# Pose, Posture, Formation and Contortion in Kinematic Systems

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**Abstract.** The concepts of pose, posture, formation and contortion are defined for serial, parallel and hybrid kinematic systems. Workspace and jointspace structure is examined in terms of these concepts. The inter-relationships of pose, posture, formation and contortion are explored for a range of robot workspace and jointspace types.

## 1 Introduction

Robot manipulators (serial, parallel and hybrid) have two spaces associated with them. Usually the *workspace* is considered to be the physical space in which the robot is embedded and through which it executes movement. At each instant each link of the robot occupies a position and orientation in the workspace. Similarly the *jointspace* is usually considered to be the more abstract space defined by the (driven) joint variables. At each instant the values of the (driven) joint variables determine the state of the robot in the jointspace.

Earlier work in this field used concepts, such as *closure* and *assembly configuration*, derived from the displacement analysis of single loop kinematic linkages and their associated serial systems [Rooney and Duffy, 1972]. More recently there has been considerable interest in parallel systems [Hunt and Primrose, 1993; Gosselin and Merlet, 1994; Duffy, 1996; Merlet,1996; Tanev, 1998], but most work to date has focussed on either the workspace or the jointspace but not usually on both. Sometimes definitions and concepts associated with both spaces have been somewhat imprecise, or confused, or even occasionally contradictory. This situation is unsatisfactory especially when the *mapping* between the spaces is to be investigated. One aim of this paper is to establish some terminology and definitions that are generic and consistent, so that they may be applied with common justification to serial, parallel and hybrid kinematic systems.

## 2 Pose, Formation and Workspace

In a kinematic system, one way to locate a rigid link in space requires the specification of a position and an orientation with respect to some (global) coordinate frame that is considered to be fixed. The *pose* of the link is defined to be the combination of its position and orientation and as the link moves it traces out a sequence of poses. A typical kinematic system consists of several interconnected links with each link having its own pose at each instant. The configuration of the complete system is therefore specified by a set of poses (one for each link) that change with time as the system moves. However, attention is usually focussed on the end-effector (for serial systems) or the moving platform (for parallel systems), and hence on the pose of just one link.

The workspace of a robot is often defined in terms of the 'reach' of the system, specified by its particular geometrical parameters. Usually the primary definition implicitly involves the orientation of the end-effector, since the orientation, size and shape of the latter determine the maximum and minimum reaches of the robot. The workspace of a 2D (3D) system is then seen as a two-dimensional (three-dimensional) Euclidean space.

However, in a 2D kinematic system the pose of a link involves three variables (two for position and one for orientation of the link), and so it should be represented by a point in a three-dimensional workspace. Moreover, this is not a Euclidean space since the orientation variable is cyclic with period $2\pi$ and distances between pairs of points in this space are not Pythagorean. Similarly, in a 3D kinematic system the pose of a link involves six variables (three for position and three for orientation of the link), and so it should be represented by a point in a six-dimensional workspace. Again, this is not a Euclidean space since the orientation variables are cyclic with period $\pi$ or $2\pi$ and distances between pairs of points in this space are not Pythagorean.

These considerations lead to the following generic definitions:

- The **workspace** of a robot kinematic system is the space defined by all those variables required to specify the position and orientation of the end-effector or moving platform with respect to a global fixed frame. This general definition of workspace refers to the type of robot kinematic system (such as serial 3R manipulator arm, or parallel 3-RPR platform) rather than to a particular system with particular geometry. The workspace of any specific robot geometry is then a subset of the general workspace with various geometrical limits imposed by the sizes and shapes of the links etc.;
- The **pose** of a link is the combination of its position and orientation specified with respect to a global fixed frame. It is just a point in the workspace;
- A **formation** is defined to be a collection of related poses for one of the links, such as the end-effector or moving platform. It is a subset of the workspace and the workspace is thereby partitioned into separate formations.

## 3 Posture, Contortion and Jointspace

An alternative way to locate a rigid link in space is to specify some of its relative spatial relations with respect to (local) coordinate frames attached to other links. For a kinematic system the joint variables determine the relative spatial relations between adjacent connected links. The *posture* of any link is defined to be the combination of (driven) joint variables that locate the link in space [Rooney and Earl, 1983], and as the link moves it passes through a sequence of postures. Thus both the pose and the posture of a link locate it in space. A typical kinematic system consists of several interconnected links, with each link having its own posture at each instant. The configuration of the complete system is therefore specified by a set of postures (one for each link) that change with time as the system moves. However, attention is usually focussed on the end-effector (for serial systems) or the moving platform (for parallel systems), and hence on the posture of just one link.

These considerations lead to the following generic definitions:

- The **jointspace** of a robot kinematic system is the space defined by all those (driven) joint variables required to specify the position and orientation of the end-effector or moving platform. This general definition of jointspace refers to the type of robot kinematic system (such as serial 3R manipulator arm, or parallel 3-RPR platform) rather than to a particular system with particular geometry. The jointspace of any specific robot geometry is then a subset of the general jointspace with various limits imposed by the ranges of motion of the joints etc.;

– The **posture** of a link is the combination of joint variable values specifying its position and orientation with respect to local frames at the driven joints. It is just a point in the jointspace;
– A **contortion** is defined to be a collection of related postures for one of the links, such as the end-effector or moving platform. It is a subset of the jointspace and the jointspace is thereby partitioned into separate contortions.

## 4 Serial, Parallel and Hybrid Kinematic Systems

The number of (driven joint) variables required for the posture of an end-effector or moving platform link is the same as the number of (position and orientation) variables required for the pose of the link. As any link moves through a sequence of poses it also moves through a corresponding sequence of postures. However, in general it is possible at each instant for each pose to correspond to more than one posture and conversely for each posture to correspond to more than one pose (Figure 1).

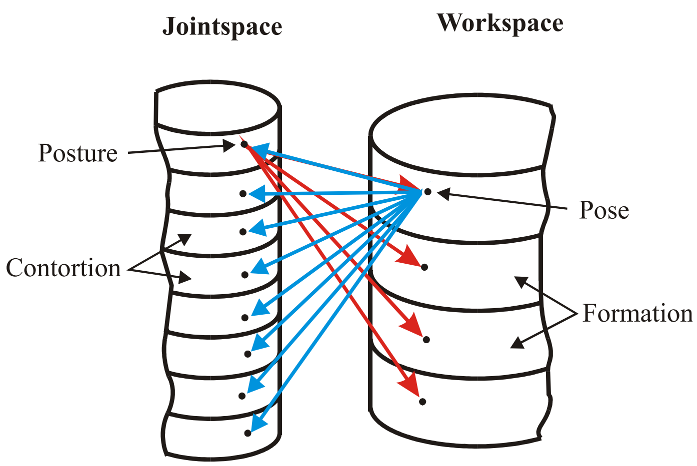



**Figure 1**. Mapping between the joint space and the workspace.

Typically, serial kinematic systems have several postures associated with a given pose, but they always have just one pose associated with any given posture. For example in 3D a general geometry 6R robot manipulator arm may have up to sixteen possible postures for a given pose [Duffy and Crane, 1980; Raghavan and Roth, 1990], and in 2D a general geometry 3-dof robot manipulator arm (with only revolute and/or prismatic joints) may have up to two postures for a given pose. To see how this is possible, consider a planar RRR serial system with the end-effector fixed in some pose. The middle revolute joint can then adopt two configurations without changing the pose of the end-effector, thereby leading to two postures for that one pose (Figure 2).

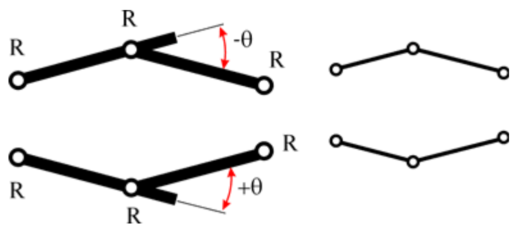



**Figure 2**. Positive and negative angle for a revolute joint, and a schematic representation.

We can show this algebraically. Thus with three revolute joints, $R_1$, $R_2$, and $R_3$ in series, there are two closure equations:

$$x = a_{12}\cos(\theta_1) + a_{23}\cos(\theta_1+\theta_2) + a_{34}\cos(\theta_1+\theta_2+\theta_3) \tag{1}$$

$$y = a_{12}\sin(\theta_1) + a_{23}\sin(\theta_1+\theta_2) + a_{34}\sin(\theta_1+\theta_2+\theta_3) \tag{2}$$

where $a_{ij}$ are the link lengths, $\theta_i (i = 1,2,3)$ are the joint angles, and ( $x, y$ ) are the coordinates of a point on the end-effector with respect to a global coordinate frame.
From eqn. (1) and eqn. (2) we obtain *two* solutions for $\theta_2$ ,

$$\theta_2 = \pm\arccos\left(\frac{\left(x - a_{34}\cos(\phi)\right)^2 + \left(y - a_{34}\sin(\phi)\right)^2 - a_{12}^2 - a_{23}^2}{2a_{12}a_{23}}\right) \tag{3}$$

where $\phi = \theta_1 + \theta_2 + \theta_3$ is the orientation of the end-effector with respect to a global coordinate frame. The two solutions for $\theta_2$ given by eqn. (3) determine exactly two postures of the RRR manipulator.

Similarly, consider a planar RPR serial system with the end-effector fixed in some pose. The prismatic joint can then adopt two configurations without changing the pose of the end-effector, again leading to two postures for that one pose (Figure 3). In this case the prismatic joint must be constructed so that it allows sliding in the negative direction and hence can have a negative value for the joint variable. Again we can show this algebraically. Thus with one revolute joint $R_1$, one prismatic joint $P_2$, and a second revolute joint $R_3$, in series, there are two closure equations. In this case, the coordinates of a point on the end-effector are:

$$x = -a_{12}\sin(\theta_1) + S_2\cos(\theta_1) + a_{34}\cos(\theta_1+\theta_3) \tag{4}$$

$$y = a_{12}\cos(\theta_1) + S_2\sin(\theta_1) + a_{34}\sin(\theta_1+\theta_3) \tag{5}$$

where $a_{ij}$ are the link lengths, $\theta_i (i = 1,3)$ and $S_2$ are the joint variables, and ( $x, y$ ) are the coordinates of a point on the end-effector with respect to a global coordinate frame.

From equations (4) and (5) we obtain two values for $S_2$,

$$S_2 = \pm\sqrt{\left[x - a_{34}\cos(\phi)\right]^2 + \left[y - a_{34}\sin(\phi)\right]^2 - (a_{12} - a_{23})^2} \tag{6}$$

where $\phi = \theta_1 + \theta_3$ is the orientation of the end-effector with respect to a global coordinate frame. The two solutions for $S_2$ given by eqn. (6) determine exactly two postures of the RPR manipulator.

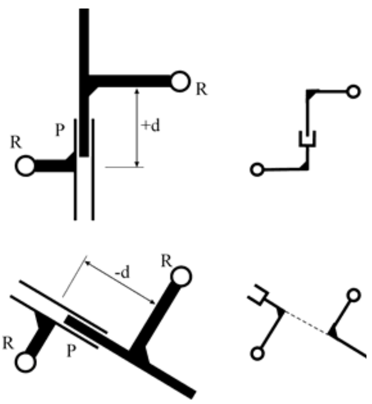



**Figure 3**. Positive and negative distance for a prismatic joint, and a schematic representation

Table 1 shows the number of postures obtained in this way for the RRR and RPR, and additionally for six other planar three-joint systems consisting of just revolute and prismatic joints. Each of the four systems containing at least two revolute joints has two postures, whereas each of the remaining four systems containing at least two prismatic joints has just one posture. In Table 1 a pose or posture is denoted by the symbol $L_j^i$, where *L* represents the end-effector link, the subscript *j* indexes the pose and the superscript *i* indexes the posture. This notation is a general one and it is also used later in Tables 2 and 3 for parallel and hybrid systems, respectively.

In contrast to the situation with serial systems, parallel kinematic systems are usually considered (correctly) to have several poses associated with a given posture, but are also considered (incorrectly) to always have just one posture associated with any given pose. For example the Stewart platform may have up to 12 possible poses for a given posture [Lazard and Merlet, 1994], or the general Stewart type platform – up to 40 possible poses [Husty, 1996].

In general a parallel manipulator system may have more than one pose for each posture and simultaneously more than one posture for each pose. This is shown schematically in Figure 1 where several postures (one from each contortion) map to each pose and where each posture maps to several poses (one in each formation). To see how this is possible, consider one of the legs of a three-legged 3-RRR parallel system with the platform fixed in some pose.

**Table 1**. Examples of different postures of planar serial manipulators.

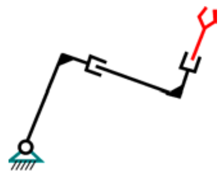
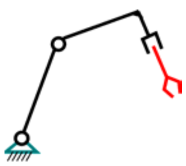
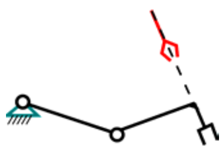
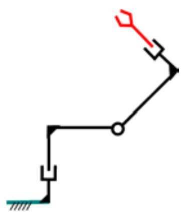
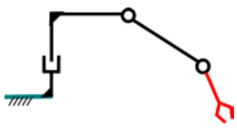
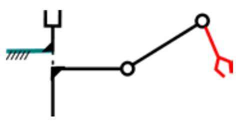
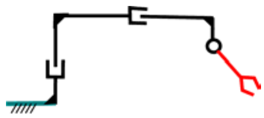
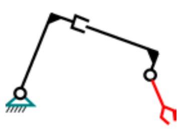
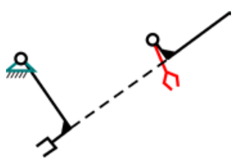
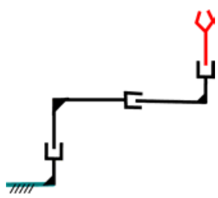
| SERIAL MANIPULATORS | | | | |
|---|---|---|---|---|
| **Jointspace** / POSTURES | | | | |
| **RRR** |  $L_1^1$ |  $L_1^2$ | **RPP** |  $L_1^1$ |
| **RRP** |  $L_1^1$ |  $L_1^2$ | **PRP** |  $L_1^1$ |
| **PRR** |  $L_1^1$ |  $L_1^2$ | **PPR** |  $L_1^1$ |
| **RPR** |  $L_1^1$ |  $L_1^2$ | **PPP** |  $L_1^1$ |

**Table 2**. Examples of different poses and postures of planar parallel manipulators.

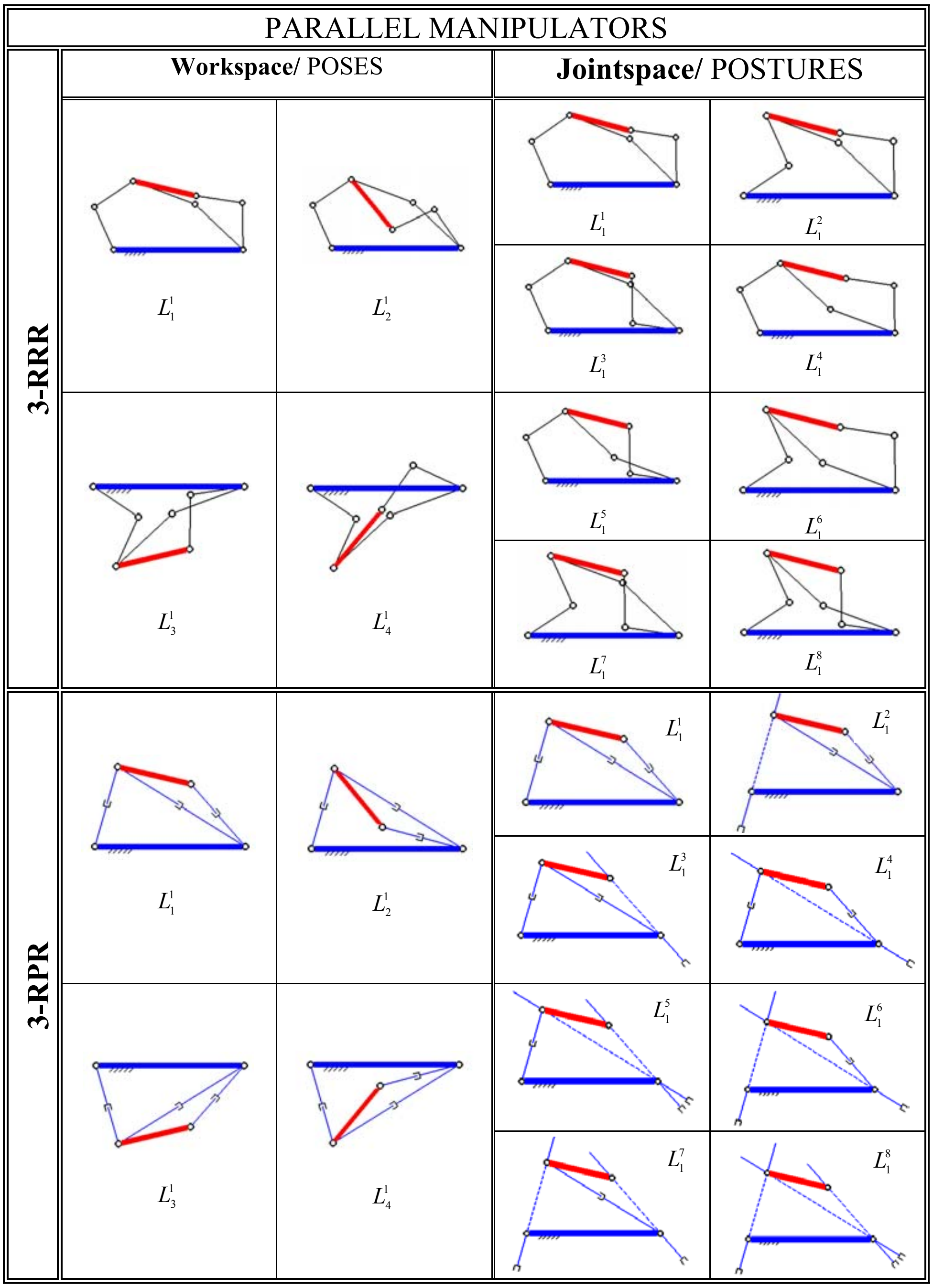
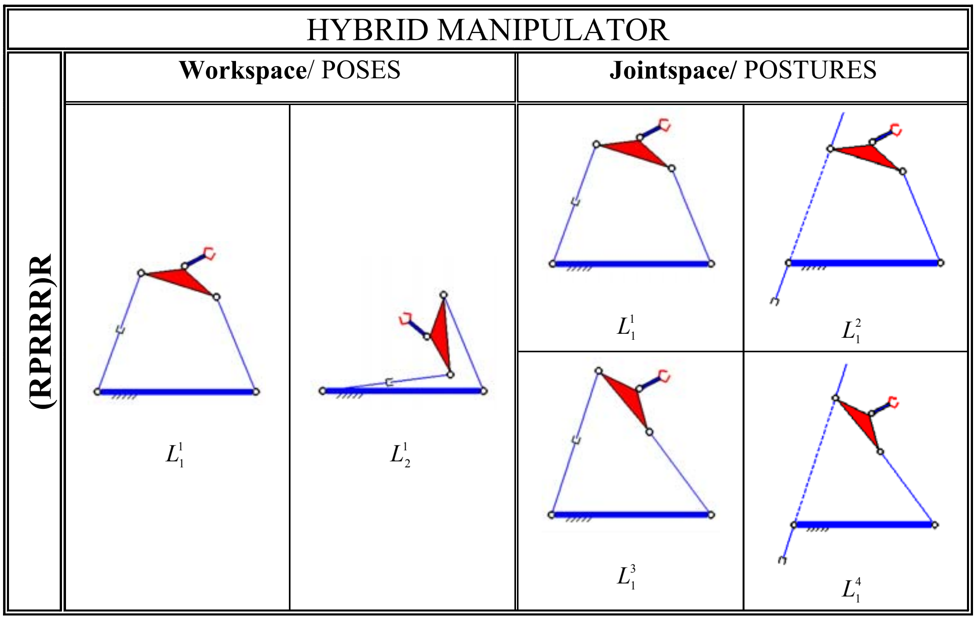


**Table 3**. Examples of different poses and postures of a planar hybrid manipulator.




As with the serial system considered earlier (Figure 2), the middle revolute joint in each leg can adopt two configurations without changing the pose of the platform, thereby leading to eight postures in total for that one pose. With general geometry this 3-RRR system also has up to six poses for each posture, as is well known [Hunt, 1983]. The upper half of Table 2 shows a particular example of a 3-RRR parallel manipulator system that has special geometry. This system has only four poses for each posture but a full complement of eight postures for each pose. (Again the $L_j^i$ notation is used here to label and distinguish the different configurations.)

The planar three-legged 3-RPR parallel system may be treated in a similar way to the 3-RRR system if the prismatic joints in the former are physically designed to allow negative-valued joint displacements. As with the serial system considered earlier (Figure 3), the prismatic joint in each leg can adopt two configurations without changing the pose of the platform, thereby leading to eight postures for that one pose. With general geometry this 3-RPR system (like the 3-RRR) also has up to six poses for each posture, as is well known [Hunt, 1983; Gosselin and Merlet, 1994]. The lower half of Table 2 shows a particular example of a 3-RPR parallel manipulator system that has special geometry [Duffy, 1996]. This system has only four poses for each posture but a full complement of eight postures for each pose. (Again the $L_j^i$ notation is used here to label and distinguish the different configurations.)

Finally, if we consider hybrid (combinations of serial and parallel) kinematic systems we find that (as with the parallel case) a hybrid system has in general more than one posture for each pose and more than one pose for each posture. Table 3 shows the (RPRRR)R hybrid system, where again the $L_j^i$ notation is used to label and distinguish the different configurations.

## 5 Workspace and Jointspace Structure

Since in general the mapping between jointspace and workspace is many-many (Figure 1) the geometrical and the topological structures of workspaces and jointspaces are often complicated. However, they may be explored in terms of pose, posture, formation and contortion. As a first step we consider briefly the RRR and RPR serial systems.

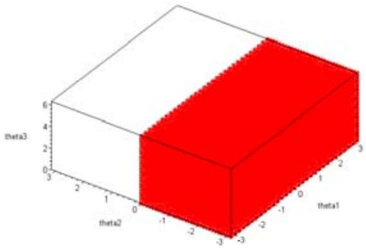



**Figure 4**. Jointspace (contortion 1) for the RRR planar serial manipulator.

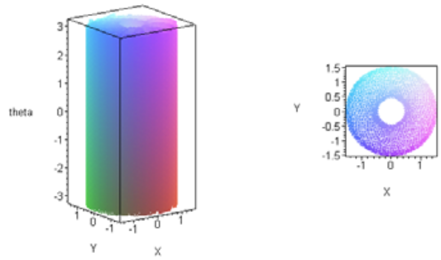



**Figure 5**. Workspace of the RRR planar serial manipulator.

Figure 4 shows one of the two contortions (of postures) in the jointspace of the RRR planar serial manipulator. Figure 5 shows the single formation (of poses) in its workspace, corresponding to both contortions in the jointspace.

Similarly, Figure 6 shows one of the two contortions (of postures) in the jointspace of the RPR planar serial manipulator, and Figure 7 shows the single formation (of poses) in its workspace, corresponding to both contortions in the jointspace..

A further paper on the structure of workspaces and jointspaces is planned.

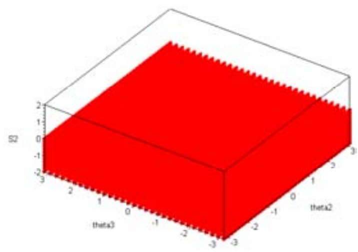



**Figure 6**. Jointspace (contortion 1) for the RPR planar serial manipulator.

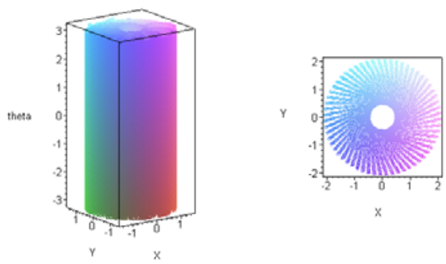



**Figure 7**. Workspace of the RPR planar serial manipulator.

## 6 Discussion and Conclusions

The paper has examined some aspects of the relationship between jointspace and workspace for robot manipulators. It has concentrated on planar kinematic systems for clarity but the approach applies equally to spatial systems. The concepts of pose and formation for workspace, and posture and contortion for jointspace have been introduced and explored for a range of serial, parallel and hybrid systems. Workspace and jointspace structures have been examined briefly, and a further paper is planned to explore this in more detail and to apply the concepts in the wider context of forward and reverse kinematics.

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