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Abstract

This paper introduces a formal conceptual framework for software development, based on a *problem-oriented* perspective that stretches from requirements engineering through to program code. In a software problem the goal is to develop a *machine*—that is, a computer executing the software to be developed—that will ensure satisfaction of the *requirement* in the *problem world*.

We regard development steps as *transformations* by which problems are moved towards software solutions. Adequacy arguments are built as problem transformations are applied: adequacy arguments both justify proposed development steps and establish traceability relationships between problems and solutions.

The framework takes the form of a *sequent calculus*. Although itself formal, it can accommodate both formal and informal steps in development. A number of transformations are presented, and illustrated by application to small examples.

Index Terms

Software engineering, problem-orientation, problem world, sequent calculus, solution, transformation

I. Introduction

> "We can start with the obvious statement that engineering is a problem solving activity." (Walter G. Vincenti, [44])

The problem-oriented approach to requirements engineering is becoming well established, especially for software intensive systems. The foundation of problem-oriented approaches has been laid in recent years, the most complete presentation being given in [25]. Software development is viewed as a *problem*, the *solution* being a *machine*—that is, a program running in a computer—that will ensure satisfaction of the *requirement* in the given *problem world* or *environment*. Because the requirement typically concerns properties and behaviours that are located in the problem world at some distance from its interface with the machine, requirements are distinguished from *specifications*.

A. Requirements and Specifications

A specification describes the behaviour of the machine at its interface with the problem world, while a requirement describes the effects that must be ensured, directly or indirectly, by that behaviour. To derive a specification from a requirement it is necessary to analyse

the given structure and properties of the problem world that will both constrain and enable satisfaction of the requirement. The ultimate justification of a proposed solution must be a convincing argument that the following proposition is true:

> "If a machine satisfying the specification S is installed in the given problem world having the given properties W, then running that machine will ensure that the problem world satisfies the requirement R."

or, more tersely, $W, S \vdash R$. The entailment denotes a problem in which S and R are given and the solution task is to obtain an S satisfying the entailment.

For any realistic problem a solution must be approached by a number of development steps: an initially vague R or W must be clarified and made more exact; a complexity may be addressed by decomposition; a known pattern of solution may be brought to bear where it is applicable; or the impact of some part of the problem world may be carefully analysed and encapsulated so that it need not be considered in later steps. We regard all of these steps, and others, as *transformations* of problems.

B. Transformations

There is a substantial body of work on development approaches based on transformation of problem or solution or both. Some transformations of the kind we define in this paper are found in such formal approaches as the development of program code from specifications in the refinement calculi of Morgan [31] and Back [2]. Some earlier work, for example the work of Burstall and Darlington [14] and Partsch [4], used transformations of program texts to improve efficiency. All these approaches focus on the later stages of development, being concerned with problems that are already cast in the form of program specifications.

Our own focus is wider. We are concerned not only with requirements engineering, where the problem-oriented approach has its origins, but more generally with *software engineering*. In our view, software engineering includes the identification and clarification of system requirements, the identification, structuring and analysis of the problem world, the structuring and specification of a hardware/software machine that can ensure satisfaction of the requirements in the problem world, the creation of the software product, and the construction of arguments, convincing to developers, customers, users and other interested people, that the system will satisfy its requirements.

Our conceptual framework takes the form of a Gentzen-style calculus [26] for software problems. The basis of a Genzten-style calculus is a *sequent*: a sequent is, simply, some well-

formed formula (wff). In the calculus a sequent can be transformed into one or more other sequents that advance the calculation towards its desired conclusion, or are, in some sense, easier to work with. The essence of a Genzten-style system is to define the set of permissible *transformations* on sequents, and the conditions under which each can be applied. Many logical frameworks—such as the propositional and predicate calculi—have Gentzen-style sequent encodings that, in turn, have useful properties, such as being amenable to computer supported transformation (for instance, [6], [17], [5], [13]).

In our framework the sequents are *software problems* cast in the general form $W, S \vdash R$. The paper shows how to represent problems as sequents, and characterises problem transformations and the conditions for their applicability.

C. Formal and Non-Formal

For software-intensive systems, development is unavoidably concerned with non-formal domains of reasoning. These include: the physical and human world; requirements loosely expressed in natural language; the capabilities of human users and operators; and the identification and resolution of apparent and real conflicts between different needs. In a software-intensive system these informal domains interact with the essentially formal hardware/software machine: an effective approach to system development must therefore deal adequately with the informal, the formal, and the relationships between them.

As Turski has pointed out [42]:

> There are two fundamental difficulties involved in dealing with non-formal domains (also known as the real world):
>
> 1. Properties they enjoy are not necessarily expressible in any single linguistic system.
> 2. The notion of mathematical (logical) proof does not apply to them.

These difficulties, well known in the established branches of engineering, have sometimes led to a harmful dichotomy in approaches to software development. Some approaches address only the formal concerns, usually in a single formal language; others address only the informal concerns, using several, often incommensurable, languages; few address both in an effective way.

The aim of the work reported here is to bring both non-formal and formal aspects of development together in a single framework. The framework is intended to provide a structure within which the results of different development activities can be combined and reconciled.

Essentially the structure is the structure of the progressive solution of a system development problem; it is also the structure of the argument that must eventually justify the adequacy of the developed system. The framework is itself formal, but it is designed to accommodate both formal and informal descriptions of machines, problem domains and requirements, and also formal and informal arguments justifying claimed relationships between development artefacts. To capture the informal nature of such arguments and the relationships they justify, we distinguish between a *transformation rule* and the *justification* of its applicability to the case in hand.

In a calculus of fully formal sequents, application of a permissible transformation is guarded by the syntactic form of the sequents involved. In our framework, the application of a transformation rule is guarded, but not necessarily by conditions requiring formal proof. Some transformations have formal guards, but some have guards that ask only for justification that the application is *adequate*. For example, a transformation in which a loosely-stated requirement is clarified may be justified by appealing to an informal statement by the customer. A transformation which depends on a behavioural property of some part of the problem world might be justified by testing. Testing is, of course, a standard way of checking functional adequacy, and serves a useful purpose in real-world software engineering: that we can use it, and other informal techniques, as justification is an important characteristic of our framework.

D. Structure of the Paper

The paper is organised in six further sections. In Section II-C, we introduce the basis of the framework, the *problem*, defining it and discussing their characteristics, with examples. In Section III we discuss the way in which problem can be related, and the role of justification in that relationship. In Section IV, we define a number of problem transformation schemata—rules that characterise class of problem transformation that can be used in the problem solving involved in software engineering. In the case study of SectionV we show software development problem can progress in our framework. There is a discussion of related work in Section VI, and we conclude the paper with discussion, conclusions, and ideas about the future of the framework in Section VII.

II. SOFTWARE PROBLEMS

We define a (software) problem as requirements in a real-world context. A context is a collections of domains $D_1, ..., D_n$ described in terms of their known, i.e., indicative, properties and interacting through the sharing of *phenomena*; the requirements $R_1, ..., R_m$ are statements of what we would like to be true of the context given a solution to the problem, i.e., optative statements. Thus, a software problem challenges us to find the solution that, in the given context, brings about the requirements. Fuller descriptions of these elements of software problems, leading to a formal definition, are given below.

A. Phenomena and Domains

In previous work, Jackson [25] describes six kinds of phenomena in two categories:

- the *individual phenomena* that include events, entities and values;
- the *relations* that include states, truths and roles.

This richness is representative of the richness found in development, and so will be useful to us.

A *domain* is a set of related phenomena that are usefully treated as a behavioural unit for some purpose. A domain has a *name* and a *description* that indicates the possible values and/or states that that domain's phenomena can occupy, how those values and states change over time, and which phenomena – shared or unshared – are produced and when. The description may be more or less precise, but should always be interpreted as in the indicative mood, i.e., as expressing fact. That domain named N has description E is written $N : E$. The name allows a domain to be referred to and enhances traceability through a development.

Associated with each domain $D = N : E$ there are three alphabets:

- the *controlled* alphabet: the phenomena shared with other domains, and controlled by D;
- the *observed* alphabet: the phenomena shared with other domains, and observed by D.
- the *unshared* alphabet: all phenomena of D that are not shared with another domain.

That $D(= N : E)$ has unshared alphabet u, controlled alphabet c and observed alphabet o is written as $D(u)^c_o$ (or, sometimes, as $N(u)^c_o$, when convenient).

More or less may be known about a domain and its alphabets, and the knowledge of a domain's description may affect what can be described of it. A domain that has been named, but of which nothing more is known has *null* description.

The language in which a description is written is not prescribed by our framework[1]; however, it should be possible for sense to be made of statements in the language. There is no *a priori* assumption of precision or lack of ambiguity or completeness in a description; a farmer's initial description of a sluice gate that appears in a field might simply be:

The sluice gate was installed three years ago.

If a richer description is required to allow a problem to be solved, then it will have to be found as part of the development. For instance, a developer with knowledge of sluice gates working on a controller for the farmer's sluice gate may be able to interpret the farmer's statement as identifying the *SGVert* model of sluice gate. The rule that accommodates such 'sense-making' is *Interpretation*; see Section IV-A.

1) Solution domains: A (software) solution domain $S = N : E$ is, simply, a named domain that solves (or is intended to solve) a problem. Within our framework, the solution's domain description stands for a physical software solution: as such it may have one of many forms, ranging from a high-level specification, a description written in Z [40], through to program code in some programming language. As a domain, a solution has controlled, observed and unshared phenomena; the union of the controlled and observed sets are called the *specification phenomena* for the problem.

As with other domains, more or less can be known about the solution domain, including the phenomena it shares with its context. Typically, of course, problem solving leads us to the discovery of the solution domain's description (including the phenomena it shares with its context).

B. Requirements

A requirement states how a solution domain should be assessed as the solution to a problem. Like a domain, a requirement is a named description, $R = N : E$. We may know more or less about a requirements description, again using *null* when we know nothing of it. A requirements description should always be interpreted in the optative mood, i.e., expressing a wish.

For a requirement R, there are two alphabets:

[1]It may also be the case that two (or more) stake-holders assign different descriptions of what is, ostensibly, the same domain. In this case, a domain will be further identified with its description 'owner', as well as the name. However, we do not pursue this complication in this paper.

- *refs*: those phenomena of a problem that are *referenced* by a requirement description.
- *cons*: those phenomena of a problem that are *constrained* by a requirement description, i.e., those phenomena that the solution domain's behaviour must influence to be a solution to the problem.

That R refers to *refs* and constrains *cons* is written R^{cons}_{refs}. If *refs* or *cons* refer to phenomena of the solution domain S, they must be specification phenomena.

C. Problems

Domains and requirements are bound together into problems using the *problem building* relation – '$\vdash$' – that separates the problem's indicative context and solution descriptions (on the left) from its optative requirement descriptions (on the right).

Let $W = D_1, ..., D_n$ ($n \geq 0$, $D_i(u_i)^{c_i}_{o_i}$) be a collection of domains, $S(u)^d_p$ a solution domain and $R = R_1, ..., R_m$ ($m \geq 0$, $R_i{}^{cons_i}_{refs_i}$) be a collection of requirements. Then $W, S \vdash R$ is the problem with context W, solution S and requirements R (which will be seen in Section IV-D to be the logical conjunction of the R_i).

Phenomena sharing in a problem is between domains in W, and between those in W and the solution S. Given D_i and S as above, $a \in c_i$, for instance, means either that:

- $a \in o_j$ for some $j \neq i$, or
- $a \in p$.

Because of this, it is not possible to determine, for a single domain outside of its problem context, its unshared, controlled and observed phenomena.

The first step in a problem-oriented development, is the synthesis of the initial problem sequent from a stake-holder's description:

Example: Consider again the sluice gate farmer, mentioned above, and let us suppose that s/he wishes to employ you to develop a sluice gate controller. S/he gives you the following starting point:

> *The sluice gate was installed three years ago. I want to be able to raise, lower and stop the gate. The gate should allow water to flow in the field for ten minutes every three hours.*

To represent the farmer's problem using our notation for problems we will need to identify some named context domains and their descriptions, named requirements and their descriptions, and place them in relation to each other and to a solution domain as a problem. Here

is a possible interpretation as a problem:

$Farmer(raise, lower, stop)^{fc}$: wants be able to raise, lower and stop the *Gate* $Gate(gp)_{cg}$: installed three years ago $Controller_{fc}^{cg}$: null	$\vdash$	$Req_{raise,lower,stop}$: The *Farmer* should be able to *raise*, *lower* and *stop* the *Gate*. The *Gate* should allow water to flow in the field for ten minutes every three hours.

Here we assume that:

- the *Farmer* controls phenomena *fc* that are shared with the *Controller*, the solution to be found;
- the *Controller* controls the *Gate* through the phenomena *cg*.

The farmer's initial description lacks detail, and so to present their description as a problem we have 'guessed' some of the details. These details include, for instance, precisely which domains are involved in the problem context along with which phenomena are shared. In any serious development context, such decisions will need validating. We will show how validation can be done within the framework later in the paper.

Other problems are not so difficult to capture:

1) The null problem: The *null problem* is the problem of which we know nothing:

$$Context : null, Solution : null \vdash Requirements : null$$

Here, *null* is used as *Context*, *Requirement* and *Solution* description to indicate that nothing is known about them. The *null* problem has a distinguished place in problem-oriented engineering; it can be thought of as the starting point of all problem-oriented development, with context, requirement and solution description being developed therefrom. We show how this might be done in Section III.

2) Specification problems: A *specification problem* is a problem that has an empty context ($\emptyset$), a solution *Solution*, of which more or less is known of its description $Desc_S$, and a single, fully known requirement, with description $Desc_R$, that references and constrains only specification phenomena (those phenomena shared with the machine).

$$Context : \emptyset, Solution_b^a : Desc_S \vdash Requirement_b^a : Desc_R$$

Example: Specification problems are typical in formal refinement settings, such as those of Morgan [31] and Back [2]. The following problem (P1) is adapted from [32]

$$P1 : \qquad \emptyset, Soln_{sq}^{rt} : null \vdash Spec_{sq}^{rt} : rt = \lfloor \sqrt{sq} \rfloor \tag{1}$$

in this case the requirement, *Spec*, constrains the value of *rt* to be the greatest integer less than the square root of *sq*. *rt* and *sq* are specification phenomena.

D. Candidate solutions and solved problems

In parallel with, or perhaps following somewhat behind, the search for a solution to a problem is the construction of an argument that justifies the fitness-for-purpose of the found solution with respect to the problem's context and requirements. Actually, the argument the developer will have to construct expresses the adequacy of a *candidate solution* with respect to a number of criteria including, but not limited to, those of fitness-for-purpose as a customer might see it; by the same token, a presented argument might be a portion of that the developer has produced. We will see how an adequacy argument is built in the next section.

Even to be considered a candidate solution, a solution domain must be compatible with its problem context, in the following sense. Let a, b, c, d, o, *refs* and *cons* be sets of phenomena with $c \cap a = \emptyset$. Any *candidate solution* $S(u)^a_b$ to a software problem with context $W = D_1, ..., D_n$, with $W(d)^c_o$, and requirements $R = R_1, ..., R_m$, with R^{cons}_{refs}, must be such that $cons \cup refs \subseteq o \cup c \cup a \cup b \cup d$.

For a problem to be solved, the customer[2] must be convinced that a candidate solution is adequate in meeting their needs, as expressed through their requirements in context, so that it can be accepted as a solution. The conceptual framework provides guidance for constructing adequacy arguments, built step-wise during development, that can be exposed to the customer to convince them of solution-hood. The actual notion of adequacy depends on the context of the development. For example, in safety-critical development, it may be that the appropriate level is somewhere close to formal mathematical proof; should the development be mission-critical, it may be that rigourous testing will be the appropriate level; should the development be entirely informal, it may be that no justification is required. This context dependence of the system we define – that adequacy of a rule application depends upon the development context – is a radical departure from traditional Gentzen-style sequent calculi.

Example: Specification problems are easy to solve within the framework, (but only if one is content with a non-computational solution *specification*). Consider, again, the specification problem, $P1$, defined above. To solve $P1$, one can simply write (cf., [32]):

$$\emptyset, Soln^{rt}_{sq} : rt := \lfloor \sqrt{sq} \rfloor \vdash Spec^{rt}_{sq} : rt = \lfloor \sqrt{sq} \rfloor$$

[2] And/or other stake-holders.

and call the problem solved. That a specification is its own solution follows from the fact that *Spec* satisfies *Spec*, for any *Spec*.

It is worth noting that, although in this case solution and requirement descriptions are the same, the solution description is indicative while the requirement description is optative. So, although descriptions may, themselves, often be expressed in indicative or optative mood, it is actually their position in the problem that defines the actual mood in which the description should be read: indicative on the left and optative on the right of the problem builder.

Returning to problem $P1$, if one needs a code solution for a problem, one must work harder:

Example: Consider again the formal specification example above; we may also write:

$$
\begin{array}{lll}
 & & [\![\textbf{var } ru; rt, ru := 0, sq+1; \\
 & & \quad \textbf{do } rt+1 \neq ru \rightarrow \\
 & & \quad\quad [\![\textbf{var } rm; rm := (rt+ru)/2; \\
P2: & \emptyset, Soln^{rt}_{sq}: & \quad\quad \textbf{if } rm^2 \leq sq \rightarrow rt := rm \vdash Spec^{rt}_{sq} : rt = \lfloor \sqrt{sq} \rfloor \\
 & & \quad\quad \textbf{else } rm^2 > sq \rightarrow ru := rm \\
 & & \quad\quad \textbf{fi }]\!] \\
 & & \quad \textbf{od }]\!]
\end{array}
\tag{2}
$$

with $P2$ giving an (essentially) executable solution for problem $P1$. The justification is:

> This solution is constructed by formal refinement, the complete development (with proofs) being contained in [32]; we note, however, that as the value of *rt* changes throughout the execution of the code, this computation must be execute atomically for correctness.

This justification would be adequate for even the most safety-critical of safety-critical developments.

III. Software Problem Transformation

Problem $P2$ is related to problem $P1$ in that, in it, there is a detailed description of the *Soln* for $P1$. Moreover, this relationship is *justified* by an argument of the formal correctness of the solution, so that we can say $P1$ is solved.

We would like to make this relationship formal so that, in this case, $P2$ is justified as $P1$ transformed, capturing the progression of $P1$'s development. *Software problem transformations* capture and formalise such relationships: a software problem transformation transforms

a single problem – the conclusion – to a set of problems – the premises – and records an argument – the *adequacy argument step* – that justifies the relationship of the premises to the conclusion.

Suppose we have problems $W, S \vdash R$, $W_i, S_i \vdash R_i$, $i = 1, ..., n$, $(n \geq 0)$ and *adequacy argument step* J, then we will write:

$$\frac{W_1, S_1 \vdash R_1 \qquad ... \qquad W_n, S_n \vdash R_n}{W, S \vdash R} \langle\langle J \rangle\rangle \tag{3}$$

to mean that:

> S is a solution of $W, S \vdash R$ with *adequacy argument* $(CA_1 \wedge ... \wedge CA_n) \wedge J$ whenever $S_1, ..., S_n$ are solutions of $W_1, S_1 \vdash R_1, ..., W_n, S_n \vdash R_n$, with adequacy arguments $CA_1, ..., CA_n$, respectively.

The search for a solution extends the tree upwards and, as in other Gentzen-style sequent calculi, problem transformation applications cascade with transformations lower in the tree producing premise problems (those above the line) that transformations at the next level up use as conclusion problems (those below the line). When there are no premise problems ($n = 0$ in the above definition) we have reached a 'leaf' node of the tree, and upwards development is complete for that part of the problem.

As the tree grows, the adequacy argument is constructed. The reader will note that we make no requirement of formality in the adequacy argument step nor, therefore, in the adequacy argument; the informality of the subject matter precludes fully formal treatment of some transformations. Indeed, in the worst case, a software problem transformation may be applied without any justification, whence we have no adequacy argument for the development as those for the premises, i.e., the CA_i, cannot be extended to the conclusion. Of course, that no justification has been given does not mean that it cannot be retrospectively added. However, a problem transformation that happens with no justification incurs the risk that no justification – and therefore no adequacy argument – will be possible. Attitudes to risk may thus influence the application of rules, and thus software development within the framework. The justification of a problem transformation is, however, necessary if we wish to argue the adequacy of a solution resulting from the development, even if the strongest possible (or practical) justification may still be quite weak.

Example: Unit testing, also called software component testing, represents the "Lowest level of testing that an AUT [Application Under Test] can undergo" [45]. Integration testing, on the other hand, demonstrates that (unit tested) software components work together in 'a correct,

stable, and coherent manner' [45]. The relationship of unit tests to integration test illustrates the relationship represented in the rule above: suppose the S_i are software components such that $W, S_i \vdash R_i$ is solved, unit testing being the appropriate 'correctness argument,' CA_i. Let S be the structure in which the software components are intended to work together[3] with R the requirement of their composition. Then J, the adequacy argument step, is integration testing; the (full) adequacy argument for S is simply the conjunction of integration testing of the composition together with unit testing of the individual components.

Example: We should write:

$$\frac{\begin{array}{l}\emptyset, Soln^{rt}_{sq} : \\ \quad\end{array} \begin{array}{l}[\![\textbf{var } ru; rt, ru := 0, sq+1; \\ \quad \textbf{do } rt+1 \neq ru \rightarrow \\ \quad\quad [\![\textbf{var } rm; rm := (rt+ru)/2; \\ \quad\quad\quad \textbf{if } rm^2 \leq sq \rightarrow rt := rm \\ \quad\quad\quad \textbf{else } rm^2 > sq \rightarrow ru := rm \\ \quad\quad\quad \textbf{fi }]\!] \\ \quad \textbf{od }]\!]\end{array} \begin{array}{l}\\ \\ \\ \vdash Spec^{rt}_{sq} : rt = \lfloor\sqrt{sq}\rfloor \\ \\ \\ \\ \\ \end{array}}{\emptyset, Soln^{rt}_{sq} : null \vdash Spec^{rt}_{sq} : rt = \lfloor\sqrt{sq}\rfloor}$$

⟨⟨This solution is constructed by formal refinement, the complete development (with proofs) being contained in [32]; we note, however, that as the value of rt changes throughout the execution of the code, this computation must be execute atomically for correctness.⟩⟩

to capture the relationship between $P1$ and $P2$ described above. Note that, as the justification of the premise problem is missing, we do not have the adequacy argument for the whole tree. The Problem Solved Rule (SectionIV-B.3) provides the required justification, as we shall see.

IV. Problem Transformation Schemata

Equation 3 (page 12) presents the generic form of a transformation. *Problem transformation schemata* capture classes of problem transformations in named rules. In a problem transformation schema *schema variables*, here written in calligraphic font, are bound to patterns that can occur both in the conclusion of the rule and its premises, thereby indicating relationships between parts of the premises and the conclusion.

In the following sections we define a number of problem transformation schemata.

A. Problem Interpretation

1) Domain description interpretation: (shortly, *domain interpretation*) is the Problem Transformation Schemata (shortly, rule) by which a (non-solution) domain description is

[3]We will see in Section IV-B.3 how such structures are represented in our framework.

interpreted, i.e., re-expressed in some way to make it more suitable for problem solving. It may be that the intention of the interpretation is to clarify a description, to introduce a domain's shared or unshared phenomena, to make it more formal, to disambiguate it, or any other of a multitude of interpretations. The rule is:

$$\frac{\mathcal{W}, \mathcal{N} : \mathcal{E}', \mathcal{S} \vdash \mathcal{R}}{\mathcal{W}, \mathcal{N} : \mathcal{E}, \mathcal{S} \vdash \mathcal{R}} \text{ [Domain Interpretation] } \langle\!\langle \mathcal{J} \text{: Explanation of why } \mathcal{E}' \text{ is better than } \mathcal{E} \text{ for the purpose in hand} \rangle\!\rangle$$

in which $\mathcal{E}'$ is the interpretation of $\mathcal{E}$, the original description of the domain named $\mathcal{N}$. The justification, $\mathcal{J}$, for domain interpretation must argue that an 'adequate' interpretation has been found; the meaning of adequate will, in general, be defined by context. The name of the rule is written above the justification.

Example: Consider again the sluice gate problem. We will assume that the developer wishes to change the *Gate*'s description to one that is more effective for subsequent development. To do this, they apply the domain interpretation rule, with $\mathcal{W}$ bound to *Farmer*, $\mathcal{N}$ to *Gate*, $\mathcal{E}$ to the description 'installed three years ago', $\mathcal{E}'$ to the description 'the *SGVert* model', $\mathcal{S}$ to *Controller*, and $\mathcal{R}$ to *Req*. Whence, with changes emboldened, the rule application gives:

Farmer: wants be able to raise, lower and stop the *Gate* *Gate*: **the** *SGVert* **model** *Controller*: *null*	$\vdash$	*Req*: The *Farmer* should be able to raise, lower and stop the *Gate*. The *Gate* should allow water to flow in the field for ten minutes every three hours.	[Domain Interpretation] ⟪The *SGVert* was the only model of sluice gate produced 3 years ago. It will be easier to work with this description as a manual describing its detailed operation exists.⟫
Farmer: wants be able to raise, lower and stop the *Gate* *Gate*: **installed three years ago** *Controller*: *null*	$\vdash$	*Req*: The *Farmer* should be able to raise, lower and stop the *Gate*. The *Gate* should allow water to flow in the field for ten minutes every three hours.	

We note the first part of the adequacy argument step given–that relating to the installation date of the *Gate*–is relatively weak: if, for instance, the *Gate* was not installed from new, the adequacy argument step may be broken. A much stronger adequacy argument step for this domain interpretation is that

The sluice gate model has been confirmed by field inspection as the *SGVert* model. which provides validation for this particular development step.

2) Requirement description interpretation: (shortly, *requirement interpretation*) is similar in form to domain interpretation, but the subject of the interpretation is the requirement:

$$\frac{\mathcal{W}, \mathcal{S} \vdash \mathcal{N} : \mathcal{E}', \mathcal{R}}{\mathcal{W}, \mathcal{S} \vdash \mathcal{N} : \mathcal{E}, \mathcal{R}} \text{ [Requirement Interpretation]} \ \langle\!\langle \mathcal{J} \text{: Explanation of why } \mathcal{E}' \text{ is better than } \mathcal{E} \text{ for the purpose in hand} \rangle\!\rangle$$

in which $\mathcal{E}'$ is the suggested interpretation of $\mathcal{E}$, the original description of the requirement named $\mathcal{N}$.

Example: Let us assume that, due to discussions with the farmer, it is clear that the *Gate* should allow approximately 45 cubic metres of water to flow in the ten minutes that the gate is open every three hour period. This extra information is useful as it would, for instance, allow a calculation of the height to which the gate must be opened to meet the requirement. Using this extra information, we may interpret the sluice gate requirement to be (changes emboldened):

Farmer: wants be able to raise, lower and stop the *Gate* *Gate*: the *SGVert* model *Controller*: the *Controller* to be found	$\vdash$	*Req*: **The** *Farmer* **should be able to raise, lower and stop the** *Gate***. The** *Gate* **should allow water to flow in the field for ten minutes (***flowRate* $\approx 4.5m^3/minute$**) every three hours.**	[Requirement Interpretation] ⟪The customer has clarified the amount of water that must flow each minute. Knowing the flow rate will allow the height to which the gate must be opened to be calculated.⟫
Farmer: wants be able to raise, lower and stop the *Gate* *Gate*: the *SGVert* model *Controller*: the *Controller* to be found	$\vdash$	*Req*: **The** *Farmer* **should be able to raise, lower and stop the** *Gate***. The** *Gate* **should allow water to flow in the field for ten minutes every three hours.**	

Here the adequacy argument step is predicated on the customer's clarification of what the requirement means.

3) Solution description interpretation: (shortly, *solution interpretation*) is, again, similar in form to domain interpretation, but the subject of the interpretation is the solution:

$$\frac{\mathcal{W}, \mathcal{S} : \mathcal{E}' \vdash \mathcal{R}}{\mathcal{W}, \mathcal{N} : \mathcal{E} \vdash \mathcal{R}} \begin{array}{l}\text{[Solution Interpretation]} \\ \langle\!\langle \mathcal{J} \text{: Explanation of why } \mathcal{E}' \text{ is better than } \mathcal{E} \text{ for the purpose in hand} \rangle\!\rangle\end{array}$$

in which $\mathcal{E}'$ is the suggested interpretation of $\mathcal{E}$, the original description of solution $\mathcal{N}$. The adequacy argument step, $\mathcal{J}$, for an application of the solution interpretation rule must again argue that an 'adequately correct' interpretation has been found.

Example: Revisiting the formal refinement above, we note that problems $P1$ and $P2$ are related through the solution interpretation rule, i.e.:

$$\frac{\begin{array}{l}\emptyset, Soln^{rt}_{sq} : \\ \quad\end{array} \begin{array}{l}[\![\textbf{var } ru; rt, ru := 0, sq+1; \\ \quad \textbf{do } rt+1 \neq ru \rightarrow \\ \quad\quad [\![\textbf{var } rm; rm := (rt+ru)/2; \\ \quad\quad\quad \textbf{if } rm^2 \leq sq \rightarrow rt := rm \\ \quad\quad\quad \textbf{else } rm^2 > sq \rightarrow ru := rm \\ \quad\quad\quad \textbf{fi }]\!] \\ \quad \textbf{od }]\!]\end{array} \vdash Spec^{rt}_{sq} : rt = \lfloor\sqrt{sq}\rfloor}{\emptyset, Soln^{rt}_{sq} : null \vdash Spec^{rt}_{sq} : rt = \lfloor\sqrt{sq}\rfloor} \begin{array}{l}\text{[Solution Interpretation]} \\ \langle\!\langle \text{The solution is a formal refinement of the specification, see [32]; we note, however, that as the value of } rt \\ \text{changes throughout the execution of the code, this computation must be execute atomically for correctness.} \rangle\!\rangle\end{array}$$

B. Problem Expansion

Our framework introduces a standard notation for the initial interpretation of the components of a problem, useful for quickly filling out the detail of a *null* description.

1) Context expansion: works in a problem's context to combine, in a given topology, a number of problem domains. The vehicle for introducing the new domains is a *Context Structure*, or *CStruct*. A *CStruct* is named, and contains the domains that will populate the context. Let $C_i = N_i : Desc_{i_{o_i}}^{c_i}$ be domains, with the c_i pairwise disjoint, and let c and o be such that $\bigcup c_i \setminus \bigcup o_i \subseteq c \subseteq \bigcup_i c_i$ and $\bigcup o_i \setminus \bigcup c_i \subseteq o \subseteq \bigcup_i o_i$[4]. Then:

$$CStructName[C_1, ..., C_m]^c_o$$

is a *CStruct* with name *CStructName*.

When a *CStruct* appears as the description of a context domain, introduced through an application of the domain interpretation rule, the problem can be rewritten as shown in

[4]I.e., a superset of the 'dangling' phenomena.

$$(a) \quad \frac{\mathcal{W}, \mathcal{C}_1, ..., \mathcal{C}_m, \mathcal{S} \vdash \mathcal{R}}{\mathcal{W}, \mathcal{N} : CStructName[\mathcal{C}_1, ..., \mathcal{C}_m], \mathcal{S} \vdash \mathcal{R}} \text{ [Context Expansion]}$$

$$(b) \quad \frac{User_{dsply}^{keyPrs}, Keybd_{keyPrs}^{input}, Scrn_{sigs}^{dsply}, DsplySys_{input}^{sigs} \vdash DsplyReq}{User_{dsply}^{keyPrs}, UIF : KbdScrn[Keybd_{keyPrs}^{input}, Scrn_{sigs}^{dsply}]_{keyPrs,sigs}^{input,dsply}, DsplySys_{input}^{sigs} \vdash DsplyReq} \text{ [Context Expansion]}$$

Fig. 1. The Context Expansion Rule: a) Definition, and b) Example

Figure 1(a). The justification for the introduction of the *CStruct* will appear when it is introduced through the domain interpretation rule, hence the rewriting of a *CStruct* does not need justification.

Example: An example of context expansion is shown in Figure 1(b), in which a *User*'s interface with the machine, *UIF*, has been detailed, through domain interpretation, as consisting of a keyboard (*KeyBd*) and screen (*Scrn*). The *CStruct* is:

$$KbdScrn[Keybd_{keyPrs}^{input}, Scrn_{sigs}^{dsply}]_{keyPrs,sigs}^{input,dsply}$$

the condition on the sharing of phenomena ensures that the expanded *CStruct* shares correctly with its environment.

2) Requirements expansion: allows requirements to be structured. The vehicle for introducing the new requirements is a *Requirement Structure*, or *RStruct*. An *RStruct* is named, and contains the requirements and names that will populate the context. Let $R_i = N_i : Desc_{i\,refs_i}^{cons_i}$ be requirements, $cons = \bigcup_i cons_i$ and $refs = \bigcup_i refs_i$. Then:

$$RStructName[R_1, ..., R_m]_{refs}^{cons}$$

is an *RStruct* with name *RStructName*.

When an *RStruct* appears as the description of a requirement, introduced through an application of the requirement interpretation rule, the problem can be rewritten as shown in Figure 2(a). As for the *CStruct*, the justification for the introduction of the *RStruct* will appear when it is introduced through the domain interpretation rule, hence the rewriting of an *RStruct* does not need justification.

Example: Most software systems have both functional and quality requirements (the latter often called non-functional requirements). In the framework, there is no expectation for quality requirements to be present in a problem. However, we consider it a worthwhile

$$(a) \qquad \frac{\mathcal{W}, \mathcal{S} \vdash \mathcal{R}_1, ..., \mathcal{R}_m, \mathcal{R}}{\mathcal{W}, \mathcal{S} \vdash R : RStructName[\mathcal{R}_1, ..., \mathcal{R}_m], \mathcal{R}} \text{ [Requirement Expansion]}$$

$$(b) \qquad \frac{Context, S \vdash Requirements : null, Qualities : null}{Context, S \vdash Req : R\&Q[Requirements : null, Qualities : null]} \text{ [Requirement Expansion]}$$

Fig. 2. The Requirement Expansion Rule (a) Definition, and (b) example application

heuristic to use the following *RStruct* as the initial requirements interpretation in each problem development, precisely so that any quality requirements are properly recorded:

$$R\&Q[Requirements : null, Qualities : null]$$

with justification that most developments have both forms of requirement. Note that the *RStruct* makes no assumptions about how the respective requirement and quality description will be written, but alerts the developer that one form is missing. If there are *no* quality requirements, this can be recorded explicitly through further development from this rule.

Given this *RStruct*, an example of its application is shown in Figure 2(b).

3) Solution expansion: Within our framework, an *Architectural Structure*, or *AStruct* combines, in a given topology, a number of extant (solution) domains with domains yet to be found. An *AStruct* can be named. For domains $C_i = N_i : Desc_{i_{o_i}}^{c_i}$, and solution domains $S_{jp_j}^{d_j}$, with the c_i and d_j pairwise disjoint, let c and o be such that

- $(\bigcup c_i \cup \bigcup d_j) \setminus (\bigcup o_i \cup \bigcup p_j) \subseteq c \subseteq (\bigcup_i c_i \cup \bigcup d_j)$; and
- $(\bigcup o_i \cup \bigcup p_j) \setminus (\bigcup c_i \cup \bigcup d_j) \subseteq o \subseteq (\bigcup_i o_i \cup \bigcup p_j)$.

Then:

$$AStructName[C_1, ..., C_m](S_1, ..., S_n)_o^c \tag{4}$$

is an *AStruct* with name *AStructName*.

In essence, by application of an architectural structure to a solution domain, we are identifying the internal structure of the solution together with some of its behaviour. There will be more or less known about the extant components, the C_i, that make up the architecture.

Solution expansion assumes that the software structure defined therein has already been identified as adequate, through solution interpretation, so that the application of solution expansion does not require any justification. Solution expansion simply moves the already

$$\frac{\begin{array}{c}\mathcal{W}, \mathcal{C}1, ..., \mathcal{C}m, \mathcal{S}2 : \mathit{null}, ..., \mathcal{S}n : \mathit{null}, \mathcal{S}1 \vdash \mathcal{R} \\ \vdots \\ \mathcal{W}, \mathcal{C}1, ..., \mathcal{C}m, \mathcal{S}1 : \mathit{null}, ..., \mathcal{S}j-1 : \mathit{null}, \mathcal{S}j+1 : \mathit{null}, .., \mathcal{S}n : \mathit{null}, \mathcal{S}j \vdash \mathcal{R} \\ \vdots \\ \mathcal{W}, \mathcal{C}1, ..., \mathcal{C}m, \mathcal{S}1 : \mathit{null}, ..., \mathcal{S}n-1 : \mathit{null}, \mathcal{S}n \vdash \mathcal{R}\end{array}}{\mathcal{W}, S : \mathit{AStructName}[\mathcal{C}1, ..., \mathcal{C}m](\mathcal{S}1, ..., \mathcal{S}n) \vdash \mathcal{R}} \text{ [Solution Expansion]}$$

Fig. 3. The Solution Expansion Rule

defined domains C_i to the environment, by expanding the problem context, whilst simultaneously refocussing the problem to be that of finding the descriptions of the domains S_j that remain. The requirement and context of the original problem is propagated to all sub-problems. The rule is shown in Figure 3.

Solution expansion is more complex than the other rules as it creates a number of premise problems, each of which requires solving, and each of which contributes its solution to the other premise problems.

Example: Consider a journal editor's problem of producing an editing tool (*EditTool*) to assist a *User*, working with a mouse and keyboard ($m\&k$) to express their *ideas*, to prepare a document *Doc* to a specific format (*form*) for a journal. For the purposes of the example, we will assume that a decision has been taken to base the *Editor* on a current LaTeX system. We will assume, therefore, that the LaTeX system has architectural structure description:

$$\mathit{LaTeX}[\mathit{LaTeX\_Env}^{\mathit{stylecmds},\mathit{EditOps}}_{\mathit{form},m\&k}](\mathit{LaTeX\_Style}^{\mathit{form}}_{\mathit{stylecmds}})^{\mathit{EditOps}}_{m\&k}$$

where

$$\mathit{LaTeX\_Env}^{\mathit{stylecmds},\mathit{EditOps}}_{\mathit{form},m\&k} : \mathit{LaTeX} \text{ version } 3.141592$$
$$\mathit{LaTeX\_Style}^{\mathit{form}}_{\mathit{stylecmds}} : \mathit{null}$$

The transformation of the architecture under the *Solution Expansion* rule is shown in Figure 4. Through it, the editor's original problem is reduced to the simpler problem of developing a style file that captures the journal's format for use in an expanded context in which the *LaTeX_Env* appears.

Co-design: Application of the solution expansion rule produces one sub-problem for each to-be-found component in the *AStruct*. When $n \geq 2$, each of the sub-problems has a different to-be-found solution domain as the solution domain, with the original context augmented

$User(idea)^{m\&k}$: uses mouse and keyboard to express their ideas
$Doc(format)_{EditOps}$: a document to be submitted
$LaTeX\_Env^{stylecmds,EditOps}_{form,m\&k}$: *LaTeX* version 3.141592
$LaTeX\_Style^{form}_{stylecmds}$: *null*

$\vdash$

$JEReqs^{format}_{idea}$: The *EditSys* should allow an author to prepare documents expressing their ideas through keyboard and mouse to make a properly formatted submission to the Journal.

[Solution Expansion]

$User(idea)^{m\&k}$: uses mouse and keyboard to express their ideas
$Doc(format)_{EditOps}$: a document to be submitted
$EditSys^{EditOps}_{m\&k}$:
$LaTeX[LaTeX\_Env^{stylecmds,EditOps}_{form,m\&k}]$
$(LaTeX\_Style^{form}_{stylecmds})^{EditOps}_{m\&k}$

$\vdash$

$JEReqs^{format}_{idea}$: The *EditSys* should allow an author to prepare documents expressing their ideas through keyboard and mouse to make a properly formatted submission to the Journal.

Fig. 4. An application of the Solution Expansion Rule to the problem of defining a document editor

by the other to-be-solved components, each with *null* description. When $n \geq 2$, therefore, Solution Expansion leaves many sub-problems to be solved 'simultaneously', the solution of each of which may influence the solution of all others, a situation that we term the *co-design problem associated with the AStruct* (*co-design problem* for short).

We give an example of the co-design problem in the case study in Section V.

Problem Solving: When there are no designable components in the architecture, i.e., $n = 0$ in Equation 4 (page 18), there are no sub-problems generated by application of the solution expansion rule. In this case, the problem has been solved by the choice of architecture. Recall that the architecture will have been introduced and justified through the solution interpretation rule, so this *problem solving* step is justified.

Example: Consider again the journal editor's problem. Let us assume that the editor is willing to re-use another journal's style, contained in the LaTeX style file `JournalStyle.sty`, for instance. In this case, the architecture to be applied under the rule is:

$$LaTeX[LaTeX\_Env^{stylecmds,EditOps}_{form,m\&k}, LaTeX\_Style^{form}_{stylecmds}]()^{EditOps}_{m\&k}$$

[Solution Expansion]

$$User(idea)^{m\&k}, Doc(format)_{EditOps},$$
$$EditSys_{m\&k}^{EditOps} : LaTeX[LaTeX\_Env_{form,m\&k}^{stylecmds,EditOps}, LaTeX\_Style_{stylecmds}^{form}]()_{m\&k}^{EditOps}$$
$$\vdash JEReqs_{idea}^{format}$$

Fig. 5. An application of the Architectural Expansion Rule to the problem of defining a document editor: a complete LaTeX solution is chosen

with

$LaTeX\_Env$	LaTeX version 3.141592
$LaTeX\_Style$	`JournalStyle.sty`

The resulting problem transformation is shown in Figure 5.

Separable problems: A problem $P = W, S \vdash R$ is said to be *n-separable* if there are partitions of W into $W_{i_{o_i}}^{c_i}$, $i = 1, ..., n$, and R into $R_{i_{refs_i}}^{cons_i}$, $i = 1, ..., n$, with the sets $(c_i \cup o_i \cup cons_i \cup refs_i)$ pair-wise disjoint. In this case P consists of n independent sub-problems, each of which is therefore independently solvable. In this case, the *AStruct*

$$nSep[](S_{1_{p_1}}^{d_1}, ..., S_{n_{p_n}}^{d_n}), \qquad \text{with the sets } (d_i \cup p_i) \text{ pair-wise disjoint}$$

is a suitable solution interpretation. For the *nSep AStruct*, solution expansion reduces to:

$$\frac{W_1, S_1 \vdash R_1 \qquad ... \qquad W_n, S_n \vdash R_n}{W, S : nSep[](S_{1_{p_1}}^{d_1}, ..., S_{n_{p_n}}^{d_n}) \vdash R} \text{ [Separable Problem]}$$

One must, of course, be careful that there is no surreptitious sharing of phenomena—one might say *covert channels*—between the partitions of W; if such covert channels exist, separation will not necessarily be justified. This is not, of course, the case with the solution expansion rule.

The case study in Section V contains an example of a 2-separable problem.

C. Problem progression

Problem progression transforms requirements on domains that are 'deep in the world' [25] into requirements on domains that are 'closer to the machine'. The idea behind problem progression is to remove a domain (that we refer to as the *to-be-progressed domain*) from the context of a problem, simultaneously rewriting a requirement description that refers to

$$(a) \qquad \frac{\mathcal{W}, \mathcal{D}(a,b) : Desc_{\mathcal{D}}, \mathcal{S} \vdash \mathcal{R}'_1, ..., \mathcal{R}'_m}{\mathcal{W}, \mathcal{D}^a_b : Desc_{\mathcal{D}}, \mathcal{S} \vdash \mathcal{R}_1, ..., \mathcal{R}_m} \text{ [Sharing Removal]}$$

$$(b) \qquad \frac{\mathcal{W}, \mathcal{S} \vdash \mathcal{R}_1, ..., \mathcal{R}_m}{\mathcal{W}, \mathcal{D}^{\emptyset}_{\emptyset}, \mathcal{S} \vdash \mathcal{R}_1, ..., \mathcal{R}_m} \text{ [Domain Removal]}$$

Fig. 6. (a) Making an assumption in the requirement equivalent to the constraints a domain, $\mathcal{D}$, places on some of its shared phenomena, a and b; requirement $\mathcal{R}_i{}^{cons_i}_{refs_i}$, becomes $\mathcal{R}_i{}^{cons'_i \cup a \cup b}_{refs'_i}$ with $Desc_{\mathcal{R}'_i}$ being "*Assuming* a, b *constrained by* $Desc_{\mathcal{D}}$, $Desc_{\mathcal{R}_i}$". Shared phenomena previously bound by a and b are, after transformation, constrained by the rewritten requirement. (b) A domain that shares nothing is removable from the context of a problem.

or constrains it to compensate. In effect, we want to define rules that allow the following transformation:

$$\frac{D_1, ..., D_{n-1}, S \vdash R'}{D_1, ..., D_{n-1}, D_n, S \vdash R} \text{ [Problem Progression]}$$

in which a solution to the premise problem is a solution to the conclusion problem. Here, we have removed a context domain, the *to-be-progressed domain*, rewriting the requirements to compensate. As there are a finite number of domains in the context of a problem, exhaustive application of such a rule – should this be possible – will eventually end with the progressing requirement being attached to the machine, i.e., it will become a specification.

The implementation of this rules is done in two steps, as shown in Figure 6:

- in the first, shown in Figure 6(a), we 'unshare' phenomena by internalising them into the to-be-progressed domain, D, making an assumption in the requirement equivalent to the constraints that the to-be-progressed domain places on its shared phenomena a and b;
- in the second, shown in Figure 6(b), a domain that shares no phenomena may be removed from the context of a problem.

Example: Consider the problem of developing a software system for an automated car body press. The problem you are given is as follows:

$$Barrier_{pos}, Operator(safe)^{pos,cmnds}, Press_{activate}, Controller^{activate}_{cmnds} \vdash Safe\_Operation^{safe,activate}_{cmnds}$$

which includes a safety barrier, *Barrier*, designed to keep the *Operator* at a safe distance from the press. It does this by constraining the *Operator*'s position, *pos*, in the following way:

Barrier: Constrains the operator's position, *pos*, to be a safe distance from the press during press operation.

The problem requirement states that:

Safe Operation: The press should respond to the commands of the operator. The operator should be kept safe at all times.

We wish to remove the *Barrier* domain from the problem context, to make the problem easier to solve. We note that the *Barrier* cannot simply be removed from the context of a problem (whilst retaining the same requirement) as doing so would lead us to the problem:

$$Operator(safe, pos)^{cmnds}, Press_{activate}, Controller^{activate}_{cmnds} \vdash Safe\_Operation^{safe,activate}_{cmnds}$$

which is a very much more difficult problem to solve in software!

Instead, through the *Sharing Removal* rule, the requirement must be rewritten to remove the shared phenomena between the *Operator* and *Barrier* giving:

$$Barrier(pos), Operator(safe, pos)^{cmnds}, Press_{activate}, Controller^{activate}_{cmnds} \vdash Safe\_Operation'^{safe,activate,pos}_{cmnds}$$

in which

Safe_Operation': Assuming the Operator is at a safe distance from the Press, the press should respond to the commands of the operator. The operator should be kept safe at all times.

As *Barrier* no longer shares phenomena with other domains, it can be removed, thus:

$$Operator(safe, pos)^{cmnds}, Press_{activate}, Controller^{activate}_{cmnds} \vdash Safe\_Operation'^{safe,activate,pos}$$

which, through requirement interpretation, can be rewritten as:

$$Operator(safe, pos)^{cmnds}, Press_{activate}, Controller^{activate}_{cmnds} \vdash Safe\_Operation''^{safe,activate,pos}$$

with

Safe_Operation'': The press should respond to the commands of the operator.

leaving a much simpler problem to solve. The last requirement interpretation is justified by the fact that the barrier always keeps the operator at a safe distance from the press.

D. Other Rules

1) Requirements conjunction: The ',' on the right of the turnstile, i.e., between requirements, is essentially equivalent to conjunction as captured by the rule:

$$\frac{\mathcal{W}, \mathcal{S} \vdash R_1 : Desc_{R_1}, R_2 : Desc_{R_2}, \mathcal{R}}{\mathcal{W}, \mathcal{S} \vdash R_1 \& R_2 : Desc_{R_1} \wedge Desc_{R_2}, \mathcal{R}} \text{ [Conjunction Introduction on the right]}$$

where $R_1 \& R_2$ is the name of the conjoined requirements, and $Desc_{R_1} \wedge Desc_{R_2}$ is the (logical) conjunction of their descriptions. The double bar means that the rule can be used in both directions, both to introduce a conjunction and to split a conjunction.

Note that, in the propositional calculus, ',' on the right is usually equivalent to disjunction, so that $P \vdash Q, R$ and $P \vdash Q \vee R$ are equivalent. In the propositional calculus, equivalence derives from the negation elimination rules, one of which is:

$$\frac{\Gamma, \neg P \vdash \Delta}{\Gamma, \vdash P, \Delta} \neg\text{-elimination on the right}$$

For us, the left- and right-hand sides of the turnstile denote indicative and optative, respectively, and movement left to right is only through the sharing removal rule, whence an assumption is added on the right.

V. Software Development under the Conceptual Framework

As stated earlier, the aim of the work reported here is to bring both non-formal and formal aspects of development together in a single framework. The framework provides a structure within which the results of different development activities can be combined and reconciled.

The structure is the structure of the progressive solution of a system development problem; it is also the structure of the adequacy argument that must eventually justify the developed system. Each is constructed in a step-wise manner: problems form the nodes in a tree with transformations extending the tree upwards from an initial problem description. This is the general form of a Genzten-style sequent calculus. The tree branches through the application of architectural expansion that allows many sub-problems to be discovered from the problem that is its conclusion. Another effect of rule application is that symbols are shared between the conclusion and premisses of a rules application; in essence, the symbols used in a problem development exist within a single name-space.

Software development under the framework proceeds by the application of problem transformations each of which develops some aspect of a software problem's description. Transformation under the framework in general allows detail of the software problem to be discovered

– the context and requirements are more known, and more is known about the structure of the solution. The choice of which problem transformation to apply at any point in a software problem development application is, however, 'tentative' as each transformation must be justified as adequate before the step can be accepted as contributing correctly to the development. Because of this, and as in mathematical proof in Gentzen-style sequent calculi, problem transformation application does not always lead to solution; step-wise transformation is prone to the discovery of 'blind alleys', entered by a poor choice of application of a particular transformation. This reflects the nature of problem solving in that, in its general form, it is exploratory of the problem and solution spaces. All that is needed, of course, is an iterative process that, given the realisation that a particular transformation (or sequence of transformations) has lead to a dead-end (or sub-optimal solution), can backtrack past the problem branches, leaving the development in a better state.

The exploration of design choices, and subsequent backtracking on failure, is a valuable component of rationale for a particular development; it can, for instance, help to prevent re-exploration of designs that did not work. For this reason, and unlike mathematical proof, we counsel that as part of the justification of the particular rule application at a point in development the reason for that choice is recorded: i.e., it is the initial choice or has been chosen because some other choice did not work. The case study illustrates how this may be done.

Of course, it is not our intention to suggest that all software developments take place within the framework; for a start, the rules as defined capture the smallest possible steps in a development. A similar situation exists within mathematics where little, if any, serious proof development work is undertaken within the confines of Genzten's systems. Even in systems in which formal proof is important – e.g., in some safety critical software development – it is often sufficient to rely on the judgement that all large steps taken could be expanded, in principle, to those existing in a Gentzen's system.

Our example, the development of part of a Package Router controller (adapted from [25], in turn adapted from [41]) illustrates the application of the conceptual framework from early requirements through to detailed design of a moderately sized example. The example's development is also used as a vehicle to discuss where larger steps can and should be taken, and numerous other issues.

A. The Package Router Problem

A package router is a large machine used by delivery companies to sort packages into bins according to bar-coded destination labels affixed to the packages. Each bin corresponds to a regional area. Packages slide from a conveyor belt under gravity through a tree of pipes and binary switches. The bins are at the leaves of this tree. The problem is to control the operation of the package router's switches so that packages are routed to their appropriate bins, obeying the operator's commands to start and stop the conveyor, and reporting any misrouted packages.

We will illustrate how the elements of the conceptual framework can fit together, and how a problem-oriented approach can lead to adequacy arguments on the package router. The order of rule application in the case study is not meant to suggest a canonical order of application.

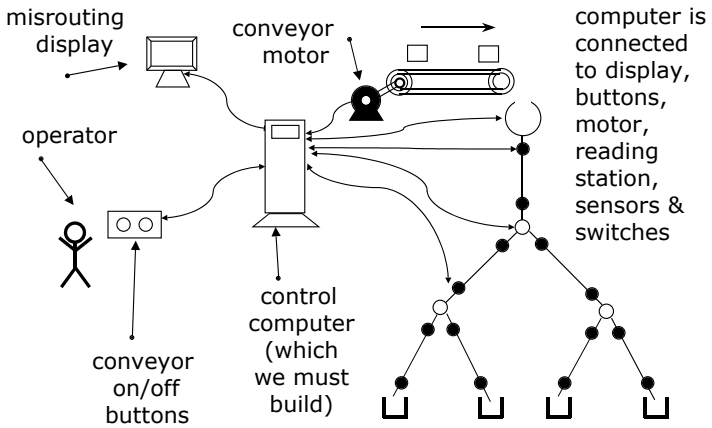


Fig. 7. The Package Router Schematic

1) The null Package Router Problem: As mentioned earlier, all problem-oriented analyses should, notionally, begin from the *null* problem. Our starting point is, therefore, the *null* problem (relabelled for the Package Router *PR*):

$$P_{null}: \qquad PRContext : null, PRSoln : null \vdash PRReq : null$$

2) Developing the null problem: From the *null* problem, problem-oriented analysis continues with an analysis of the customer provided documentation. Here we assume this to consist of the schematic shown in Figure 7 and some other natural language description (shown later). We will consider the elements of this schematic to indicate what needs to be considered as the domains and phenomena of the problem. The schematic must be understood as a proposed arrangement, and perhaps even to be approximate; for instance, there are only three switches, four bins and two packages indicated, which is unlikely for a real system. We

must, therefore, recognise that any early analysis will be no more than an initial attempt at capturing the problem, with iteration being possible should it not lead to a suitable problem expression. Our approach is simply to consider each element of the schematic as one or more domains of the problem, and to take the hard-wired connections as indicative of the connectivity, i.e., the shared phenomena, between domains.

Of course, making this decision raises a number of issues: in modelling the context we always have a choice between aggregating or separating the descriptions of relevant parts of the world; but which is the most fruitful choice for problem analysis? In general, we want to aggregate and model as a single domain identified parts of the world with pertinent behaviours, unless such parts are causally independent. For instance, through aggregation, we would consider the conveyor belt together with the motor. The question here is not one of the correctness of the captured problem – whether separate or joint need not affect the correctness of the description(s) – but complexity: a joint domain description may, for instance, lead to simpler analysis by ignoring, or rather the coalescing of, the failure modes of each component. When reliability, safety, etc, of the system are not issues, then, aggregation of causally related domains may be preferable.

Secondly, an issue arises in that a language for the description of domains should be found. Initially, it may be sufficient to consider iconic representations of domains, as is done below; this choice may have some advantages:

- icons may be accessible both to customer and developer;
- icons may be a good starting point for traceability throughout the development process.

Working from the schematic, we propose (a) the following *CStruct* as representative of the problem's context:

$$\begin{aligned} &\mathit{PackageRouterCStruct}[\mathit{Operator}, \mathit{Display}, \mathit{On/Off\ Buttons},\\ &\mathit{Bin}[1], ..., \mathit{Bin}[\#\mathit{bins}], \mathit{Switch}[1], ..., \mathit{Switch}[\#\mathit{sw}], \mathit{Reading\ Station},\\ &\mathit{Conveyor\ Motor\ and\ Belt}, \mathit{Package}[1], ..., \mathit{Package}[\#\mathit{pks}]] \end{aligned}$$

with domain descriptions being shown in the first column of Figure 8, and the inferred sharing

of phenomena between domains and solution as shown in the following table.

		CONTROLLED								
		Operator	*Display*	*On/Off B*	*Bin[i]*	*Switch[j]*	*R S*	*C M and B*	*Package[id]*	*PRSoln*
O	*Operator*									
B	*Display*									$\neq \emptyset$
S	*On/Off B*	$\neq \emptyset$								
E	*Bin[i]*								$\neq \emptyset$	
R	*Switch[j]*								$\neq \emptyset$	$\neq \emptyset$
V	*R S*								$\neq \emptyset$	
E	*C M and B*								$\neq \emptyset$	$\neq \emptyset$
D	*Package[id]*					$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$		
	PRSoln			$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$			

The entry in column c, row r of the table indicates what is known about the phenomena controlled by domain c that are observed by domain r. For instance, the *Operator* controls (some non-empty set of) phenomena that are observed by the *On*/*Off Buttons*, but we do not know, at this point in the development, the identities of the phenomena through which the sharing is accomplished. Empty entries indicate no sharing (the diagonal is empty because sharing is irreflexive).

The first transformation we apply is a domain interpretation of the *PRContext* domain's *null* description, which leads to Problem $P0$:

$$\frac{PRContext : PackageRouterCStruct, PRSoln : null \vdash PRReq : null}{PRContext : null, PRSoln : null \vdash PRReq : null}$$

[Solution Interpretation]
⟨⟨The *PackageRouterCStruct CStruct* is an initial attempt to model the customer schematic. In the *CStruct* we allow many *Bins* and *Switches*, and do not limit the number of *Packages*, even through there are a fixed number of these in the schematic.⟩⟩

The reader will note that all schematic context domains appear in the *CStruct*, but that the *Bins*, *Switches* and *Packages* are distinguished by an identifier: although there will be a known fixed number of *Bins* ($\#bins$) and *Switches* ($\#sw$) during operation, the number of *Packages* ($\#pks$) that will flow through the system is unbounded (or at least *a priori* unknowable). The result for the framework of this multiplicity, as we shall see, is that certain transformations will need to be applied many times, once per each instance of the domain. We see this as providing no conceptual difficulties within the framework, although it motivates a number of presentational conventions (such as the use of indices to distinguish domains).

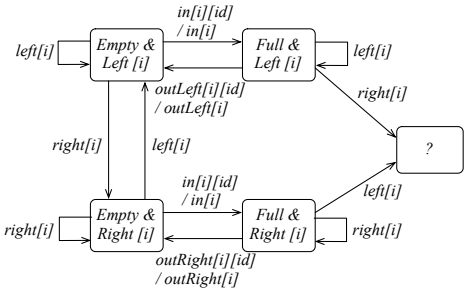
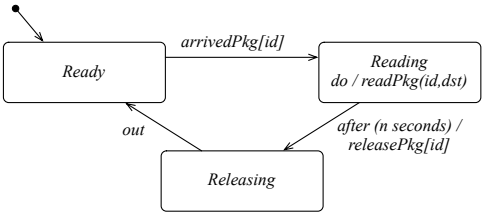
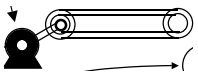
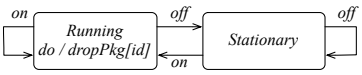
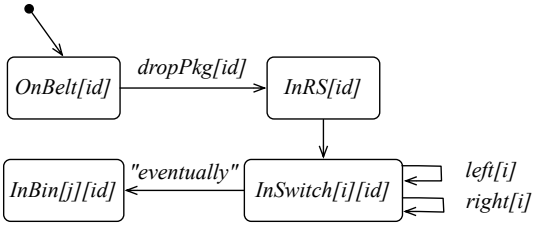
	$P0$ **descriptions**	$P1$ **descriptions**
Operator		"The operator uses the on/off buttons to control the conveyor through the controller."
Display		"The display indicates the state of the system."
On/Off Buttons		"The on/off buttons connect the Operator to the machine."
Bin[j] $1 \leq j \leq \#bins$		"*Bin*[j] has a sensor that fires when a package enters it."
Switch[i] $1 \leq i \leq \#sw$		
Reading Station		
Conveyor Motor and Belt		
Package[id] $1 \leq id \leq \#pks$		
PRSoln	*null*	*null*
PRReq	*null*	*null*

Fig. 8. $P0$ and $P1$: The iconic (column 1) and accumulated (column 2) domain interpretations for the Package Router problem

3) Further problem-oriented analysis: To be able to further understand the problem, it is important to have more detailed descriptions. In this section, we show how such analysis could be conducted within the framework focussing, for brevity, on the behaviours of the *Operator*, the *Conveyor Motor and Belt* and the *Package* domains. (A more complete analysis of the other domains is given in [36].)

Throughout the development, the domain description languages we use are English and the UML 2.0 variant of statecharts [34].

a) Operator: We will suppose that the customer has described the behaviour of the Operator as follows:

"The operator uses the on/off buttons to control the conveyor through the controller."

That this description is not formal does not pose any problems, and although it would not be difficult for a formal description of the *Operator*'s behaviour to be developed–perhaps as a simple statechart–such formalisation adds little to the development while possibly detracting from the traceability and accessibility of other non-technical stake-holders. We will therefore work with this informal description, as far as possible. Other domains, *viz*. *Display*, *On*/*Off Buttons*, and *Bin*s, will similarly be informally expressed.

Using domain interpretation, we may replace the iconic representation of the *Operator* in $P0$ with the natural language description (the result being shown in the third column of Figure 8), update the table of phenomena to reflect the new knowledge of the phenomena shared between the *Operator* and the *On*/*Off Buttons* (by adding on_o, off_o phenomena), and provide the following adequacy argument step for the transformation:

$J1$.*Operator*: The customer has described the *Operator* in this way.

b) Conveyor Motor and Belt description: In contrast to the human *Operator*, we will describe the causal domain *Conveyor Motor and Belt* more formally. The knowledge for a semi-formal description might be available in a manual of operation, for instance. Suppose the manual contains the (semi-)formal description of the *Conveyor belt and Motor*'s behaviour shown in Figure 9, where *on* and *off* are commands controlled by the *PRSoln* solution domain and observed by the *Conveyor Motor and Belt* domain, then a domain interpretation application can be made.

The justification for applying the domain interpretation rule using the statechart of Figure 9 is:

$J1$.*CMB*: for this interpretation step, we have provided a description of the *Conveyor Motor and Belt* domain.

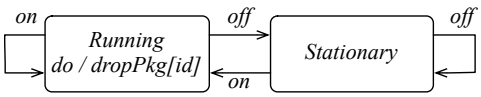


Fig. 9. *Conveyor Motor and Belt* behaviour.

c) The Package Domains: The *Package* domains also need to be described as does their interaction with other domains. The description of the package labelled *id*, *Package*[*id*], is, simply, that it comes from the conveyor belt into the reading station and slides, under gravity, through the switches until it *eventually* comes to rest in one of the bins. The statechart of Figure 10 shows this behaviour and, for instance, identifies *dropPkg*[*id*] as the event that accompanies the drop a package from the conveyor into the reading station.

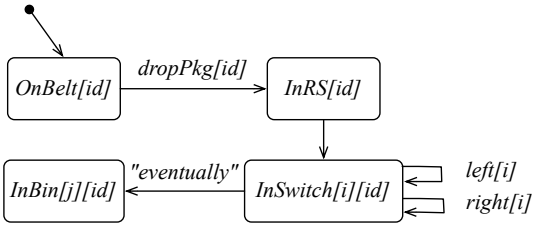


Fig. 10. The behaviour of *Package[id]*.

Aside: An approach to development risk: The description for a package has been arrived at through an analysis of the problem; there has been, for instance, no customer input. During development, we could simply assume that the description in Figure 10 is adequate in capturing the essential behaviour of the domain and leave the analysis at that. However, we may also wish to consider that subsequent development may commit resources – developer time, for instance – on the adequacy of this description, that such resources have an opportunity cost, and so one would wish that such a description was more than just a guess. Dealing with risk is properly part of development processes, and so should have a place within our framework.

We cannot, of course, mitigate development risks through the application of the ideas of the framework. However, that risk has been considered is properly a part of the adequacy argument for a development, and we can record its consideration (and the steps (if any) that have been taken to mitigate it) in the adequacy argument.

Risk management is a complex topic and a more detailed discussion is beyond the scope of this paper. One option would be to ask the customer to validate the statechart description as the basis for further development. For now, we choose just to accept the risk recording the fact in the the domain interpretation rule's justification.

Returning to the *Package* domains, we can apply the domain interpretation rule repeatedly[5] with justification:

> $J1.Pkg$: for this interpretation step, we have provided descriptions of the $Package[id]$ domain. There is a risk that this description is wrong. We have chosen to accept this risk.

to arrive at the description in Figure 10.

4) The other domains: From the iconic representation of Problem $P0$ we may merge eight domain interpretation steps into a single step, the resulting Problem $P1$ being shown in Figure 8. More detail of the shared phenomena is also given by the analysis, as indicated in the following table where some of the entries have been filled in (only phenomena we are going to use in the following have being detailed):

		CONTROLS								
		Operator	*Display*	*On/Off B*	*Bin[j]*	*Switch[i]*	*R S*	*C M and B*	*Package[id]*	*PRSoln*
O	*Operator*									
B	*Display*									$\neq \emptyset$
S	*On/Off B*	$\{on_o, off_o\}$								
E	*Bin[j]*								$\neq \emptyset$	
R	*Switch[i]*								$\neq \emptyset$	$\neq \emptyset$
V	*R S*								$\neq \emptyset$	
E	*C M and B*								$\{dropPkg[id]\}$	$\{on_c, off_c\}$
S	*Package[id]*					$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$		
	PRSoln			$\{on_b, off_b\}$	$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$			

5) Requirements Analysis: We will assume justified transformations have added adequate descriptions for each domain and continue the development by considering the problem's requirement. Suppose that the customer describes the requirement as:

> "The problem is to control the operation of the package router so that packages are routed to their appropriate bins or reported as misrouted, and to obey the operators commands to start and stop the conveyor."

[5]Once for each of the $\#pks$ packages.

so that, using the requirements interpretation rule, the *null* requirement description of Problem $P1$ can be interpreted by this description with justification:

$J2.Req$: This is the initial customer-provided description.

leaving us with Problem $P2$ derived from Problem $P1$:

$\vdots$

PRReq : The problem is to control the operation of the package router so that packages are routed to their appropriate bins or reported as misrouted, and to obey the operators commands to start and stop the conveyor

$\vdots$

Now, let us suppose that further discussion with the customer reveals that this requirement has two components:

Obey Operator: the controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*.

and

Route & Report: the controller should control the package router so that packages are routed to their appropriate bins or reported as misrouted: a package with identity *id* and destination *dst* arriving at the reading station ($arrivePkg[id]$) should reach the bin corresponding to its destination ($bin[id][j]$, where $Bin[j]$ corresponds to *dst*); otherwise misroute information message

"misroute of package *id*"

should be displayed.

We may encode these requirement parts as the *RStruct*:

$$RStruct[Obey\ Operator_{on,off}^{Running,Stationary}, Route\ and\ Report_{dst}^{bin[id][j]}]$$

and interpret the current requirement accordingly, leading to a new Problem $P3$, with justification:

$J3.ReqSplit$: *Obey Operator* and *Route and Report* have been identified and validated by the customer as the requirements for the system.

applying it through the *RStruct* rule (no justification necessary) to give problem:

$$P4: \quad \begin{array}{l} \mathit{Operator}, \mathit{Display}, \mathit{On/Off\ Buttons}, \\ \quad \mathit{Bin}[1], ..., \mathit{Bin}[m], \mathit{Switch}[1], ..., \mathit{Switch}[n], \mathit{Reading\ Station}, \\ \quad\quad \mathit{Conveyor\ Motor\ and\ Belt}, \mathit{Package}[1], ..., \mathit{Package}[k], \mathit{PRSoln} \\ \quad\quad\quad \vdash \mathit{Obey\ Operator}, \mathit{Route\ and\ Report} \end{array}$$

An aside on quality requirements: It is worth noting at this point that there are no quality requirements for this problem. In a mission critical situation, such as our imagined problem context, quality requirements for system availability, for instance, would typically be made and the lack might alert the developer to the need for a discussion with the customer of their availability requirements. It may be that there are requirements of "high availability", perhaps "5 9s available" (i.e., 99.999% availability or, maximum, 5 minutes per year downtime).

*QR*1: The system should be 5 9s available.

There are two parts to the justification of the interpretation that adds *QR*1 to the requirements. The first is shared with all interpretations, i.e., that the description is an adequate description of what the customer wanted, which can be validated in consultations with the customer. This is simply associated with the step and can be ignored when validation is provided.

The second part stems from the fact that it cannot be known whether the solution satisfies a functional or a quality requirement until the solution is built. Hence, at this point in the development, the best we can do is to ensure that future design choices, such as the choice of high-level architecture or components, *do not make it impossible* to satisfy such requirements. A simple example of what we mean is that one should not choose a processor that is known to be very unreliable as the basis of the system, as such a processor is unlikely to meet the quality requirement; however, even the choice of a very reliable processor does not guarantee that the quality requirements will be met, as there are many other choices that could impact the satisfaction of the quality requirements.

Many techniques exist to help with such requirements; for instance, in the safety-critical context, it may be appropriate to conduct a preliminary safety analysis (PSA) on a step [30]. A PSA can help identify that a design choice will lead to an unsatisfiable safety requirement.

Although quality requirements are very important for the development of systems, for brevity, we suppose that there are no quality requirements related to our problem.

6) Separating problems: At this point we note that the requirement description we have arrived at has two parts that only interact through phenomena shared between the *Conveyor*

Motor and Belt and the *Package* domains. It is now our intention to find two sub-problems that can be solved independently of each other, the solutions to which contribute to a solution to the whole problem. For brevity, we focus only on *Obey Operator* as the progressing requirement; the development for *Route and Report* is similar.

We wish to be in a position to 'break the link between' the *Conveyor Motor and Belt* and the *Package* domains so that we have two independent sub-problems that can be solved using the separable problem rule.

Now, from the previous domain descriptions, the *Conveyor Motor and Belt* controls and shares only phenomenon $dropPkg[id]$ with $Package[id]$. We may therefore apply the sharing removal rule[6] to the requirement:

> *Obey Operator'*: Assuming the constraints on $dropPkg[id]$ given by the state machine of Figure 10, the controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*.

with the $dropPkg[id]$ phenomenon becoming unshared in the *Package* domains. (*Route and Report* is similarly restated leading to *Route and Report'*.)

As $Package[id]$ behaviour places no constraints (i.e., neither requires nor refuses) the $dropPkg[id]$ phenomena, we may make a simple interpretation of this requirement, using the added assumption to expand how the conveyor belt and motor work as *Obey Operator'*:

> *Obey Operator''*: The controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*,

which is the original *Obey Operator* requirement. The application of the sharing removal rule does not require a justification and leads to problem:

$$
\begin{aligned}
P5: \quad & Operator, Display, On/Off\ Buttons, \\
& Bin[1], ..., Bin[m], Switch[1], ..., Switch[n], Reading\ Station, \\
& Conveyor\ Motor\ and\ Belt(dropPkg[1], ..., dropPkg[k]), \\
& Package[1](dropPkg[1]), ..., Package[k](dropPkg[k]), PRSoln \\
& \vdash Obey\ Operator'', Route\ and\ Report'
\end{aligned}
$$

with sharing given by the following table (note that $Package[id]$ no longer shares phenomena with *Conveyor Motor and Belt*):

[6] Applied once for each possible *id*.

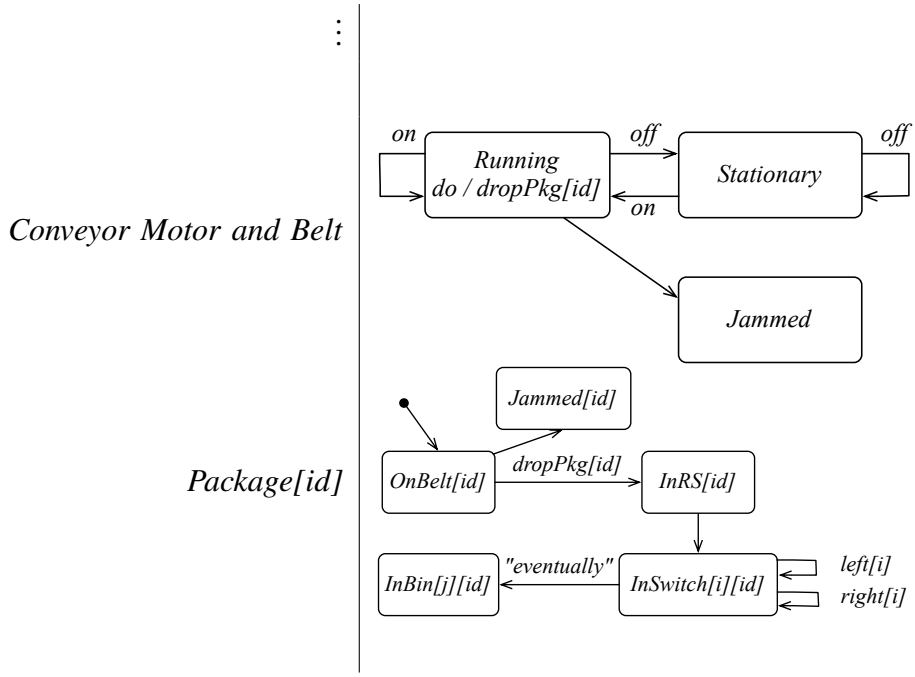


Fig. 11. Supposing the conveyor can be jammed by a package.

		CONTROLS								
		Operator	*Display*	*On/Off B*	*Bin[j]*	*Switch[i]*	*R S*	*C M and B*	*Package[id]*	*PRSoln*
O	*Operator*									
B	*Display*									$\neq \emptyset$
S	*On/Off B*	$\{on_o, off_o\}$								
E	*Bin[j]*								$\neq \emptyset$	
R	*Switch[i]*								$\neq \emptyset$	$\neq \emptyset$
V	*R S*								$\neq \emptyset$	
E	*C M and B*									$\{on_c, off_c\}$
S	*Package[id]*					$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$		
	PRSoln			$\{on_b, off_b\}$	$\neq \emptyset$	$\neq \emptyset$	$\neq \emptyset$			

An aside on development iteration: It is instructive to see how the development changes should the conveyor belt be able to be jammed by packages. So, for the purposes of argument, suppose that the conveyor can be jammed, i.e., it has behaviour shown in Figure 11.

A moment's thought will convince the reader that, when the belt can jam, the *Obey Operator* requirement simply cannot be satisfied: when jammed, an on_o event will not result in the conveyor belt state *Running*. If the belt can jam, then, we would need to revisit

the requirement – by consulting with the customer – finding a more permissive (original) requirement description, such as:

> *No Jam Obey Operator: When not jammed*, the controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*.

In this case, replaying the sharing removal above gives:

> *No Jam Obey Operator'*: Assuming the behaviour given by the state machine of Figure 11, *when not jammed*, the controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*.

or, more simply

> *No Jam Obey Operator: When not jammed*, the controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*.

as the assumption is made redundant by the '*When not jammed*' guard.

We return to the original problem.

7) Choosing a solution architecture: In separating the requirement into two parts, it is our intention to deal with them as separate sub-problems. To do this, we introduce an architecture for the solution, through solution interpretation using an *AStruct*, and expand it using solution expansion. Here, as there are two independent sub-problems, the *PRSoln AStruct* is

$$2Sep[](CC^{ccc}_{cco}, RRC^{rcc}_{rco})^{cc}_{co}$$

such that:

- the *Conveyor Controller CC* is the solution component for the *Obey Operator''* requirement; and
- the *Route and Report Controller RRC* is the solution component for the *Route and Report'* requirement.

The justification for this choice of architecture needs to argue that the architecture is adequate:

> $J6.Architecture$: This architecture is adequate as there are two independent sub-problems.

Application of the solution interpretation rule leads us to update our problem to $P6$:

$$\vdots$$
$$PRSoln \quad 2Sep[](CC^{ccc}_{cco}, RRC^{rcc}_{rco})^{cc}_{co}$$
$$\vdots$$

with a separable problem rule application leaving us with:

$$\frac{\begin{array}{ll} P7: & Operator, On/Off\ Buttons, \\ & Conveyor\ Motor\ and\ Belt, \\ & CC \vdash Obey\ Operator'' \end{array} \qquad \begin{array}{ll} P8: & Display, Bin[1], ..., Bin[\#bin], Switch[1], ..., Switch[\#sw], \\ & Reading\ Station, Package[1], ..., Package[\#pks], \\ & RRC \vdash Route\ and\ Report' \end{array}}{\begin{array}{l} Operator, Display, On/Off\ Buttons, \\ Bin[1], ..., Bin[\#bin], Switch[1], ..., Switch[\#sw], Reading\ Station, \\ Conveyor\ Motor\ and\ Belt(dropPkg), Package[1](dropPkg), ..., Package[\#pks](dropPkg), \\ 2Sep()[CC^{ccc}_{cco}, RRC^{rcc}_{rco}] \\ \vdash Obey\ Operator'', Route\ and\ Report' \end{array}}\ [\text{Separable Problem}]$$

with sharing defined as in the previous table.

8) Problem progression: We now focus our attention on the simpler of the sub-problems, $P7$: that of interpreting the operator's commands as made explicit through the buttons, and of controlling the conveyor through the motor. The first progression removes the *Operator* domain. To develop the problem further, we perform a number of problem progressions, shown in the following cascaded development, to arrive at the specification problem of the *CC* component (the premise of the whole cascaded development):

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{CC^{on_c,off_c}_{on_b,off_b} \vdash CC\ Spec}{CC^{on_c,off_c}_{on_b,off_b} \vdash Obey\ CC''}\ \begin{array}{l}[\text{Requirement Interpretation}] \\ \langle\langle \text{In the } CMB \text{ domain, } on_b \text{ results in state } Running, \ off_o \text{ results in state } Stationary\rangle\rangle\end{array}}{Conveyor\ Motor\ and\ Belt(on_c, off_c), CC(on_c, off_c) \vdash Obey\ CC''}\ \begin{array}{l}[\text{Domain Removal}] \\ \langle\langle CMB \text{ shares no phenomena}\rangle\rangle\end{array}}{Conveyor\ Motor\ and\ Belt_{on_c,off_c}, CC^{on_c,off_c}_{on_b,off_b} \vdash Obey\ CC'}\ [\text{Sharing Removal}]}{Conveyor\ Motor\ and\ Belt, CC_{on_b,off_b} \vdash Obey\ CC}\ \begin{array}{l}[\text{Requirement Interpretation}] \\ \langle\langle \text{In the } On/Off\_Buttons \text{ domain, } on_o \text{ results in an } on_b \text{ event, and the an } off_o \text{ results in a } off_b \text{ event}\rangle\rangle\end{array}}{On/Off\ Buttons(on_b, off_b), Conveyor\ Motor\ and\ Belt, CC_{on_b,off_b} \vdash Obey\ CC}\ \begin{array}{l}[\text{Domain Removal}] \\ \langle\langle On/Off\ Buttons \text{ share no phenomena}\rangle\rangle\end{array}}{On/Off\ Buttons^{on_b,off_b}, Conveyor\ Motor\ and\ Belt, CC_{on_b,off_b} \vdash Obey\ Buttons'}\ [\text{Sharing Removal}]}{On/Off\ Buttons, Conveyor\ Motor\ and\ Belt, CC \vdash Obey\ Buttons}\ \begin{array}{l}[\text{Requirement Interpretation}] \\ \langle\langle \text{In the } On/Off\_Buttons \text{ domain, } on_o \text{ results in an } on_b \text{ event, and the an } off_o \text{ results in a } off_b \text{ event}\rangle\rangle\end{array}}{Operator(on_o, off_o), On/Off\ Buttons_{on_o,off_o}, Conveyor\ Motor\ and\ Belt, CC \vdash Obey\ Buttons}\ \begin{array}{l}[\text{Domain Removal}] \\ \langle\langle Operator \text{ shares no phenomena}\rangle\rangle\end{array}}{Operator^{on_o,off_o}, On/Off\ Buttons_{on_o,off_o}, Conveyor\ Motor\ and\ Belt, CC \vdash Obey\ Operator''}\ [\text{Sharing Removal}]$$

where

Obey Operator	The controller should obey the operator's commands: an on_o command from the operator should result in the conveyor belt state *Running*; an off_o command from the operator should result in the conveyor belt state *Stationary*
Obey Buttons	Assuming on_o/off_o events occur, an on_o should result in the conveyor belt state *Running*; an off_o button press should result in the conveyor belt state *Stationary*
Obey Buttons′	An on_b event should result in the conveyor belt state *Running*; an off_b event should result in the conveyor belt state *Stationary*
Obey CC	Assuming on_b/off_b event occur, an on_b should result in the conveyor belt state *Running*; an off_b button press should result in the conveyor belt state *Stationary*
Obey CC′	An on_b event should result in the conveyor belt state *Running*; an off_b event press should result in the conveyor belt state *Stationary*
Obey CC″	Assuming the *Conveyor Belt and Motor* state results in *Running* when an on_c is received, and results in *Stationary* when a on_c events is received, an on_b event should result in the conveyor belt state *Running*; an off_b event press should result in the conveyor belt state *Stationary*
CC Spec	on_b event should result in on_c; an off_b event should result in off_c.

It would be a simple matter to apply the solution interpretation rule to complete the development of the *CC* component: this interpretation could provide code or, more simply, use *CC Spec* as a solution description of *CC*. In the latter case, the justification would be trivial: as discussed earlier on, a specification always satisfies itself.

The solution of the other architectural component, *RRC*, would, clearly, be more involved, and we do not do it here.

B. The Software Development Structure

We have solved the problem down to the specification of one of the two architectural components, at this point it is worth considering the design process that has led us here. The steps we have performed, arranged as a tree, are shown in Figure 12. The traditional Genzten-style presentation is textual, as in the above cascaded rule application. The tree presentation clarifies the structure of the development and its argument by abstracting the details of the transformation. The tree consists of a number of nodes, the named problems we have derived, and arcs representing how those problems are related. One node is inverted to indicate a solved problem. The tree branches because we have argued that the solution should consist of two components.

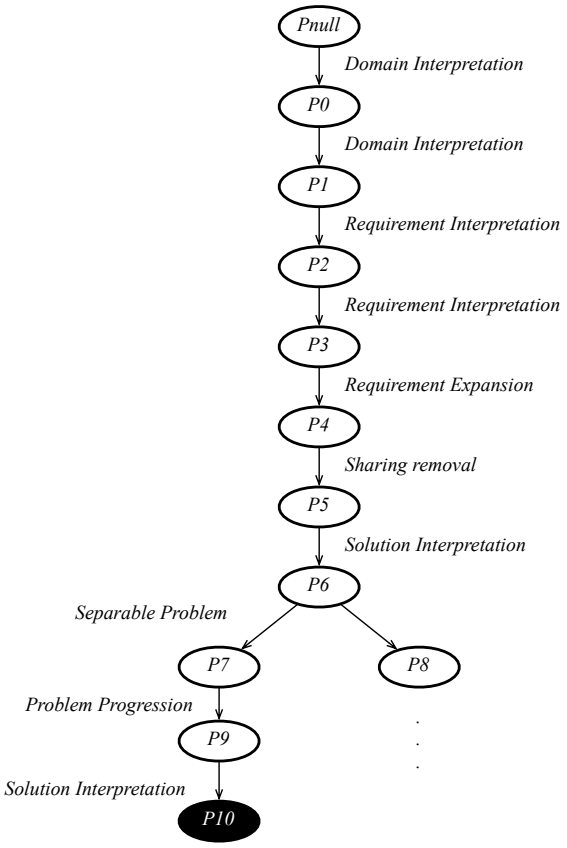


Fig. 12. The structure of the Package Router Problem solving process: $P6$ is solved.

This form of representation is reminiscent of statecharts, showing the flow of development and the steps taken, and suggests that the structure of development—the relationship between the steps, rather than the steps themselves—is worthy of record. We say more about this observation in Section VII.

Of the development itself; in the case study we focussed on the design of the simplest component. More work would be necessary for the very detailed step-by-step presentation, but we would hope that higher-level abstractions are possible that would mitigate the need for extreme detail.

VI. Related Work

The notion of problem on which our framework is based derives from the reinterpretation of a proof obligation in [24], [47], there used as one of the criteria for the completeness of requirements engineering: being able to discharge the obligation $W, S \vdash R$ determines that a solution specification S has been found that, in the context W, provably satisfies requirement

R. For our framework, we identify and separate out this 'problem component' of the proof obligation, re-expressing it in a proof-theoretic form to give our syntactic problem sequent $W, S \vdash R$. This is our point of departure from the work reported in [24], [47]. Whereas in that work, $W, S \vdash R$ is an end point in some process leading to context, requirement and solution description, in our approach we wish to *synthesise* $W, S \vdash R$ problem sequents, and to base design and development processes on their synthesis. A problem sequent $W, S \vdash R$ has first class status within our Gentzen-style sequent calculus: it is the object manipulated by the calculus.

Our framework shares much with the conceptual basis of [47]. First, it allows for description grounded in the language of the domain. In particular, it does not prescribe a single description language in which designations are expressed. Second, the control of phenomena is an important part of our framework: a solution machine must correctly control and observe the phenomena that its environment exposes. Our framework assumes that within each problem each phenomenon has a single controller (either in the environment or the machine).

Other related work is the Problem Frames approach, sketched in [24] and more fully developed in [25]. This approach provides the developer of a software intensive system with a focus that remains for as long as possible in the problem domain, rather than allowing it to move early to the solution domain. The modelling of the intellectual tools that Problem Frames provide to the developer has also motivated our work. However, with the Problem Frame focus on the problem world, the developer's choices are limited to those that exist there: useful solution notions, such as architecture, are therefore missing or at best treated only implicitly.

The idea of transforming requirements through to specification appears in many approaches: in goal-based approaches, for instance [46], [43], high-level goals, which may be regarded as requirements deep in the world, are successively decomposed and refined until operationalised requirements are specified for a machine or other agent to satisfy. Similarly, in scenario-based approaches [38], [1], [16] refinement is often required in the move from high-level scenarios, often capturing business processes within an organisation, through to low-level scenarios expressing the direct interaction between a software system and its actors. Our problem progression rule is based on idea expressed in [25, Page 103]. There, a transformation of requirements is hypothesised to exist that will keep a problem's solution invariant while domains are removed from the problem. More recently, the authors (with others) [22], [37],

Li *et al.* [29], and Seater *et al*. [39] have explored what the detail of such a transformation would look like.

Transformations of the nature we define here are often found in the formal approaches to software development, such as the transformation of specifications through to program code in the refinement calculi of Morgan [31] and Back [2] mentioned earlier. We illustrate how such formal approaches might be included within our framework in Section IV-A.3. Another aspect of these formal refinements is shared by our framework: the partiality of their application. For instance, the refinement of the specification $w : [pre, post]$ by the specification $w : [pre', post]$ is only possible when the condition $pre \Rightarrow pre'$ is satisfied – this is precisely the *weaken precondition* refinement rule ([32]) that guarantees a correct refinement only when the refining specification has a weaker precondition.

Approaches to software and software development based on logics and calculi have been the subject of computer science for many years, and much has been learned about the logics, calculi, and their derivatives, that are best suited to describe software. The many notable examples include Floyd-Hoare logic [23], the Refinement Calculi [32], [2], the Temporal Logic of Actions [28], and the Duration Calculus [15]. We hope that, with this work, we add something to this list that stretches someway further towards the customer.

VII. Discussion and Conclusions

The main contribution of this research is a framework for software problem analysis that allows software problems to be described, and software solutions and adequacy arguments to be incrementally built. We have shown how various formal and informal elements of software development fit into the framework and how they are related therein. This has included:

- the identification and clarification of system requirements;
- the understanding and structuring of the problem world;
- the structuring and specification of a hardware/software machine that can ensure satisfaction of the requirements in the problem world; and
- the construction of adequacy arguments that show (but, in general, do not prove) the system will satisfy its requirements.

Problem-oriented approaches to requirements engineering have suffered from being difficult to combine with models of software system development that iterate between problem and solution domains. Bass, Clement and Kazman [3], for instance, observe that early trade-offs between qualities must be made if a system architecture is to be chosen, with the choice

of architecture driving the discovery of other system requirements. Other authors [27], [41], [33], [20] make similar observations. This is an important issue, and one that we have tried to address in our framework.

In previous work of the authors and their colleagues, explicit consideration has been given to problem and solution spaces as 'equal partners' in software engineering [21], [12]. Hall *et al* add to the Problem Frames approach a notion of architectural service [19], and Rapanotti *et al* [37] add a notion of architectural decomposition, to allow solution options to be explored. This view, balancing problem and solution, raises the question of the most appropriate way to relate the spaces and has led us to the conceptual framework, discussed in this paper, for problem-oriented *software engineering*. Here we have simultaneously generalised and simplified these earlier approaches through the definition of a basic rule: we allow the application of an architectural structure in the solution space structure to influence the problem space quite directly.

The system we have defined is a Gentzen-style sequent calculus. Proof in Gentzen-style sequent calculi appears to lend itself to computer support. PVS [35] is a good example. Work on capturing proof patterns in Gentzen-style sequent calculi has produced the important class of tactic languages, that includes that of PVS and [18]. Does our Gentzen-style sequent calculus have an associated tactic language? If so, it would be a language for capturing patterns of software design steps. This deserves investigation.

As to the Four Dark Corners paper [47], although the precise nature of the relationship between the two approaches can only be clear when we have finalised the details of our approach, we are led naturally to the conjecture that whenever (all) the conditions of [47] hold for $W, S \models R$ then $W, S \vdash R$ will be derivable within our framework. This result will, most likely, be our aim for arguments of soundness of our framework[7]. At some point we hope also to consider completeness; but we believe that this must depend on a more exact notion of adequacy than we have so far been able to formulate.

Of the relationship in the opposite direction, that a solution is found within our framework does not necessarily mean that it will satisfy the conditions of [47], because:

- we do not limit ourselves to the development of solution specifications, i.e., the relationships between events at the machine interface that mediate problem and solution spaces;

[7]And, although the arguments may not be trivial, our 'problem solved' rule in Section IV-B.3 goes most of the way, we think.

rather our scope spans the full problem and solution space, including computational artefacts such as code; and

- we ask only for solution adequacy, not a proof of correctness.

Finally, we note that our concept of problem has no inherent bias towards software as a solution, although it is entirely appropriate for it. Bleistein *et al.* [7], [11], [8], [9], [10] apply the problem-oriented approach to modelling and tracing from business strategy thorough to concrete system requirements, introducing process models in the form of Role Activity Diagrams. This leads us to conjecture an extension of problem-orientation to other areas of problem solving and thus, following Vincenti's observation quoted earlier, to other areas of engineering.

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