Mathematical resilience: what is it and why is it important?

Book Section

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Many people have difficulties learning mathematics for many different reasons. However many people, otherwise good learners, find mathematical tasks difficult, to the point that they steer clear of any engagement with mathematics. Such people may exhibit a high degree of phobia or anxiety, but many of them simply avoid any situation that might involve mathematical reasoning; we would say too many. This chapter is about what might be done to stop young learners from developing a need to disengage from mathematics.

Working explicitly to develop ***mathematical resilience*** (Johnston-Wilder and Lee 2010) seems to offer a way of recognising the issues that many people encounter when learning mathematics and enabling them to succeed despite those issues. The construct mathematical resilience indicates a positive approach to mathematics that allows people to overcome affective barriers presented when learning mathematics. Mathematical resilience can be developed by anyone; there are clear indicators from literature and from our research about how this can be done. Learners who display mathematical resilience will continue despite feeling 'stuck'; they will persevere by recruiting resources to help them. These resources may be their peer group or possibly on-line resources or an adult who will listen or the even their textbook. Above all, they will not accept the state of 'being stuck' as a permanent position. They will have a growth or incremental theory of learning (Dweck 2000) and therefore know that they can learn more mathematics, provided they find the support they need.

What is mathematical resilience?

Mathematical resilience (Johnston-Wilder & Lee 2010) is a pragmatic, mathematised understanding of the well-established concept resilience. It has been shown (e.g. Kooken et al. 2013) that in order to be resilient mathematically a student must understand the need to struggle mathematically, hold a growth theory of learning and have resources available to them to support their learning. Therefore mathematical resilience describes that quality by which some learners approach mathematics with agency, persistence and a willingness to discuss, reflect and research.

Far too many students, particularly in US and UK, leave compulsory education without effective numeracy. Although reasons for this are complex, a major contributory factor is the quality of mathematics teaching (e.g. Nardi and Steward 2003; Ofsted 2008). We use 'teaching' here and not 'teachers' deliberately; all teachers we have talked with want the best for their students. However many teachers become convinced (Harlen 2005) with some justification, that examinations are the measure of success and that the best way to enable their pupils to pass these examinations is to adopt a style of teaching that privileges instrumental over relational understanding (Skemp 1972) and which equates to a focus on formulas and algorithms (Stigler and Hiebert 2009). Thus memorising is valued over depth of understanding and 'teaching to the test' over enabling pupils to see the power and connectivity of mathematics.

Stigler and Hiebert (2009) showed that mathematics teachers see their role as path-smoothing (Wigley, 1992); instead of allowing students to develop strategies to overcome any obstacles they encounter when learning mathematics. Teachers tend to feel they have failed if their students get 'stuck' and seek to provide help promptly. As a consequence, students develop 'learned helplessness' and assume that inability to immediately see the way forward with a mathematics problem indicates they 'cannot do mathematics'. Many mathematics lessons use a *restricted practice* of teacher exposition of a single isolated technique followed by pupil completion of exercises practising the technique, the exercises being aimed at helping pupils remember how and when to use that technique (Nardi & Steward 2003, Ofsted 2008).Teachers feel they must adopt an algorithmic approach, taking their students step-wise through a process that leads to a solution. 'Remember this!" they say, "and you will not get stuck". Now the path is smooth but there are huge demands on memory and limited scope to adapt ideas. Students do not positively experience the state of 'being stuck' which Mason (1988) regards as a precursor to real learning.

Narratives from people who exhibit mathematics phobia (e.g. Ashcraft, 2002, Newman, 2004 and Hoffman 2010) clarify that the way mathematics is often taught in mathematics classrooms could be termed an unwitting form of cognitive abuse. Boaler (2009), Jain & Dowson (2009) and Baloglu & Koçak (2006) all indicate that certain common ways of working in mathematics classrooms cause anxiety, sometimes extreme, amongst learners. Instances given include asking learners to perform tasks that require rapid feats of memory, requiring pupils to memorise formulae without understanding and divorcing mathematical ideas from the reality they model so powerfully. In this sense, mathematical resilience is a positive adaptation to enable success (Newman 2004). If learners are to engage willingly with mathematics, they must learn that they can struggle through problems, deal with barriers and misunderstandings and think through mathematical ideas, thus they need mathematical resilience.

All learning requires resilience; we contend that the resilience required for learning mathematics is a particular construct because of various factors including: the type of teaching often used when teaching mathematics, the nature of mathematics itself and pervasive beliefs about mathematical ability being 'fixed'. The idea of 'mathematical resilience' offers a positive construct that can help those who are supporting learners of mathematics have a vision of what they are seeking to develop in students and thus to make positive pedagogical choices about the way that they help learners. We understand supporters of those who are learning mathematics to include teachers but also parents or workplace colleagues, anyone who undertakes to be alongside someone seeking to learn mathematics.

The quest for a definition of the construct ***mathematical resilience*** (Johnston-Wilder and Lee 2010) is no simplistic task. There is extensive literature concerning psychological resilience, and how young people 'at risk' of educational failure succeed despite overwhelming trauma or disadvantage. Resilience is *"ordinary magic …made up of ordinary rather than extraordinary processes" (*Masten 2001 p.227). Studies in educational settings in the latter half of the twentieth century focused on students who despite being classified as 'at risk' succeeded academically. This led to the development of the term 'resilience' to describe this invulnerability (Garmezy 1985). The emphasis in these enquiries was on the circumstances that prevented some students succumbing to the risks that appeared to encompass them. This in turn resulted in ideas for helping children to become resilient. Resilience is not a once for all attribute. "Children may be more or less resilient at different points in their lives depending on the interaction and accumulation of individual and environmental factors." (Howard et al. 1999 p.310). Benard (1991) believes that schools could foster resilience by paying attention to social competence, problem-solving skills, critical consciousness, autonomy and a sense of purpose in students. Carr and Claxton (2002) view resilience as a key learning disposition and define it as "the inclination to take on (at least some) learning challenges where the outcome is uncertain, to persist with learning despite temporary confusion or frustration and to recover from setbacks or failures and rededicate oneself to the learning task." (Carr and Caxton 2002 p.12)

Working to develop resilience must involve changes to mindsets (Yeager and Dweck 2012). Many common occurrences in the learning of mathematics foster an entity theory of learning. Such things as the common practice of 'setting' or 'streaming' assume that 'ability' can be accurately measured and changes very little. Although students are sometimes moved between sets or streams, this system assumes that students 'ability' is both measureable and fixed. In addition a prevalent marking system which gives students a mark out of ten, proves to those with an entity theory of learning that they can or cannot 'do' mathematics. When advice on improving is given this indicates an expectation that everyone can improve. The on-going work of Dweck and her colleagues shows how holding an incremental or growth view of learning allows students to engage with pedagogies that enable deep learning in mathematics, allow students to understand mathematics and also increase their scores in mathematics:

> " if students can be redirected to see intellectual ability as something that can be developed over time with effort, good strategies, and help from others, then they are more resilient when they encounter the rigorous learning opportunities presented to them" (Yeager and Dweck 2012 p. 306)

Best practice comes back to the ideas that other work has identified as constituting effective teaching (e.g. educational psychology, educational counselling, teacher professional development). Thus, achievement in school is higher when: teachers teach for mastery; curricula are relevant to students' present and future needs; authentic assessment practices are used; democratic classrooms are created where students contribute to the rule-making and governance; rational, humane and consistent behaviour management techniques are adopted; teachers are warm, approachable, fair and supportive and a range of ways of being successful are made available to students (Porter & Brophy, 1988; Nuthall & Alton-Lee, 1990).

Furthermore speaking or otherwise communicating is an important part of developing mathematical resilience; becoming able to articulate mathematical ideas, concepts and reasoning has a profound effect on the way that learners see themselves (e.g. Lee 2006, Mercer and Littleton 2007, Vygotsky, 1981). An individual takes on the identity of a mathematician (Holland et al. 1998) by learning how to talk like a mathematician. Giving learners the opportunity to 'talk like a mathematician' means that they become someone who 'knows and can do mathematics'; that is, they begin to see themselves as mathematically resilient. Supporters of learners of mathematics seeking to build mathematical resilience will encourage collaborative working where learners support one another in learning, enabling pupils to both understand and articulate their own knowledge and needs in the process of learning and to support others.

Developing Mathematical resilience

There are many ways that mathematical resilience can be developed. There are changes that can be made to pedagogy in schools that are known to make a difference to young people. However if mathematical resilience was not developed in school, it can be developed later. We have shown that three groups of people can use ideas about mathematical resilience to improve learners' attitudes to mathematics and thus increase the likelihood that they will learn and use mathematical ideas.

Teaching for Mathematical resilience

Currently mathematical resilience is not developed purposively; it seems to occur by happenstance where it is developed at all. The term 'mathematical resilience' is useful in drawing teachers' attention to the harm that can be perpetrated and towards using pedagogies that can mitigate any harm that has already been done. As students study mathematics for longer in many countries, teaching in ways that result in young people avoiding mathematics is increasingly untenable. It is not hard to find ideas for developing mathematical resilience; journals aimed at mathematics teachers provide many such ideas. Many mathematics teachers work hard in their classrooms to enable their pupils to enjoy and succeed in mathematics in ways that are known to develop resilience. They encourage their pupils to work collaboratively and to explore, articulate and make connections between areas of mathematics and how it is used in the world outside school. They see understanding as important, rather than speed of recall. Teachers who help their students develop resilience do not pretend that mathematics is easy, or attempt to remove the challenges that mathematics presents. They acknowledge that students will need to struggle to overcome the barriers that most people find when learning mathematics. They help their students find their own ways to overcome these barriers, because they believe that every person can improve their understanding of mathematics given the right support from resources such as the teacher themselves, other students, the internet and so on.

Teachers must be re-assured that teaching for understanding and using mathematics lessons to discuss and collaborate in working for deep learning of mathematical ideas will not harm examination results and has the added advantage that their students will be more resilient, i.e. prepared and able to learn and to use mathematical ideas beyond school.

Coaching for Mathematical resilience

Many schools have a shortage of well-qualified mathematics teachers and are judged on their pupils' examination results. This has often resulted in unsustainable stress placed on teachers who remain in post. Thus, many mathematics learners do not receive high quality teaching designed to enable them to persist with their mathematical learning. One place where such difficulties need to be modified urgently is the workplace, particularly where apprentices are being trained. Any adult in the workplace can be called upon to coach another in using mathematics and the more they encourage explicitly the development of mathematical resilience the more likely they are to truly enable mathematical thinking in those that they coach.

We offered, through the auspices of Asdan, working with the Progression Trust, a training course for people whose role was to help apprentices develop mathematical ideas in the workplace. The role of a coach for mathematical resilience is to help learners to stay longer in what we termed their learning 'growth zone'. The coaches were themselves coached in using language in such a way that the students could recognise and articulate the degree of challenge they are facing; encouraging increasing independence and agency; modelling resilient approaches to mathematics, being part of a community of practice and knowing how to access help (Johnston-Wilder et al. 2013).

Many of the course participants suffered mathematics anxiety themselves; the initial meetings were dedicated to helping them understand that they could bring the resilience that they used in many areas of their life to bear on mathematical learning. They had to begin to explore resilient behaviours before they could feel safe enough to begin to think mathematically. The differing roles of the two tutors on the course were important. One was a mathematics education specialist and the other a coaching specialist; their collaboration ensured the participants were supported as they overcame their own difficulties in approaching mathematics. Time is needed to develop a community of practice; by week three, the group was beginning to function effectively. An effective coach knows and understands mathematical resilience because they have worked through their own anxieties and developed a more positive stance towards mathematics in a safe and collaborative environment. Once this process has been worked through they can coach learners to develop a resilient attitude towards mathematics. Participants should be exposed to mathematical ideas, despite the fact that this is likely to cause discomfort. Only in these circumstances will they learn to manage their own reactions and thereby reflect on how to help someone else.

Parenting for mathematical resilience

Many parents have developed an anxiety which has made them feel helpless in the face of their own children's difficulties for all the reasons discussed above. Left unrecognized, this anxiety is likely to be passed onto their children. 'Heather', a young mother, experienced many negative events whilst at school. Here is one of Heather's descriptions:

> "we were doing maths we were working on long division, i got a red cross on some of my work when i checked on the calculator the answer was correct, (mmm) this puzzled me so when i questioned my teacher she said it was because my working out was wrong and made me feel stupid in front of my peers when i explained (translated) my methods."

Heather, had demonstrated mathematical resilience as a child; she was willing to 'have a go', and puzzled by the teacher's marking. The teacher's response in this situation undermined Heather; she experienced loss of social esteem, became unwilling to ask questions in the future and thus lost much of her innate mathematical resilience. This is her description of the result when she tried to help her daughter

> "throughout the years when i've worked with my daughter on maths homework its been a combination of falling out, shouting, tears , avoidance feeling stupid on both parts her putting herself down me trying to restore her confidence but both of us to[o] tense and stressed for it to make any difference."

Heather had resources from her daughter's school; she had the expectation that 'she should be able to help' as she did with other subjects; but her own self-image was of total failure in relation to mathematics. Heather sought help from us; she was offered a resource, in the form of a mathematics dictionary. We encouraged her to be aware of her own everyday use of mathematics, and to focus her attention on her role as 'curious, supportive, listening mother' rather than

inadequate mathematics ‘expert’. With no extra mathematical knowledge, the change was dramatic.

> “one day after she came home from school she curiously asked me for the text book they had been working from that day in school, she confidently flicked the page open to an angles section where there was a page full of lines in all directions, and said "we were doing this today but i don't get it " so i said well lets have a look then, with a few minutes she understood, it turned out that all that really confused her was the layout of the page as it was full of lines set up in twos connecting at one point and were heading in all sorts of directions”

Heather’s response is one of supportive curiosity and listening, of modelling resilient behaviours, of talking things through and using resources available, encouraging persistence in thinking with a dash of curiosity and support, along with an expectation that given time a plan of action could be put in place. Thus the home situation is no longer stressful and becomes one of growth in small steps. Instant recognition and answers were no longer expected, greater benefit could be obtained by applying mathematically resilient behaviours.

Developing a culture that promotes mathematical resilience

Learners of mathematics often expect to be T.I.R.E.D. (Nardi and Stewart 2003); where expectations are raised and students are challenged and supported appropriately (Lee & Johnston-Wilder 2012, Watson & DeGeest 2005), they meet, often exceed, the challenges set. Mathematically resilient students succeed despite barriers, they are adaptive; able to cope with ambiguity; expect problems and challenges and expect to meet them successfully. They solve problems logically and flexibly; look for creative solutions to challenges; are curious and learn from experience; have an internal locus of control; are aware of their feelings; have a strong social network and are able to ask for help. Mathematically resilient role-models will not respond ‘oh I can’t do maths’ but ‘that is an interesting question – how shall we find out?’. A learning culture that emphasises inclusion, group work, a growth theory of learning, struggle, persistence and curiosity fosters mathematical resilience and therefore a willingness to take part in mathematical learning. It is also a culture where the learners are doing the work and experiencing the positive emotions of challenges met, peers supported, being listened to, being part of a purposeful community and learning.

Conclusion

Mathematically resilient learners have the following four characteristics:

- belief that brain capability can be grown; that intelligence is not fixed but that with support and effort everyone can get smarter at mathematics.
- understanding of the personal value of mathematics; that it is worth making the effort.
- understanding of how to work at mathematics, that it requires perseverance, curiosity and struggle.
- awareness that support is available from peers, other adults, ICT, internet, etc.

A resilient stance towards mathematics can be developed in learners through a strategic and explicit focus on the culture of learning mathematics, within both formal and informal learning settings.

Part of the solution lies in developing mathematical resilience outside the mathematics classroom in all adults and children. If learners are surrounded by adults who find mathematics is something to be anxious about, is different and that normal ways of learning cannot be used for mathematics, then it is unsurprising that learners also develop negative attitudes to mathematics. We have shown that small changes make a difference. Instead of the supporting person taking the stance of the one who must show the ‘correct’ way to the answer, a supporter can model resilient behaviour by being the one who asks pertinent questions or encourages articulation and listens to explanations, who suggests resources that might help or who simply struggles alongside. Instead

of modelling avoidance and fear, the resilience that many people demonstrate in areas of their lives can be brought to mathematical learning, emotions can be managed and difficulties can be overcome. And mathematics can be something to grow together.

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