"Clebsch took notice of me": Olaus Henrici and surface models

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The (Danish born) German mathematician Olaus Henrici (1840–1919), having spent a short time as an apprentice engineer, began his mathematical studies in 1859 in Karlsruhe where he came under the influence of Clebsch, as he later recalled:

> "Of greater importance to me was the fact that Clebsch took notice of me. He induced me to devote myself exclusively to Mathematics. During the three months summer vacation in 1860 I remained in Karlsruhe earning a little money by private teaching. I was honoured by seeing much of Clebsch. Practically every morning I called for him at 10 o'clock for a long walk during which much Mathematics was talked. It was only later that I realised how very much I had learned during these lessons without paper or blackboard." [1, p.71]

With recommendations from Clebsch, Henrici went to Heidelberg to study with Hesse and in 1863 he took a PhD in algebraic geometry before moving to Berlin to attend the lectures of Weierstrass and Kronecker. Unable to make a living in Germany, he moved to London in 1865 to work with a friend on some engineering problems. The enterprise was not successful so he turned to mathematics tutoring and continued with his mathematical research. Through Hesse he obtained an introduction to Sylvester, and through Sylvester he got to know Cayley, Hirst and Clifford. In 1870 he succeeded Hirst as the Professor of Pure Mathematics at University College, and in 1880, on the death of Clifford, he took over the chair of Applied Mathematics. Four years later, he was appointed as the founding professor of Mathematics and Mechanics at the newly formed Central Technical College where he established a Laboratory of Mechanics, a position he retained until he retired in 1911.

A proponent of pure (projective) geometry and a leading figure in the British movement against the teaching of Euclid (his textbook [2] was satirized by Charles Dodgson [3, pp.71–96]), Henrici produced a number of models of geometrical surfaces, several of which he exhibited in front of the London Mathematical Society (LMS). He promoted the use of models in teaching, encouraging students to construct geometrical models for themselves [4]. (An evocative description of the student workshop at UCL in 1878 is given in [7].) He played an active part in the great exhibitions in London in 1876 [5] and Munich in 1893 [6]–he was part of a three-man British committee for the latter (the others were Greenhill and Kelvin)—and his models feature prominently in both.

Henrici's "Professorial Dissertation for 1871-72" was entitled "On the Construction of Cardboard Models of Surfaces of the second Order" [8, p.161], and he gave some of these cardboard models to Clebsch (who was by then in Göttingen). It was one of these models—constructed from semi-circular sections—that in 1874 inspired Clebsch's student Alexander Brill to make similar models of his own ([8, p.159]) which he exhibited in 1876 with acknowledgement to Henrici. (These models would provide the starting point of the famous Brill mathematical model business which was run by Alexander's brother, Ludwig.)

Three of the most important of Henrici's surface models were those of the third order surface $xyz = (\frac{3}{7})^3(x + y + z - 1)^3$, the moveable hyperboloid of one sheet, and Sylvester's 'amphigenous' surface[1]. The first of these, in which the 27 lines (all real) form three groups of nine coincident lines, was initially constructed in cardboard by Henrici who showed it at the LMS in 1869. A plaster model lent by Henrici was displayed at the Science Museum in London where it later became a source of inspiration for the artist E. A. Wadsworth who used it in his 1936 poster advertising the South Kensington Museums.

The moveable hyperboloid of one sheet originated in 1873 as a problem set by Henrici for one of his students. Henrici had expected the construction he had defined to be rigid and was surprised when it was not the case. It turned out not to be difficult to understand why the surface was moveable, and Henrici was led to establish the theorem: "If the two sets of generators of a hyperboloid be connected by articulated joints wherever they meet, then the system remains moveable, the hyperboloid changing its shape" [9]. The properties of the surface became more widely known through a Cambridge Tripos question set by Greenhill in 1878, the solution of which was published by Cayley [10]. Since then the surface has been shown to have applications in connection with the motion of a gyratory rigid body, and it is still relevant in research today [11].

Of all Henrici's surface models, the most ambitious was undoubtedly the model of Sylvester's amphigenous surface. This 9th order surface emerged out of Sylvester's great paper proving Newton's rule for the discovery of the imaginary roots of a polynomial which Sylvester had published in 1864 [12]. After a long and convoluted algebraic argument in which he had derived the equation of the surface, Sylvester had shown that when a particular plane touches the surface along a particular curve, it divides each half of the space separated by the surface into three distinct parts. And, as Henrici observed, it is this property which connects the surface in a remarkable a manner with theory of binary quintics and by which Sylvester had shown how to decide whether the roots of a fifth degree equation are real or imaginary [13], [6, p.173–175]. In March 1865 Sylvester discussed the possible construction of the surface with Hirst and a mechanician at the Royal Society but shortly afterwards told Hirst that he "had thought a good bit upon this wonderful surface since last seeing you ... [its] form ... seems to be gradually growing up in my mind but it requires a prodigous effort beyond my present powers of conception to realise it in its totality" [15, pp.184–185]. There is no record of a model of the surface having been made at this time and it seems that one was not produced until December 1870, when Henrici "exhibited a large model of Dr Sylvester's amphigenous surface" [13] in front of the LMS. Since Sylvester had found a much simpler proof of Newton's rule—one which did not involve the amphigenous surface—in the summer of 1865 [14], it is likely that he then lost

[1] 'Amphigenous' is a botanical term which means growing all round a central point.

interest in trying to construct the surface, his interest being rekindled only when he met Henrici. A model of the surface was exhibited by Henrici in 1876, where it was singled out for comment by H. J. S. Smith [5, p.52], and again in 1893, but it appears not to have survived.

Henrici's work on these models all contributed to his growing reputation amongst British mathematicians, and in 1874 it formed part of the citation for his election to the Royal Society. Further, it is notable that a modelling club was established in Cambridge by Cayley and others (including Maxwell as "the custodian of the models") [16, 331] in the aftermath of the British Association for the Advancement of Science annual meeting in Bradford in 1873, the first such meeting attended by Henrici. Geometry had occupied a prominent position at the meeting, and Klein too was among the attendees. The club took an active part in the 1876 exhibition, although it seems to have faded soon after. Sylvester maintained his interest in models and in Oxford in 1887, four years after his return from the United States, he put on a course entitled "Lectures on Surfaces, illustrated by plaster, string and cardboard models" [17, p.229], although it did not draw much of an audience, presumably due to the fact that the subject was not part of the students' examination requirements.

In 19th century Britain Henrici was one of the leading proponents for surface models and he did much to stimulate an interest in them, both in his students and in his peers. His German origin and education, particularly his tutelage under Clebsch, enabled him to act as a bridge between British and German mathematicians interested in models. It is no coincidence that Britain was the largest foreign contributor to the Munich exhibition.

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The fourth dimension: models, analogies, and so on

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The way to the geometry of a four-dimensional space was not straightforward. In principle, such a geometry was possible after the elaboration of solid geometry in analytic form. But there were still some reservations against such a geometry due to the fact that geometry was understood as the study of space and space was considered as three-dimensional. In Moebius' *Barycentrischer Calcul* (1827), we find several instances where he assures his reader that four-dimensional space could not exist and also H. Grassmann stated in the introduction to his *Lineale Ausdehnungslehre* (1844) that his new science is not bound by any restriction concerning dimensions whereas geometry could not go further than dimension three. Around 1850, we find several cautious attempts to transcend this restriction (Cauchy, Cayley, ...) in speaking of pseudo-points and things like that[1]. An important step forward was taken by C. Jordan in his long paper *Essai sur la geometrie a n dimensions* (1875 - a short overview of its content was published before in 1872) in which Jordan developed the geometry of linear (sub-)spaces of an n-dimensional space. But this could still be criticized as being algebra in geometric disguise. Note that neither Jordan nor someone else before tried to give an intuitive picture of a geometric object in the four-dimensional space at all.

Aroud 1880, the problem of determining the number of regular polytopes in four-dimensional space became rather popular. This is the analog of Euclid's result on today so-called Platonic solids (book XIII, theorem 18a); it was clear that this is a genuine geometric question. In order to arrive at its solution, it is definitely important to have an insight into the structure of those hyper-solids. A rather complete and convincing purely synthetic solution was given by William Irving Stringham in his dissertation (1879) under the supervision of J. J. Sylvester (then at John Hopkins in Baltimore). After having received his degree, Stringham went to Germany to stay with F. Klein in Leipzig, where he gave a talk on his result in Klein's seminar - once again with a lot of picture. After receiving a call from Berkeley, he returned to the States in the same year. Stringham demonstrated that in four dimensions there are six regular polytopes. We cite them here as the hyper-simplex, the hyper-cube, the hyper-octahedron, the 24-cell, the 120-cell and the 600-cell (Stringham had a somewhat awkward terminology of his own, which

[1] A collection of interesting texts can found in Smith 1959, 524-545.