A systematic numerical and experimental study into the mechanical properties of five honeycombs*

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ABSTRACT

Honeycombs are engineered cellular materials that often show superior specific strength, stiffness and energy absorption compared to solid materials. As a consequence they have found numerous applications across engineering fields. The development of additive manufacturing (AM) technologies has initiated an abundance of studies into novel honeycombs as historic manufacturing constraints are lifted. Investigations have been focused on improving or tailoring a given property but very few have focused on isotropy, and little has been done to bring together different patterns under the same manufacturing and experimental conditions. In this study, AM has been used to manufacture nominally identical honeycombs based on differing unit cells, in a range of orientations and densities. Elastic and plastic properties for the hexagon, triangle, square, re-entrant and double-V honeycombs have been obtained through mechanical testing. The plastic properties of these honeycombs have been modelled for all possible in-plane loading directions using minimal computational resources. The effect of orientation and density has been presented, confirming the level of in-plane isotropy for dense honeycombs with regards to Young’s modulus, Poisson’s ratio, yield strength and compressive strength. Insights have also been gained into how these properties vary with relative density. These results provide a basis for comparison with future work on honeycombs.

1. Introduction

Engineering honeycombs are a class of cellular solids comprised of a two-dimensional lattice that has been stacked into the third dimension [1]. High specific stiffness and specific energy absorption has led to widespread use in engineering, from lightweight sandwich panels in aviation [2] to explosion protection in military vehicles [3]. The effective properties of these cellular materials are attributed to the shape and configuration of their cells as well as to the properties of the materials from which they are constructed. They are often referred to as meta-materials due to the engineered effective properties that are not available with naturally occurring materials. Initially they consisted of periodic arrangements of hexagonal, square or triangular cells however in recent years the development of additive manufacturing (AM) technologies has led to a wave of novel honeycombs being developed [4]. This novelty results from variations in the intricate geometric designs made possible by AM, and the growing range of available materials. Included in this growing range of honeycombs is a class of cellular solid with negative Poisson’s ratio, known as auxetic meta-materials [5, 6, 7]. Much attention has been given to this interesting class of honeycombs as negative Poisson’s ratio is rare in natural materials.

Qi et al. [4] carried out an extensive review of 239 publications reporting studies into the mechanical properties of a broad range of classical and novel honeycombs. The review was focused on the compressive strength, Young’s modulus and specific energy absorption of honeycombs. These properties were determined using a range of experimental, analytical and theoretical methods. The review demonstrates how different design strategies have been employed to expand the range of available properties and how these compare with “classical honeycombs”. It also demonstrates the difficulty in making reliable comparisons between the results of different studies due to the diversity in manufacturing and analysis employed across the studies.

Studies such as [8, 9], which are based solely on numerical or theoretical models are common for honeycomb materials because testing can be expensive and time-consuming. However, physical testing is essential in order to validate assumptions made in numerical or theoretical models. Experiments also provide the means to investigate properties that are difficult to define theoretically, such as plastic behaviour [10]. This additional time and expense often limits the scope of experimental studies where gaps in experimental data can be filled via simulations, allowing for a complete understanding of the properties of a honeycomb. For these reasons a combination of methods is desirable.

Samples produced via AM processes can be very sensitive to the parameters of the manufacturing process, such as material, orientation, tool-path, tooling, layer height and environmental conditions [11, 7]. Therefore to enable reliable comparison of different structures it is important that they are manufactured using comparable processes and parameters. Where studies report testing of physical samples produced via AM, details of the manufacturing processes or print parameters are not always given [12, 13], making replication and comparison with different studies difficult. Other studies include tests on the relevant classical honeycomb...
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Figure 1: Hexagon (a), square (b) and triangle (c) lattices. $l$ is held constant and the vertex to vertex cell wall lengths, $l_h$, $l_s$, and $l_t$, are varied to achieve equivalent densities

yet it is not always clear how or if efforts have been made to ensure comparability with regards to toolpath [14, 15]. Where toolpaths are considered, print quality can vary due to the complexity of the geometry [16] and it can be difficult to see how toolpaths could be adapted to provide equivalence in alternative honeycomb designs [17].

Isotropy is often overlooked when investigating the effective mechanical properties of honeycombs. Many studies are concerned with out-of-plane mechanical properties where investigations into different loading situations fall naturally outside the scope of the study [18, 19, 20], however in-plane properties can vary significantly with different loading situations. For applications where the directions of forces cannot be assumed, such as lightweight sandwich panels in aviation and explosion protection in military vehicles [2, 3], isotropy (or the degree of anisotropy) is an important consideration. Some honeycombs, such as the hexagon-based honeycombs, are assumed to be fully isotropic, and other honeycombs may be isotropic for small deformations but anisotropic for large deformations [21]. Orthogonal tests have been carried out for a variety of studies [22, 23, 24, 25] giving some insight into how the properties vary with rotation however for certain honeycombs, targeted orientations based on rotational symmetries are required to draw conclusions. A simple example is the square lattice which is identical in the x and y directions but is highly anisotropic [1]. Examples of full rotational analysis, which determines a honeycomb’s response to forces from all possible directions, is rare. Qi et al. [4] reviewed only 4 such studies: Taylor et al. [26] carried out rotational finite element analysis on hierarchical re-entrant honeycombs with different mass distributions. The isotropy of these honeycombs were considered and compared to equivalent hexagonal honeycombs. Zhu et al. [21] and Zhu et al.[27] proposed modified hexachiral structures to improve the isotropy of standard hexachiral structures at large deformations. Finite element (FE) simulations were performed on unit cells at intervals of 5° to investigate the improvements in isotropy with regards to Poisson’s ratio. Peng and Bargmann [28] proposed a novel hybrid auxetic honeycomb with enhanced stiffness. Numerical rotational analysis was carried out at a resolution of 6° with results given for normalised Young’s modulus and Poisson’s ratio.

Since the review by Qi et al. [4], Moat et al. [29] carried out experimental investigations into the isotropy of aperiodic honeycombs. AM samples were produced at multiple orientations based on the rotational symmetry of the lattice. These samples were tested in compression along with hexagonal and square honeycombs. This kind of systematic rotational analysis is rare, and has not yet been carried out for all classical honeycombs in order to validate assumptions about the isotropy of mechanical properties. As shown in the review by Qi et al. [4], studies into the mechanical properties of honeycombs tend to focus on one or two mechanical properties, tested using only a small number of samples, or via theoretical or analytical models. Experimental studies are important for determining actual properties of 3D printed models where it is the final method of manufacture [5] and published values for the mechanical properties of 3D printed material often introduce error when used for simulations [30]. There has been no systematic attempt to experimentally validate test results for classical honeycombs, manufactured using equivalent methods, against theoretical or numerical models, for a broad range of mechanical properties.

The aim of this study is to compare the mechanical properties of a range of classical and auxetic honeycombs with a focus on isotropy, whilst ensuring that AM samples have been produced using the same method, material and print parameters. By comparing the experimental data from compression tests with data obtained from linear elastic simulations, a basis for comparing equivalent novel honeycombs, both periodic and non-periodic [31, 32], is presented. By placing importance on equivalence, any comparisons are based on the structure of the honeycomb and not anomalies introduced by the method of manufacture. The experimental data is used to validate the modelling whilst the modelling is used to create more detailed data sets and give a better indication of the orientational dependence of the mechanical properties of the honeycombs.
2. Materials and methods

2.1. Selection of samples for rotational analysis

Five honeycombs were selected for this study, three classical: hexagon (Figure 1(a)), square (Figure 1(b)) and triangle (Figure 1(c)); and two auxetic: re-entrant (Figure 2(a)) and double-V (Figure 2(b)). For the hexagon, square and triangle lattices, only two parameters are required to specify the cell configuration, the cell wall thickness (t) and the edge length (l). These parameters are shown schematically for each lattice in Figure 1. The auxetic lattices both require two additional parameters. The re-entrant cell is defined by the re-entrant angle (θr) and the cell height (2l) (Figure 2(a)), whereas the double-V cell is defined by the width of half a cell (l/2), the internal angle (θ1) and the external angle (θ2) (Figure 2(b)). These auxetic parameters can be used to tune the mechanical properties of the honeycomb, however in this study this is not investigated and the chosen parameters of θr = 30°, θ1 = 21° and θ2 = 60° are held constant throughout.

For manufactured samples, lattice rotations were selected based on the rotational symmetry (Cn) of the lattice, with n = 6 for the hexagon and triangle, n = 4 for the square and n = 2 for the re-entrant and double-V. If θ is the fundamental domain 360°/n, it was considered that testing any rotation past θ would be superfluous. It was also noted that a reflection along the y-axis, with 0° in the y-direction, would be equivalent to rotating the finished sample around a central axis in the y-direction. Moreover, only samples within a rotation of θ/2 are unique since rotating a lattice by 0° + φ is equivalent to a rotation of 360° − φ when viewed from the rear and 360° = nθ = 0°. As any rotation by a multiple of the fundamental domain yields the same pattern, it can be said that rotations of θ + φ and θ − φ are equivalent. Samples were then selected at multiples of θ/10, making sure that samples were not duplicated, giving a total of five samples per lattice, each with the pattern at a different orientation.

Figure 2: Parameters used to construct auxetic honeycombs: re-entrant (a) and double-V (b)

The relative density of a given lattice is dependant on t and l, for this study it was decided to keep t constant (t = 0.5 mm) and vary l to achieve the required density for all samples. This method is used to correct for error associated with the differing toolpaths produced when varying l. However, the size and number of cells captured within a patch varies between samples of differing density.

2.2. Selection of samples for density analysis

In order to investigate how the effects of density on mechanical properties compare between modelling and experiments, samples at 25%, 35% and 45% dense were produced. Samples were only manufactured in two orientations for this part of the study, 0° and θ/2. As described in section 2.1, the required density was achieved by varying the cell wall length.

2.3. Production of samples

Fused deposition modelling (FDM) is an extrusion based AM system where thermoplastic polymers are extruded from a nozzle. The nozzle lays down material following a toolpath and the model is built up layer by layer. FDM can produce cost effective models quickly, however the method often introduces isotropy into models [33]. Moreover, any printed specimen may exhibit different mechanical properties depending on toolpath and print parameters such as layer height [11]. Datasheet values for mechanical properties do not always yield equivalent results when used for FEA models and compared with experiments [30]. For this reason, results are normalised and appropriate comparisons can be made.

3D models of the samples were created using a bespoke python code utilising FreeCAD’s python package [34]. After selecting the type of lattice, cell wall thickness, target density (ρt) and rotation, the cell wall length is varied so that the required density is achieved. Lattices are rotated within the defined sample area, cells are trimmed and then scaled until the total area of cells (Ajt) and sample cross-sectional area (Ajt) is such that 100Ajt/Ajt = ρt.

Figure 3 shows stages of sample production using the re-entrant pattern as an example. First an array of cell centre points is created based on the cell parameters and the sample size. The array is rotated to the specified angle. The coordinates of the cell vertices are then defined by their orthogonal distances from the centre point as a function of l, and a cell is created at each centre point with vertices rotated to the specified angle. Examples of rotated samples can be seen in Figure 3(a). Figure 3(b) shows how the cell vertex coordinates are redefined so that the cell wall is the required thickness (t). Figure 3(c) shows how the required density is achieved by varying l. Top and bottom surfaces are added to ensure forces are applied uniformly, this can be seen in Figure 3(d). Figure 3(e) shows the 3D model of the sample in FreeCAD and Figure 3(e) shows the finished sample printed in PLA.

Samples were produced with dimensions 50 x 50 x 50 mm in accordance with ASTM D1621 [35]. Cura 4.12 [36] was used to slice the model for printing. At 0.5 mm wall thickness it was found that Cura generated toolpaths that produced cell walls where pores were not visible and lines of extrusion appeared to be bonded together, all types of unit cell were also constructed in the same way. It was decided that any toolpath that required single extrusions for cell walls with discontinuities at cell vertices would not produce comparable structures due to the difference in quantity and
position of such discontinuities. It must be noted that no investigations into the structural integrity of varying toolpath has been carried out as focus was on comparability. Varying cell wall thickness greatly affected the finished sample with respect to wall bonding and toolpath, for this reason the target density was achieved by varying the cell wall length and not the cell wall thickness. Figure 4 shows examples of these toolpaths for triangular honeycombs (Figure 4(a)) and hexagonal honeycombs (Figure 4(b)). Each cell is composed of one continuous toolpath with each cell wall composed of two lines of extrusion, one from each neighbouring cell. With this configuration, discontinuities at vertices caused by single line walls are avoided. All principle tensile and compressive forces are carried axially along fibres [33]. The samples were then printed on an Ultimaker S3 FDM printer with a nozzle diameter of 0.4 mm in white polylactic acid (PLA). All samples were built up of 0.2 mm layers to achieve the required sample thickness of 50 mm.

2.4. Mechanical testing

Compression tests were carried out on an Instron 6800 mechanical testing machine with a 50 kN load cell. All test were conducted at a constant strain rate of 0.5 mm/min to a maximum strain of 25% and were controlled using crosshead displacement measurement. Strain was measured using digital image correlation (DIC) for greater accuracy and to negate the need to account for machine compliance. DIC is a full-field optical strain measurement method where images acquired during tests are compared with an undeformed image and local strain maps are produced by tracking subsets of pixels [37]. These subsets must contain random features so that the algorithm can achieve a unique solution, this was achieved by applying a random speckle pattern to the samples with black spray paint. The experimental setup can be seen in figure 5. Images and data where collected at a rate of 1 Hz. For these experiments, LaVision DaVis 10.2.1.81611 [38] software was used to apply live virtual strain gauges where subsets of pixels at each end of the strain gauge are tracked in real time removing the need to process entire images. Figure 6 shows an image of these strain gauges with the red line corresponding to the measurement of axial
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Using this method, full rotational analysis of honeycombs becomes viable. Although these honeycombs are periodic and analysis can be carried out on a unit cell [1], this alternative method is used for consistency with the physical testing and to enable future comparisons with other novel, non-periodic honeycombs [40].

Numerical simulations of the honeycombs were carried out by element-based assignment of material properties using FEniCS [41], [42], an open source finite element solver. Here the structures are treated as composites consisting of material and void, the void being assigned as weak isotropic material, in this case $E = 100 \text{ Pa}$. Digital representations of the manufactured samples were created with identical size and parameters. The material assignment process is described in detail within recent works by Imediegwu et al. [40]. The honeycombs were simplified to 2-dimensional lattices with displacements obtained using Hooke’s law for plain strain conditions. An effective elasticity matrix for each honeycomb was obtained from simulations. The resulting compliance matrix was transformed for each degree of rotation and the effective Young’s modulus and Poisson’s ratio extracted. By using this method, re-meshing of the sample is not required for each rotation and the rotational analysis can be automated.

3. Results

In combination, the data from the compression tests and from the linear elastic simulations provide a rich resource from which to draw conclusions about the behaviour of the honeycombs investigated in this study. Discussion of the results focuses on the isotropic properties of the honeycombs, by considering the effect of rotation, and on the effect of the density of the structures. In this and following sections, all reference to mechanical properties relate to the effective mechanical properties of the honeycomb unless otherwise indicated.

3.1. The effect of rotation on effective mechanical properties

Figure 7 shows stress strain curves for an example of each 45% dense honeycomb in their stiffest orthogonal orientation. It can be seen that the square at $0^\circ$ is the stiffest with the double-V at $90^\circ$ being the least stiff. Along with varying elastic responses, it can be seen that the plastic response for each honeycomb varies significantly. The square, triangle and re-entrant show clear occurrences of drops in stress, whereas the hexagon and double-V consistently deform smoothly towards densification. The double-V shows 3 distinct features, the initial linear elastic region, a second linear region with a shallower gradient and a plateau. The sample deformed smoothly up to 25% strain with no fluctuations. The re-entrant and triangle show a clear first local maxima followed by steep reductions in force indicating loss of structural integrity. The force then drops with no distinct plateau and several sharp fluctuations are observed.

The hexagon has a less defined local maximum and a smooth flat plateau showing no signs of the sudden drops in stress.
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observed in other samples. The square at 0° does not show a clear first local maximum, the force instead climbs gradually with small oscillations, there is also no significant drops in stress until 20% strain. Although the triangle has a greater elastic modulus, yield strength and compressive strength than the double-V, it absorbs less energy up to a strain of 25%. This is observed by comparing the areas under the two curves [43]. The holistic characteristics of these results are repeatable however specific manifestations are different.

The extremes of Poisson’s ratio obtained experimentally and are presented in Figure 8. The hexagon and triangle show the least spread in Poisson’s ratio, with the hexagon showing a spread of 0.15 and the triangle showing a spread of 0.04. Compared to the square, re-entrant and double-V honeycombs showing spreads of 0.74, 0.89 and 0.32 respectively, this suggests higher levels of anisotropy in the square, re-entrant and double-V compared to the hexagon and triangle honeycombs. Both the double-V and re-entrant honeycombs are reported to exhibit auxetic behaviour [44, 45], and this is also found here. Auxeticity is determined by the angles $\theta_1, \theta_2, \text{and } \theta_3$ for honeycombs with lower densities, however, it is worth noting that the double-V is only slightly auxetic at this density. The re-entrant shows the greatest spread of results, varying between -0.24 and 0.65 average Poisson’s ratio. The square at 45° has the highest Poisson’s ratio with a value of 0.78.

Figure 9 shows polar plots of the mechanical properties vs. orientation found through mechanical testing and modelling for normalised Young’s modulus ($E_{np}^*$) and Poisson’s ratio. The Young’s moduli ($E_{np}^*$) of the modelling and measurements have been normalised to the Young’s modulus of the square at 0° ($E_{00}^*$) obtained from modelling and measurement respectively. This is analogous to the performance ratio in [4] whereby here, all honeycombs are compared to the performance of the square at 0° using the equation

$$E_{np}^* = \frac{E_{np}}{E_{00}}.$$  

For each honeycomb, an image is included that shows the patterns aligned with 0° on the polar plots. Experimental results are represented as solid points, and equivalent values due to the rotational symmetries discussed in Section 2.1 being represented as hollow points. Modelling results are plotted as closed curves, and the isotropy of the honeycomb can be visualised by noting that a circular plot would indicate an isotropic material and any deviation from a perfect circle indicates anisotropy. In all cases, it can be seen that properties obtained from modelling are a good approximation to those obtained through experiments in both normalised magnitude and shape of the distribution.

The modelling results predict isotropy for the hexagon for 45% dense honeycombs, evidenced by the plots for Young’s modulus and Poisson’s ratio being relatively circular. The experiments follow a circular trend with results fluctuating slightly. The results obtained through modelling are lower than those obtained experimentally for normalised Young’s modulus and higher for Poisson’s ratio. It can be seen that the hexagon appears less isotropic when considering yield strength and compressive strength. The results here form a more hexagonal shape with 6 maxima at 0°, 60°,
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Figure 9: Rotational plots of normalised Young's modulus ($E^*_\phi/E^*_\text{sq}$), Poisson's ratio, yield strength and compressive strength for five honeycombs. Solid blue lines represent modelling results, solid circular markers represent results obtained experimentally and empty circular markers represent experimental results plotted at equivalent rotations due to rotational symmetries. For comparison, all axis for a particular property have the same scale.

$120^\circ, 180^\circ, 240^\circ$ and $300^\circ$ replicating the 6-fold symmetry of the pattern.

The square lattice is highly anisotropic, with a maximum normalised Young’s modulus at $0^\circ$ and minimum normalised Young’s modulus of 0.25 at $45^\circ$ along with similar patterns at equivalent angles based on the 4-fold symmetry of the system. The modelling again provides a good approximation for the Young’s modulus in both shape and normalised magnitude. As with the hexagon, modelling values are slightly lower than those measured experimentally.
The modelling values are again slightly higher for Poisson’s ratio. The anisotropy is less profound when considering yield strength and compressive strength, but still exhibits the same symmetry as the Young’s modulus with maxima at 0°, 90°, 180° and 270° and minima at 45°, 135°, 225° and 315°. The square at 0° had the highest measured yield strength whereas the triangle had the highest average yield strength due to higher isotropy.

For the triangle honeycomb, modelling predicts isotropy with regards to modulus and Poisson’s ratio. Furthermore the experimental data supports this with modulus results fluctuating slightly around a circular trend. The values for Poisson’s ratio and Young’s modulus are very close for modelling and experiments with the plot for experiments taking a slightly more hexagonal shape aligning with the rotational symmetry of the structure.

The modelling provides a good approximation for the shape of the Young’s modulus plot for the re-entrant, however the maxima at 90° and 270° are lower than measured experimentally with a normalised value of 0.53 obtained from modelling and 0.77 measured experimentally. The modelling estimates the Poisson’s ratio accurately for positive values, however the negative values are slightly lower than experimental values dropping to -0.21 at 0° and 180°, and -0.38 at 90° and 270°. Measured values are -0.03 and -0.24 at the same angles. As with the square, the re-entrant is highly anisotropic with the anisotropy being less profound when considering yield and compressive strength. The 2-fold symmetry of the pattern is clearly visible in all plots.

For the double-V, the modelling replicates the Young’s modulus very well with only the experimental value for 90° and 270° deviating notably. For these angles, modelling predicts a value of 0.47 and a value of 0.59 was obtained through experiments. The double-V shows the greatest discrepancy between modelling and experimental values for Poisson’s ratio. The overall shape of the two plots are similar, however modelling predicts minima at 90° and 270° of -0.2 whereas a Poisson ratio of approximately 0 was measured experimentally. There is a similar inconsistency for the maxima at 45° and 135° where modelling predicts a value of 0.17 whereas a value of 0.31 was obtained experimentally.

These results can also be compared with theoretical models of isotropy. For example, Zhu et al [46] introduced an isotropy factor I to quantify the level of isotropy, it is defined as

\[ I = \frac{|v_{\text{max}} - v_{\text{min}}|}{|v_{\text{max}} + v_{\text{min}}|} \]

where \(v_{\text{min}}\) and \(v_{\text{max}}\) are the minimum and maximum effective Poisson’s ratio for one lattice at one density for all orientations. Isotropy factors for experiments and simulations can be found in table 1. The re-entrant and the double-V have a maximum isotropy factor of 1 as they vary from positive to negative Poisson’s ratios. Modelling predicts an isotropy factor of 0.04 for the triangle, this is in close agreement with the value of 0.06 obtained experimentally. This reflects the isotropy seen for these patterns in Figure 9. Modelling predicts a lower isotropy factor of 0.02 for the hexagon whereas the experiments produce a higher factor of 0.16. As expected, the isotropy factor for the square is significantly higher with values of 0.91 and 0.70 obtained from experiments and modelling respectively. These differences between modelling and experimental results for the hexagon and square are not apparent in Figure 9.

### 3.2. The effect of density on effective mechanical properties

Figure 10 shows polar plots of normalised Young’s modulus and Poisson’s ratio for the five honeycombs. As with Figure 9, in these polar plots, points represent experimental values and closed curves represent values from simulation. Each plot shows results for honeycombs with relative densities of 0.25, 0.35, and 0.45 with a darker line indicating a higher density. As with Figure 9, the Young’s modulus of the hexagon honeycomb appears to increase for relative densities of 0.25 and 0.35 calculated from experimental and modelling results (based on [46]).

### Table 1

<table>
<thead>
<tr>
<th>Density</th>
<th>Method</th>
<th>Hexagon</th>
<th>Square</th>
<th>Triangle</th>
<th>Re-entrant</th>
<th>Double-V</th>
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<tbody>
<tr>
<td>0.25</td>
<td>Experimental</td>
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<td>0.02</td>
<td>0.00</td>
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<td>N/A</td>
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<td>0.00</td>
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<tr>
<td>0.35</td>
<td>Modelling</td>
<td>0.04</td>
<td>0.77</td>
<td>0.04</td>
<td>1.00</td>
<td>0.40</td>
</tr>
</tbody>
</table>

As expected, the isotropy factor for the square is significantly higher with values of 0.91 and 0.70 obtained from experiments and modelling respectively. These differences between modelling and experimental results for the hexagon and square are not apparent in Figure 9.

### Table 2

<table>
<thead>
<tr>
<th>Density</th>
<th>Method</th>
<th>Hexagon</th>
<th>Square</th>
<th>Triangle</th>
<th>Re-entrant</th>
<th>Double-V</th>
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<td>0.77</td>
<td>0.04</td>
<td>1.00</td>
<td>0.40</td>
</tr>
</tbody>
</table>

This table shows isotropy factors for honeycombs with relative densities of 0.25 and 0.35. The anisotropy of the hexagon honeycomb appears to increase for relative densities of 0.25 and 0.35 with modelling results forming a more elliptical shape. The same is true for the Poisson’s ratios where modelling gives isotropy factors of 0.04 and 0.07 for 0.25 and 0.35 relative densities. The Poisson ratio also decreases with relative density with a maximum of 0.59 and 0.81 at 0.45 and 0.25 relative density respectively. At 0.35 relative density, the modelling results for Poisson’s ratio are lower than experimental results whereas at 0.25 relative density the modelling results are higher.

On inspection of the normalised Young’s modulus results obtained from modelling of the square, it can be seen...
Figure 10: Rotational plots of normalised Young’s modulus ($E_y/E_{y,0}$) and Poisson’s ratio for the honeycombs are shown in the first two columns. Plots have varying axis scale and Young’s moduli have been normalised to the square at 0°. Solid lines represent modelling data and small circular markers represent data obtained through experiments. The darkness of the colour indicates the density of the sample with the darkest being the most dense. Relative densities of 0.25, 0.35 and 0.45 are shown. Plots of normalised Young’s modulus ($E_y/E_{y,45}$) for two rotations are shown in the last two columns. Data has been normalised with the maximum density for the respective data set. Large circular markers correspond to modelling data, small orange markers correspond to experimental data and the solid line is a polynomial fitted to modelling data. Lattices are indicated using the colour scheme introduced in previous plots.
that a similar shape is created for all densities, displaying 4-fold symmetry with significantly varying magnitude. The Poisson’s ratio however varies very little with density with the modelling plots for 0.25 and 0.35 relative densities indistinguishable when viewed at this scale. The isotropy factor approximated by modelling increases to 0.90 at 0.25 relative density, making it more consistent with the experimental value of 0.92. In all three density cases, the modelling predicts the Poisson’s ratio reasonably well, with higher values for the higher densities and lower values for 0.25 relative density.

At lower densities, the modelling results form a less circular shape for the triangle, displaying an apparent 4-fold symmetry. The Poisson’s ratio showed very little variation with density when considering experimental and modelling results. Anisotropy is also predicted by the modelling for the Young’s modulus at a relative density of 0.25, with minima at 0°, 90°, 180° and 270°, and maxima at 45°, 135°, 225° and 335°. The experimental results for the 0.25 and 0.35 samples are remarkably similar for Poisson’s ratio and the values fluctuate around the modelling results.

The shape of the plots obtained by modelling for the re-entrant are repeated for all densities. The Young’s modulus follows the same 2-fold symmetrical shape, increasing in magnitude with density whereas the Poisson’s ratio only changes slightly when negative at 0°, 90°, 180° and 270°. The modelling gives accurate approximations for Poisson’s ratio at 0° and 180° yet slightly lower values at 90°, 270°.

The double-V shows the most variation with density with regards to shape and symmetries. Modelling plots for normalised Young’s modulus not only show a change in shape but the angles of the maxima change from 0° and 180° at a relative density of 0.25 to 90°, 270° at 0.45. On inspection of the modelling plots for Poisson’s ratio, the magnitude and the shape change significantly. For honeycombs with a relative density of 0.45, values change from positive to negative whereas for all other densities, values are negative for all rotations. Approximated values from modelling are lower than experimental result however the approximations are more accurate for samples with a relative density of 0.25. At this density, the shape of the plot has changed significantly and has transitioned to one with 2-fold symmetry from one with 4-fold symmetry at a relative density of 0.45.

Plots of normalised Young’s modulus ($E_{np}^*$) versus relative density are also shown in Figure 10. For these plots, results for Young’s modulus ($E_\theta$) have been normalised with the Young’s modulus at a 0.45 relative density ($E_{0.45}$) for their respective pattern and angle. As a result, no quantitative comparisons can be made between the plots. Angles have been chosen based on the availability of experimental data for comparison; experiments were only carried out at angles of 0° and $\theta/2$. Modelling and experimental results have been normalised independently using the equation

$$E_{np}^* = \frac{E_\theta^*}{E_{0.45}}$$

Solid lines represent polynomials fitted to modelling results, large points represent modelling results and small orange points represent experimental results.

A quadratic polynomial provides a reasonably accurate fit for the modelling results for the hexagon at 0° and 30°. The experimental results are also very close to those obtained from modelling. A straight line has been fitted to the results for the square at 0°, however there is significant scatter. Modelling predicts the normalised Young’s modulus at a relative density of 0.35 fairly accurately whereas the value for 0.25 is significantly lower with a value of 0.3 as opposed to a value of 0.5 obtained experimentally. Young’s modulus appears to rise with density in steps rather than a straight line. For the square at 45°, a quadratic polynomial fits the results well. Approximations provided by the modelling are lower than those obtained experimentally which also display a more linear trend. Straight lines have been fitted to the results for the triangle at 0° and 30° with little scatter.

Modelling matches experimental results remarkably closely for 0°, but for 30° gives lower Young’s modulus values at a relative density of 0.25. The line fitted to the results for the re-entrant at 0° is almost straight however there is some scatter in results. Modelling predicts experimental results for this orientation well, with values slightly lower for a relative density of 0.25 and slightly higher for 0.35. A quadratic polynomial fits the results for the re-entrant at 90° closely. The modelling also matches with experimental values extremely well with values just slightly higher at a relative density of 0.35. A quadratic polynomial has been fitted to the results for the-double-V at 0° and 90°, with only the result for a relative density of 0.45 deviating from the line for both orientations. Modelling approximations are fairly close to the experimental results for 0°, being only marginally lower. For a relative density of 0.25 at 90°, the modelling results predicts experimental values accurately, whereas values predicted for Young’s modulus are lower than obtained experimentally for 0.35 relative density.

4. Discussion

Results from this detailed analysis and comparison of the mechanical properties of periodic honeycombs has provided confirmation for some established expectations about these structures, and has contradicted some expectations.

4.1. Manufactured samples

The dimensions of the sample and pattern parameters, along with the toolpaths and print parameters appear to produce reliable samples without noticeable defects whilst displaying repeatable properties. Mechanical properties also follow the trend expected of structures manufactured out of ductile materials. In Figure 11 yield stress has been plotted against Young’s modulus for honeycombs with relative densities of 0.45. The data forms the expected diagonal trend of increasing yield stress with modulus typical of materials that deform plastically [47].

Figure 12 shows a plot of yield stress versus Young’s modulus for all honeycombs at two rotations with varying
density. As with Figure 11 an approximate linear relationship of increasing yield stress with Young’s modulus can be observed. The relationship is more profound when only considering one honeycomb in one rotation.

4.2. Mechanical testing

DIC has proven to be a valuable tool to measure the strain in samples to large deformations, particularly for obtaining transverse strain and consequently Poisson’s ratio. Calculating Poisson’s ratio using DIC can be difficult as strains are derivatives of measured displacements, consequently Poisson’s ratio is the division of two derivatives making results sensitive to noise. Displacements are often smaller in the transverse direction so noise is augmented and Poisson’s ratio calculations can vary significantly [48]. Poisson’s ratios have been successfully measured with DIC and validated with approximations from modelling. Figure 8 shows any variations to be insignificant when considering the amplitude of the fluctuations compared with the magnitude of the measurements. These measurements have been carried out using live strain gauges where analysis is only performed on a small subset of the total image in real time, thus, removing the need for time consuming processing of images.

By centering a pattern within the honeycomb and rotating it within fixed sample boundaries, rotational analysis was carried out experimentally at a resolution of between 7.5° and 22.5° for six patterns. This was made possible by selection of samples based on rotational symmetries, automated generation of 3D CAD models and production of samples using FDM. Using these methods it is feasible to carry out full rotational analysis experimentally, enabling investigations into the isotropy of honeycombs for large deformations.

4.3. Isotropy

Standard theory for investigating the mechanical properties of honeycombs, as introduced by Gibson and Ashby [1], predicts isotropy for hexagon honeycombs with a relative density under 0.3. Modelling and experiments conducted in this investigation also suggest near isotropy for higher relative densities of 0.35 and 0.45, with the isotropy factor decreasing with density and ranging between 0.02 and 0.04 for modelling and between 0.1 and 0.16 for experiments. The isotropy factor increases to 0.07 for modelling results with a relative density of 0.25. The isotropy factor also decreases with density for the triangle, ranging between 0.06 and 0.09 for experiments and 0.04 and 0.15 for modelling. A possible reason for this negative correlation is as a result of partial cells at the sample boundaries. These exist as a result of cropping a pattern to achieve the required size and relative density. For the chosen modelling technique, partial cells do contribute to the overall structure, but differently to full cells which is in contrast to modelling using a unit cell [1]. As density is reduced, the proportion of the structure containing partial cells is increased providing an increase in the inequality between the mechanical properties in orthogonal directions. The results also show no signs of correlation between the directional mechanical properties and the rotational symmetry of the pattern. This is because the effective mechanical properties are obtained from analysis on one orientation, with directional properties achieved through transformation.
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of the compliance matrix. In reality, the models used for simulations have 2-fold or 4 fold symmetry, and this is displayed in the polar plots. The isotropy factor appears to increase with density for experiments however this is assuming the maximum and minimum Poisson’s ratio have been captured. For experimental samples, it is also worth noting that partial cells at the side boundaries do not contribute to the structure, and depending on orientation, partial cells at the top of the sample may weaken or strengthen the structure. The top and bottom plates also have the effect of constraining the structure and therefore may increase the effective Young’s modulus. It is apparent from studying Figure 9 that the level of isotropy is not always consistent when considering yield strength and compressive strength and the shapes of the plots take on different forms as with the hexagon, square and triangle.

The isotropy factor is appropriate for quantifying isotropy as it takes into account the magnitude of the Poisson’s ratio as well as the spread of results. One draw back is that it returns 1 when Poisson’s ratio varies between positive and negative values as with the auxetic honeycombs making comparisons impossible using this method. Furthermore, isotropy factors for experiments carried out on the lower densities are not valid as the maxima did not occur at 0° or θ/2° for all but one density of double-V honeycomb.

Figure 11 provides an additional form of visualising isotropy. The hexagon and triangle honeycombs show considerably less spread of data than others again demonstrating their higher isotropy.

4.4. Density

Cellular solids can be grouped into two categories, bending-dominated and stretching-dominated. This categorisation depends on how the structure deforms under an applied load. For bending-dominated structures, struts rotate around the joints and bending occurs whereas for stretch-dominated structures, struts are loaded axially. The dominating deformation mode is not only dependent on the structure itself but on the direction from which it is loaded [49]. The square honeycomb not only shows very high anisotropy, but very different trends in Young’s modulus with increasing density for different rotations. Trend lines for the lattices at 0° and 45° have been plotted in Figure 13, all results have been normalised to the square at 0°.

As expected, the square at 0° follows a more linear trend than at 45° because the square lattice is stretch-dominated at 0° (bending moments are equal to zero) and bending-dominated 45°. The density is dependent on the thickness of the cell wall (t) and the length of the cell wall (l). Theory predicts that the effective Young’s modulus of stretching-dominated patterns such as the square at 0° and triangle at all angles have a linear relationship with \( t \) and the modulus for the base material, whereas bending-dominated patterns do not [1] [45]. Increasing density by varying cell size and fixing sample size however produces scatter in results. Effective properties are determined largely by the quantity of cell walls aligned in the loading direction and increasing density does not always add to this. This may be a possible reason for the stepped appearance of the plot for normalized Young’s modulus. Large increases in effective modulus are also experienced when the cell walls lie on the sample boundary leading to anomalies where a slight increase in density causes a drop in Young’s modulus, this is evident in Figure 10. Although inconsistencies from varying cell size and fixing sample size can be avoided by analysis of the unit cell and varying cell wall thickness, simulations would not be as representative of the physical behaviour observed in the experiments and the method would not be transferable to non-periodic honeycombs.

The more linear trend of Young’s modulus versus density for the triangle is to be expected considering the lattice is stretch-dominated so almost all in-plane forces are axial and therefore stresses are increased linearly with the thickness of the cell walls for a given force. The isotropy factor remains almost constant for all densities with the exception for a density of 0.25 where large fluctuations can be seen in the polar plot for Poisson’s ratio. This could be due to the increased proportion of the structure taken up by partial cells at the boundaries as theorised earlier.

It has been shown that density has a large effect on auxeticity at different orientations. Experimental results for a relative density of 0.45, show that re-entrant and double-V are only slightly auxetic. Poisson’s ratio is determined by the geometry of the unit cell, however for bending dominated structures at higher densities, \( \nu \) may reach a value where the
dominant deformation mode is shearing opposed to bending. This has been demonstrated experimentally for hexagons in previous studies [50]. The re-entrant and double-V have similar Poisson’s ratios for orthogonal orientations at some densities yet when full rotations are considered, Poisson’s ratio’s can vary vastly.

4.5. Equivalence of modelling and experimental techniques

The effects of boundary cells on effective properties are likely to be different for experiments and simulations. The necessity for top and bottom plates on manufactured samples may also influence the effective properties of each honeycomb inconsistently for different patterns, rotations and densities. The Datasheet value for the Young’s modulus of the 3D printed PLA in the direction of the principle stress has been used for simulations. In reality the printed PLA will vary with full thickness and databases values often differ greatly from printed samples [30]. Inevitable residual stresses will be present in samples due to the FDM process [51], the effects of these are hard to quantify within the scope of this study. To reduce the need for computational resources, simulations have been simplified to a 2-dimensional problem in plain strain conditions. In fact, the out of plane stresses are more complex than assuming plane strain conditions as the discontinuities of stress at cell wall interfaces can affect the effective properties of each honeycomb differently [52]. Despite these assumptions and simplifications, modelling has provided reasonably approximations for normalised Young’s modulus and Poisson’s ratio.

5. Conclusions

Targeted and systematic rotational analyses of three classical and two auxetic honeycombs has been carried out experimentally on 3D printed PLA samples. Samples were produced using FDM and had comparable toolpaths to ensure consistency throughout the range of patterns investigated. Values for Young’s modulus, Poisson’s ratio, yield strength and compressive strength were measured. Full rotational analysis was carried out using an adaptation to asymptotic expansion homogenisation (AEH) and values for Young’s modulus and Poisson’s ratio were obtained. When Young’s modulus was normalised against the square honeycomb at 0°, modelling provided a good approximation for the results found experimentally. Polar plots in Figures 9 and 10 illustrate the isotropy and rotational symmetries of the various honeycombs and isotropy factors were comparable in most cases. The Poisson’s ratios determined from modelling predicted those found experimentally and in all cases the shapes of the plots were similar. Investigations were carried out experimentally for three different densities with a reduced set of rotations. Simulations provided full rotational analysis on a range of densities. Modelling and experimental results were normalised against their respective maximum and compared. A close correlation was found between results obtained from experiments and simulations. It was found that Poisson’s ratio remained relatively constant with density for a given orientation and significant variations were observed for others. The re-entrant honeycomb showed variations when loaded orthogonally but showed very little variation for intermediate rotations whereas the contrary was found for the square. The triangle showed very little variation with density for all rotations however the double-V showed variations with rotation and density, with maxima occurring at different angles with changes in density.

By comparing the experimental data from compression tests with data obtained from linear elastic simulations, a basis for comparing equivalent novel honeycombs has been presented and gives a clear indication of the orientational dependence of the mechanical properties of the honeycombs. Moreover, comparisons can be made with any novel honeycomb, including non-periodic honeycombs where no unit cell can be defined and effective properties must be modelled using a representative patch. This study has shown that it is feasible to carry out rotational analysis experimentally to a reasonable resolution by automatically generating rotated samples and printing using FDM. AEH can also be used to carry out full rotational analysis in a very short time, just over an hour for the 45% dense honeycombs in this study. This can be used to direct experimental studies. This study validates long held assumptions about the isotropy of dense honeycombs and also sheds light on how auxetic honeycombs behave when not loaded orthogonally and for varying density.

References

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URL https://doi.org/10.1016/j.compositesci.2018.08.096


URL https://doi.org/10.1016/j.polymertesting.2018.05.020


URL https://doi.org/10.1016/j.ijimpeng.2019.04.010


URL https://doi.org/10.1016/j.compositesb.2019.10.7219


URL http://dx.doi.org/10.1016/j.ijmecsci.2014.02.011


URL https://doi.org/10.1016/j.ast.2020.106107


URL http://dx.doi.org/10.1016/j.tws.2017.06.008


URL https://doi.org/10.1016/j.compositesb.2019.107219


URL https://doi.org/10.1016/j.matdes.2023.111922


URL https://doi.org/10.1016/j.ijmecsci.2020.106021


URL https://doi.org/10.1016/j.rimat.2022.100293


URL http://dx.doi.org/10.1108/13552540210441166

[25] FreeCAD, FreeCAD.

URL https://www.freecad.org/


URL https://www.freecad.org/

[27] Ultimaker, Cura.

URL https://ultimaker.com/software/ultimaker-cura


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Declaration of Interest Statement

**Declaration of interests**

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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