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Comparison of spherical and aspherical intraocular lenses with decentration and tilt error using a physical model of human contrast vision and an image quality metric

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Results: With increasing decentration, the spherical IOL shows a significantly smaller loss of quality for both apertures compared to the aspherical lens. With an aperture of 4.5 mm, the image quality of the aberration-corrected IOL is better for small decentration and tilt. The loss of quality of the spherical IOL increases with increasing tilt in both directions. In contrast, the image quality of the aspherical IOL is reduced under decentration for certain tilt values. For ACD - 0.1 mm, both IOLs behave similarly to the in-focus situation. For ACD + 0.1 mm, the influence of tilt without decentration is small for both IOLs. With increasing decentration, the quality loss of the aspherical IOL is similar to that in-focus and greater than that of the spherical lens.  
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Title:
Comparison of spherical and aspherical intraocular lenses with decentration and tilt error using a physical model of human contrast vision and an image quality metric

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Abstract

Purpose: In this study, two intraocular lenses (spherical IOL SA60AT and aspherical IOL SN60WF) are examined in an eye model under conditions of misalignment (defocus, decenteration and tilt). The lenses are rated using the contrast sensitivity function (CSF) based on Barten's physical model. The square root integral (SQRI) method is used as a quality criterion comparable to the subjective image quality assessment of the human eye.

Methods: The IOLs to be tested are decentered from 0 to 1 mm and tilted from -5 to +5 degrees in the Navarro eye model (optimized for far-point 6 m and pupil aperture 3 mm). The defocus of the IOLs is ±0.1 mm at the anterior chamber depth (ACD). The optical modulation transfer function (MTF) is simulated with a ray tracing program. The SQRI is calculated using this MTF and the Barten CSF model (for in-focus at aperture 3 and 4.5 mm and for defocus at 3 mm).

Results: With increasing decenteration, the spherical IOL shows a significantly smaller loss of quality for both apertures compared to the aspherical lens. With an aperture of 4.5 mm, the image quality of the aspherical IOL is better for small decenteration and tilt. The loss of quality of the spherical IOL increases with increasing tilt in both directions. In contrast, the image quality of the aspherical IOL is reduced under decenteration for certain tilt values. For ACD - 0.1 mm, both IOLs behave similarly to the in-focus situation. For ACD + 0.1 mm, the influence of tilt without decenteration is small for both IOLs. With increasing decenteration, the quality loss of the aspherical IOL is similar to that in-focus and greater than that of the spherical lens.

Conclusion: In general, under the same conditions the spherical SA60AT displays a lower tolerance in loss of quality of subjective vision with lens alignment errors, in comparison to the aspherical SN60WF, limited by certain combinations of decenteration and tilt according to this study. This study shows a way to evaluate IOLs based on the subjective visual performance of the eye.
Introduction

Since the introduction of aspherical intraocular lenses in cataract surgery, the image quality of pseudophakic eyes has been evaluated in clinical studies and investigations by comparison with spherical lenses were performed [1, 2]. Most studies compared visual acuity at high contrast, mesopic and photopic contrast sensitivity, or modulation transfer functions (MTF). The MTF is primarily investigated either experimentally [3-7] or theoretically [8-10] using eye models. In order to compare different intraocular lenses (IOLs), quality criteria describing the form of the MTF and hence the image quality, have been introduced. Common criteria are modulation values at specified spatial frequencies (e.g. 100 line pairs/mm according to ISO 11979-2), the Strehl ratio as area under the MTF curve relative to the aberration-free MTF curve, or the representation of the MTF itself as a function of spatial frequency. Further criteria include, for example, RMS values from wavefront analysis, spot diagrams or the mean (e.g. RMS) image spot radius and others. Common to all criteria is that they only assess the optical quality of the IOL in the eye. However, in addition to the aberration of the optical image on the retina, the visual performance of the human eye is also limited by the discrete arrangement of receptors on the retina and by neural processing. Normally, this visual performance is described by the contrast sensitivity function (CSF), which is measured psychophysically in the human eye using sinusoidal patterns of differing contrast and spatial frequency [11]. Barten has proposed a complex theoretical model of the CSF [12], and a simple model fitted to psychophysical data from average normal observers [13]. The experimental model has been used to evaluate IOLs, for example in [14, 15]. The validity of the theoretical model has been investigated in vertebrates by Jarvis et al. [16, 17], and it is also used in DICOM (Digital Imaging and Communications in Medicine), Part 14 for image quality assessment [18, 19].

Turuwhenua [15] has investigated the effects of decentration and tilt (also referred to here as displacement) of intraocular lenses for two pupil apertures (2 mm and 4 mm) on MTF calculations using the experimental model. However, in contrast to Barten's physical model, the experimental model does not take into account the pupillary aperture in calculating the CSF or the extrafoveal vision resulting from displacement of the IOL. In this study two commercial lenses (spherical AcrySof® IQ SA60AT and aspherical AcrySof® IQ SN60WF from Alcon Pharma GmbH, Germany) are used in
a pseudophakic eye model and the subjective vision compared with Barten's physical
model. The IOLs are evaluated taking decentration and tilt into account for two
apertures, assessing the resulting extrafoveal image. The square-root-integral
(SQRI) method, which correlates well with subjective image evaluation in human
eyes, is used as a quality criterion. This method measures the image quality in terms
of perceptible difference thresholds (just-noticeable differences) [13, 20].

Material and methods

Pseudophakic eye model: The optical performances of spherical (SA60AT) and
aspherical (SN60WF) Intraocular lenses were simulated in an eye model with an
aspherical cornea according to Navarro [21] (shown in Table 1). The Navarro model
has a positive spherical aberration (SA) of +0.24 μm (RMS), resulting mainly from the
corneal surface [22]. Table 2 shows the technical parameters of the two commercial
lenses having the same powers of 22 D [9]. SA60AT is an asymmetrically biconvex
lens and SN60WF is an asymmetrically biconvex lens with an aspherical posterior
surface. All spherical lenses such as the SA60AT, have a positive SA and so are
unable to compensate for the natural positive SA of the corneal surface. The
aspherical SN60WF has a negative SA of 0.20 μm and this can contribute to
improving the image quality of the eye model with a large pupillary aperture [23]. The
two IOLs were placed in the position of the natural human lens at the given anterior
chamber depth (ACD). The pseudophakic eye model was simulated with the ZEMAX
raytracing program (Version 2010). For an emmetropic eye, the vitreous chamber
depth of the model was optimized for minimal wavefront aberration at the far-point
(6.0 m) with a 3.0 mm pupil aperture. The aperture is fixed at the ACD position in
front of the IOL. The simulation software’s standard optimization algorithm is used,
with mean error minimization (root mean square) of the aberration of the model for all
light rays transmitted through the pupil on the optical axis at the retinal plane (the
number of rays is chosen to give an uncertainty of less than 1% in the result). The
wavelength is λ = 555 nm (green). As a result, the best monochromatic optical
performance for the selected lens is achieved and its imaging properties can be
compared with different optical designs. The optical imaging property is indicated by
the diffraction-limited modulation transfer function (MTF).
Model of human eye vision: Barten’s theoretical model is based on a threshold function \( m_t(u) \) limited by the internal noise of the visual system. This function specifies the relationship between the minimum luminance amplitude to the average luminance of a sinusoidal grating at a 50% recognition probability and corresponds to the estimate detection threshold of 75% of an observer with the two-alternative choice method in measurements of presented stimuli [12]. This model describes the optical and neuronal factors separately, and its inverse according to Barten is the contrast sensitivity function (CSF) with the spatial frequency \( u \) in cpd (cycles per degree):

\[
CSF(u) = \frac{1}{m_t(u)} = \frac{MTF_{opt}(u)/k}{\sqrt{\frac{2}{T} \left( \frac{1}{X^2} + \frac{1}{X_{\text{max}}^2} + \frac{u^2}{N_{\text{max}}^2} \right) \left( \frac{1}{\eta \cdot p \cdot E} + \frac{\phi_0}{1 - e^{-u/u_0}} \right)}}
\]

with parameters \( k = 3 \) (signal-noise-ratio), \( T = 0.1 \) sec (integration time), \( X_{\text{max}} = 12 \) deg (integration area), \( N_{\text{max}} = 15 \) cycle (number of integration cycles), \( \eta = 0.03 \) (quantum efficiency), \( \phi_0 = 3 \cdot 10^{-8} \) sec-deg (spatial density of photon noise), \( u_0 = 7 \) cycle/deg (spatial frequency threshold) und \( p = 1.247 \cdot 10^6 \) photons/sec/deg\(^2\)/Td (photon conversion factor) for \( \lambda = 555 \) nm and photopic vision [12]. Here, an angular size of \( X = 2 \) deg for the object is used, corresponding to the object size according to Digital Imaging and Communications in Medicine (DICOM) [18]. The pupil light intensity \( E \) is:

\[
E(L) = \frac{\pi \cdot d^2}{4} \cdot L \cdot \left( 1 - (d/9.7)^2 + (d/12.4)^4 \right)
\]

where the pupil diameter \( d \) is in mm and the object luminance \( L \) in cd/m\(^2\). The term in brackets stands for the Stiles-Crawford effect. \( MTF_{opt} \) corresponds to the optical and retinal transfer process: \( MTF_{opt}(u) = MTF_m(u) \cdot MTF_r(u) \). In this study, \( MTF_m \) is the MTF from simulation calculation in the pseudophakic eye model and \( MTF_r \) describes the MTF of the retinal scan [12, 17]: \( MTF_r(u) = \exp(-2(\pi \cdot \sigma \cdot u)^2) \). The parameter \( \sigma = 2.5 \cdot 10^{-3} \) cells/deg is the standard deviation of the adjacent line spacing of hexagonally arranged foveal photoreceptors [12].

Barten’s model can be extended for extrafoveal vision: an object point off the fixation axis produces an image point at an angle \( e \) (eccentricity) on the retina, outside the center of the fovea (Fig. 1), and the contrast sensitivity decreases with eccentricity.
The parameters in equation 1 must then be replaced by the functions of $e$: $\phi_0(e)$, $\omega_0(e)$, $\eta(e)$, $X_{\text{max}}(e)$ and $\sigma(e)$ [12, 16], as described in the appendix. With displacement of the IOL, the surface intensity of the image point is also distorted and imaged off-axis with an eccentricity $e$. The eccentricity is specified here as:

$$e = \frac{180}{\pi} \cdot \frac{d_R}{f_{NP}}$$

in degree, where $d_R$ is the distance of the principal ray from the optical axis, and $f_{NP}$ is the distance between the posterior nodal point of the optical system and the retina. Both values can be taken from the simulation program.

**Subjective image quality:** The subjective assessment of an image generated by an external optical system can be evaluated in terms of the contrast sensitivity of the observer’s eye. The square root integral method (SQRI) gives a good agreement with the human perception of the quality of an image compared to a reference image.

$$\text{SQRI} = \frac{1}{\ln 2} \int_{u_{\text{min}}}^{u_{\text{max}}} \left[ \frac{\text{MTF}(u)}{\text{CSF}(u)} \right]^{1/2} \frac{du}{u},$$

where the SQRI is specified in terms of the number of perceptible distinction thresholds in units of jnd (just noticeable differences) [13, 20]. $\text{MTF}(u)$ is the transfer function of the optical system under investigation and $\text{CSF}(u) = 1/m_t(u)$ the contrast sensitivity function of the eye. A loss of quality of 3 jnd corresponds to a noticeable modification, and of 10 jnd to a significant modification of the reference image [13]. In this study, the inner optics of the eye have been evaluated with $\text{MTF}(u) = 1$ (i.e. no external optical system). The integration limits are then determined by the CSF.

**Experimental procedure:** The IOL under test is decentered and tilted in a plane relative to the reference axis of the front surface, in an eye model optimized for a 6 m far point. The tilt of the IOL is varied from -5 to +5 degrees in 1 degree steps, with a decentration from 0.0 mm to 1.0 mm in steps of 0.2 mm, for pupil apertures of 3.0 mm by ISO 11979-2 and 4.5 mm as example of a greater aperture value. The (one-dimensional) $\text{MTF}_m$ in this plane is evaluated, together with the principal ray distance $d_R$ and thereby the eccentricity $e$. Results are shown for selected lens displacements with defocusing of the lens of ±0.1 mm on the ACD. For all results, the CSF is calculated with the simulated MTF$_m$ and the eccentricity $e$ for photopic vision at a light intensity of $L = 250 \text{ cd/m}^2$. The MTF$_m$ is based on a Fourier transform (FFT) with
a sampling rate of 512 x 512 and is calculated for the optics, taking into account aberration and diffraction limits [24, 25]. The spatial frequency grid is 0.35 cpd. The SQRI method is approximated using the trapezium rule, the integration limit of $u_{\text{min}} = 0.35$ cpd is given by the MTF$_m$ and $u_{\text{max}}$ is the cutoff spatial frequency of the diffraction-limited MTF$_m$. From the results, example images of the USAF target charts (black and white, 678 x 678 pixels) have been simulated for visual comparison. At a distance of 6 m and a viewing angle of $X = 2$ deg, the chart would be approximately 21 cm in height. The image generated by the raytracing program (with 2D-MTF$_m$) is Fourier transformed and filtered in the Fourier domain using the Barten model (see, e.g. [25]). The (two-dimensional) Barten model is used as a filter with $\text{MTF}_{\text{opt}}(u) = \text{MTF}_r(u)$. The 2D-CSF is rotationally symmetric and the spatial frequency $u$ is the Euclidean distance between the corresponding spatial frequencies in the x and y image directions. The inverse FFT then yields the image according to human visual performance.

**Results**

**Simulation:** Table 3 presents the parameters of the pseudophakic eye model after optimization for minimal wavefront aberration with a pupil aperture of 3.0 mm, together with the SQRI results, for the two lenses under test. The diffraction-limited SQRI value represents the aberration-free optical design of the model. Comparing SQRI values in percentage terms, at 98.7% for the SA60AT, and 99.9% for SN60WF, the pseudophakic eye is almost perfectly optically optimized to the IOL. Fig. 2 shows an example of the CSF according to Barten's theoretical model for both lenses with 4.5 mm aperture. Barten's simple empirical model is shown for comparison.

**Presentation:** The SQRI values as a function of IOL tilt are given in Fig. 3 (SA60AT and SN60WF) for a pupil aperture of 3.0 mm. In Fig. 4 the aperture is 4.5 mm with all other conditions the same. The IOL decentration is shown as a parameter to the right of each family of curves. For negative decentration values, the curves would be reflected about the 0-degree vertical axis as a result of symmetry. For each curve the average gradient of the SQRI values as a function of tilt over the range -2 to +2 degrees is shown. Table 4 shows the average gradient of the SQRI values as a function of decentration (without tilt) over the range 0.2 to 1.0 mm. Fig. 5 shows the differences between the SA60AT and SN60WF SQRI values for the same
displacement values. Fig. 6 shows the results under defocusing in the eye model for a 3.0 mm pupil aperture. Simulated images of the USAF Target Chart are shown in Fig. 7.

**Lenses with aperture 3 mm:** Under decentration without tilt, the spherical lens exhibits an average loss of quality of 28 jnd/mm, which is smaller than the 43 jnd/mm for the aspherical lens (Table 4). For decentrations of 0.2 to 1.0 mm, the average gradient of the SQRI values for the spherical lens, ranging from -0.1 to -1.3 jnd/deg (Fig. 3, left) is similarly smaller in magnitude than that of the aspherical lens (Fig. 3, right), which ranges from 1.2 to 1.4 jnd/deg, with a maximum of 2.2 jnd/deg for a 0.4 mm decentration. According to Table 3, the SQRI values for both lenses when correctly positioned are almost identical, but they change by differing amounts under displacement. The spherical IOL shows different, but generally decreasing, tendencies of the SQRI values with increasing positive and negative tilt. By contrast, the aspherical lens displays a smaller loss of quality with positive tilt than with negative tilt, and its SQRI values can be higher than those of the spherical IOL with the same amount of decentration. The maximum difference in SQRI within the measured tilt range is approximately 10 jnd at 1.0 mm and +5 degree (Fig. 5, left). With increasing decentration however, the range of tilt values for which the loss of quality of the aspherical lens dominates over that for the spherical lens, increases.

**Lenses with aperture 4.5 mm:** Under decentration without tilt, the spherical lens exhibits an average loss of quality of 21 jnd/mm, which is significantly smaller than the 48 jnd/mm for the aspherical lens (Table 4). For decentrations of 0.2 to 1.0 mm, the average gradient of the SQRI values for the spherical lens, ranging from -0.2 to -1.4 jnd/deg (-0.1 to -1.3 jnd/deg for the 3.0 mm lens) (Fig. 4, left) is similarly significantly smaller in magnitude than 4.6 to 1.7 mm SQRI gradient values for the aspherical lens (Fig. 4, right). Admittedly, at 82.1 jnd (98.8 jnd at 3.0 mm), the SQRI of the 4.5 mm spherical lens without displacement is smaller than the 102.7 jnd (99.8 jnd at 3.0 mm) of the aspherical lens under the same conditions. However, once the decentration reaches 0.4 mm (without tilt) the SQRI value of 79.6 jnd for the aspherical IOL is lower than the 82.1 jnd maximum value for the spherical IOL, and with increasing decentration or negative tilt the loss of quality of the aspherical IOL becomes greater compared to the spherical IOL. In the positive tilt direction, the aspherical lens dominates, with less loss of quality. The maximum SQRI difference
compared to the spherical lens at the same decentration is approximately 24 jnd at
0.2 mm, +2.5 degree (Fig. 5, right). In the comparison of apertures in Fig. 5, the
influence of aspherical IOLs on the image quality can be seen at larger apertures.
Their SQRI values remain high as the aperture changes, at least for small
displacements. For the standard aperture of 3 mm, the difference is noticeably
smaller.

Lenses when defocused: The spherical IOL with no displacement has an SQRI value
of 93.6 jnd when defocused by 0.1 mm in both directions (98.8 jnd in focus) (Fig. 6,
left), and the aspherical IOL 94.6 jnd (99.8 jnd in focus) (Fig. 6, right). An average
loss of quality as a function of decentration (without tilt) is given in Table 4.
Comparing both lenses shows that the spherical IOL has smaller values when
defocused by +0.1 mm (Hyperopia) and -0.1 mm (Myopia). With increasing
decentration the gradients of the SQRI values for the spherical lens alternate in sign
according to focus direction, but are more or less unchanged in magnitude at 1.0 to
1.5 jnd/deg. The SQRI gradient for the aspherical lens is 1.1 to 1.7 jnd/deg (1.2 to
2.2 jnd/deg in focus) with a positive gradient. The SQRI curves behave differently
according to the direction of the defocusing: with defocusing of +0.1 mm, the SQRI
values of the spherical lens are evened out over the range of tilt values compared to
the in-focus values (Fig. 3, left). In the -0.1 mm defocus direction, the behaviour of
the curves for both lenses with decentration is the same as in focus. In the +0.1 mm
defocus direction, the curve for the aspherical IOL without decentration is smoothed
out as a function of tilt, just the same as for the spherical IOL. As the decentration
increases however, the behaviour of the curves for the aspherical IOL is comparable
to that when in focus (Fig. 3, right).

Images under human eye vision: Fig. 7, left shows the simulated images of the USAF
Target Chart for the spherical SA60AT (A) and aspherical SN60WF (B) lenses with
aperture 4.5 mm and without displacement in the eye model. The marked area
(number 5) shows the bar spacing for a visual acuity of 1. The difference in SQRI
values is approximately 21 jnd (82.1 jnd for SA60AT, 102.7 jnd for SN60WF) and the
SA60AT shows a blurry image in comparison to that of the SN60WF. Fig. 7, right
shows an image comparison showing the asymmetric behaviour with tilt (Fig. 4, right)
of the aspherical SN60WF with decentration 0.4 mm and tilt -3 deg (C) and +3 deg
(D) for the same aperture of 4.5 mm. The difference in SQRI values is approximately
25 jnd (68.7 jnd for -3 deg, 93.9 jnd for +3 deg). Since the displacement of the IOL is in the vertical plane, the horizontal bars should be noted. The difference in SQRI values between image D and image B is approx. -9 jnd, and between images D and A approx. +12 jnd. Comparing the images visually, only a very slight difference in sharpness can be seen between B and D (< 10 jnd difference), whereas the difference in sharpness between A and D (> 10 jnd difference) is more distinct. This suggests that a difference of 10 jnd is just perceptible and can be taken as a threshold value. The gray background of the images is the result of the bandpass behaviour of the CSF.

Discussion

In this study, two commercial intraocular lenses (spherical SA60AT and aspherical SN60WF) were investigated in a pseudophakic eye model under conditions of misalignment (defocus, decentration and tilt). The lenses were assessed using the contrast sensitivity function for photopic vision based on Barten's physical model. The square root integral (SQRI) method was used as a quality criterion comparable to the subjective image quality assessment of the human eye. The study also shows an alternative method of evaluating intraocular lenses using the SQRI criterion, avoiding the need to consider the various representations of optical image quality such as MTF, spot diagrams and others, or the influence of various imaging errors.

Simulation: Various formulae are available for calculating intraocular lenses for the pseudophakic eye - for example, SRK/T, Haigis [26], Hoffer Q [27], Holladay [28] and Barrett [29]. Raytracing programs are also used [30, 31]. Usually individualized constants are used to calculate the refraction-corrected pseudophakic eye under normal conditions. In this study, the eye model was therefore optimized for each of the IOLs to be tested with a standard aperture (3.0 mm by ISO 11979-2) in order to compare the lenses with the best possible IOL adjustment. However, eye models predict a much better optical quality than human eyes, in which aberrations (astigmatism, coma, trefoil) can occur and, furthermore, in which there is a difference between the visual and optical axes [32]. The simulated contrast sensitivity function and quality values from the SQRI method are therefore overestimated compared to real eyes (see also the comparison of the CSF with Barten's simple model in Fig. 2).
According to Barten [13] change of 3 jnd in the SQRI value represents a noticeable, and 10 jnd a significant, loss of quality, determined experimentally by a standard observer comparing an image with a reference image. The simulation uses a one-dimensional MTF in the plane of the IOL shift with monochromatic light, corresponding to contrast measurement in the eye using a sinusoidal grating. The comparison of the losses of quality in the results presented here is therefore difficult and selected simulated images were used for visual comparison of the image quality (Fig. 7). This result shows that a loss of quality of approximately 10 jnd is noticeable, and from approximately 20 jnd is clearly apparent. The study by Turuwheenua [15] that inspired this work optimized the optical design for two distinct pupil apertures (2 mm and 4 mm) using Barten’s experimental model for the optical system of the eye. However, the experimental model does not take into account the effect of the eccentricity e on the retinal and neuronal processing of the human visual system. In this study, the optical design was only optimized for the standard aperture (3 mm) and the apertures under test (3 mm and 4.5 mm) were considered using the theoretical model which does include the effects of eccentricity on human visual perception. Only a limited comparison of the results is therefore possible. This study did not take into account the fact that a real pseudophakic eye with a displaced IOL can, to a certain extent, compensate for this by changing the eye position, and thus adjusting the visual axis in order to image an object on the foveal plane. In this study, the ACD of the eye model was taken to be the position of the IOL under consideration, since the postoperative ACD is usually not known and has to be estimated prior to implantation of an IOL. In general, the postoperative ACD of a pseudophakic eye is greater. Furthermore, the displacement of the IOL was only examined in one meridional plane. Different results can be expected for different orientations of the decentration and tilt.

Comparison of the lenses: Any lens that is offset from or tilted with respect to its optical axis will adversely affect the image quality. A displacement can have various causes: incorrect fixation of the lens in the capsular bag, contraction of the capsular bag, rupture of the zonular fibers, or a secondary cataract. The extent of the loss in quality depends on the amount of displacement and on the type of intraocular lens. The spherical (SA60AT) and aspherical (SN60WF) IOLs exhibit different behaviours in the subjective visual performance, as expressed in the SQRI values. With increasing decentration and a small tolerance in the tilt of the lens, the spherical IOL
shows a better image quality compared to the aspherical IOL. This is also confirmed by other authors [33, 34]. With a larger aperture and small decentration, the image quality of the aberration-corrected SN60WF is superior to the SA60AT. However, the correction for spherical aberration is optimized for arrangements of the lens in the eye without alignment errors and the image quality decreases more rapidly with displacement than with the standard lens. The asymmetrical behavior between positive and negative tilt of the aspherical lens (SN60WF) can also be seen in Turuwhenua [15]. With decentration, and using the rotation convention chosen here, the loss in quality with positive tilt is smaller than with decentration and zero or negative tilt. The tilt compensates for the decentration of the lens and vice-versa. This is a result of the optical properties of the lens, since the eccentricity changes almost linearly with displacement and the SQRI values also decrease with increasing eccentricity. This is shown in Fig. 3 for an example of tilt with a 0.4 mm decentration, without consideration of the extrafoveal vision in the simulation calculations. As the lenses are defocused, the loss of contrast increases. With a hyperopic eye (ACD + 0.1 mm), the loss of quality as a function of tilt for both lenses is reduced; for the SA60AT this is clear throughout the selected range of decentration, and for the SN60WF only for small decentrations. By displacement of the IOL, and thereby longer extrafoveal ray paths, the focused image normally located behind the retina in the hyperopic eye is imaged in the retinal plane and the loss of quality is thus compensated (with a total loss of approximately 10 jnd compared to the refraction-corrected eye). For a myopic eye (ACD - 0.1 mm), with an overall loss of quality due to defocusing, the subjective visual performance when the lens is displaced displays otherwise the same behaviour as the emmetropic eye.

Conclusions: In general, under the same conditions the spherical SA60AT displays a lower tolerance in loss of quality of subjective vision with lens alignment errors, in comparison to the aspherical SN60WF, limited by certain combinations of decentration and tilt according to this study. Commercial lenses have been examined in comparative studies and their optical design shows different imaging properties [3, 5, 9, 34]. The results of this work can therefore vary with the use of other lenses. For determining the quality of intraocular lenses, the MTF can also be specified experimentally, or with individual simulated eye models of patients, and with Barten's model, human visual performance specified in terms of the CSF. In contrast to technical quality criteria, criteria such as the discrete arrangement of the retinal
receptors, neuronal processing, extrafoveal imaging, and also photopic and mesopic vision and field of view can be included in the assessment. While not part of this study, in addition to monofocal lenses other types such as toric, multifocal, extended depth of focus (EDOF) lenses or diffractive lenses can also be examined in the pseudophakic eye using the SQRI method.

Appendix

According to Barten [12] the CSF according to equation (1) must be modified for extrafoveal vision. With a ganglion cell density of \( N_{g0} = 12,450 \text{ cells/deg}^2 \) (147,000 cells/mm\(^2\)) [17] in the centre of the retina, the parameters are:

\[
N_g(e) = N_{g0} \left\{ \frac{0.85}{1 + (e/0.45)^2} + \frac{0.15}{1 + (e/3.3)^2} \right\}, \tag{A.1}
\]

\[
\phi_0(e) = \phi_0 \cdot N_{g0} / N_g(e), \tag{A.2}
\]

\[
u_0(e) = \nu_0 \cdot \left( \frac{N_g(e)}{N_{g0}} \right)^{0.5} \left\{ \frac{0.85}{1 + (e/4)^2} + \frac{0.13}{1 + (e/20)^2} + 0.02 \right\}^{-0.5}, \tag{A.3}
\]

\[
\eta(e) = \eta \cdot \left\{ \frac{0.4}{1 + (e/7)^2} + \frac{0.48}{1 + (e/20)^2} + 0.12 \right\}, \tag{A.4}
\]

\[
X_{\text{max}}(e) = X_{\text{max}} \left\{ \frac{0.85}{1 + (e/4)^2} + \frac{0.15}{1 + (e/12)^2} \right\}^{-0.5}, \tag{A.5}
\]

\[
A(u,e) = \frac{1}{X^2} + \frac{1}{X_{\text{max}}^2(e)} + \frac{(0.5 \cdot X)^2 + 4 \cdot e^2}{(0.5 \cdot X)^2 + e^2} \cdot \frac{u^2}{N_{\text{max}}^2}, \tag{A.6}
\]

\[
\sigma(e) = 1/\sqrt{7.2 \cdot \sqrt{3 \cdot N_g(e)}}, \tag{A.7}
\]

Equation (A.6) replaces the first bracketed term of the discriminant in equation (1). The eccentricity \( e \) must be given in degree.
### Table 1: Optical design parameters of the Navarro eye model [21].

<table>
<thead>
<tr>
<th>Surface</th>
<th>Anterior cornea</th>
<th>Posterior cornea</th>
<th>Anterior lens</th>
<th>Posterior lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius in mm</td>
<td>7.72</td>
<td>6.5</td>
<td>10.2</td>
<td>-6.0</td>
</tr>
<tr>
<td>Conic constant Q</td>
<td>-0.25</td>
<td>0</td>
<td>-3.1316</td>
<td>-1.00</td>
</tr>
<tr>
<td>Cornea</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anterior chamber</td>
<td>0.55</td>
<td>3.05</td>
<td>4.0</td>
<td>16.4</td>
</tr>
<tr>
<td>Refractive index n</td>
<td>1.3761</td>
<td>1.3374</td>
<td>1.420</td>
<td>1.336</td>
</tr>
</tbody>
</table>

### Table 2: Optical design parameters of the tested IOL (R<sub>a</sub> and R<sub>b</sub>: anterior and posterior radii of curvature of the IOL; Q<sub>a</sub> and Q<sub>b</sub>: anterior and posterior conic constants; CT: central thickness; n: refractive index (wavelength λ = 555 nm); P: optical power; 2<sup>th</sup>, 4<sup>th</sup> and 6<sup>th</sup> a.c. and p.c: anterior and posterior aspheric polynomial coefficients of the curvature) [9].

<table>
<thead>
<tr>
<th>Model</th>
<th>R&lt;sub&gt;a&lt;/sub&gt; in mm</th>
<th>Q&lt;sub&gt;a&lt;/sub&gt;</th>
<th>R&lt;sub&gt;b&lt;/sub&gt; in mm</th>
<th>Q&lt;sub&gt;b&lt;/sub&gt;</th>
<th>CT in mm</th>
<th>n</th>
<th>P in dpt</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA60AT</td>
<td>16.379</td>
<td>0</td>
<td>-25</td>
<td>0</td>
<td>0.668</td>
<td>1.554</td>
<td>+22</td>
</tr>
<tr>
<td>SN60WF</td>
<td>19.583</td>
<td>0</td>
<td>-20</td>
<td>-33.227</td>
<td>0.633</td>
<td>1.554</td>
<td>+22</td>
</tr>
<tr>
<td></td>
<td>2&lt;sup&gt;th&lt;/sup&gt; a.c.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt; a.c.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6&lt;sup&gt;th&lt;/sup&gt; a.c.</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2&lt;sup&gt;th&lt;/sup&gt; p.c.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt; p.c.</td>
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<tr>
<td></td>
<td>6&lt;sup&gt;th&lt;/sup&gt; p.c.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA60AT</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SN60WF</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2.5 · 10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>-1.7 · 10&lt;sup&gt;-5&lt;/sup&gt;</td>
<td>-8.7 · 10&lt;sup&gt;-7&lt;/sup&gt;</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Parameters of the pseudophakic eye model after optimization for minimal wavefront aberration with ZEMAX at far object distance 6 m and standard diameter of the pupil of 3.0 mm. The SQRI values are given for optimization and diffraction limited imaging.

<table>
<thead>
<tr>
<th>Model</th>
<th>Vitreous chamber in mm</th>
<th>Nodal point - retina f&lt;sub&gt;NP&lt;/sub&gt; in mm</th>
<th>SQRI in jnd</th>
<th>Diffraction limit SQRI in jnd</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA60AT</td>
<td>18.423</td>
<td>16.048</td>
<td>98.8</td>
<td>100.1</td>
</tr>
<tr>
<td>SN60WF</td>
<td>18.546</td>
<td>16.132</td>
<td>99.8</td>
<td>99.9</td>
</tr>
</tbody>
</table>

### Table 4: Average gradient values of SQRI of the decentration from 0.2 to 1.0 mm with a tilt of 0 deg for the tested IOL and the pupil aperture.

<table>
<thead>
<tr>
<th>Model</th>
<th>Lens defocus in mm</th>
<th>Average gradient (3.0 mm) in jnd/mm</th>
<th>Average gradient (4.5 mm) in jnd/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA60AT</td>
<td>0.0</td>
<td>-28</td>
<td>-21</td>
</tr>
<tr>
<td></td>
<td>+ 0.1</td>
<td>-22</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- 0.1</td>
<td>-27</td>
<td>-</td>
</tr>
<tr>
<td>SN60WF</td>
<td>0.0</td>
<td>-43</td>
<td>-48</td>
</tr>
<tr>
<td></td>
<td>+ 0.1</td>
<td>-39</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- 0.1</td>
<td>-43</td>
<td>-</td>
</tr>
</tbody>
</table>
**Figures:**

**Fig. 1:** Definition of the decentration and tilt of the IOL and the resulting off-axial image point with the eccentricity $e$ ($N, N'$: front and rear nodal point of the optical system, $d_R$: image point distance of the principal ray from optical axis, $f_{NP}$: distance from the nodal point $N'$ to the retina).

**Fig. 2:** Contrast sensitivity as function of spatial frequency in cycles per degree by Barten's theoretical model. The CSF shows the spherical lens SA60AT (solid line) and the aspherical lens SN60WF (dashed line) in a Navarro model eye at a pupil size at 4.5 mm (viewing distance 6 m, optimized at 3.0 mm). The simple Barten model (dotted line) is a fitted model with empirical data. Both models under the same conditions (object angle 2 deg, luminance 250 cd/cm²).
**Fig. 3**: SQRI as a function of the decentration and tilt with spherical SA60AT (left) and aspherical SN60WF (right) in the eye model with a pupil aperture of 3.0 mm. The parameters to the right of each plot indicate the decentration of the IOL. For comparison, the gray dashed line shows the SQRI curve without extrafoveal image (eccentricity e = 0 deg) using an example decentration of 0.4 mm.

**Fig. 4**: SQRI as a function of the decentration and tilt with spherical SA60AT (left) and aspherical SN60WF (right) in the eye model with a pupil aperture of 4.5 mm. The parameters to the right of the diagram indicate the decentration of the IOL.

**Fig. 5**: Difference in SQRI between the aspherical SN60WF and spherical SA60AT at aperture 3.0 mm (left) and 4.5 mm (right) under the same decentration and tilt.
**Fig. 6:** SQRI as a function of the decentration and tilt with spherical SA60AT (left) and aspherical SN60WF (right) under defocusing in the eye model with the pupil aperture 3.0 mm. The parameters to the right of the diagram indicate the decentration of the IOL. The solid line shows the axial displacement of the lens with ACD + 0.1 mm, the dotted line with ACD - 0.1 mm.
Fig. 7: Simulated images of a USAF target chart for the IOL in the eye model with a pupillary aperture of 4.5 mm and an object angle of 2 deg. Left: Comparison of the lenses in the refraction-corrected eye model (A, B). Right: Comparison with negative (C) and positive (D) tilt with decentration of the aspherical IOL. The marked area shows the distance between the bars under a visual acuity of 1.


Figure 3

The graph shows the relationship between the tilt of the lens in degrees and its SQRI value in jnd (just noticeable difference) for different lens diameters. The graph includes data for both spherical (sph. IOL) and aspherical (asph. IOL) lenses, with specific values for lens diameters ranging from 0.0 mm to 0.4 mm.

- For sph. IOL (SA60AT), the SQRI values are plotted for lens diameters of 0.0 mm, 0.2 mm, 0.4 mm, 0.6 mm, 0.8 mm, and 1.0 mm. The tilt values range from -5 to 5 degrees.

- For asph. IOL (SN60WF), the SQRI values are plotted for lens diameters of 0.0 mm to 0.4 mm. The tilt values range from -5 to 5 degrees.

The graph indicates that as the tilt of the lens increases, the SQRI value also increases, with a notable increase for larger lens diameters and higher tilt values.
Figure 6

The figure shows the relationship between SQRI value in jnd and tilt of lens in degrees for different lens distances and combinations.

- For SA60AT, 3.0 mm, +0.1 mm, the SQRI value decreases with increasing tilt.
- For SA60AT, 3.0 mm, -0.1 mm, the SQRI value also decreases with increasing tilt.
- For SN60WF, 3.0 mm, +0.1 mm, the SQRI value decreases with increasing tilt.
- For SN60WF, 3.0 mm, -0.1 mm, the SQRI value decreases with increasing tilt.

The graph indicates that the SQRI value decreases with an increase in tilt for all conditions tested.