Nonlinear Dynamical Processes in Musical Interactions: investigating the role of nonlinear dynamics in supporting surprise and exploration in interactions with digital musical instruments

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Abstract

Nonlinear dynamical processes play a central role in many acoustic instruments, yet they rarely feature in digital instruments, and are little understood from an interaction design perspective. Such processes exhibit behaviours that are complex, time-dependent, and chaotic, yet in the context of acoustic instruments can facilitate interactions that are explorable, learnable and repeatable. This suggests that these processes merit deeper investigation for digital music interaction design.

Two studies are presented which investigate user interaction with nonlinear dynamical musical tools. A lab-based study used four purpose-built digital musical instruments to test interaction designs featuring nonlinear dynamical processes. Evaluations with 28 musicians demonstrated the potential for these processes to provoke creative surprises, and support exploration without a corresponding loss of control. A subsequent ethnographically-informed study with 24 musicians linked these findings to a mode of engagement which we term 'edge-like interaction'. Edge-like interactions draw on the complex, unpredictable behaviours found in nonlinear dynamical processes close to critical thresholds, facilitating creative exploration.

The two complementary studies provide evidence both for the existing importance of nonlinear dynamical processes in musical interactions with acoustic interactions, and their potential for deployment in the development of new creative digital technologies, musical or otherwise.

1. Introduction

Musical interaction is a challenging but productive field of enquiry for human-computer interaction (HCI). It provides valuable examples of long-term creative tool engagement and can be useful in highlighting specific differences between interaction in the physical domain of acoustic instruments, and interaction in the comparatively recent domain of digital musical tools (Holland et al., 2013). This article explores the

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role of nonlinear dynamical processes in interactions with musical instruments. These processes are viewed as essential elements in interactions with acoustic instruments that are notably absent from most digital musical interfaces. For the purposes of this article, nonlinear dynamical processes are considered to be real-time processes whose evolution over time is determined by a nonlinear function of the current system state and user inputs. Such processes are at the heart of many acoustic instruments (Smith, 2010; Fletcher, 1999; Rodet and Vergez, 1999), and are particularly significant in sustained tone instruments such as blown and bowed instruments (Fletcher, 1999).

Despite their ubiquity in acoustic instruments, nonlinear dynamical processes are rarely investigated in terms of their relevance for interaction design. We argue here that although real-time interactions with these processes can be confusing, unstable, difficult, and unpredictable, they can nevertheless be learnable, controllable, subtle, capable of repeatable effects, and open to deep exploration and development over a lifetime. Further these latter properties are fundamentally bound up with the unstable and unpredictable aspects of interacting with these processes.

Two recent studies are presented that investigate the roles that nonlinear dynamics play in existing musical interactions, and the roles that they might play in digital musical interactions, and in other creative areas of HCI. They show how nonlinear dynamical processes are leveraged in musical interactions, how unpredictable and unstable aspects are productively engaged with and how exploration of the interaction itself becomes an essential part of musicians' creative practices.

The first study explores the use of nonlinear dynamics in four specifically designed digital musical interfaces, investigating how users respond differently to the presence or absence of nonlinear dynamical processes, with a particular focus on attitudes to control, unpredictability, repeatability, and exploration. The findings suggest that nonlinear dynamical processes can be used to devise interfaces that hit a "sweet spot" between deterministic control on the one hand and unpredictability and a scope for exploration and discovery on the other.

For the second study, 24 interviews were conducted with free improvising musicians. These interviews explore participant attitudes to their own instruments and investigate attitudes to unpredictability and exploration in relation to the specific nature of these instruments in their musical practices. Free improvised musical practice provides an insightful area for investigating creative tool interactions, and has been drawn upon in music and HCI research by a number of authors (Bowers and Villar, 2006; Bowers and Hellström, 2000; Martin et al., 2016; Eldridge, 2008; Winkler, 2001). Tools are often used in unusual ways, modified, augmented or built from scratch (Bailey, 1992). Practitioners often seem to deliberately create or seek out unstable interactions with their instrument (Kopf, 1986; Warburton, 2001; Unami, 2005), and place an explicit focus on *exploration* as an end in itself, rather than a means toward other ends (Prévost, 2011). A range of examples of nonlinear dynamical interactions are traced from these interviews, and a specific approach to engaging with these nonlinear dynamical behaviours is elaborated, referred to here as "edge-like interactions" due to the focus on exploring behaviours at critical thresholds.

We will argue that these edge-like interactions provide a key demonstration of how nonlinear dynamical processes can be explored creatively, how they can provide new perspectives on gesture, and particularly the timing of gestures. We show how the rich range of behaviours found close to critical thresholds in nonlinear dynamical processes can be useful for the design of digital tools that seek to support rich, deep, creative interactions that allow for the kind of exploration over long time periods found in acoustic musical instruments.

Section 2 provides sets the context for this research, linking it to work on parameter mapping in digital musical interfaces and highlighting the importance of nonlinear dynamics in acoustic instruments. We also show how nonlinear dynamics can be linked to creative interactions in the non-musical domains of painting and computer games. Section 3 explores specific examples of interactions with with simple mathematical systems nonlinear dynamical systems, showing how critical thresholds can yield surprises, and can become sites for exploration. Section 4 introduces the rationale behind both the lab-based study and the interview-based study, which are then presented in full in sections 5 and 6 respectively. Section 7 brings together the findings from both studies and explores wider implications for interaction design and HCI.

2. Background

From an interaction perspective, the use of nonlinear dynamical processes may seem counter-intuitive: why introduce notoriously complex, unstable, unpredictable systems into musical interactions, where surely nuanced and detailed control is essential?

The limitations of conventional views of control in musical tools, and the benefits of complex interactions, were explored by Hunt and Kirk (Hunt and Kirk, 2000) in their study of mapping techniques. They concluded that while very simple one-to-one correlations between human actions and sonic parameters allowed for a greater degree of accuracy in simple tasks, more complex musical tasks were generally facilitated by more complex many-to-many mappings. While the more complex mappings were less accurate in initial tests, they were seen as allowing for a more "subconscious" level of control, and were seen as more fun, and having greater long term potential. Kvifte similarly suggests that complex mappings are "more interesting and rewarding to use than systems of simple one-to-one mappings," (Kvifte, 2008, p. 353). Hunt and Kirk themselves associate the complex cross mappings with a holistic mode of engagement, which they contrast with a more analytical approach facilitated by simpler one-to-one mappings. They link the former very clearly with exploration, suggesting that this kind of an engagement will allow users to discover "hidden relationships" between parameters. This is contrasted with the focus on goal-oriented, sequential thinking in analytic modes of engagement.

Menzies (2002) takes this research further, exploring linear dynamical control systems, thereby incorporating time-based processes in the interaction. The author points to the complexity of dynamical processes that we engage with in our day to day lives — e.g. inertia, gravity, friction — suggesting that we are generally "riding the system" rather than dictating behaviours completely. Despite highlighting the significance of nonlinear processes in acoustic instruments, the article is limited to implementations of linear dynamical processes in digital musical systems.

Investigating nonlinear dynamical control relationships therefore presents a progression of this logic. These processes bind input and output parameters together into an interrelated whole that makes the systems necessarily complex from a mapping perspective, and incorporate the time-based aspects of Menzies' dynamical processes. The kinds of unpredictability supported by nonlinear dynamical processes can be important drivers in creative engagements. Existing examples can be found in digital musical research and practice: Beyls (1991), Kiefer (2014), Ikeshiro (2013), and Bowers and Hellström (2000) highlight the potential value of nonlinear dynamical processes in musical interactions, and provide examples of digital implementations of such systems. A range of musicians have also directly implemented nonlinear dynamical systems as synthesis procedures (Choi, 1994; Warthman, 1995; Slater, 1998; Dunn, 2007; Ikeshiro, 2013), where the orbits of particular trajectories in the systems are used as audible oscillators.

This article provides empirical support to these creative uses of nonlinear dynamical processes, unpicking how they affect the nature of creative interaction, and how this can be drawn upon in musical interactions and potentially creative interactions in other HCI domains. Nonlinear dynamical processes themselves are considered in more detail in this following section, both in terms of their mathematical behaviours, and their applied use in the study of musical instruments and musical practices.

From a certain perspective, interactions with acoustic instruments can be thought of as simple: there is a clear link between human action and sonic result, and the relationship between the two generally corresponds to our everyday experience of the physical world. When examining the specific nature of these relationships however—and particularly when attempting to create digital models—interactions with such instruments appear rich and complex. Bowed and blown instruments in particular provide very complex relationships between input and output elements. As Maganza et al. (1986) point out, the study of wind instruments is essentially a branch of hydrodynamics. It has frequently been observed in the digital musical instrument research community that if instruments such as the violin or saxophone were created today, they would likely be rejected by designers as overly complicated (McDermott et al., 2013).

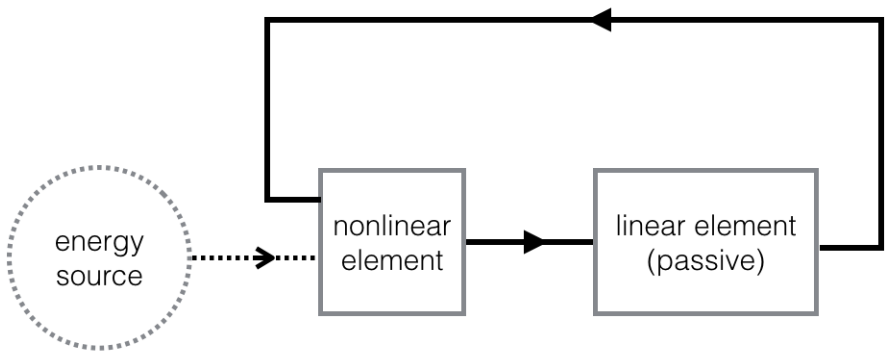

Figure 1: A simplified block diagram description of musical instruments as a coupling of a nonlinear element with a passive linear element, as exemplified by brass, woodwind or bowed instruments (based on Woodhouse et al (McIntyre et al., 1983)).

Figure 1 shows a simple block diagram commonly used to represent most woodwind, brass and bowed instruments (McIntyre et al., 1983; Rodet and Vergez, 1999; Smith, 2010; Kergomard, 2016). The nonlinear element may be part of the bow-string interaction, the behaviour of a reed, the behaviour of an air-jet across a narrow aperture, etc. The linear element represents the response of the string or tube, typically a characteristic resonance that may be altered by the musician by adjusting the effective length of the string or tube. The output from the linear element feeds back into the nonlinear element — e.g. waves reflecting on a string to interact again with the bow, or pressure waves in a tube interacting with the reed — creating a nonlinear dynamical system. Fletcher (1999) makes a useful distinction between "essentially nonlinear" instruments and "incidentally nonlinear" instruments (ibid. p. 725). All sustained tone acoustic instruments are associated with the former (particularly bowed and blown instruments), whilst the latter is associated more closely with struck or plucked instruments (piano, bells, gongs). "Essentially nonlinear" instruments provide clear examples of the critical role played by nonlinear dynamics in musical interactions. While saxophonists and violinists may not necessarily be aware of the nonlinear feedback loop at the core of their instruments, they will certainly have experienced and navigated these processes as part of their everyday musical practices.

Thinking through input-output mappings in such acoustic instruments is therefore complicated. As noted by Hunt and Kirk (2000), there is no straightforward answer to the question "what aspect of the sound is controlled by my breath pressure?" Breath pressure can alter the sounding pitch, timbre, noisiness, and volume of a particular sound. At certain points, a minuscule change to breath pressure may cause a complete jump in pitch and volume, whereas at another point, even fairly drastic changes may not alter the sound to a significant degree. The system exhibits hysteresis, meaning that the state of the inputs at any given point in time is not in itself enough to determine the sounding result. The system has a memory, which has been influenced by prior inputs[1]. An input's trajectory over time and its rate-of-change therefore become significant aspects of the interaction.

An interesting case study that bridges the gap from the behaviour of acoustic systems, to nonlinear dynamics in electronic tools, is provided by microphone-loudspeaker feedback (or guitar pickup-loudspeaker). This interaction can very clearly be understood as a nonlinear dynamical process, with the feedback loop being made very obvious, and distortion providing the nonlinear function. Feedback has been a popular technique that has been explored in various forms of contemporary music (Sanfilippo and Valle, 2012; Waters, 2007; Scipio, 2003).

Considering interaction from the perspective of nonlinear dynamical processes can be instructive in other domains. Two examples are considered here to show how nonlinear dynamical aspects can be drawn on in the diverse activities of painting and playing computer games, and how surprise and exploration can be key elements in both activi-

[1] Overblowing on a reed instrument provides a simple example of this phenomenon. A musician can provide a particular breath pressure to the instrument, which results in a particular tone; when the pressure is increased past a particular threshold, the tone leaps to a higher harmonic. The breath pressure may then be reduced below the threshold to the original pressure level,without the tone dropping back to the original lower harmonic, demonstrating that the same input given at different times leads to very different outputs (McIntyre et al., 1983).

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The behaviour of the bristles of a paintbrush is nonlinear and dynamic (Chu and Tai, 2002): as pressure is applied to the brush the bristles act like springs, interacting with each other to determine where and how they come into contact with the canvas. The paint itself is also a highly complex material, the modelling of which is often based around the nonlinear dynamical Navier-Stokes equation (see Curtis et al. (1997) or Laerhoven and Reeth (2005) for example models). The behaviour of non-newtonian materials such as oil and acrylic paints provides further complexities to these nonlinear dynamical behaviours (Chhabra, 2010). A simple swipe of the paintbrush against the canvas is therefore a highly nuanced interaction, with both the current and past brush position and pressure bound up with the complexities of the bristles, paint, and canvas. As with acoustic musical instruments, the subtleties of these complexities are intimately bound up with the nature of creative practices with these mediums. The nonlinear dynamical nature of the interaction makes the pacing and timing of gestures into critical aspects and allowing for endless variation, refinement and exploration.

Physics based computer games are necessarily dynamic in nature, and various nonlinearities can be identified that contribute to engaging behaviours. As an example driving games can be considered. In general, cars in driving games are not under the direct control of the user, in that the user doesn't specify the vehicle coordinates or even its velocity. Rather, acceleration is applied to begin to increase or decrease the speed, and directional controls are used to turn the steering wheel. However, whether the car successfully turns left or right is determined not only to these current inputs, but also by the current state of the car. If the car is turned too quickly for example, it may skid or "drift" - the more positive term used in many games where skidding becomes an encouraged element[2]. In the skid, the user may be orienting the wheels to the left, despite the car's movement in the other direction, and the car itself may be moving almost sideways[3]. Again the timing of the user's actions are critical: easing off the accelerator just before turning into a corner may yield vastly different results to ploughing through at full speed. Nonlinear dynamics are also exploited to creative ends in encouraging stunts, as in games such as Grand Theft Auto or Burnout. Engagement with the dynamics becomes a creative pursuit, where gracefulness, drama, chance and even humour may play a part.

3. Surprise and Exploration with Nonlinear Dynamical Systems

Nonlinear feedback systems have the potential for a wide range of behaviours, even where the initial equations are relatively simple. In chaotic systems very small adjustments in input parameters can lead to highly divergent outputs. Perhaps more significantly from an interaction perspective, the potential for hysteresis in nonlinear dynamical systems means that the behaviour of the system is dependent not only on the

[2] e.g. Need For Speed: Underground (https://www.youtube.com/watch?v=tSXp4cV9x5g), and the Grid series (https://www.youtube.com/watch?v=EZ8D6qnqPq0)

[3] See also Linson (2011), who explores drifting as an example of an interaction that is intuitive to experts and counter-intuitive to beginners, examining the ramifications for the design of digital musical instruments.

current input, but also on the current state of the system (Lakshmanan and Rajasekar, 2003, p. 23), and therefore prior inputs to the system can also be highly significant in determining the output.

The logistic, Lorenz and Duffing systems are all standard examples that have been explored thoroughly in the literature around nonlinear dynamics. This section looks at specific phenomena in these nonlinear dynamical equations. A range of complex, interactive behaviours that emerge close to critical bifurcation thresholds are described to highlight how interactions with these systems might be both surprising, and open to creative exploration. This section may be skipped for those less interested in the specific behaviours of the mathematical systems.

3.1. Orbit hopping in the Lorenz attractor

The standard formulation of the Lorenz attractor is shown below (Equation 1. The system can be thought of as interactive, where the user controls the values of the three coefficients σ, ρ and β. When ρ is pushed past a certain threshold, trajectories may move back and forth between the two spiral attractors shown in Figure 2 in a deterministic but (in practice) unpredictable manner. When ρ is then reduced sufficiently, the trajectory will settle into a particular orbit around one of the two spiral attractors. Which attractor the trajectory settles to is governed by the state of the specific trajectory under consideration at the point where the system bifurcates, and therefore, the previous inputs to the system. When interacting, a user can effectively attempt to hop a particular trajectory from one spiral attractor to the other by increasing ρ past the critical threshold, allowing the trajectory to cross to the opposite attractor, and then reducing ρ sufficiently that the trajectory stays in the attractor.

$$\begin{aligned} \frac{dx}{dt} &= \sigma(y-x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z \end{aligned} \tag{1}$$

This provides an example of hysteresis. While hysteresis may occur in less perceptually obvious situations (e.g. the variations in scratch tones in a bowed interaction), it can also manifest in more dramatic ways, such as in the clarinet overblowing example discussed above, where a performer can increase breath pressure to the point where the system jumps to a higher harmonic, and then reduce the pressure again retaining the higher harmonic. Thus, for a particular given input from a musician—however precise—there are different possible sonic results depending on past activity.

3.2. Period doubling and intermittency

The logistic system provides an example of increasing complexity close to a definitive threshold, beyond which chaotic behaviour may occur (at $r = 3.56995...$ in Equation 2).

$$\frac{dx}{dt} = rx(1-x) \tag{2}$$

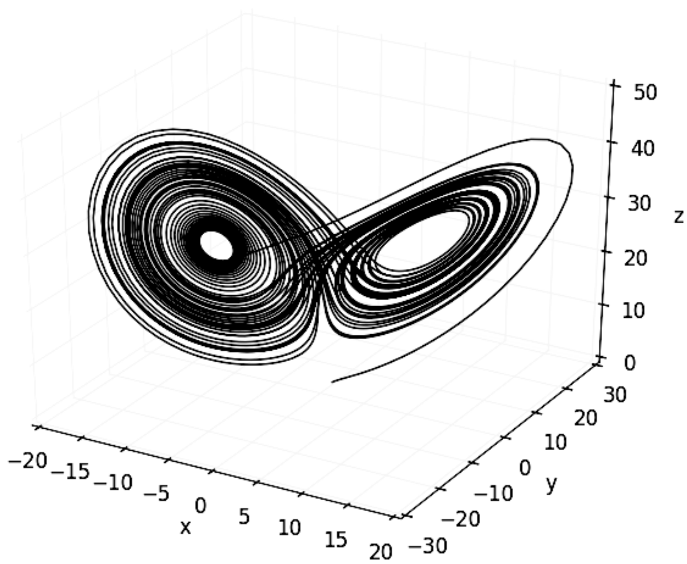

Figure 2: Lorenz attractor with $\rho = 28, \sigma = 10, \beta = 8/3$

This is an example of the period doubling route to chaos, where there are a rapid series of period doubling bifurcations, close to the threshold of chaos for the control parameter, r. As the critical threshold is approached, the period doubling bifurcations are closer together (Lakshmanan and Rajasekar, 2003, p. 97), presenting a highly sensitive control region just prior to the onset of chaos. The period doubling route to chaos can be found in a variety of musical situations, such as wind instruments (Gibiat and Castellengo, 2000), bowed interactions (Inácio, 2008, p. 135–137) and the voice (Neubauer et al., 2004).

A second route to chaos is termed *intermittency* (Lakshmanan and Rajasekar, 2003, p. 132). As a system parameter is pushed close to a threshold for chaotic behaviour, the system may exhibit intermittency behaviour, where almost periodic trajectories are interrupted by occasional, unpredictable bursts of chaotic behaviour (ibid. p. 130, and see also Pomeau and Manneville (1980) for intermittency in the Lorenz system). As the threshold for chaos is approached, the chaotic bursts become more frequent.

3.3. Complex behaviours following the disappearance of an attractor

Grebogi et al. (1987) describe the behaviour of trajectories as chaotic attractors undergo sudden discontinuous changes in response to alterations to system parameters. When a chaotic attractor is destroyed by moving past a critical threshold—as seen in a saddle-node bifurcation for example (Kapitaniak et al., 2000)—a particular trajectory may remain in the region of the chaotic attractor for an indeterminate amount of time before jumping to another attractor (potentially stable or chaotic):

> “they are initially attracted to the phase-space region formerly occupied by the attractor [...] they then bounce around in this region in a chaotic

> fashion, which, for most purposes, is indistinguishable from the behavior of orbits on the chaotic attractor; finally, after behaving in this way for a possibly long time, they suddenly move away from the region of the former attractor (never to return) and approach some other attractor. The length of time an orbit spends on the remnant of the destroyed chaotic attractor depends sensitively on its initial condition" (ibid. p. 5366)

The authors also describe situations where a trajectory may intermittently fluctuate unpredictably between the old region and the new region made available by crossing the critical threshold.

The examples given above provide only a small selection of possible behaviours that exist around critical thresholds. They nevertheless give an idea of the richness and complexity of interactions that may exist, the role of hysteresis, and the necessary difficulty in having complete and accurate control close to these thresholds. These examples of complex interactions are demonstrated in mathematically simple nonlinear dynamical systems, but it seems likely that the complexity of interaction will be magnified when considering the more complicated situations found in real acoustic instruments. Further, the examples above are generally only considering changes to a single system parameter. As (Lakshmanan and Rajasekar, 2003, p.130) point out in relation to the Duffing oscillator, "one can change not only the parameter [γ] but also [ω], for example, in [Equation 4 (see below)]. Then a very complex picture of bifurcations phenomena, present even in such a simple nonlinear system as the Duffing oscillator, can be realized". These properties show how such systems can yield surprises and how they permit significant exploration in even very narrow control regions.

4. Methodology Overview

The following sections outline two studies that were conducted to explore how musicians encounter and engage with nonlinear dynamics. The first study examined this question in relation to purpose-built digital musical systems, while the second examined the question in the context of musicians' existing practices with their instruments.

This provides a mixed-methods approach that triangulates between lab-based studies where the specific implementations of nonlinear dynamical systems can be controlled and understood, and ethnographically-informed interviews that draw on the rich reality of musical practice. Each method can help to fill out aspects that are lacking in the other, and overlapping results between them strengthen the findings of both studies. As the roles played by nonlinear dynamics in musical instruments and interactions are relatively poorly understood at present, this is seen as preferable to attempting very specific, quantitative studies on the one hand, or very broad, ethnographic studies on the other.

The lab-based study is presented first in Section 5, followed by the ethnographically-informed study in Section 6. Results from both studies are then discussed more generally in Section 7.

5. Lab-based study

The first study was conducted in a lab-based setting, allowing particular implementations of nonlinear dynamical processes to be tested from a user interaction perspective. The study provides a mixture of quantitative and qualitative perspectives on musical engagements with these particular implementations. Aspects of this study have been reported in Mudd et al. (2015a), but are examined here in more detail, and reinterpreted in the light of the ethnographically-informed study described below in Section 6.

5.1. Lab-based study methodology

Four digital musical interfaces were specifically designed for this study and presented to 28 participants of differing musical backgrounds. These interfaces were designed to test a key aspect of interaction with nonlinear dynamical processes: their behaviour close to critical thresholds. As the examples in Section 3 have shown, these boundary points, where the system may change abruptly, appear to play a significant role in user interactions with such processes.

This study investigates whether the nonlinear dynamical nature of these boundary points is significant, or whether boundary points created by other means would have a similar effect on user interaction. Each of the interfaces created for the study can be divided into three elements: a set of physical MIDI inputs (two dials and a fader), an audio engine, and a mapping relating the input dials to the parameters of the audio engine. The two independent variables in the study are, firstly, the nature of the mapping (continuous or discontinuous), and secondly, the nature of the audio engine (which can either include nonlinear dynamical processes or not). Table 1 shows these combinations.

The study looks at how changes to these variables affect the ways that musicians perceive and engage with with the interfaces. In particular, the study looks at:

1. perceptions of control,
2. whether participants feel as though they can repeat/re-access particular sounds and behaviours,
3. perceptions of unpredictability,
4. whether participants feel that they can explore and discover new sounds or behaviours.

Each participant was required to spend a period of 4–8 minutes practising with a given interface before making a short recording of 1–4 minutes, which they were asked to think of as a performance: an attempt to create music that they would stand behind aesthetically. Participants could determine their own durations within these limits, but the software restricted the ability to progress to the next stage before the minimum times had elapsed. Data from the input dials and slider was logged from both practise and recording sessions. Participants then answered a range of Likert scale questions providing feedback on their experience of using the particular interface, before repeating the process with the remaining interfaces (counterbalanced by randomising the order).

Interface	nonlin. dynamics	mapping	audio engine
1	Yes	Continuous	Resonated Duffing Osc
2	Yes	Discontinuous	Resonated Duffing Osc
3	No	Discontinuous	Resonated Osc
4	No	Continuous	Audio Sample-Based

Table 1: The four interfaces used in this study

Once the process had been completed with all four interfaces, participants provided information on their musical background, and conducted a short, semi-structured interview, allowing them to expand on and contextualise their Likert responses, to comment more fully on their experience with each interface, and to relate these experiences to their own musical practices. This methodology was tested with 4 participants in an initial pilot study (Mudd et al., 2015b).

5.2. *The interfaces*

Each of the four bespoke interfaces generated sound in real-time based on the user input from the fader and dials. They represent the four possibilities presented by the different states of the two independent variables: the inclusion of nonlinear dynamical processes as a core element of the audio engine, and the continuous or discontinuous nature of the mapping from the dials/fader to the audio engine (see Table 1). The inputs are 7-bit MIDI, allowing 128 discrete states for each input.

5.2.1. *Interface 1: Nonlinear dynamics with continuous mappings*

Both interfaces 1 and 2 include a nonlinear dynamical process at the core of the sound generation system. They are based around the damped forced Duffing oscillator (Guckenheimer and Holmes, 1983; Thompson and Stewart, 1986), shown below in Equations 3 and 4 as a discrete map. This equation is implemented as a discrete map at audio sample rate (44.1 kHz) and coupled with a set of resonators such that the x_n term is passed through the filter bank, and the output of the filter bank is used in its place in the above equation. This combination of a nonlinear function coupled with a linear resonator bears a close resemblance to the structure of many acoustic instruments (McIntyre et al., 1983) and hence to many physical models (Smith, 2010). The specific structure of interface 1 is shown in Figure 3. The mappings from the three inputs are all continuous functions, controlling γ, α, and the filter gain and resonance.

$$x_{n+1} = y_n \tag{3}$$

$$y_{n+1} = -\delta y_n - \beta x_n - \alpha x_n^3 - \gamma cos(\omega t) \tag{4}$$

5.2.2. *Interface 2: Nonlinear dynamics with discontinuous mappings*

Interface 2 differs from interface 1 only in terms of the mapping from the MIDI controls to the system parameters: interface 1 uses continuous mappings, whilst interface 2 uses discontinuous mappings that cause jumps in the parameters at particular

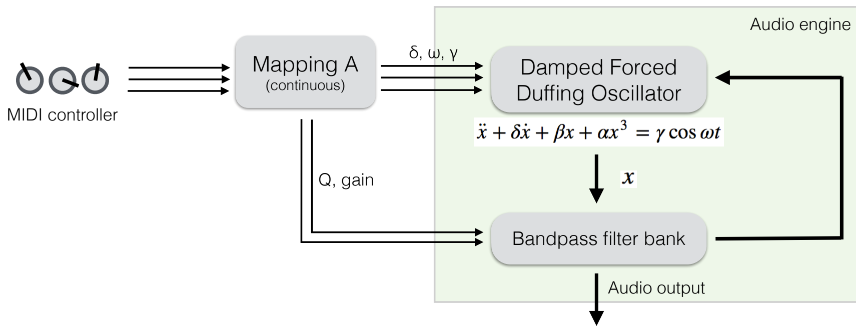

Figure 3: Interface 1 is comprised of a damped forced Duffing oscillator coupled with a bank of linear resonators. The user interacts with the system via three MIDI controls.

points. In other respects this interface is the same as interface 1, and is therefore a nonlinear dynamical process controlled via discontinuous mappings as shown in Table 1.

The mappings from the three inputs use lookup tables to provide discontinuities at various points in the dials' ranges (Figure 4 provides an example). The mapping is continuous in certain regions, but abruptly jumps from one continuous region to another at certain thresholds. The mappings are many-to-many, in the sense used by (Hunt and Kirk, 2000), as multiple inputs may influence a single parameter, and multiple parameters may be influenced by a single input.

5.2.3. Interface 3: Static system with discontinuous mappings

Interface 3 is very similar to interface 2, but with the Duffing system removed as shown in Figure 5. There are no nonlinear dynamical processes and the audio engine non-dynamical in that its behaviour no longer depends on prior states, only the current input (the term *static* is used here to indicate the absence of nonlinear dynamical processes). The result is a single oscillator passed through a bank of narrow bandpass filters. The discontinuous mapping is retained from interface 2, controlling changes to the bandpass filter frequencies. Although the system is in some ways similar to interface 2 and to a lesser extent interface 1 in terms of the processes involved, the range of possible sounds is very different due to the removal of the nonlinear dynamical processes. Although the sinusoidal oscillator is relatively unchanged by the filtering (other than volume changes), the discontinuous changes to the bandpass filter frequencies causing

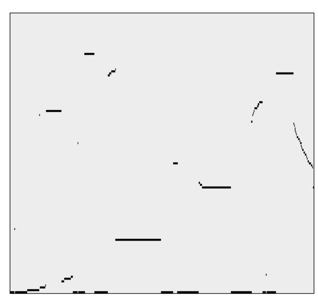

Figure 4: Lookup table that maps input dial 1 to γ in a discontinuous manner in interface 2. The mixture of stable areas, abrupt changes and semi-linear areas simulates some of the interactions that can be experienced with nonlinear dynamical systems, but without a dynamical component.

resonant clicks.

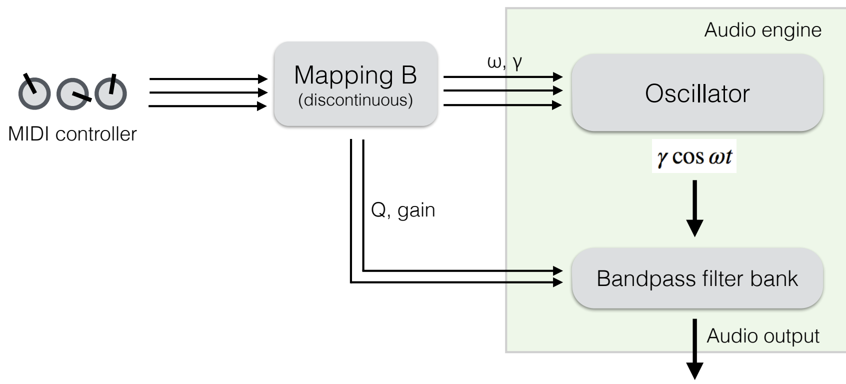

Figure 5: Interface 3. In the third interface, the Duffing oscillator and feedback are removed, leaving an oscillator and resonant filter bank. The discontinuous mapping is otherwise preserved from interface 2.

5.2.4. Interface 4: Static system with continuous mappings

Interface 4 attempts to preserve the sound world of the Duffing systems and is therefore based around a two minute audio file recorded from interface 1. The system is therefore not a nonlinear dynamical system, but retains a very similar sound world to interfaces 1 and 2. The inputs are mapped continuously to positions in the sample, playback rate and overall volume. Granular processes are used to separate the control of playback rate from the control of the pitch of the sample.

5.2.5. Data collection and analysis

Data from the input dials and fader was logged from both the practice and recording sessions. After using each interface, participants completed a short questionnaire that asked them to rate their level of agreement with the following four statements on a five-point Likert scale (emphasis on the key terms added here for clarity):

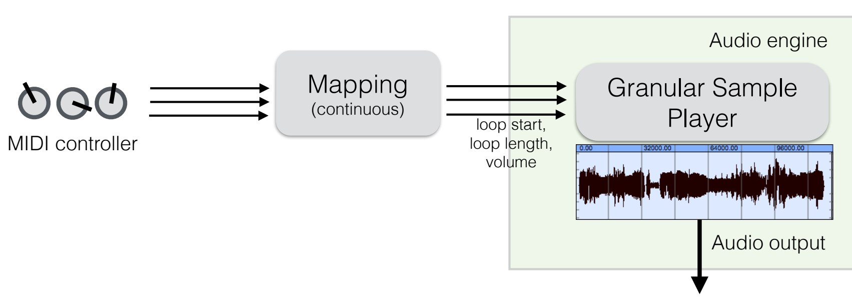

Figure 6: Interface 4. The fourth interface uses a recording of interface A in order to create a system that sounds very similar but behaves very differently.

1. "I felt in **control** of the sound"
2. "I found it straightforward to **recreate** particular sonic events"
3. "I was often **surprised** by the instrument's response"
4. "I feel that there are many areas that I could still **explore** and discover"

After completing this process with all four interfaces, participants provided information on their musical backgrounds, and took part in a short, semi-structured interview. This interview allowed the musicians to expand on and contextualise their Likert-scale responses, to comment more fully on their experiences with each interface, and to relate these experiences to their own musical practices

5.3. Lab-based study results

Analysis of the questionnaire responses highlighted two important results, shown in Figure 7. Firstly, that the interfaces incorporating nonlinear dynamical processes were seen as significantly more surprising, and providing a greater scope for exploration and discovery (determined by Wilcoxon signed-rank tests with $Z = 77$, $p < 0.01$ and $Z = 80$, $p < 0.025$ for surprise and exploration/discovery respectively). Secondly, that continuous rather than discontinuous mappings correlated to a sense of control and a sense that particular events could be easily recreated ($Z = 104$, $p < 0.01$ and $Z = 197$, $p < 0.01$, respectively).

These results support the idea that the presence of nonlinear dynamics can enable surprising, explorative interactions. The negative results are perhaps more illuminating however: discontinuous mappings had no significant affect on the participants' sense of surprise ($Z = 149$, not significant) or exploration ($Z = 227$, n.s.), and — perhaps more importantly — that the presence of nonlinear dynamical processes did not significantly affect perceptions of control ($Z = 254$, n.s.) or repeatability ($Z = 365$, n.s.).

This suggests that although nonlinear dynamics can be productively unruly and unpredictable, this is not to say that they are inherently uncontrollable. This ties in closely with the descriptions of blown and bowed acoustic instruments as nonlinear dynamical processes: they can be very difficult and unpredictable initially, but that doesn't mean that they are uncontrollable.

The interviews provide some useful contextualisation for these results, particularly in painting a more nuanced picture of how surprise and unpredictability may manifest in musical interactions, and how this may affect engagement. Several participants felt that certain surprises were arbitrary, and therefore frustrating and limiting. One participant made a particularly useful distinction:

> "What I want is a surprise that leads somewhere, rather than a surprise that's a dead end."

This distinction may help to explain the fact that the interfaces with discontinuous mappings were not seen as especially surprising or explorable compared to those with continuous mappings. Although the abrupt changes in the output at specific points in the inputs may be initially surprising, the threshold crossing will nevertheless be

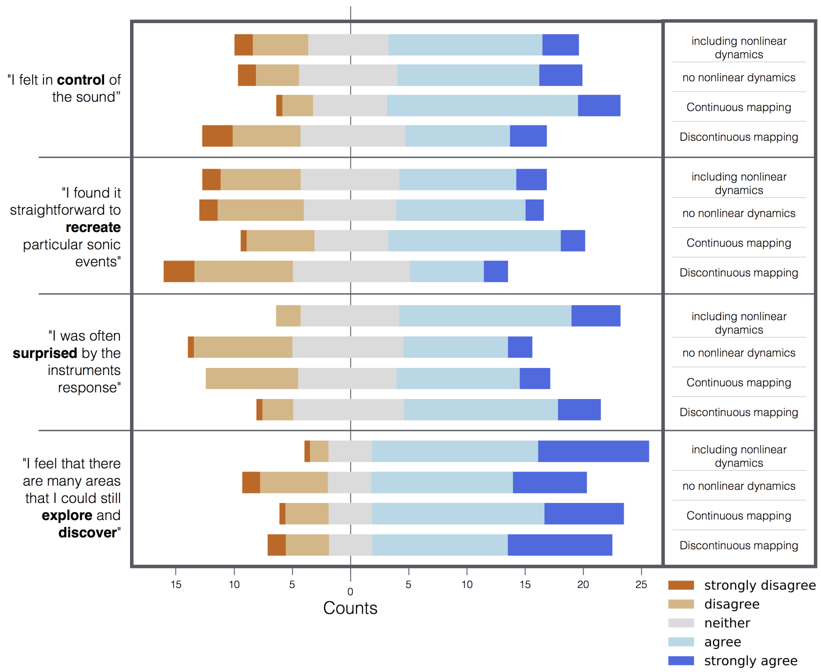

Figure 7: Participant agreement with four statements grouped according to whether they included nonlinear dynamics or not, and whether they used a continuous or discontinuous mapping. The key findings can be seen in the divergence between responses relating to *control* and *recreating* events for the alterations to the mapping, and the divergence between responses relating to *surprise* and *exploration* for the inclusion or removal of nonlinear dynamics.

exactly the same each time, and therefore cannot be explored in the same way as a boundary in a nonlinear dynamical system. The discontinuous boundaries merely limit the bounds of different areas of more continuous interaction. Several participants explicitly reported that the abrupt changes inhibited their musical exploration:

- “when I got to a certain point when I felt like I was attuned to searching and playing with the sounds one tiny movement of the hand would sort of kill it or disrupt it.” (interfaces 2 and 3)
- “it was in a sense not satisfactory either because during the recording I made I fell onto a thing that [...] was really not very nice, I was very surprised about, so I was a bit disappointed because I was just happy exploring something nice and it completely fell off!” (interface 2)
- “each one of the gestures seemed to have a limited range, whereas the other ones, micro movements could create some very interesting relationships” (interface 2)

Thresholds in nonlinear dynamical processes can behave very differently. The changes that occur when thresholds are passed (and the locations of the thresholds themselves) are contingent on the state of the system as determined by prior inputs, and therefore open to considerable exploration. The complexity of behaviour often found close to critical thresholds may also be an important factor in signposting the proximity of a boundary, so that as a user can, to an extent, anticipate the boundary even though they may not be able to anticipate the behaviour around and beyond that boundary. Nonlinear dynamical processes therefore seem to support productive surprises in musical interactions, supporting interactions that are both deterministic, controllable and repeatable in a certain sense, but are nevertheless chaotic, explorable, and difficult to predict completely.

The insights into surprise and exploration in musical interactions established in this first study prompted the desire to investigate instrumental interactions more directly in existing musical practices. The ethnographically-informed study presented in the following section therefore examines attitudes to surprise and exploration in relation to nonlinear dynamical processes in the existing interactions between musicians and their instruments.

6. Ethnographically-informed study

A set of semi-structured interviews were conducted with 24 musicians whose practices incorporate free improvisation in some way. Free improvisation provides a site where exploration is often placed at the centre of the practice, whether this is in private rehearsals or in public performances (Bailey, 1992; Prévost, 2011; Cardew, 1971). The first author has spent a significant amount of time engaged in free improvised music as a performer, concert organiser, and researcher. While this precludes the possibility of a more disinterested, external standpoint, the experience of having spent time embedded in the community helps to provide insights that can be beneficial in navigating interviews with improvisers and interpreting the data.

Thematic analysis has been used to code and analyse the interview data, with themes and codes being determined in part by the specific interest in surprise, exploration and nonlinear dynamics, and in part influenced by the interview subjects responses.

The findings support and contextualise the results of the lab study, showing how nonlinear dynamical processes can play a role in surprise and exploration. The interviews provide concrete examples in participants' practices of unpredictable interactions with nonlinear dynamical processes in acoustic and electronic instruments, and examples of how interaction close to critical thresholds can lead to the discovery of new areas for exploration. A distinction is drawn between surprises that result from nonlinear dynamical processes and surprises that come about as the result of chance processes. A mode of interaction referred to here as "edge-like" is characterised from the participant descriptions of exploring unpredictable, unstable areas of interaction close to critical thresholds. Participants provided qualitative descriptions of working in this way, giving insights into how they use these threshold regions in their musical practices.

6.1. Participants

The 24 participants were recruited for the study in order to include a broad range of tools and instruments (shown in Table 2). 22 of the 24 participants were London-based musicians. Whilst this was partly a practical aspect of conducting the study, London contains a highly diverse range of free improvisation practices, and incorporates a wide variety of attitudes and approaches to instrumental interactions.

6.2. Interviews

Semi-structured interviews were conducted individually with each participant, focusing on their particular performance practices. The structured questions listed below attempt to draw out attitudes to surprise, exploration and control, and to encourage the participant to consider these elements in relation to their specific musical tools and instruments. Interviews were conducted in person, or via remote video connection. Audio from the sessions was recorded and transcribed for subsequent analysis.

1. What tools and instruments do you use in your practice?
2. Could you describe the role of exploration in your performance?
3. Are you often surprised by your instrument/tools (as appropriate)?
4. Is there anything that has been in your mind during this interview that has not been said, or anything that you wish to add?

Where possible, a video or audio recording of the participant performing was found in advance, and certain sections identified where it seemed likely that the participant was dealing with material that was in some way surprising, or otherwise seemed to be exploring some kind of unknown territory. This provided a very concrete situation for the participant to talk about, and even if the musical sections selected proved not to include surprising situations, they could still provide significant insights into the participant's thought process when playing.

Participant	Acoustic Instruments
A	Objects
B	Piano / objects
C	Cello
D	Trombone
E	Saxophone
F	Violin
G	Double bass
H	Double bass
I	Viola
	Electronic Instruments
J	Laptop / samples
K	Electronics / theremin / radios
L	Laptop / radios / samples / electronic objects
M	Modular synthesiser
N	Modular synthesiser / laptop
O	Sine tones (laptop)
P	Digital feedback networks
	Mixed Electronic and Acoustic
Q	Flute / electronics
R	Objects / voice / effect pedals
S	Saxophone / objects / electronics
T	Piano / samples / objects / effect pedals
U	Wine glasses / objects / effect pedals
V	Objects / laptop sampling
W	Violin / drums / laptop sampling
X	Electric guitar / feedback / objects

Table 2: List of tools used by the ethnographically-informed study participants, categorised as acoustic, electronic or mixed for each participant.

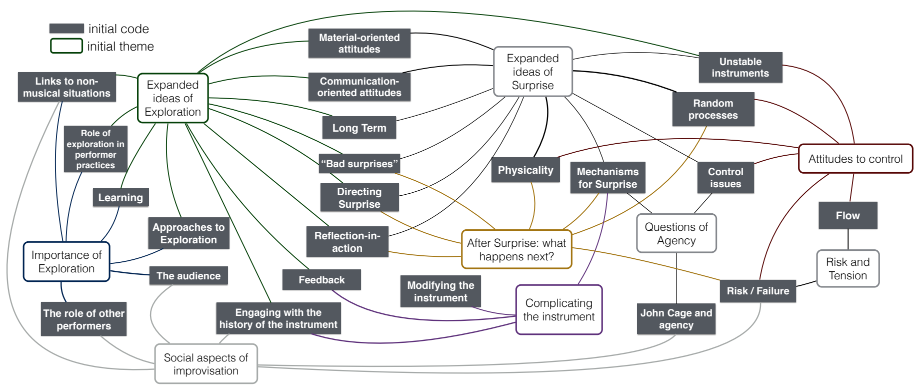

Figure 8: Initial coding map showing initial codes and themes.

6.3. Thematic analysis: themes and codings

The interview data was coded with a view to investigating issues around surprise and exploration and relating these aspects back to the nature of the musical tools employed by the various participants. In this sense, the approach to thematic analysis is deductive, in the sense used by Braun and Clarke (2006): codes and themes are developed with particular areas of interest in mind. However, the semi-structured nature of the interviews enabled participants to discuss topics beyond these central areas, allowing for unforeseen considerations to be explored. Figure 8 shows the range of topics that emerged from the initial coding[4].

The thematic analysis provides a range of perspectives on how exploration and surprise relate to the specific nature of the instruments and tools being considered. Themes relating to exploration and surprise are examined below, and a particular approach to engaging with nonlinear dynamical aspects of musical instruments, *edge-like interaction*, is identified and characterised.

6.4. Exploration and Surprise

The interview data provides both support for the lab-based findings—that nonlinear dynamical processes can provide scope for both surprise and exploration without seeming to have a negative impact on a sense of control or repeatability—and a broader context for understanding these results. Data from the interviews relating to exploration and surprise is shown below, and related back to the participants' relationships with their instruments.

6.4.1. Exploration and nonlinear dynamics

Participants were explicitly asked how exploration features in their musical practices. The responses were grouped into four categories as shown in Table 3. Most

[4]A more complete breakdown of the codes and themes, along with examples of each can be found in Section 5.3 of the first author's doctoral thesis (Mudd, 2017)

participants (15 of 24) felt that exploration was a key aspect in their practices, both in performances and rehearsals. Four participants placed some importance on exploration, but were unconvinced about whether it was genuinely possible to set out to explore and find new things, or that the explorative aspect of their practice was a consequence of following a different train of thought rather than a direct intention at the outset. For two participants the question did not seem to fit, or their answers didn't provide a clear position on exploration. Finally, three participants deliberately distanced themselves and their practice from the term, all of whom detailed a more compositional approach to performance and improvisation (e.g. participant W described performance as a "crystallisation of previous exploration").

The importance attached to exploration by the participants is not particularly surprising given the emphasis placed on *searching* by established improvisers such as Eddie Prévost and Cornelius Cardew, both of whom appear to have influenced many of the participants.

Table 3 lists these contrasting attitudes to exploration alongside the instruments that the participants play. It is notable that all of the wind instrument players (flute, trombone, and two saxophones) are in the top category (exploration plays an important part in their performance practice), along with five out of six (83%) of the bowed instrumentalists (cello, viola, violin, two double basses). Eight out of nine (89%) of the participants who had reservations about the term 'exploration' used electronics in some way, compared to only six out of the fifteen who placed exploration at the centre of their practice (40%).

It is tempting to conclude, in the context of this research, that there is an association between the "essentially nonlinear" character of bowed and blown instruments (see Section 2 above) and a focus on exploration, that the instruments in some way suggest or reward exploratory approaches or conversely that players interested in exploration are drawn to instruments of this kind. This is consistent with the data from the lab-based study which showed that nonlinear dynamical interactions appeared to afford exploration and discovery. There are however, a wide range of other social and cultural factors that may account for this association, and the specific sampling of participants from experimental musical communities for the study will likely mean that the results may be specific to attitudes in that community.

6.4.2. Surprise and nonlinear dynamics

The study participants provided a wealth of examples of situations in which they were surprised by their instrument in some way. These examples were coded together under the heading "mechanisms for surprise". As with the attitudes to exploration described above, many participants reported enthusiasm for being surprised not only in practice or rehearsal situations, but also in performance. The examples have been split into three categories: surprise as a result of the nonlinear dynamical properties of the instrument (Table 4), surprise due to a lack of physical control or due to an inability to understand the complexity of a tool being used (Table 5), and surprise due to random (or effectively random) processes (Table 6). These categories are not mutually exclusive and some examples are in more than one category. None of the surprises noted here were brought up as troubling or problematic by the participants, but were generally seen in a positive or, at worst, neutral light.

Exploration is important		
Participant	Instruments and Tools	Category
C	Cello	Acoustic
D	Trombone	Acoustic
E	Saxophone	Acoustic
F	Violin	Acoustic
G	Double bass	Acoustic
H	Double bass	Acoustic
I	Viola	Acoustic
J	Laptop, samples and objects	Mixed
L	Laptop, radios and samples	Electronic
O	Sine tones	Electronic
P	Digital feedback networks	Electronic
Q	Flute and electronics	Mixed
S	Saxophone	Acoustic
T	Piano, samples, objects and pedals	Mixed
X	Electric guitar, feedback and objects	Mixed
Exploration is important with some caveats		
Participant	Instruments and Tools	Category
B	Piano and objects	Acoustic
K	Electronics, theremin and radios	Electronic
M	Modular synthesiser	Electronic
N	Modular synthesiser and laptop	Electronic
The importance of exploration is unclear		
Participant	Instruments and Tools	Category
A	Objects	Acoustic
R	Objects, voice and pedals	Mixed
Distanced themselves from the term		
Participant	Instruments and Tools	Category
U	Wine glasses, objects and pedals	Mixed
V	Electronic objects and laptop sampling	Electronic
W	Violin and laptop sampling	Mixed

Table 3: The importance of exploration for participants using different tools, broken into four categories

The categorisation is intended to illuminate commonalities and contrasts between the categories. A significant difference between the examples of surprise that emerge from chance processes, and those that emerge from nonlinear dynamical processes, is how performer *control* is manifested in relation to the surprises. With the examples of chance processes provided by participants, little can be done to deliberately develop, expand on, or engage with the surprise-producing mechanism. The nature of the surprising interaction is effectively uncoupled from the actual details of the performer actions, and subtle variations in how these actions are conducted will not be reflected in the outcomes (e.g. in the use of radios, randomised audio file selection).

By contrast, the surprises resulting from nonlinear dynamical interactions, can generally be developed, varied, potentially repeated. The surprising situation may be harnessed to the point where it is no longer a surprise (e.g. participants K, L, Q, P, T and X working with feedback processes), or developed to the point where there is a strong sense of control but still within a volatile situation (e.g. the unpredictable bowed string and reed interactions described by participants C, D, E, F, G, I and W). The contrast between nonlinear dynamical surprises and chance surprises supports the point made above in relation to the lab-based study, that nonlinear dynamics can be both unpredictable and explorable, whilst still being controllable in some sense. The examples of nonlinear dynamical surprises in Table 4 all seem to fit in well with the participant quote from the lab study noted in Section 5.3: “what I want is a surprise that leads somewhere, rather than a surprise that’s a dead end.” This is also a key aspect of the “edge-like” interaction in the following section.

Many of the situations in which mental and physical restrictions lead to surprises can also be explored and developed. There is a considerable overlap between surprises in this category and in the nonlinear dynamical category. For example, participant R, working with voice, talks about repeating a single word or phrase until exhaustion necessarily forces changes and developments in the sound produced. However, the behaviour of the voice relies significantly on nonlinear dynamical processes (Neubauer et al., 2004), so the specifics of the changes in the voice, and the potentially surprising behaviours might stem from the nonlinear dynamical nature of the situation, rather than simply the physical limitation.

6.5. Small-scale surprises

The term surprise was sometimes seen as ambiguous, and participants would generally have to unpack what it meant before agreeing that it played a part in what they do. The word ‘unpredictable’ often proved a better fit for situations where the participant would have a relatively clear idea of what might emerge from a particular action or process, but some aspect or small detail might not be determined. An example brought up by participants engaged with bowed interactions (e.g. I, B, G) shows this well: when starting to bow a string or another object, even relatively experienced participants confessed that they may not know exactly how the string will initially react, which harmonics will be prominent, how noisy or how clean the note will be, and so on. But it is perhaps too strong to term the result a surprise, despite the somewhat unpredictable outcome. Once the interaction has begun, the participants could then work with whichever specific sounds and behaviours emerged, and could begin to guide the sound in a much more focused way. This kind of interaction can be linked to other attitudes expressed by the participants, particularly a view of the instrumental interaction as a 'dialog' or 'conversation'.

6.6. "Edge-like" interactions

A particular interaction strategy—"edge-like" interactions—cuts across many of the themes shown in Figure 8:

- expanded ideas about surprise and exploration,
- particular attitudes to control,
- the deliberate complication of the instrument or the interaction with the instrument,
- risk and tension,
- questions of agency.

The term "edge-like" was explicitly introduced by participant K, but fits descriptions given by many of the other participants (C, E, F, G, K, L, M, N, P, Q and T). Participants expressed a particular interest in pushing their tools into states where control becomes difficult; where the tools exhibit a range of unpredictable and emergent behaviours, and have the potential for relatively abrupt changes or even failure. Participant K describes exploring "edges" as a key aspect of their practice:

> "I would go to this border of feedback — I use it until now generally — so you put everything on the edge, and this is where things start to happen, [...] this is where pleasant surprises start to happen" (participant K).

The description of "pleasant surprises" appears to mimic the participant comment from the lab-based study noted previously regarding the desire for surprises that lead somewhere, rather than surprises that are dead-ends. Edge-like situations were often evoked by participants through a range of metaphors and analogies such as riding a crazy horse, surfing, managing currents of water, stretching an elastic band, fighting, and the more general notion of *pushing* tools and instruments towards critical thresholds. Some example descriptions are given below.

- "it's a bit like being a jockey riding a crazy horse, because the potential of it, it's a very powerful tool, so you have to be very careful [...] It's more like going into such a state and, yeah, riding the horse and going to search a place with this horse." (participant K).

- "it's more like a feedback thing where you're surfing with it and you're playing with the edges, and it might fall out underneath you but then you can get out there [...] you have to be always moving through the piece, but your goal is to sort of try to keep it, so you're surfing, but it's shifting, you're like surfing something that's going down rapids. So it's moving, but you have to stay afloat" (participant L).

Surprise through nonlinear dynamical processes (1)

Participant(s)	Mechanism for surprise
Q	Adding feedback to extend the flute - adds a behaviour that develops “almost independent of your action thereafter”.
L	Cross modal feedback between a sound-making light sensor, and a light-making sound sensor.
C	Bowing the cello very close to the bridge, and playing very high harmonics across multiple strings - it “just lifts off”.
F	“Wolfe note” on the cello - a note that may or may not sound or is harder to get to sound cleanly.
D	Twisting the embouchure to produce unpredictable multiphonics on the trombone that are “unexpected and uncontrollable” and that just “come and go”.
T	Feedback with a toy voice modulator.
T, V	Motorised objects moving by themselves.
S	Pushing at extremes of the saxophone: exploring very loud or very quiet areas where it is difficult to control accurately.
K	No-input feedback setup with a button that needs to be held to get the sound: muting and unmuting is part of a feedback network, so does more than merely mute and unmute a sound as the feedback takes time to develop. Finds the threshold with it where it does interesting things.
F	Adding a spring or a paperclip between violin strings and bowing it producing unpredictable harmonics. They will also fall off the string at some unpredictable point.
G	Bowing a double bass right on top of the bridge whilst rapidly moving a single finger which is lightly touching the string.
E	Working with high frequency, high intensity sounds on the saxophone where the harmonics are all very close together and “fizz about”. Then attempts to add other layers in, e.g. adding voice, or “bending through” the harmonics: “interferences and dynamics and distortion”.
V	Throwing two magnets in the air (separately), for the sound of them connecting in the air (or not).
W	Deliberately using too little rosin on the violin bow to create less stable higher harmonics.
I	Uncertainty about bowing a string for the first time: can get different harmonics, a pure tone, a gravely noise.
M	Balancing a network of interlinked LFOs to create unpredictable fluctuations that are hard to control directly.
P	feedback networks where the parameters for the processes found at each node respond to the audio being generated.
X	Electric guitar feedback.

Table 4: Mechanisms for surprise that involve nonlinear dynamical processes

Surprise through physical and mental limitations (2)

Participant(s)	Mechanism for surprise
C	Uncertainty due to fatigue: contrasting genuinely stumbling rhythms with faked stumbling rhythms.
S	Pushing at extremes of the saxophone: exploring very loud or very quiet areas where it is difficult to control accurately.
E	Working with high frequency, high intensity sounds on the saxophone where the harmonics are all very close together and “fizz about”. Then attempts to add other layers in, e.g. adding voice, or "bending through" the harmonics: "interferences and dynamics and distortion".
R	Vocal repetition of a single word or phrase until it necessarily starts to change from exhaustion.
O	Unpredictability from the imprecision of the sine tone interface.
L	CDJs - physical uncertainty in the movement of the CDJ turntable interface that are incredibly sensitive.
N	Refreshing the modular setup regularly: repatching, adding new modules, removing modules and getting back to a point where the whole thing isn’t known too well - a regular cycle.
W	Detuning the violin before performance to explore unexpected microtonal elements - the harmony therefore can’t be easily known beforehand.
J	Limited understanding of particular granular synthesis processes - they are not understood and may do unexpected things (e.g. combinations of grain rate, grain size, traversal rate, envelope settings and so on).
L, J, T	Dipping into samples and not knowing what will be there.
L	Huge amount of long samples available in performance (although they are categorised as much as possible).
R	Dipping into records - looking for a particular loop, but with the knowledge that something completely different may come out.

Table 5: Mechanisms for surprise that rely on physical or mental limitations

Surprise through (effectively) chance/random processes (3)

Participant(s)	Mechanism for surprise
L	Actual randomisation of sound selection (Ableton Live follow actions: audio files are chosen at random by the software).
L, K	Radios - often with the signal interrupted by other devices.
L, J, T	Dipping into large audio files without knowing in advance what will be there.
L	Huge amount of long samples available in performance precluding the possibility of knowing for sure what each one will sound like before triggering them.
V	Fizzing tablets in water: participant can never be sure how intense they will be or how long they will fizz for.
T, V	Motorised objects moving by themselves.
W	Performing with recorded excerpts of earlier work on shuffle on a CD (not done by the actual participant, despite them being sympathetic to this approach).
R	Dipping into records - looking for a particular loop, but with the knowledge that something completely different may come out.
N	Stochastic synthesis process (using an implementation of Xenakis' Gendyn algorithm) for fluctuating LFO signals.
K	Overloading a drum machine arpeggiator algorithm ("if you start to press buttons too quickly it would get lost").

Table 6: Mechanisms for surprise that involve chance elements

- "I would have for example another envelope plugged into feedback. And that would be pushing into self-oscillation which could go really crazy, but you kind of try to retain it within, so it doesn't spill out [...] like if you have water so it doesn't spill outside of that scope." (participant M).
- "it's like with [a] current of water: I kind of like controlling that current and it can spill out sometimes, and maybe if it was a current of colour it might spill onto the canvas and do something [...] I somehow feel more comfortable, or I feel better in that territory." (participant M).
- "it's like how far can you pull this invisible rubber band before it snaps. And you can almost feel the tension." (participant F).

Participants E, F, M and Q used the vaguer term "pushing" to describe similar situations, where the participant can push towards thresholds with different behaviours encountered at these thresholds. The participants provide a range of motivations for working in such states:

- finding new sounds, behaviours, and territory for exploration,
- learning more about how their tool functions and behaves,
- explicitly enjoying the fact that the situation is beyond their comprehension,
- reducing their own agency, and sharing agency with the tool,

- adding a degree of tension and risk to the situation.

The characterisation of edge-like interactions introduced in this section appears to provide a concrete link between the specific properties of nonlinear dynamics on the one hand, and specific approaches to creative exploration on the other. This is discussed in more detail below, in conjunction with the results of the lab-based study.

7. Discussion

The lab-based study suggests that the properties of nonlinear dynamical processes are a resource for creative exploration in digital instruments, and that these processes can be helpful in designing interfaces that behave in unpredictable ways whilst ultimately remaining controllable and learnable. The ethnographically-informed study provides concrete examples from musical practice of engagements with nonlinear dynamical processes for surprising and explorable interactions. This is particularly clear in the characterisation of edge-like interactions that emerges from the participant interviews. Taken together, the results of the two studies demonstrate ways in which interaction designers can usefully incorporate nonlinear dynamical processes into musical tools to facilitate engaging interactions.

Broader implications for musical interaction emerging from these two studies are discussed in this section. These concern respectively:

- the importance of timing as a resource for exploration in nonlinear dynamical interactions (Section 7.1),
- the scope for deep exploration even in restricted input ranges (Section 7.2),
- different conceptions, and manifestations, of surprise (Section 7.3),
- surprise in relation to learning and enjoying situations that are not fully understood (Section 7.4)

These implications are considered in terms of their relevance both for musical interaction, and for HCI more generally.

7.1. Timing and gesture

A key aspect of what makes nonlinear dynamical interactions explorable in musical practice may be the importance of *timing* in determining the relation between input gesture and sounding outputs. This may provide a useful perspective on gestural interaction more generally. With nonlinear dynamical processes, there can be a significant amount of subtlety and possible variation available even in an interaction limited to a low resolution, single dimension of input, with the subtleties of timing affording a wide variety of outcomes based on only changes in the speed of movement. For example, accelerating quickly at the start of an input gesture to a nonlinear dynamical system, and slowing towards the end of the gesture may mean that the output of the system at the end point of the gesture can be very different to the output of a system at this point with a gesture that started out slowly and sped up towards the end. A simple microphone-loudspeaker feedback example can demonstrate this. If a performer is using a microphone to create controlled feedback tones, and moves a microphone from point A to point B, lingering in a particular position in the middle of the gesture may allow a particular harmonic to feedback sufficiently that it remains dominant when the microphone arrives at point B. A faster gesture may not allow this harmonic to become strong enough, and the dominant harmonic at point B is therefore potentially different. Thus, interaction designers may benefit from considering the potential of nonlinear dynamics in situations where they are seeking to support explorative gestural interactions with low resolution and low dimensional inputs.

7.2. Deep exploration

The studies presented here suggest that participants had contrasting understandings of the term exploration. A conventional idea of exploration might involve rapidly covering as wide area as possible as quickly as possible. In describing the motivation behind working at unstable edge points, participant K, in the ethnographically-informed study, expressed the desire to "go deep." This model of exploration appears to involve digging into the detail of a particular aspect and uncovering successively deeper degrees of subtlety and variation within that narrow area. The participant explicitly contrasts this with the former model of exploration, invoking *The Travels of Marco Polo*. The explorer's travels, and his descriptions of them, move from novelty to novelty ceaselessly, which can be interesting initially, but rapidly becomes fatiguing.

These considerations provide additional insight into why the rich variety found close to critical thresholds in nonlinear dynamical systems may be interesting to musicians. The edge-like interactions discussed above in Section 6.6 seem to connect closely with this "deep" model of exploration. This variety contrasts with the fixed nature of the boundary points in the discontinuous—but not dynamical—mappings to the digital musical interfaces used in the lab-based study that seemed to frustrate participants. The smallest of changes in either parameter setting or timing close to critical thresholds in nonlinear dynamical processes can open up very different behaviours and areas for exploration.

7.3. Different conceptions of surprise

The interviews in both the lab-based study, and the ethnographically-informed study highlight how surprises that stem from interactions with instruments may vary significantly in terms of scale, and in how they are observed and engaged with by the musicians.

Many participants in the lab-based study found abrupt, arbitrary surprises frustrating and limiting. A distinction was made between surprises that lead somewhere, and surprises that are dead ends, that close down possibilities. This may help to explain the fact that the discontinuous mappings were not seen as significantly more surprising or explorable than the continuous mappings: the abrupt changes in the output at given points in the input may be initially surprising, but since they are exactly the same each time, they do not promote exploration, they merely limit the bounds of different areas. Several participants explicitly reported that the abrupt changes inhibited their musical exploration. With nonlinear dynamical interactions however, boundaries may shift depending on the timing of events and on the state of the other inputs, and may change qualitatively, or disappear completely. Moreover, the bifurcation points are not necessarily abrupt discontinuities. The intermittency example given in Section 3 shows how the disappearance of an attractor may affect trajectories in different ways depending on the specific nature of the trajectory. The transition points between two states may become resources in themselves, providing a range of new behaviours to explore.

Whilst there is likely a subjective dimension to whether a given surprise is seen as leading somewhere, the fact that the nonlinear dynamical processes were correlated not only with surprise, but also with the possibility of exploration and discovery suggests that they may facilitate surprises that lead somewhere and can be developed and engaged by musicians and creative practitioners.

7.4. Surprise and learning

Participant C in the ethnographically-informed study suggested that being in situations that are unknown is in itself a valuable experience, that engaging with processes that are not fully understood is itself deeply rewarding. This echoes the value placed on learning in improvisation by several other participants (E, F, G, I, K, O and X). Exploration is seen as "a self development, and a learning [...] and to keep this sort of open-ended thing that can never quite be pinned down as to what is actually going on." (participant X). While such learning likely stems from a range of factors beyond the interaction with the instrument itself (e.g. interactions with other players, different venues, and so on), the potential for this kind of long-term engagement and exploration with a particular tool may be facilitated through interactions with nonlinear dynamical processes.

The surprises collected in Table 4 that emerged due to nonlinear dynamical interactions can be contrasted with the surprises that emerged from chance processes in Table 6. Surprises due to deliberately random processes may not reveal anything new, and are unlikely to change someone's understanding of a situation. The use of radios for example, as discussed by participants L and K may lead to very unexpected sonic outcomes through the interaction, but they do not generally inform the understanding of the interaction with the system under consideration. Similarly with the process of dipping into audio files, or records as described by participants L, J, T and R, the result may be surprising, and the result can be explored and developed, but nothing is revealed about the nature of the system or the interaction through this surprise. Further, the fact that there *is* a surprise, is itself not a surprise: random processes are usually deliberately invoked and the outcome is already known not to be known. By comparison, the surprises articulated by participants G and C in relation to their bowed instruments, although potentially more subtle, are intrinsically linked to the complex nature of the interaction being engaged. With these instruments it may not be so simple to encounter a surprising result, particularly if the performer has worked with the instrument for a significant number of years. The emergence of a surprise may therefore itself be a surprise. Participant G describes an approach that has much in common with the idea of deep exploration traced above, where surprises are unearthed through extended exploration of minute aspects of the interaction. The hidden nature of these unpredictable regions is therefore different in kind from chance surprises that can be easily and deliberately provoked, as they reward long-term exploration.

8. Limitations of the studies

The lab results provide insights into engagements with a very specific implementation of a nonlinear dynamical process. While the ethnographically-informed study provides wider context on engagements with nonlinear dynamics more generally, such processes may be implemented in a limitless number of ways, not all of which may facilitate the kinds of explorative interactions that are investigated here. Further, the design of a musical interface requires many other design decisions, and the design elements under consideration cannot easily be isolated and tested independently of the other elements; oversimplifying a musical device risks rendering it "unmusical" (Stowell and McLean, 2013). The interfaces used in this study therefore represent only one possible implementation of the nonlinear dynamical components, and they are a single factor amongst many in influencing the nature of the interaction. Other factors could be contrasted with nonlinear dynamics, other than the simple discontinuous mappings used in this study. For example, a similar study that explored stochastic processes may help to further draw out what is particular about nonlinear dynamics, and support the discussion regarding more nuanced conceptions of surprise.

The influence of the particular sounds and musical behaviours afforded by the interfaces has been noted above. Although care was taken to keep the interfaces used in this study within a relatively consistent sound world, this overall sound world was selected relatively arbitrarily, and the study could be re-run with very different kinds of musical affordances. At the heart of this problem is the fact that music can be many different things to different people, and there is no simple way of designing an interface that does not emphasise or prioritise certain musical approaches over others.

A useful direction for future studies would be to examine how views change when using the tools for prolonged periods. Participants were asked to judge the scope for exploration and discovery in a particular interface after only five to twelve minutes of use. It would therefore be beneficial to examine how attitudes towards these elements differ over longer periods of time. It would also be very useful to see how perception of surprise develops over longer periods, to see whether the nonlinear dynamical processes can still produce unexpected results after several days, weeks or months.

9. Conclusions

Nonlinear dynamical processes are common in the physical world, but at present are rarely considered in relation to interaction design in digital musical contexts. The studies presented in this article demonstrate the value of such processes in musical interactions, and suggest that they may play a valuable role in other domains of creative interaction.

The lab-based study demonstrated that these processes can facilitate interactions that are unpredictable and explorable, but may still be seen as controllable and usable. The results of the ethnographically-informed study show the importance of these factors in the real-world experiences of improvising musicians. The study also shows how nonlinear dynamical processes can be central to the creative *exploration* of musical tools, whether acoustic or electronic. The examination of nonlinear dynamics processes in interaction has provided detailed examples of how interface complexity can be a virtue in creative HCI applications, particularly the focus on timing. Even low-dimensional, low-resolution inputs (such as a dial) can be investigated over long time periods, yielding productive surprises and stimulating creative responses.

The results also provide insights into the nature of surprise in creative interactions: in terms of how users can interact with the surprise-making mechanisms, how they can learn from these surprises, the different scales of surprises from significant to relatively minor, and in terms of whether these surprises can lead users to new territory to explore.

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