Evaluating the learning of number concepts through measures: A design-based research project

A presentation for BSRLM Spring Conference 2021
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Focus of Presentation

1. Rationale/outline/context of thesis
2. Methods and approach used in Phase 1
3. Emerging themes: methodological and pedagogical
4. Feedback and questions
Rationale

• Children’s understanding of the multiplicative relationship is identified as being an indicator of progress in mathematics (Siemon et al. 2008; Sieglar et al. 2012; Nunes et al. 2012).

• The multiplicative relationship is explicit in the descriptions of learning within the new Curriculum for Wales 2022 framework. The principles of progression for Mathematics and Numeracy focus on five proficiencies: Conceptual Understanding, Communication using Symbols, Fluency, Logical Reasoning, Strategic Competence (Hwb, 2020).


The number system is used to represent and compare relationships between numbers and quantities.

<table>
<thead>
<tr>
<th>Progression step 1</th>
<th>Progression step 2</th>
<th>Progression step 3</th>
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</thead>
<tbody>
<tr>
<td>I have explored forming a quantity in different ways, using combinations of objects or quantities.</td>
<td>I have explored and can use my understanding of multiplicative relationships to multiply and divide whole numbers, using a range of representations, including sharing, grouping and arrays.</td>
<td>I have extended my understanding of multiplicative reasoning to include the concept and application of ratio, proportion and scale.</td>
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<tr>
<td>I have experienced grouping and sharing with objects and quantities, and I can group or share small quantities into equal-sized groups.</td>
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Social constructivism (Ernest, 1998): a philosophy of mathematics as a socially constructed body of knowledge.

Learners interpret and generalise their mathematical experiences to form internalised concepts.

Mediation within a social context is necessary; through social interaction/representation the interpretation can be scrutinised and may be subject to reformulation.

How do learners develop understanding of multiplication and division as concepts?

Concrete to Abstract
/Spataneous to Scientific concepts

Abstract to Concrete
i.e. Scientific concept first?

Typically the approach in most curricula


Measures as a context for learning number

Davydov (1990) believed that all concepts in school mathematics are scientific and that, from the outset, mathematics should be taught in a way that develops scientific concepts and awareness of the concept itself. He believed in children should progress from abstract to concrete (learning scientific concept and then specific concrete examples of it).

He believed that the way children learn the scientific concepts should reflect the way the concept has evolved.

As number concepts have evolved as a way of quantification, particularly through measures, Davydov (1990) created a curriculum that focused on the development of number concepts through measurement of continuous quantities such as length, area, volume and capacity.

Schmittau (2003, p.227) points out that a flaw in starting number concepts with counting discrete objects is that it will ‘ground children in their spontaneous notions of number’. This will make fractions and irrational numbers more difficult to learn.


Measures as a context for learning the multiplicative relationship

Davydov (1992) sees multiplication as a change in the system of units.

How could tasks using measures as a context support the learning of the multiplicative relationship for young learners?

A design-based research project to develop and evaluate materials for teaching multiplicative reasoning through measures.

<table>
<thead>
<tr>
<th>Proposed sub-question</th>
<th>Method of exploration</th>
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<tbody>
<tr>
<td>What are teachers' and learners' prior experiences of teaching and learning multiplicative reasoning?</td>
<td>Review of secondary sources, semi-structured interviews and observation in school.</td>
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<tr>
<td>What are learners' prior experiences of learning number and measures?</td>
<td>Review of secondary sources, semi-structured interviews and observation in school.</td>
</tr>
<tr>
<td>How can tasks using measures be developed to introduce and consolidate multiplicative reasoning, taking into account learners' and teachers' prior experiences?</td>
<td>Initial trial, implementation and iteration phase in one school involving teaching, teacher and learner feedback and analysis through semi-structured interviews and observation.</td>
</tr>
<tr>
<td>What is the impact of learning multiplicative reasoning through measures on learners?</td>
<td>Pre and post assessment. Learner and teacher feedback through semi-structured interview. Observation.</td>
</tr>
<tr>
<td>What are teachers' and learners' views on teaching/learning multiplicative reasoning through measures using the materials developed?</td>
<td>Semi-structured interviews.</td>
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</table>
Phase 1: Developing tasks

Interview with teachers and observation of learning environment

Pre-assessment: semi-structured interviews with learners (8) and pre-assessment tasks

Enactment of tasks: audio recording and my reflection

Semi-structured interviews with learners (8)
Interview with teachers (4, nursery to Year 2 teachers) and observation of learning environment

Concrete, Visual, Abstract is an approach promoted by maths co-ordinator and teachers noted the importance of all learners having practical experiences and visual support; this was evident in the set up classrooms.

Teachers promoted ‘every day’ and ‘real-life’ contexts for maths.

Teachers’ discussion of learning multiplication and measures reflected the yearly expectations in the numeracy framework, a statutory document introduced by Welsh Government in 2013 to raise standards, in particular:

- The discussion around learning of the multiplicative relationship focused on multiplication as repeated addition, the commutative law and division as sharing.
- The discussion around measures focused on development of understanding of language, non-standard measures and standard measures.

‘With measure as well, you know, we’d use things like Duplo, you know, to measure length initially and giving them the choice as well, so you know saying we need superhero capes, what do you want to use to measure, and if the cubes are smaller, well let’s see what the difference is, and just getting them to use lots of non-standard units first of all’

‘We’re doing measurement this week, but the children need to learn if I’m measuring water, it means I need a measuring jug and I measure in litres and millilitres, if I’m measuring time I need a clock or a stopwatch and they get them so muddled up because the language is so so similar.’
Pre-assessment *

Measuring length of a heavy object with a restricted number of plastic rods so that its length could be reproduced elsewhere:

*Learners did not consistently recognise the need to use the same unit repeatedly (1 of 4 pairs iterated)*

Reproducing the amount of liquid in one container to ensure the same amount of liquid in different container (with the use of a small or large cup)

*All groups completed the task successfully, repeating use of larger cup showing understanding of unit being used (e.g. 2 and half large cups)*

Measuring the same length twice using different sized straws, where one straw is twice the length of the other

*Learners used both straws independently of their relationship*

Identifying how many of a set of 4 unifix cubes would be equal to a length of 20 unifix cubes

*All learners approached task initially using addition (e.g. counting all 20, counting 4 then counting another 4 etc. to get to 20) – suggesting they saw the unit as the unifix cube rather than the set of 4*

## Enactment/Trial of tasks

<table>
<thead>
<tr>
<th>Task</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring the same length twice using different sized straws, where one straw is twice the length of the other</td>
<td>Relation to pre-assessment; to reinforce relationship between size of unit and number of times it is used</td>
</tr>
<tr>
<td>Identifying which cups were needed a given amount of times</td>
<td>Relation to pre-assessment; to reinforce relationship between size of unit and number of times it is used</td>
</tr>
<tr>
<td>Identifying how many of a very small container make up a large jug, with the introduction of an intermediate larger cup</td>
<td>To establish use of intermediate unit</td>
</tr>
<tr>
<td>Finding how many pancakes could be made from a quantity of flour if one cup could make a particular amount</td>
<td>To reinforce that a unit can represent a number other than 1</td>
</tr>
<tr>
<td>Finding how many different sized Cuisenaire rods made up a fixed length</td>
<td>To bridge between standard units (cm) and multiplicative relationships</td>
</tr>
<tr>
<td>Finding how many spoonfuls of a liquid would be contained in a bottle, with the introduction of an intermediate measure</td>
<td>To incorporate standard units (ml) into tasks involving intermediate units</td>
</tr>
<tr>
<td>Using bags of sugar cubes as an intermediate measure to make up a specific weight</td>
<td>To incorporate standard units (g) into tasks involving intermediate units</td>
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</table>
## Data collected and analysis methods

Reflective notes and 6 hours of audio data – transcribed and analysed thematically using codes that were both inductive and deductive (Xu and Zammit, 2020)

<table>
<thead>
<tr>
<th>Behaviour (what is done)</th>
<th>Learner examples</th>
<th>Teacher examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>counting in ones, counting in multiples, suggesting idea, relation to context</td>
<td>questioning, relation to context, suggesting idea</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Emotion (what emotion may be expressed)</th>
<th>Learner examples</th>
<th>Teacher examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>surprise, enjoyment</td>
<td>N/A – considered through reflective notes</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Awareness of</th>
<th>Learner examples</th>
<th>Teacher examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>change in unit, a specific multiplicative relationship, additive relationship, composite units</td>
<td>N/A – considered through reflective notes</td>
<td></td>
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</table>


Emerging findings: from audio

Early tasks supported awareness of relation between quantity and size of unit, e.g.

‘Well with the smaller straw, you'd think that there would be less amount because the big straw is bigger than it but because it's little, there'll be more, like the number, there'll be a bigger number than with the bigger one’

‘We're going to have more being scooped up with this one because it's a bigger, it can fit a bigger amount in it ‘

Across all files (n=6) awareness of multiplicative relationships is evident.

‘quarter would make about six and we could count in sixes then’

‘Anyone know their six times tables?’

Awareness of change in unit is also suggested, e.g.

‘What if we poured the water into the (large) cup…Because you could see how much (little) cupfuls’

‘We could count how many of these (little cups) make up one (large cup)’
## Emerging findings: from post-task interviews

<table>
<thead>
<tr>
<th>Task</th>
<th>Helped me learn the most maths</th>
<th>Made me think the most</th>
<th>Most fun</th>
<th>Suggestions for improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straws and string</td>
<td>Learner 4 – learnt how to measure</td>
<td>Learner 2 – alludes to double/half relationship Learner 3 – because not using standard measure Learner 5 (suggested in what is said) Learner 6 Learner 7 Learner 8</td>
<td>Learner 1 Learner 2 Learner 7</td>
<td>Learner 3 notes different sized straws Learner 8 – stretchy</td>
</tr>
<tr>
<td>Jugs and little cups for rabbits</td>
<td>Learner 1 Learner 2 Learner 7</td>
<td>Learner 2 Learner 8</td>
<td>Learner 2 Learner 8</td>
<td>Language implication from Learner 2 comments</td>
</tr>
<tr>
<td>How many pancakes? Flour and cupfuls.</td>
<td>Learner 1 Learner 3 Learner 7</td>
<td>Learner 1 Learner 3 Learner 7</td>
<td>Learner 1 Learner 3 Learner 7</td>
<td></td>
</tr>
<tr>
<td>Cuisenaire – how many lengths?</td>
<td>Learner 1 Learner 2 Learner 3 Learner 5 Learner 6 Learner 7 Learner 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medicine for dog – how many spoonfuls?</td>
<td></td>
<td>Learner 4 Learner 7</td>
<td>Learner 2 notes use of line Learner 3 notes size of bottle Learner 5 notes order</td>
<td></td>
</tr>
<tr>
<td>Weight – sugar cubes and weighing sugar</td>
<td>Learner 1 Learner 5 Learner 6</td>
<td>Learner 2 (suggested in what is said) Learner 7 (suggested in what said)</td>
<td>Learner 5</td>
<td></td>
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</tbody>
</table>
A few thoughts…

• Learners enjoy tasks involving continuous quantities; they expressed excitement and enjoyment across all tasks, e.g.

'I love working with liquids' 

'Working with flour is so satisfying'

• Cuisenaire was identified as helpful; it was associated with learning most maths. Perhaps because it allowed for generation of easily visible relationships and generation of multiple multiplicative relationships.

Or it may be because of when it was introduced?

• The relation of a task to a particular context (e.g. how many rabbits can I feed) seemed to be 'dropped' by some learners. They associated the tasks more with the activity than the context.
Considerations…

Restricting the amount of small units available to necessitate the use of an intermediate (composite) unit can also mean that establishing an initial visual representation of that composite relationship is more difficult. Learners’ comments ‘being able to see all the cups’ suggest they value this initial establishment of a relationship, yet the tasks which learners (and coding) suggest support more learning, or made them think more involved restriction.

The type of quantity and the way it is measured can also impact on the initial visual representation of composite relationships: a learner comments on a weighing task that it is easier to see when things are equal, and this could also be said for Cuisenaire. Liquids and equality relationships are more difficult to see unless the same sized containers are used.
Further design considerations...

Structural design – to what extent do the tasks support learners in bridging between their experiences of multiplicative relationships with discrete objects and the experience of seeing the multiplicative relationship as a change in unit with continuous quantities? Straws and Cuisenaire supported this more.

Structural design – to what extent should the inclusion of standard units should be considered; the use of standard measures could enhance the notion of a change in unit

Technical design – to what extent should the designs involve restriction of items? What continuous quantities work particularly well? How can I (manageably) source containers with particular relationships and to what extent is accuracy (e.g. cupful) going to be important?
DIOLCH - THANK YOU