Can quantum teleportation be used for information transfer for the quantum internet?

Student Dissertation

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EMA Part 1

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A Report submitted as the examined component of the Project Module SXP390.

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- stand-alone equations, and explanations of symbols and constants used in equations.
Abstract

Quantum teleportation, the replication of information from one location to another over unlimited distances, has been successfully demonstrated in recent years. This opens the way for a quantum internet linking together multiple remote quantum devices over a quantum network. In this paper, we provide a brief theoretical background of quantum teleportation, whereby two or more particles are in a particular total quantum state such that the measurement of a property, e.g., the polarization of a photon, of one of these particles will provide the certainty that the other particles in this total quantum state will also have the same value of this property, regardless of how far away these other particles are. We survey recent experimental tests of quantum teleportation, describing the progressive increase in the teleportation distance for a qubit, a quantum bit, which is a superposition of two states, e.g., 0 and 1. We highlight some key challenges brought by quantum teleportation, specifically those arising from the ‘no-cloning theorem’, which states that it is impossible to create a copy of an unknown quantum state, and decoherence, which degrades the quality of quantum states due interactions with the environment. These challenges currently limit teleportation to a single, point-to-point, communication link with distances up to 1,400 km. We discuss recent technological developments which attempt to address these challenges and show that the necessary technologies and standards for a quantum network are currently in the early stages of maturity. We conclude that quantum teleportation can be used for information transfer for a quantum internet, but that given the challenges presented by quantum teleportation for a multi-node remote user quantum network, the likely availability of an initial teleportation-capable quantum internet is in the ten-to-fifteen-year timescale. Finally, we propose areas for further research.

Total word count for Abstract: 290 words
Preface

This report is a literature review project constituting the main element of my End of Module Assignment for the Open University’s SXP390 module, my final third-year module of a B.Sc. (Hons) Physics undergraduate degree.

The intended audience for this project are readers who have studied quantum mechanics at third-year undergraduate degree level or above.

I have written the report in first person plural, in line with SXP390 guidelines, but the report is wholly my own work.

For ease of readability, I have not repeatedly stated explanations of terms for a polarized single-photon qubit. Except where otherwise indicated, the state of a polarized single-photon qubit is represented as

$$|\psi\rangle_1 = \alpha|V\rangle_1 + \beta|H\rangle_1,$$

where the subscript, 1, is for photon 1, and

\(\alpha\) and \(\beta\) are complex numbers such that \(|\alpha|^2 + |\beta|^2 = 1\), and

\(V\) and \(H\) represent vertical and horizontal polarizations, respectively.

All experimental results shown in figures use Poissonian count rate distribution for error bar calculation, unless otherwise indicated.

Tahir Ahmed

August 2021
### List of abbreviations

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<tr>
<td>BBO</td>
<td>Beta-barium borate</td>
</tr>
<tr>
<td>BSM</td>
<td>Bell State Measurement</td>
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<tr>
<td>EIT</td>
<td>Electromagnetically Induced Transparency</td>
</tr>
<tr>
<td>EPR</td>
<td>Einstein-Podolsky-Rosen</td>
</tr>
<tr>
<td>EOM</td>
<td>Electro-optical modulator</td>
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<td>OU</td>
<td>Open University</td>
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<td>PBS</td>
<td>Photon Beam Splitter</td>
</tr>
<tr>
<td>PPL</td>
<td>Point-to-point link</td>
</tr>
<tr>
<td>PROMPT</td>
<td>Provenance, Relevance, Objectivity, Method, Presentation, Timeliness</td>
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<tr>
<td>QEQI</td>
<td>Quantum Entanglement and Quantum Information</td>
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<td>QKD</td>
<td>Quantum Key Distribution</td>
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<td>QR</td>
<td>Quantum Repeater</td>
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<td>SPDC</td>
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<td>TMA</td>
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Chapter 1 Introduction

1.1 Background

The current internet is based on the interconnectivity of disparate classical computer systems which pass data over communication networks spanning the globe. Reliable, secure, and quick transfer of information over long distances is crucial to the satisfactory working of the internet.

One of the key outcomes of quantum mechanics is quantum teleportation, first proposed nearly 30 years ago (Bennett et al., 1993). Quantum teleportation relies on quantum entanglement whereby two or more particles are in a particular total quantum state, called a Bell state or an Einstein- Podolsky-Rosen (EPR) state, such that the measurement of a property, e.g., the polarization of a photon, of one of these particles will provide the certainty that the other particles in this total quantum state will also have the same value of this property, regardless of how far away these other particles are.

Using quantum teleportation, the measurement of the properties of several particles which are in an entangled state can be used to replicate information of a system at one location to that of another system at a different, remote location, opening the possibilities for a quantum internet within the foreseeable future.

In this project, we summarise the theory underlying quantum teleportation, we survey some significant experimental tests which teleported the polarization, qubit, states of single photons. We discuss the challenges brought by quantum teleportation for a quantum network. We show that the standards and technologies needed for a quantum internet are in their early stages of maturity. We propose likely timescales for achieving an initial teleportation-capable quantum internet and provide areas of current research which may contribute to the development of future teleportation technologies. Finally, we suggest further research.
1.2 Objectives

1 – Describe quantum teleportation theory.

2 – Survey recent experimental tests of quantum teleportation.

3 – Critically assess the challenges of, and opportunities for, using quantum teleportation to enable the quantum internet.

4 – Draw some conclusions about the feasibility and timescales for a quantum internet using quantum teleportation.

1.3 Scope of work

This project focuses on the mechanisms and challenges of quantum teleportation for a quantum internet, focusing primarily on teleportation of single-photon qubits over a quantum channel. As such, networking protocols, equipment components, set-up configurations, quantum computers, encryption techniques and the current internet are not discussed in detail.

1.4 Methodology

We have used

- OU Library Search, and Google, with different levels of detail using general and specific search criteria, including Boolean terms, to retrieve information from databases, journals, articles, conference proceedings and websites,

- PROMPT to evaluate and shortlist peer reviewed papers, focusing on those less than 5 years old. We also selected some very recent papers that haven’t yet been peer reviewed, because they might inform on future research directions,

- the reference lists included in the papers summarised in TMA01, TMA02 and TMA03,

- the papers and books referenced in the QEQI material provided as part of the SXP390 module.
Chapter 2 Quantum teleportation theory

2.1 Quantum teleportation theory

Fig. 1 - Quantum teleportation, shows how an arrangement for three particles comprised of 2 pairs of entangled particles can be used to replicate polarization information for a photon, photon 1, ‘the unknown photon’, held by Alice in one location, to another photon, photon 3, held by Bob in another location.

The teleportation process (Bolton and Macintosh, 2009, pp. 193-196), in summary, is:

- photon 1 is the photon whose state is to be teleported, held by Alice,
- photon 2 and photon 3 are an entangled photon pair, photon 2 sent to Alice and photon 3 sent to Bob, over a quantum channel (not shown),
- Alice entangles photon 1 and photon 2 using a beam splitter, carries out a Bell State Measurement (BSM) on photon 1 and photon 2, and notifies Bob of her result,
- Bob carries out a transformation which replicates photon 3’s state to that of Alice’s photon 1, thus achieving quantum teleportation.
These steps in more detail are as follows:

The state of photon 1, can be represented by a state vector $|\psi\rangle_1$, a superposition of states $|V\rangle_1$ and $|H\rangle_1$:

$$|\psi\rangle_1 = \alpha|V\rangle_1 + \beta|H\rangle_1,$$

Photons 2 and 3 are a pair of maximally-entangled photons produced in one of four Bell states:

$$|\Psi^\pm\rangle_{23} = \frac{1}{\sqrt{2}} (|V\rangle_2 |H\rangle_3 \pm |H\rangle_2 |V\rangle_3),$$

$$|\Phi^\pm\rangle_{23} = \frac{1}{\sqrt{2}} (|V\rangle_2 |V\rangle_3 \pm |H\rangle_2 |H\rangle_3),$$

where $\frac{1}{\sqrt{2}}$ is the normalization constant.

If, say, photons 2 and 3 are in an entangled state represented by $|\Psi^-\rangle_{23}$, then all three photons are represented by the state vector:

$$|\Psi\rangle_{123} = |\psi\rangle_1 |\Psi^-\rangle_{23},$$

which, when expanded, gives

$$|\Psi\rangle_{123} = \frac{\alpha}{\sqrt{2}} (|V\rangle_1 |V\rangle_2 |H\rangle_3 - |V\rangle_1 |H\rangle_2 |V\rangle_3) + \frac{\beta}{\sqrt{2}} (|H\rangle_1 |V\rangle_2 |H\rangle_3 - |H\rangle_1 |H\rangle_2 |V\rangle_3).$$

Alice uses a photon beam splitter (see section 3.1 pp. 15-16 for details) to entangle photons 1 and 2. Their combined state vector is a superposition of one of the four Bell states:

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|V\rangle_1 |H\rangle_2 \pm |H\rangle_1 |V\rangle_2),$$

$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|V\rangle_1 |V\rangle_2 \pm |H\rangle_1 |H\rangle_2).$$

The state vector for the three photons can then be represented by

$$|\Psi\rangle_{123} = \frac{1}{2} [ |\Psi^-\rangle_{12} (\alpha |V\rangle_3 - \beta |H\rangle_3) + |\Psi^+\rangle_{12} (\alpha |H\rangle_3 + \beta |H\rangle_3) + |\Phi^-\rangle_{12} (\beta |V\rangle_3 + \alpha |H\rangle_3)]$$

which is a linear combination of photon 1 and 2’s four Bell states with coefficients that are the states of photon 3.

Using the photon detectors, Alice takes a combined BSM, of photons 1 and 2, causing the collapse of the state vector of these two photons onto one of the four Bell states given by $|\Psi^\pm\rangle_{12}$ and $|\Phi^\pm\rangle_{12}$. Photon 3 immediately collapses onto one of the four terms of the state vector $|\Psi\rangle_{123}$, each with a probability of $\frac{1}{4}$. 

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Alice informs Bob of the result of her BSM via the classical communication channel, e.g., using 2 binary digits (bits) to represent the outcome. Bob carries out a transformation on photon 3 corresponding to the 2 bits. This transforms Bob’s photon 3 into the exact initial state of photon 1 originally held by Alice (Table 1 - Replication).

| Alice’s BSM outcome of photons 1 and 2 | Alice’s 2 bits for BSM | State of Bob’s photon 3, \(|\phi\rangle_3\), after collapse of \(|\psi\rangle_{123}\) | Matrix form of \(|\phi\rangle_3\) | Transformation matrix | After transform, the state of Bob’s photon 3, \(|\phi\rangle_3\), replicates that of Alice’s photon 1, \(|\psi\rangle_1\) |
|----------------------------------------|------------------------|-------------------------------------|----------------|---------------------|---------------------------------|
| \(|\psi^-\rangle_{12}\) 00            | \(-\alpha |\nu\rangle_3 - \beta |H\rangle_3\)          | \([\begin{smallmatrix} -\alpha \\ -\beta \end{smallmatrix}]\) | \([\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}]\) | \([\begin{smallmatrix} -\alpha \\ -\beta \end{smallmatrix}] = [\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}]\) |
| \(|\psi^+\rangle_{12}\) 01            | \(-\alpha |\nu\rangle_3 + \beta |H\rangle_3\)          | \([\begin{smallmatrix} -\alpha \\ +\beta \end{smallmatrix}]\) | \([\begin{smallmatrix} -1 & 0 \\ 0 & 1 \end{smallmatrix}]\) | \([\begin{smallmatrix} -\alpha \\ +\beta \end{smallmatrix}] = [\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}]\) |
| \(|\Phi^-\rangle_{12}\) 10            | \(\beta |\nu\rangle_3 + \alpha |H\rangle_3\)          | \([\begin{smallmatrix} +\beta \\ +\alpha \end{smallmatrix}]\) | \([\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}]\) | \([\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}] [+\beta \\ +\alpha \end{smallmatrix}] = [\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}]\) |
| \(|\Phi^+\rangle_{12}\) 11            | \(-\beta |\nu\rangle_3 + \alpha |H\rangle_3\)          | \([\begin{smallmatrix} -\beta \\ +\alpha \end{smallmatrix}]\) | \([\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}]\) | \([\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}] [-\beta \\ +\alpha \end{smallmatrix}] = [\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}]\) |

Final state of Bob’s photon 3, \(|\phi\rangle_3\)

\(|\phi\rangle_3 = \alpha |\nu\rangle_3 + \beta |H\rangle_3 = |\psi\rangle_1\)

Table 1 – Replication

2.2 The no-cloning theorem

It can be proven mathematically that making a copy, or clone, of a particle’s state is not possible, leading to the no-cloning theorem:

‘The linearity of operators in quantum mechanics forbids the cloning of quantum states’ (Bolton and Macintosh, 2009, p. 191).
All communication channels involve the transmission of signals through a medium, such as free space and optical fibre. Signals are subject to corruption and deterioration due to factors including background noise, collisions with other particles, and the inverse square law, placing a practical limit on the length of communications links.

In the classical internet (Fig. 2 - Classical internet), transmission over long distances is achieved using repeaters and switches:

- repeaters to receive a signal which is then copied, amplified and retransmitted thus extending the overall transmission distance, and
- switches to route the signal to the intended final recipient device,
- communication distance is extended by the addition of extra routers and switches.

For a quantum internet using teleportation, the no-cloning theorem prevents information of an entangled particle to be copied, limiting the quantum channel to that of a single point-to-point link (PPL). Storage of information is also not possible.

2.3 Decoherence

In theory, the final state of Bob’s teleported photon is an exact replica of that of Alice’s initial photon. In practice, teleportation is subject to factors such as signal deterioration, causing gradual degradation of qubit quality, or decoherence. Each interaction of a photon with its environment degrades the quantum state’s superposition, ultimately resulting in the quantum state becoming a classical state. Decoherence limits how long qubits can be reliably used in teleportation experimental hardware to the order of milliseconds (Humphreys et al., 2018, pg. 4), hence placing limits on the quantum channel distance.
2.4 Fidelity

Fidelity, $F$, is a measure of how closely the final state of Bob’s photon after teleportation matches the initial state of Alice’s photon before teleportation. $F$ has values between 0 and 1, where 0 represents no replication and 1 represents total replication. It can be shown that, for quantum teleportation of a single spin-$\frac{1}{2}$ particle such as polarized light, the classical limit, the limit at which the quantum system behaves like a classical system, is $F = \frac{2}{3}$ (Massar and Popescu, 1995, pg. 3). Successful teleportation requires the fidelity to be above this classical limit, i.e., $F > \frac{2}{3}$. $F$ is often represented as 0.67, 66.7%, to 2 or 3 s.f. or in similar formats.

2.5 Teleportation in practice

The unlimited teleportation distance of the quantum channel described by theory is, in practice, limited to a few hundred kilometres over a single PPL due to the combined effects of the no-cloning theorem and decoherence.
Chapter 3 Quantum teleportation experiments

3.1 Entanglement methods

Spontaneous Parametric Down Conversion (SPDC) (Fig. 3 - SPDC) is a typical method for generating entangled photons (photons 2 and 3 in Fig. 1).

SPDC uses a nonlinear crystal to split a photon of higher energy into pairs of photons of lower energy, consistent with the laws of conservation of energy and conservation of momentum. A typical apparatus has a laser beam passing through a crystal such as beta-barium borate (BBO). The emerging photon pairs have trajectories which are along the edges of two cones, one vertically polarised, the other horizontally polarised. The intersection of the cones is where the two emergent photons may exist simultaneously, resulting in entanglement.

A PBS is a half-silvered mirror which splits each of two incident beams of light into two, reflecting one half and transmitting the other half (Fig. 6 Photon beam splitter). A frequency filter (not shown) ensures only photons of the same frequency are incident on the mirror. The input spatial wave functions, \(|a\rangle\) and \(|b\rangle\), of the two incident photons overlap, rendering the output spatial wave functions, \(|c\rangle\) and \(|d\rangle\), indistinguishable after passing through the splitter,
resulting in four possible output mode combinations given by the four Bell states $|\Psi^{\pm}\rangle_{12}$ and $|\Phi^{\pm}\rangle_{12}$.

![Figure 4 - Photon beam splitter](Bolton and Mackintosh, 2009, p. 199, Fig 7.13)
<table>
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<th>Alice’s entanglement method for photons 1 and 2</th>
<th>Observable teleported</th>
<th>Number of dimensions teleported</th>
<th>Type of quantum channel</th>
<th>Quantum channel length</th>
<th>Reference</th>
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<td>SPDC. Pulse of ultra-violet light through β-barium borate crystals</td>
<td></td>
<td>Linear and circular polarization of spin-( \frac{1}{2} ) photons</td>
<td>Single-photon qubit</td>
<td>Not stated</td>
<td>Not stated</td>
<td>(Weinfurter et al., 1997)</td>
</tr>
<tr>
<td>2004</td>
<td>SPDC. Pulse of ultra-violet light through β-barium borate crystals</td>
<td></td>
<td>Linear and circular polarization of spin-( \frac{1}{2} ) photons</td>
<td>Single-photon qubit</td>
<td>Optical fibre</td>
<td>600 m</td>
<td>(Ursin et al., 2004)</td>
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<tr>
<td>2012</td>
<td>SPDC. Pulse of near-infrared light through β-barium borate crystals</td>
<td></td>
<td>Linear polarization of spin-( \frac{1}{2} ) photons</td>
<td>Single-photon qubit</td>
<td>Free space</td>
<td>143 km</td>
<td>(Ma et al., 2012)</td>
</tr>
<tr>
<td>2017</td>
<td>SPDC. Pulse of ultraviolet light through bismuth borate crystals</td>
<td></td>
<td>Linear polarization of spin-( \frac{1}{2} ) photons</td>
<td>Single-photon qubit</td>
<td>Free space</td>
<td>500 km - 1,400 km</td>
<td>(Ren et al., 2017)</td>
</tr>
</tbody>
</table>

Table 2 - Significant experiments
3.2 Quantum teleportation experiments

A selection of significant quantum teleportation experiments is shown in Table 2 – Significant experiments. All the quantum teleportation experiments described in this chapter used SPDC and PBS for entanglement of photons.

Quantum teleportation was first achieved in 1997 (Weinfurter et al., 1997), successfully teleporting a single-photon qubit with 0°, ±45° and 90° linear polarizations, and circular polarization.

In 2004, a single-photon qubit was teleported using a 600 m optical fibre quantum channel located in a sewer pipe under the River Danube, Vienna, Austria (Ursin et al., 2004) (Fig. 5 – 600 m teleportation).

SPDC was used to entangle photon 2 (to Alice) and photon 3 (to Bob). A PBS entangled photons 1 and 2 (Alice). The BSM result was conveyed to Bob via a classical microwave channel. If necessary, Bob applied a transform (feed-forward) to ensure that photon 3’s state exactly replicated that of photon 1’s initial state. Feed-forward entails the application of a $\pi$ phase shift between the vertical and horizontal components of photon 3 using an electro-optical modulator (EOM).

Three input states for photon 1’s polarization were used, 45° linear, left-handed circular, and horizontal. Each measurement run lasted 28 hours. The authors’ wording regarding fidelity results was unclear, but they claim that the classical limit was surpassed. It was difficult for us to verify this claim because detailed result data was unavailable. The article was peer-
reviewed, published in a reputable journal (Nature) and has been cited several times (225 in Scopus, 205 in Web of Science), indicating general acceptance of the claims.

In 2012, quantum teleportation of a qubit was performed between two locations in the Canary Islands, La Palma and Tenerife, separated by 143 km (Ma et al., 2012).

A quantum channel using optical point-to-point transmission over free space was used to transmit a pair of SPDC entangled polarized photons, photon 2 to La Palma (Alice), photon 3 to Tenerife (Bob). A third polarized photon, photon 1 (Alice), was introduced to photon 2 and a BSM was performed on photons 1 and 2 in La Palma. The BSM result was sent to Tenerife (Bob) via the classical channel. If Bob’s photon 3 was not already in the same state as the initial state of Alice’s photon 1, a feed forward transformation was applied to photon 3 to ensure that photon 3’s state replicated photon 1’s initial state.

The SPDC arrangement ensured that only entangled photons in the $|\Psi^-\rangle_{23}$ state were used in the experiment. The PBS apparatus used for entangling photon 1 and 2 limited the output Bell states to $|\Psi^\pm\rangle_{12}$ only. A total of 605 fourfold coincidence counts were made over a period of 6.5 hours, covering ‘without feed forward’ and ‘with feed forward cases, each with different sets of input states for photon 1, $|\Phi\rangle_1$, where

$$|\Phi\rangle_1 = \alpha |V\rangle_1 + \beta |H\rangle_1,$$

In the ‘without feed forward’ cases, photon 1 was put into four possible states:

$$|\Phi\rangle_1 = |H\rangle,$$

$$|\Phi\rangle_1 = |V\rangle,$$

$$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) = |P\rangle,$$

and

$$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle - i|V\rangle) = |L\rangle,$$ where $i = \sqrt{-1}$, such that $|\Phi\rangle_1$ is phase-shifted.

The BSM outcome used was $|\Psi^-\rangle_{12}$.

In the ‘with feed forward’ cases, photon 1 was put into two possible states:

$$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) = |P\rangle_{FF},$$

and

$$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle) = |R\rangle_{FF},$$

such that the required $\pi$ phase shift between the $|H\rangle$ and $|V\rangle$ components of photon 3 resulted in a 90° polarization rotation. A $\pi$ phase shift would have had no effect where the
input states were $|\Phi\rangle_1 = |H\rangle$ and $|\Phi\rangle_1 = |V\rangle$ and so these cases were deemed to be included in ‘without feed forward’ cases. The BSM outcome used was $|\Psi^+\rangle_{12}$.

Fig. 6 – 143 km teleportation, shows the fidelity achieved for each input state for both cases. The grey rectangles represent the ‘without feed forward’ cases showing an average fidelity of $F_{av} = 0.863 \pm 0.038$. The red rectangles represent the ‘with feed forward’ cases with fidelities $F = 0.760 \pm 0.050$ and $F = 0.800 \pm 0.037$ for $|P\rangle_{FF}$ and $|R\rangle_{FF}$, respectively. All these fidelities are above the classical limit, indicated by the horizontal black line.

According to the article’s supplementary information, sources of fidelity error included bad weather conditions which affected the quantum channel despite the use of specially-designed noise reduction and signal boosting equipment to reduce decoherence.

The authors proposed that their experiment laid the foundation for future applications such as quantum networks using satellite-based quantum teleportation.

In 2017, a satellite-based teleportation experiment was conducted (Ren et al., 2017), between a ground transmitter in Ngari, Tibet (Alice) and the Micius satellite (Bob) (Ren et al., 2017). The distance between the transmitter and satellite varied from about 500 km to 1,400 km due to Micius’s orbit. SPDC was used for entangling photons 2 and 3, and a PBS for the BSM of photons 1 and 2. A 0 or $\pi$ phase shift was applied as necessary to photon 3. Successful teleportation over a maximum quantum channel distance of 1,400km was achieved.

A total of 911 fourfold coincidence counts were made over 32 Micius orbits covering six different input states for photon 1:
$|\Phi\rangle_1 = |H\rangle$,  
$|\Phi\rangle_1 = |V\rangle$,  
$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) = |+\rangle$,  
$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle - |V\rangle) = |--\rangle$,  
$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle) = |R\rangle$, and  
$|\Phi\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle - i|V\rangle) = |L\rangle$.

Fig. 7 – 1,400 km teleportation, shows the fidelity achieved for each input state.

The average fidelity, $F_{\text{av}} = 0.80 \pm 0.01$ (not shown) is above the classical limit, indicated by the horizontal dashed red line.

The article’s supplemental ‘Methods’ document states that sources of fidelity error included double-pair emission at the SPDC, partial photon distinguishability and dark background count. The experiment was subject to atmospheric disturbance and geometric loss due to the varying angular separation between the transmitter and Micius.

These experiments demonstrate that quantum teleportation is achievable across distances of up to about 1,400 km.
Chapter 4 The challenges of quantum teleportation

The experimental tests described in Chapter 3, and many other tests, have proven that teleportation is achievable. But the challenge remains of overcoming the quantum channel distance limitations and decoherence.

4.1 Quantum channel distance

The current teleportation distance record for single PPL quantum channels is 1,400 km for free space (Ren et al., 2017) and 44 km for fibre (prepublication, awaiting peer review) (Valivarthi et al., 2020).

Quantum channel distance could be extended by connecting many single PPLs using a ‘quantum repeater’ (QR), a device which uses ‘entanglement swapping’ to overcome the no-cloning problem (Fig. 8 – Quantum repeater).

Initially, two EPR pairs of particles are generated (1st: EPR Generation). One particle of the first EPR pair is sent to Alice, the other to the QR, and one particle of the second EPR pair is sent to the QR, the other to Bob.

A BSM conducted at the QR creates entanglement between Alice’s and Bob’s particles (2nd: BSM Process).

In theory, many QRs could be used to increase the quantum channel distance indefinitely, but in practice, each QR and PPL introduces additional decoherence, with the further challenge of co-ordinating and sequentially synchronising the BSMs across all QRs within
sub-second timescales before the states of Alice’s and Bob’s particles decohere. These challenges could be addressed if qubits could be stored in a ‘quantum memory’.

### 4.2 Quantum memory

A quantum memory provides the ability to reliably store quantum states for later retrieval and processing. For single-photon qubits, one approach is Electromagnetically Induced Transparency (EIT) (Ma, Slattery and Tang, 2017).

With EIT, a polarized photon is absorbed into an atom or ion in a gas or solid, raising its energy level from some initial state. Stimulated emission via a control laser pulse changes the state to a lower energy level from which transitions to even lower energy levels are forbidden. The qubit information contained in the photon’s polarization is held in the spin state of the atom.

The atom is effectively ‘trapped’ for the time it is prevented from absorbing energy from its environment, for example by being kept at extremely low temperatures or held motionless by superconducting magnets. But the atom or ion will eventually decohere.

For retrieval, the atom’s spin state is transferred back to a photon with the same energy and polarization as the original photon, through absorption and stimulated emission, again using laser pulses.

The EIT approach has several variants applicable for different storage mediums: cold or warm gas, solid-state, and rare-earth doped solids.

The storage times for EIT are between $1.5 \times 10^{-3}$ ms to $10^3$ ms (Ma, Slattery and Tang 2017, pg. 6, Table 1) with efficiency varying between 12% to 87%, where efficiency is defined as the ratio of the number of retrieved photons to that of those absorbed.

Our view is that these storage times might be sufficient for small numbers of connected repeaters, but they are still too low to provide the timely entanglement swapping needed for a quantum internet having a large number of repeaters.

### 4.3 Entanglement purification

Entanglement purification, also known as entanglement distillation, is a technique for obtaining a small number of highly-entangled states from a large number of less entangled states. Fig. 9 – Entanglement purification theory, shows the scheme for two pairs of photons in less entangled states, Pair 1 and Pair 2.
One of each of two pairs of entangled photons are sent to Alice, $a_1$ and $a_2$ from Pair 1 and Pair 2, respectively. The other photons of the entangled pairs are sent to Bob, $b_1$ and $b_2$, from Pair 1 and Pair 2 respectively.

Alice and Bob each input their photons into a PBS, and keep only those cases where one, and only one, photon is detected in their respective output modes. They individually conduct a polarization measurement of their photons using $\pm$ bases where

$$|+\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle), \text{ and}$$

$$|−\rangle = \frac{1}{\sqrt{2}} (|H\rangle − |V\rangle)$$

and compare their results over the classical communication channel. Depending on the results, Alice performs a transform on her photon $a_3$, if necessary, to ensure that her photon $a_3$ and Bob’s photon $b_3$, are entangled.

Fig. 10 - Entanglement purification results, shows that the relative fractions of the $H/H$, $V/V$, $+/+$, and $−/−$ states increased after purification (Fig. 10c and Fig. 10d) compared with prior to purification (Fig. 10a and Fig. 10b).
The authors claim a fidelity of 92% could be obtained from two pairs of low-entanglement photons, where each pair has a fidelity of 75%.

Our view is that the gain in increased fidelity is at the expense of conducting additional ‘rounds’ of entanglement swapping, potentially prohibitively limiting the number of repeaters required and increasing the complexity in efficiently managing entanglement generation and co-ordinating BSMs across these additional repeaters.

4.4 Higher dimensions

Teleportation of a photon’s polarization typically uses a qubit containing two dimensions, horizontal and vertical. But if more information pertaining to the polarization, i.e., higher dimensions, could be used, then decoherence could possibly be reduced.

In 2019, a team from the University of Science and Technology of China (USTC) conducted quantum teleportation tests for a single photon in a 3-dimensional quantum state, a qutrit, (Luo et al., 2019) in a state $|\phi\rangle_a$, where

$$
|\phi\rangle_a = \alpha_0 |0\rangle_a + \alpha_1 |1\rangle_a + \alpha_2 |2\rangle_a
$$

where subscript, a, is for photon a, and $|1\rangle$, $|2\rangle$, and $|3\rangle$ are encoded by 3 different paths of the photon, and

$\alpha_0, \alpha_1$ and $\alpha_2$ are complex numbers such that $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 = 1$. 
The qutrits for Alice’s photon, photon a, and those from the entanglement source, photons b and c, were each created by passing one of the two output paths from a PBS into another PBS resulting in three output paths, 0, 1, 2 for each of photons, a, b, c (Fig. 11 – Luo’s team’s experimental set-up).

A 3-dimensional BSM of Alice’s photon a’s qutrit with that of photon b’s qutrit resulted in 9 possible outcomes, each of equal probability of $\frac{1}{9}$, rather than the 4 possible outcomes, each of equal probability of $\frac{1}{4}$, from a BSM for a 2-dimensional scheme.

Successful teleportation of 12 different qutrit states of Alice’s photon a was achieved, with an average fidelity of $F = 0.75 \pm 0.01$ (Fig. 12a – Qutrit experiments, Luo’s team’s results). The average fidelity is shown by the solid red/grey horizontal line, the classical limit for teleportation is labelled as ‘Qutrit limit’ with value $F = 0.67$ is shown by the dashed horizontal blue line.

Another team from USTC used an alternative experimental set-up involving 3 pairs of 2 entangled photons (Hu et al., 2020) to obtain fidelity results for 10 different qutrit states for Alice’s photon (Fig. 12b - Qutrit experiments, Hu’s team’s results).
Luo’s team’s results show a higher average fidelity, $F = 0.75 \pm 0.01$, than Hu’s team’s results of $F = 0.685 \pm 0.027$, indicated by the horizontal green bar in Fig. 12b. Both teams’ average fidelity results are above the classical limit of 0.667.

The red horizontal dashed lines, labelled as ‘Classical limit’ in Fig. 12a, unlabelled in Fig. 12b, with value $F = 0.5$, represent a totally non-quantum system and are not needed for our discussion here.

Also, both teams claim that their methods justify extension to teleportation of higher dimensional states, i.e., greater than 3, but neither team proved their claims in their experiments.

During the peer review of their respective papers, Luo’s team argued that Hu’s team had an insufficient number of measurements to support the fidelity of their results, although each team agreed that the other had teleported a qutrit (Young, 2019).

These experiments demonstrate successful teleportation of a qutrit, but they require highly complex, co-located, short-distance equipment configurations. Whether long-distance qutrit teleportation is achievable and whether high-dimensional teleportation reduces decoherence remain open questions.
Chapter 5 Discussion

5.1 What is a quantum internet?

The classical internet evolved over many years into the technology that we use today. This evolution consisted of parallel advances in computing technology and advances in discussion, debate and eventual agreement on the necessary standards, protocols, and frameworks, such as the generally adopted TCP/IP 5-layer Reference Model (‘Internet protocol suite’, 2021). These standards describe how heterogenous, disparate computing systems should interoperate in a reliable, secure, and efficient manner.

The quantum internet appears to be following a similar evolutionary development process.

On the one hand, quantum technologies are being explored and extended, whilst on the other, possible frameworks and standards are being proposed.

One suggestion (Wehner, Elkouss and Hanson, 2018), Fig. 13 - Quantum internet stages, shows an undefined timeline for the progressive increase in the functionality of quantum technologies, which build on the initial ‘trusted repeater networks’ stage consisting of separate point-to-point quantum channels.

Subsequent stages require progressive advances in application protocols and technical capability. The model is further expanded by including example protocols and technologies for each stage, e.g., the ‘fault-tolerant few qubit networks’ stage will require clock-synchronisation and distributed quantum computing.

Another suggestion (Kozlowski, Dahlberg and Wehner, 2020) builds on the fact that teleportation requires both a quantum and a classical channel, and therefore the quantum internet and the classical internet will need to co-exist. Fig. 14 - Quantum and classical internet, shows ‘quantum data planes’, i.e., nodes, in the network which provide quantum
repeater and quantum switching functionality as well as the coordination of entanglement swapping between quantum and classical links.

A 2020 workshop held by the U.S. Department of Energy discussed a roadmap for a nationwide quantum internet. The workshop compared the TCP/IP 5-layer Reference Model with a proposed Quantum Internet Network Architecture (Fig. 15 – Comparison of TCP/IP and proposed quantum internet network architectures).

The ‘Entanglement Generation and Memory-assisted Distribution’ layer would produce long-distance entanglement between nodes using entanglement swapping. The ‘Coexistence with Classical Networks’ layer would transmit qubits using teleportation.

To date, a fully functioning quantum internet has yet to be developed and there is no clear or agreed roadmap.
Teleportation over long distances continues to be a major technological challenge.

Our review of current research and experiments indicates that the current level of maturity of the quantum internet is as per the ‘trusted repeater network’ stage in the model described in Fig. 13 - Quantum internet stages.

5.2 Teleportation roadmap

The enabling technologies for a teleportation-based quantum network, i.e., quantum memories, entanglement swapping, etc., have to some extent been built and tested. The real challenge is to improve and combine these technology components to deliver an efficient and reliable quantum network consisting of many nodes and links, such as that shown in Fig. 14 - Quantum and classical internet.

We propose that these improvements in teleportation technology will be achieved within the next ten to fifteen years through an incremental and scalable series of milestones where each milestone builds on the technical capabilities delivered in earlier milestones (Fig. 16 - Quantum teleportation roadmap).

<table>
<thead>
<tr>
<th>Milestone</th>
<th>Description and key features</th>
<th>Network configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Single PPL - quantum channels - Ground-to-satellite/optical fibre, distance – 1,400 km</td>
<td>Linear</td>
</tr>
<tr>
<td>M2</td>
<td>Two PPLs with single Quantum Repeater - entanglement swapping</td>
<td>Linear</td>
</tr>
<tr>
<td>M3</td>
<td>Three PPLs with two Quantum Repeaters - entanglement swapping, entanglement synchronisation, entanglement purification</td>
<td>Linear</td>
</tr>
<tr>
<td>M4</td>
<td>Six PPLs with three Quantum Repeaters and one Quantum Switch - entanglement swapping, entanglement synchronisation, entanglement purification, entanglement routing</td>
<td>Star</td>
</tr>
</tbody>
</table>

Figure 16 – Quantum teleportation roadmap
We provide a description of the key features for each milestone and give examples of current research that may contribute to future technological developments.

5.3 Milestone M1 (achieved)

Milestone M1 consists of teleportation over a single PPL quantum channel using fibre-optic cable and free space links of up to 1,400 km. Milestone M1 has already been achieved over the last few years as described in Chapter 3.

5.4 Milestone M2 (next 5 years)

Milestone M2 would teleport a qubit over two PPLs connected by a QR with entanglement swapping capability, supported by a quantum memory with storage times that preserve the states of Alice and Bob’s single photons as well as those needed for entanglement swapping for a pair of photons. We expect these storage times to be in the order of several seconds.

A single solid-state quantum memory demonstrated up to 75 seconds for storage of a single-qubit state and up to 10.2 seconds for a two-qubit entangled state (Bradley et al., 2019). Further work will be required to incorporate this qubit and entanglement storage capability with teleportation.

Entanglement between two quantum memories separated by optical fibre with lengths of up to 50 km has been achieved, where each quantum memory had a storage time of 70 µs (Yu et al., 2020, p. 244).

5.5 Milestone M3 (5 – 10 years)

Milestones M3 enhances milestone M2 by incorporating an additional QR in the quantum channel. This would require

- quantum memory storage times to be in the order of several minutes,
- entanglement distribution - the synchronisation of BSMs across the network,
- entanglement purification - to counteract transmission losses.

Protocols for entanglement distribution have recently been investigated, including those for a homogenous repeater chain, a quantum channel where the QRs are equally spaced (Dai, Peng and Win, 2020). Using a homogenous repeater chain, the authors develop a theoretical model which optimizes the sequence of entanglement swapping in a quantum network so as to minimise the overall transmission time.

For entanglement purification, algorithms have been proposed which seek to optimise the fidelity of the target states whilst minimising the number of purification rounds (Ruan et al., 2021).
5.6 Milestone M4 (10 – 15 years)

Unlike the previous milestones which have a linear network configuration connecting two remote users, Alice and Bob, Milestone M4 uses a star network configuration connecting multiple remote users, of which only two form the Alice-Bob teleportation pair. Routing of entanglement swapping through alternative paths using a quantum switch would be required.

A mathematical analysis, using Markov-chains, for a quantum switch was conducted for simultaneous entanglement-swapping in a quantum network consisting of multiples pairs of users (Pant et al., 2019).
Chapter 6 Conclusion

6.1 Summary

We have described the basic theory of quantum teleportation theory and have shown that many experiments have proven the validity of teleportation of single-photon qubits over quantum channels comprised of single point-to-point links having distances up to 1,400 km, limited by signal transmission losses and corruption, which also degrade the quality of the teleported state, known as decoherence. The no-cloning theorem shows that copying of quantum states is not possible, and combined with decoherence, presents significant challenges for extending quantum teleportation further than the current distance limit. We have described some of these challenges and have shown that the current standards and technologies for a quantum internet are in the early stages of maturity. We have proposed a ten to fifteen-year timescale in which these challenges could be overcome in an incremental series of milestones which progressively develop teleportation technologies through consideration of current research.

6.2 Future research

The next milestone, M2, is a fundamental step towards a teleportation-capable quantum internet. M2 requires entanglement swapping capability and photonic qubit memory storage of several seconds, each of which has been achieved separately. We propose that the next key research area is in how to combine these separate functions to deliver teleportation over a small-scale quantum network.

6.3 Achievement of project objectives

Our project objectives have been fully met:

1 – Describe quantum teleportation theory. Chapter 2.

2 – Survey recent experimental tests of quantum teleportation. Chapter 3.

3 – Critically assess the challenges of, and opportunities for, using quantum teleportation to enable the quantum internet. Chapter 4.

4 – Draw some conclusions about the feasibility and timescales for a quantum internet using quantum teleportation. Chapter 5.
References


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