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Chapter 10 Promoting a positive learning environment

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Affect, or the emotions and feelings that people experience, is a real and significant issue when people are learning mathematics. When people feel included and supported in a positive environment they think more, work harder and enjoy being there. Taking the time and trouble to empathise with your pupils and to establish a positive learning environment will mean that they are more willing to engage, and will lend more attention to learning mathematics, which will lead both to better results for your pupils and to more enjoyment for you in your job.

Task:

Think about a time when have you seen pupils pay attention and enjoy a lesson. What contributed to that feeling of enjoyment? Now think about a time when the pupils did not seem to be engaged, even if they were not actively misbehaving. What was different about the pupils' experiences and learning outcomes in these two situations?

Establishing a positive environment in the mathematics classroom requires establishing ground rules, such as showing respect for each other and valuing contributions of all, and also making sure that the environment is safe and physically welcoming. Importantly it also requires that you, as teacher, pay close attention to affect and offer empathy in order to create a positive ethos that allows pupils to grow their learning without anxiety or other negative feelings. A positive environment will help pupils develop the emotional resources and mathematical resilience that enables them to overcome the barriers that learning mathematics presents for many people. This chapter shows how working with mathematics activities in certain ways can help you provide a positive learning environment for all your learners.

Affect – the emotional side of learning mathematics and why it is important.

You often do not have to look far to find people who have a negative view of learning mathematics. Many adults today are comfortable saying “I can’t do maths!” but would be embarrassed to say “I just can’t read!” Many young people in school do not enjoy their mathematics lessons, which can cause them to misbehave or disengage in less obvious ways. In a key piece of research Nardi and Steward (2003) investigated these deliberately invisible pupils and found that they saw their work in mathematics lessons as being TIRED – tedious, isolated, rote learning, elitist and depersonalised. Over time repetitive and alienating teaching methods, and high-stress practices such as always demanding responses at speed, can contribute to learners developing a more extreme negative emotional response to mathematics. This is known as mathematical anxiety (Ashcraft and Krause 2007) which can have long term effects, and even be contagious. It is therefore important for all teachers to be aware that pupils can become anxious when learning mathematics, to take steps to help them deal with challenge and difficulty in a positive way, and to help them overcome this barrier to success.

Mindsets affect learning

Many of the ideas which lead to pupils feeling excluded from mathematics can be the result of holding a fixed mindset. Dweck (2000) found that pupils will hold one of two mindsets:

A fixed mindset: pupils believe their intelligence or abilities are fixed traits. There is a ceiling that they cannot go beyond. This may result in pupils competing to show that they are better than others or not trying as they worry that their abilities may be found to have a low ceiling. Pupils with this way of thinking believe that inherent talent alone creates success; if they have to make an effort this indicates that they “can’t do it”.

A growth mindset: pupils believe that their most basic abilities can be developed through dedication and hard work and that struggling to understand and solve problems is good as it allows their abilities to grow. This view creates a love of learning and an understanding that it is worth making an effort to grasp and comprehend, as that leads to success.

Task:

Your pupils will learn about mindsets through what you say and what you do. Reflect on how far you encourage a growth mindset by:

- Focusing on pupils’ hard work and understanding and making links between this and success.
- Treating mistakes, difficulty and failures as something that benefits learning rather than reflecting badly on pupils’ abilities.
- Giving pupils relevant tasks, not rote learning or procedures to follow, clear feedback that shows how to improve and time to make that improvement (see Chapter 2).

Mathematical Resilience

Mathematical Resilience (MR) is a positive stance towards mathematics which pupils can be helped to develop. MR helps pupils to overcome the barriers that can be present when learning mathematics, rather than to avoid or feel anxious about them, and to feel that they can be a successful learner of mathematics.

A classroom environment (Lee and Johnston-Wilder, 2013) that helps pupils to develop MR:

- Helps every pupil understand that their intellect is not fixed but can be grown (Yeager & Dweck 2012), and that every time they try out something they make new connections in their brains and strengthen existing ones.
- Includes and values all pupils. This means offering work that all pupils can see the value of and can learn from and also giving each pupil a real voice in the classroom, for example by offering a choice of work.
- Acknowledges that mathematics involves struggle but that success is very possible. Rather than asking pupils to memorise “easy” ways to solve problems, which they often forget how to use, MR teachers help pupils make connections between ideas and nurture understanding.
- Shows pupils how to get help and support with their learning. Pupils with MR tend to work collaboratively. They know they can get help from and give help to other people and that in

doing so their understanding will grow. They get help from the internet or from books and if one explanation does not work will look for another.

Nurturing mathematical resilience means teaching in a way that is ALIVE, that is *accessible, linked, inclusive, valuable and engaging*. ALIVE teaching means that everyone is included, engaged and making links and connections. Pupils come up with their own ideas that are listened to and valued.

We will discuss some specific ALIVE activities in the next section but first we will look at some general ideas that can be used to bring your classroom ALIVE.

What can be done to bring your classroom ALIVE?

Be less helpful: Letting pupils struggle can go against your initial instincts as a teacher but overcoming barriers and understanding mathematics will (eventually) lead to more enjoyment for your pupils. Being too helpful removes temporary anxiety but can encourage longer-term helplessness and teacher dependence. Being less helpful means that pupils will have to persevere, but they will also have the positive experience of puzzling out a solution or making connections for themselves.

Build confidence: When pupils truly believe they have the ability to successfully complete a task they are more motivated to persevere through difficulties. Bandura (1977) calls this optimistic self-belief self-efficacy and suggests four ways to build pupils' self-efficacy: exposing them to experiences where they expend effort and overcome struggle; letting them see role-models (such as older 'mentor' pupils) who have achieved mastery despite adversity; working in a social environment where mistakes are seen as a natural and important part of learning; and helping them interpret their emotions during struggle as normal and not signs that they cannot "do" mathematics.

Understand and manage emotions: Some pupils will already be anxious about engaging with mathematics when they arrive in your classes. You can help them to understand and manage any negative emotions by using the Growth Zone (GZ) Model (Johnston-Wilder and Lee, 2013). The GZ Model has three zones, the green or comfort zone, the amber or growth zone and the red or anxiety zone. Explain to your pupils:



Figure 1 The Growth Zone

The green zone can be a good place to be, here you are comfortable with what you are doing. You may be consolidating ideas that you have just learned and developing automaticity and fluency. However, if you always stay in the green zone you will not be learning anything new.

The amber or growth zone is where you are learning new ideas and concepts or making new connections. The ideas you are learning here are new, so you will feel uncertain here. Learning can feel risky and uncomfortable, you are likely to make mistakes or get stuck, which is what happens when you are learning. You may experience good stress, which makes you feel excited and gives you that good feeling of success when you succeed despite barriers and problems.

The red zone is to be avoided but is often experienced by pupils. Here you will feel you just cannot understand. You feel anxious and this means your brain cannot think. When you experience these feelings you need to have ways to get back to green or yellow, to comfort and safety. This may mean for example: working with someone you can question and who will make suggestions but never take away your success by giving the answer, going online and practising the ideas on a suitable website using instant feedback to explore ideas or going right back and describing what you have done already, articulating simpler ideas can help you make progress. Remind your pupils that it is their job to work out what might help them learn. It remains the teacher's job to engineer a safe classroom, and give pupils the ideas, encouragement and time to find out how best they can learn.

Task:

Can you think of a time when you were working in the green zone? How did this make you feel? Think now of a time you felt you were in the red zone. How did you feel then? Did the way you felt change the way you felt about those ideas in mathematics or yourself as a mathematician?

Making Mathematics Activities Come ALIVE

There are lots of ways of promoting affect in your classroom. Amongst other ideas you have already seen how some teachers use ICT (chapters 4 and 5), group work (chapter X), investigations (chapter 8) and activities and games (chapter 9) to promote mathematical thinking, inclusion and enjoyment. The rest of this chapter contains some example mathematical activities that are ALIVE: accessible, linked, inclusive, valuable and engaging. All of these activities are ALIVE, but in the discussions we focus on one of these characteristics at a time.

Making activities that are *accessible*:

Mathematical activities that are accessible for all pupils mean they are constructed in such a way that all pupils can access and 'solve' them. At the same time such activities offer the opportunity for pupils to extend their thinking and reasoning to a high level (McClure, 2012 for NRICH). Such "low threshold high ceiling" tasks do not put a ceiling on the learning of the pupils as closed tasks may do and allow for differentiation by outcome. Other characteristics of "low threshold high ceiling tasks" are: they offer multiple methods, pathways and representations, inquiry opportunities are included, asking the problem before teaching the method, adding a visual component, asking pupils to convince and reason and to be sceptical (Boaler, 2016, p.90).

The next activity is "low threshold high ceiling" because the mathematics and reasoning starts simply but can get very complex. The mathematical topic is exploring variables (the numerical quantities that will vary) and constants (the quantities that will stay the same), which pupils often struggle with, using an example from city life: a shopping mall.

Activity 1: Identifying constants and variables



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Tell your pupils:

- Imagine you are a professional mathematician and you are working on developing a mathematical model to describe the dynamics of a shopping mall such as Westfield shopping centre near London. You first have to identify all the variables (quantities that vary) and constants (quantities that stay the same) playing a role in the shopping centre.
- Make a list of all the 'players' or 'elements' in this setting. For example the car park, the security people or the number of shops on the first floor.

Write suggestions on the board and ask the pupils to identify which are variables and which are constants. Could they be both? If so, what would this depend on?

Ask pupils, in their groups, to pick four variables and think how they relate to the constants. There are no right or wrong answers here. For example, the number of security people could vary depending on how many visitors (buyers) there are at any given time, or on the number of shops or cars.

Finally, ask the pupils to find quantifiers they could use to describe the relationship between the variables and constants they identified and to write these as mathematical statements. For example, 'one security guard for every ten shops'; written as a mathematical statement would be: 'number of security guards = number of shops/10.

Think about other variables and constants and ask them to practice using accurate algebraic notation.

Consider where and how this could be extended to make more connections to higher level mathematical ideas. For example by considering discrete versus continuous variables and non-linear relationships that can be explored such as economies of scale.

Making activities that are *linked*:

Helping pupils know that they have a deep understanding of mathematical concepts requires activities that help them make connections between mathematical ideas. Doing this will help pupils give a secure understanding of the mathematics which is an important aspect of a positive learning environment. Research (Askew et al., 1997) suggests that teachers who make connections in their teaching are more successful than those who do not.

A disconnect between topics or aspects of a topic can easily happen when the learning in that topic is spread over several years, pupils fail to see connections, rely too much on memorisation and their knowledge becomes fragmented. An example of this is learning about factors, multiples, HCF (highest common factor) and LCM (lowest common multiple) of numbers, which is often taught in year 7 but the ideas are extensively applied much later in when learning algebra.

The next activity helps pupils to make connections by noticing the differences and similarities between what a factor is, between factors of numbers and factors in expressions, and to think about the mathematical thinking processes involved in finding factors of numbers and expressions.

Activity 2: Finding factors of numbers and expressions

Tell your pupils to work in small groups or pairs and to list the various factors of the following numbers and expressions.

- 60
- $3xy$
- 15
- $12x^2y^3$
- $3x^4 - 27x^4$
- $x^2 + 2x$
- $2x^2 - 8x + 8$

Then ask them to discuss the following questions:

- Why do you think some numbers only have two factors?
- What is the same and what is different in your answers to those questions?
- How did you find that answer? ·
- What is the same or what is different in your process of thinking? Did you all use the same method? Did all the methods work well with all questions?

- Can you now define or describe what a factor is, with examples of where and how it can be found.

Next pupils have to find the lowest common multiples (LCM) and highest common factors (HCF) of a mixture of numbers and expressions. Ask your pupils to think about and make notes on the methods they have used.

Find common factors and multiples of the following:

- 48 and 72
- x^2 and $3xy$
- $\sqrt{18}$ and $\sqrt{32}$
- $(a - b)^2$ and $(a - b)^3$
- $(a^2 - b^2)$ and $(a^3 - b^3)$

Then ask the pupils to discuss the following questions:

- What is the same and what is different in your answers to those questions?
- How did you find each answer?
- What is the same or what is different in your process of thinking? Did you all use the same method? Did all the methods work well with all questions?
- Can you convince other pupils or your partner that these methods are mathematically correct?
- Can you now define or describe what the HCF and LCM are, with examples of where and how they can be found?

Making activities *inclusive*:

When activities are inclusive, all pupils feel involved, and all pupils can contribute to the learning activities. By all pupils here we mean those who feel that mathematics is not for them, that it is boring and that they just cannot “do it”, as well as those that have special educational needs when learning mathematics.

Using mathematical activities that involve using senses can help all pupils engage with the mathematics. For example “hearing” the patterns in a sequence by sounding them out; “feeling” the length, width or circumference of objects, or manipulating cubes to experience patterns, “seeing” how to multiply two brackets together by cutting up bits of paper (a^2 squares, strips of length a , and unit squares) and fitting them together according to the brackets.

The following activity asks the pupils to form graphs from their own bodies. In this way they can see and feel the graphs as well as relate the meaning of the graphs to ideas they experience.

Activity 3: Body Maths – including pupils through feeling and seeing data

Ask your pupils to check their shoe size. If you can use UK shoe sizes they will be probably range from 3 – 12, whilst European adult shoe sizes range from 35 – 45. First draw and label an axis, for example with chalk along the longest side of the classroom, or perhaps better outside in the playground. If using Euro sizes, discuss why you have not started at zero.

Ask the pupils to stand in a line of people with the same shoe size in the correct place on the axis. Then discuss the following questions:

- What do mathematicians call what they have made with their bodies?
- What is the modal shoe size? Explore ideas about bi or tri-modal distributions.
- How can you find the median?
- Can you find the mean shoe size? How can we do that efficiently? Perhaps use this opportunity to help the pupils realise that grouping data and multiplying makes finding the mean easier.
- What is the range of shoe sizes? Show me the range.

Now make a different graph that shows different statistics. Ask the pupils to stand in height order perpendicular to the shoe size labels, or you can remove the shoe size labels. Then tell the pupils to step forward a number of paces equivalent to their shoe size or if using Euro sizes the number of paces their shoe size is more than 35. They will make a scatter diagram. Now find the line of best fit by using a piece of string or rope. Ask two pupils to hold the rope in position.

Now:

- Walk towards a pupil with small feet and who is not very tall. Ask the pupils what does it mean if you are in this position?
- Choose several pupils away from the line, what does this show? Give the pupils time to think and try out answers so that they see that someone has small feet for their height or larger feet than might be expected for their height.
- Now ask what does the line show?

If you can, take a photo of the bar chart and the line of best fit, preferably from some height e.g. a second floor window, so that the pupils can see their graphs. Allowing your pupils to make, feel and see graphs in this way helps them to understand the ideas behind the mathematics whilst making sure that everyone is included and sharing the learning experience.

Follow up by asking the pupils to think of examples of their own “body maths” graphs in pairs or small groups. Ask each of them to describe their ideas and then take a vote on which graph the class wants to make. Then let them make it. Encourage them to think of scatter graphs, with good correlation, for example height plotted against armspan or leg length and then draw them. This time they could, perhaps, have axes drawn on a wall and “plot” themselves using sticky notes. Then ask pupils to suggest and test out a pair of variables that would give rise to a graph with no correlation.

Making activities *valuable*:

Building mathematical resilience demands that pupils see the point of doing mathematics, that it is valuable to them (Kookken et al., 2016). As an adult who is able to use and control mathematical ideas you will be able to appreciate how mathematics is relevant to internet shopping and the GPS tracker on your phone, or how it might be applied to model traffic flow or stop an epidemic. However, to your pupils mathematics might be more like a chameleon that merges into the background.

Many of your pupils will not have role models who estimate their shopping bill to make sure they have not overspent or who consciously work through offers to check one is better than another. They will not naturally see the value in this. And if they have not experienced success in mathematics examinations so far it is unlikely they will understand the value in trying hard for an examination.

Value for many pupils has to come through links to ideas that are interesting to them, that cause them enjoyment and consider as worth their attention. The next activity simply requires something from the news or from the pupils' daily interests that is interesting for the pupils. This may be a sports competition, the Olympics or the football World Cup, a newsworthy event such as Tim Peake's space adventure or an event in school such as Fairtrade Fortnight or in a pupil's life such as taking part in a national taekwondo competition.

Activity 4 – Asking Valuable Questions

You could choose the topic in two ways.

1. You can suggest the topic yourself. For example: if you want your pupils to explore statistics and you know they are football mad then suggest the football World Cup, or you are finishing a topic on circles and the Space Station appears in the news, then suggest that.

Or

2. You can ask your pupils for suggestions of topics that interest them and which might have some mathematics in them. Write on the board the topics that the pupils suggest then take a vote on the topic which they are going to use to explore the *mathematics*.

The questions must come from the pupils, not from you. When they ask the questions and they will value the answers. Ask the pupils to work in pairs to devise some mathematical questions about the topic. You may need to ask the class what makes a question mathematical and discuss that mathematics deals with numbers and variables but also with relationships between things.

Ask the pupils to write one question they would be really comfortable answering, one that will take some work and some struggle to answer and one that they may not be able to answer. Link to the growth zone model and help your pupils remember that they have to take a risk if they want to learn. Ask the pupils to post their questions on the board, perhaps on sticky notes so they can be shared and moved about easily.

The pupils then choose a question that interests them and get to work answering it. They can challenge themselves or stay safe it is up to them. If the question they have is too easy and is not challenging them or too hard and the challenge is too much, they should negotiate with you to choose another question. The pupils will have to do some research, either by going on line or consulting reference books. You can supply some useful information if you know the topic in advance or provide access to the internet to search.

Finish this learning episode by asking the pupils to present their findings either by making posters and displaying them or by asking the pupils to tell the class what they have found and what mathematics they used in their explorations.

Making activities *engaging*:

When pupils are engaged they are learning efficiently and will experience enjoyment. Polya (1963), a prominent mathematician and educator, stated 'For efficient learning, the learner should be interested in the material to be learned and find pleasure in the activity of learning' (Polya, 1963, p.608). Several teaching approaches can be used to make mathematics learning engaging. For example: games for learning can make exploring mathematical ideas both interesting and pleasurable. Other ideas that engage pupils are asking them to think beyond the doing and working collaboratively as many learners feel safer and learn better when learning in groups (Markus & Kitayama, 1991).

Thinking beyond the doing

By asking pupils to think and reflect on how they solve or 'do' mathematics problems they will deepen their understanding, move beyond simply following procedures and algorithms. One way of making pupils 'think beyond the doing' is by asking them to do the reverse of what they are normally asked to do: instead of answering questions, ask them to make up questions (and work out the answers as well); instead of giving them instructions on how to use or do something, ask them to write the instructions.

Collaborative learning

Asking pupils to work cooperatively or collaboratively can really help their learning and engagement with mathematics (Slavin et al., 2003). Being part of a team can give pupils the confidence and motivate them to tackle more challenging problems, whilst group discussions force them to think through ideas and to clarify these (William, 2011).

The next activity will engage learners by using collaborative learning and making them think beyond the doing by asking them to write instructions on how to use a clinometer.

Activity 5: writing instructions for using a clinometer

Tell your pupils:

A clinometer is a tool to help surveyors measure the height of tall structures such as trees, landmarks or buildings. Imagine you are the development team of a company that produces such clinometers and your team is asked to write an easy-to-use manual on how to use it. Remember that your clients are surveyors who are quite knowledgeable about mathematics and trigonometry, so feel free to include mathematical explanations on how and why a clinometer can be used for working out heights. You might have to make one first and experiment with it so you understand well how it works before you write your manual.

These are some questions that might help you with your task:

- What are the components of this instrument and how does it work? What is the mathematics involved? It might help to make a drawing of what you think happens.
- How can you use a clinometer to work out the heights of a tall object? What is the mathematics involved? It might help to draw what you think happens.
- What happens if you stand on a hill and the object you are measuring is at the top of the hill? Do you need a different method or not?
- What is the potential for error, and how might you estimate error bounds?

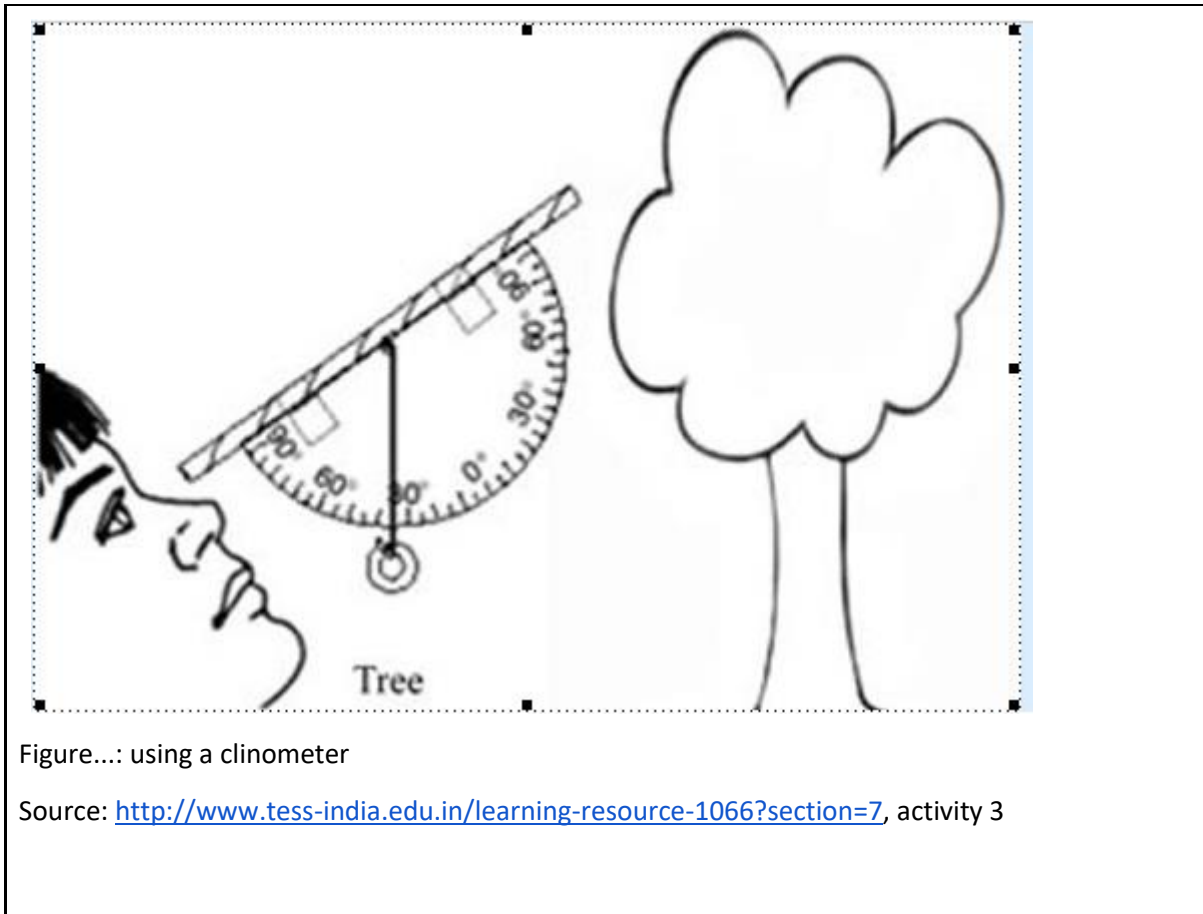


Figure...: using a clinometer

Source: <http://www.tess-india.edu.in/learning-resource-1066?section=7>, activity 3

Task:

How ALIVE are your lessons? Are you using tasks that are accessible, linked, inclusive, valuable and engaging? Reflect on how often you think you will be able to teach lessons that are ALIVE.

It is most likely that the honest answer is “some of the time”. Teaching like this takes time, planning and energy and experienced teachers would struggle to teach like this “most of the time”. However, ALIVE teaching is very rewarding for you and your pupils. As your pupils become able to access the ideas, know that all mathematical ideas link together, that they are included in valuable work and that they can be engaged in mathematical thinking, both your pupils and you as their teacher will come to enjoy these lessons.

If you answered “not at all”, what is stopping you? Perhaps you feel constrained by schemes of work to teach in a certain way? It is possible to work around most constraints. Decide on one idea you would like to try and ask your mentor or head of department if you might try something different. If you feel this idea is successful, then say so and ask if you can try another. Perhaps volunteer to talk about what you find out at a department meeting. Others in the department may be willing to try out ALIVE ideas as well.

Summary

This chapter has explored what makes for a positive learning environment, and how it can be achieved in the mathematics classroom. Although it is important that everyone in that environment shows respect for each other and values the contributions of all, a positive learning environment for

mathematics requires more than that. It involves helping your pupils deal with the emotional side of learning which in the case of mathematics all too often means helping them overcome avoidance and anxiety. A positive learning environment requires building the pupils' mathematical resilience by:

- Acknowledging that *all* pupils can grow their capabilities with the right help and support
- Helping pupils understand that learning mathematics requires struggle but no-one need struggle alone
- Ensuring all pupils see value in the work they are doing for them, and thus feel included in the community of those learning mathematics.
- Encouraging pupils to understand how they can get the support they need, whether that is working collaboratively, seeking support from books or the internet or quietly reflecting by themselves.

This chapter has also demonstrated how working with ALIVE mathematics activities can help you establish and support a positive learning environment for all your learners.

Acknowledgements

Activities 2 and 5 were adapted from:

TESS-India, <http://www.tess-india.edu.in/> (<http://creativecommons.org/licenses/by-sa/3.0/>)

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