idea of the Polar Planimeter – a name invented by Amsler to indicate that the instrument turns around a fixed point, i.e. the pole, of the instrument (see Fig. 2).

FIGURE 2. Amsler's polar planimeter

The instrument was simple, inexpensive and easy to use: three properties any object needs to become commercially successful. Of the three inventors, Amsler's enterprise (founded as early as the end of 1854) was the most prosperous one; the total number of previously produced planimeters was surpassed within only three years, and the number of planimeters manufactured in the decade 1854-1864 increased by an amazing 3500! Thus, by several small changes and one really big change, the final instrumental form of mechanically evaluating integrals was reached after five decades, and by 2001, one and a half centuries later, when production of mechanical planimeters ceased (with one exception that can be ignored in terms of numbers), the total number of mechanical integrating instruments, 95% or more of them planimeters, had reached an estimated 1.5 million!

References

[1] J. Fischer, *200 Jahre Planimeter. Ein bayerischer Vermesser und seine geniale Idee. 1814-2014. Catalogue of the exhibition, May 5 until September 15, 2014 in Munich*, München: Landesamt für Digitalisierung, Breitband und Vermessung, 2014.

[2] T. Gonnella, *Teoria, e Descrizione d'una macchina colla quale si quadrano le superficie piane*, Antologia **18** [Firenze, April-June] (1825), 122–129 + table with 6 figures.

[3] T. Gonnella, *Opuscoli matematici nei quali si tratta I. Di alcuni perfezionamenti del telescopio newtoniano. II. Delle formule che determinano rigorosamente l'andamento dei raggi luminosi a traverso d'un sistema qualunque di lenti. III. D'una macchina per quadrare le figure piane*, Firenze: Mazzoni, 1841.

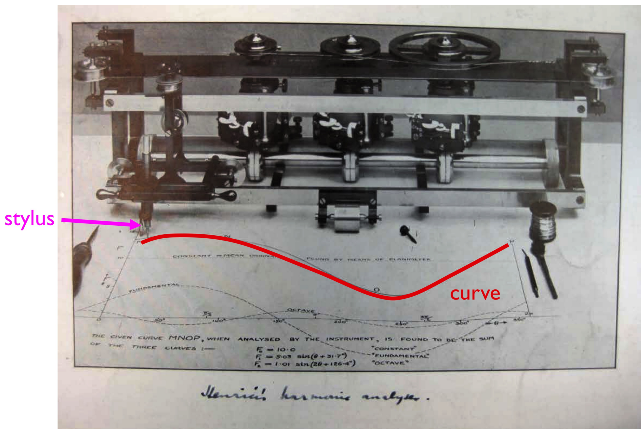
"An exquisite machine": Olaus Henrici's harmonic analyser

JUNE BARROW-GREEN

In May 1894, Robert Ball, the Lowndean Professor of Astronomy and Geometry at Cambridge, described Olaus Henrici's new harmonic analyser as "an exquisite machine"[3], and he went on to expound on the machine's usefulness, especially in connection with the production of tide tables. Ball was reporting on a soirée held at the Royal Society at which Henrici's new machine was being exhibited. It was not Henrici's first harmonic analyser – he had had his first one constructed in 1889 – but it was a revised version of the new one he had designed in 1893 together with his assistant at the Central Technical College, Archibald Sharp, and which had been built by the renowned instrument maker Coradi of Zurich. Henrici's analyser was also displayed at the *conversazione* held in June 1896 at the Central Technical College[3] where a visitor reported that

> "[...] the smooth working of the latest form of Prof. Henrici's harmonic analyser, led the engineer to speculate on the time when all calculations, however complex, would be done by turning a handle, and when the brain would be left quite free to think and originate." [2]

As can be seen in the figure below, the analyser consists of multiple pulleys and glass spheres – rolling sphere integrators – connected to measuring dials. The image of a curve is placed under the device and the user moves a mechanical stylus along the curve's path tracing out the wave form. The resulting readings on the dials give the phase and amplitude of up to 10 Fourier coefficients.



Henrici had been led to the construction of his first harmonic analyser by the work of W. K. Clifford [7], his colleague at University College London (UCL), who in 1873 had provided "a beautiful graphical representation of Fourier's Series" [11, p. 113]. However, this 1889 machine did not work as well as Henrici hoped, not least because it gave only one Fourier coefficient at a time, and because the mechanism required to produce the simple harmonic motion introduced too much friction. He exhibited a revised version of this analyser at the German Mathematical Society's *Mathematical Models, Apparatus and Instruments Exhibition* held at the *Technische Hochschule* in Munich in 1893. Henrici was the lead organiser for the British exhibits, his German background combined with his interest in mathematical models and instruments making him a natural choice for the role.[4] He

[3] Olaus Henrici (1840-1918) was educated in Germany before making his career in London, first at University College, and then at the Central Technical College. See [4, p. 22].

[4] Britain provided more exhibits than any other country apart from Germany, with Henrici providing several geometrical models as well as his analyser. The other British organisers were Lord Kelvin and George Greenhill.

also wrote an article on harmonic analysers for the catalogue [9, p.125–136]. The exhibition had been due to take place in Nuremberg in 1892 but had had to be postponed to 1893 due to an outbreak of cholera but the exhibits had been sent in 1892 which explains why Henrici's new harmonic analyser, the one developed with Sharp, was not displayed.

Henrici was not the first to produce a harmonic analyser, credit for that goes to William Thomson (later Lord Kelvin) who in 1876 produced a rudimentary design for a machine which was fully realised in 1878. But Thomson's analyser, although capable of a high degree of accuracy, was large and difficult to manoeuvre. Henrici's intention, as described by him in the Munich catalogue [9, p. 134–135], was to produce a machine that was cheap, easier to handle and more portable than Thomson's, and which would be appropriate for applications where less accuracy was required. However, although the Henrici-Sharp analyser was certainly smaller than Thomson's, it wasn't cheap. In 1894, an analyser with five integrators was priced at £60 (c. £7,000 in 2017). The point was not lost on potential users. In 1920, Vannevar Bush observed that "The Coradi analyser is probably the most convenient machine, [...] very few of these instruments are in use, however, because of their cost." [5, p. 903]. In 1936 Bush again commented on the convenience of Henrici's machine but this time in the context of the relationship between the invention and the usage of such machines:

> "It is not much exaggerated to state that as many forms [of the harmonic analyser] have been invented as there are actual instruments in present use. Perhaps this is not undesirable, for it is certainly much more pleasant to invent a device of this nature than it is to operate the finished product. The writer pleads guilty to having invented several, none of which are in use. [...] The most convenient and precise is the Henrici-Coradi." [6, p. 659]

Bush's remark notwithstanding, some of Henrici's analysers were used, although it would seem not many. Felix Klein acquired one in 1894 [15], and in 1901 described it in his lecture course printed as *Anwendung der Differential- und Integralrechnung auf die Geometrie: eine Revision der Prinzipien* (1902) later edited and published as *Präzisions- und Approximationsmathematik* (1928).[5] References to the analyser appear in the literature elsewhere in Germany and in France but evidence for its actual use is sparse.

One person who did make good use of the analyser was the American physicist and astronomer Dayton Miller (1866-1941) who between 1908 and 1916 successfully used it for acoustic experiments [14]. Miller also encouraged his students to use it as evidenced by the following publications:

> Robert Sherwood Shankland, *The Dispersion of X-rays* (1933). Master's thesis. Included curve fitting with the Henrici analyser.
>
> Leslie Foldy, *The use of the Henrici Harmonic Analyzer to Obtain Frequency Spectra of Pulses* (1944). Report.

[5]For an English translation of the relevant pages, see [10, p. 79–82].

Miller corresponded with Gottlieb Coradi (1847-1929) about the analyser and in a letter to Miller, dated 30 November 1916, Coradi revealed that the analyser had not been the commercial success that he had hoped:

> "[...] it is really regrettable that this Analyzer of which you are proofing [sic] the high value in extent practical application is still nearly unknown. I am therefore greatly please[d] with your pamphlet books and photos and shall take the liberty to mention these when occasion occurs. The totality of work and idea of your researches is admirable and of most interesting results and I am very grateful to you as to one of the few of the possessors of a Henrici-Analyzer which have been kind enough to give me an idea of the work done with the apparatus. If this would have been more regularly the case, I would probably have had more practical success with the Analyzer when I would have been able to bring the different kinds of application to that general knowledge by means of notes in my catalogue and description." [8]

Another person who used the analyser for acoustic work was Carl Seashore who included a picture of it as the frontispiece to his book *Pyschology of Music* (1938) where it was described as "a symbol of the science of music".

Although Henrici's academic reputation, built while he was at UCL, was as a geometer, he had a background in engineering as well as in mathematics – he began his working life as an apprentice in an engineering works and later studied engineering under Ferdinand Redtenbacher (1809-1863), the founder of scientific mechanical engineering, at Karlsruhe. Even as a geometer he was an enthusiast for practical work, arranging for a workroom for his geometry students at UCL and producing many models of geometrical surfaces himself. With his move in 1884 to the Central Technical College and the setting up of his Mechanics Laboratory, he had the opportunity to become more focussed on applications, and in 1894, as well as producing a new version of his second analyser, he produced a report on planimeters for the British Association for the Advancement of Science [12]. Viewed from this perspective, Henrici's harmonic analyser – "Perhaps the most strikingly original piece of work done by Henrici" [13, p. xlvi] – can be seen as a natural synthesis of his mathematical and engineering talents, and despite the fact that it was not as widely used (or sold) as Henrici (or Coradi) had hoped, its ease of use and efficiency of design meant that it maintained a justifiably high status for over forty years.

References

[1] Anonymous, *The Royal Society Soirée*, Nature **48** (15 June 1893), 160.

[2] Anonymous, *Science and society at the Central Technical College*, Nature **54** (25 June 1896), 186.

[3] R. S. Ball,*The Month's Science*, The Graphic (12 May 1894).

[4] J. E. Barrow-Green, *"Clebsch took notice of me": Olaus Henrici and surface models*, Oberwolfach Reports **12** (2015), 2788–2791.

[5] V. Bush, *A Simple Harmonic Analyser*, Journal of the American Institute of Electrical Engineers **39** (1920), 903–905.

[6] V. Bush, *Instrumental Analysis*, Bulletin of the American Mathematical Society **42** (1936), 649–669.

[7] W. K. Clifford, *Graphic Representation of the Harmonic Components of a Periodic Motion*, Proceedings of the London Mathematical Society **5** (1873), 11–14.
[8] G. Coradi, *Letter from G. Coradi of Zurich to Miller*, `https://artscimedia.case.edu/wp-content/uploads/sites/175/2016/03/14222324/let-24-txt.pdf`
[9] W. Dyck, *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente*, München: K. Hof- u Universitätsbuchdruckerel von Dr. C. Wolf & Sohn, 1892.
[10] F. Klein, *Elementary Mathematics from a Higher Standpoint. Volume III. Precision Mathematics and Approximation Mathematics*, Berlin & Heidelberg: Springer, 2016.
[11] O. Henrici, *On a new harmonic analyser*, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **38** (1894), 110–121.
[12] O. Henrici, *On planimeters*, Report of the Sixty-fourth Meeting of the British Association for the Advancement of Science (1894), 496–523.
[13] M. J. M. Hill *Professor Olaus Henrici*, Proceedings of the London Mathematical Society **17** (1918), xlii–xlix.
[14] D. C. Miller, *The Henrici harmonic analyzer and devices for extending and facilitating its use*, Journal of the Franklin Institute **182** (1916), 285–322.
[15] `http://modellsammlung.uni-goettingen.de/index.php?r=11&sr=49&m=557&lang=en`

Mathematical machines in tide analysis and prediction (1876-1950)

Marie-José Durand-Richard

This talk presents three mechanical devices – the tide gauge, the harmonic analyser and the tide predictor – which were progressively introduced during the 19th century for recording, analyzing and predicting tides. It shows how different professional and academic cultures collaborated until the 20th century to establish a new control of ocean navigation, essentially with the colonial expansion of Great Britain and France.

I. Newton's and Laplace's theorization of tides. The phenomenon of tides has been observed for a very long time. Semi-diurnal tides, with two High and two Low Waters per day, were the best known on the Atlantic coasts, but no explanation was given other than the influence of the Moon. The first really efficient theorisation of tides was given by I. Newton (1643-1727). He applied the Three-Body Problem to the Water – assuming it covered the whole globe –, the Earth, and successively the Moon and the Sun. The main attraction is that of the Moon because of its proximity to the Earth. The Sun, because of its mass, also has a significant attraction. Newton's theory was correct in its basic principle, but it neglected the effect of the rotation of the Earth. So, some inequalities remained unexplainded, with theory also disagreeing with observation on some points.

The dynamical theory of P. S. de Laplace (1749-1827), was achieved in his impressive *Mécanique Céleste* (1799-1825). He formulated the differential equations describing the motion of the fluids attracted by the Sun and the Moon, but still with the simplifying hypothesis that water covered the complete surface of the Earth. Laplace identified three periodical species of tides – annual, diurnal and semi-diurnal – for each of the two attracting bodies, the nature of each being dependent upon the astronomical elements involved in its mathematical formulation. Laplace also gave a very general formulation of the elevation of a molecule at the