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Supplementary Material

SCALING UP SELF-SUSTAINED SMouldERING OF SEWAGE SLUDGE FOR WASTE-TO-ENERGY

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1 Present address
S.1. Supplementary Information on the Experimental Setup

All experiments followed the protocols for convective and conductive ignition from (Rashwan et al., 2021a) and (Rashwan et al., 2016), respectively, and Fig. S.1 details the setups used. Every experiment fostered robust ignition; inferred from high temperatures near the heater and no fuel remaining upon excavation (Torero et al., 2020). Therefore, the experimental results were not affected by the differences in setups and procedures, but instead driven by the key experimental conditions varied: reactor radius (DRUM vs. LAB), fuel concentration, fuel moisture content (MC), and applied air flux (Table 1). Each experiment number indicates a unique sludge sample, where the MC% and ash% are means from at least triplicates (EPA Method 1684). The sludge was combined primary and secondary sludge at the end of sludge treatment after polymer addition and centrifugation (Rashwan et al., 2016). All experiments with > 70% MC indicate that virgin sewage sludge was used. The mixtures for DRUM S4 and LAB S4, S11(1-2), and S12(1-2) were prepared by drying the sludge in an oven at 105 °C until the desired MC was achieved (where ~3.2% was the lowest MC practical). The mixtures for DRUM S5 and LAB S5(1) were prepared by mixing batches of virgin sludge and sand and drying the batches by tumbling them within a commercial mortar mixer until the desired MC was achieved. The higher heating value (IKA C 200 bomb calorimeter) and C | H | N | S | O_{diff} (Flash EA 2000NC CHNS-O elemental analyzer, where oxygen was the difference), were measured for six samples and varied between 16.3-18.5 MJ kg_{dry}^{-1} and 45-52% | 7-8% | 6-7% | 0-1% | 33-41%, respectively. The fixed carbon and volatile matter were measured from a triplicate on one sewage sludge sample as 12% and 7%, respectively (following ASTM-D5832-98, where fixed carbon was the difference).
Fig. S.1.  Experimental setups used for: (a) LAB S12(1) and S12(2); (b) all DRUM experiments, (c) LAB S7, S11(1), and S11(2); (d) LAB S4, S5(1), S5(2), S6; and (e) LAB S8.  (c) used a conductive ignition procedure and (a), (b), (d), and (e) used a convective ignition procedure.
S.2. Analytical Method to Model Axial Heat Transfer in the Cooling Zone

Following the methodology from Rashwan et al. (2021a), the one-dimensional, transient, LTNE solution from Kuznetsov (1994) was used to describe the axial heat transfer in the cooling zone. This methodology does not fully resolve the dynamics in the cooling zone but highlights the key effect of non-uniform air flux on axial convective heat transfer. As the methodology is detailed in (Rashwan et al., 2021a), only the governing equations, key assumptions, and solution are summarized.

Assuming the sand and air may be considered as a continuum, the governing energy equations in both phases are (Kuznetsov, 1994):

\[
\phi \rho_g C_p \frac{\partial T_g}{\partial t} + \dot{m}_g^\prime C_p \frac{\partial T_g}{\partial x} = \phi k_g \frac{\partial^2 T_g}{\partial x^2} + h_{sg} a_{sg} (T_s - T_g) \quad (S.1)
\]

\[
(1 - \phi) \rho_s C_p \frac{\partial T_s}{\partial t} = (1 - \phi) k_{sapp} \frac{\partial^2 T_s}{\partial x^2} - h_{sg} a_{sg} (T_s - T_g) \quad (S.2)
\]

where radiation is embedded in the solid conductivity following the Rosseland approximation, \( k_{sapp} = k_s + 16 \sigma d_p T_s^3 / 3 \) (Zanoni et al., 2017) and the specific surface area assumed the sand grains where spherical \( a_{sg} = 6(1 - \phi)/d_p \) (Kuznetsov, 1994; Zanoni et al., 2017). The interfacial heat transfer coefficient, \( h_{sg} \), measured from Zanoni et al. (2017), was used, which is applicable for many smouldering systems and valid for \( Pr = 0.72, 0.5 \leq Re \leq 31 \), and \( 0.125 \leq d_p \leq 2.000 \text{ mm} \):

\[
Nu = h_{sg} d_p / k_g = 0.001 \left( Re^{1.97} Pr^{1/3} \right) \quad (S.3)
\]

The initial condition assumed the entire reactor reached the peak smouldering temperature (Kuznetsov, 1994):
\[ T_g(x, 0) = T_{\text{peak}} \]  \hspace{1cm} (S.4)

Equation (S.4) effectively assumes that the smouldering intensity along the centreline was steady throughout propagation. This is not generally the case, but is observed in many applied smouldering studies using sufficiently robust smouldering (e.g., (Fabris et al., 2017; Rashwan et al., 2016; Yermán et al., 2015; Yermán et al., 2016; Zanoni et al., 2019c)). Furthermore, as the model was one-dimensional, the unburned edges seen in Figs. 5 and 7(c) were not modelled.

The boundary conditions were (Kuznetsov, 1994):

\[ T_g(0, t) = T_{\text{amb}} \]  \hspace{1cm} (S.5)

\[ \frac{\partial T_g}{\partial x}(\infty, t) = 0 \]  \hspace{1cm} (S.6)

Equation (S.5) assumed ambient air was injected driving cooling, which reflects the conditions in all experiments in Table 1 after the heater was turned off. Equation (S.6) assumed that the air temperature was constant far ahead of the cooling front. This condition is often assumed to develop analytical solutions investigating other aspects of smouldering, like the propagation velocity or temperatures within the reaction zone (e.g., (Aldushin et al., 1999; Dosanjh et al., 1987)).

To make Eqs. (S.1-S.2) non-dimensional, a small dimensionless parameter was defined (Kuznetsov, 1994):

\[ \delta = \phi \rho_g \dot{m}_g n_{g}^2 C_{p_g}^3 / h_{sg} a_{sg} (\rho C_p)_{\text{bulk}} k_{\text{bulk}} \]  \hspace{1cm} (S.7)
where the volume-averaged bulk heat capacity and thermal conductivity were described as: 

\[ \phi \rho_g C_{pg} + (1 - \phi) \rho_s C_{ps} = (\rho C_p)_\text{bulk} \quad \text{and} \quad \phi k_g + (1 - \phi) k_{sapp} = k_{bulk}, \]

respectively (Kuznetsov, 1994). By balancing conduction, storage, and convection, the axial characteristic heat transfer length, \( x_c \), and time, \( t_{c,x} \), were identified (Kuznetsov, 1994):

\[
(\rho C_p)_\text{bulk} \frac{\Delta T}{t_{c,x}} \sim \dot{m}_g'' C_{pg} \frac{\Delta T}{x_c} \sim k_{bulk} \frac{\Delta T}{x_c^2} \quad \text{(S.8)}
\]

From Eq. (S.8), \( x_c = k_{bulk}/\dot{m}_g'' C_{pg} \) and \( t_{c,x} = (\rho C_p)_\text{bulk} k_{bulk}/(\dot{m}_g'' C_{pg})^2 \), and the non-dimensional length and time were defined as \( \xi = x/x_c \) and \( \tau = t/t_{c,x} \) (Kuznetsov, 1994). The cooling velocity, \( v_{cool} \), is defined from dividing the characteristic length from time (Kuznetsov, 1994):

\[
v_{cool} = \dot{m}_g'' C_{pg} / (\rho C_p)_\text{bulk} \quad \text{(S.9)}
\]

The cooling velocity describes how fast the cooling front progresses. As shown in Eq. (S.9), \( v_{cool} \) is directly proportional to \( \dot{m}_g'' \), and is therefore very sensitive to non-uniform air flux (see Figs. 1 and 7).

Though radial heat transfer is not included in the governing Eqs. (S.1-S.2), the characteristic radial heat transfer time, \( t_{c,r} \), was identified for comparative purposes below in Section 3.2 by taking the reactor radius, \( r_o \), as the characteristic length, assuming the medium was isotropic, and balancing radial bulk energy diffusion and storage:

\[
(\rho C_p)_\text{bulk} \frac{\Delta T}{t_{c,r}} \sim k_{bulk} \frac{\Delta T}{r_o^2} \quad \text{(S.10)}
\]
Therefore, \( t_{c,r} = r_a^2 \left( \rho C_p \right)_{bulk} / k_{bulk} \). The dimensionless temperature was described as:

\[ \theta = \frac{T - T_{peak}}{(T_{amb} - T_{peak})} \]

and the degree of LTNE was assumed small and treated as a perturbation (Kuznetsov, 1994):

\[ \theta_s = \theta_g + \delta \Delta \theta \quad (S.11) \]

Kuznetsov (1994) obtained the solution to Eqs. (S.1-S.2) by using Laplace transform methods. Extra details on the solution and intermediate steps are provided in (Rashwan et al., 2021a). The transient solution for the non-dimensional air temperature is (Kuznetsov, 1994):

\[
\theta_g(\xi, \tau) = \frac{1}{2} \operatorname{erfc} \left( \frac{\xi - \tau}{2 \sqrt{\tau}} \right) + \frac{1}{2} \exp \xi \cdot \operatorname{erfc} \left( \frac{\xi + \tau}{2 \sqrt{\tau}} \right) \quad (S.12)
\]

and temperature difference between phases is (Kuznetsov, 1994):

\[
\Delta \theta = \frac{1 - \Lambda_2}{4 \tau \sqrt{\pi \tau}} \left\{ \left( \xi + \tau \right) \exp \left[ - \left( \frac{\xi - \tau}{2 \sqrt{\tau}} \right)^2 \right] + \left( \xi - \tau \right) \exp \left[ \xi - \left( \frac{\xi + \tau}{2 \sqrt{\tau}} \right)^2 \right] \right\} + \\
\left( \Lambda_1 - \Lambda_2 \right) \left\{ \frac{1}{2} \exp \xi \cdot \operatorname{erfc} \left( \frac{\xi + \tau}{2 \sqrt{\tau}} \right) \quad (S.13) \right\}
\]

\[
- \frac{1}{2 \sqrt{\pi \tau}} \left[ \exp \left\{ - \left( \frac{\xi - \tau}{2 \sqrt{\tau}} \right)^2 \right\} + \exp \left\{ \xi - \left( \frac{\xi + \tau}{2 \sqrt{\tau}} \right)^2 \right\} \right] \right\}
\]

where \( \Lambda_1 = \left( \rho C_p \right)_{bulk} / \phi \rho_g C_p_g \), \( \Lambda_2 = k_g \left( \rho C_p \right)_{bulk} / \rho_g C_p_g k_{bulk} \).

Equations (S.12) and (S.13) can then be used with Eq. (S.11) to describe the non-dimensional solid temperature. Following (Rashwan et al., 2021a, b), the temperature dependent thermophysical properties were assumed constant from averaging over \( T_{amb} \) to \( T_{peak} \), e.g., \( \left( \rho C_p \right)_{eff} = \int_{T_{amb}}^{T_{peak}} \rho C_p(T) dT / (T_{peak} - T_{amb}) \) (see Table S.1).
As Eqs. (S.11-S.13) describe one-dimensional LTNE heat transfer within the reaction-less cooling zone, they do not resolve the multidimensional aspects of non-uniform air flux (Rashwan et al., 2021a). Therefore, Eqs. (S.11-S.13) only resolve key aspects of axial heat transfer in the cooling zone, not radial heat transfer. However, Fig. 4 shows that the main physics driving cooling along the centreline are well-captured with this one-dimensional simplification, as all curves reasonably match the key cooling trends.
Table S.1. Temperature-Dependent Model Input Parameters.

<table>
<thead>
<tr>
<th>Par.</th>
<th>Details</th>
<th>Value(^1)</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{\text{amb}})</td>
<td>Varied slightly between experiments</td>
<td>16-27</td>
<td>°C</td>
<td>Measured</td>
</tr>
<tr>
<td>(d_p)</td>
<td></td>
<td>2.00E-3</td>
<td>m</td>
<td>a</td>
</tr>
<tr>
<td>(\rho_s)</td>
<td></td>
<td>2650</td>
<td>kg m(^{-3})</td>
<td>a</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Constant in all experiments</td>
<td>0.37</td>
<td>-</td>
<td>a</td>
</tr>
<tr>
<td>(\sigma)</td>
<td></td>
<td>5.67E-8</td>
<td>W m(^{-2}) K(^{-4})</td>
<td>b</td>
</tr>
<tr>
<td>(M)</td>
<td></td>
<td>0.0290</td>
<td>kg mol(^{-1})</td>
<td>b</td>
</tr>
<tr>
<td>(R)</td>
<td></td>
<td>8.31</td>
<td>m(^3) Pa K(^{-1}) mol(^{-1})</td>
<td>b</td>
</tr>
<tr>
<td>(C_{p_s}(T_s))</td>
<td>1.75(T_s) + 340.32</td>
<td>1280</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>a</td>
</tr>
<tr>
<td>(C_{p_g}(T_g))</td>
<td>(-3 \times 10^{-5}(T_g^2) + 0.2261T_g + 940.35)</td>
<td>1053</td>
<td>J kg(^{-1}) K(^{-1})</td>
<td>a, b</td>
</tr>
<tr>
<td>(\mu_g(T_g))</td>
<td>(-9 \times 10^{-12}(T_g^2) + 4 \times 10^{-8}T_g + 6 \times 10^{-6})</td>
<td>2.47E-5</td>
<td>Pa s</td>
<td>a, b</td>
</tr>
<tr>
<td>(k_{s,\text{app}}(T_s))</td>
<td>(16\sigma d_p(T_s^3)/3 + 6.38 \times 10^{-4}T_s + 0.0915)</td>
<td>0.548</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>a</td>
</tr>
<tr>
<td>(k_g(T_g))</td>
<td>(-1 \times 10^{-8}(T_g^2) + 8 \times 10^{-5}T_g + 4.3 \times 10^{-3})</td>
<td>4.42E-2</td>
<td>W m(^{-1}) K(^{-1})</td>
<td>a, b</td>
</tr>
</tbody>
</table>

\(^1\) Temperature dependent values are examples over the centreline \(T_{\text{amb}}\) to \(T_{\text{peak}}\) measured in DRUM S5.

\(a\) (Zanoni et al., 2017)

\(b\) (Bergman et al., 2011)
S.3. Supplementary Information the Results

Figure S.2 captures the smouldering front distortion at one axial location during the early development of the front distortion when the front propagated up to 0.100 m in DRUM S8 and S9, 0.265 m in LAB S11(2), and 0.210 m in LAB S8. As discussed in (Rashwan et al., 2021a), non-uniform air flux develops throughout smouldering propagation, so Fig. S.2 only provides a snapshot into its development. All temperatures in Fig. S.2 are plotted until they equalled the temperature of the nearest TC measurement towards to the multipoint tip (the measurement point closest to the column centre), and the multipoint tip is plotted until its maximum value. This timeframe highlights the preheating distortion ahead of smouldering when each TC measurement was least influenced by conduction along the multipoint TC. After this time, the multipoint TCs towards the wall were influenced by conduction along the probe from the centre and therefore do not accurately capture the temperatures in the fuel bed at those times. During the front distortions captured in Fig. S.2, the multipoint TCs towards the column centre were also influenced by conduction along the probe from the hotter positions towards the wall, thereby dampening the apparent smouldering front distortions. However, though the temperature histories in Fig. S.2 contain some error due to this instrument interference, they may still be compared against each other to infer general front distortion differences between these experiments.

All experiments in Fig. S.2 experienced the same smouldering front distortion from non-uniform air flux (shown in Fig. 7) as the positions towards the wall heated before the positions towards the centre. From Fig. S.2, the smouldering front appears most distorted along the edge in DRUM S9 (Fig. S.2(b)), least distorted in LAB S11(2) (Fig. S.2(c)). The
severity of DRUM S9 aligns with the discussion in Section 3, and LAB 11(2) was a dry experiment with a relatively low air flux in the LAB column, which is therefore expected to have the least distortion of all experiments in Fig. S.2. However, the comparison between DRUM S8 (Fig. S.2(a)) and LAB S8 (Fig. S.2(d)) is complicated for a few reasons. First, the multipoint TCs captured the front development at different stages of propagation, i.e., DRUM S8 shows the distortion during the ignition procedure when the front only travelled 0.100 m (when the heater was still on, during the early stages of distortion) and LAB S8 shows the front distortion later in propagation when the smouldering front travelled 0.210 m (after the heater was turned off, during the later stages of distortion). Since DRUM S8 measured the smouldering front distortion during the ignition procedure, a significant crust (and associated air channelling) may have not yet developed. Furthermore, the multipoint TCs in DRUM S8 only capture 0.08 m of the 0.3 m radius, so the full distortion in DRUM S8 was not captured (where the entire distortion across the column was captured in LAB S8). That is, the heating front towards the column centre in DRUM S8 was further distorted. Therefore, though Figs. S.2(a) and S.2(d) appear similar, DRUM S8 probably experienced greater overall distortion than LAB S8 (which aligns with the propagation velocity discrepancy observed in Fig. 5 and discussed in Section 3). Altogether, Fig. S.2 provides good evidence that non-uniform air flux affected all experiments, and its severity is a function of the specific experimental conditions.
Fig. S.2. Multipoint thermocouple (TC) histories when the smouldering front propagated up to 0.1 m in (a) DRUM S8 and (b) DRUM S9; 0.265 m in (c) LAB S11(2); and 0.21 m in (d) LAB S8. The positions in the legend note the TC distance from the centre. The heater was turned off before the LAB experiments multipoint TCs peaked and after the DRUM multipoint TCs peaked. All plots note the time since the beginning of the experiment but focus on the 30 minutes prior to the multipoint tip peaking (the position closest to the reactor centre).
Figure S.3 shows crust formations from three LAB experiments where smouldering was weak (LAB S5(2), Fig. S.3(a)) and robust (LAB S6 and S8, Figs. S.3(b) and S.3(c), respectively). All crust formations were similar to those observed in sewage sludge experiments at similar high MC, fuel concentration, and applied air flux performed in the DRUM, i.e., DRUM S6, S7, and S8 in Fig. 5 (though the crust in LAB S8 was mostly localized to a fraction of the radius rather than a ring around the column wall). However, all LAB experiments in Fig. S.3 facilitated faster propagation velocities (0.40 – 0.49 cm min$^{-1}$) than the DRUM experiments at the same conditions (0.24 – 0.27 cm min$^{-1}$). This discrepancy is hypothesized to have resulted from more severe non-uniform mass flux in because of the larger $t_{c,r}/t_{c,x}$ in DRUM than the LAB (further discussed throughout Section 3.2).

![LAB crust formations in (a) LAB S5(2), (b) LAB S6, and (c) LAB S8.](image)

Figure S.4 presents an example of sand and sewage sludge mixtures that were poorly mixed (used in LAB S5(2), Fig. S.4(a)) and well-mixed (used in LAB S6, Fig. S.4(b)). As seen above, the LAB S5(2) mixture had unmixed ‘clumps’ of sewage sludge compared to LAB S6, which did not have any of these ‘clumps’. These ‘clumps’ effectively increased
the fraction of water bound tightly in their centre and reduced the available surface area for smouldering. As a result, under the same fuel concentration and air flux conditions with slightly higher MC, LAB S6, S7, and S8 were more robust than LAB S5(2), which is inferred from their higher peak temperatures and faster propagation velocities (detailed in Table 1). Note that all high MC LAB experiments were similarly well-mixed to LAB S6; LAB S5(1) was the only poorly mixed LAB experiment.

Fig. S.4. Example of sewage sludge and sand mixtures from (a) poor mixing in LAB S5(2) and (b) good mixing in LAB S6.

Figures S.5 shows the centreline TC histories from the remaining key experiments discussed in Section 3.
Fig. S.5. DRUM centreline temperature histories from (a) S4, (c) S5, (e) S6, and (g) S8 and DRUM wall temperature histories from (b) S4, (d) S5, (f) S6, (h) S8, and (i) S9, which all had significant crusts. The thermocouple (TC) layout was common for all experiments, where P1 and P2 indicate the air plenum temperatures below the fuel bed, see Fig. S.1.

Figure S.6 shows the centreline peak temperatures measured from select LAB and DRUM experiments at comparable conditions with roughly equal dry fuel concentrations. Figure S.7 presents the heating rates from these same experiments (except LAB S12(2)) until each TC’s respective peak temperature in Fig. S.6 throughout time, position, and temperature. LAB S12(2) is omitted from Fig. S.7 because its short pack height and experimental interruptions (noted in Table 1) led to non-representative heating rates. The heating rates throughout time and position anchored the maximum heating rate at time = 0 and position = 0, respectively. All heating rates were from TCs embedded in the fuel bed that reached their peak temperatures after turning off the heater, so to minimize the influence of ignition on these results. However, the TCs nearest to the heater (shown as the lighter lines in Fig S.7) generally exhibited higher heating rates, which was due to minor, persistent ignition effects. Furthermore, the drying/condensation processes ahead...
of smouldering appear as small peaks in the heating rates at low temperatures (< 100 °C). However, the heating rates at these low temperatures due drying/condensation were not plotted in LAB S7, because the conductive ignition method drove fast drying processes ahead of smouldering that were not comparable with the other experiments in Fig S.7, which all used convective ignition methods (see Fig S.1). See (Rashwan, 2015; Rashwan et al., 2016) for example temperature-time profiles that show these fast drying/condensation heating rates in other sewage sludge smouldering LAB experiments that used conductive ignition. Taken together, Figs. S.6 and S.7 highlight how the smouldering peak temperatures and heating rates all decreased more in the DRUM than LAB experiments with increasing MC%, due to non-uniform air flux and non-uniform reactions (i.e., like the smouldering propagation velocities shown in Fig. 5).

Fig. S.6. Centreline mean peak temperatures from well-mixed sewage sludge experiments with similar applied mass fluxes and concentrations of dry fuel (i.e., LAB S4, S5(1), S6, S7, S8, and S12(2) and DRUM S4, S5, S6, S7, and S8 – see Table 1 for experimental conditions), where the error bars approximate one standard deviation of experimental variability.
Fig. S.7. Heating rates presented over: (a-c) time, (d-f) position, and (g-h) temperature. The time in each experiment was adjusted in each curve to the time of its specific maximum heating rate (i.e., \( \text{Heating Rate Time}(t) = \text{Measured Time}(t) - \text{Time of Maximum Heating Rate} \)) and the position was estimated using each experiment’s centreline propagation velocity in Table 1 (i.e., \( \text{Position}(t) = \text{Heating Rate Time}(t) \times v_{\text{oxid}} \)). The legends in (d), (e), and (f) are common to the other graphs in the same column, i.e., (a) and (g), (b) and (h), (c) and (i), respectively (see Fig. S.1 for extra details on the layouts). For clarity, every 3rd data point is plotted, and each frame notes the experiments’ moisture contents (MC).
Similar to (Rashwan et al., 2021a), Fig. S.8 shows the LAB and DRUM sewage sludge experiments developed relatively similar peak temperature profiles across the radius. However, the temperatures near the wall were much cooler in the experiments that fostered an unburned crust. The approximate range of crust edges are noted with vertical dashed lines in Fig. S.8; therefore, all temperatures left of the leftmost dashed line were relatively unaffected by the extinction near the walls.

Fig. S.8. Peak temperatures across radius showing edge effects in sewage sludge experiments with radial measurements. The dashed vertical lines show the approximate range of the edge of the crust observed in the experiments, which varied between experiments and along their circumferences. Therefore, all crust observed extended at least from \(0.9 r_0^{-1}\) to the wall (from the rightmost vertical line to 1). The error bars note the minimum and maximum values observed away from each experiments’ initial effects (after turning off the heater when the smouldering front propagated \(\geq 0.1\) m from the heater) and end effects (when the front was \(\geq 0.01\) m from the end of the fuel bed).
Compared to DRUM S6-S9 in Fig. 5, Fig. S.9 shows that DRUM S10 resulted in a comparatively thin crust formation, though smouldering propagation suffered the most severe effects from non-uniform air flux (discussed in Section 3). This thin crust is hypothesized to primarily be due to the high air flux facilitating more robust smouldering locally near the wall (i.e., the effect of non-uniform reactions was not as severe) (Rashwan et al., 2016; Zanoni et al., 2019a, b) and perhaps also influenced by the heater, which was remained on until the highly distorted front reached ~0.4 m along the wall (Fig. S.10).

Figure S.10 shows the effect of non-uniform air flux on the temperatures in DRUM S10. As mentioned above, the wall TCs show that the smouldering front nearly reached 0.4 m up the wall when the heater was turned off but only reached ~0.2 m up the half-radius and between 0.1 and 0.2 m along the centre. These different front positions indicate that the front was extremely distorted, though smouldering occurred across nearly the entire radius (inferred from the high wall temperatures in Figs. S.8 and S.10 and the thin crust formation in Fig. S.9). Furthermore, 40 minutes after the heater was turned off (at 150 minutes, in Fig. S.10), the air temperatures above the fuel bed (TCs 8a-11a and 6b-8b in Fig. S.10) began to climb when the front only reached 0.2 m along the centreline (~37% of the final pack height). Therefore, air channelling was probably most severe after this time and slowed subsequent propagation, especially along the centreline, thereby weakening smouldering. From another perspective, this air channelling is responsible for the long tailing behaviour observed in the mass loss profile in Fig. 6 (see further discussion in Section 3).
Fig. S.9. Photo showing a thin crust formation after DRUM S10.
Fig. S.10. Temperature histories from DRUM S10. Note that P1 and P2 indicate plenum temperatures in the air space below the fuel bed and ‘a’, ‘b’, and ‘c’ indicate the centreline, half-radius, and wall thermocouples (TCs), respectively (see the experimental layout in Fig. S.1).

S.4. Making Mass Loss and Time Non-dimensional for Fig. 6

The mass loss and time from LAB S7, LAB S8, DRUM S9, and DRUM S10 were made non-dimensional by normalizing the mass to the amount of combustible material in the column, and the time to the ignition time \(t_{ig}\) and the end of propagation time \(t_f\) for robust smouldering. The mass loss was therefore made non-dimensional by:

\[
\text{non} - \text{dimensional mass loss} (t) = \frac{\text{mass loss} (t) - \text{mass loss} (t_{ig})}{\text{expected mass loss} (t_f) - \text{mass loss} (t_{ig})} \quad (S.14)
\]

the expected mass loss \(t_f\) summed the sewage sludge moisture content, the combustible dry fraction of the sewage sludge, and the moisture content of the sand (which was measured as \(\sim 0.028\%\)). The time was made non-dimensional as:
non-dimensional time = \frac{t - t_{ig}}{t_f - t_{ig}} \quad (S.15)

t_{ig} was identified from the inflection point in the mass loss data (\max d^2 m/ dt^2) when the first TC in the pack started showing signs of ignition (rapid temperature increase). 

\( t_f - t_{ig} \) was approximated as:

\[ t_f - t_{ig} = \frac{(x_f - x_0)}{0.13 + 0.08u_g} \quad (S.16) \]

\((x_f - x_0)\) was the pack height observed upon excavation (as the pack slightly collapsed as sewage sludge was smouldered from the sand pore space) and the denominator in Eq. (S.16) is the linear, empirical approximation for robust, fuel-limited smouldering from (Rashwan et al., 2016), where \(u_g\) is the applied Darcy flux. In LAB S8, the non-dimensional mass loss is plotted from non-dimensional time = 0.13 onward, to avoid the initial physical vibrations associated with ignition.
S.5. References