Aesthetical Entanglements in Mathematics Learning

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In this paper we develop a perspective on the diverse aesthetics historically associated with mathematics, inspired by Rancière's approach to aesthetics and politics. We call 'Silencing Aesthetics' a dominant aesthetic that Rota has characterized as a "copout (...) intended to keep our formal description of mathematics as close as possible to the description of a mechanism." The challenge this paper attempts to explore is how to question silencing aesthetics to make space for generative ones. Our efforts have focused on setting up and studying inclusive and pluralist 'Studios', gathering craftworkers, anthropologists, mathematics educators and mathematics enthusiasts. We include here a case study based on a conversation among basket weavers, anthropologists, and mathematics educators, focused on the artisanal and mathematical nature of knots.

Keywords: Aesthetics, basketry, knots, mathematics, materiality.

Mathematical aesthetics

The literature on mathematical aesthetics is not easy to summarize or recapitulate (Sinclair, Pimm, & Higginson, 2006 pp. 1-17; 224-254), but we can succinctly identify certain themes that have been focal for it:

- Discussion about the nature of feelings evoked by mathematical work, as well as those accompanying the perception of mathematical diagrams and symbols, which may include, among others:
  - Feelings of pleasure elicited by the senses of beauty, elegance, productivity, harmony, perfection, and the like.
  - Feelings of melancholy, detachment, coldness, austerity, and estrangement.
- The psychology of mathematical discovery and the roles of the unconscious.
- Differences and commonalities between mathematics and the arts.
- Criteria for what makes mathematical things to be beautiful or ugly, and degree of uniformity of the corresponding aesthetic judgements among professional mathematicians.
- Mystical and uncanny dimensions of mathematics.
- Whether the appreciation of mathematical aesthetics is restricted to a small elite endowed with the 'math gene' or is accessible to everyone.

Adding to the pioneering work of de Freitas and Sinclair (2014), this article will develop a perspective inspired by Rancière's (2000, 2010) approach to aesthetics and politics. We think that this perspective has the potential to illuminate anew mutual implications between mathematical aesthetics and mathematics learning. Rancière proposed that we all live in consensual worlds delineated by certain "distributions of the sensible", which are "systems of self-evident facts of perception based on the set horizons and modalities of what is visible and audible as well as what can be said, thought, made, or done” (Rancière, 2000 p. 12). A distribution of the sensible is a demarcation of that which “truly” exists against a receding background of the unreal and meaningless, together with ways of behaving, perceiving, and valuing that are concordant with it. These ways of behaving, perceiving, and valuing constitute the aesthetics of those who take part in a prevalent consensus. In times of political turmoil, the consensus validating a certain distribution of the sensible comes under scrutiny and questioning, making possible the opening of new spaces of legitimacy for some of those who had had no part in the prior consensus, which had been relegated to noise and meaninglessness (Rancière, 2004 p. 225). ‘Dissensus’ is the term used by Rancière for such political questioning. Dissensus is not equivalent to ‘disagreement’ since disagreements are common within a consensual communitarian perspective consistent with a certain distribution of the sensible. Rather, dissensus refers to a struggle towards removing a veil of noise and meaninglessness that had hitherto rendered certain aspects of reality indiscernible.

The main examples chosen by Rancière in his writings on aesthetics were taken from art and literature. We surmise that Rancière’s distrust of disciplinary traditions, including the ones permeating the natural, social, and mathematical sciences, prompted him to eschew elaboration on their aesthetical dimensions and moved him to rather focus on their hierarchical demarcations of “who can talk about what”. Several artists and craftspeople we have been working with have found formal mathematics inaccessible or alienating, in school and even now. This is despite engaging in work that is mathematically rich and powerful, where mathematics is conceived in its broadest sense. This silencing of their voice (by the discipline of a discipline) resonates with Rancière’s example of the ‘disjunction between the arms and the gaze’ of a woodworker (Rancière, 2006) and “that a discipline is always much more than an ensemble of procedures which permit the thought of a given territory of objects … it supposes a cut in the common fabric of manifestations of thought and language.” (Rancière, 2006 p. 8). While fully acknowledging the political power of disciplinary enculturation in the cutting between “what is visible and what is not, what can be heard and what cannot, what is noise and what is speech” (Rancière, 2004 p. 225), we think that it is important to study their diverse aesthetics and how these change across communities and historical eras, with an eye to envisioning how they can be challenged and commingled with each other. The anchor of this paper is a case study based on a recorded conversation between basket weavers and mathematics educators. We explore how the work of this kind of pluralist Studio — intermingling mathematics and craftwork — may contribute towards breaking-through the silencing power often irradiated by prevalent mathematical aesthetics.

The case study presented in the next section is based on a conversation in one of the studio sessions sponsored by the *Forces in Translation* project. This project (https://forcesintranslation.org/about/) focuses on interactions between basket weaving, anthropology and mathematics. It includes basket makers, anthropologists and mathematics educators exploring, through in-person and online studio sessions, how different basket weaving techniques and their cultural traditions interplay with the understanding of mathematical ideas, such as spatial relationships, surfaces, curvature, growth, and forces at play, such as tension, friction and compression that hold together complex structures.

Crafting mathematics – a conversation between basketry and mathematics

This selected conversation interlaced knot tying as a craft and knot theory as a branch of mathematics. We conjecture that the transitions to and from knot tying and knot theory involve navigating across distinct distributions of the sensible and their aesthetics. For knot tyers, knots are to be made and valued according to their frictional strength, easiness of unknotting, and other qualities critical to their use by fisherman, packers, etc. Within the consensual aesthetics of knot tyers, many knots studied in (mathematical) knot theory are not even knots, and vice versa. The two distributions of the sensible share some common entities, such as certain knots, e.g., the *trefoil* or *overhand,* but they are immersed in distinct dialects, patterns of expectations, bodily skills, and ways of making sense of them. The aesthetics of a distribution of the sensible is inseparable from the corresponding history of communities and practices.

Learning knot tying or knot theory, both entail the encounter with an unfamiliar aesthetics, which represents a political engagement in the sense of 'bringing to reality' materials and events that had previously been invisible or occluded by irrelevant noise. Experiences with knot making of either kind, however, are not deterministic, in the sense that each of them can become sources for diverse aesthetical orientations, including the possibilities of coming to appreciate the same knots and techniques as useless, inspiring, too easy, too difficult, powerful, very strange, being good at them, and so forth.

Case Study in Six Scenes

Scene 1

Geraldine:	I suppose a hitch is not a knot, is it?
Stephanie:	Yes it is [showing a tied rope with a branch holding it, see Figure 1]. Yeah.
Geraldine:	It's not quite a knot, it's a hitch.
Stephanie:	It's a Lark Head's knot.
Geraldine:	Well, if you take it [the branch] off, it's not a knot. It's one of those.. not knots [showing a rope forming a circle held by her hands, see Figure 2] (...) And a knot is meant to be a knot that doesn't come undone [showing a knot that stays as such when pulled apart, see Figure 3].

Figure 1

Figure 2

Figure 3

Commentary of Scene 1

According to Wikipedia, "a hitch knot is a type of knot used to secure a rope to an object or another rope. It is used in a variety of situations, including climbing, sailing, and securing loads" (https://en.wikipedia.org/wiki/List_of_hitch_knots). In effect, the hitch knot shown by Stephanie in Figure 1, ties a rope to a branch. Stephanie names it "Lark's Head", which is a type of hitch knot also called "Cow Hitch". Hitch and Lark's Head knots have definite places in distributions of the sensible inhabited by knotters, although their places are not necessarily uniformly agreeable among them. Geraldine argues that “it is not quite a knot”. Her argument for the dubious status of the hitch, is that if the branch is removed, then the rope gets unknotted. In Figure 2 she shows the status of the unknotted rope as a circle. The latter is meant to be a “not knot”, referring to what in knot theory is called ‘unknot’, which plays a role among mathematical knots analogous to zero among integers. In Figure 3, Geraldine shows a knot that, in contrast, does not get undone if a branch previously entangled with it, is taken off from it. Given that its name suggests that it is not a knot, the unknot appears to have an odd role in knot theory. It is not unlike the dubious role that, for centuries, zero had among numbers. Scene 1 reflects an entanglement of distributions of the sensible lived-in by knotters and knot theorists. This entanglement will be further articulated in Scene 2.

Scene 2

Geraldine:	One, two [counting two rope “contacts”, or crossings, in a knot similar to the one shown in Figure 1]. I think that's because it's not a knot.
Stephanie:	But it is called a knot by knotters? (...)
Geraldine:	Yeah, they're called knots by knotters, but not by mathematicians.

Commentary on Scene 2

Geraldine counts two crossing traversed by the rope, which suggests to her another argument for why it is not a knot: in knot theory the minimal number of crossings for a non-trivial knot (other than the unknot) is three. Scene 2 shows that the entanglement we alluded to in Scene 1 is not a matter of confusion, but of which distributions of the sensible are to be adopted, that is, the ones of knotters or mathematicians. Geraldine’s statement emphasizes the number of crossings, which is a principal topic in knot theory. At the same time, her holding of the rope in Figure 2 emphasizes another aspect, namely, that in knot theory, knots — including the unknot — do not have loose ends, which can be achieved by fusing the two ends of a knotted rope.

Scene 3

Geraldine:	It's just that lovely half hitch. I really like these half hitches, because this, this one… this one is two linked half hitches[see Figure 4]. And it's the same as the looping. It's the looping. I mean, that's, but it's two half hitches.

Commentary on Scene 3

In the introductory section, we proposed that the ways of behaving, perceiving, and valuing concordant with a certain distribution of the sensible constitute its aesthetics. In Figure 4, Geraldine shows a rope forming two half-hitches, while she tells the rest of the group that they are “lovely” and that she “really likes them”. As a basket weaver, Geraldine uses looping techniques to create beautiful baskets of an extraordinary variety, and it turns out that two half hitches are “the same as the looping”. The immense creative value that looping has for Geraldine is also ascribed to two half hitches; they are prominent in her aesthetic as a master of looping. We chose to highlight in this commentary the emotional investment that Geraldine expresses. This is not meant to imply that emotional values, so central to any aesthetics, are to be seen only in this scene. Anything said or shown in the six scenes is animated by emotional values. We elaborated on them here just because it is particularly salient and explicit.

Figure 4

Figure 5

Figure 6

Scene 4

As the group is discussing the number of crossings for certain knots, Mary demonstrates what she thinks with an overhand knot.

Mary: If you actually sort of suspend it in space, then it only touches at two points [Figure 5], but if you pull it up and make it flat, it touches at three points [Figure 6].

Commentary on Scene 4

Mary shows that the number of crossings of a knot depends on how one holds it, illustrating her point with an overhand knot as she makes it feature two or three crossings. This opens a huge topic: What is a crossing? How come that knot theory stipulates that an overhand knot with joining ends (i.e., a trefoil) has three or more crossings when one can suspend it in space with only two? Or, as it will later transpire in the discussion, with only one or with none? How is a crossing different from a "touching"? This topic will be further elaborated by Charlotte in Scene 5.

Scene 5

Charlotte: Yeah, well, I was interested in the idea of crossings and the relation between the three dimension and two dimensions, it's related to what Mary just said [in Scene 4]. I was trying to making sense of the crossings in three dimension and then this one is here. [see Figure 7, yellow arrow indicating crossing being referred to] (...) but then when once you flatten the knot, then it becomes two [see Figure 8].

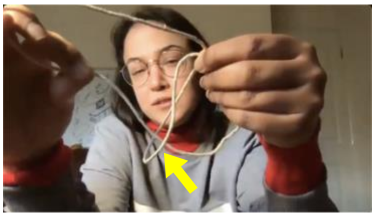

Figure 7

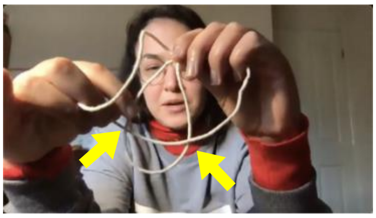

Figure 8

Figure 9

Commentary on Scene 5

In Figure 7, Charlotte holds a trefoil in such a way that the string touches itself at a point down below. But as she turns the loop 90 degrees to flatten the knot, in Figure 8, that single touch in the lower section becomes two. She is operating all the time with three dimensional materials, but she envisions a transformation from three dimensions to two, which coincides with what Mary had referred to in Scene 4 as "flattening."

Scene 6

Hilary: [Following up from Charlotte’s comment in Scene 5] I make knots in willow for my frame baskets. So the frame of it is a knot, usually the hoop frame, (...) So I'm looking for that when I make the hoop, I'm looking for the point at which that is getting to stay upright, which is about there [See Figure 9], where I can press down on it, and it's going to be, it sort of balances on itself (...) it's heavier at one end, so you've got to find that balance point.

Commentary on Scene 6

Hilary shows in Figure 9 the kind of overhand knots that she makes with willow to create the hoop frame of a basket. The hoop frame not only fits, approximately, a 2-dimensional plane, but it also must “balance on itself”. Balancing is a matter of helping the material find a gravitational symmetry letting the hoop “stay upright”. Hilary elaborates on how critical sensing the materiality of a knot is to basket weaving. Could we say that this emphasis on materiality marks a radical difference between the aesthetics of basket weaving and the aesthetics of knot theory? We think it does not. Even the very flat knot diagrams, customary in knot theory, preserve three dimensional features as they show points of over and under passing. Such crucial under/over reflects material impenetrability. Knot theory’s distribution of the sensible does not involve the melding of two filaments or strings into one, in other words, actual flattening.

Discussion

> Knots seem to refuse to be seen from one particular point of view or perspective.
> (de Freitas & McCarthy, 2014 p. 45)

We have set up and studied inclusive and pluralist ‘Studios’, gathering craftworkers, anthropologists, mathematics educators and mathematics enthusiasts. Many of these Studios were open to the public and to family groups as well. The six scenes of our case study, which are part of a much longer conversation, reflect the messy work of a group of participants with diverse backgrounds and ages, as they navigate and comingle various aesthetics, sharing the notion that no single aesthetic has ultimate and dominant value. Scenes 1 and 2 reflect that there are different distributions of the sensible consensually adopted by, in this case, knotters and knot theorists, such that that which exist for them is perspectival, interrelated, and sensible. Scene 3 suggests that materials and techniques allocated in a communitarian distribution of the sensible have multiple and intense powers of emotional attachment. Scenes 4 and 5 evoke how grappling with the idea of ‘crossing’ is a matter of degrees and nuance dependent on the careful attention to materials and diagrams. Scene 6 highlights the multiple lives of knots as they basket weave materials and patterns.

The conversation that we have portrayed entangles different aesthetics, including ones from knot-tying, knot theory, and basket weaving. Other aesthetics surrounding knots have been studied by anthropologists:

> The knot is ascribed more than functional value in the Pacific as it becomes the object of meditative thought and holds together through binding not two things but two concepts: that of the visible, and that of the invisible whose momentary entanglement facilitates temporal concepts of genealogy and remembrance. (Küchler, 2003 p. 207)

In this discussion we want to elaborate on the significance of aesthetical entanglements in mathematics learning, because they may help question a prevalent and widespread aesthetic of mathematics that works by silencing those who fail to appreciate it, a community that includes the majority of students. This aesthetic is not an honest one either. The actual practice of mathematicians is much more similar to that of craftspeople or students, though that is occluded in much of how the subject is presented and taught. The mathematician Rota (1997) has described two interrelated aspects for what we call 'Silencing Aesthetics': 1) Sudden light in the darkness and 2) Attributions of mathematical beauty to cover up the messiness of mathematical enlightenment. They are two sides of the same impetus. Regarding the former, he writes:

> All the effort that went in understanding the proof of a beautiful theorem, all the background material that is needed if the statement is to make any sense, all the difficulties we met in following an intricate sequence of logical inferences, all these features disappear once we become aware of the beauty of a mathematical theorem (...) [all that will remain] is the image of a flash of light of insight, of a sudden light in the darkness (ibid, p. 179)

Rota views veiling the messiness inherent in enlightenment, which is regulalrly hidden when mathematicians formally present their work, as motivated by powerful aesthetical and political forces:

> The term 'mathematical beauty', together with the light-bulb mistake, are tricks that mathematicians have devised to avoid facing up to the messy phenomenon of enlightenment. The comfortable one-shot but misleading idea of mathematical beauty saves us from having to deal with the messy situation of a concept having degrees. All talk of mathematical beauty is a copout from confronting the logic of enlightenment, a copout that is intended to keep our formal description of mathematics as close as possible to the description of a mechanism. This copout is a step in a cherished activity of mathematicians, that of building a perfect world immune from the messiness of the ordinary world, a world where what we think should be true turns out to be forever true, a world that is to be kept free from the disappointments, the ambiguities, the failures of that other world in which we are forced to live. (ibid, p. 182)

Numerous efforts have been conducted to *unveil* the messiness of mathematical enlightenment. A large literature from research in mathematics education is dedicated to this and, most notably from philosophy, is *Proofs and Refutations* (Lakatos, 1976), and from cognitive linguistics *Where Mathematics Comes From* (Lakoff & Núñez, 2000). However, these untidy accounts of mathematical work are often marginalized as belonging either to historical epochs prior to the development of contemporary mathematics, or to students who are still far from mastering the subject. The underlying message for many students who seldom experience flashes of light of mathematical insight, while often sensing unapproved messiness in their understandings, is that they lack abilities necessary to enjoy mathematics and appreciate its alleged aesthetics.

This paper attempts to explore how we could question silencing aesthetics to make space for generative ones. Given the dominant cultural images of mathematics, it is a complex issue. Aesthetical entanglements such as the one reflected in our case study, in which knot theory and knot tying aesthetics are compared on an equal footing, each equally legitimate despite their radical differences, may help in grasping an aesthetics of knot theory that is neither mechanistic, nor flawless.

The episode was not, for the most part, a matter of dissensus: none of the participants were prone to argue that knots used by fishermen should be considered knots by knot theorists, or that the latter should discriminate knots according to their frictional strength. These would be dissensual claims to the extent that they attempted to subvert the distribution of the sensible consensually adopted by each of the communities. We can recognize many points of contact between knot theory and knot tying, such as the trefoil being an overhand knot with its ends joined. But these points of contact constitute, in fact, rich counterpoints to be explored. There is nothing absolute about defining knots, say, as not having loose ends. The issue can be highlighted by noticing that without friction knots with loose ends can always be unscrambled onto a linear rope, which would make knot theory pointless.

The possibilities for aesthetical entanglements in mathematics learning are boundless. As another example, we have conducted a study with 10-year-olds at an afterschool program at an art museum (Nemirovsky, 2018), which involved adjoining aesthetics surrounding fabric bowls, basketry, and Gaussian curvature. The overall idea is to liberate space and time for the artisan, the practitioner, the child and the lay citizen to partake in the sharing of the sensible and to become “a deliberative citizen” (Rancière, 2000 p. 12).

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