Planning mathematics lessons

Book Section

Chapter 1

Planning mathematics lessons

ROBERT WARD-PENNY AND CLARE LEE

For an emerging mathematics teacher, planning is fundamental. A good lesson plan can provide a foundation for both effective teaching and successful learning. Planning can also help to tackle many of the concerns and fears that teachers have as they start their careers; in the words of the author Alan Lakein, ‘planning is bringing the future into the present so that you can do something about it now’.

As people think and organise their thoughts in different ways, planning is a personal process. You must develop techniques and habits that support your own teaching, and this chapter is intended to set this process in motion. It contains a number of practical activities which you can use to develop your planning skills whilst you begin your teaching. As you read through this chapter, you might find it useful to have to hand a copy of any pro forma documents which your institution or mentor have provided, and if possible, some examples of lesson plans which you have already written.

BEGINNING TO PLAN

There is no one correct way of planning a lesson. Some teachers find it useful to start with the learning objective, or aim of the lesson, and then sketch out some rough ideas before starting to construct a formal lesson plan. For instance, you might start with a large piece of paper and write down everything that comes to mind about the mathematical concept or procedure that

you are teaching. You can then pick out the most relevant aspects and appropriate resources for your particular class and draw a path through the ideas (Figure 1.1); this path forms the basis for a more formal lesson plan.

Task

Sketch an informal planning diagram for teaching coordinates, and draw a path through it in a similar manner to Figure 1.1. Do you think you would find this approach useful when starting to plan a lesson?

One of the most common ideas currently used in planning mathematics lessons is the three-part lesson: starter, main and plenary. This is a useful starting point, as it reminds you that pupils' attention spans are limited, and that moving between tasks can maintain pupils' focus and bolster their learning. However, it is intended as a structure, not a straitjacket; for instance you might find that a longer lesson requires a mini-plenary in the middle, or two iterations of each phase. On the other hand, extended work such as investigations may call for a more holistic approach; examples of such tasks are offered in Chapter 8.

[Insert Figure 1.1 here]

[caption]*Figure 1.1* Informal planning diagram for 'adding fractions'

Task

Observe a range of experienced teachers and focus on the ways in which they structure their lessons. How do they use or adapt the idea of the three-part lesson? How do they alter the

structure of their lessons to suit different classes?

PLANNING EFFECTIVE STARTERS

An opening or introductory activity can serve a number of different roles in a lesson:

- **Linking back** – start a lesson by reflecting together on what your pupils already know. If a class is in the middle of a series of lessons, you might choose to briefly recap the previous lesson. For instance, you might ask pupils to work in groups to create a spider diagram of everything they know now about 'area'. This information can then be added to throughout this and subsequent lessons.
- **Looking forward** – begin with a problem which the pupils cannot yet solve efficiently. For a lesson on the nth terms of sequences, you might start with some simple linear sequences. Can the pupils find the tenth term of each sequence? What about the millionth term? Returning to the same problem at the end of the lesson can help pupils explicitly recognise their own progress and the purpose of the learning.
- **Mental/oral starters** – a good opportunity to develop each pupil's facility with mental mathematics. This might take the form of a generic game such as bingo or 'Countdown', or be tailored to support the learning objectives. For instance, if you were teaching a lesson on straight-line angles you might present a 3 × 3 (or 5 x 5) grid of numbers, where four (or twelve) pairs of numbers add up to 180 – can the pupils find which number is left over? Similarly, a mental starter on multiplying fractions can support a lesson on tree diagrams and pre-empt difficulties.
- **Real world starters** – a starter involving manipulating numbers drawn from the real world. For example, pupils may work in groups to answer 'how many toilet rolls do

you think the UK uses each year?' or 'how many letters do you think fit in a post-box?' and then defend their responses. Questions can also arise from recent headlines or from the calendar, for instance: 'how many gifts were given in total during the song "The Twelve Days of Christmas"?'

- **Focusing starters** – starter activities can also be used to help manage behaviour. Having an activity such as a puzzle ready at the start of a lesson can direct pupils' attention as they enter the room. Pupils could use five minutes at the start of the lesson to work in pairs and see how many playing cards they can make into a stable house, or play a reaction testing game on the interactive whiteboard, generating numbers to be used later in the lesson. In both cases, pupils who arrive and settle promptly benefit from doing so.

Task

Choose a mathematical topic which you might soon have to teach, for example calculating the mean from a set of discrete data. Which of these types of starter do you think would be most appropriate for this topic? Sketch out two or three ideas for starters which you might like to try for yourself.

Now Try This

Drawing on the ideas offered above, your own ideas and your observations, write down a list of at least five different starter activities. When you are in school, try these out in the classroom and make notes. What worked well? What was less successful, and why? Did any types of activity particularly suit different groups?

PLANNING FOR LEARNING

The *learning objective* or *learning intention* is central to planning mathematics lessons. You will usually draw this from a medium- or long-term planning document. However, you should also think about *learning outcomes* or *success criteria* in your planning. These will help you to structure and differentiate pupils' learning in more detail. As an example, consider this learning objective: *be able to visualise and use plans and elevations of 3-D objects*. You might choose to deconstruct this objective into three learning outcomes. Can you:

- identify 3-D shapes when given plans and elevations?
- draw plans and elevations of basic 3-D shapes?
- work with plans and elevations that include hidden (dotted) edges?

This list offers a logical structure for progression within the lesson: you might start with a whole-class exercise where the teacher projects plans and elevations of real-world objects onto the board for the pupils to identify; move onto a worksheet with simple 3-D shapes from which the pupils have to draw plans and elevations; and then conclude with a practical exercise where pupils make a structure with blocks and then sketch the corresponding plan and elevations.

Task
Starting with the learning objective '*understand Pythagoras' theorem and be able to find missing side lengths in right-angled triangles'*, write down a set of outcomes for your pupils that would indicate a clear progression towards a good understanding of, and ability to use, Pythagoras' Theorem. How might you develop these outcomes into activities for the main part

of a lesson?

PLANNING FOR VARIETY

If you ask your pupils what they want from a ‘good’ lesson they are likely to mention variety, group work and choice. It can be challenging for a new teacher to plan for variety, since transitions can be difficult stages to manage. One rule of thumb is to change the activity roughly every 15 to 25 minutes, unless what the pupils are doing demands more time. Signal the change before it happens, for example by saying ‘in 5 minutes I will want you to put those blocks away and get some spotty paper’ or using something like a countdown clock on your interactive whiteboard to keep both you and the pupils to time.

Variety in lessons involves a balancing act between several aims. For example, you will need to balance:

- giving your pupils time to build their understanding of mathematical concepts *with* setting aside time for consolidation and practice;
- encouraging the pupils to think and reflect individually *and* allowing them to talk through ideas with others;
- providing ways for your pupils to see, feel and touch *with* requiring them to read diagrams and develop their ability to visualise.

Task

Think back over several lessons that you have observed or taught and think about the ways that pupils worked in those lessons. How did the teacher plan for variety?

It is likely that many of your lessons so far have included individual work using textbooks or worksheets, partly because that is the way mathematics has been taught in the past, but also because that might feel safe while you learn to keep control in your classes. The chapters in this book are full of practical suggestions which can help you introduce variety into your teaching. However, you can also vary the way in which you use the textbook itself.

MAKING THE MOST OF TEXTBOOKS

Textbooks are a common feature of mathematics classrooms, and they can be a valuable source of practice material. Over time, however, pupils will tire of simply working through lengthy exercises, so it is worth considering different ways in which you might plan to use a textbook.

- **Not every question** – which questions do learners need to attempt to work towards the learning outcome? Would it suffice to only do the odd questions, or the prime numbered questions? Perhaps the pupils could decide how confident they are and choose themselves, for instance, selecting five questions from a set of ten.
- **Reverse engineering** – start by considering with the pupils how the questions are graded. What makes question 2f *(*simplify $a^2b \times ab^2$) more challenging than question 1d (simplify $a^2b^3 \times a^4b^2$)? Where do they think most people will make mistakes? Which question is the hardest, and why?
- **Do it yourself** – get the pupils to write a textbook page for themselves. How will they introduce the topic? Will they include examples? What questions will they include, and why?

Many modern textbook series come with teachers' guides and linked multimedia resources.

These might also provide you with ideas to support your planning.

Task
Find a mathematics textbook and choose a page at random. What do you think are the strengths and weaknesses of this page? If you were planning to use this in the classroom, how might you adapt it?

PLANNING APPROPRIATE PLENARIES

Although plenaries usually happen at the end of a lesson, they can be used whenever the teacher intends to pause in order to focus and consolidate pupils' learning. Just as with the starter activity, it is possible to reach this goal in many ways.

- **What can you do now that you couldn't at the start of the lesson?** – you might tie this in to the starter (see 'looking forward' above) or end by looking at a question which the pupils can now attempt.
- **Coming soon** – look ahead to the next lesson or section of the lesson. You might even set this up as a challenge; if you've just looked at linear sequences, offer the pupils a quadratic sequence and challenge them to find and justify the next three terms by the start of next lesson.
- **Why does this matter?** – an excellent opportunity for exploring the relevance of mathematics in the wider world. For instance, you might finish off a lesson on formulae by looking at examples that pupils have already met in science and technology lessons, or conclude a lesson on probability by looking at the national lottery.

- **Pupil-led plenaries** – occasionally ask pupils to prepare a one-minute presentation about what they think they have learnt. Another version of this is to hide the learning objective at the start of the lesson and ask pupils to guess what it was at the end of the lesson. This type of activity can promote reflection in your pupils as well as giving you feedback on your own teaching.
- **Plenary games** – finish the lesson with a game where the pupils are required to use the skills they have just developed. For instance, a lesson on coordinates could end with a consolidating game of 'battleships'.

Now Try This
Try each of these approaches at least once. What other ideas have you come across for focusing and consolidating pupils' learning at the end of a lesson?

PLANNING TIMINGS

We have now discussed a range of activities that can make up the starter, main and plenary parts of a lesson. Whatever activities you choose, timing is crucial. Some teachers find it easier to write actual times, such as 10:15 a.m. on their plans, rather than the timings of activities. This allows them to quickly check their progress during the lesson. Another tip is to plan a 'trapdoor' for each lesson; identify in advance one activity which you can drop without interrupting the flow of the learning. It is also worth having an 'extra' or extension activity planned in case the pupils work much more quickly than you expect, or your use of Assessment for Learning shows that you need to move the pupils on more quickly than you had anticipated.

Once your activities and timings are in place, it is worth adding in some more details. What

resources or questions (Chapter 7) do you need to prepare in advance? If you have a teaching assistant, how are you going to direct them? If you are going to explain a new mathematical concept, what misconceptions are likely to arise, and how will you address them? If you are going to demonstrate a mathematical procedure or technique, do you have plenty of examples pre-prepared? This list is far from complete and might already seem daunting but considering these sorts of issues at the planning stage can forestall problems and make the practice of teaching much easier. It is also valuable to consider in advance how you are going to 'signpost' the lesson for the pupils.

SIGNPOSTING

Sometimes pupils can appear to be involved in a sort of magical mystery tour, where one thing just follows another and there are no clues to help them know what they will learn from what they do. Signposting helps the pupils understand why they should concentrate on an activity and what they will learn by doing so. It sets out the direction of the lesson by looking back to what has happened and achieved before; showing the pupils where they are now and looking forward to what will happen next, or pointing towards alternative routes.

For example, after a first activity you might restate the learning objective or learning intention, ask the pupils to reflect briefly on their learning from the activity and then say what is to be done next. 'Signpost' moments allow you to give pupils timings for different parts of the lesson, so that pupils can plan how they use their own time to maximise their learning. Good signposting will encourage pupils to be able to say: 'we've just spent time doing . . . in order to . . . now I'm . . .'

Task

Consider how you might approach a lesson on negative numbers, or another topic of your choice. On the signposting questions diagram (Figure 1.2), note down some comments or questions which you might use to 'signpost':

- the learning the pupils have done up to now (where they have come from);
- what the next learning activity will be (where they are going);
- what options there might be for future or other learning (where you would go if you went down one of the other roads).

[Insert Figure 1.2 here]

[caption]*Figure 1.2* Signposting questions

PLANNING HOMEWORK

Homework is a valuable part of the learning process which should not be wasted. It is worth spending some time experimenting with different types of homework activities. After a lesson on circle theorems, for instance, you might ask the pupils to write two exam-style questions on a piece of A4 paper with the answers on the back. This could form the basis of the starter for the next lesson, where pupils could swap the sheets and see if they come up with the same answers. Alternatively, you might change the form of presentation involved in the homework. Could the pupils produce an ICT-based presentation or a poster for homework? You might even experiment with essay-style responses to a mathematics lesson, such as 'write half a page on how decimals are used in the real-world'.

Now Try This
Try setting an ‘unusual’ homework task, making sure that your expectations for the task are clear to the pupils. After the homework is submitted, you might like to discuss with the pupils how they feel about the different types of homework they are set by each subject. How might their comments inform your future practice?

Some mathematics teachers and schools challenge the traditional form of homework even further by using a ‘flipped classroom’ or ‘flipped lesson’ model. This involves each pupil covering the instruction-based part of the lesson at home, perhaps by reading through their textbook or watching a short video, and then completing a few relatively straightforward problems. They then have the opportunity at school to practice what they have learnt and apply it in more depth, or to seek help if necessary. This strategy can present a number of advantages, as it can free up lesson time for more pupil-teacher interaction and higher-order thinking. However, it may also require more advance planning and additional resources, and it is critical that you make your expectations clear to your pupils, as well as what the consequences will be if they come to your lesson unprepared. You might also want to consider whether a flipped classroom approach would work equally well for all topics in the curriculum.

DIFFERENTIATION

Each of your pupils will arrive to your mathematics lesson with different levels of prior knowledge, understanding and confidence. Sometimes such differences between pupils are explicit, such as when a pupil has come from a different school and has followed a different curriculum, but more often differences in mathematical understanding are less pronounced or

even hidden. If you want all of your pupils to make good progress each lesson, it is centrally important to plan so that each learner can move on with their mathematics, starting from where they are at the time. This practice of tailoring learners' experiences to ensure that each pupil makes progress is called differentiation.

In practice, differentiation can be candid or camouflaged. In its most extreme case, you might need to plan and teach separate but parallel lessons to make sure that every learner makes progress (for instance, during a GCSE revision session where half the class needs to review one topic, and the other half needs to revisit another.) However, you are more likely to differentiate your teaching in smaller ways, perhaps by targeting your questions during an explanation phase, matching the level of challenge to the pupil, or by leaving an extension challenge on the board which extends the learning, makes a connection to a previously visited topic, or requires a higher-level thinking skill such as justifying or proving. Alternatively, you might plan to vary the amount of scaffolding or support that you provide for your pupils, starting some off with a worksheet where the first five equations with letters on both sides have the first line of working completed, or by allowing some of the class to use calculators to find missing angles on a straight line but challenging the others to work out their values using only mental mathematics.

However you plan to differentiate, it is important to check that the resulting differences in the pupils' experiences are meaningful ones. Traditionally a lot of differentiation has simply involved different rates or quantities or work; everyone works from the same exercise but more confident learners might complete the first eight questions, instead of the first six. This type of differentiation has its place, but it often results in students just doing more work, instead of more *challenging* work, and it conflates speed with capability. One way to avoid pupils wasting

time is to plan different starting points in an exercise, saying for instance that pupils who are cautious with this mathematical concept or method should start at question one, whilst those who feel more confident can start at question five. You can also plan differentiated homework in this way, so that each learner can choose between two options, such as six easier questions or four more challenging ones.

It is not unusual though to find that the needs of your pupils go beyond the range of a single worksheet or textbook exercise, so you may need to go further and have multiple sets of resources available. We have seen a number of successful lessons where learners can choose and move between two or three different tasks levelled using labels such as ‘cautious’, ‘confident’ and ‘super confident’.

Open-ended and investigative tasks regularly lend themselves to differentiation, as long as you make your expectations clear to each pupil. For instance, the task ‘create a shape with an area of $100cm^2$’ could result in a rich diversity of outcomes, but if you do not insist that each pupil shows you what they are really capable of, you could end up with a classroom of squares! Similarly, if you are setting out on a statistical project, you might plan to start by outlining in broad terms what a project might include if it was meeting or exceeding the learning objectives set out at the start of the unit.

Differentiation is a huge topic, but an important one, and one which will be returned to many times in this volume. For instance, Chapter 2 will discuss how formative assessment can help you identify the needs of your pupils and decide how to differentiate, Chapter 6 will touch on how you might group pupils to support progress for all, and Chapter 8 will talk more about investigations and open-ended tasks. Chapter 10 will also discuss how to build a classroom

environment where it is OK for each learner to admit their mathematical strengths and weaknesses.

Task

Observe two or three mathematics lessons from different teachers, and make note of the ways in which each teacher differentiates their teaching to make sure that every pupil has the chance to make progress during the lesson.

- Is the differentiation explicit or subtle?
- In what ways does the teacher support pupils who are struggling?
- In what ways does the teacher extend or stretch pupils who are doing well?

Finally, think about a lesson which you are due to teach soon. How could you adapt the methods that you have noticed to differentiate your own teaching more fully?

INTERROGATING YOUR PLAN

At this point of the planning process you will have written down ideas: for a range of activities, suggested timings and signposts and any other notes which are individual to your class. However, before finishing it is worth going over the plan and checking it.

Checking your plan allows you to know that it is as good as it can be. You could consider the *types of activity* involved. At each stage, what are the pupils doing and what is the teacher doing? Are the pupils are simply sitting and listening for a long time, do you need to plan other activities to promote variety? Another way of interrogating a plan is to consider the *types of*

thinking involved. Are the pupils simply practising a single technique throughout the lesson, or is there space for creative or higher-order thought?

Read through your plan as if you were a particular pupil or group of pupils. How would you respond to this lesson if you were:

- an introverted pupil with low self-confidence?
- a high attaining pupil who completes written work quickly?
- a pupil who cannot concentrate for long periods of time?
- a pupil who struggles to draw charts and diagrams neatly?

You will undoubtedly be able to think of more 'types' of pupils to add to this list, and this is another good opportunity to check that you have planned for differentiation. Make sure that your plan takes account of any relevant special educational needs, and that it addresses any behavioural concerns; if you were a pupil who got bored and disrupted lessons, at which point of the lesson would you be most likely to cause trouble? Finally, if you are a student teacher and have been set any specific targets by your mentor, it is worth interrogating the plan to ensure that you have integrated opportunities to show how you are addressing those targets.

Task

If possible, look back at a lesson plan that you have previously written. Is there sufficient variety in the types of activity planned? Did you integrate different types of thinking? If you have already taught this lesson, does reading through the plan as if you were a pupil help you understand why different parts of the lesson were more or less successful?

PLANNING TO EVALUATE AND IMPROVE

It is easy to neglect lesson evaluations when you are working in school, as they seem less immediately pressing than many other jobs. However, failing to learn from bad lessons may doom you to repeat them, whilst you will want to fix the good ideas in your mind. Evaluation can help you manage personal challenges; it is much more productive to identify problem areas to be developed than to simply label the lesson ‘bad’ and move on.

Start your evaluation by focusing on the learners. Did the learners achieve the objectives/intentions of the lesson? How do you know? If a large enough number of learners did not meet the objective, you might decide to use a different approach or explanation in a subsequent lesson. If all of the learners achieved the objective of the lesson, you might need to consider the level of challenge and the pace of lessons for this group.

You should also evaluate your own choices. Which parts of the lesson went well? Can you explain why they went well? Can this approach be used elsewhere, or developed further? Equally, which parts of the lesson were not as successful? If you were to teach this lesson again, what would you change? How could you support your answers with evidence?

Finally, make sure that you keep a record of ideas that work well. You might find it useful to start a notebook of successful starter and plenary ideas, drawing on your colleagues’ experiences as well as your own. Some teachers keep lessons which they know have worked particularly well for them in the past, so that they build up a library of resources and outline plans which can be adapted year after year.

PLANNING IN PRACTICE

This chapter has talked about how you might plan a single mathematics lesson, but in practice

you will ultimately need to plan sequences of lessons. Before you begin to plan any individual lesson, consider how your plan will fit into a sequence of lessons, noting how new learning will relate to both the mathematics your pupils have learnt before, and also what topics are intended to come next.

You will also need to be mindful of your school's approach to teaching mathematics, as this can significantly affect how mathematics departments structure units or topics. Many schools have a prescribed 'spiral' mathematics curriculum, where each year the pupils revisit topics at a higher level of challenge, and each teacher has a fixed number of lessons to cover an agreed list of topics before moving on. Others though follow a 'mastery' based approach, which usually means teachers have more lessons to present fewer mathematical concepts and procedures in a multiple ways, with the intention of promoting deeper understanding. Likewise, some departments use pre-tests to discover pupils' prior knowledge, whilst others only set tests at the end of each unit. When working in a school you will need to find out about, and work to meet the expectations of your own department.

This chapter has offered you a number of tools and techniques which you might choose to use in your planning and evaluation. It is now your responsibility to select and develop methods which work for you in your own context. Proper planning can prevent poor performance; excellent planning can help a lesson shine.

SUMMARY

In this chapter we invited you to think about the process of constructing a lesson plan and discussed a number of elements of lesson planning:

- using diagrams to define the overall lesson structure;
- observing structures used by experienced teachers;
- selecting ‘effective starters’ for each lesson;
- defining the learning outcomes;
- thinking about variety and the use of plenaries;
- thinking about timing and signposting;
- including homework in your plan;
- differentiating your planning to meet all learners’ needs;
- interrogating, evaluating and improving your plan.