Corrigendum

Corrigendum to "The Use of Latin-Square Designs in Educational and Psychological Research"

[Educational Research Review 24 (2018) 84–97]

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DOI of original article: https://doi.org/10.1016/j.edurev.2018.03.003

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DOI of this corrigendum:

A Latin square is a matrix containing the same number of rows and columns (*k*, say), such that the cell entries consist of a sequence of *k* symbols (for instance, the integers from 1 to *k*, or the first *k* letters of the alphabet) inserted in such a way that each symbol occurs only once in each row and only once in each column of the grid. Mathematicians refer to *k* as the *order* of the Latin square. For example, Fig. 1 shows a Latin square with four rows and four columns that contains the integers from 1 to 4. Richardson (2018) reviewed the use of Latin squares in research design and data analysis. He concluded that the judicious use of Latin-square designs was a powerful tool for experimental researchers. This corrigendum identifies a substantive error in Richardson's account and provides a clarification of the underlying issue.

(Insert Figure 1 about here)

Bugelski (1949) suggested that Latin squares could be particularly useful in controlling sequence or carryover effects in repeated-measures designs. Fig. 1 can be interpreted as applying to this situation. Assume that the integers refer to the four conditions of interest, that the columns refer to the order of administration of the conditions, and that the rows refer to participant groups who receive the conditions in different orders. In this example, each consecutive pairing of two conditions occurs exactly once. Mathematicians refer to such Latin squares as "row-complete", whereas educational and psychological researchers refer to them as "digram-balanced". Williams (1949) described a method for constructing row-complete Latin squares for any even value of *k*.

The situation for odd values of *k* has proved to be much more difficult, and the results for odd values of *k* are less well-known outside of the mathematical world. In particular, Richardson (2018, p. 90) stated: "Row-complete Latin squares with odd numbers of rows and columns do not exist." If true, this would entail that some alternative methods would need to be devised for controlling sequence or carryover effects in repeated-measures designs involving odd numbers of conditions. Nevertheless, this is the substantive error to which we wish to draw attention. In fact, row-complete Latin squares do exist for certain odd orders, namely if *k* is not a prime number.

A *prime* number is a positive integer greater than 1 that can only be divided evenly by itself and 1. By definition, even numbers greater than 2 are not prime numbers, in that they can be divided by 2. Positive integers greater than 1 that are not prime numbers are referred to as *composite*, in that they can be expressed as the product of two or more smaller positive integers (for instance, $35 = 7 \times 5$). It is well-known that most odd positive integers are composite, in the sense that the percentage of odd positive integers less than *N* which are composite approaches 100% when *N* is sufficiently large. Mendelsohn (1968) identified the first row-complete Latin square of an odd order greater than 1, for which $k = 21$. Subsequently, several mathematicians provided constructions for row-complete Latin squares for other odd values of *k* such as 9, 15, 25, 27, 33, 39, 55, and 57, as well as other specific classes of odd numbers (see Higham, 1998, for details). It should be noted that these values are all composite numbers, not prime numbers.

Higham (1997, 1998) provided a procedure for constructing row-complete Latin squares for values of *k* that are odd composite numbers greater than 9, and he gave a worked example for $k = 35$. As noted earlier, Williams (1949) described a procedure for constructing row-complete Latin squares for all even values of *k*. Moreover, Archdeacon, Dinitz, Stinson, and Tillson (1980) presented a row-complete Latin square when $k = 9$, and there are no odd composite numbers less than 9. Higham concluded that row-complete Latin squares existed for all values of *k* that are composite numbers, both odd and even. The question remains open as to whether there exists a row-composite Latin square when *k* is a prime number greater than 2.

Where does this leave researchers who wish to devise experiments in which the number of conditions is an odd prime? One solution would be to include an additional condition so that the total number of conditions is even and then to apply the construction described by Williams (1949). Alternatively, as Richardson (2018) explained, researchers have developed algorithms for constructing *pairs* of Latin squares which control sequence or carryover effects if used in combination. These apply to research designs involving odd numbers of conditions and so will apply in particular to designs where the number of conditions is a prime.

John T.E. Richardson would like to apologise for any inconvenience caused.

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Figure caption

Fig. 1. A 4 × 4 Latin square.

Figure 1

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1