity is governed by the difference between the impurity of the parent node and the weighted sum of the impurities of the child node. This is necessary because the goal of these decision trees is to split the data as homogeneously as possibly before recombining it into the random forest. Further information about this split can be found in the documentation of the algorithm.

This combination of the decision trees is where XGBoost (XGB) and Random Forest Regression (RFR) differ. The RFR uses a technique known as ‘bagging’ while the XGB library uses a technique known as ‘boosting’.

Bagging (Bootstrap Aggregating) allows for the training of the multiple decision trees, which are subsequently used for predictions on the new values. The predictions are governed by averaging the outputs of the decision trees.

Meanwhile, Boosting utilises weak learners and ensures that outliers are not missed. For instance, consider a sampling size of 500 samples for the first decision tree. For the purposes of explanation, consider that it performs well on 400 of these samples. Consequently, the next decision tree skews towards the 100 samples which performed poorly, therefore allowing the model to learn from its mistakes. This process is governed by a lot of hyperparameters, all of which are listed in the XGBoost documentation.

3.3.3 Implementation of regression models

The Random Forest Regressor (RFR) and XGBoost (XGB) methods have both been implemented in Python on the dataset that has been described previously. All the CSV files are loaded onto a pandas dataframe and cleaned. Cleaning the CSV files involves stripping the header from each file and the indices so as not cause conflicts while training. These cleaned CSV files need to be appended row-wise and not column-wise for training. While it is possible to append the data column-wise, this method is harder to train for machine learning because sampling into training and test data is harder, given that the training dataset is in different columns and needs to be distinguished from the NaNs. This would result in 45 input vectors (for each experiment) and 45 output vectors that consequently need to be combined.
It is also imperative to add at this stage that all dataframes were re-sampled and trimmed to have the same number of entries (set to 9.5k) to not bias the model strongly towards dataframes that have more values. This value was chosen because the median length of the files is approximately 9.4k. Although these models excel at removing bias, this data sanitization step is recommended to further bolster the bias removal.

Additionally, the time column is not needed, since the displacement values acts as a surrogate for time. These displacements are linked to their unique force values in the adjacent cell, thus ensuring that the combined dataset behaves like a quasi time-series dataset.

For any of the models used, a plethora of hyperparameters are available for tuning and can be benchmarked against many metrics, such as the MAE (Mean Absolute Error), the RMSE (Root Mean Squared Error), the MSE (Mean Squared Error), or the R2 score. However, it must be noted that these metrics are not sufficient in isolation without looking at the predictions generated by them and a holistic idea of the whole model must be visualized before finalizing a model’s hyperparameters. For instance, the results from an RFR model with 5 trees and 5 leaves has good metrics (R2 score of greater than 0.9), but the predictions generated by these are mainly step-wise functions (fig. 3.12). This figure clearly shows that while the R2 score for this RFR model is over 0.9, the predictions generated by this are inaccurate since force behaves like a step function of displacement.

![Figure 3.12: Predicted output from RFR model with 5 trees and 5 leaves contrasted with actual data from an experiment ‘unknown’ to the model.](image)

Unfortunately, increasing the number of trees and leaves available comes with its own risk. This could lead to a risk of overfitting to the data. This would result in the model performing well on the training data and not so well on the validation data or new data.

To optimize the search for the best hyperparameters, a 5-fold cross-validation grid
search was implemented. This sklearn module searches the whole sample space for the hyperparameters defined and searches for the best solution parameters that reduce the error metrics while not overfitting to the data. Additionally, it also performs a 5-fold cross-validation (CV) to ensure there is no overfitting. The 5-fold CV splits the whole dataset into 5 ‘folds’ (or subsets) and tests the hyperparameters 5 times, by switching which fold is the validation dataset and which one is the training dataset. Two error metrics are minimized here to reduce the risk of over-fitting even further: the MAE and the R2 score. Note that a high R2 score is desirable, hence $(1 - R^2)$ is minimized. The relevant code snippet is given in Appendix A.2.

Coupled with the Grid Search and 5-fold CV, the best hyperparameters for the RFR model were found to be 25 for the number of decision trees, with their maximum depth being limited to 10. These results serve as a learning checkpoint for how the analysis should proceed with XGB as well. Using this as a stepping stone, the following hyperparameters, based on a new grid search, were found to work best for XGB:

- Max Depth: 10
- Number of Gradient boosted trees (N estimators): 50
- Learning Rate: 0.1
- Subsample: 1.0

The results from these two models (RFR and XGB) are now shown in the next section.

### 3.3.4 Results and discussion of regression models

The RFR model with the best hyperparameters has been tested on the ‘hidden’ test datasets that were withheld from the model from the beginning. This is separate to the training and validation data. The withheld tests were of pin displacement rates (in mm/min) 0.3333, 0.6667, 1.1111. A snippet of these results for this model’s prediction and the actual values is shown in fig. 3.13. Fig. 3.14 compares the predictions of this model to the same tests that RFR models were contrasted with previously. It can be seen that the results from the RFR model match well with the hidden experimental data. A detailed summary of the performance of this model is also presented for a more quantitative evaluation in table 3.2.
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Figure 3.13: Final RFR model compared to experiment with pin displacement rate 0.3333 and 1.1111 mm/min.

Figure 3.14: Final XGB model compared to experiment with pin displacement rate 0.3333 and 1.1111 mm/min.
### 3.3. SRTT Rate Dependency Evaluation

<table>
<thead>
<tr>
<th>Metric</th>
<th>Pin displacement rate (mm/min)</th>
<th>Random Forest Regressor</th>
<th>XGBoost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.3333</td>
<td>11.93622</td>
<td>9.37769</td>
</tr>
<tr>
<td>MAE</td>
<td>0.6667</td>
<td>78.39881</td>
<td>82.48919</td>
</tr>
<tr>
<td></td>
<td>1.1111</td>
<td>49.86155</td>
<td>54.13499</td>
</tr>
<tr>
<td></td>
<td>0.3333</td>
<td>0.99963</td>
<td>0.99966</td>
</tr>
<tr>
<td>R2</td>
<td>0.6667</td>
<td>0.98464</td>
<td>0.98293</td>
</tr>
<tr>
<td></td>
<td>1.1111</td>
<td>0.99335</td>
<td>0.99214</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of both XGB and RFR on unknown data.

The general performance metrics of these models are shown in fig. 3.15 and fig. 3.16. The scatter plots for predictions vs true values show a much narrower spread initially (below approximately 500 N) than they do later. This suggests that the models are able to predict well for the elasticity region, and the differences in performance arise in the plasticity region. The MAE for the whole region is 31.95 for the RFR model and 31.68 for the XGB model.

Similarly, the residual binning plots are helpful in noticing the deviations in predictions. The percentage of residuals within 1 standard deviation are 77.40% for the RFR model and 77.46% for the XGB model. The percentage within 2 standard deviations is 95.08% for the RFR model and 95.23% for the XGB model. These metrics suggest that the models perform largely similarly.

![RFR: Actual vs. Predicted Values](image1)

![Residual Plot](image2)

(a) Actual vs Predicted values. (b) Residual Bin Plot.

Figure 3.15: Performance metrics for RFR model.
Thus, it would be wise to conclude that both models perform well for this material. This study has provided future researchers with decent starting points for these models for SRTT and the methodology to prepare the datasets. It can be observed from the performance metrics that the XGB tends to perform marginally better than the RFR model. However, the RFR model works better on the hidden data overall. In light of these observations, it would be prudent to conclude that either model should be sufficient for predicting force outputs from new tests on this material.

3.3.5 Commentary on rate dependency

Two different approaches to evaluate the rate dependency for the SRTT were presented in this section. One approach was to contrast the tests with one another, while the other approach was training machine learning models and using them as a tool to evaluate rate dependency.

This testing methodology of various displacement rates is an alternative to testing the same displacement rate multiple times. This would have provided with a smaller sample size, but with more testing at discrete points. This methodology was not adopted in this study, since this (multiple testing at the same rate) does not allow for a wider sample size. However, considering the importance of multiple testing, it can be seen that the step sizes between the experiments were kept small to analyse for any discrepancy in the data. This serves, as evidenced by the heatmaps and the ML models, as a satisfactory compromise between testing at multiple rates and testing the SRTT at many more displacement rates but with a wider step size between the tests.

The heatmaps provided a comparative contrast of all the tests, with the maximum deviation between tests found when tests were contrasted against tests performed at over 1.0 mm/min and below 2.0 mm/min. As mentioned previously, there remains merit in assuming that the differences be attributed to testing anomalies. Additionally, the machine learning models have also shown significant promise on their predictions generated on the hidden test data. This is despite the fact that some of these tests (such as the 0.3333 and 0.6667 mm/min) did not compare well with the faster displacement rates (above 1.15 mm/min) very well.

More insights can be gathered in this data once it is converted to stress-strain values and proper material insights are gathered. Consequently, to interpret the results of this
test, the data needs to be converted now. This has been done with the help of inverse FEA, as delineated in the next section.
3.4 Inverse FEA

3.4.1 Background on inverse FEA

A brief introduction to all the various inverse analysis procedures was introduced in section 1.7. This section describes the background and rationale for the methods adopted for this study. For the inverse finite element method adopted, the model was fixed (non-updating) and parametric variation was performed to benchmark against the experimental studies.

An important finding from the study by Rouse et al. [75] was that they link their force-displacement data to the stress-strain curves after obtaining optimised material parameters via inverse FEA. While more about this is discussed in section 3.4.4, it is important to note that the SRTT requires inverse FEA for data interpretation, hence the need to perform this analysis. Additionally, Rouse et al. use only the first 1.6 mm of the experimental data for inverse FE model calibration [75].

Chapter 1 also summarised the two optimisation techniques that could potentially be used for the calibration of the inverse FE model: the Nelder-mead method and the differential evolution method. A combination of the two methods, with the DE being run initially followed by the NM method, was suggested as a viable approach. A potential combination procedure is detailed in algorithm [1].
Algorithm 1 Proposed inverse FE optimisation procedure

Require: min(RMSE)

Ensure: Material parameters stay within physically possible limits

Generate Finite Element (FE) model upto 1.5 mm displacement
Initialise Differential Evolution (DE) optimisation
Run FE model with initial parameters

while min(RMSE) ≠ True do

Initialise Abaqus
Check FE output file
Calculate RMSE with experimental file
if min(RMSE) = True then
    Terminate optimisation
else if min(RMSE) ≠ True then
    Store residual Value for optimisation algorithm
    Generate new material parameters
    Initialise Abaqus
    Generate new FE model with new material parameters
    Run FE model with new material parameters
end if
end while

Get new material bounds limit from above DE runs
Initialise gradient-free optimisation
Repeat optimisation Loop for gradient-free optimisation
Store finalised material values from both optimisations
Run full FE model with both values
Compare RMSE and choose the best material model values
The two methods, NM and DE, are applied using the ‘Optimize’ library in SciPy. This method should theoretically ensure that the best use of resources is made and the strengths of both these methods are effectively utilised. The optimisation methods mentioned here now need to be applied to FEA. This is detailed in the next section.

3.4.2 Application of inverse FEA

Abaqus was used for the FE modelling. A new .inp (‘input’) file was generated for each iteration of an inverse analysis. This was done via a Python script that updated itself with the new material parameters at every iteration.

The python script accommodates the re-creation of all the steps involved in the .inp file, including the submission of the job. For the accessibility of future researchers, the whole optimisation code is given in Appendix A.3. The code in the appendix has the capability to perform Differential Evolution optimisation as well as Nelder-Mead optimisation and is commented appropriately.

The material model used is the Ramberg Osgood model. This is because all the prior studies for the SRTT have also extensively made use of this material model. The material model is given as:

\[ E \varepsilon = \sigma + \alpha \left( \frac{\sigma}{\sigma_0} \right)^{n-1} \]

where, \( E \) is the elastic modulus, \( \varepsilon \) is the strain, \( \alpha \) is the yield offset, \( \sigma \) is the stress, \( \sigma_0 \) is the yield stress, and \( n \) is the hardening exponent.

The step was chosen to be Dynamic but implicit. The explicit step took too long (more than quadruple the run-times) to be computationally advantageous for convergence of the optimisation. The step size was allowed to be automatic, but with a minimum step size imposed to be 0.02. Additionally, the dynamic step also had ‘NLGEOM’ enabled, thus allowing for geometric non-linearity to occur as the ring deforms.

The contact properties were chosen to be ‘Tangential’ with a penalty friction formulation and friction coefficient of 0.2. However, it should be noted that the friction coefficient does not seem to have an effect on the test, based on all prior research for the SRTT (see ). The contact between the pin and the ring made use of a ‘surface-to-surface’ contact in the analysis, with the pin’s outer surface set to be the master surface and finite sliding allowed for this contact. This allows for the arbitrary motion between the pin and the ring as the test progresses.

The full model was not simulated to save computational time, since inverse FE models need to run multiple times to converge. Instead, the symmetry of the ring was exploited and a 1/8 model was simulated. This is also evidenced to be sufficient by prior literature in the SRTT. Fig. showcases these boundary conditions imposed for symmetry. Fig. showcases the FE model with the loading of the pin and the central element of the ring marked, the latter of which will be used to extract the equivalent gauge area and equivalent gauge length parameters later (see section for usage).
3.4. INVERSE FEA

Figure 3.17: FE model of the SRTT showcasing the boundary conditions imposed for symmetry.

Figure 3.18: FE model of the SRTT showcasing the loading direction of the pin and the central element of the ring.

The ring mesh was chosen to be C320R, a brick type quadrilateral element, while the pin was chosen to be a rigid body (no deformation allowed) with a mesh element of R3D4. A mesh size of 0.2 mm was chosen for the ring after a mesh sensitivity analysis. Given that the inverse FE optimisation would run for hundreds of cycles, it was deemed that the 0.2 mm mesh size would be better suited over the 0.25 mm mesh size, even though
the different between the force values was just 10 N at pin displacement 4.0 mm. The mesh sensitivity analysis graph is shown in fig. 3.19.

Figure 3.19: Mesh sensitivity analysis of the FE model, with the runtime for each relevant simulation given on the x-axis as well.

The next section delineates the results obtained from the inverse FE modelling, issues encountered, and how those issues were fixed.

3.4.3 Results of inverse FEA

The initial DE optimisation was run up to 1.5 mm of pin displacement. This DE analysis did not converge fully despite over 1300 iterations (1325 performed), which took over two weeks of simulation time. With the FE analysis outputting at intervals of 0.1 s, each iteration for a DE step takes between 12 and 15 mins to complete. However, this approach proved to be misguided since the material was consequently calibrated only for the 1.5 mm of pin displacement. Running the FE simulation for the full length of the pin displacement led to convergence errors. Thus, it was found that optimising for only the initial part of the pin displacement was a misguided approach and significant amount of pin displacement needed to be taken into account.

To rectify this, another DE-NM optimisation cycle was run, but with the pin displacement set to 4.5 mm. Each DE-driven optimisation took between 40 and 45 minutes to complete. After over 750 iterations, which took approximately 3 weeks, the DE analysis had not converged. The bounds that were set for this analysis and the subsequent NM analysis are shown in table 3.3.
3.4. INVERSE FEA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DE (low)</th>
<th>DE (up)</th>
<th>NM (low)</th>
<th>NM (up)</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>185.00</td>
<td>210.00</td>
<td>190.00</td>
<td>210.00</td>
<td>209.03</td>
</tr>
<tr>
<td>Yield Strength</td>
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<td>290.00</td>
<td>288.68</td>
</tr>
<tr>
<td>Hardening Exponent</td>
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<td>8.00</td>
<td>6.00</td>
<td>6.80</td>
<td>6.00</td>
</tr>
<tr>
<td>Yield Offset</td>
<td>0.10</td>
<td>0.80</td>
<td>0.14</td>
<td>0.18</td>
<td>0.1507</td>
</tr>
</tbody>
</table>

Table 3.3: Material parameter bounds and final values from optimisation on inverse FEA with 4.5 mm pin displacement.

Unfortunately, after these analyses had converged, it was found that the initial bounds set for DE were too narrow in their scope. This error was discovered because the strain measurement system for the uniaxial tests was found to be off by a factor of almost 25%. The required yield strength bounds, based on a yield strength of over 500 MPa obtained in the uniaxial tests (see 2.4), were set too low. This needed to be addressed, and the optimisation bounds need to be widened to accommodate this oversight and error.

It is necessary to delineate these findings so that these mistakes are not repeated. The crucial learning from these simulations were that the inverse optimisation must be run for a full plasticity regime for proper convergence and that the optimisation bounds must be kept as wide as possible, to account for oversights that were encountered here.

Unfortunately, at this juncture, due to the lack of computational resources and time, running a DE analysis with such extremely wide bounds would result in an optimisation routine that would take months to converge. Thus, it was considered pragmatic to run only an NM optimisation with the bounds being informed by the uniaxial tensile tests that were performed. The bounds that were set and the results obtained are shown in Table 3.3. Fig. 3.20 shows the variation of the RMSE residual, that converges to a value of 12.6615, with the optimisation terminating at 316 iterations. With each iteration of the NM optimisation taking between 40 and 45 minutes, the whole optimisation procedure took almost 10 days to finish.

Fig. 3.21 shows the variation of all the parameters over the whole optimisation cycle. It can be seen from the images that yield strength is the first to converge, dropping directly to its converged value. This is followed by the elastic modulus, which makes use of the full sample space before converging. Similar bound exploration is seen in hardening exponent and yield offset as well before they converge.

The bounds were chosen as such due to a couple of factors - the result from the uniaxial test plus, for the yield strength specifically, the manufacturer tested proof strength value of 480 MPa. An astute argument can be made here that the bounds were set too low and should’ve been given a wider exploration scope to explore the solution space further. However, this increases the pitfalls of the NM optimisation, namely its tendency to be stuck in local minima of the solution space.
### Table 3.4: Material parameter bounds and final values from NM optimisation on inverse FEA with 4.5 mm pin displacement.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NM (low)</th>
<th>NM (up)</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>190.00</td>
<td>220.00</td>
<td>190.15</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>450.00</td>
<td>590.00</td>
<td>450.00</td>
</tr>
<tr>
<td>Hardening Exponent</td>
<td>1.00</td>
<td>25.00</td>
<td>6.73</td>
</tr>
<tr>
<td>Yield Offset</td>
<td>0.01</td>
<td>0.99</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Figure 3.20: Residual tracking from corrected NM optimisation with 4.5 mm pin displacement.
3.4. INVERSE FEA

Figure 3.21: Results from corrected NM optimisation with 4.5 mm pin displacement.
Consequently, the full SRTT simulation was run (up to 5.0 mm pin displacement) and compared with the experimental dataframe. This is shown in fig. 3.22. The converged values result in a simulation that matches well with the experimental dataframe. While the most deviation is found in the elastic region initially, the plastic region tends to match well with the experimental dataframe. From the simulation, the von Mises stress and true strain (‘SMISES’ and ‘LEMAX’ options in Abaqus) experienced at the centre of the ring can also be extracted and compared with the uniaxial tensile tests. This is shown in fig. 3.23. The match with the yield region is not completely satisfactory, but this is a persistent issue that has been highlighted in previous SRTT studies as well [73–75].

Figure 3.22: Visualising the full optimised simulation’s force-displacement values against the experimental SRTT dataframe.
3.4. INVERSE FEA

Figure 3.23: Visualising the full optimised simulation’s stress-strain curve against the uniaxial tests.

The learnings and implications of this inverse FEA analysis are delineated next.

3.4.4 Discussion of inverse FEA results

The inverse FEA were initially supposed to make use of gradient-free (Nelder Mead) and population-based (Differential Evolution) optimisation routines. However, due to an oversight on the amount of displacement used for calibration and the error in uniaxial tensile tests that was discovered later, most of the DE analyses could not be used for the final calculations. This, however, is a remark on how computationally exhaustive the DE optimisation routine is. While it is potentially beneficial when used for materials that are completely unknown, it is not recommended that this be the first approach if time is of primary concern.

The NM optimisation routine performed extremely well for all the analyses and was often extremely quick in comparison to the DE routine for finding the converged values. The converged values seem to perform well with the SRTT experiment’s force-displacement graph and also when the stress-strain values are compared with the uniaxial tensile tests. However, this conversion from force-displacement values to stress-strain values is an extremely cumbersome procedure that relies completely on FEA. This is delineated in the next section.
3.5 SRTT: Data conversion

3.5.1 Current practices on data conversion

The current practice of converting the force-displacement data from the SRTT to the required stress-strain values relies on obtaining the equivalent gauge area (or gauge section) and equivalent gauge length after performing FEA on the rings. The equivalent gauge section is obtained by dividing the reaction force observed at the loading pin by the stress experienced at the central element of the ring (which was shown in fig. 3.17). Similarly, the displacement of the loading pin is divided by the strain experienced at the central element of the ring to obtain the equivalent gauge length. Once these values of equivalent gauge area and gauge length are obtained, these are used in the force-displacement values that are obtained in the SRTT experiment.

The force values from the SRTT experiment are divided by the equivalent gauge area to obtain the corresponding stress values. Similarly, the displacement values from the SRTT experiment are divided by the equivalent gauge length values to obtain the corresponding strain values. Thus, the current approach can be described sequentially as:

1. Perform SRTT inverse FE simulation.
2. From the simulation, obtain (a) the stress-strain values (\(\sigma_{sim}\) and \(\varepsilon_{sim}\), respectively) at the central element of the simulated ring, and (b) the force-displacement (\(F_{sim}\) and \(d_{sim}\), respectively) values observed at the simulation’s loading pin.
3. Divide the \(F_{sim}\) by \(\sigma_{sim}\) to obtain the equivalent gauge area (\(A_{eq}\)) and the \(d_{sim}\) by \(\varepsilon_{sim}\) to obtain the equivalent gauge length (\(l_{eq}\)).
4. Divide the experimentally observed force and displacement values (\(F_{exp}\) and \(d_{exp}\), respectively), which were obtained from the loading pin, by the equivalent gauge area (\(A_{eq}\)) and equivalent gauge length (\(l_{eq}\)), respectively.
5. The first division \(F_{exp} \div A_{eq}\) results in the stress (\(\sigma\)) values, and the second division \(d_{exp} \div l_{eq}\) results in the strain (\(\varepsilon\)) values.

The reliance on the FEA for data conversion makes this test greatly inaccessible, since it demands prior knowledge of the material and accurate model setup before data can even be interpreted. Not to mention the time it takes for setting up an FE model, which can compound in even greater inaccessible for this test if results are needed quickly.

The authors of the original study delineate the nature of the graphs that should be observed for the equivalent gauge section and the gauge length when plotted against the reaction force on the x-axis. The values obtained for the current study’s simulation are detailed in the next section.

3.5.2 Implementation of data conversion

Fig. 3.24 and 3.25 showcase the variation of the equivalent gauge area and the equivalent gauge length with respect to the reaction force observed at the pin in the FEA. It can be seen that the equivalent gauge area rises sharply as the test commences before settling in
a relatively horizontal trend until around 100 N. A gradual ascent commences after that, with a linear ascent observed after around 270 N. In contrast to this, the equivalent gauge length decreases sharply as the test commences and has a relatively horizontal trend until around 100 N as well. A gradual descent is observed until 270 N as well, which is followed by a steady-state for the rest of the test. These trends are extremely similar to those observed in the prior SRTT studies by Kazakeviciute et al.\textsuperscript{[73, 74]} and Rouse et al.\textsuperscript{[75]}.

Figure 3.24: Equivalent gauge section variation for the SRTT.

Figure 3.25: Equivalent gauge length variation for the SRTT.
These equivalent area and length values can now be used to obtain the stress-strain values now, which is shown in fig. 3.26—3.28. The first figure overlays a plethora of displacement rates’ stress-strain curves on top of each other. The experiments match well initially for the elastic region, but the divergence in stress-strain values becomes more pronounced as the experiment goes on, with the 2.0 mm/min having the lowest yield value and the 3.0 mm/min experiment having the maximum value. The addition of the uniaxial tensile test values to this plot is shown next in fig. 3.27. Only initially does the elastic part of the SRTT match with the uniaxial test, with the values quickly diverging from the uniaxial test almost immediately. This highlights the shortcomings of the FEA approach and how the material model and this conversion approach limits the potential of the SRTT.

![SRTT: Overlaying most converted results](image)

Figure 3.26: Overlay of most stress-strain curves from various experiments at different pin displacement rates.
3.5. SRTT: DATA CONVERSION

Figure 3.27: Comparing some converted SRTT experiments to the uniaxial tests.

All conversion values can be visualised in the big plot shown in fig. 3.28, with the graphs arranged in a grouping of ascending pin displacement rates (left to right and top to bottom order). The comparison of the rates between 1.2 to 2.0 mm/min is especially striking, with both the rates (1.2 and 2.0) matching closely only with each other but demonstrating a much lower yield strength of around 400 MPa. These test rates also performed poorly in the inter-test comparison, where the force-displacement data was compared. Given that this is observed only for a few tests in this range (and not all), it would be prudent to say that this is a testing anomaly for these tests, likely due to some loading issues when these tests were performed. The tests otherwise match well with each other only initially (elastic region).

A short commentary on these conversions is now presented in the next section.
Figure 3.28: Stress-strain curves from various experiments at different pin displacement rates.
3.5.3 Commentary on current data conversion

These differences can be evaluated against each other, just like the force-displacement values were compared in heatmaps.

This is shown in figs. 3.29 and 3.30. The minimum MAE deviation is observed to be 0.396% and the maximum was observed to be 14.896%. The minimum was observed for the comparison between the 0.1 and 0.3 mm/min pin displacement rate experiments, while the maximum was observed between the 1.4 and 2.0 mm/min pin displacement rate experiments. The mean value for all the inter-test comparisons was found to be 9.938% and a standard deviation of 5.955.

This can be better visualised in the shaded plot (3.30) that highlights the experiments that have an MAE deviation of higher than 5%. It is observed that the differences between the experiments are higher now than they were when the force-displacement values were compared.

Figure 3.29: Heatmap of all converted SRTT experiments (stress-strain values) performed with colour bar scaled from min to max of MAE % value. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
Figure 3.30: Heatmap of all converted SRTT experiments (stress-strain values) performed with colour bar active only for experimental comparisons that yield an MAE % score of less than 5%. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
The difference in these magnitudes can also be better understood visualising the difference between the MAE % obtained in the force-displacement comparison and the new stress-strain comparison. This is done by taking the difference of these matrices (force-displacement matrix subtracted from stress-strain) to indicate how much more error this conversion provides. This is shown in fig. 3.31. This figure allows for a better visual indicator of this conversion has affected the differences in the SRTT experiment. It can be seen that the files which were not comparing well in the force-displacement graphs have an even worse comparison after the conversion is applied. This is not a commentary on the data, but on the conversion method itself.

Figure 3.31: Heatmap showing the percentage change in MAE between force-displacement and stress-strain experiments, before and after applying conversion. The blue end of the colour map indicates decreased MAE, while the yellow end of the colour map indicates increased MAE. Provides insights into the accuracy of the conversion method.

It can be seen with the data so far that the SRTT data hinges a lot on the FEA and the conversion factors it generates. An astute recommendation, relevant to similar experimental programs, that can be made here to minimize this MAE% is that one must perform inverse FEA on all the experiments that are generated to obtain individual conversion parameters.
However, for such a large experimental dataset, this would be impractical considering the amount of time it would take to do this would span over many months, if not years, for the current computational resources available for this study. Thus, to address this problem, direct conversion relationships which can provide the yield strength and elastic modulus are evaluated next.
3.6 SRTT: Existing direct data interpretation

3.6.1 Comparison of test data with existing methodologies

The study by Calaf et al. [84] was an important step for the advancement of both the RHTT and the SRTT. They derive the following relationships:

\[
E = 123 \cdot \text{Slope}_{ini} \tag{3.2}
\]

\[
\sigma_y = 2.136 \cdot P_y \tag{3.3}
\]

\[
\text{offset} = t \left(0.0205 \frac{D_{ext}}{t} - 0.13\right) \tag{3.4}
\]

where, \(E\) is the elastic modulus, \(\text{Slope}_{ini}\) is the initial slope of the force-displacement curve, \(\sigma_y\) is the yield strength, \(P_y\) is the yield load, \(\text{offset}\) is the offset line analogous to the offset method for calculating yield strength in a uniaxial test, \(t\) is the ring thickness, and \(D_{ext}\) is the external ring diameter. The offset is used to calculate the \(P_y\).

These relationships are applied to the SRTT test dataset obtained from this test matrix. The yield strength values are obtained by the 0.2% offset method from the converted SRTT data, and the linear portion of the converted SRTT data is used to calculate the elastic modulus. The results are tabulated in table 3.5 along with their standard deviations for all the experiments. It can be seen that the converted SRTT values are better at predicting the yield strength (closer to uniaxial data) as compared to the values from the conversion relationship given in equation 3.3. Unfortunately, the \(\text{offset}\) parameter often yielded a negative value for this experimental dataset, which is why an offset value of 0.15 mm was used, since this was recommended by the authors [84]. The elastic modulus values differ for both the values (equation and converted SRTT’s linear region).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculated value from Calaf et al. [84]</th>
<th>From Converted SRTT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_y) (MPa)</td>
<td>206.97; 25.57</td>
<td>487.98; 29.89</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>77.16; 10.89</td>
<td>97.09; 14.27</td>
</tr>
</tbody>
</table>

Table 3.5: Evaluation of the SRTT parameters based on the relationships obtained from RHTT study by Calaf et al. [84].

The results are also visualised in fig. 3.32 and 3.33. The values are plotted against all the experiments in ascending order of pin displacement rate. It can be seen that the yield strength values differ much more with each other as compared to the elastic modulus values. However, even within the differences of the yield strength values, no clear trend emerges between how the values derived from equation 3.3 differ to those found from the converted SRTT values. Similarly, no clear trend is found for the elastic modulus values as well. Additionally, as mentioned previously, the elastic modulus values for both (from eqn 3.2 and linear portion of converted SRTT values) differ from the uniaxial values.
However, for both the graphs, it can be seen that there is a strong scatter for both the parameters. A likely reason for this is the conversion obtained via the inverse FEA. The scatter is likely to reduce as inverse FEA is performed for all the 48 tests, thus providing with a truly unique stress-strain output. However, given the time and cost limitations imposed inherently by inverse FEA, this would not be pragmatic.

Figure 3.32: Comparison of proof strength (at 0.2% strain) from converted SRTT graphs and via the method by Calaf et al. X-axis ticks represent the SRTT displacement rates sorted in ascending order. Rates are given in mm/min.
Figure 3.33: Comparison of elastic modulus from converted SRTT graphs and via the method by Calaf et al. [34]. X-axis ticks represent the SRTT displacement rates sorted in ascending order. Rates are given in mm/min.
The elastic modulus and yield strength values obtained directly from the SRTT stress-strain curves are shown in isolation fig. 3.34 and 3.35, respectively. The elastic modulus variation does not show a significant trend for the pin displacement rates, but the yield strength shows a somewhat linearly increasing relationship and should be investigated further for new conversion relationships.

Figure 3.34: Visualising the elastic modulus change in the stress-strain curves of the SRTT. X-axis ticks represent the SRTT displacement rates sorted in ascending order. Rates are given in mm/min.
Figure 3.35: Visualising the yield strength change in the stress-strain curves of the SRTT. X-axis ticks represent the SRTT displacement rates sorted in ascending order. Rates are given in mm/min.
The impetus to build relationships between known materials and test outputs is not novel. For instance, just like Calaf et al.’s study, Gussev et al. [153] test a variety of materials to build a correlation between the RHTT and the conventional tensile tests. The demand is clearly there, but dedicated studies oriented specifically towards SRTT are needed.

To alleviate this problem and to bookend this matrix of tests, the next section (3.7) summarises the methodology adopted in obtaining new conversion relationships and data exploration for the tests that have been performed in this chapter.
3.7 SRTT Evaluation of new conversion relationships

To contribute to the matrix of tests and to analyse the nature of the relationship between the curves obtained in this testing program, it is best to first visualise the force-displacement curves obtained from the tests and the converted stress-strain curves. This is done via k-means clustering.

K-means clustering is a popular unsupervised learning algorithm used to partition a dataset into K clusters, where K is a predefined number. The algorithm works by iteratively assigning data points to the closest cluster centroid and then recalculating the centroids based on the new assignment. The goal of the algorithm is to minimize the sum of squared distances between each data point and its assigned centroid. This was implemented via the SciKit-Learn [150].

This clustering, shown in fig. 3.36, allows for the visualisation of the regions of interest and observance of any interesting patterns in the data. Particularly, the variation of stress with respect to the force is a seemingly interesting clustering. Given the fact values used for the calculation of the elastic modulus depend heavily on the slope of the stress-strain curves, the first three regions seem critical in that cluster with an extremely linear relationship. The same can be said for the displacement-strain clustering, with this clustering having 6 linear regions. The displacement-stress clustering, similarly, has 4 linear regions, but the first one has the least spread in data. The clustering also provides a valuable insight into the force-strain clustering as well, with the first two clusters being largely linear.
Figure 3.36: K-means clustering with 7 clusters (each represented by a separate colour) used on SRTT data to find patterns within force-strain, force-stress, displacement-stress, and displacement-strain data.
At this juncture, there are a few pathways to help find a relationship between the test outputs (force-displacement curve) and the stress-strain values—

1. The elastic modulus value from the analytical solution could be usable. That could be used as one approach to obtain that value after testing has been performed. However, the slope of the force-displacement curve can be found independently and that can be related to the elastic modulus as well.

2. Currently, there is no way to estimate the yield strength apart from inverse analysis. However, as seen in fig. 3.36, the inflection point in the force-stress, displacement-stress, and the force-strain graphs suggest that when the force-displacement curve of the test output changes its nature, a similar change is observed in the stress and strain parameters as well. This could be a good starting to point to explore with the help of a bilinear fitting technique.

3. As it stands, even with the elastic modulus and yield strength obtained, there is no way to visualize these curves in the form of a stress-strain graph. However, there is no need for this apart from doing a double confirmation on the elastic modulus values obtained via method 1 listed above.

Thus, the following scenarios need to be tested for elastic modulus:

1. Analytical solution.
2. Slope of F-D curve.
3. Elastic modulus from converted stress-strain curve obtained from RFR model.

For the yield strength, meanwhile, the following avenues need to be explored:

1. Bilinear fitting on the F-D curve and finding a conversion factor.
2. Yield strength from converted stress-strain curve obtained from RFR model.

All approaches for both the parameters are now evaluated in the next couple of sections.

### 3.7.1 SRTT: Obtaining elastic modulus

**Analytical solution**

A great value is to be found in running the analytical solution script for all the experiments and visualizing that at this stage, with the elastic modulus (E) set as unknown and have that be optimized as well via SciPy. This was performed and the (mean) values of the force correction factor parameters and elastic modulus are listed below, along with their standard deviations:

1. \( E: 200.3707, 0.4461 \)
2. \( a_1: 11.9152, 1.5789 \)
3. \( a_2: 0.0608, 0.2190 \)
4. \( a_3: 16.3458, 1.9466 \)
5. \( a_4: 0.0799, 0.5122 \)
The correction factor ‘fp’ thus takes the form of the equation (3.5) shown below. It is also depicted in fig. 3.37, where it showcases a linearly increasing trend with respect to the displacement.

\[
fp(u_v) = 11.9152 \cdot exp(0.0608 \cdot u_v) + 16.3458 \cdot exp(0.0799 \cdot u_v)
\]  

(3.5)

Figure 3.37: Correction factor variation with respect to analytical displacement.

**Slope of F-D curve**

To consider the initial slope of the graph, a linear fitting was performed up to the first 0.15 mm of pin displacement. To negate any loading effects, the dataframe displacement values below 0.05 mm were cut off.

A mean slope value of 658.219 was obtained with a standard deviation of 81.675. An example image is shown in fig. 3.38, where the orange line indicates the linearly fitted slope, while the blue line represents the experimental data.
A question now arises of comparison of these slope values to find a conversion factor. They could be compared against the uniaxial tensile test value or the analytical solution value for each curve. To remove any ambiguity, both were tested.

Considering the uniaxial tensile test value of 208.21 GPa, a multiplication factor of 317.474 was obtained to multiply the slope with. The standard deviation was 35.731.

Considering the analytical solution value for each slope’s corresponding file, a multiplication factor of 305.516 was obtained to multiply the slope with. The standard deviation was 34.341.

From converted Stress-strain curve

As seen in fig. 3.33 where the Calaf et al.’s conversion factor was compared against the slope from stress-strain curves obtained via inverse FE, these values tend to under-predict the elastic modulus. However, if these elastic modulus values are compared with the analytical solution values for their corresponding files, a conversion factor of 2.111 is obtained. That is, the values from the stress-strain curve of the elastic modulus need to be multiplied by that factor. The standard deviation is 0.328, which is significantly lower (percentage-wise) than the slope from F-D curve method.

A better understanding of these values obtained via the inverse FE can be seen in fig. 3.39 and 3.40. The graph is divided into 4 quadrants based on the pin displacement rates. The first quartile has tests from 0.2 to 0.3333 mm/min of pin displacement, the second quartile has tests from 0.35 to 0.85 mm/min of pin displacement, the third quartile has tests from 0.9 to 1.4 mm/min of pin displacement, and the fourth quartile has the remaining tests from 1.45 to 4.0 mm/min of pin displacement.

It can be seen in the first image that no quadrant has a low standard deviation, but the 3rd quadrant of tests is overall lowest in magnitude. This data visualization is helpful in recommending good test practices in the end. The scatter plot (fig. 3.40) is also quite
useful, with green lines indicating 1 standard deviation and black lines indicating 2 units of standard deviation. No clear trend seems to be emerging here.

Figure 3.39: Bar plots of elastic modulus obtained from converted SRTT, with all 48 tests divided into 4 quadrants (ascending order of pin-displacement rate).
Figure 3.40: Scatter plot for elastic modulus obtained from converted SRTT. Green and black lines are 1 and 2 units of standard deviation, respectively.
Choice of method

Clearly, the analytical solution, when used wisely, seems like the logical winner here in terms of the approach to be used. However, it must be borne in mind that the analytical solution must be compared to the actual experiment values for proper calibration and elastic modulus value calculation. Additionally, a sufficiently large bound space must be given for the optimisation.

The stress-strain curve’s elastic modulus seems like the second-best choice, given the low variance in the conversion factor. However, it is very computationally expensive to do an inverse FE analysis every time a test is performed. Thus, for double verification, it would seem that using the slope of the initial part of the force-displacement curve is a good choice. The multiplication factors obtained seem generally satisfactory, but the choice of the elastic modulus for comparison (analytical or uniaxial) would be case-dependent. If the uniaxial test’s data is available, that factor (317.474) should be taken, but in the absence of those, the factor calibrated against the analytical solution (305.516) remains the second-best choice.

3.7.2 SRTT: Obtaining yield strength

Bilinear fitting F-D curve

Since the value around 200 N seems to be a region of interest because most force-displacement curves obtained in this study have an inflection at that point, a bilinear fitting around that point may provide better insights into the yield strength. Thus, a bilinear fit was performed for all the experiments (example image shown in fig. 3.41).

![Bilinear fitting F-D curve](image)

Figure 3.41: Demonstrating bilinear fitting for SRTT.

Since there is no direct comparison for the yield strength, this again can be compared with the uniaxial tensile test values for a conversion factor, or with each SRTT’s converted
stress-strain curve. Comparing with the uniaxial tensile tests, a multiplication factor of 2.365 is obtained for the inflection point, with a standard deviation of 0.149. Comparing with each test’s converted stress-strain curve, however, yields a multiplication factor of 2.237 for the inflection point, with a standard deviation of 0.069.

From converted Stress-strain curve

As seen in fig. 3.35, the mean yield strength obtained from the conversion of force-displacement curve to stress-strain curve via inverse FE was 487.979 MPa with a standard deviation of 29.889 MPa. While the variance is high, the values seem to be much more in line with those predicted by the uniaxial tensile test. Additionally, a linearly rising trend was also observed from that image.

A better understanding of these values obtained via the inverse FE can be seen in fig. 3.42 and 3.43. It can be seen that no quadrant has a low standard deviation, with the 1st quadrant of tests being the lowest in magnitude. This is in contrast to the elastic modulus’ bar graph shown earlier in fig. 3.39. Same quartile regions are used here as were used for the elastic modulus’ bar graph.

The scatter plot is also quite useful, with green lines indicating 1 standard deviation and black lines indicating 2 units of standard deviation. It does seem that the yield strength values seem to be rising as the test displacement rate rises. This information is extremely useful in proposing good test practices for the SRTT. However, as mentioned previously, it should be noted that these converted values are obtained from the inverse FEA performed on one SRTT. Before making strong conclusive recommendations for a conversion relationships, it would be prudent to perform more inverse FE analysis for all the tests individually.

However, it does does seem that good testing practices can be proposed. Low deviation in both the 2nd (0.35 to 0.85 mm/min pin displacement rates) and 3rd (0.9 to 1.4 mm/min pin displacement rates) quartiles was observed from the bar graph. Additionally, these testing rates (from both the quartiles) can also be seen to be near the mean of the whole testing matrix. Thus, this range of pin displacement rates may be proposed for SS316L as recommended range of test rates. Caution should be exercised however, because, as mentioned previously, these converted values are obtained from inverse FEA performed on one SRTT.
Figure 3.42: Bar plots of yield strength obtained from converted SRTT, with all 48 tests divided into 4 quadrants (ascending order of pin-displacement rate).
Figure 3.43: Scatter plot for yield strength obtained from converted SRTT. Green and black lines are 1 and 2 units of standard deviation, respectively.
Choice of method

The inflection point serves as a good point for the determination of the yield strength and is recommended for future testing purposes. However, again the question arises of which factor to choose for comparison against this inflection point: the one obtained from the uniaxial tests or the converted SRTT. If the uniaxial test data is present, it is recommended that a multiplication factor of 2.365 be used. Otherwise, the multiplication factor of 2.237, which was calibrated against the converted SRTT, be used. However, a caveat here is that the stress-strain converted values are inherently difficult to obtain for the SRTT presently.

3.7.3 Discussion of SRTT data conversion

A clear observation that can be made here is that there exist ways to empirically determine elastic modulus and yield strength from the SRTT. However, given the limited material testing, these approaches should be kept as a validation method after performing inverse FE and then training an RFR (or equivalent) model. Training a general-purpose model with more materials would be a good to ensure robustness and understand the test better.
3.8 Commentary on SRTT experiments and analysis

This chapter contained the following:

1. The 48 experiments that were performed (including 3 extra tests for testing the machine learning model).
2. The data cleaning methodology.
3. Gradient boosting training and validation of model
4. Inverse Finite Element Analysis routines and discussion of results.
5. Conversion of SRTT test data to stress-strain curves via inverse FE.
6. Comparison of converted SRTT data to existing relationships.

The performed experiments were cleaned and prepared for univariate machine learning models and two models were contrasted. Additionally, an inter-test comparison was also performed to obtain the differences found between each test.

Inverse analysis was subsequently performed based on optimisation routines. However, errors were encountered during this optimisation due to the optimisation bounds being too narrow initially. This was addressed and the Nelder-Mead optimisation (gradient-free method) was used in the end.

This optimisation resulted in the equivalent gauge section area and gauge length values, which were used to obtain the stress and strain respectively. This was done by dividing the force from the tests by the equivalent gauge section area, and the displacement from the tests was divided by the equivalent gauge length. This was found to be a cumbersome method given the amount of time it takes, thereby making the SRTT largely inaccessible.

However, some direct conversion relationships exist and these were examined for this experimental dataset. The results were not satisfactory and new direct conversion methodologies were examined. Based on the results, a physics-driven approach that makes use of the analytical solution was proposed, in addition to a linear-fitting approach to the experimental dataset for determining the elastic modulus. For the yield strength, a bilinear fitting method was proposed as well.

However, as mentioned in the previous section (3.7.3), there remains a strong impetus to test more materials and train a more general-purpose multivariate machine learning model for more tests, therefore bolstering the current conversion relationships.
Chapter 4

DIC on Small Ring Tensile Test

The previous chapter analysed a matrix of Small Ring Tensile Tests performed on SS316L. However, given the nascency of this testing methodology, no dedicated measurement devices exist.

To address this, Digital Image Correlation (DIC) is leveraged in this chapter to explore potential sites on the ring that can be used to position an extensometer. The rings have been painted with a randomised speckle pattern that allows for the tracking of the ring’s displacements, which can potentially inform an extensometer design.

While DIC is integral to inform this extensometer design, a physical extensometer might be more useful than DIC by providing the SRTT with more accessible data. Given how involved DIC can be with its processes (sample preparation, image cleaning, and so on), a physical design would provide a far quicker result for the SRTT, thereby making the test much more accessible and easier to adopt.
4.1 Analysis of the DIC data

The procedure of DIC in 2D was briefly introduced in chapter 2. However, this chapter now provides context to all the methodologies introduced by showcasing the capability of DIC on the small ring tensile test.

Nikon D810 cameras were used for this analysis, coupled with a macro lens. The lens was Nikon 200 mm f/4. The macro capabilities of this lens also translate to a shallow depth of field, which is approximately 3 mm. The shallow depth of field is advantageous for 2D DIC, since it helps effectively isolate the regions of interest. Consequently, considering the operating f-stop (f/4) and focal length (200 mm) of the lens, the aperture diameter would be 50 mm, thus permitting a lot of light for DIC. The camera resolution was set to 4928 x 7380, with each DIC analysis calibrated to scale for each specimen. To maintain consistency in the experiments, the ISO was set to 100 with the exposure time set to 1/3 of a second.

An example image is shown again in fig 4.1.

![Example image of the SRTT images used in DIC](Figure 4.1: Example image of the SRTT images used in DIC)

DIC analysis is performed on three experiments to analyse the following scenarios:

1. Does the DIC y-displacement data match the crosshead displacement data?
2. How does the ring’s x-displacement data, measured by DIC, correlate with other test outputs?

3. Is it possible to put an extensometer on the ring at any angle (30, 45, or 60 degree from the vertical) and obtain any coherent results?

DIC was performed on all the 48 experiments and made open-source for future researchers [140]. Given the relatively rate-independent nature of these tests, 3 different displacement rates were chosen for DIC.

The images from the experiment with pin displacement rate 1.1111, 0.6667, and 0.4000 mm/min were chosen. These experiments were chosen because of their clearly visible speckle patterns.

These images were converted from their original mosaicked colour .NEF format (proprietary Nikon format) to greyscale .TIF format. This was done via the methodology laid out in the study by Forsey et al. [154] and the code provided by them for the image conversion. Once these images were converted, these were loaded in DaVis 10.

The choice of the step size and the subset size is a matter of trial and error, and it depends on the area that is being analysed. The subset size referring to the window size that is being analysed, and the step size referring to the overlap of pixels after the window shifts in a single pass of the analysis in the specified region of interest. Since these values differ for each region that is being analysed, they are mentioned separately in each section as appropriate. The introduction to subset and step size was already provided in section 2.3.1.

To illustrate the rationale for choice of step size and subset size in the context of these experiments, and how it’s a matter of trial and error, consider an image from the SRTT shown in fig. 4.2. The superimposed grid is a result of the subset size 39 and step size 13, which is used in section 4.3 and 4.4. It can be seen in the image’s speckle pattern that there are grid areas that are almost completely black (yellow ellipses), while some areas are completely white (red ellipses). Ideally, each cell of the grid pattern would have some black paint on the white base coat. Since this is a random speckle pattern and this is not always achieved, there needs to be a balance in the step size and subset size that achieves this as much as possible. Consider the yellow ellipse that has 4 cells marked from 1 through 4. It can be seen that each cell has at least some white undercoat visible, thus allowing for the DIC tracking to work. Thus, it can be seen that the choice of the subset and step size is a matter of trial and error that is heavily dependent on the specimen that is being analysed, with the aim to minimize the cells that are dominated completely by either white or black colours (again, as evidenced by the red ellipses).
Figure 4.2: Illustrating the subjectivity that arises in the choice of step size and subset size in DIC analysis in the context of the SRTT.
However, some parameters are constant for all analyses. For instance, the correlation type between the image datasets is relative to the first image. This provides a neutral and unaffected frame of reference that minimises error propagation that would arise from correlating on successive images. The first image of all experiments was captured before the experiment began.

Additionally, the number of maximum iterations for DIC calculation are also kept constant at 50. A second order shape function was not used, since this resulted in much more extra computational time (around 100% more) without any added benefit in accuracy, and would sometimes fail to converge as well.

All images have been sharpened first with the help of a double Laplacian filter before doing any analysis. This is shown below:

\[
\begin{pmatrix}
-1 & -1 & -1 \\
-1 & 8 & -1 \\
-1 & -1 & -1
\end{pmatrix}
\]

A Laplacian filter is able to detect the edges in the image space and increase the contrast, thereby sharpening them. This provides the analysis an enhanced speckle pattern separation, thereby, hypothetically, for better DIC processing.

Section 4.2 now elaborates on the DIC done on the plate affixed to the rig to analyse its motion in the y-direction. Lastly, it should be again noted here that all data is made open-source at [155].
For better comparison with the y-displacement values obtained from the testing rig’s crosshead displacement, a small plate was affixed to the bottom grip of the SRTT setup. This plate’s movements were tracked via DIC. This DIC analysis was given a subset size of 51 and a step size of 17.

The results of the plate’s y-displacement DIC data and total plate displacement via DIC (x and y-displacements combined) are shown in fig. 4.3. It can be seen that the variation of these displacements is largely linear with respect to the crosshead displacement. An initial offset is observed for the total displacement as the test begins, which lasts up to approximately 0.091 mm of crosshead displacement, while the y-displacement component is linear after approximately 0.3 mm of crosshead displacement.

![Figure 4.3: y-displacement (from DIC) on plate compared to total plate displacement with the observed crosshead displacement on the x-axis.](image)

This can also be visualized alongside the crosshead displacement. Fig. 4.4 showcases the total DIC plate displacement, the y-displacement (DIC) of the plate, and the crosshead displacement. It can be seen that the y-direction DIC displacement overlaps with the total DIC displacement after the initial offset for the latter lasts up to 50 seconds approximately. This suggests that the y-direction DIC displacement is dominant after this time.
4.2. SRTT: DIC ON AFFIXED PLATE

The difference between the maximum point of the measured displacements, which is at the end of the test, was found to be approximately 0.3 to 0.5 mm for all the analyses. A good match is found initially, but the plate’s total displacement and crosshead displacement values slowly start to diverge from each other as the experiment progresses, as seen in fig. 4.4. On average, for the 3 experiments tested, the maximum deviation was found to be 7.5%.

The total displacement on the plate converges well with the y-displacement, giving more confidence that the y-displacement is the primary form of displacement for the bottom half of the SRTT setup. The average absolute difference between the y-displacement and total displacement is 0.04 mm for this DIC analysis. The total displacement matches well with the y-direction DIC displacement after around 100 seconds in this analysis.

Fig. 4.5 also showcases the variation of both x and y directions, contrasted with total plate displacement. The variation of x-displacement is largely minimal, with the values starting at 0.394 mm and ending at 0.248 mm at the end of the experiment. As seen in the previous image (fig. 4.4), the y-direction DIC displacement matches well with the total displacement after around 100 seconds, which corresponds to approximately 0.6667 mm of crosshead displacement.
The next section intends to track the x-direction displacement at the 0 degree (horizontally parallel) direction in the ring.
4.3 SRTT: DIC on 0-degree points

Given the geometry of the ring, it can be divided into two sides: left and right. While this can be further divided based on top-bottom symmetry, for the purpose of analysis this has been not done for tracking the displacements in this section. This is because this division does not leave the processed DIC dataset with enough image data, since the 0-degree region is quite minimal. A subset size of 43 was used with a step size of 17.

The 0 degree points that have been extracted for analysis from the ring are visualized below in fig.4.6.

![Visualizing selected points](image)

Figure 4.6: Visualising DIC points for 0-degree (horizontal axis and ring edge region) analysis.

The edge points of the ring were chosen for analysis, as opposed to the points in the middle. The rationale for choosing the points at the edge is that this is more pragmatic, since it would serve as a surrogate for a potential extensometer design that would clamp at the edges at diametrically opposite ends of the ring (the 0-degree regions). The x and y-displacements for these selected points are visualised in fig. 4.7 and 4.8.
Figure 4.7: X and Y DIC displacement components and their variation with respect to time. Displacement values are for the 0 degree points on the left arm of the ring.

Figure 4.8: X and Y DIC displacement components and their variation with respect to time. Displacement values are for the 0 degree points on the left arm of the ring and capped at 600s.

In the first image (fig. 4.7), the y-displacement DIC data has a linearly rising trend with respect to time. The x-displacement DIC data rises much more sharply with respect to time, but then starts to gradually taper after around 600 seconds.

The second image, fig. 4.8, showcases the variation only up to 600 seconds (or around 3.5 mm of total plate DIC displacement) since this is where the inflection in x-displacements is observed. It can be seen that beyond that point the variation in x-displacement is not so linear. This is because the ring collapses and becomes sufficiently elongated that the
4.3. SRTT: DIC ON 0-DEGREE POINTS

...sides are parallel to each other and only y-displacement is primarily active.

To analyse this data further, these x and y-displacements can be plotted against the observed total plate displacement, which was delineated in the previous section (4.2). These have been visualized in scatter plots below (fig. 4.9 and 4.10). Both the displacement values are limited up to 600 seconds, or around 3.5 mm, since the x-displacement DIC values tend to taper after this. These scatter plots help visualize the linear trend of both the images independently.

![Graph 4.9](image1)

**Figure 4.9:** Scatter plot of x-displacements at 0-degree on the ring (x-axis of graph) vs the displacements on the affixed plate (y-axis of graph).

![Graph 4.10](image2)

**Figure 4.10:** Scatter plot of y-displacements at 0-degree on the ring (x-axis of graph) vs the displacements on the affixed plate (y-axis of graph).
A stronger linearity is observed for the y-displacement than it is observed for the x-displacement. Given the tapering nature of the x-displacement later in the test, only a limited range (up to 3.5 mm) is visualized. Nonetheless, this x-displacement data is analysed as well along with the y-displacement data.

This is done via a simple linear regression analysis. The results of this are visualized below in Figure 4.11 and 4.12. It can be seen that both the regression curves match well with the observed displacements, respectively. However, with an R-squared value of 0.99946, the y-displacement data has a stronger correlation throughout the whole region as compared to the x-displacement regression line, which has an R-squared value of 0.99670. The slope of the x-displacement line was observed to be 1.579 with an intercept of 0.056. Meanwhile, for the y-displacement, the slope was observed to be 2.058 with an intercept of -0.368.

![Linear Regression Model (Left Arm)](image)

Figure 4.11: Linear regression on x-displacement data at 0 degree points of the left arm of the ring. Y-axis of the graph has the total DIC plate displacement.
An argument can be made here that the use of the linear regression fitting for the x-displacement component, instead of a second or third-order fitting, may not be appropriate. To address this argument, consider the images shown in fig. 4.13, which showcase the various fitting results for the left arm for limited x-displacement data and full x-displacement data. It can be seen that the second and third-order polynomials fit better for the limited x-displacement as compared to the first-order linear regression. The R-squared value for these fits is also higher, at 0.99949 and 0.99968 for the second and third-order fit, respectively. However, if the full DIC x-displacement data is considered, as shown in the fig. 4.13 (a), it can be seen that the onset of failure, as the ring collapses completely, does not fit well any of the fitting data. The R-squared values are still high at 0.99354 and 0.99626 for the second and third-order fit, respectively. However, since the SRTT is a relatively new form of test, the region beyond the ultimate tensile strength is not well understood. Consequently, the variations in the x-displacement that occur nearer to the end of the test are unpredictable and not suitable for further analysis, since fit quality deteriorates at the end of the test.
Figure 4.13: (a) Different order polynomial regression for full DIC x-displacement data at 0 degree points of the left arm of the ring; (b) Different order polynomial regression on limited DIC x-displacement data at 0 degree points of the left arm of the ring. The x-axis of the graphs has the total DIC plate displacement.
4.3. SRTT: DIC ON 0-DEGREE POINTS

Nonetheless, the set of linear fitting analysis is extended to the right arm of the ring as well, and the images are shown in fig. 4.14 and 4.15.

**Figure 4.14:** Linear regression on x-displacement data at 0 degree points of the right arm of the ring. Y-axis of the graph has the total DIC plate displacement.

**Figure 4.15:** Linear regression on y-displacement data at 0 degree points of the right arm of the ring. Y-axis of the graph has the total DIC plate displacement.

It can be seen that the fit is much worse for the right arm than it was for the left arm. This is possibly due to the misalignment of the ring as it was loaded. While every possible effort was made to keep the rings centred, the loading was not always perfect. This was reflected in the DIC data as well. The slope for the right arm’s x-displacement is 1.537 with an intercept of 0.553 mm. For the y-displacement, the slope is 1.909 with an intercept of 0.351 mm. Again, the fit is much better for the y-displacement than it is for
the x-displacement values. The average slope for the y-displacement points, thus, is 1.983 with an intercept of -0.017. The R-squared value for the fitting on the y-displacement is 0.99751 while for the x-displacement it is observed to be 0.97888.

These fitting methodologies provide valuable insight for the data by providing with an approximate multiplication factor, which would be the slope of the fitted line. Given the higher R-squared value for the y-displacements, this is preferred over the x-displacement values. However, given the luxury of data availability to do DIC on other areas of the ring, more areas can be analysed for DIC. This has been done in the next section.
4.4 SRTT: DIC on 30-degree points

After preliminary visual exploration of all the images acquired from the tests, it was
determined that a 30-degree point could potentially be a good starting point for the
analysis of a potential extensometer position. This is because a 45 degree point may
sometimes be obscured by the grips while testing the ring, especially as the test progresses.
An example of this from one of the experiments is shown in fig. 4.16. As the test progresses,
there is a greater chance of this point being obscured by the grip’s shadow in the bottom
right quadrant, as shown in fig. 4.17. The reference circle shown in the previous image
(4.16) is also shown in 4.17. It can be seen that as the test progresses, the 45 degree region
is more covered than the 30 degree region. This issue is also encountered in the 30-degree
analysis and has been discussed later in this section.

Figure 4.16: Illustrating various angles on the SRTT DIC for potential extensometer
sites. Reference circle is a piece of paint on the ring.
Figure 4.17: Illustrating various angles on the SRTT DIC for potential extensometer sites once the ring has deformed nearer to the end of the test. Reference circle is a piece of paint on the ring.
The quadrants of the ring are labelled in a counter-clockwise manner. The quadrants start from the top right section of the ring. A subset size of 39 was used with a step size of 13. The x, y, and total displacements are visualised in fig. 4.18. The gap in the data observed in the 4th quadrant is due to DIC not working. This visualisation illustrates the importance of two things: a) a need for uniform lighting and speckle pattern, and b) the effect of ring placement while loading it in the rig. The first part is likely the reason for the failure of DIC after approximately 600 seconds, since this time is approximately when a shadow is being cast on the ring by the grip. This shadow effect can be visualised in fig. 4.19. This image has been taken at a time step of approximately 725 seconds in the experiment.
Figure 4.18: All DIC displacement components (x, y, and total) for all 4 quadrants’ 30-degree regions.
4.4. SRTT: DIC ON 30-DEGREE POINTS

These quadrant displacement components can be grouped together for a better data interpretation. This can be visualised in fig. 4.20. It can be seen in the image that, just like the 0-degree analysis, strong linearity is found in the y-displacements but the same cannot be observed for the x-displacements. Additionally, the values from quadrant 4 (bottom right area) area seem greatly affected by the initial loading conditions. This quadrant 4 issue is also supported by its erratic x-displacement behaviour. The total displacement values, similar to the y-displacement DIC values, are linear in nature as well once the ring is loaded properly.

Isolating only the values from the first quadrant for data visualisation (fig. 4.21) provides a better visual aid for exploratory data analysis. This image showcases that the x-displacement does not have a linear nature as compared to the y-displacement DIC values. The total DIC displacement values are affected by this non-linear nature of values of x-displacement and are not as linear as the y-displacement DIC values.
SRTT (0.4 mm/min) DIC: Evaluating displacement discrepancy for each quadrant

Figure 4.20: Segregating each displacement component from all quadrants (y-axis of graph) and contrasting against the total plate DIC displacement (x-axis of graph).

Comparison of x-displacement in all quadrants

Comparison of y-displacement in all quadrants

Comparison of total displacement in all quadrants

Time (s)
Figure 4.21: All DIC displacement components from quadrant 1 (y-axis of graph) compared to the plate’s total DIC displacement (x-axis of graph).

Given the y-displacement values of the DIC exhibit strong linearity upon visual inspection, a regression analysis was performed on them against the plate’s displacement values. This was done for the full range of the y-displacement values and for values up to 4.5 mm only as well. Results of this are visualized in fig. 4.22 and 4.23. The first image shows the linear fitting for the full displacement data, while the second one shows the fitting only up to 4.5 mm of displacement. Both images exhibit a strong linear fitting with good R2 scores. These are also tabulated in table 4.1.
CHAPTER 4. DIC ON SMALL RING TENSILE TEST

Figure 4.22: Visualising results from regression analysis on the full range of y-displacement values at 30-degree points on the ring. Y-axis of the graph is the total plate DIC displacement and x-axis is the y-displacement (via DIC) at the 30 degree region.
4.4. SRTT: DIC ON 30-DEGREE POINTS

Thus, it can be seen that the y-displacement has a good displacement fit. Taking the mean of all the slopes of all quadrants for the full range of the y-displacements results in a slope of 2.0035 with an average intercept of 0.038. This shows that the data correlates extremely well when averaged out, and the individual discrepancies that are seen in the previous plots are merely artefacts of the experiment and the ring loading.

The importance of these findings, along with previous sections, is now discussed in the next section (4.5).
Table 4.1: Values from regression analysis on y-displacement values (via DIC) obtained at 30° points on ring samples.
All the findings from the regression analysis on all the processed DIC data are delineated in table 4.2.

<table>
<thead>
<tr>
<th>Experiment (mm/min)</th>
<th>Region</th>
<th>Slope</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0°</td>
<td>1.983</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>2.003</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>30° (limited y-disp)</td>
<td>1.981</td>
<td>-0.033</td>
</tr>
<tr>
<td>0.6667</td>
<td>0°</td>
<td>1.955</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>1.975</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>30° (limited y-disp)</td>
<td>1.994</td>
<td>-0.026</td>
</tr>
<tr>
<td>1.1111</td>
<td>0°</td>
<td>1.871</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>1.889</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>30° (limited y-disp)</td>
<td>1.894</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of findings from regression analysis performed on processed DIC data.

It can be seen that the prediction from the 30 degree regions is consistently closer to a factor of 2 (the slope), whether considering the full range of y-displacements or a limited range of displacements (up to 4.5 mm).

Additionally, the value of the intercepts is also largely negligible. This is a promising finding since this suggests that a ‘claw’ like extensometer, that grips rings at 30-degree in all 4 quadrants would prove to be a valuable surrogate for capturing the displacement rates. This would remove the requirement for DIC calculations and would provide for quick displacement calculations, thereby making the test more reliable and more accessible for future researchers.

However, it should be noted that the fundamental nature of the small ring tensile test may warrant re-examination. Consider fig. 4.24. This figure has the force-displacement graphs for the crosshead displacement, total plate displacement DIC data, x-displacement for 30° DIC, y-displacement for 30° DIC, and total displacement for 30° DIC.

It can be seen that the force-displacement curves from DIC are not totally in-line with the curves obtained from the force-displacement curve plotted from the crosshead (‘X-head’) displacement. Only the total plate displacement DIC data is close to the crosshead displacement data. However, after closer scrutiny, it can be seen that quadrant 2 and 3 have much smoother transitions for the y-displacement and total displacement in the 30-degree regions. This is promising because the shortcomings observed, especially in quadrants 1 and 4, can be attributed as an artefact of the DIC on that arm.
Figure 4.24: Force-displacement plots using the DIC displacement data. The ‘X’, ‘Y’, and ‘Total Disp’ refer to the displacements obtained from 30-degree points of the DIC. Each quadrant refers to the physical position of these points on the ring. The total plate displacement is given by ‘Plate Disp’ and the crosshead displacement is given by ‘X-head Disp’.
A well-designed claw-like extensometer would be able to mitigate these discrepancies and should be able to provide more robust displacement data. Theoretically, this claw-like extensometer would grip the ring at 4 points (at 30 degree points) and take care of initial adjustments, that hinder DIC right now, by calculating relative displacements and adjust the shift in each side of the ring dynamically. Regardless of the limitations of the analyses showcased here, there exists a promising avenue in designing an extensometer for the SRTT.

A further question that also arises here is why were these values of DIC displacement data not used for stress-strain conversion in the previous chapter. Fig. 4.25 and 4.26 showcase the answer to this question. The first image showcases the stress-strain curve with the strain generated from total plate DIC displacement. The second image showcases the same, but with the strain generated from the mean of all quadrants’ y-displacement DIC data at $30^\circ$ points.

The strain is obtained by dividing the DIC displacement values in both the images by the equivalent gauge length obtained in chapter 3. The stress values are obtained by dividing the load of the experimental data by the equivalent gauge section area obtained in the previous chapter.

Figure 4.25: Stress-Strain curve for 0.4 mm/min test, with strain obtained from total plate displacement via DIC.
Figure 4.26: Stress-Strain curve for 0.4 mm/min test, with strain obtained from average of all y-displacement DIC values at 30° points.
4.5. SRTT: DISCUSSION OF DIC RESULTS

The plate DIC gives an elastic modulus of 145.28 GPa and a yield strength of 488.94 MPa. The 30° DIC, meanwhile, yields a negative elastic modulus due to its inclined slope and a yield strength of 604.52 MPa.

These values showcase two things:

1. The plate DIC can be used in place of the cross-head displacement and results in slightly better results in terms of the elastic modulus and yield strength.

2. The 30-degree DIC values need more calibration during the test to account for the ring shifting during the initial phases of the test.

The last point about the 30° DIC highlights the need for a robust extensometer design. An extensometer that would be insensitive to slight misalignments during the loading of the ring, since DIC seems especially susceptible to it. This potential extensometer could also, theoretically, take into account the relative motion between all the 4 points in all 4 quadrants to account for the shift that occurs at the beginning of the test.

Additionally, the extensometer design would also benefit from a device that can be a better surrogate for the y-displacement than the crosshead. That is, something like the affixed plate on which DIC was performed.

A final point that can be made about these findings is that the DIC procedure can be improved further by removing the Laplacian filter that was applied initially. It was later discovered that this filter tends to slightly reduce the quality of the image. While the filter seemingly works upon visual inspection, it is not recommended to use this in future studies since it introduces rounding errors that arise due to intensity adjustment.

A summary of all the findings from this chapter is presented next.
4.6 Summary of findings

DIC was leveraged as a tool in this chapter to explore potential extensometer positions for the SRTT. As mentioned previously, a good extensometer would make the test more accessible and results could be generated much more easily as compared to DIC.

Three avenues were explored:

1. DIC on a plate affixed to the moving part of the SRTT rig.
2. DIC at the 0° points of the ring.
3. DIC at 30° points of the ring.

It was found that the plate’s DIC displacement values correlate closely to the crosshead displacement values. The difference between them was minimal, and it is recommended that a proper tracking device be used to track the driving rig’s displacement in future SRT experiments.

Considering the ring geometry, the y-displacement values for the 0° points correlated well with the total plate DIC displacement, but the 30° points consistently performed better.

Stress-strain curves were also briefly discussed and these highlighted how the plate DIC-based extensometer would be a valuable addition to the SRT, but the 30° claw-like extensometer would need to dynamically adjust for initial shifts in the rings during the loading stage.

To summarise, the following two propositions can be made:

1. An extensometer to track the driving pin’s displacement is recommended.
2. A claw-like extensometer that attaches to all 4 points of the ring at 30° values is a strongly recommended avenue to explore.
Chapter 5

Combination of the Small Ring Test and the Small Punch Test
5.1 Combination Impetus and Information

This chapter addresses the final research question of this study: is there any feasibility of combining the Small Ring Test (SRT) and the Small Punch Test (SPT)?

The combination is proposed by extracting the discs for the SPT from the blank space of the rings extracted for the SRT. Based on the thickness of the ring at 2 mm and the required disc thickness of 0.5 mm, it was found that two discs can be extracted (considering loss of material to machining processes).

The potential benefits of this would be immense savings in the amount of material used. Volumetrically considering three uniaxial specimens of size 100 x 40 x 6 mm (used in chapter 2), this combination would provide materials savings of 97.89% for a master ring sample of inner diameter 10 mm, outer diameter 12 mm, and thickness 2 mm. Fig. 5.1 aids the visualisation of this hypothesis. The red region represents the master ring, from which all samples are to be extracted, including the ring for the SRT. The blank space between the ring and the disc (approximately 2 mm) is the residual material and is showcased with a yellow colour. The purple colour represents the discs that are extracted for SPT, each of which is 8 mm in diameter and 0.5 mm in thickness.
Figure 5.1: Schematic of the hypothesis, showcasing the combination of the SRT and the SPT. All dimensions are in mm and representative only for visualisation purposes. Not to scale.
Additionally, given the possibility of performing different types of tests on the SRT and SPT, different tests can be performed on the same sample. The SRT is capable of evaluating creep \[24\] and tensile data \[22\], while the SPT is capable of evaluating creep, tensile, and fracture mechanical properties \[31\].

Lastly, there is a potential to evaluate the grain sizes from the residual material between the ring-disc blank space. This has also been evaluated in this chapter. This residual material could also potentially be used for hardness testing. While not showcased here, there is also a potential to perform metallographic examination on the material extracted.

To address this research question, three avenues are explored:

1. Combining the SRT (tensile) and the SPT (tensile) for SS-316L.
2. Combining the SRT (creep) and the SPT (tensile) for Nimonic-75 (N-75).
3. Combining the SRT (tensile) and the SPT (tensile) for a plate with weld deposits.

These avenues are explored in the next few sections, but before exploring the hypothesis’ results, the disc extraction for the SPT from within the rings is briefly explained in the next section.

### 5.1.1 Disc extraction from within SRT samples

The ring samples for SRTT were 2 mm thick, with an outer diameter of 12 mm and an inner diameter of 10 mm. These are the same dimensions as used for the SRTT in chapter 3. With a required thickness of 0.5 mm from the discs, 4 discs can be extracted theoretically if one is to assume no loss of material due to machining. The dimension of the discs is set by the European Standards \[30, 31\] (see 1.5.1 for background on this test). However, this is not possible with the current available manufacturing processes within the University. EDM (Electric Discharge Machining) is used to extract the rings and the discs. Given the limitations of EDM (Electric Discharge Machining) based on the cutting wire thickness, two discs were extracted from each ring sample for comparison. The EDM wire had a thickness of 0.25 mm (smallest possible size in the department with the machines there was access to).

For 2 discs of 0.5 mm thickness, 1.0 mm of disc material is needed. Considering the 0.25 mm wire thickness, extraction of 2 discs causes a material loss of 0.5 mm (2 x 0.25 mm). Thus, for the ring of 2 mm thickness, this means that out of the thickness of the material available, 1.5 mm thickness is used up to extract 2 discs. Thus, with the remainder of the material (0.5 mm) after these 2 discs are extracted, a third disc could not be extracted. Another challenge was also the narrow circumferential allowance. The rings had an inner diameter of 10 mm, while the discs had to be 8 mm in diameter. This did not leave a lot of room for error.

First, the master ring sample was extracted from the parent block of material. Once the ring is extracted from the parent material, the ring is fully extracted by machining the blank space so that a ring of 10 mm inner diameter is separated. Next, from this blank, the discs are extracted for the SPT. The EDM wire of 0.25 mm, mentioned previously, was used for all machining processes. The jig used for the initial ring extraction is shown in fig 5.2.
5.1. COMBINATION IMPETUS AND INFORMATION

Figure 5.2: Fixture used for extraction of the full ring space. The big circular metal clamped at both ends diametrically is the area from where specimens are extracted. A circumferential cut can be seen in this metal for an SRT specimen.
As mentioned previously, this allows for the first step of extraction: extraction of the initial master ring space. Subsequently, the blank space of this ring is placed in an EDM machine (wire thickness 0.25 mm) to machine out the blank space. This is shown in fig 5.3. The thin vertical wire seen in this image is used to cut out the discs from the blank space of the ring. The biggest challenge for the manufacturing department was to align the EDM wire for this disc machining due to its thinness. This rig allows for the extraction of the blank for discs’ machining, which is then split into 2 discs.
Figure 5.3: Fixture used for the extraction of discs from within the ring. The magenta rectangular box encloses the EDM wire.
5.2 Combining SRT and SPT: Stainless Steel (Grade 316L)

5.2.1 Small Ring Tensile Test

Test Background

Chapter 3 evaluated, in detail, the feasibility of the small ring test over a multitude of pin displacement rates. It was demonstrated in that chapter that the small ring test could potentially be fit for purpose for miniature-specimen tensile test.

In this section, five ring samples are evaluated. The rings are made of SS316L and discs for SPT have been extracted from the blank space of these rings. The displacement rates tested are (mm/min): 0.08, 0.1, 0.3, 0.5, and 0.8. These have been labelled (in the same order) as B1 to B5, respectively.

Similar data cleaning techniques as described for the normally extracted rings were used here (delineated in section 3.2.1) with the Convolution Smoother being chosen for these rings. The choice for Convolution Smoother and the background is presented in 3.2.1. The results are presented in the next section.

Test Results: Hypothesis Rings

The loading and testing of the rings is the same as was done for the rings in chapter 3. Fig. 5.4 below compares the five displacement rates with each other. The tests perform extremely well and no significant deviation is found. The MAE % between the tests is also visualized in a heatmap in fig. 5.5. The maximum MAE is found between the tests with the rate 0.3 mm/min and 0.8 mm/min, and is found to be 2.313%. The minimum was found to be between 0.5 mm/min and 0.8 mm/min, with a value of 0.446%.
5.2. COMBINING SRT AND SPT: STAINLESS STEEL (GRADE 316L)

Interpretation of Results

To interpret the results of the rings from which discs have been extracted (that is, the discs to evaluate the combined testing methodology), these rings are also compared to...
their equivalent rate counterparts from the normally extracted rings (that is, the rings with no discs extracted from their blank space for SPT), and no significant deviation is found in those either. A select few results are shown in fig. 5.6. Detailed differences are eventually delineated in the results table (see 5.1).

Figure 5.6: Comparing results from rings with discs extracted to their normally extracted counterparts.

Another test to check for discrepancy between the samples tested in this batch and the rings extracted normally (that is, rings with no disc extraction) is to use the machine learning model trained earlier on this dataset (see 3.3.3 for details on the model). The results of this have been visualized in fig. 5.7. ‘RFR’ in the legend box indicates the Random Forest Regression model. This is generated by providing the displacement values as the input vector to the RFR model, and the force values are generated from that. The MAE for B2 (0.1 mm/min) ring is 10.69; for B4 (0.5 mm/min) ring the MAE is 31.42; and for the B5 (0.8 mm/min) ring the MAE is 35.72. With the MAE being less than 2%, it would be prudent to say that there is an excellent match between the rings extracted normally and the rings that had discs extracted from their blank space.
5.2. COMBINING SRT AND SPT: STAINLESS STEEL (GRADE 316L)

5.2.1 Small Punch Test

Test Background

The small punch tests are carried out on the rig described in chapter 2. The output from the test is the force-displacement graph, just like the SRTT, since the SPTT is a fixed displacement-rate driven test. That is, the rate of displacement of the punch is kept constant throughout the test. The CEN standards and the revised standards by

![Force predictions (rings with discs extracted data)](image)

Figure 5.7: Testing the machine learning model on rings B2, B4, and B5.

Based on the conversion methods described at the end of chapter 3, the values of force at the inflection point are given in Table 5.1. The mean value for the elastic modulus is 202.26 GPa with a standard deviation of 2.67 GPa and the mean yield strength value is 439.49 MPa with a standard deviation of 19.58 MPa. Based on this table, the yield strength values differ by 5.52% with the rings tested in chapter 3 and the elastic modulus differs by 0.94%.

<table>
<thead>
<tr>
<th>Ring</th>
<th>Rate (mm/min)</th>
<th>Force (N) (bilinear fit)</th>
<th>Yield Strength (MPa) (force x 2.237)</th>
<th>Elastic Modulus (GPa) (analytical solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.08</td>
<td>198.24</td>
<td>443.46</td>
<td>201.24</td>
</tr>
<tr>
<td>B2</td>
<td>0.1</td>
<td>198.83</td>
<td>444.78</td>
<td>208.37</td>
</tr>
<tr>
<td>B3</td>
<td>0.3</td>
<td>195.97</td>
<td>438.38</td>
<td>200.85</td>
</tr>
<tr>
<td>B4</td>
<td>0.5</td>
<td>205.62</td>
<td>459.97</td>
<td>201.71</td>
</tr>
<tr>
<td>B5</td>
<td>0.8</td>
<td>184.12</td>
<td>411.88</td>
<td>201.11</td>
</tr>
</tbody>
</table>

Table 5.1: SRTT converted values from empirical formulas derived in chapter 3.

5.2.2 Small Punch Tensile Test

Test Background

The small punch tests are carried out on the rig described in chapter 2. The output from the test is the force-displacement graph, just like the SRTT, since the SPTT is a fixed displacement-rate driven test. That is, the rate of displacement of the punch is kept constant throughout the test. The CEN standards and the revised standards by
Bruchhausen et al. recommend a punch tip radius of 1.25 mm, and the manufactured punch had a tip radius of 1.22 mm [30, 31].

While the CEN standards [30] propose a methodology for the calculation of tensile parameters, there are numerous propositions to alternative formulas as well. Some of these have been described in extensively in section 1.5.1 including the revised standard by Bruchhausen et al. [31]. To obtain the 0.2% proof strength ($R_{p0.2}$) from the SPTT, Bruchhausen et al. propose is a bilinear function fit if specimen deflection is obtained and a trilinear fit if the punch displacement is used. Since the latter is used in this study, a trilinear fit is recommended.

As a brief reminder, the UTS (Ultimate Tensile Strength) can also be obtained via 2 different methods [31]. The 2 approaches listed below:

\[
R_m = \beta R_m \frac{F_m}{h_0 u_m} \quad (5.1)
\]
\[
R_m = \beta R_m \frac{F_i}{h_0^2} \quad (5.2)
\]

The first equation corresponds to the first method of obtaining the UTS (M1) makes use of the maximum force reached in the test $F_m$ and $u_m$ is the corresponding displacement. The second equation corresponds to the second method of obtaining the UTS (M2) and makes use of the force at a specific location, $i$, which is geometry dependent. For the tests performed in this study, the value of $F_i$ must be taken at punch displacement 0.645 with a $\beta$ value of 0.179 which is valid for both the equations. All these values are taken from the new standard laid out by Bruchhausen et al. [31].

The results from the normally extracted discs and from the discs extracted from the ring-disc combination are presented next.

Test Results: Normal Discs

This section delineates the results from the normally extracted discs, that is, the discs that are extracted via normal machining procedures and do not make use of the ring-disc combination hypothesis. This set of control tests were performed on SS316L discs at various punch displacement rates. A full list of the punch displacement rates tested is enumerated below, with the ‘D’ representing the Disc number:

- 0.1 mm/min (D1)
- 0.3 mm/min (D2, D3, D4)
- 0.5 mm/min (D5)
- 1.0 mm/min (D6)

The results from this first set of tests are illustrated in fig.5.8. Discs D3 and D4 are not shown since they are the same punch displacement rates. It can be seen that the force-displacement values do not match initially but adopt a largely matching for the force-displacement profiles after the dip in the graph at around 400 to 450 N. Table 5.2 meanwhile details the results for all the discs and the values obtained via the trilinear fitting.
5.2. COMBINING SRT AND SPT: STAINLESS STEEL (GRADE 316L)

Figure 5.8: Select results from normally extracted discs for SPTT.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Rate</th>
<th>( v_a ) (mm)</th>
<th>( f_a ) (N)</th>
<th>( R_{p0.2} ) (MPa)</th>
<th>UTS (M1, MPa)</th>
<th>UTS (M2, MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.1</td>
<td>0.311</td>
<td>145.86</td>
<td>266.52</td>
<td>275.59</td>
<td>278.18</td>
</tr>
<tr>
<td>D2</td>
<td>0.3</td>
<td>0.350</td>
<td>144.34</td>
<td>265.19</td>
<td>260.89</td>
<td>247.99</td>
</tr>
<tr>
<td>D3</td>
<td>0.3</td>
<td>0.157</td>
<td>146.90</td>
<td>272.66</td>
<td>299.58</td>
<td>369.22</td>
</tr>
<tr>
<td>D4</td>
<td>0.3</td>
<td>0.233</td>
<td>142.12</td>
<td>261.73</td>
<td>290.91</td>
<td>218.02</td>
</tr>
<tr>
<td>D5</td>
<td>0.5</td>
<td>0.222</td>
<td>149.13</td>
<td>282.33</td>
<td>300.55</td>
<td>330.47</td>
</tr>
<tr>
<td>D6</td>
<td>1.0</td>
<td>0.290</td>
<td>155.71</td>
<td>294.79</td>
<td>293.71</td>
<td>290.03</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>273.87</td>
<td>286.87</td>
<td>288.89</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td></td>
<td></td>
<td>11.46</td>
<td>14.22</td>
<td>49.99</td>
</tr>
</tbody>
</table>

Table 5.2: All results from normally extracted discs for SPTT.

At this stage, it must also be noted that a slight kink was observed in all the tests performed, regardless of the material chosen. Despite repeated testing on over 30 discs to isolate the problem by tweaking the testing fixtures, this problem consistently persisted. Thus, it was attributed that this kink in the graphs is merely an indicator of the disc cracking. This does happen for many materials and is not unusual, as indicated by the standard laid out by Bruchhausen et al. They mention that the instantaneous drop in force, that leads to these kinks (or ‘pop-ins’), is a material artefact that is attributed to cracking.

The presence of the kink in the graph does not hinder the results from this study, since as long as the results consistently match between the discs extracted from the parent sample and the discs extracted from the ring’s blank space, there is confidence in stating that this ring-disc combination methodology could work. However, it can be seen that
there is quite the spread in the values obtained from the discs extracted via normal EDM procedure. Additionally, some UTS values extracted via the first method do not seem physically viable (determined via the maximum force) since some of these values are less than the corresponding 0.2% proof strength values.

Test Results: Hypothesis Discs

For the purposes of the hypothesis testing, 5 rings were used. As mentioned previously, each ring of 2 mm thickness would yield two discs of 0.5 mm thickness. Thus, this results in 10 discs to be tested. Each batch of samples is given an index ranging from B1 to B5, with the discs extracted from the top labelled as D1 and the bottom ones labelled as D2. The reference of top and bottom is given by the ring’s speckle pattern. The discs extracted from the same side as the side which was painted for the ring in SRTT are referred to as the ‘top’ discs, while the discs below it are referred to as the ‘bottom’ discs. Fig. 5.9 helps visualise this extraction of samples. The specimen with the speckle pattern is the ring that will be used for SRT. Samples that are extracted from the top, that is, the same side as the ring’s speckle pattern (labelled as ‘1’ in the image) are the D1 discs and the discs on the bottom are the D2 discs.

![Image](image-url)

Figure 5.9: Visualising the extraction of SPT specimens from the SRT ring’s blank space. The speckle pattern specimen is the ring for SRTT, while the inner circle with ‘1’ etched on it is the disc D1 for SPTT.

The punch displacement rates used for the hypothesis testing are listed in table 5.3 along with the results obtained from the trilinear fitting.
Table 5.3: Results from trilinear fitting on SPTT on discs extracted from within a ring’s blank space.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Rate</th>
<th>( v_a ) (mm)</th>
<th>( f_a ) (N)</th>
<th>( R_{p0.2} ) (MPa)</th>
<th>UTS (M1, MPa)</th>
<th>UTS (M2, MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1D1</td>
<td>0.1</td>
<td>0.32</td>
<td>131.02</td>
<td>267.33</td>
<td>281.57</td>
<td>299.23</td>
</tr>
<tr>
<td>B1D2</td>
<td>0.6</td>
<td>0.37</td>
<td>106.82</td>
<td>221.53</td>
<td>275.74</td>
<td>254.15</td>
</tr>
<tr>
<td>B2D1</td>
<td>0.2</td>
<td>0.34</td>
<td>111.32</td>
<td>231.43</td>
<td>282.65</td>
<td>276.19</td>
</tr>
<tr>
<td>B2D2</td>
<td>0.9</td>
<td>0.36</td>
<td>114.46</td>
<td>236.96</td>
<td>295.76</td>
<td>286.97</td>
</tr>
<tr>
<td>B3D1</td>
<td>0.3</td>
<td>0.33</td>
<td>105.89</td>
<td>215.65</td>
<td>284.02</td>
<td>268.34</td>
</tr>
<tr>
<td>B3D2</td>
<td>1.0</td>
<td>0.27</td>
<td>110.77</td>
<td>223.99</td>
<td>299.76</td>
<td>305.91</td>
</tr>
<tr>
<td>B4D1</td>
<td>0.4</td>
<td>0.35</td>
<td>110.00</td>
<td>222.16</td>
<td>309.03</td>
<td>325.13</td>
</tr>
<tr>
<td>B4D2</td>
<td>0.8</td>
<td>0.33</td>
<td>110.00</td>
<td>224.92</td>
<td>295.88</td>
<td>306.14</td>
</tr>
<tr>
<td>B5D1</td>
<td>0.5</td>
<td>0.34</td>
<td>120.34</td>
<td>245.99</td>
<td>284.62</td>
<td>270.10</td>
</tr>
<tr>
<td>B5D2</td>
<td>0.7</td>
<td>0.36</td>
<td>140.83</td>
<td>288.65</td>
<td>294.37</td>
<td>298.83</td>
</tr>
</tbody>
</table>

Mean: 237.86 290.34 289.10
Std. Dev.: 22.19 9.66 20.62

The material parameters are calculated from the trilinear fit method as described in the previous section (5.2.2).

A good visualization of these tests can be appreciated in fig. 5.10 and 5.11. The numbers in the brackets in the legend box denote the punch displacement rate (in mm/min). It can be seen that there is a much closer match between the tests in the discs that are extracted from the top as compared to the discs extracted from the bottom. This could likely be due to the increased punch displacement rates, but this is not a definitive conclusion.
Figure 5.10: Results from SPTT on the top discs (D1) extracted from within the rings.

Figure 5.11: Results from SPTT on the bottom discs (D2) extracted from within the rings. The punch displacement rates are in mm/min and are depicted in the brackets in the legend box.
Additionally, it can also be noticed that there is a slight kink in all the graphs for SPTs shown in this chapter, regardless of the extraction method of the discs.

Comparing the discs from both the extraction methods, it can be seen that the proof strength ($R_{p0.2}$) (see values in table 5.2 and 5.3) does not have a very good match between the methodologies, but the ultimate tensile strength is more closely matched.

Rig Compliance Issues

At this point, it is worth noting that all the proof strength results from the SPT differ greatly from those obtained in the SRT and the uniaxial tests, suggesting an issue in the compliance of the rig, likely due to the inversion method of its construction and machine issues. As explained in section 2.2.2, the SPT rig was inverted by 180° due to the machine able to perform only tensile mode testing. Since the traditional SPT setup requires a compression mode of testing (punch penetrating the disc), the rig was inverted so that the punch is beneath the disc and the upward motion of the punch (tensile mode of the machine) results in the disc being penetrated from the bottom (as shown in fig. 2.12 in 2.2.2). This whole rig inversion setup was also loaded as a single piece due to limited access to the machine.

Another reason why this could not be dependent on the extraction methods and how this does not negate the ring-disc combination hypothesis is shown in fig. 5.12 and 5.13. These figures are the heatmaps of all the SPTT performed and is done in the same as was done for SRTT in 3.3.1. That is, for the generation of the heatmap, the larger data array in the comparison was re-sampled to the size of the smaller data array and the respective error metrics were computed for the force values. A mean R2-score of 0.9488 (standard deviation of 0.0602) and a mean MAE % of 4.979% (standard deviation of 3.311 %) was found. The good R2-score (of almost 95% similarity) and the low MAE % suggest that this issue in the parameters could not be attributed to the hypothesis of ring-disc testing. Additionally, multiple uniaxial tests (of approximate size 100 mm) were successfully performed by other researchers on the machine, suggesting that the crosshead measurement of the Zwick machine was adequate. Thus, an only likely reason why all the SPTT proof strength results do not match with SRTT or uniaxial tests is the SPT rig and its overall performance. The unusually long load-chain for the machine could be a strong contributing factor as well, alongside the inversion method used for the disc penetration.
Figure 5.12: Heatmap of all SPTT experiments’ force-displacement values, with the colour bar scaled from min to max of R2 score. Experiments are grouped by extraction method (normally extracted versus extracted from blank space of ring) and sorted by ascending rates of punch displacement rate. The ‘B’ represents the Batch of the hypothesis test (given in 5.2.2) and ‘Norm’ represents the normally extracted discs. Punch displacement rates are shown in parentheses in tick labels in mm/min.
5.2. COMBINING SRT AND SPT: STAINLESS STEEL (GRADE 316L)

Figure 5.13: Heatmap of all SPTT experiments’ force-displacement values, with the colour bar scaled from min to max of MAE %. Experiments are grouped by extraction method (normally extracted versus extracted from blank space of ring) and sorted by ascending rates of punch displacement rate. The ‘B’ represents the Batch of the hypothesis test (given in 5.2.2) and ‘Norm’ represents the normally extracted discs. Punch displacement rates are shown in parentheses in tick labels in mm/min.

The trilinear fitting method does not seem to work adequately on the 0.2% proof strength results. The $R_{p0.2}$ had a value of 513.89 MPa for the uniaxial tests, of 487.98 MPa for the normally extracted rings, and 439.49 MPa for the rings with discs extracted from them. Meanwhile, the trilinear fitting provided a value of 273.87 MPa for the normally extracted discs and a value of 237.86 MPa for discs extracted from the ring’s blank space. As mentioned previously, this discrepancy cannot be attributed immediately to the extraction method of the discs extracted normally, as shown by the heatmaps above. Fig. 5.14 also showcases the results of SPTT (normally extracted discs and discs extracted from within the rings) overlaid on top of each other. A similar trend is observed for all discs tested. The dip in the graph at around 400-450 N is also consistent for all discs, regardless of the extraction method.
Consequently, a quad-linear fit was evaluated, and it was found that it yields a closer match with the expected results (of the SRTT and uniaxial tests), suggesting that the punch displacement recordings obtained were a significant contributor to this issue, along with the rig compliance issues. As mentioned previously, another reason for the mismatch in the results obtained via the trilinear fit could possibly be the slack in the system due to the long loading chain and the inversion of the SPT rig.

An example image of this fitting is shown in fig. 5.15. A disadvantage of this fitting technique (and rig issues) is that the ultimate tensile strength cannot be obtained, since the conventional relationships do not work for this rig. The conventional relationships for the UTS (Method 1 and Method2) are dependent on the maximum force and the force at 0.645 mm of punch displacement, and there is no alternative method to obtain them in the current SPT rig. Table 5.4 and 5.5 show the results from the quad-linear fitting on the parent (normally extracted) discs and the hypothesis discs, respectively. With this quad-linear fitting, the mean value of the proof strength for the discs extracted normally is 510.61 MPa and that of the discs extracted from the rings (for the ring-disc hypothesis) is 459.73 MPa. The difference between these two sets of values is approximately 10.51%. Previously, with the trilinear fitting values, this difference was 7.61% approximately, with the mean value from the normally extracted discs again being the higher one (see previously mentioned table 5.2 and 5.3 for these values).
5.2. COMBINING SRT AND SPT: STAINLESS STEEL (GRADE 316L)

Figure 5.15: Example of quad-linear fitting.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Displacement rate (mm/min)</th>
<th>$f_b$ (N)</th>
<th>$R_{p_{0.2}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>245.66</td>
<td>505.11</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>250.17</td>
<td>519.74</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>270.14</td>
<td>561.33</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>242.95</td>
<td>505.11</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>232.84</td>
<td>484.07</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>245.63</td>
<td>488.77</td>
</tr>
</tbody>
</table>

Mean: 510.61
Std. Dev.: 30.48

Table 5.4: Quad-linear fit: Parent discs, SPTT. $f_b$ represents the fitting force obtained.
Table 5.5: Quad-linear fit: Hypothesis discs, SPTT. $f_b$ represents the fitting force obtained.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Displacement rate (mm/min)</th>
<th>$f_b$ (N)</th>
<th>$R_{p0.2}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1D1</td>
<td>0.1</td>
<td>222.21</td>
<td>438.90</td>
</tr>
<tr>
<td>B1D2</td>
<td>0.6</td>
<td>221.55</td>
<td>458.61</td>
</tr>
<tr>
<td>B2D1</td>
<td>0.2</td>
<td>223.12</td>
<td>452.57</td>
</tr>
<tr>
<td>B2D2</td>
<td>0.9</td>
<td>248.67</td>
<td>496.10</td>
</tr>
<tr>
<td>B3D1</td>
<td>0.3</td>
<td>242.31</td>
<td>483.42</td>
</tr>
<tr>
<td>B3D2</td>
<td>1.0</td>
<td>249.91</td>
<td>498.57</td>
</tr>
<tr>
<td>B4D1</td>
<td>0.4</td>
<td>240.72</td>
<td>478.80</td>
</tr>
<tr>
<td>B4D2</td>
<td>0.8</td>
<td>230.75</td>
<td>458.73</td>
</tr>
<tr>
<td>B5D1</td>
<td>0.5</td>
<td>234.83</td>
<td>457.64</td>
</tr>
<tr>
<td>B5D2</td>
<td>0.7</td>
<td>207.82</td>
<td>423.18</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>459.73</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td></td>
<td>26.08</td>
</tr>
</tbody>
</table>

Interpretation of Results

Largely, the trends of the tests seem to be the same. It is likely that with proper design and manufacturing of the rig components and extensive testing, there could be an even closer match in results than what has been demonstrated here. With a difference of 10.51% in the values between the two in the quad-linear fitting, the values are relatively close, but there is still a significant difference between them. It should also be noted that the difference in the trilinear fitting previously was 7.61%. Given that the difference between both the sets of discs persists, this can strongly be attributed to the design of the SPT rig and the manner in which the long loading chain of this setup affects these results. Additionally, no adverse comment can be made about the discs’ extraction (top or bottom) being significant, since no clear trend has emerged here that would indicate that one set of disc performs different to the other for the same ring. This comment is bolstered by the heatmaps shown in fig. 5.12 and 5.13 in the previous section, which also suggested a good match between the tests objectively, with an R2 score of almost 0.95 and an MAE % of less than 5%. Thus, the results from both sets of discs do match with each other, but it is the fitting methodology, due to the rig issues, which hinders

Thus, although the quad-linear fit does not work perfectly for the discs, it yields much closer results to the SRTT and uniaxial tests (see 5.2.4 for comparison). Consequently, the quad-linear fitting methodology has been used for all the SPTT results in the next sections where SPTT is performed.

Considering the limited scope of this study, this suggests there may be a promising avenue to explore in this direction. However, it must also be acknowledged here that the rig compliance issues do pose an issue for this test matrix and for future small punch testing as well. A quad-linear fit is used in the next sections where SPTT is performed, but the fact that a quad-linear fit must be used instead of a trilinear fit remains a weak point in this hypothesis.
5.2.3 Material Information

The material used for this hypothesis was the same that was used for the uniaxial tests and the large matrix of SRTT in the previous chapter (see 2.4.1 for material information). However, this hypothesis also allows for grain size measurement from the residual space between the ring and the disc. Example micrographs are shown in fig. 5.16 and fig. 5.17 for the master extraction sample and the residual space, respectively. The grain sizes were observed to be largely similar for both the materials, with all the 5 master samples having a grain size of approximately 57 µm (±8 µm) and the 5 residual samples having a grain size of approximately 52 µm (±4 µm). Additionally, Vickers hardness testing was also performed on these samples. Recall that the Vickers hardness (Hv 0.3) for the original material was approximately 225. For the parent material here, this was found to be approximately 260.4 (±14) while for the residual space it was found to be approximately 216.6 (±4).

Figure 5.16: Example micrograph of the material from which the whole master sample (ring and disc both) are extracted.
Figure 5.17: Example micrograph of the residual between the ring and the disc.
5.2.4 Commentary on Test Combination

A comparison of all the key values and the tests they are obtained from is presented below in Table 5.6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Elastic modulus (GPa)</th>
<th>Yield Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial sub-size</td>
<td>208.21</td>
<td>513.89</td>
</tr>
<tr>
<td>SRTT (normal)</td>
<td>200.37</td>
<td>487.98</td>
</tr>
<tr>
<td>SRTT (hypothesis)</td>
<td>202.26</td>
<td>439.49</td>
</tr>
<tr>
<td>SPTT (normal)</td>
<td>—</td>
<td>510.61</td>
</tr>
<tr>
<td>SPTT (hypothesis)</td>
<td>—</td>
<td>459.73</td>
</tr>
</tbody>
</table>

Table 5.6: Comparison of values obtained from all tests.

One astute commentary on these results is that while these results don’t match with the uniaxial tests perfectly, they do tend to match well with each other.

From the results, it is evident that the SRTT performs much better and is much more consistent in its results as compared to the SPTT. Although, the remark against latter could also be attributed to the rig compliance issues. While the SRTT results do differ with each other, it should be noted that these tests differed with each other in the inter-test comparison in the normally machined rings as well (chapter 3). Additionally, the difference here is smaller than the maximum difference observed in that comparison. Thus, this ring combination holds promise and should be considered further.

The findings suggest that the proposed approach of combining the tests is a promising avenue for future investigations. Notably, the observed discrepancy between the test results may be attributed to the limitations of the employed testing methodologies. Further investigation through extensive testing could potentially address these limitations. However, due to external factors, particularly with regard to the scheduling and availability of resources required for conducting the SPTT, a thorough analysis could not be conducted.

There clearly exists a scope for improvement and much is left to be desired, especially on the results of the small punch tests, but that does not take away from the fact that this avenue needs to be explored further. The grain sizes of the parent material and the residual space were also found to be in close agreement, although the hardness values for the former (260.4 Hv) were not as close to the manufacturer’s reference value (225 Hv). The residual space’s hardness value (216.6 Hv), meanwhile, deviated significantly with respect to the parent material but was in closer agreement to the manufacturer’s reference value. Regardless of their agreement, it can be said that hardness testing can potentially be performed on the residual space of the ring-disc extraction.

To address this research hypothesis further, this hypothesis was augmented by a creep test on SRT equipment and the extracted discs were used for tensile tests on Nimonic-75, a standard reference material [156]. This is demonstrated in the next section (5.3.1).
5.3 Combining SRT and SPT: Nimonic-75

5.3.1 Small Ring Creep test

Test Background

A standard benchmark test on Nimonic-75 (N75) was performed. This standard test requires loading the rings for 600 hours at 600°C at 160 MPa load \(^{156}\). The required conversion formulas for the ring are calculated by the formula given by Hyde TH et al. \(^{72,157}\) and Hyde CJ et al. \(^{76}\):

\[
\sigma_{ref} = \eta \left( \frac{PR}{b_0d^2} \right) \quad (5.3)
\]

\[
\dot{\varepsilon}(\sigma_{ref}) = \frac{d}{4\beta R^2} \Delta \quad (5.4)
\]

where, \(\eta\) and \(\beta\) are conversion factors, \(P\) is the applied load, \(b_0\) is the ring depth, \(d\) is the ring width, \(\sigma_{ref}\) is the reference stress required for the creep test, \(R\) is the radius of the circular ring, \(\dot{\varepsilon}(\sigma_{ref})\) is the strain rate at the corresponding stress, and \(\Delta\) is the displacement rate.

With the help of the constants from the literature by Hyde CJ et al. \(^{76}\), for a load of 160 MPa, this required stress is translated to an equivalent load of 142.988 N for Ring P and a load of 139.858 N for Ring H.

Test Results

The rings were loaded on a Walter+Bai AG LFMZ-20kN machine. The raw displacement-time graphs are shown in fig. 5.18 and the temperature logs are shown in fig. 5.19. It can be seen from the displacement profiles that the trends match initially for both the rings, before diverging later in the test after approximately 2.0 mm of crosshead displacement. Although, the difference between the both remains largely constant. The x-axis of time is in seconds, with a magnitude of 1e6. The temperature logs indicate how the test temperatures varied throughout the test. It can be seen that the hypothesis sample had its temperature slightly more elevated than the parent sample (by around 2.5°C). Efforts were made to make sure this does not happen, as indicated by the sharp dips and rises when the furnace temperature was adjusted when this was noted. This temperature differential could be a contributing factor as to why the H ring displaces more than the P ring.
5.3. COMBINING SRT AND SPT: NIMONIC-75

Figure 5.18: Displacement profiles for the creep tests with respect to time (x-axis of graph).

Figure 5.19: Temperature logs for the creep test.

The displacement-time graph can be used to calculate the displacement rate. The
displacement rate is obtained by calculating the displacement observed over the time intervals and obtaining the displacement change per second. This can subsequently be used to calculate the strain rate and the strain as well with the help of equation 5.4. The results for this are shown in fig. 5.20 and 5.21. It can be seen that both the profiles follow a similar trend. A sharp decrease in strain rate and displacement rate is observed after initial loading, after which they remain largely constant.

Figure 5.20: Variation of displacement rate (y-axis of graph) with respect to time (x-axis of graph).
5.3. COMBINING SRT AND SPT: NIMONIC-75

Figure 5.21: Variation of strain rate (y-axis of graph) with respect to time (x-axis of graph).

This strain rate can now be used to calculate the strain vs time graph, which is shown in fig. 5.22. This profile is largely similar to the displacement vs time profile shown previously in fig. 5.18. Additionally, fig. 5.23 showcases the logarithm of the strain (y-axis) with respect to the logarithm of time (x-axis). In the logarithm graph, it can be seen that there is an initial mismatch in both the samples, which starts converging after around 400k seconds (or around 112 hours). This convergence then holds for the rest of the test for both the samples.
CHAPTER 5. COMBINING SRT AND SPT

Figure 5.22: Variation of strain (y-axis of graph) with respect to time (x-axis of graph).

Figure 5.23: Variation of natural logarithm of strain (y-axis of graph) with respect to logarithm of time (x-axis of graph).
5.3. **COMBINING SRT AND SPT: NIMONIC-75**

Interpretation of Results

The European Commission has a certification released for N75. While it is not possible to determine the 2% and 4% strain for this type of test due to its biaxial loading nature, it is possible to benchmark this test against the minimum creep strain rate at 400 h mark.

The certification standard \[158\] requires a creep deformation rate of $72e-6 \text{ h}^{-1}$ with an uncertainty limit of $5e-6 \text{ h}^{-1}$. This translates to $2e-8 \text{ s}^{-1}$ with an uncertainty limit of $0.139e-8 \text{ s}^{-1}$.

Unfortunately, for both the tests, this is not matched. For the ring H, the creep rate at that time is $9.232e-8 \text{ s}^{-1}$ and for the ring P it is $9.496e-8 \text{ s}^{-1}$. However, it should be noted that the effective load for the test does tend to change, as pointed out by Hyde et al. \[76\] due to the changing geometry of the ring. This should be taken into account as a limitation of the test while interpreting these results.

Additionally, the deformation behaviour usually obtained by these tests is also of a different nature entirely. Consider the ideal deformation response graph for this material from a uniaxial tensile creep test, shown in fig. 5.24 and the deformation behaviour obtained above in fig. 5.22. It can be seen that the nature of both the graphs is different. While the benchmark test has a largely horizontal primary creep stage, it is quite the opposite for the SRCT. The same goes for the secondary stage, as a linearly increasing region is obtained in fig. 5.24 but the rise in the SRCT is much slower.

Another important variable to consider in interpreting these results is the influence of grain size and morphology on creep behaviour. Although there is grain morphology data provided, there is a possibility it has a role in the observed discrepancies. Grain size and morphology can significantly affect dislocation motion and consequently, the creep rate. Different resistance levels to dislocation movement can manifest due to grain boundaries, consequently affecting the overall creep characteristics. The unique combination of grain size, morphology, and, most importantly, the biaxial nature of the Small Ring Creep Test may all contribute to the variation from the established benchmarks.
Figure 5.24: Creep curve from one of the certification tests in the benchmark standard. Image adapted from [159].
Thus, while these results do not seem to match with the uniaxial creep test results, it should be noted that they do tend to match with each other. This mismatch, thus, is a remark on the testing methodology and not the hypothesis of ring-disc combination.

The discs extracted, meanwhile, are also tested with SPTT in the next section (5.3.2).

### 5.3.2 Small Punch Tensile Test

#### Test Results

The same quad-linear fitting as adopted for SS316L discs was used here, since the same rig (with rig compliance issues) was available for testing.

Two discs were extracted from each ring. These were each tested at a punch displacement rate of 0.3 mm/min (top disc, D1) and 0.6 mm/min (bottom disc, D2), with ‘H’ again being the label for the discs that were extracted from within the blank space of a ring that was in SRCT and ‘P’ referring to the disc extracted via normal EDM procedures.

The combined results for this are shown in fig. 5.25 where it can be seen that tests match extremely well with each other. There are also not many oscillations initially, as there were for the SS316L discs. The dip in the graph also appears much later, at over 750 N.

![All N-75 SPTT graphs](image)

**Figure 5.25: All SPTT results on N75.**

The required values for comparison are found in the same as before with the quad-linear fitting method. An illustrative image for this testing programme is shown in fig. 5.26.
CHAPTER 5. COMBINING SRT AND SPT

Figure 5.26: SPTT quad-linear fitting results on B1 Disc for N75 at 0.3 mm/min.

The values obtained are listed in Table 5.7. It can be seen in the table that the bottom discs tend to have much higher values than the top discs. This is different to the results observed for the SS316L discs, where the disc’s location dependency was found to be of no significance.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Displacement rate (mm/min)</th>
<th>$f_b$ (N)</th>
<th>Yield strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-D1</td>
<td>0.3</td>
<td>259.28</td>
<td>492.84</td>
</tr>
<tr>
<td>H-D1</td>
<td>0.3</td>
<td>240.77</td>
<td>452.23</td>
</tr>
<tr>
<td>P-D2</td>
<td>0.6</td>
<td>291.12</td>
<td>546.79</td>
</tr>
<tr>
<td>H-D2</td>
<td>0.6</td>
<td>280.35</td>
<td>537.15</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>507.25</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td></td>
<td>38.45</td>
</tr>
</tbody>
</table>

Table 5.7: Quad-linear fit values for N75 SPTT.

Interpretation of Results

The SPTT showed that the bottom discs were more biased towards predicting higher values than the top discs. Between each disc (top or bottom), there seems to be a close match with the parent disc and the hypothesis disc, that is, the bottom discs of parent and hypothesis compare well with each other and so do the top discs. However, this trend is closer for the bottom discs than it is for the top discs. With a quad-linear fit giving a difference of 19 N for the top discs, the bottom discs are much closer with a difference of 9 N.
5.3.3 Commentary on Test Combination

Based on the results of the SRCT and the SPTT, it can be seen that the SRCT tests have the minimum strain rates in the same order of magnitude as the benchmark expects it to be. However, it is also showcased that the strain vs time graphs do not exhibit the same behaviour as the benchmark expects it to. Given that both the rings match with each other in terms of trends and minimum strain rate, this is a remark on the testing itself instead of the hypothesis. If anything, this bolsters the hypothesis and puts more impetus on refining the SRCT for proper data interpretation and conversion.

For the SPTT, as mentioned before, the bottom discs perform much better than the top discs. This was not observed for the SS316L discs and should thus be noted here. Unfortunately, due to the lack of material, a uniaxial specimen could not be machined for a comparative result.

This testing combination, however, does go on to show that two types of tests (creep and tensile) can be performed from the same set of material. While all these materials were extracted from materials that were ‘as-received’ from the manufacturer, there is a benefit to be found whether this testing hypothesis extends to welds or other unknown materials as well.

For this purpose, an ex-service plate of unknown material history was used to test this hypothesis. This is now described in the following sections, starting from section 5.4.1.
5.4 Combining SRT and SPT: Plate with Weld Deposits

5.4.1 Impetus and Methodology

The idea behind this thesis of combining 2 different tests from a single sample was lastly tested on the plate with different materials and unknown material history. 3 samples have been extracted from the weld and the bottom parent material region in the form of squares: A, B, and C. The original material was AISI Stainless Steel 316 Grade. The material deposited on top of the plate as a weld was deposited via hardfacing, with the material being 75Ni13.5Cr2.7B-3.5Si.

The plate is shown in fig. 5.27. The ‘C’ region square is above the ‘B’ region square in the image. Discs for SPTT have been extracted solely from the blank space of the ring due to material limitations. For completeness, it must be noted that the circular holes shown in the image (fig. 5.27) were originally extracted to perform creep tests via the SPT, but this plan did not come to fruition and these samples were not tested. As mentioned previously, the squares A, B, and C were machined through the disc till the bottom of the plate. This provides with the parent material samples as well.

![Figure 5.27: Visualising the tested plate with welds on top.](image)

The extraction of specimens from the ring’s blank space is similar to the prior sections (see fig. 5.9 for the visualisation shown in previous sections). Discs extracted from the same side of the speckle pattern of the ring for SRTT are the D1 discs, while the discs below that are the D2 discs. Samples are extracted from the top (where the weld is
deposited) and the bottom (the plate’s parent material) as well. All samples are machined for all the three A, B, and C regions and the top and bottom sides of those regions as well.

The results from both the SRTT and SPTT are discussed next.

5.4.2 Small Ring Tensile test

Test Background

The following pin displacement rates have been tested in SRT for both the top (weld region) and the bottom (parent material) extracted specimens. No difficulties in the test were expected because of prior experience with SRTT, as evidenced by chapter 3.

- Region A: 0.3 mm/min
- Region B: 0.5 mm/min
- Region C: 0.7 mm/min

Test Results

A significant difference in test results was observed in the rings tested from the weld (top region) in comparison to the rings extracted from the parent material (bottom region).

The experimental force-displacement curves obtained from these tests are shown in fig. 5.28 and 5.29. It can be seen that a closely matched set of results is obtained for the rings extracted from the bottom of the plate. The only difference is that the ring from region C was allowed to go to failure. Given the similar nature of this graph to the ones obtained on 316L, it was determined that there may be some merit in comparing these tests to the ones obtained from those eventually. Rings extracted from the bottom region seem to encounter failure at over 2600 N of load.

The second figure showcases the results from the top parts of the regions. The behaviour is quite brittle and failure is reached much sooner than the rings from the bottom region. Compared to over 8 mm of pin displacement observed for the bottom regions, the rings from the top region also fail before 1 mm of pin displacement is achieved. The rings extracted from the weld also have a largely inconsistent failure threshold. All the rings extracted from the weld region failed, and their failure loads are enumerated below:

1. Region A Top Ring: 402.52 N
2. Region B Top Ring: 358.16 N
3. Region C Top Ring: 284.43 N
Figure 5.28: SRTT on specimens extracted from the plate’s bottom (parent) side.

Figure 5.29: SRTT on specimens extracted from the plate’s top (weld) side.

The interpretation of these results is discussed next.
5.4. COMBINING SRT AND SPT: PLATE WITH WELD DEPOSITS

Interpretation of Results

To analyse whether the RFR model trained on SS316L would work here, it was used again to predict the force outputs and was compared with the actual force outputs obtained. This is shown in fig. 5.30. It can be seen that the predictions are quite off due to the differences in the elasticity regions. The inflection point from the RFR model is at around 200 N while that from the disc C (bottom) is at around 100 N. This is at around 200 N. A likely reason for this is that the elastic properties of both the materials are different. This goes on to show how hard a generic model can be to train and the need for materials to be tested.

![Force Predictions from RFR](image)

**Figure 5.30:** SRTT on specimens from location region compared to the RFR model trained on SS316L.

Thus, the empirical relationships derived in chapter 3 were used to find the yield strength and the elastic modulus. The results for this are shown in table 5.8. An example image for the bilinear fitting is also shown in fig. 5.31. The table shows that there is a variance in the inflection point for all the rings, and consequently, the yield strength as well. The elastic modulus for the region A plate is much higher than the rest of the rings at 208.89 GPa as well. The yield strengths, meanwhile, range from 226.76 MPa for the disc C (bottom) to 293.07 MPa for the disc B. The former’s bilinear fitting is also shown in the image (5.31). Given that this material was not ‘as-received’, these results and their deviations are not unreasonable.
### Table 5.8: Yield strength and elastic modulus values for rings extracted from bottom of tested plate.

<table>
<thead>
<tr>
<th>Ring</th>
<th>Rate (mm/min)</th>
<th>Force (N) (bilinear fit)</th>
<th>Yield Strength (MPa) (force x 2.237)</th>
<th>Elastic Modulus (GPa) (analytical solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bot-A</td>
<td>0.3</td>
<td>119.87</td>
<td>268.15</td>
<td>208.89</td>
</tr>
<tr>
<td>Bot-B</td>
<td>0.5</td>
<td>131.01</td>
<td>293.07</td>
<td>200.01</td>
</tr>
<tr>
<td>Bot-C</td>
<td>0.7</td>
<td>101.37</td>
<td>226.76</td>
<td>200.56</td>
</tr>
</tbody>
</table>

Figure 5.31: SRTT bilinear fit on inflection point for the tested plate.

Unfortunately, given the extremely brittle nature of the rings extracted from the top of the plate (weld material), it was not possible to run the same analyses on those rings. This is because no inflection point was observed for these rings and the conversion formulas (derived in section 3.6) assume the presence of an inflection point. The absence of this could potentially be attributed to the brittleness of the material.

The next section (5.4.3) discusses the small punch tensile test results from the discs that were extracted from the blank spaces of these rings.

### 5.4.3 Small Punch Tensile Test

#### Test Background

The punch displacement rates for the discs extracted are listed in table 5.9.
5.4. COMBINING SRT AND SPT: PLATE WITH WELD DEPOSITS

<table>
<thead>
<tr>
<th>Region</th>
<th>Disc (top and bottom)</th>
<th>Punch Displacement Rate (mm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>1 (bottom only)</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>2 (bottom only)</td>
<td>0.7</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5.9: SPTT punch displacement rates for specimens extracted from welded plate.

Unfortunately, due to an unforeseen accident in the workshop, the discs extracted from the top weld region in ‘B’ were lost and could not be tested. However, the bottom discs from ‘B’ were retrieved and tested, but the results of this could not be compared with that of the discs from the top (weld) region.

Test Results

The tested rates have already been listed in table 5.9 previously. The graphs from these tests are shown in fig. 5.32 for the top weld region discs, and in 5.33 for the bottom region discs.

The behaviour of the discs from the top weld region is different from the bottom region. There is a much steeper rise observed in their force values, which changes to a relatively gradual rise after around 400 N. This is followed by an abrupt failure at around 700 N.

The bottom discs, meanwhile, can be seen to behave much more in line with the prior testing done for SS316L and N-75 in terms of the test’s force-displacement profile. Just like N-75, a dip in the graph is observed here as well at around 700 N for all the tests. However, this dip is not as pronounced. It should also be noted that a couple of discs had a machine error for the bottom discs from region A, which resulted in the logging to start prematurely. This can be seen in the graph as well.
Figure 5.32: SPTT on specimens extracted from the plate’s top (weld) side. Numbers in brackets correspond to the punch displacement rate.

Figure 5.33: SPTT on specimens extracted from the plate’s bottom (parent) side. Numbers in brackets correspond to the punch displacement rate.
It can be seen that there is consistency in the results obtained from each section, that is, results from the top (weld) section are largely similar to each other and the results from the bottom (parent) section are largely similar to each other as well.

The parameters are obtained from these sets of discs via the quad-linear fitting method again. A couple of example plots to illustrate this methodology for this set of experiments are shown in fig. 5.34 and 5.35. The first image showcases the fitting for the top disc, while the second images depicts the fitting for the bottom disc.

Figure 5.34: SPTT quad-linear fit on disc 1 from top of region C.
Figure 5.35: SPTT quad-linear fit on disc 1 from bottom of region B.

The quad-linear fitting results for these sets of experiments are delineated in table 5.10 and 5.11 for the top and bottom regions, respectively. The mean and standard deviations for these results are also computed and shown in table 5.12. For enhanced comparison, the results from the disc 1s and disc 2s are also listed separately.

<table>
<thead>
<tr>
<th>Disc</th>
<th>Displacement rate (mm/min)</th>
<th>$f_b$ (N)</th>
<th>$R_{p0.2}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-D1</td>
<td>0.2</td>
<td>188.99</td>
<td>362.09</td>
</tr>
<tr>
<td>A-D2</td>
<td>0.6</td>
<td>197.65</td>
<td>378.69</td>
</tr>
<tr>
<td>C-D1</td>
<td>0.3</td>
<td>197.07</td>
<td>362.93</td>
</tr>
<tr>
<td>C-D2</td>
<td>0.5</td>
<td>170.35</td>
<td>301.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>507.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev.</td>
<td></td>
<td>38.45</td>
</tr>
</tbody>
</table>

Table 5.10: Quad-linear fitting results on SPTT on discs extracted from top (weld) region.
5.4. COMBINING SRT AND SPT: PLATE WITH WELD DEPOSITS

<table>
<thead>
<tr>
<th>Disc</th>
<th>Displacement rate (mm/min)</th>
<th>$f_b$ (N)</th>
<th>$R_{p0.2}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-D1</td>
<td>0.2</td>
<td>262.82</td>
<td>513.78</td>
</tr>
<tr>
<td>A-D2</td>
<td>0.6</td>
<td>234.90</td>
<td>459.21</td>
</tr>
<tr>
<td>B-D1</td>
<td>0.4</td>
<td>254.23</td>
<td>433.53</td>
</tr>
<tr>
<td>B-D2</td>
<td>0.7</td>
<td>267.00</td>
<td>472.98</td>
</tr>
<tr>
<td>C-D1</td>
<td>0.3</td>
<td>232.68</td>
<td>428.51</td>
</tr>
<tr>
<td>C-D2</td>
<td>0.5</td>
<td>257.92</td>
<td>465.79</td>
</tr>
</tbody>
</table>

Table 5.11: Quad-linear fitting results on SPTT on discs extracted from bottom (parent) region.

<table>
<thead>
<tr>
<th>Disc</th>
<th>$f_b$ (N)</th>
<th>Yield strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Top</td>
<td>188.27, 11.34</td>
<td>351.12, 34.64</td>
</tr>
<tr>
<td>All Bot</td>
<td>250.43, 14.38</td>
<td>450.98, 30.02</td>
</tr>
<tr>
<td>All Bot D1</td>
<td>249.24, 15.92</td>
<td>458.94, 42.47</td>
</tr>
<tr>
<td>All Bot D2</td>
<td>253.27, 15.20</td>
<td>465.33, 6.87</td>
</tr>
</tbody>
</table>

Table 5.12: Mean and standard deviations from quad-linear fitting results on SPTT on discs extracted from weld plate.

The next section provides an interpretation of these values.

**Interpretation of Results**

It can be seen that the values for the top discs (from the weld region) are much lower than the ones from the bottom region. These findings are consistent with the observations from the small ring tests.

However, the yield strength obtained from the SPT discs is notably higher than the values predicted by the small ring tensile tests. In fact, the yield strength values obtained from the SPT discs in the bottom region are more than double those from the rings, which suggests unsatisfactory performance.

Another challenge encountered was the prediction of the samples extracted from the weld-plate. The samples tend to fail much quicker and unpredictably. The failure seems to be more brittle even visually, as seen in fig. 5.36 in contrast to the discs extracted from the bottom, as seen in fig. 5.37. The discs from the bottom region, in comparison, exhibited a much more ductile failure mode. This is shown by the whole circular region coming off once the disc fails, while the brittle failure for the top discs (fig. 5.36) shows minimal punch penetration in comparison.
Figure 5.36: Visualizing failure of discs extracted from top region.
5.4.4 Commentary on Test Combination

Overall, the results of the SRTT and SPTT tests on the tested plate demonstrate the potential of this method for real-world applications. However, further research is needed to develop this method into a more widely applicable and standardised technique, especially with better SPTT equipment.

Nonetheless, the study reveals promising aspects of miniaturised specimen testing, such as the good repeatability between specimens within each test programme and the successful implementation of tests without the need for uniaxial specimens.

A short commentary on all the results can now be found in the next section (5.5).
5.5 Overall summary of SRT and SPT combination

This chapter presented a comprehensive analysis of various tests aimed at assessing the feasibility of combing the SRT and the SPT. Preliminary results were also evaluated for a single specimen of Stainless Steel (Grade 316L). Although the results were not a perfect match for the SRT and SPT, they promised an avenue of materials testing worth exploring [160]. The mismatch was attributed to rig compliance issues, especially for the SPT [160], but this issue was not fully explored until this full study.

While the results for the small punch test on SS316L were not as closely matched as they were in all the other departments, there was still merit to be found in those tests. With the results from the trilinear and quad-linear fitting demonstrating errors of 10% or lesser, there is demonstrably more scope for improvement as compared to the small ring test, which matched very closely for the elastic modulus and was a good match for the proof strength as well.

As shown by the small punch tests on the N-75 discs, there is a possibility to find a close match between the discs extracted via normal EDM procedures and discs extracted from within the blank space of a ring. However, it should be noted that the disc 1 had a difference of 8.24% and the disc 2 had a difference of 1.76%.

The testing on the tested plate with weld deposits did not show the promise of using the small punch test as expected. In isolation, the SPTT outputs closely matched with one other for both the regions (weld and parent regions). However, as noted previously, these outputs did not match well with the small ring tensile test.

Although, given that the outputs matched well with each other in a testing regime, it suggests that there may be some merit in exploring this avenue further.

The small ring tensile tests, overall, have shown an excellent promise for the combination hypothesis regardless of the machining procedures. This remains true even for the rings extracted from the plate with different materials. The results are largely consistent and repeatable

However, the conversion relationships are still nascent, given the recency of this testing methodology.

Thus, it would be prudent to say that the hypothesis of the specimen combination holds merit, but requires much more testing on more materials before commercial adoption. These chapters, so far, have served as a strong starting point in this new research direction.

Given the amount of material savings that may be possible, and with the adoption of more tests in the small punch test (such as fatigue testing), it is strongly recommended that this research direction be explored extensively in the future.
Chapter 6

Discussion of All Results
6.1 Overview of Work Done

Before presenting the conclusions, a recap of the thesis is presented now, chapter by chapter. The following work was presented in the first chapter:

1. Commentary on the existing miniaturised-specimen test techniques.
2. Commentary on current state of the inverse analysis for parametric identification.
3. The need for more miniaturised-specimen test techniques.
4. The plan to combine the Small Ring Test (SRT) and the Small Punch Test (SPT).

The following work was presented in the second chapter:

1. The design and construction of the SRT and SPT rigs.
2. Capability for 3D DIC in restricted spaces for the SPT, with the assistance of partial mirrors.
3. The uniaxial tensile test results.
4. The analytical solution for the SRT and how the parameters differ from existing literature.

The following work was presented in the third chapter:

1. Small Ring Tensile Test (SRTT) programme on SS316L was introduced, with tests performed at varying pin displacement rates.
2. A data cleaning methodology was examined for the testing programme.
3. Rate dependency was analysed for the SRTT performed.
4. Inverse Finite Element Analysis (FEA) was performed for the SRTT to obtain the equivalent gauge section and the equivalent gauge length, which were subsequently used to obtain the stress-strain curves.
5. Converted SRTT values were contrasted with values obtained from existing conversion relationships.
6. New data-driven conversion relationships were devised.
7. Discussion on the need for a general-purpose regression model that produces the full stress-strain curves from the SRTT outputs.

The following work was presented in the fourth chapter:

1. Collection of Digital Image Correlation (DIC) images and their pre-processing from the SRTT matrix (performed in the previous chapter) was introduced.
2. The need for a custom extensometer for the SRTT was presented and how DIC would help achieve this goal.
3. DIC analysis was performed on an affixed plate on the SRT rig to analyse the y-displacement as a surrogate for the crosshead displacement.
4. DIC analysis was performed at 0 degree (horizontally extreme) points of the ring and the x and y-displacements were monitored and evaluated.

5. DIC analysis was performed at all 30 degree points of the ring (30° on either side of the horizontal axis) and the variation of x, y, and total displacements in that region was evaluated.

6. The feasibility of a custom SRT extensometer based on the DIC data.

The following work was presented in the fifth chapter:

1. The combination methodology of the SPT and SRT was introduced, along with the extraction procedures.

2. SPTT (Small Punch Tensile Test) was performed on discs extracted from the parent block of material and from material extracted from the blank space of rings.

3. Results were showcased for SRTT on rings which had discs extracted from them and were contrasted with SRTT from chapter 3.

4. Results were showcased for SPTT on discs extracted normally and discs extracted from within the rings for Stainless Steel Grade 316L (SS316L).

5. The rig compliance issues that were found and how a trilinear fit was unsuitable and a quad-linear fit was evaluated.

6. Small Ring Creep Test (SRCT) on Nimonic-75 (N75) was performed, with 1 ring undergoing normal machining procedures and the other ring having discs extracted from the blank space.

7. SPTT on the N75 discs that were extracted, which were in turn compared to SPTT performed on discs extracted via normal machining procedures.

8. Analysis of the weld region deposited on top of an SS316 plate, with SRTT and SPTT being combined for the weld region.

9. Samples were extracted from the bottom (parent) region of the plate with SRTT and SPTT performed on them as well.

10. All results were compared with each other.

Considering the summary of the work done, an overall discussion of all the results is presented next.
6.2 Discussion

This thesis set out to critically evaluate miniaturised-specimen testing techniques, specifically the small ring test and the small punch test, with more emphasis being given to the former due to its nascency. The overarching research question was to evaluate whether there is merit in combining the SRT with the SPT from a single specimen. Multiple research objectives were laid out to answer this research question.

The combination hypothesis can be understood better with the help of fig. 6.1 again. The uniaxial reference tests used in this study had the dimensions 100 x 40 x 6 mm. The master ring sample for this combined hypothesis would require a cylinder of height 2 mm and diameter 12 mm. For the 3 tests that can be extracted from this (1 ring, 2 discs), this would result in volumetric material savings of around 97.89%. This high percentage of savings in material is extremely useful when the amount of material available for testing is limited. Potentially, this combined methodology, once perfected, can help effectively analyse in-service components in industries (in the nuclear or aerospace industry, for instance) with minimal structural interference. Additionally, this methodology also helps in handling of radioactive material, given the low overall activity of the resultant specimens. The development of new alloys, which may be expensive to produce, could also be accelerated and be more cost-effective. Essentially, this combination elevates the benefits offered by miniaturised-specimen testing techniques by threefold. Thus, the need for this methodology is greatly underscored by the benefits it can provide.
Figure 6.1: Schematic of the hypothesis, showcasing the combination of the SRT and the SPT. All dimensions are in mm and representative only for visualisation purposes. Not to scale.
The first research objective concerned the small ring test’s reliability and repeatability, and was evaluated thoroughly in chapter 3 for the tensile tests. Currently, the SRTT relies on rigorous finite element analysis and inverse parametric identification (if no material data is available) for the conversion of the test output (force-displacement data) to conventional stress-strain data \[73 - 75\]. A fair question is why the SRTT was chosen and not the SRCT (Small Ring Creep Test). The reason for this was that the machine available (Instron 8862 UTM) had data logging issues and would unpredictably shut down when used for long periods of time. Additionally, the Walter+Bai AG LFMZ-20kN machine that was eventually used for SRCT on N75 was not available for extended periods of time. Thus, the SRCT could not have been used extensively and the SRTT was chosen for this purpose.

Addressing this research objective for the SRTT was judged to be a good first step to answer the research question, since the combination of the tests hinges on the efficacy of the SRT. With 48 tests performed at various pin displacement rates on SS316L, a wide range of data was collected and inter-test comparison was performed. This matrix of SRTT was cleaned with the help of ‘Convolution Smoothing’ technique to prepare the data for machine learning and inverse finite element analysis. The inter-test comparison via heatmaps showed how most tests had a close match with each other, with a mean MAE (Mean Absolute Error) deviation of 4.95%. Additionally, the mean R2 score was also found to be 0.95632. The visualisation of the R2 score in fig. 6.2 provides a better understanding of the test comparison. Not all tests compared well with each other, as the image suggests, with the tests with an R2 score of less than 0.9 being highlighted.
Figure 6.2: Heatmap of all SRTT experiments’ force-displacement values, with the colour bar active only for experimental comparisons that yield an R2 score of less than 0.9. Experiments are compared in ascending order of pin displacement rate, which is shown in tick labels in mm/min.
Some discrepancy is found for faster testing rates (between 1.15 mm/min to 1.8 mm/min) when contrasted with the slower testing rates (below 0.45 mm/min). This discrepancy between some tests was attributed to be an artefact of testing, given how this trend is not true for all the displacement rates and no consistent rate-dependent trend is observed. For instance, consider the 1.5 mm/min test, one of the faster displacement rates, compared to the 0.2 mm/min test, one of the slower displacement rates. The R2 score for this comparison is 0.97804 while the MAE is 2.77902%. The artefact of testing could possibly be related to the loading of the ring, especially the out-of-plane alignment of the ring when being set up for the test. This refers to the alignment along the z-axis, if the front face of the ring has x-axis in the horizontal and y-axis on the vertical. Unfortunately, this cannot be thoroughly resolved with the DIC images that have been captured, since 2D DIC is unable to resolve for the z-direction. Since this is a first of its kind study that evaluates the SRTT in such depth, this is only a hypothesis for the data discrepancy.

To evaluate these tests further, two machine learning regression models (Random Forest Regression and XGBoost) were also trained on 45 of the 48 tests (3 tests were hidden from the model for evaluation). These Random Forest Regression (RFR) and XGBoost (XGB) models were trained with appropriate hyperparameters. 5-fold cross-validation was performed on the hyperparameter space to choose the model’s hyperparameters. This 5-fold cross-validation ensured that the training data were split into 5 subsets (or ‘folds’) and each hyperparameter was tested five times, with one subset used as the validation set in each iteration [150]. This approach helps assess the model’s performance across different data subsets and aided in effective hyperparameter tuning. The RFR model performed well on the hidden test data, with the maximum MAE being 78.4 (or 3.1%) for the 1.1111 mm/min pin displacement rate SRTT.

The combined evaluation of the whole SRTT matrix via inter-test comparison and machine learning showcased that there is no significant rate dependency in the whole testing programme for SS316L. A good level of repeatability was found for this matrix of tests, which corroborates the conclusions drawn by Rouse et al. [75]. This data were also made open-source for future researchers [140].

Prior to this thesis, Rouse et al.’s study on the SRTT for Ti-6Al-4V was the only study that extensively evaluated this testing methodology [75], and this thesis furthers the knowledge domain. However, as mentioned previously, the SRTT hinges on FEA or inverse FEA (depending on availability of material data) to convert the test output to conventionally used stress-strain data. This study originally proposed a combination of the Differential Evolution (DE) and Nelder-Mead (NM) optimisation methods for inverse FEA, but eventually only the NM optimisation was used. This was deemed sufficient since Brown’s PhD thesis rigorously demonstrates the efficacy of this optimisation alone [125] and the pitfalls of the NM optimisation were avoided by proper initial seeding of values.

Inverse analysis is an extremely computationally expensive method of converting the test outputs to conventionally used outputs. Thus, there is a strong impetus to analyse conversion relationships. These conversion relationships add tangible value to the SRTT, since this allows for a more accessible form of testing that does not rely on rigorous computational modelling. To put things in perspective, doing inverse FEA with just NM optimisation on one test took approximately 10 days to finish. Thus, for all the 48 tests, it would take over a year to perform inverse analysis.

To establish new conversion relationships that directly help convert the force-displacement data to the stress-strain data, inverse analysis was necessary, since this provides a bench-
mark comparison. With the help of the NM optimisation, the inverse analysis was performed (using Ramberg-Osgood material model) and equivalent gauge area and equivalent gauge lengths were obtained for the SRTT. For the ease of the reader, the image is shown in again in fig. 6.3. The RMSE (Root Mean Squared Error) between the experimental SRTT data and the simulation data was 12.66, which equates to an RMSE of 1.05%. This is a highly satisfactory match for the inverse FEA.

![Comparison of optimized simulation and experiment](image)

**Figure 6.3:** Visualising the full optimised simulation’s force-displacement values against the experimental SRTT dataframe.

Presently, the data conversion utilises the equivalent gauge area and equivalent gauge lengths obtained from the optimised simulation. The equivalent gauge area is obtained by dividing the reaction force observed at the loading pin in the simulation by the stress experienced at the central element of the ring in the simulation. Similarly, the displacement of the loading pin in the simulation is divided by the strain experienced at the central element of the ring in the simulation to obtain the equivalent gauge length. These equivalent values are subsequently used on the experimental dataset, with the force values being divided by equivalent gauge area to obtain the stress values and the displacement values being divided by equivalent gauge length to obtain corresponding strain values. The conversion was applied to all the 48 tests and the stress-strain data was consequently obtained.

Although the results obtained from the inverse FEA shown are very promising, the procedure for conversion is extremely time-consuming. Thus, in service of the research question of the SRT and SPT combination, a direct relationship for the SRT would significantly increase the appeal of the SRT by making the test much more accessible.

The conversion relationship proposed by Ktari et al. [96] was the only direct conversion relationship that exists for the SRTT. Applying this relationship did not result in satisfactory values for the yield strength and elastic modulus. However, it should be noted that the converted SRTT data (via inverse FEA) does not result in satisfactory values for
the elastic modulus.

To address the issue of direct conversion relationships, especially for the elastic modulus, the analytical solution for the SRTT was critically evaluated (see [2.5] for the mathematical solution). Based on the work done by Rouse et al. [75] where the analytical solution is presented, a python script was developed (see [A.1] for the code) to obtain the force correction parameters and the elastic modulus for unknown materials. The force correction parameters are necessitated because the analytical solution assumes a 2D ring and these correction parameters account for the 3D effects.

A couple of notable observations were made from utilising this analytical solution:

- The force correction parameters obtained from this study differ from those obtained by Rouse et al. [75].
- The recommendation by Rouse et al. [75] to analyse the pin displacement up to 1.6 mm of displacement was not feasible for the SRTT performed in this study. The recommendation made by Kazakeviciute et al. [73, 74] for utilising the pin displacement up to 0.5 mm is more relevant here, since the linearity in the curve is lost after that point.

The mismatch with the correction parameters obtained from Rouse et al. [75] and the ones obtained in this study could likely be due to the fact that different pin displacements were used. Additionally, the materials used are also different in both the studies, thus suggesting that this analytical solution must be calibrated for different materials. It turns out that the analytical solution resulted in an excellent match for the elastic modulus for the whole SRTT matrix. With the reference uniaxial tensile test’s elastic modulus of 208.21 GPa, a mean elastic modulus of 200.37 GPa from the analytical solution provided the strongest match. Thus, it is recommended that the analytical solution be used for analysing the SRTT data. Moreover, using the analytical solution establishes a robust starting point for physics-driven machine learning models when more materials are tested. This would provide more insight into the SRTT and help bolster confidence in the SRTT.

For the yield strength, it was found that the inflection point in the force-displacement curves of the SRTT is a good starting point for establishing a conversion relationship. A multiplication factor of 2.237 was obtained; when multiplied with the force at the inflection point, this yields the corresponding yield strength values.

As mentioned previously, the SRTT is a nascent testing methodology. This critical evaluation of the SRTT showcases that this testing methodology has merit, and it is worthwhile to pursue the research question further. Were the results of the SRTT not satisfactory, it would have been conclusive to say that this combination (of SRT and SPT) is not worthwhile to pursue. Thus, this underscores the importance of the work done for the SRTT. Additionally, the work presented here is an important stepping stone for future researchers to derive a physics-driven machine learning model after a sufficiently diverse set of material data is collected. The merit for such a model is that this would help devise robust conversion relationships and would eventually negate the reliance on computationally expensive inverse FEA, thereby making this testing methodology much more accessible and quick to perform and analyse.

In efforts of making the SRTT an even more accessible form of testing, DIC (Digital Image Correlation) was used. All the tested rings were painted with a speckle pattern for DIC analysis. This was in service of the DIC-driven approach to analyse the deformations
on the ring’s surface, which could then be used to analyse potential sites on the ring’s surface for a physical extensometer. This was another research objective in service of the research question, and it aimed to uniquely leverage DIC to recommend a potential extensometer design in efforts of increasing the accessibility for the SRTT.

A physical extensometer would remove the need for computation for DIC and relying on the machine’s crosshead data for displacement. To evaluate any potential sites for the extensometer, a plate was affixed on the moving pin’s rig to track the total and y-displacement of the plate via DIC. The sites on the ring were the horizontally extreme points of the ring (also known as the 0° points) and the 30° points on either side of the axis of the ring. 45° points were not chosen because in some tests, these points were physically obscured by the SRT testing rig.

The most promising results were obtained for the y-displacement values at the 30° points of the ring. Linear regression on these points consistently suggested a multiplication factor of 2 for these y-displacement values. Multiplying these y-displacement values with this factor (shown in table 4.2) would return the plate DIC displacement values. This promising finding suggests that a claw-like extensometer that grips the ring at 30° points could potentially be used in the future. The findings of this study for a potential extensometer design and site are another step towards standardising the SRT overall. Thus, this serves to further the research question by addressing the issue of accessibility for the SRT even more, while also demonstrating successful efforts towards test standardisation.

While miniaturised-specimen testing techniques have been subject to criticism for their efficacy, continuous work over the decades has ensured that using small specimens could be pragmatic. The SPT is an example of such a testing technique. Ever since its proposal in 1981 by Manahan et al. [32], this test is now codified in the European Code of Practice [30, 31]. This forms the other half of the SPT and SRT combination hypothesis. Using the SPT is a pragmatic choice given the confidence in this testing technique worldwide that has been accumulated over years of experience. Thus, the analysis of the SPT is not as comprehensive in this study as was for the SRT.

To test the hypothesis of the combination of SRT and SPT, SS316L was used initially. Discs were extracted via normal EDM procedures on the parent block of material and tested. Discs were also extracted from the blank space of the ring (that was used for SRTT) and tested. The results of these sets of discs were compared with each other. A good match was found between both the results, however, the recommended fitting technique (trilinear fitting on the force-displacement curve) was found to be unsatisfactory. This was attributed to the rig’s performance issues in the SPT.

The machine on which the SPT was performed had access issues, which required that the SPT rig be loaded as a singular piece. As mentioned in 2.2.2, the designed SPT rig had to be loaded as a single piece since the testing machine did not have the manoeuvrability required to load the dies and the punch separately. This access issue to the machine was compounded by its ability to perform tests only in the ‘tensile’ mode, as opposed to the required ‘compression’ mode, which is needed for the punch to penetrate the disc. To address these issues, the SPT rig was completely inverted and loaded into the machine for testing. This loading is again visualised in fig. 6.5 which is aided by fig. 6.4. In both the images, it is notable how the punch is beneath the dies and the designed SPT rig is loaded as a single piece with the top SPT rig slotting into the driving screw-head and pulling the punch upward, thereby penetrating the disc. The standard SPT setup requires a 45° chamfer on the bottom die (that receives the disc) as the punch drives into the disc.
in a rate-controlled manner. Since this set-up was completely inverted, this resulted in
the punch being at the bottom of the whole setup and the die with the chamfer was at
the top. This is again aided by a cross-section visualisation in fig. 6.6. This non-standard
form of testing was likely to be the root of the rig’s performance issues.

Figure 6.4: SPT grip: Assembled but not loaded in the machine. The dies are labelled,
and the punch is loaded and labelled as well. The top SPT rig slots into the driving
screw-head that pulls the punch upward, thereby penetrating the disc. The mounts are
also labelled for more visualisation of the mechanism. Note the presence of the punch
beneath the dies, indicating the inversion of the rig.
Figure 6.5: SPT grip: Loading into the machine. It can be seen that the whole structure is supported by the mounts at the bottom rig. The SPT rig is loaded as a single unit into the testing machine.
An astute question which arises is whether the mismatch of the results from the trilinear fitting could be attributed to the hypothesis of the ring-disc testing. Given that the overall force-displacement profiles from these sets of tests, that is, profiles from the SPTT performed on discs extracted normally and from the discs extracted from the blank spaces, were largely similar, the failure of the trilinear fitting could not be attributed to the ring-disc hypothesis. This is evidenced by the overall comparison heatmaps that were
shown in 5.2.2 and are shown again below in fig. 6.7 and 6.8. A mean R2-score of 0.9488 (standard deviation of 0.0602) and a mean MAE % of 4.979% (standard deviation of 3.311 %) was found. The good R2-score (of almost 95% similarity) and the low MAE % suggest that this issue in the parameters could not be attributed to the hypothesis of ring-disc testing.

![R2 Score Heatmap](image)

Figure 6.7: Heatmap of all SPTT experiments’ force-displacement values, with the colour bar scaled from min to max of R2 score. Experiments are grouped by extraction method (normally extracted versus extracted from blank space of ring) and sorted by ascending rates of punch displacement rate. The ‘B’ represents the Batch of the hypothesis test (given in 5.2.2) and ‘Norm’ represents the normally extracted discs. Punch displacement rates are shown in parentheses in tick labels in mm/min.
Figure 6.8: Heatmap of all SPTT experiments’ force-displacement values, with the colour bar scaled from min to max of MAE %. Experiments are grouped by extraction method (normally extracted versus extracted from blank space of ring) and sorted by ascending rates of punch displacement rate. The ‘B’ represents the Batch of the hypothesis test (given in 5.2.2) and ‘Norm’ represents the normally extracted discs. Punch displacement rates are shown in parentheses in tick labels in mm/min.
Additionally, these rig compliance issues also hindered the development of a robust stereo (3D) DIC system for the SPT. The proposed 3D DIC system was an evolution of the system proposed by Vijayanand et al. [54]. The proposed system (shown again in fig. 6.9) made use of partial mirrors and cameras mounted on moving stages, with the stages being used to calibrate the cameras based on a fixed pattern on the die and subsequently the stages were moved to focus the cameras on the loaded sample. As mentioned in 2.3.2.

![Image of 3D DIC camera setup](image)

Figure 6.9: Full 3D DIC camera setup mounted on the stage, along with the partial mirror between the cameras to enable imaging in both the cameras. The furnace with the SPT rig is kept open for visualisation purposes.

Given these rig compliance issues, a quad-linear fit was evaluated in place of the recommended trilinear fit [31]. It is important to acknowledge that this is not conventional and goes against the recommendations of the standard. However, the results from the trilinear fit and the quad-linear fit match well with each other for discs extracted from within the ring and discs extracted by conventional machining procedures. Thus, this is a remark on the testing apparatuses and not on the hypothesis of specimen combination. Should the hypothesis have failed, the disc results should not have matched with each other.

For the rings used in SRTT, the results matched extremely well for the rings extracted via conventional procedures and for the rings that had discs extracted from their blank space. The mean MAE was found to be less than 2% for the rings, further augmenting the confidence in this SPT and SRT combination hypothesis.

Considering this point, and being cognizant of the pitfalls of the SPT rig’s issues, this hypothesis was extended further to Nimonic-75. Creep tests were performed on the rings and tensile tests were performed on the discs. N75 is a benchmark creep reference material, thus making it an obvious choice for the first small ring creep tests (SRCT) [156].

The SRCT’s strain versus time profile did not match well with that of the conventional uniaxial creep test. However, the MSR (minimum creep strain rate) from the rings was in the same order of magnitude as that of the uniaxial creep tests. For the ring extracted with conventional procedures and the ring which had discs extracted for SPT, the MSR was found to be 9.496e-8 s\(^{-1}\) and 0.232e-8 s\(^{-1}\), respectively. The expected value was 2e-8 s\(^{-1}\). However, this is again a remark on the testing methodology and not on the hypothesis, given the close match in results for both the tested rings. This further highlights the work
needed for the SRT. A quad-linear fit was again utilised for the discs, and an extremely close match was found for the discs that were compared.

Lastly, this hypothesis was extended to a plate of Stainless Steel (Grade 316) with weld deposits on top. The plate had unknown material processing history and only the material information was known. Master ring samples had been extracted from the top (weld) region and the bottom (parent) region. Results of the SRTT performed revealed a remarkable difference in the behaviour of the rings extracted from the weld region. The failure was largely inconsistent, and the rings were extremely brittle. However, the rings from the parent (bottom) region behaved similarly to the rings from SS316L. The yield strength from the rings ranged from 226.76 to 293.07 MPa. Meanwhile, the elastic modulus ranged from 200 to 208 GPa. Given that this parent material was not ‘as-received’ from a manufacturer, these results, and their deviations, are not unreasonable. The results do show that a full mechanical test can be performed with a limited amount of material.

The discs showed a similar behaviour to the rings in terms of brittleness in the discs extracted from the weld (top) region; and similar behaviour to discs made from SS316L for discs from the parent (bottom) region. The SPT and SRT results do not match with each other, suggesting unsatisfactory performance. However, given the lack of a comparison test, it could not be verified which values were correct. While the hypothesis of the ring-disc combination performed sufficiently well to warrant further research, it fell short when utilised on a plate of unknown material processing history with weld deposits on top.

Criticism is justified for the SPT’s results and its performance due to the rig’s compliance issues. However, given the results matching well with each other (for discs extracted normally and for discs extracted from within the ring) for SS316L and N75, there is an encouraging research avenue to explore for the ring-disc combination. Additionally, the rings also match extremely well with each other for tensile as well as creep tests.

This thesis has showcased robust and significant results for the small ring tensile test with a data-driven and physics-driven approach, which has been further augmented by the proposal of an extensometer for the same. Given the amount of varied testing theoretically possible with this hypothesis (tensile and creep for SRT; tensile, creep, and fatigue for SPT), not to mention the metallographic analysis from the residual space between the ring and the discs, it would be wise to cautiously conclude that this hypothesis was successful and a robust stepping stone has been established for future researchers to explore this research gap further.
Chapter 7

Conclusions and Future Work


7.1 Conclusions

The key findings of this study can be summarised as follows:

**First:** This study showcased how to not perform 3D DIC for the Small Punch Test (SPT) when the space is constricted. However, proper rig design and manufacturing could potentially solve this issue, as demonstrated by Vijayanand et al. [54].

**Second:** This study provided a thorough critique of the analytical solution of the small ring tensile test (SRTT). The relevant parameters and methodology were also thoroughly explained. Additionally, force correction parameters were also found for SS316L. Notably, these differ from the parameters derived by Rouse et al. [75].

**Third:** This study analysed the rate dependency of SS316L on the SRTT. 48 tests were performed at various pin displacement rates and no significant rate dependency was found. Inter-test comparison was used to evaluate this, in conjunction with regression models which made use of Random Forests and XGBoost.

**Fourth:** Inverse FEA was performed to convert all the SRTT force-displacement data to corresponding stress-strain data. This inverse analysis was performed with the help of optimisation techniques (Nelder Mead). This data was used in conjunction with existing conversion relationships (by Ktari et al. [96]) to evaluate the SRTT. It was found that the SRTT constantly under-predicts the yield strength and elastic modulus from existing conversion relationships. Thus, a new physics-driven approach was utilised to obtain new conversion relationships. These factors allow the obtaining of the elastic modulus and yield strength directly from the force-displacement data.

**Fifth:** A potential extensometer design for the Small Ring test (SRT) was evaluated. This was done with the help of digital image correlation (DIC) data as a tool. Potential extensometer sites were explored based on this image data, and it was found that a claw-like extensometer that grabs the ring at 30° on either side of the horizontal axis should be considered. This extensometer would track the y-displacement of the ring throughout the test. An extensometer would make the SRT data more reliable, since the current practices rely on the crosshead displacement for evaluation.

**Sixth:** A combination of the SRT and the SPT was evaluated, with the discs for the SPT extracted from the blank space of the ring for the SRT. Promising results were found for the SPT. However, the SPT data did not match well with the uniaxial data, regardless of the location of extraction. This was attributed to rig compliance issues. Although the data for the SPT did not match well with the uniaxial data, a quad-linear fitting was evaluated (as opposed to the trilinear fit recommended by the standards [30, 31]) and this provided a good match with the uniaxial test results. Additionally, the discs extracted from within the ring (that is, for the combination hypothesis) had comparable results with the discs extracted normally. The results not matching well with the uniaxial test via a trilinear fit is a commentary on the rig and not on the hypothesis.

This combination hypothesis was extended to a creep test on the SRT and the blank space discs undergoing a tensile test on Nimonic-75. The small ring creep test results matched well with each other, and so did the SPT results.
Lastly, this combination hypothesis was also extended to a plate with hardfacing weld deposited on top of it. Three samples were extracted at three different locations. The SRT could not reliably inform the results for the weld section due to its brittle nature, but the results for the parent material were promising. Discs were also extracted from the blank space of these rings. While the disc results matched well with each other, the yield strength predicted by these discs did not match with the results from the small ring test.

Thus, it could be concluded that while the ring-disc combination did not work all the time, there is a promising potential to explore this avenue further given the that the hypothesis works well for the ring-disc extracted specimen and the normally extracted specimen. A strong benefit of exploring this avenue further is that this combination would allow for 3 tests to be performed on a very small amount of material (227 mm$^3$). Three equivalent uniaxial tensile tests (same as the ones used in this study, of size 100 x 12 x 3 mm), would need a material volume of 3600 x 3 = 10,800 mm$^3$. The potential material savings from this combination are apparent, providing a strong impetus to research this further.
7.2 Recommended future research

This study has provided a solid foundation for future researchers to analyse the small ring test and the small punch test in combination with one another. This study has also shown the promise that lies in the materials savings that could be obtained, should this combination hypothesis be successfully tested further. A few recommendations from this study are:

**First**: Similar 3D DIC setup as proposed in this study should be performed for restricted spaces. Robust rig design and manufacturing should be taken care of, as was done by Vijayanand et al. This could potentially provide with the capabilities to do 3D DIC on small punch creep tests.

**Second**: A general-purpose regression model should be trained after performing the SRTT on different materials. This would provide with more insights in the SRTT, and potentially help discover new conversion relationships as well.

**Third**: An extensometer should be constructed to physically test the proposed extensometer. This claw-like extensometer would make the SRTT much more accessible and easier to perform, as compared to a technique like DIC.

**Fourth**: The hypothesis of the ring-disc combination should be tested robustly with an SPT rig that has proper rig compliance. This would allow for the hypothesis to be extensively tested on different materials, while also comparing with the uniaxial tests.

**Fifth**: Different test types should be tested for this hypothesis of ring-disc combination. That is, since both the tests should theoretically be able to perform creep, tensile, and fatigue (only on SPT) tests, the combination of these 3 tests should be evaluated.
Bibliography


[43] Bin Yang, Wen-Qi Sun, Wen-Chun Jiang, Ming-Lei Wang, Ming-Chao Li, and Jing-Kai Chen. Comparative study of the tensile properties of a 1.25 Cr-0.5 Mo steel characterized by the miniature specimen and the standard specimen. *International Journal of Pressure Vessels and Piping*, 177:103990, 2019.


Appendix A

Relevant Code
A.1 Analytical Solution

Full analytical solution script for the analytical solution discussed in chapter 2 (section 2.5) is given below. Python 3.10 was used.

```python
import math
import time
import sympy as sym
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.integrate as spi
import sympy.utilities.lambdify as lamb

# Calculate second moment of area

def calculate_dimensions(a, b, t):
    
    Calculate the second moment of area of the ring from the given dimensions.

    Args:
    a (float): The horizontal (minor) axis of ring (internal)
    b (float): The vertical (major) axis of ring (internal)

    Returns:
    float: The second moment of area of the ring.

    
    b1 = b + t
    a1 = a + t
    I = math.pi * (b1 * a1 ** 3 / 4 - b * a ** 3 / 4)
    return I

# Calculate the integrals required for the ring analysis from the given ring dimensions.

def calculate_integrals(a, b):
    
    Calculate the integrals required for the ring analysis from the given ring dimensions.

    Args:
    a (float): The horizontal (major) axis of the ring.
    b (float): The vertical (minor) axis of the ring.

    Returns:
```

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9 float items: all integrand results of $A1$, $A2$, $A3$, $A4$, $A5$, $A6$, $B1$, $B2$, $B3$.

theta = sym.symbols('theta')  # Symbolic theta

# Parameters defined in the solution. Do not change!
Rhat = a*b*sym.sqrt(1/(a*a/2) + (b*b/2) + ((a*a - b*b)*sym.cos(2*theta))/2)
gamma = sym.sqrt(b*b - 2*b*Rhat*sym.cos(theta) + Rhat**2)
kappa = sym.atan((1/sym.sin(theta))*(b/Rhat - sym.cos(theta))
phi = sym.atan((-1*b*(b/(a*a))*sym.tan(theta))

# Integration limits
upper_limit = math.pi/2
lower_limit = 0

# Integral expressions
a1_exp = Rhat
a2_exp = Rhat*gamma*gamma*sym.cos(kappa)*sym.cos(kappa)
a3_exp = Rhat*gamma*gamma*sym.sin(kappa)*sym.sin(kappa)
a4_exp = Rhat*gamma*gamma*sym.sin(2*gamma)
a5_exp = 2*Rhat*gamma*sym.cos(gamma)
a6_exp = 2*Rhat*gamma*sym.sin(gamma)
b1_exp = Rhat*sym.sin(phi)*sym.sin(phi)
b2_exp = Rhat*sym.cos(phi)*sym.cos(phi)
b3_exp = Rhat*sym.sin(2*phi)

# Convert each symbolic expression to a numerical function and use quad for integration
A1 = spi.quad(lamb(theta, a1_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
A2 = spi.quad(lamb(theta, a2_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
A3 = spi.quad(lamb(theta, a3_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
A4 = spi.quad(lamb(theta, a4_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
A5 = spi.quad(lamb(theta, a5_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
A6 = spi.quad(lamb(theta, a6_exp), lower_limit, upper_limit, epsrel=1e-6)[0]

B1 = spi.quad(lamb(theta, b1_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
B2 = spi.quad(lamb(theta, b2_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
B3 = spi.quad(lamb(theta, b3_exp), lower_limit, upper_limit, epsrel=1e-6)[0]
epsrel=1e-6)[0]

return A1, A2, A3, A4, A5, A6, B1, B2, B3

E = 200*1000  # Multiply with 1000 since we need in MPa
# Remove the 1000 above if using parameters by Rouse et al.
# Since it somehow works only when in GPa for that

# Correction parameters from Rouse et al
a1 = 0.986
a2 = 0.103
a3 = 3.567e-6
a4 = 4.995

# Ring dimensions
a_new = 5.0  # Horizontal, major axis
b_new = 5.0  # Vertical, minor axis
v = 0.3  # Poisson’s ratio
t = 1.0  # ring thickness
d = 2.0  # ring depth
A = 2  # Cross-section area

# Create empty lists to store the values
u_v_values = []
u_h_values = []
a_values = []
b_values = []
p_mod_vals = []
p_raw_vals = []
fp_vals = []
s = time.time()  # Start time

# Initialize the force array
P = np.linspace(0.1, 500.1, num = 250)

# Begin loop
for (i, value) in enumerate(P):
    I = calculate_dimensions(a_new, b_new, t)  # Update moment of inertia
    A1, A2, A3, A4, A5, A6, B1, B2, B3 = calculate_integrals(
        a_new, b_new)  # Get new integrals
    u_v = value*( (A2 / (E*I)) - (A5**2 / (4*E*I*A1)) + ( (B1 * (3 + 2*v)) / (A*E) ) )
    u_h = value*( (A5*A6) / (4*E*I*A1) ) - (A4 / (2*E*I) ) +
        ( (B3 * (1 + 2*v)) / (2*A*E) ) )
    if u_v > 0.5:
        break
    else:
A.1. ANALYTICAL SOLUTION

\[
fp = a1 \times \exp(a2 \times u_v) + a3 \times \exp(a4 \times u_v) \quad \rightarrow \quad \text{Correction factor}
\]

\[
\text{pmod} = fp \times \text{value} \quad \# \quad \text{Modify force value}
\]

\[
\text{p_mod_vals} . \text{append}(\text{pmod}) \quad \# \quad \text{Modified value appended}
\]

\[
\text{p_raw_vals} . \text{append}(\text{value}) \quad \# \quad \text{Just the raw value appended}
\]

\[
u_v . \text{values}. \text{append}(u_v) \quad \# \quad \text{Store y-displacement}
\]

\[
u_h . \text{values}. \text{append}(u_h) \quad \# \quad \text{Store x-displacement}
\]

\[
\text{fp_vals} . \text{append}(fp) \quad \# \quad \text{Store correction factor fp}
\]

\[
a\_\text{old}, \quad b\_\text{old} = a\_\text{new}, \quad b\_\text{new}
\quad \# \quad \text{Update the ring dimensions now}
\]

\[
b\_\text{new} = b\_\text{old} + \text{np.abs}(u_v)
\]

\[
a\_\text{new} = a\_\text{old} - \text{np.abs}(u_h)
\quad \# \quad \text{Append the displacement values to the lists}
\]

\[
a\_\text{values}. \text{append}(a\_\text{new})
\]

\[
b\_\text{values}. \text{append}(b\_\text{new})
\]

\# Set a limit for disp extraction from dataframe
\[
dlim = u_v . \text{values}[1]
\]

\[
dlim\_\text{pts} = \text{len}(u_v . \text{values})
\]

\[
\text{df_for_fp} = \text{pd.read_csv}(/ \text{experimental/csv/file/here}/) \quad \# \quad \rightarrow \quad \text{Experimental CSV}
\]

\[
\text{df_for_fp_trimmed} = \text{df_for_fp}[\text{df_for_fp[}'\text{Displacement}'\text{]} \leq dlim] \quad \# \quad \rightarrow \quad \text{Extract only upto dlim mm}
\]

\[
disp_for_fp = \text{df_for_fp_trimmed[}'\text{Displacement}'\text{]} \quad \# \quad \rightarrow \quad \text{Store in a separate disp array}
\]

\[
disp\_\text{interp_for_fp} = \text{np.linspace}(\text{disp_for_fp.min}(), \text{disp_for_fp.max}(), \text{num}=\text{dlim}_\text{pts}) \quad \# \quad \rightarrow \quad \text{Interpolate dlim_pts number of points}
\]

\[
\text{force\_interp_for_fp} = \text{np.interp}(\text{disp\_interp_for_fp}, \text{disp_for_fp}, \text{df_for_fp_trimmed[}'\text{Force}'\text{]}) \quad \# \quad \rightarrow \quad \text{Intpolate dlim_pts number of force points}
\]

\[
\text{print}("\text{Runtime:}, \quad (\text{time.time()}-\text{s})")
\]

\[
\text{Pmax\_exp} = \text{df_for_fp[}'\text{Force'}\text{].max()} \quad \# \quad \rightarrow \quad \text{Get max force from dataframe}
\]

\[
\text{anal_norm_force} = \text{p_mod_vals}/\text{Pmax\_exp} \quad \# \quad \rightarrow \quad \text{Normalize Analytical force (modified values)}
\]

\[
\text{df\_norm\_force} = \text{force\_interp_for_fp}/\text{Pmax\_exp} \quad \# \quad \rightarrow \quad \text{Experimental force}
\]

\[
\text{disp\_ratio} = \text{np.divide}(\text{u_h_values}, \text{u_v_values}) \quad \# \quad \rightarrow \quad \text{Calculate displacement ratio}
\]

# Plotting code not included
# Following things were plotted:
# 1) Expt vs Analytical (without correction factor fp)
# 2) Expt vs Analytical (with and without correction factor fp)
A.2 Random Forest Regression Models

Grid Search and model evaluation for Random Forest Regression code described in chapter 3. Python 3.10 was used for this code.

```python
# Define the hyperparameter grid to search
param_grid = {
    'n_estimators': [5, 10, 25],
    'max_depth': [5, 10, 25],
}

# Initialize the model
regressor = RandomForestRegressor()

# Scoring function
def r2_mae_scorer(y_true, y_pred):
    r2 = r2_score(y_true, y_pred)
    mae = mean_absolute_error(y_true, y_pred)
    return (1 - r2) + mae

# Make scorer creates scoring function to pass to our search
scorer = make_scorer(r2_mae_scorer, greater_is_better=False)

# Perform the grid search with 5-fold cross-validation
grid_search = GridSearchCV(regressor,
                            param_grid,
                            scoring=scorer,
                            cv=5)
grid_search.fit(X_train, np.ravel(Y_train))

# Get the best hyperparameters from the grid search
best_params = grid_search.best_params_

# Train the model using the best hyperparameters
regressor = RandomForestRegressor(**best_params)
regressor.fit(X_train, np.ravel(Y_train))
```
A.3 Inverse FEA

The code used for inverse FE optimisation is given below, and the procedure was described in chapter 3. Note that the code used for the generation of the new .inp file is not given here. Python 3.10 was used. Also note that Abaqus uses Python 2.7 and future researchers must be aware of this when working with Abaqus-Python for scripting the new .inp files.

```python
import time
import numpy as np
import scipy.optimize as optimize
import pandas as pd
import math
from sklearn.metrics import mean_squared_error
import subprocess
from scipy.optimize import differential_evolution

# Replace full path with your experimental CSV data
expt_path = r'C:\Users\expt.csv'

# Replace full path for the simulation CSV
# I usually use the name "rp_output_function.csv"
sim_path = r'C:\Users\sim.csv'

# Find the closest values to the requested displacement value

def find_neighbours(value, df, colname, rescol):
    '''
    This function helps us find the closest values to the requested displacement values.
    Closest because sometimes we won’t find an exact match.
    '''
    exactmatch = df[df[colname] == value]
    if not exactmatch.empty:
        resval = df[rescol].iloc[exactmatch.index[0]]
    else:
        # We could go for the lower neighbour,
        # but if it does not exist it will throw up an error
        lowerneighbour_ind = df[df[colname] < value][colname].
            => idxmax()
        # Best be safe and go for the upper value!
        upperneighbour_ind = df[df[colname] > value][colname].
            => idxmin()
        resval = df[rescol].iloc[upperneighbour_ind]
    return resval

# Function to Load the expt dataframe
```

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```python
# Load experiment CSV
def load_expt(path):
    expt_df = pd.read_csv(path)
    return expt_df

# Load simulation CSV
def load_sim_data(path):
    sim_df = pd.read_csv(path, names = ["Time", "Displacement", "Time2", "Force"], delimiter=',')
    # Some data cleaning from the Abaqus file
    # Drop the repeated Time column
    sim_df = sim_df.drop(sim_df.columns[2], axis = 1)
    # Exploiting 1/8th symmetry, so multiply disp x 2
    sim_df['Displacement'] = sim_df['Displacement'].apply(lambda x: x*2)
    # And multiply force x 4
    sim_df['Force'] = sim_df['Force'].apply(lambda x: x*4)
    # Find 0s in Force column since these indicate no loading
    tosub = sim_df['Force'].ne(0).idxmax() - 1
    # Subtract those displacement, time, and force values below
    sim_df.Time = sim_df.Time - sim_df.Time.iloc[tosub]
    # Trim dataframe
    sim_df = sim_df.iloc[tosub:]
    # We need to re-index otherwise Pandas freaks out
    sim_df = sim_df.reset_index(drop=True)
    return sim_df

def df_trim_and_extract(df, arr):
    empty_arr = []
    for i in arr:
        empty_arr = np.append(empty_arr, find_neighbours(i, df, 'Displacement', 'Force'))
    return empty_arr

# Find the limit of the displacement values we can load
# This is based on the simulation data
# Do this because you are limited by sim data
def find_sim_lastdisp(df):
    lastelem = df.Displacement.iloc[-1]
    tosubtract = lastelem % 0.1
    lastelem = lastelem - tosubtract
    return lastelem
```
# No need to load the full array for comparison right now
df_sim = load_sim_data(sim_path)
expt_df_2mm = load_expt(expt_path)
#expt_df_4mm = load_expt(expt_path) #We will use this only if we get good residual to proceed further

# We are limited by a sim, so finding where Sim displacements end and returning here
displacement_limit = find_sim_lastdisp(df_sim)

# Array to iterate over and extract Displacement-force values periodically, 0.1 right now
arr = np.arange(0.0, displacement_limit, 0.1)
# Array of forces at 0.1 intervals (experiment)
expt_force_arr = df_trim_and_extract(expt_df_2mm, arr)
# Array of forces at 0.1 intervals (simulation)
sim_force_arr = df_trim_and_extract(df_sim, arr)

masterjobname = ' ' #Job name over which we will iterate
abqfileloc = ' ' #In case the path names are different. Comment it out otherwise

#########
# Important!
# File names below!
#########
readfile_hard = 'hardening_exponent.csv' #IMPORTANT. CSV FILE WITH HARDENING PARAM.
readfile_yield = 'yield_offset.csv' #IMPORTANT. CSV FILE WITH YIELD OFFSET.
readfile_elasticmod = 'elastic_modulus.csv' #IMPORTANT. CSV FILE WITH ELASTIC MODULUS.
readfile_yieldstr = 'yield_strength.csv' #IMPORTANT. CSV FILE WITH YIELD STRENGTH.
outfile_hard = 'hardening_exponent_tracker.csv' #IMPORTANT. CSV FILE TO TRACK HARDENING PARAM.
outfile_yield = 'yield_offset_tracker.csv' #IMPORTANT. CSV FILE TO TRACK YIELD OFFSET.
outfile_elasticmod = 'elastic_modulus_tracker.csv' #IMPORTANT. CSV FILE TO TRACK ELASTIC MODULUS.
outfile_yieldstr = 'yield_strength_tracker.csv' #IMPORTANT. CSV FILE TO TRACK YIELD STRENGTH.
residual_filename = 'optim_residual_tracker.csv' #IMPORTANT. CSV FILE TO TRACK RESIDUAL COST FUNCTION VALUE

# New parameters will be added by the function below
# This function will change the parameters based on the excel
def csvreader(filename, parameter, recordfile):
    with open(filename) as f:
        for (i, line) in enumerate(f):
            oldparams = line
    newparams = str(parameter)
    with open(filename) as f:
        filedata = f.read()
    filedata = filedata.replace(oldparams, newparams)
    with open(filename, 'w') as f:
        f.write(filedata)
    with open(recordfile, 'a') as f:
        f.write(newparams + "\n")

def residual_writer(parameter, recordfile):
    parameter = str(parameter)
    with open(recordfile, 'a') as f:
        f.write(parameter + "\n")

"""
Now we have the 2 most important functions— the residual calculation and the optimization
"""

# Note: Abaqus_jobsub.bat is a batch script
# This .bat file creates a new .inp file and submits it
# Abaqus_outputreader.bat is a batch script
# This .bat file reads the output from the previous batch script
# Need to keep it separate because Abaqus Python is funny
# And (sometimes) does not like it when you do these 2 things concurrently

# BEGIN FUNCTION FOR RES CALC
def residual_calc(x):
    # Remember— expt—df and sim—df have discretized values, so
    # we can directly compare them.
    # We DO need an rp_output_function file

    # FIRST TASK— CHANGE MATERIAL KEY!
    #
    # SECOND TASK— INVOKE ABAQUS PYTHON SCRIPT TO RUN 1
    # SIMULATION.
    #
    # THIRD TASK— INVOKE ABAQUS PYTHON SCRIPT TO FETCH ODB
    # OUTPUT
    #
# sim_res = load_sim_data(abqfileloc) #to get new simulation data
#
# GET− difference between 2 arrays of FORCE reading. RMSE.
# DECLARE − COUNTER variable to check iterations and file appending
# WRITE− COUNTER variable and r−value to a CSV file (append it)

# Task 1 below
s = time.time()
csvreader(readfile_hard, x[0], outfile_hard)
csvreader(readfile_yield, x[1], outfile_yield)
csvreader(readfile_yieldstr, x[2], outfile_yieldstr)
csvreader(readfile_elasticmod, x[3], outfile_elasticmod)

# Task 2 below
subprocess.call([r'C:\Users\Abaqus_jobsub.bat'])

# Task 3 below
subprocess.call([r'C:\Users\Abaqus_outputreader.bat'])

# Task 4 below
# Need to load new sim data to sim_force_arr
df_sim = load_sim_data(sim_path)

# We are limited by a sim
# Find where Sim displacements end and return below
displacement_limit = find_sim_lastdisp(df_sim)
arr = np.arange(0.0, displacement_limit, 0.1)
sim_force_arr = df_trim_and_extract(df_sim, arr)

# Task 5 below
# Get residual and return it
# Using RMSE
# You can use any metric you want
residual = math.sqrt(mean_squared_error(exp_force_arr,
                                        sim_force_arr))
residual_writer(residual, residual_filename)
print('!!!!!!!!!!!!!!!!!!')
print('Iteration done, time taken for sim and CSV: ', ((
          time.time()−s)/60))
print('!!!!!!!!!!!!!!!!!!')
return residual

# BEGIN OPT ROUTINE
def optimization_routine():
x0 = np.array([5.5, 0.16, 400, 210])
bnds = ((1.0, 25.0), (0.01, 0.99), (210, 550), (190, 220))

# Uncomment below to use Population-Based Differential Evolution
#result = differential_evolution(residual_calc, bnds,
updating='immediate', x0=x0)

# Below is Nelder–Mead
result = optimize.minimize(residual_calc, x0, method='Nelder–Mead', bounds=bnds)

# summarize the result
print('Status: %s' % result['message'])
print('Total Evaluations: %d' % result['nfev'])

# CALL OPT ROUTINE. RUNS FULL FILE.
optimization_routine()