A computer-assisted proof of dynamo growth in the stretch-fold-shear map

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Introduction

•Our work is on a functional linear operator called the Stretch-Fold-Shear (SFS) operator [1] which arises from a model of dynamo growth.

•The existence of an eigenvalue of this SFS operator S of magnitude greater than one ensures dynamo growth.

•Aim of this research is to prove such existence using a computer-assisted proof.

The SFS Operator

•$F = \{c(x)$ complex-valued function of a real variable $x \in [-1,1]\}$

•$S: F \to F$

•$Sc(x) = e^{\frac{i\alpha(x-1)}{2}} c\left(\frac{x-1}{2}\right) - e^{\frac{i\alpha(1-x)}{2}} c\left(\frac{1-x}{2}\right)$

•$\alpha \geq 0$(real parameter) is the shear parameter.

Mathematical Tools

•Computer-assisted proof

•Interval arithmetic

•Function ball

•Julia Programming Language

Our Work

Theorem (Computer assisted):

Let $\alpha \in [0,5]$. Then there exists an eigenvalue-eigenfunction pair $(\lambda, c(x))$ of the operator S satisfying the following:

1. For $\alpha \in [0,1.5705]$, $|\lambda| < 1$
2. For $\alpha \in [1.571, 5]$, $|\lambda| > 1$
3. For $\alpha \in [1.5705,1.571]$, $|\lambda| \in [0.99624, 1.00374]$.

Graphical Investigations:

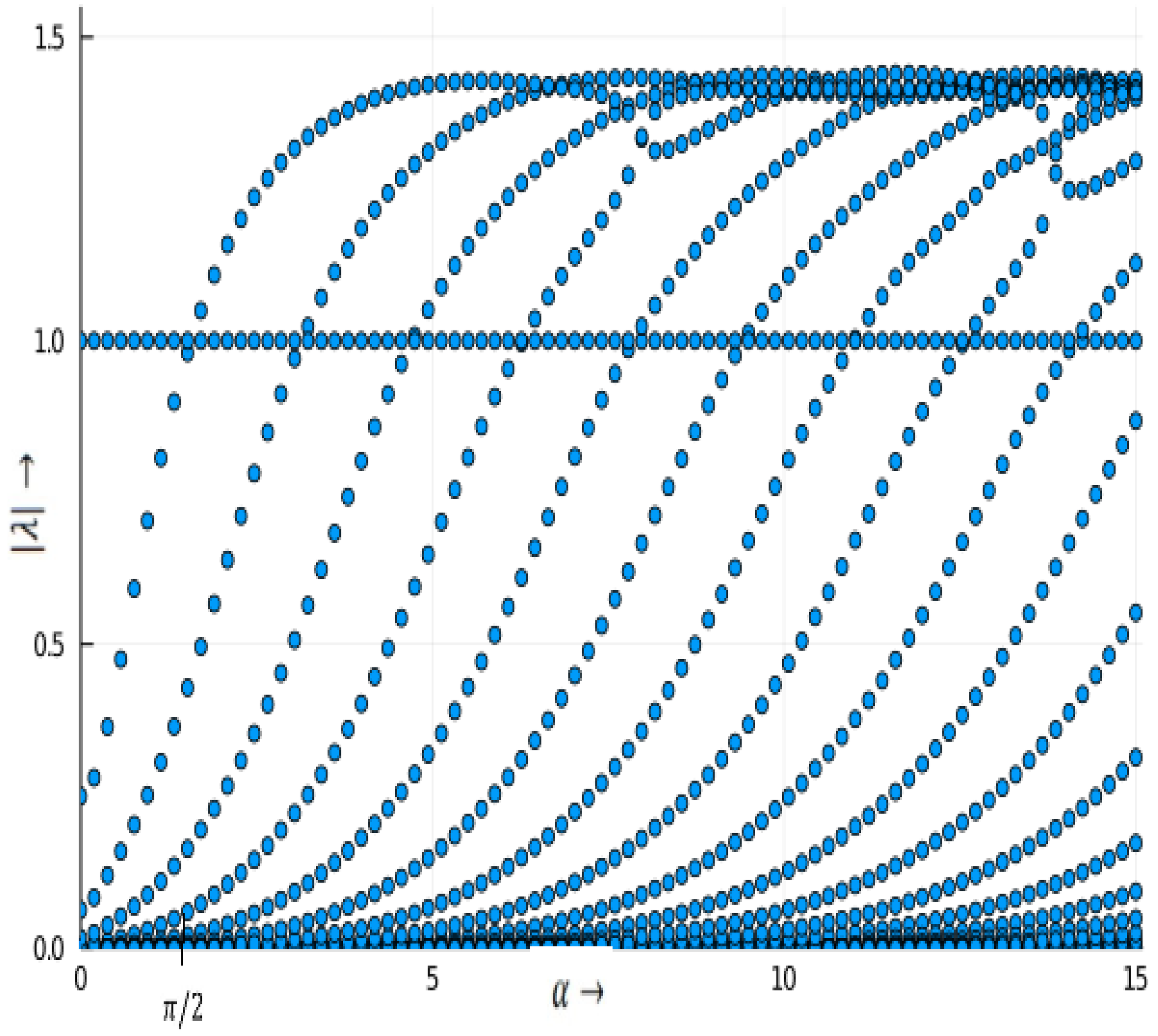

Figure 1: (Reproducing a graph from Gilbert's paper) Modulus of Eigenvalues of S.

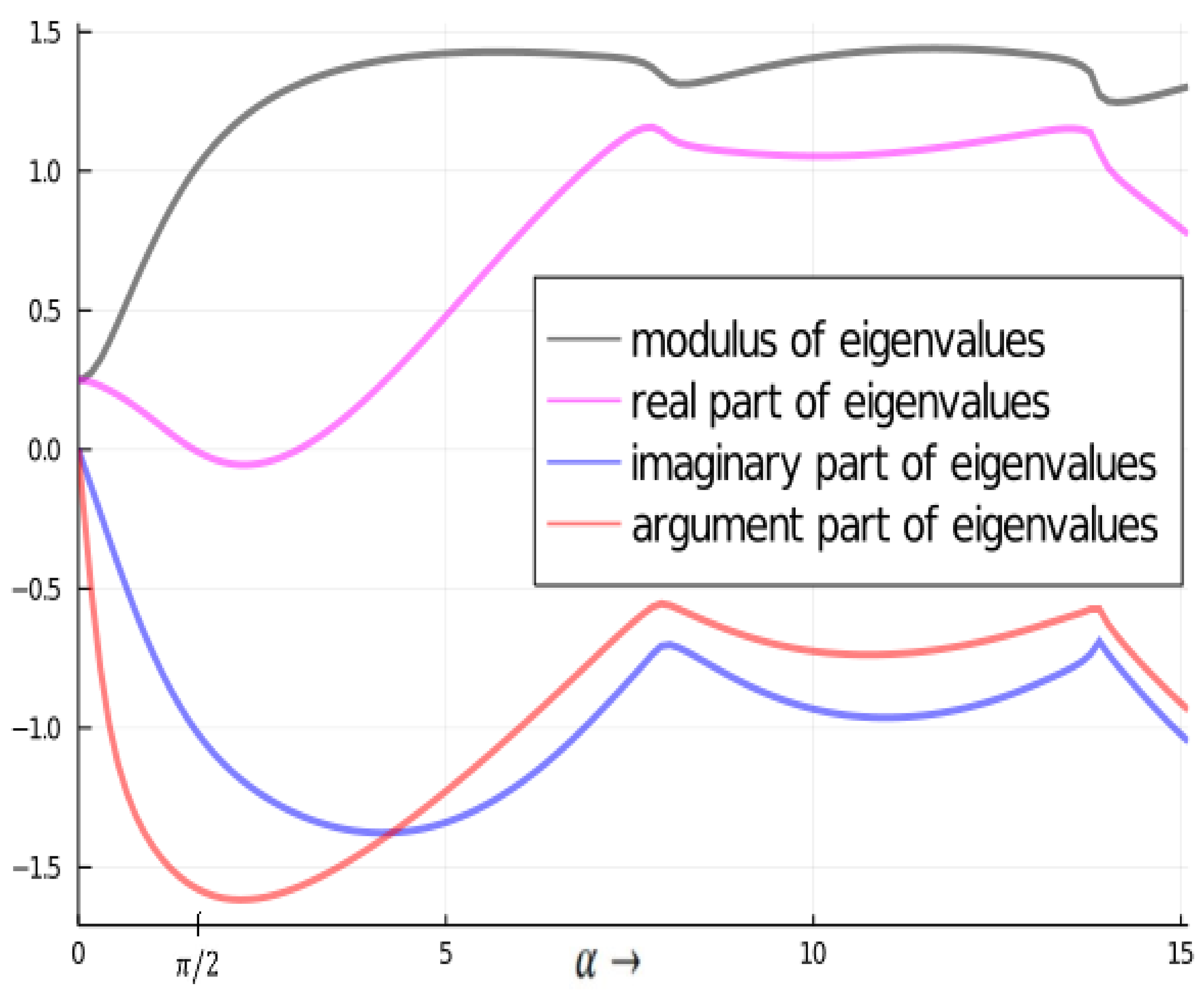

Figure 2: Investigating the first family of eigenvalues of S.

Conclusion

- To prove the theorem a computer-assisted proof is used to find rigorous bounds around a function ball of each approximately calculated eigenvalue-eigenfunction pair by implementing interval arithmetic in Julia.
- In future we will make α_{max} as large as possible, hopefully $\alpha_{max} = 10$.
- We are also investigating the eigenvalues (carefully around $\alpha = \frac{\pi}{2} \approx 1.5705$) graphically up to $\alpha_{max} = 15$.

Literature Cited

[1] A. D. Gilbert, "Advected fields in maps - I. Magnetic flux growth in the stretch-fold-shear map," Physica D, Elsevier, vol. 166, pp. 167–196, 2002.