A MULTITAPER-RANDOM DEMODULATOR MODEL FOR NARROWBAND COMPRESSIVE SPECTRAL ESTIMATION

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***Abstract*—The *random demodulator* (RD) is a *compressive sensing* (CS) system for acquiring and recovering bandlimited sparse signals, which are approximated by multi-tones. Signal recovery employs the discrete Fourier transform based *periodogram*, though due to bias and variance constraints, it is an inconsistent spectral estimator. This paper presents a *Multitaper* RD (MT-RD) architecture for compressive spectrum estimation, which exploits the inherent advantage of the MT spectral estimation method from the spectral leakage perspective. Experimental results for sparse, narrowband signals corroborate that the MT-RD model enhances sparsity so affording superior CS performance compared with the original RD system in terms of both lower power spectrum leakage and improved input noise robustness.**

***Index Terms*—compressive sensing, random demodulator, Multitaper, Slepian sequences, eigenspectra**

I. Introduction

In *cognitive radio* (CR) networks, one of the key tasks is to identify the presence of spectrum holes by using spectrum sensing [1], which is extremely challenging due to the very high sampling rates involved [2]. Signals in wireless networks are characteristically sparse in the frequency domain because of spectrum underutilization [3], and this has given impetus to *compressive sensing* (CS) techniques [4], [5] which can efficiently sense either sparse or compressible signals by sampling them at sub-Nyquist rates.

One specific CS technique is the *random demodulator* (RD) [6] which has been proven to be effective for signals categorised by being bandlimited, periodic and frequency sparse and so as a consequence, are able to be approximated by a finite tone model [7]. Furthermore, if a multiband signal has a small total active bandwidth [8], then it too can be approximated by tones so enabling the RD to be applied in CR spectrum sensing applications.

The RD performs signal recovery of frequency sparse signals by using as its sparsity basis, the discrete Fourier transform (DFT) based *periodogram* spectrum estimation method [7]. The finite length of real signals however, causes sidelobe leakage which leads to a biased spectral estimate. Moreover, the *periodogram* has inherent variance limitations because it does not decrease as the signal length increases [8], [9], so it not a consistent spectrum estimator. In contrast, the *Multitaper* (MT) spectral estimation technique uses *Slepian sequences* [1], [3], [8], [10] to afford a propitious trade-off between spectral bias, variance and resolution, exhibiting maximal energy concentration within a given bandwidth, centred about a particular frequency. For these reasons it has emerged as a viable sensing technique for CR applications, with the added attraction being its suitability for multiband signals, since by its nature, it assumes signals have energy concentrated within specific frequency bands. It needs to be emphasised the MT method only addresses spectral leakage due to the periodogram and not energy emanating from the tone grid as a consequence of tone-model mismatch [11]. From a CR perspective, it is also important to stress the aim is to recover a signals *power spectral density* (PSD) in order to identify occupied bands rather than the reconstruction of the original signal.

This paper investigates seamlessly integrating the MT spectrum estimator into the RD model for enhanced CS performance, particularly for frequency sparse, narrowband signals, such as textitamplitude modulated (AM) and chirps, which are widely encountered in telecommunications applications, radar, and geophysics [7], [11]. For these signals, the proposed MT-RD architecture consistently provides superior CS performance in comparison with the original RD in terms of both spectral leakage and robustness to input *signal to noise ratio* (SNR). This superior CS performance is ascribed to the enhanced signal sparsity the inclusion of the MT estimator gives the RD model due to the maximal energy concentration within a given bandwidth and ensuing sidelobe suppression. For completeness, the performance of the MT-RD architecture is also examined for higher-order modulated multiband signals like *quadrature phase shift keying* (QPSK), which are defined in current CR network standards [12]–[14].

The remainder of the paper is organized as follows. Section 2 presents a synopsis of RD-related and other CS designs and implementations, while Section 3 details the role of MT within the MT-RD model. A critical results discussion is provided in Section 4, with some concluding observations being made in Section 5.

II. Previous Work

Since its introduction, various enhancements to the original RD architecture [6], [7] have been proposed which generally involve either modifying the underlying RD structure or applying different spectrum estimation techniques to overcome the innate drawbacks of the periodogram. These enhancements however, tend to come at the cost of either increased system complexity [2], [15] or the requirement to have *a priori* knowledge about the input signal [16], [17]. Alternative RD-based designs including the *compressive multiplexer* (CM) [18] and *polyphase* RD (PRD) [19] address some of the limitations of the RD relating to non-ideal chipping sequences and inaccuracies in the impulse response of the low-pass filter (integrator) [20], [21]. These structures however impose additional hardware complexity. From a spectral estimation perspective, the MT method is considered in combination with *singular value decomposition* (SVD) [22]. The basic premise however, is of multiple RD structures equal to the number of Slepian sequences employed, which given the high cost of the MT and SVD, can impose a significant computational overhead on the model. In [23], the MT spectral estimator is applied instead of the periodogram along with CS greedy algorithms to achieve improved signal recovery, though only sinusoidal and narrowband modulated signals were considered, while complexity implications were not analysed. Similarly in [24], Slepian sequences were used to construct matrices for improved sparse signal recovery, but without analysing either leakage performance or the complexity cost incurred.

III. MT-RD Compressive Sensing Framework

The operation of the RD structure shown in Figure 1, can be explained as firstly sampling the input $x(t)$ at the Nyquist rate to yield a discrete-time vector $x(n)$ of length N before applying matrix Φ to obtain the measurements [25]:

$$y = \Phi x = \Phi F^{-1} f \tag{1}$$

where is an *NxN* random matrix exhibiting low coherence [25] and *F* the DFT sparsity matrix. Recovery of the frequency vector f is achieved by applying *l1-norm minimization* [26].

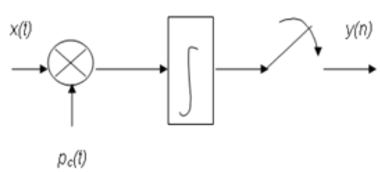

Fig. 1: The original RD block diagram

Using the periodogram incurs a number of drawbacks which gave impetus to investigate embedding the MT method into the RD model. The MT technique applies multiple windows to $x(n)$ to determine a corresponding set of Fourier Transforms. These windows are orthonormal discrete prolate spheroidal (Slepian) sequences (tapers) with their respective DFT having maximum energy concentration for a given bandwidth resolution β [8], [10]. The degree of concentration is reflected by the corresponding sequence eigenvalues [1], [10] with the number of sequences depending on N and β. The Fourier eigenspectra of $x(n)$ are determined from:

$$f_k^l = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} v^l(n) x(n) \omega_N^{kn} \tag{2}$$

where v^l is the eigenvector representing the *lth* Slepian sequence, $\omega_N = e^{\frac{-j2\pi}{N}}$ and f_k^l is the *kth* frequency of vector f^l. Equation (2) can be expressed in vector form as:

$$f_k^l = F V^l x \tag{3}$$

V^l is a *NxN* diagonal matrix, whose main diagonal contains the v^l eigenvector.

If there are *L* Slepian sequences, then a weighted -based Fourier transform can be formed from the *L* eigenspectra as follows:

$$f_{MT} = \frac{\sum_{l=0}^{L=1} \lambda_l f^l}{\sum_{l=0}^{L-1} \lambda_l} \tag{4}$$

where λ_l is the eigenvalue associated with the *lth* Slepian sequence. An attractive property of f_{MT} is that each eigenspectrum contributes according to the degree of energy concentration it exhibits, which depends upon its corresponding eigenvalue. Expressing (4) in vector form:

$$f_{MT} = FSx = (FS)x = F_{MT} x \tag{5}$$

where S is a *NxN* diagonal matrix whose numerical values depend only on the Slepian sequences and their associated eigenvalues, while F_{MT} is derived by windowing F with S. Using (5), (1) can be expressed as:

$$y = \Phi x = \Phi(F_{MT}^{-1}) f_{MT} = \Theta_{MT} f_{MT} \tag{6}$$

So the recovery process for the MT-RD model uses $\Theta_{MT} = \Phi(F_{MT}^{-1})$ instead of Θ. An alternative and insightful interpretation of this is to rewrite (5) as:

$$f_{MT} = F(Sx) = F x_{MT} \tag{7}$$

where x_{MT} is the input signal windowed by the diagonal matrix S. By using (7), (1) can recover f_{MT} as follows:

$$y_{MT} = \Phi x_{MT} = (\Phi S)x = \Phi_{MT} F^{-1} f_{MT} = \Theta f_{MT} \tag{8}$$

where $\Phi_{MT} = \Phi S$ is the measurement matrix Φ modified by S. (8) reveals that the MT-RD architecture not only modifies *F*, but can also be used for MT-based spectrum estimation by employing the modified measurement matrix Φ_{MT} to sample x and then recover f_{MT}.

Both interpretations of the MT-RD model exploit the enhanced MT properties integrated into the model via S, which scales either Φ or F, so both retain their respective characteristics of randomness and sparsity. Due to its diagonal structure, S does not affect the low coherence of Φ, while creating a modified basis in which signals exhibit greater sparsity so improving the CS performance. Furthermore, the S matrix values can be embedded into the initialisation parameters of the MT-RD model and stored in either Φ_{MT} or F_{MT}, so the overall model complexity is not augmented. Crucially, MT-RD is signal independent so no extra burden is imposed on the signal providers, which from a CR system perspective, is an advantageous feature.

IV. Results Discussion

To critically evaluate the new MT-RD CS model, a series of experiments were undertaken upon a MATLAB-based simulation platform using an HP Pavilion G6 Notebook with 2.4GHz Intel Core-i5 and 4GB RAM. Three dissimilar test signals of length 300ms were considered reflecting the signal modulation types typically used in CR network standards [17, 18], namely: *i)* A band-pass signal centred at 200Hz of 40Hz bandwidth was AM modulated by a 1.2KHz carrier and approximated with 76 tones, out of a total number of 512, spaced at 1Hz. *ii)* A chirp signal centred at 1.3kHz with a chirp rate of approximately 134Hz/sec and a sampling frequency 1.4 times higher than Nyquist which corresponds to a signal length of 1024 samples. *iii)* a QPSK modulated signal centred at 1kHz and 2.5kHz, with a bit rate of 215bps and sampling rate 1.3 times Nyquist, corresponding to 2048 samples. While the first two test signals are narrowband, the third was selected to investigate the performance of MT-RD on wideband signals.

Both the AM and chirp signals were sub-sampled at rates between 70%, and 8.75% of the Nyquist rate which corresponded to 512 and 64 samples respectively. For the QPSK signal, the rates were set between 65% and 8.125% of Nyquist, corresponding to 1024 and 128 samples respectively. To analyse the robustness of the MT-RD structure, additive white Gaussian noise was added to give an input SNR between 15dB and 5dB to each test signal, with the number of Slepian sequences set to $L = 10$

The results for the chirp and AM signals are displayed in Figures 2 and 3 respectively, where the total percentage energy lying outside the bands of interest is termed the PSD *spectral leakage* and is measured at various sub-Nyquist sampling rates. The graphs reveal that for both signal types there is a reduction of PSD *spectral leakage* even for sampling rates below 20% of Nyquist. The MT-RD structure is also more robust to noise, since the performance improvement is consistent across the full input SNR range. For example, for the chirp signal in Figure 2, the average spectral leakage reduction achieved is 21%, 20%, 19% and 16% at input SNR levels of 15dB, 10dB, 8dB and 5dB respectively, while for the AM chirp signal (Figure 3), the corresponding leakage reductions are 16%, 17%, 23% and 19%. This reflects the key maximal energy concentration property [1], [3], [10] of Slepian sequences which is crucially retained in both the Fourier eigenspectra in (2) and weighted Fourier transforms f_{MT} in (8). Applying the MT-RD model to such narrowband signals can be viewed as enhancing signal sparsity by reducing the spectral leakage, so there are fewer significant frequency components. This correspondingly improves CS performance with the corollary being a more accurate spectral estimate.

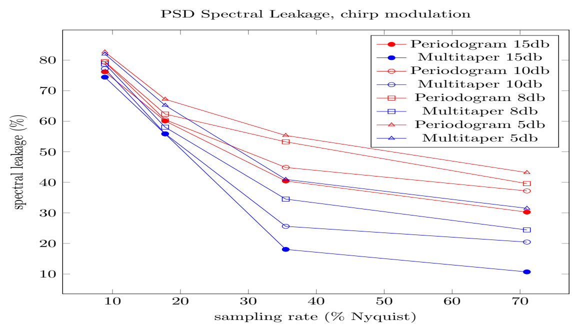

Fig. 2: PSD spectral leakage performance of the *MT-RD* and periodogram for the chirp test signal and various input SNR.

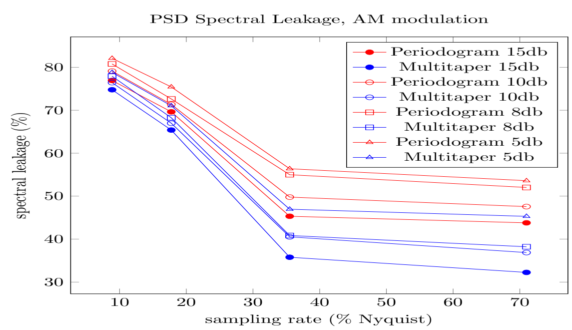

Fig. 3: PSD spectral leakage performance of the *MT-RD* and periodogram for the AM test signal and various input SNR.

To substantiate this superior CS performance, Figures 4 and 5 compare the recovered PSD from MT-RD and periodogram respectively, for both the chirp and AM test signals. The sampling rate and input SNR were arbitrarily chosen at 35% of Nyquist, though other practical values are equally applicable. To corroborate the noise robustness of the MT-RD model, the input SNR was set to just 3dB, rather than 8dB. The resulting spectral leakage is visually much less pronounced in both MT-based test signal PSD, reduced by 20% and 17% for chirp and AM signals respectively, to underscore the MT-RD theory presented in Section 3.2. Figure 6 contrasts the signal sparsity of the MT-RD and periodogram for an AM modulated signal, across a range of input SNR in terms of the number of the active frequency components. The sampling rate was set to 32.5% of Nyquist, though other sampling rates are equally applicable. For illustrative purposes, the active frequency threshold was set at 20dB relative to the maximum energy value, though an arbitrary threshold value is equally valid. The curves reveal the MT consistently enhances

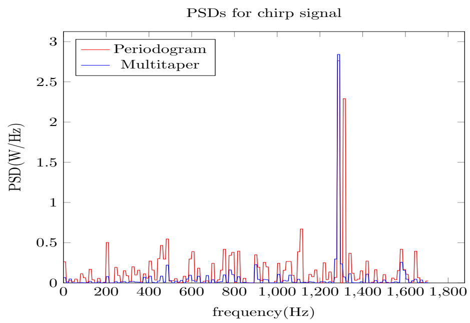

Fig. 4: Recovered normalized MT-RD and periodogram-based PSD of a chirp test signal at 35% Nyquist rate and 3dB input SNR.

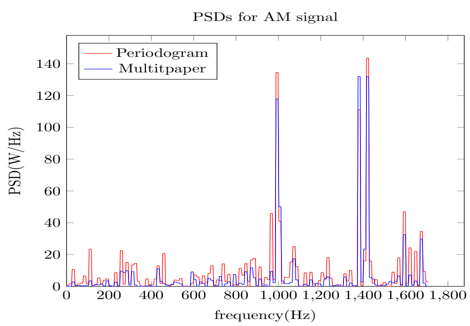

Fig. 5: Recovered normalized MT-RD and periodogram-based PSD of a chirp test signal at 35% Nyquist rate and 3dB input SNR.

signal sparsity (fewer active frequencies) compared with the periodogram, to underscore the fact that the modified basis function discussed in Sections 1 and 3 gives narrowband-type signals greater sparsity. Finally, the corresponding CS

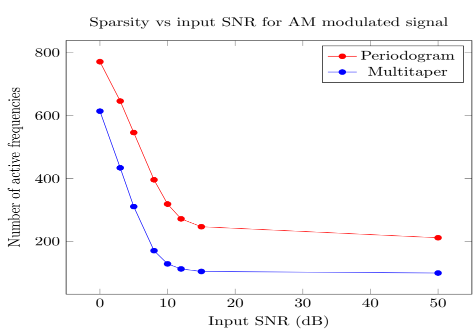

Fig. 6: Effect of MT method on signal sparsity verses input SNR for an AM modulated signal.

results for the QPSK signal are displayed in Figures 7 and 8. The sampling rate was set to 35% of Nyquist, though other rates can be applied. The reason for the apparent unevenness in the MT-RD results, with sometimes a small improvement obtained, allied with occasional minor degradations, is that QPSK signals are characterised by a main lobe being centred about the carrier frequency, with side lobes distributed across the spectrum. These side lobes are part of the true signal spectrum rather than spectral leakage so there are instances where the MT-RD model is unable to increase sparsity and thus overall CS performance. For QPSK and other wideband signal types, adopting a *pre-colouring* strategy within the RD model is a more pragmatic CS solution [16], [17].

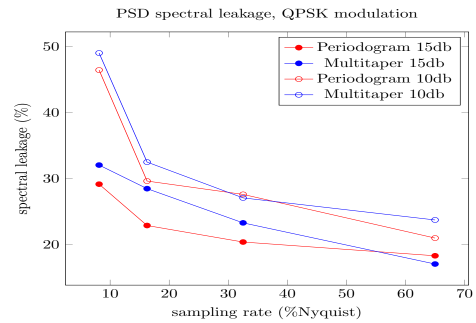

Fig. 7: The effect of MT method on PSD spectral leakage of QPSK test signal for various input SNR.

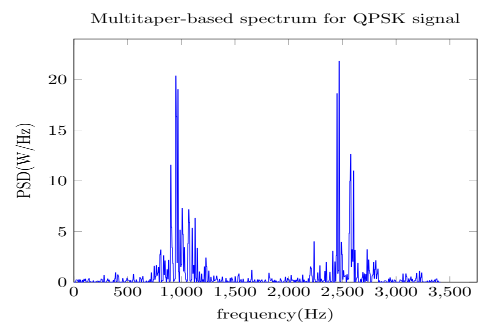

Fig. 8: Recovered normalised PSD of QPSK test signal for MT-RD.

V. Conclusion

This paper has presented a novel *multitaper-random demodulator* (MT-RD) design for compressive spectrum estimation. The model efficaciously exploits the superior spectral leakage properties of the MT spectral estimator in comparison to the Fourier-based *periodogram* for sparse, narrowband, signals, while providing analogous performance to the original RD for high-order modulated signals like QPSK. Experimental results using narrowband amplitude and chirp modulated test signals, confirm the MT-RD model consistently provides enhanced signal sparsity, outperforming the original RD model in terms of both spectral leakage and robustness to input noise. Future work will critically evaluate the MT-RD design for other modulation and access schemes including 16/64QAM and OFDM, as well as comparing the efficacy of the new model against both the compressive multiplexer and modulated wideband converter CS designs.

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