Visual structures of embedded shapes

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Abstract. Shape computations recognise parts and create new shapes through transformations. These elementary computations can be more than they seem, inducing complicated structures as a result of recognising and transforming parts. This paper introduces, what is perhaps in principle, the simplest case where the structure results from seeing embedded parts. It focusses on lines because, despite their visual simplicity, if a symbolic representation for shapes is assumed, lines embedded in lines can give rise to more complicated structures than might be intuitively expected. With reference to the combinatorial structure of words the paper presents a thorough examination of these structures. It is shown that in the case of a line embedded in a line, the resulting structure is palindromic with parts defined by line segments of two different lengths. This result highlights the disparity between visual and symbolic computation when dealing with shapes – computations that are visually elementary are often symbolically complicated.

Keywords: Shape grammars, shape structure, embedding, visual palindromes.

1 Introduction

Representations of shapes conventionally employ a structure of parts. In computer-aided design (CAD) these parts might be points and labels, lines and curves, planes and surfaces or bounded solids; higher order structures arise from assemblies of these parts. However, the cultural, artistic and design applications of shape often require a diversity in part descriptions with different designers or artists entertaining different descriptions. What is the origin of these diverse structures and what are their characteristics? This paper explores these questions by proposing that recognition of parts induces a part structure, and presents an analysis of the resulting structures, with part recognition formalised in shape rules.

Shape structures can range from a predetermined structure, as is the case for CAD models, to random assignation of parts. In terms of the cultural, artistic and design perspectives, relevant part structures for shapes seem to be in the middle of this range, with structures defined differently in different contexts. Shape computations formalise these structures in shape grammars where rules recognise parts and create new shapes through transformations. But such computations can be more than they seem, inducing complicated structures from seeing (recognising parts) and moving (transforming parts)

[1]. This paper examines shape computations and the properties of part structures, the complexity of which can be surprising, even for elementary shape computations. It develops arguments presented by Stiny [2], which explore the diversity of part structures necessary to support shape computation.

As an illustrative example consider the lattice patterns in Fig 1. Such patterns are common in the Republic of Korea, where they are used to ornament doors and windows. They have an intricate visual structure and can be viewed in different ways. For example, the pattern in Fig 1c could be recognised as the motif for the patterns in Figs 1a and b. Alternatively, all three patterns can be viewed as compositions of overlapping and abutting polygons, such as squares, rectangles, and crosses. The different ways of viewing the patterns give rise to different part structures which are not predetermined as required in a CAD model, nor are they random. The part structures are a consequence of seeing and change dynamically according to parts recognised by the viewer.

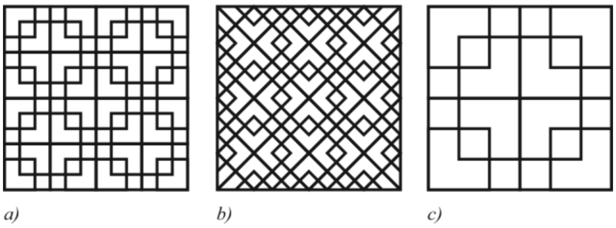

Fig. 1. Korean lattice patterns

Shape rules can be used to formalise the different ways the patterns are viewed, and application of rules in a shape computation gives rise to different structures according to the different parts recognised. For the patterns in Fig 1, this can be illustrated by considering the two simple configurations, highlighted in Fig 2. The first consists of two adjacent squares of equal size while in the second the squares are of different size.

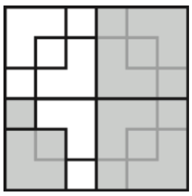

Fig. 2. Configurations embedded in a Korean lattice pattern

The shape rule in Fig 3a is a shape identity rule [2] that recognises squares, and it can be applied under Euclidean transformation and scale to the patterns in Fig 1 to identify embedded squares in the patterns. Application of an identity rule does not visually transform a shape, but it does impose structure in order to accommodate embedded parts [3-4]. Focussing, on the configurations identified in Fig 2, applying the rule to different embedded squares gives rise to different part structures as illustrated in Figs 3b-e.

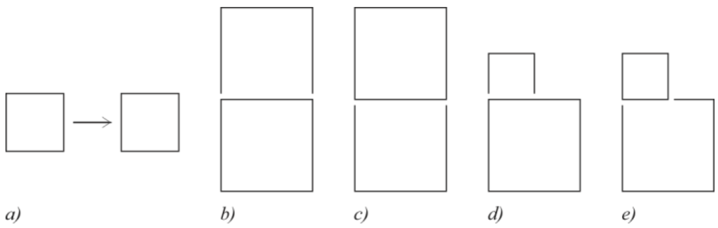

Fig. 3. a) Identity shape rule, b-e) Part structures resulting from rule application

Visually, two squares are immediately apparent in the configurations, but in order to make both the squares apparent symbolically a more complicated structure is necessary. For the first configuration, composed of two adjacent squares of equal size, this structure is trivial and requires only that the common edge is included in the description of the part, as illustrated in Fig 4a. For the second configuration, composed of two adjacent squares of different size a more complicated part structure is required, in order to account for the edge of the small square embedded in the edge of the large square, as illustrated in Fig 4b. This structure supports application of the shape rule to either of the squares apparent in the configuration, but large and small squares have different structures necessitating two versions of the rule to accommodate them.

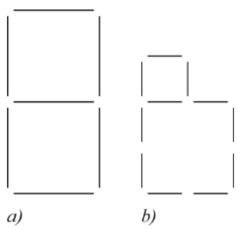

Fig. 4. Part structures that accommodate recognition of two squares

The shape structures illustrated in Fig 4 are examples of a phenomenon which occurs when rules, even elementary rules, act on shapes, to recognise and transform parts, e.g. [5-7]. This paper explores this phenomena with reference to the class of shapes exemplified by the two configurations illustrated in Fig 2, and the part structures that result from recognising their embedded parts. It focusses on applications of the identity rule in Fig 3a to shapes composed of two adjoining squares which share a common edge and a common end point, as illustrated in Fig 5.

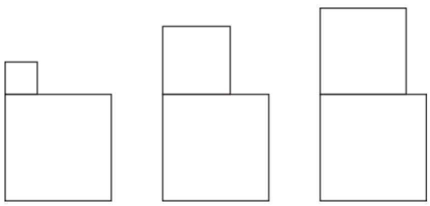

Fig. 5. Example shapes, composed of two adjoining squares

The rule, applied under Euclidean transformations and scale, can select either of the squares, and each selection gives rise to a different part structure, according to the decomposition of the common edge. Visually, this is an elementary problem. But finding a symbolic description of the shape that accounts for both part structures is more of a challenge. The shared edge is the key characteristic of this particular shape computation, and consequently the problem is equivalent to finding part structures that accommodate the elementary operation of embedding a line inside another line. The paper identifies structures that account for this embedding relation, thereby providing symbolic representations of shapes that account for visually apparent parts.

2 Lines embedded in lines

Lines embedded in lines can give rise to more complicated structures than might be intuitively expected. This is illustrated in Fig 6, where the part structures that arise when two squares share a common edge are identified. Here, the lengths of the edges are denoted l and n, with $n < l$, and the squares adjoin such that an edge of the smaller square is embedded in an edge of the larger whilst sharing an end point. In these examples a part structure has been identified that accommodates all applications of the rule in Fig 3a, and also

1) accounts for the smaller edge embedded in the longer edge
2) retains the symmetric properties of the edges/squares.

The triangles are included on the common edge to illustrate part structures while highlighting their symmetry. Each triangle correlates with a line segment embedded in an edge, and these are subdivided into finer structures, representing lines embedded in lines. Embedded lines associated with triangles are symmetrical, and their subdivision into embedded parts is symmetrical; in this sense, the triangles represent the structure of the edges as visual palindromes.

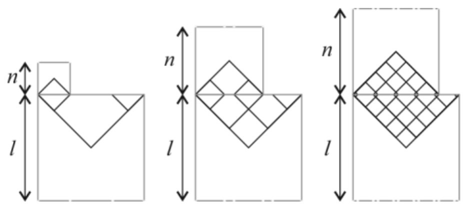

Fig. 6. Examples of part structures resulting from embedded lines

The structures of the decomposed line segments are also illustrated in the decomposition of the top edge of the smaller square and the bottom edge of the larger. In each example, the edges of the larger square and the smaller square have a different part structure, but shape computations that result from applying the rule in Fig 3a can accommodate this by having two versions of the rule.

In Fig 6, the structures shown are the simplest that allows a short edge to be embedded in a long edge while retaining the symmetric properties of the two squares. Intuitively, it might be expected that embedding a short edge as part of a longer edge would simply result in a decomposition of the longer edge to accommodate the shorter. But this intuitive decomposition only occurs in the first of these examples. In the other examples a finer decomposition of both edges is necessary, and the reason for this is illustrated in Fig 7, where a process of deriving the part structures of the edges is presented. This process involves resolving the symmetries of the visual palindromes corresponding to the part structure of the edges.

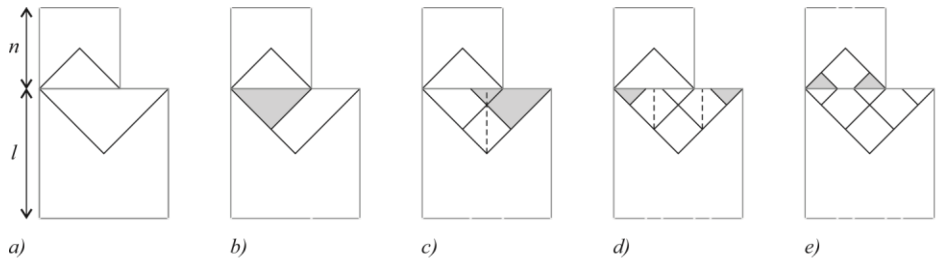

Fig. 7. Deriving the part structure of embedded lines by resolving symmetries

Fig 7a illustrates the high-level structure of the edges where, to account for the symmetric properties of the squares, each edge is identified as a visual palindrome, represented by a triangle. Fig 7b illustrates the embedding of the shorter edge in the longer; the structure of the longer edge now incorporates an embedded line that is the length of the shorter edge, represented by the triangle highlighted in grey. This new structure breaks the symmetry of the longer edge, which is addressed in Fig 7c by reflecting the smaller triangle in the illustrated axis of symmetry. A new triangle is defined by the overlap, and this represents further subdivision of the visual palindrome; it is this emergent form that requires a finer decomposition of the edges than might be intuitively expected. Fig 7d resolves the symmetry of the longer edge by reflecting the emergent triangle in the illustrated axes of symmetry. Finally, in Fig 7e, the structure of the shorter edge is subdivided according to the structure of the longer edge. The resulting part structure accounts for the symmetric properties of both squares, and allows the edge of the smaller square to be embedded in the edge of the larger square. The result is a periodic palindromic structure where the shorter edge can be described by the string *uvu* and the longer edge can be described by the string *uvuvu*, where *u* and *v* represent line segments of different lengths, determined by the ratio of the lengths of the edges, *l* and *n*, as illustrated in Fig 8.

Fig 8 explores the different part structures that arise when two squares share a common edge. Here, the arrangement of the squares is constrained such that the edge of the smaller square is embedded in the larger, with both sharing an end point, and *l*, the edge length of the larger squares, is kept constant while *n*, the edge length of the smaller squares, increases from Fig 8a to 8h. Again, triangles are included to illustrate the part structures, whilst highlighting their symmetry, and this structure is also reflected in the decomposition of the top edge of the smaller square and the bottom edge of the larger.

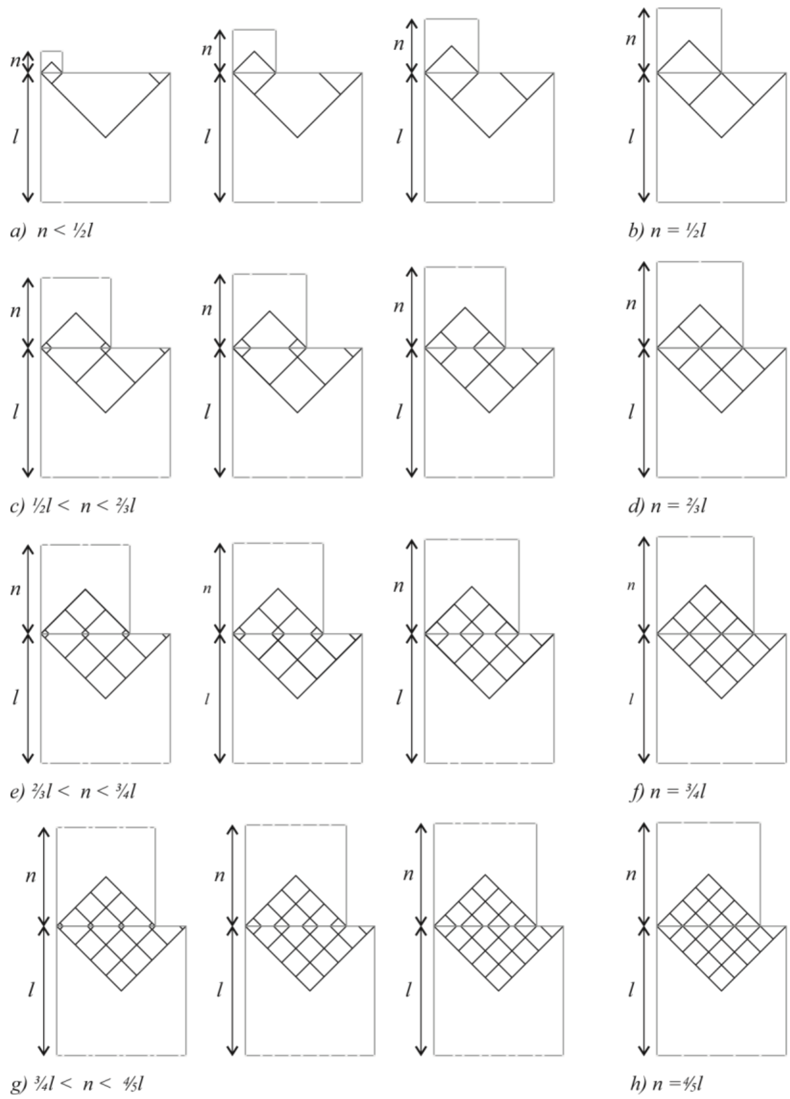

Fig. 8. Part structures resulting from two squares sharing a common edge

In Fig 8a, $n < ½l$ and embedding the shorter edge in the longer edge results in the part structure that is intuitively expected: the structure of the shorter edge remains unchanged and the structure of the longer edge includes the shorter edge as an embedded part. As a result, the structure of the shorter edge can be described by the string *u*, where *u* represents a line of length of $l_u = n$, and the structure of the longer edge can be described by the string *uvu* where *v* represents a line segment of length $l_v = l - 2n$.

Increasing the edge length of the smaller square results in an increase in the length l_u, and a decrease in the length l_v. Specifically, as $n \rightarrow ½l$, $l_u \rightarrow ½l$ and $l_v \rightarrow 0$, and, in Fig 8b, when $n = ½l$, $l_v = 0$ and the longer edge can be described by the string *uu*.

In Figs 8c-h, $n > ½l$ and the embedded shorter edges overlap resulting in the emergence of more complicated structures, as illustrated in Fig 7. When $n > ½l$ embedding the shorter edge in the longer edge results in a decomposition of both edges, and as n increases the symbolic descriptions of the resulting part structures can be categorised according to the following cases:

- In Fig 8c, $½l < n < ⅔l$, the short edge can be described by *uvu* and the long edge by *uvuvuv*. As $n \rightarrow ⅔l$, $l_u \rightarrow ⅓l$ and $l_v \rightarrow 0$
- In Fig 8d, $n = ⅔l$, $l_u = ⅓l$ and $l_v = 0$, the short edge can be described by *uu* and the long edge by *uuu*
- In Fig 8e, $⅔l < n < ¾l$, the short edge can be described by *uvuvu* and the long edge by *uvuvuvu*. As $n \rightarrow ¾l$, $l_u \rightarrow ¼l$ and $l_v \rightarrow 0$
- In Fig 8f, $n = ¾l$, $l_u = ¼l$ and $l_v = 0$, the short edge can be described by *uuu* and the long edge by *uuuu*
- In Fig 8g, $¾l < n < ⅘l$, the short edge can be described by *uvuvuvu* and the long edge by *uvuvuvuvu*, and as $n \rightarrow ⅘l$, $l_u \rightarrow ⅕l$ and $l_v \rightarrow 0$
- In Fig 8h, $n = ⅘l$, $l_u = ⅕l$ and $l_v = 0$, the short edge can be described by *uuuu* and the long edge by *uuuuu*

The pattern identified here continues, tending towards the limiting case where $n = l$ and the two squares are the same size, with the edges of both squares represented by a single line, as illustrated in Fig 4a. But, as $n \rightarrow l$, $l_u \rightarrow 0$, and the part structure of the edges gets get finer and finer with the number of line segments increasing. This structure is always defined according to line segments of two alternating lengths, and it can always be described as a periodic palindrome over two atoms, *u* and *v*. In general, the structure of the shorter edge can be described by the string $(uv)^k u$, and the structure of the longer edge can be described by the string $(uv)^{k+1}u$, where *u* and *v* represent lines of length l_u and l_v, respectively, and *k* is a positive integer.

3 The combinatorial structure of embedded lines

The part structures that result from embedding one line in a second line can be further explored symbolically, by considering the combinatorial structure of words [8]. In the study of the combinatorics of words, a finite set of symbols is said to be an *alphabet*, denoted Σ. *Words* are sequences (either finite or infinite) of letters from Σ, for example $A = a_1 \dots a_n$ and $B = b_1 \dots b_m$, are words over Σ if the characters a_i and b_j are members of Σ, for some integers n and m, with $1 \leq i \leq n$ and $1 \leq j \leq m$. Two words, A and B, are equal, denoted $A = B$, if $n = m$ and $a_i = b_i$ for $1 \leq i \leq n$, i.e. $a_1 = b_1$, $a_2 = b_2$, … , $a_n = b_m$. The empty word is composed of no letters and is denoted ε.

The set of all finite words, denoted Σ*, is generated by Σ under an associative operation defined by concatenation. For example, if $A = a_1 \ldots a_n$ and $B = b_1 \ldots b_m$, are words over Σ, then they are members of Σ*, and the word $A \cdot B = AB = a_1 \ldots a_n b_1 \ldots b_m$ is also a member of Σ*. The set of all finite words is a free monoid with ε as the identity element under concatenation, since $A \cdot \varepsilon = A = \varepsilon \cdot A$. Repeated self-concatenation is denoted by superscript. For example, $A^3 = A \cdot A \cdot A = AAA = a_1 \ldots a_n a_1 \ldots a_n a_1 \ldots a_n$. The length of a word A is denoted $|A|$, and is recursively defined so that the ε has length 0, and for any word A in Σ* and any letter a in Σ, $|Aa| = |A| + 1$.

A word that is embedded as part of a second word is either a factor, a prefix or a suffix. For two words A and B, these are defined as follows. A is a *factor* of B if the words X and Y exist in Σ* such that $B = XAY$. A is a *proper factor* of B if $A \neq B$. A is a *prefix* of B if there exists a word X in Σ* such that $B = AX$. A is a *proper prefix* of B if $A \neq B$. Similarly, A is a *suffix* of B if there exists a word X in Σ* such that $B = XA$. A is a *proper suffix* of B if $A \neq B$. A is a *border* of B if it is both a prefix and suffix of B, and it is a proper border if $A \neq B$.

The part structures identified visually in Fig 8 arise readily in combinatorics, when words with identified parts are equated with each other, as summarised in the following lemmas, reported in [9], and illustrated in Figs 9 and 10. The first of these is concerned with the structure that arises when two words with distinct parts are equated with each other, and the second is concerned with the structure that arises when two such words share a common part, identified as the prefix of one word and the suffix of the other.

Lemma 1. If A, B, C and D are words in Σ*, such that $AB = CD$ and $|A| \leq |C|$, then there exists a word V in Σ* such that $C = AV$ and $B = VD$. This is illustrated in Fig 9, where words are represented by rectangles.

Proof. Let $m = |A|$, $n = |B|$, $p = |C|$ and $q = |D|$. Given that $AB = CD$ and $m \leq p$, C can be expressed as $C = c_1 \ldots c_p = a_1 \ldots a_m c_{m+1} \ldots c_p = A\, c_{m+1} \ldots c_p = AC'$ where $C' = c_{m+1} \ldots c_p$. Also $n \geq q$, and B can be expressed as $B = b_1 \ldots b_n = b_1 \ldots b_{n-q} d_1 \ldots d_q = b_1 \ldots b_{n-q} D = B'D$ where $B' = b_1 \ldots b_{n-q}$. Since $AB = CD$, this gives $AB'D = AC'D = AVD$, and Lemma 1 holds with $V = b_1 \ldots b_{n-q} = c_{m+1} \ldots c_p$. □

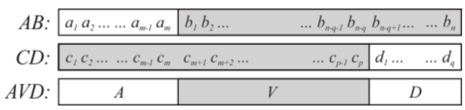

Fig. 9. Illustration of Lemma 1, with words represented by rectangles

Lemma 2. If A, B and C are words in Σ*, such that $AB = BC$ and $A \neq \varepsilon$, then $A = UV$, $B = (UV)^k U$ and $C = VU$ for some words U, V in Σ*, and some integer $k \geq 0$. This is illustrated in Fig 10, where words are represented by rectangles.

Proof. Let $m = |A| = |C|$ and $n = |B|$. If $n \leq m$, then from Lemma 1, $A = BV$ and $C = VB$, and Lemma 2 holds with $U = B = a_1 \ldots a_n = c_{m-n+1} \ldots c_m$, $V = a_{n+1} \ldots a_m = c_1 \ldots c_m$, and

$k = 0$, as illustrated in Fig 10a.

If $m < n$, then again by Lemma 1, $B = AB'$ for some word B' in Σ^*, where $B' = b_{m+1} \ldots b_n$. This gives $A^2B' = AB'C$, and therefore $AB' = B'C$. $A \neq \varepsilon$ so that $|B'| < n$ and Lemma 2 follows from induction on n, as illustrated in Fig 8b, with $U = a_1 \ldots a_{n-km} = c_{n-km+1} \ldots c_m$, $V = a_{n-km+1} \ldots a_m = c_1 \ldots c_{n-km}$ and k is the number of steps necessary to subdivide B into parts of length less than m, and is given by $\lceil n/m \rceil - 1$, where $\lceil n/m \rceil$ is the ceiling function, giving the smallest integer greater than n/m. □

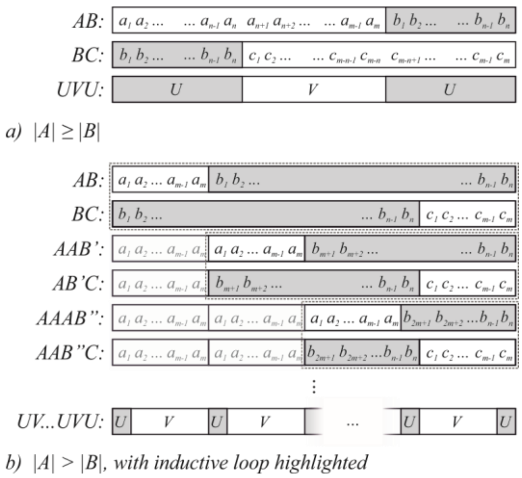

Fig. 10. Illustration of Lemma 2, with words represented by rectangles

The combinatorial structures that result from Lemma 2 are analogous to the edge structures identified in Fig 8. This is because the symmetry of the squares requires that edges with identified parts be equal to their mirror images, which themselves contain the same parts. But, this condition of symmetry is stronger than the conditions under which Lemma 2 holds, and the structures of embedded lines can be further explored by considering words under similar symmetry constraints.

As one-dimensional strings, the symmetry group of words is defined by one-dimensional translation and reflection. Translation results in periodic words, where an infinite sequence $S = s_1s_2 \ldots$ is called periodic if there exists an integer $p \geq 1$, called a period, such that for each $n \geq 0$, $s_{n+P} = s_n$. A finite word A in Σ^* is called periodic if there exists an integer $p \geq 1$, such that A is a prefix of an infinite sequence with period p. The period of A is the smallest such integer. For example, the word *abaaba*, has periods of 3, 5, and 6, and its period is 3. Lemma 2 shows that equal words with common parts are intrinsically periodic in nature, with a period of $|U|+|V| = m$, and the same is true for lines embedded in lines, as illustrated in Fig 8.

Reflection of a string can result in palindromic words. If $A = a_1 \ldots a_m$ is a word over Σ^*, then $\overline{A} = a_m \ldots a_1$ is the word obtained by reflecting A, i.e. reading A backwards.

A *palindrome* is a word that is equal to its reflection: A is a palindrome if $A = \overline{A}$, so that $a_1 = a_m$, $a_2 = a_{n-1}$, … . For example, *rotavator* is an example of a palindrome, and trivially, the empty word ε and all words of length 1, are also palindromes.

In Fig 8, the symmetry of the squares requires that the decomposition of edges are palindromic, which in Lemma 2, equates to the requirement that B is palindromic and that $W = AB = BC$ is also palindromic. This requirement imposes further constraints on the structure of embedded parts. In particular, A and C are reflections of each other because if $W = \overline{W}$ then $AB = \overline{C}\overline{B}$ and $B = \overline{B}$ gives $A = \overline{C}$. Also, U and V are palindromic because if $B = \overline{B}$ then $(UV)^kU = \overline{U}(\overline{V}\overline{U})^k = (\overline{U}\overline{V})^k\overline{U}$ so that $U = \overline{U}$ and $V = \overline{V}$.

In light of these conditions, Lemma 2 accounts for the structures observed in Fig 8, with words and their parts analogous to lines and their parts. To make this explicit, let W represent a line of length l, A represent a line of length m, and B represent a line of length n. Embedding B in W, such that W and B retain their reflective symmetry, gives rise to a palindromic periodic structure,

$$W = AB = UV(UV)^kU = (UV)^{k+1}U$$

where U is a line of length l_u, V is a line of length l_v and k is given by $\lceil n/m \rceil - 1$. The period of this structure is $l_u + l_v = m$, and given that $W = AB = (UV)^{k+1}U$ and $B = (UV)^kU$, the lengths l and n can be written as

$$l = (k+2)l_u + (k+1)l_v$$
$$n = (k+1)l_u + kl_v$$

and it follows that

$$l_u = (k+1)n - kl$$
$$l_v = (k+1)l - (k+2)n$$

For example, if $n = \frac{5}{8}l$, then $m = \frac{3}{8}l$ and $k = \lceil 5/3 \rceil - 1 = 1$, $l_u = \frac{1}{4}l$, $l_v = \frac{1}{8}l$. This confirms observations of Fig 8c, where the edge of the larger square is composed of three line segments of length l_u and two line segment of length l_v, so that $l = 3l_u + 2l_v$, and the edge of the smaller square is composed of two line segments of length l_u and one line segment of length l_v, so that $n = 2l_u + l_v$.

In Fig 8, part structures are differentiated according to values of k, with $k = 0$ in Figs 8a and b, $k = 1$ in Figs 8c and d, $k = 2$ in Figs 8e and f, and $k = 3$ in Figs 8g and h. The limits identified for any given value of k result from definitions of l_u and l_v, so that as $n \to (k+1)l/(k+2)$, $l_u \to l/(k+2)$ and $l_v \to 0$. This was identified in Fig 8, where in Fig 8a, as $n \to \frac{1}{2}l$, $l_u \to \frac{1}{2}l$ and $l_v \to 0$, and in Fig 8b, $n = \frac{1}{2}l$, $l_u = \frac{1}{2}l$ and $l_v = 0$. Similarly, in Fig 8c, as $n \to \frac{2}{3}l$, $l_u \to \frac{1}{3}l$ and $l_v \to 0$, and in Fig 8d, $n = \frac{2}{3}l$, $l_u = \frac{1}{3}l$ and $l_v = 0$.

Also, as $n \to l$, $m = l - n \to 0$, so that $k = \lceil n/m \rceil - 1 \to \infty$, and the part structure of the edges get finer and finer with the number of line segments always increasing. But, at the limiting case where $n = l$, this structure disappears and $l_u = l$. This final point highlights the difference between words and shapes.

Shapes are defined on the continuum, e.g. a line can take a length of any real value, and they can always be decomposed into finer and finer parts. Conversely, words have a fixed granularity defined by a single letter, and consequently can only take integer lengths, and have a maximum decomposition into individual letters. Despite this difference, consideration of words is useful for understanding the part structure of embedded shapes, but it should be noted that the resulting conclusions cannot in turn be applied to words.

4 Discussion

This paper has examined the part structures arising from an elementary shape computation, namely application of a shape identity rule which recognises embedded squares. This rule, illustrated in Fig 3a, was applied to a range of shapes composed of two squares adjoined at a common edge and sharing a common end point, and it was shown that in order for a symbolic shape structure to accommodate all possible applications of the rule, the shapes require a periodic palindromic structure, the nature of which depends on the relative positioning of the two squares.

The diversity and complexity of possible part structures necessary to accommodate application of elementary shape rules is in marked contrast to applying symbolic surrogates for shape rules in a purely combinatorial manner where parts that are created remain separate and do not merge and disaggregate. There are several consequences of the complexity and diversity of part structures required by shape computation. Fitting these structures together in a coherent way so that a rule (and its induced parts) progresses seamlessly to the next rule (and its corresponding structures), is a problem in its own right. One lesson of this paper is that it might be expedient to 'lose' structures between rule applications in a dynamic reinterpretation of structure. Carrying the baggage of past structures and ensuring the consistency among these structures has marginal benefit in design where current novel and innovative interpretations are emphasised. Although interpretation history (though tracking past structures) seems central for the cultural context of shape designs this is not necessarily an argument for maintaining an accumulating part structure which mirrors history especially when that structure quickly becomes unwieldy. Alternatively the generative reconstruction of possible histories as required may be more effective.

The diversity of part structures induced by shape rules is greater in shape computations than in their symbolic mirrors. The extent of this diversity reflects the different perceptions and interpretations of shapes adopted by different participants in the cultural, artistic or design environment. However, with the possibility of diverse and complicated part structures arising from each rule (to recognise or transform a shape), this paper identifies a significant combinatorial challenge in creating coherent part structures. Continual reinterpretation becomes a critical strategy in shape computation. This necessity may not be apparent when shapes are represented symbolically and rules comprise algebraic operations on these symbols. CAD representations of shapes which work with such symbolic operations are fundamentally constrained in their flexibility to generate designs according to artistic and cultural drivers. These constraints increase as the differences between parts decreases. Although part structures induced by elementary shape rules present finer divisions as differences between the embedded line and its host decreases, the structure remains finite, increasing in complexity until embedding reaches equality and the part structure flips to identity, the shape itself.

In general, the complexity and diversity of part structures is extensive. This paper systematically explored what is perhaps the simplest case, when the shape consists of a line segment and an embedded line with an identity rule which can be applied, under Euclidean transformations and scale, to each of the lines. Research is ongoing to investigate the complexity and diversity of part structures for more complicated shapes and other rule schemas beyond identity. For example, the diversity in part structures consequent on rule application is extended in the elementary embedding of two lines in a third, as exemplified in the configuration highlighted in Fig 11, where three squares share a common edge. The resulting part structures do not all appear to be extensions of the finite structures of parts for one line embedded in another. For certain dispositions and sizes of two or more embedded lines it appears that finite part structures are not always possible. This further divergence between the properties of visual shape descriptions and those used in conventional computations which are finite and symbolic, will be examined in detail in a subsequent paper.

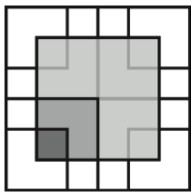

Fig. 11. Configuration embedded in a Korean lattice pattern

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References

1. Prats, M., Lim, S., Jowers, I., Garner, S. W., Chase, S. C.: Transforming shape in design: Observations from studies of sketching. Design Studies, 30(5), 503-520, (2009)
2. Stiny, G .: Shape: Talking about Seeing and Doing. MIT Press, Cambridge, (2006)
3. Stiny, G.: Useless rules. Environment and Planning B Planning and Design, 23, 235–7, (1996)
4. Jowers, I., Prats, M., McKay, A., Garner, S.: Evaluating an eye tracking interface for a two-dimensional sketch editor. Computer-Aided Design, 45, 923–936, (2013)

5. Earl, C. F.: Shape boundaries. Environment and Planning B Planning and Design, 24, 669-687, (1997)
6. Krishnamurti, R.: The maximal representation of a shape. Environment and Planning B Planning and Design, 19, 267-288, (1992)
7. Krstic, D.: Shape decompositions and their algebras. Artificial Intelligence for Engineering Design, Analysis and Manufacturing, 19, 261-276, (2005)
8. Lothaire, M.: Combinatorics on Words (Second Edition), Cambridge University Press, (1997)
9. Lyndon, R.C., Schützenberger, M.P.: The equation $a^M = b^N c^P$ in a free group. Michigan Mathematical Journal, 9, 289-298, (1962)