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Hypernetworks for Policy Design in Systems of Systems of Systems

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Abstract. Hypernetworks generalise networks and hypergraphs, allowing relations between many things to be modelled by *hypersimplices* with richer structure than hypergraph edges. They provide a way of integrating bottom-up and top-down micro, meso and macrolevel dynamics in multilevel systems. They provide a natural way of representing social structures, enabling polices to be tested by computation and big data before they are implemented.

Keywords: networks, hypergraphs, hypernetworks, modelling, Q-analysis, Galois lattice, policy, social informatics, policy informatics, gangs

1 Introduction

Social networks have been intensely studied since the nineteen sixties when computers enabled increasingly large social systems to be studied [1]. However most social systems involve networks of networks, and the interaction of many agencies and recent work has begun to explore the properties of coupled networks. To date the focus has been on *binary relations* between pairs of things analysed using network theory. Surprisingly the possibility of n-ary relations between any n things has received less attention, even though they are ubiquitous in all systems.

Hypergraphs [2, 3] provided an early attempt to model relations between more than two things. They mark a big step forward in the study of higher relations but they are set-theoretic and lack the representational power needed for complex systems.

A further step forward is to use *simplices* rather than hypergraph edges to represent related entities [4, 5, 6, 7]. The vertices of simplices are *ordered*, and they have a multidimensional connectivity structure. Algebraic topology provides much useful theory for representing coupled dynamical subsystems. Even so, the orientation of simplices and related algebraic operations in simplicial complexes have limited representational power for social systems.

In contrast, hypernetworks [8, 9] have much richer structure to represent social dynamics. Hypernetworks are formed from *relational simplices*, otherwise called *hypersimplices*, in which the relational structure is explicit. Like the formation or breaking of links in networks, the formation and disintegration of hypersimplices represent the discrete dynamics of systems. They generalise the dynamics of network formation, when vertices join or leave the system, and links are formed or cease to exist. The formation or disintegration of a hypersimplex is a structural *event*. Such events mark time in systems, where this is related to but different from clock time. For example time series are usually (more or less) continuous mappings of simplices to numbers in clock time, but the formation or disintegration of the simplices are discrete events as the structure of the system changes.

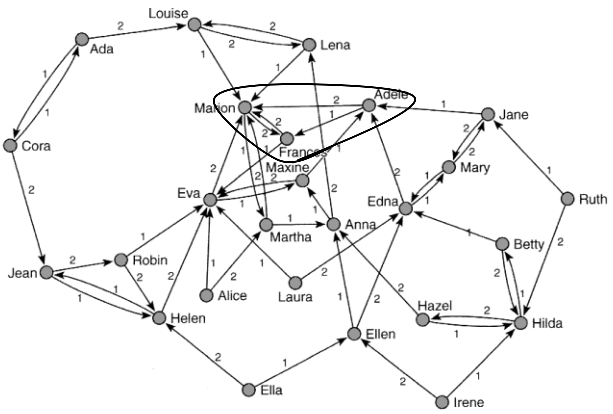
Hypersimplices are *wholes* formed their vertices as *parts*. This part-whole structure allows simplices at one level to become vertices at higher levels in multilevel systems, and the mappings defined on the simplices aggregate or disaggregate accordingly. This gives a way of representing and integrating the bottom-middle-top up-down-diagonal dynamics of complex multilevel systems of systems of systems.

The *state* of a system at any instance in time is represented by its multilevel simplices and patterns of numbers defined on them. Policy *goals* can be defined to be desirable system states, and policy can be defined as the decisions and actions intended to move the system onto trajectories that will achieve the goals. This paper will present hypernetworks as a generalisation of networks and show how they can be used for modelling complex systems in the context of policy and designing the future.

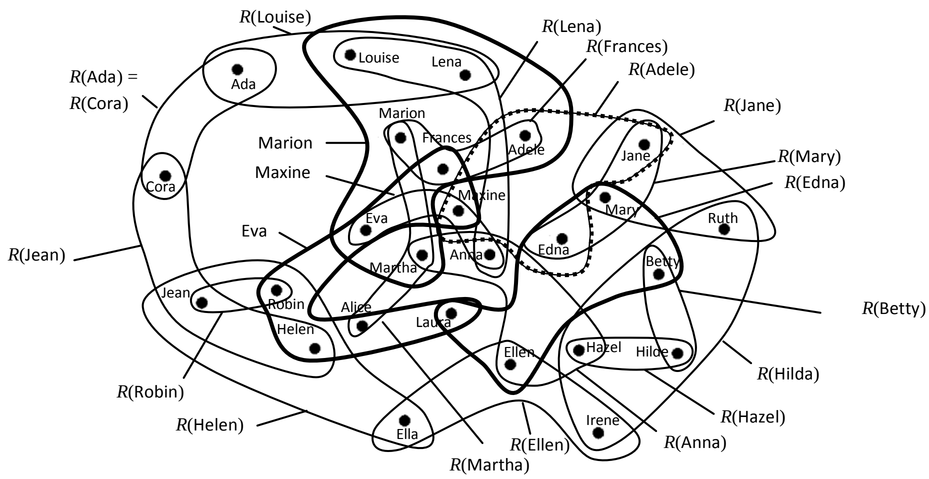
2 Graphs, networks, hypergraphs, and simplicial complexes

A *graph* is a set of points called *vertices* or *nodes* and a set of pairs of vertices called *edges* or *links*. Let V be a set of vertices, $E = \{ (v, v') \mid v$ and v' belong to $V \}$. Then $G = (V, E)$ is a graph. An edge is *directed* or *oriented* if $(v, v') \neq (v', v)$. A graph with directed edges is called a *digraph*. A *network* is a digraph with mappings assigning numbers to the vertices and the edges, $f : V \rightarrow \mathbb{R}$ and $f : E \rightarrow \mathbb{R}$ where $\mathbb{R}$ is a number system such as the integers, the rationals or the reals. In many networks $f(v, v')$ represents a *weighting* or a *flow* from v to v'. A graph or network is *bipartite* if its vertex V set can be partitioned into two sets A and B, $V = A \cup B$ and $A \cap B = \emptyset$, such that every edge can be written as (a, b) where a belongs to A and B belongs to B. The literature often uses the terms graph and network interchangeably.

Graphs and networks are used to represent relationships between things. Let R be a relation between the sets A and B. Let $V = A \cup B$ and $E = \{ (a, b) \mid a \text{ is } R\text{-related to } b \}$. Then (V, E) is a graph. When numbers are assigned to its vertices and edges it is a network. For example, an airline network has vertices airports and edges the pairs of airports related by having direct flights between them. Then $f_-(a)$ can be the number of people leaving a and $f_+(b)$ can be the number of people arriving at b with $f(a, b)$ being the number of people flying from a to b in a given time.



(a) Network of girls' preferred dining partners. (Source: de Noy *et al* [10]).



(b) Hypergraph of girls' preferred dining partners

Fig. 1. Networks and hypergraphs

The *degree* of a vertex a in a graph is the number of edges (a, b) in the graph. The *out-degree* of a vertex a in a directed graph is the number of edges (a, b) in the graph and the *in-degree* of a vertex b is the number of edges (a, b). For example Figure 1(a) shows twenty six girls with links to their favourite two dining partners. The degree of Adele is 5, and her in-degree is 3. The out-degree of all the girls is 2.

The edges of graphs are restricted to having two vertices. Hypergraphs remove this restriction. Let R be a relation between set A and B. In principle, given a set B, any class of subsets of B is a *hypergraph*. Let $R(a_i)$ be the set of all members of B that are R-related to a_i, where a_i is in A, $R(a_i) = \{ b_j \mid \text{for all bj with } a_i \; R \; b_j \}$. Then $H_A(B, R) = \{ R(a_i) \mid \text{for all } a_i \text{ in } A\}$ is a hypergraph. Figure 1(b) shows the hypergraph of the relation $H_{Girls}(Girls, R_{dining\_prepference})$. The hypergraph edge $R(g_i)$ is the set of all girls whose first or second choice of dinner companion is girl g_i.

In the network representation of the dinner partners data, the in-degree is a measure of the popularity of a girl. For example, Eva and Marion each have six girls who like their company, and Edna and Hilda have four. By comparison none of the girls has Ella, Irene, Laura, or Alice as their first two preferences. The hypergraph representation adds information to this by explicitly listing the girls. Also the hypergraph representation has a different kind of connectivity through the intersections of the edges, which may have more than two vertices.

Hypergraphs provide a way of modelling n-ary relation when $n > 2$. However, hypergraphs lack the structure needs to make some fundamental distinctions. For example, consider the relation between the set of words and the alphabet. Then, for example $R(\text{dog}) = \{\text{d, o, g}\}$ and $R(\text{god}) = \{\text{g, o, d}\}$. But set-theoretically $\{\text{d, o, g}\} = \{\text{g, o, d}\}$, so $R(\text{dog}) = R(\text{god})$. This can be overcome by using simplices.

Let V be a set of vertices. The sequence of vertices $\langle v_0, v_1, \ldots, v_p\rangle$ is defined to be an abstract *p-simplex*. A p-simplex has a *geometric realisation* as a polyhedron in an n-dimensional space, $n \geq p$. For example, the geometric representation for $\langle v_0, v_1, v_2\rangle$ is a triangle in 2-D space, that for $\langle v_0, v_1, v_2, v_3\rangle$ is a tetrahedron in 3-D space, and so on. The simplex $\langle v_0, \ldots , v_p\rangle$ is a *p-dimensional face* of the simplex $\langle v'_0, \ldots , v'_q\rangle$ if $\{v_0, \ldots , v_p\} \subset \{v'_0, \ldots , v'_q\}$. For example the tetrahedron $\langle v_0, v_1, v_2, v_3\rangle$ has four triangular faces, $\langle v_0, v_1, v_2\rangle$, $\langle v_0, v_1, v_3\rangle$, $\langle v_0, v_2, v_3\rangle$ and $\langle v_1, v_2, v_3\rangle$. A set of simplices with all its faces is called a *simplicial complex*.

The geometric realisation of a single vertex, $\langle v_1\rangle$ is a point, and the geometric realisation of a 1-dimensional simplex $\langle v_1, v_2\rangle$ is a line oriented from v_1 to v_2. Thus every network is a 1-dimensional complex, and simplicial complexes generalise networks to higher dimensions.

The sets of girls identified as hypergraph edges in Figure 1 can also be viewed as the vertices of simplices. Let two simplices be q-near if they share a q-dimensional face. The transitive closure of the q-nearness is the *q-connectivity* relation. It partitions sets of simplices into *q-connected components*. A listing of the q-connected components is called a Q-analysis, which can be succinctly summarised by a *skyscraper diagram* (Fig. 2). At $q = 1$ five distinct components emerge. The first is for Ellen who is liked by Ella and Irene at the bottom left of Figure 1. The second is Hazel, Betty and Hilda which form a group at the bottom right of Figure 1. Next comes Edna with Mary, Jane and Adele at the top right of Figure 1. Ada and Cora form a small components at the top left of Figure 1. The largest component clusters around the popular Eva and Marion at the top centre of Figure 1. Q-analysis reveals structure that is not obvious in the network representation, and also provides a more tractable way of computing Galois pairs of simplices for bipartite relations [2, 9].

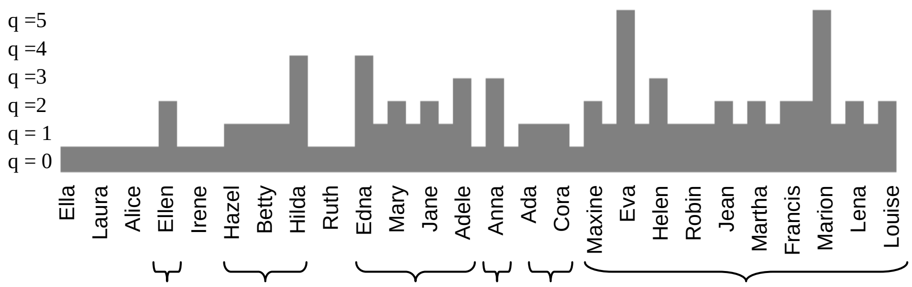


Fig. 2 The Q-analysis skyscraper diagram for the girls' dining preference relation

Figure 3 shows five faces, f_1, f_2, f_3, f_4, and f_5 made of the shapes s_1, s_2, s_3, s_4, s_5 and s_6. These face shapes can be represented by the simplices $\sigma(f_1) = \langle s_1, s_3, s_5, s_6\rangle$, $\sigma(f_2) = \langle s_1, s_4, s_5, s_6\rangle$, $\sigma(f_3) = \langle s_2, s_3, s_5, s_6\rangle$, and $\sigma(f_4) = \langle s_2, s_4, s_5, s_6\rangle$, $\sigma(f_5) = \langle s_2, s_3, s_5, s_6\rangle$.

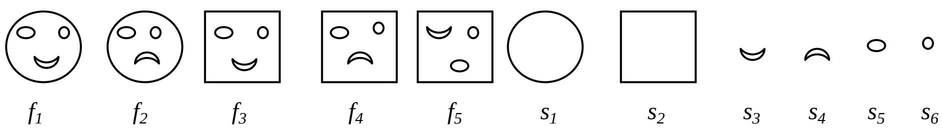


Fig. 3. Faces made up from shapes

Using simplices also has problems. For example, $\sigma(f_3) = \langle s_2, s_3, s_5, s_6\rangle = \sigma(f_5)$, and the simplex representation cannot discriminate them. Let R be a relation on an ordered set of four vertices $\langle x_1, x_2, x_3, x_4\rangle$, defined as: place x_2 under the centre of x_1; place x_3 above and to the left of the centre of x_1; place x_4 above and to the right of the centre of x_1. Let $\langle x_1, x_2, x_3, x_4; R\rangle$ be defined to be a *relational simplex*, or *hypersimplex*. Now $\sigma(f_3) = \langle s_2, s_3, s_5, s_6; R\rangle$ while $\sigma(f_5) = \langle s_2, s_5, s_3, s_6; R\rangle$, so that $\sigma(f_3) \neq \sigma(f_5)$.

A hypernetwork is defined to be a collection of hypersimplices. Generally hypernetworks support patterns of numbers that represent properties and flows through the system being modelled. The relationship between the structures defined here is given in Figure 4.

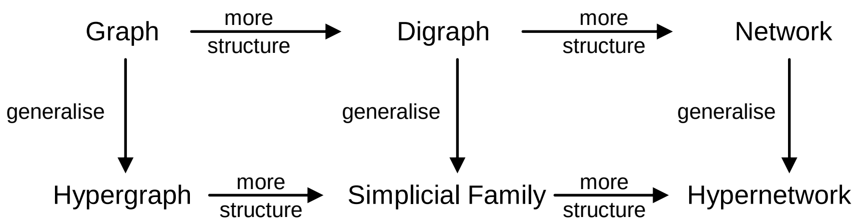


Fig. 4. The relationship between graphs, network, hypergraph and hypernetworks

3. Multilevel structure

Hypersimplices provide a method of representing multilevel structure. Imposing an *n*-ary relations on sets of elements creates objects at higher levels. *E.g.* the three blocks *a*, *b*, and *c* are assembled by *R* into a structure, R: $\{a, b, c\} \rightarrow \langle a, b, c ; R \rangle$ and given the *name* arch. If the elements exist at, say, *Level N* then the structured object exists at a higher level, say *Level N+1*. Here the higher level structure has an *emergent property* not possessed by its elements, namely there is a 'gap' between the assembled blocks.

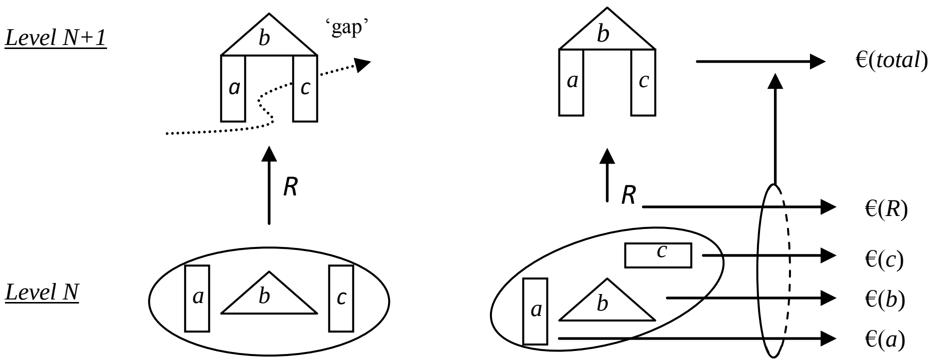


Fig 5. Aggregating *Level N* parts to *Level N+1* wholes, and aggregating numbers

One of the challenges of modelling complex systems is to integrate micro and macro theories of behaviour. From a policy perspective, the target systems almost always include people as social and economic systems. Social systems have structures at many levels, from the individual at micro levels through organisations and institutions at meso levels to nations and international structures at macro levels. Alongside this are economic considerations with individual's costs and benefits at the microlevel aggregated through social groups at meso levels to the costs and benefits perceived by the Finance Ministry, European Bank or the World Bank at macro level. Alongside this there are considerations of the particular system being managed, such as 'health', 'welfare', 'transport', 'environment, 'food', 'education', 'housing', and so on. Each of these has its micro, meso and macro levels. How can the impact of policy be understood across these entangled multilevel systems of system of systems?

4. Hypernetworks and Policy

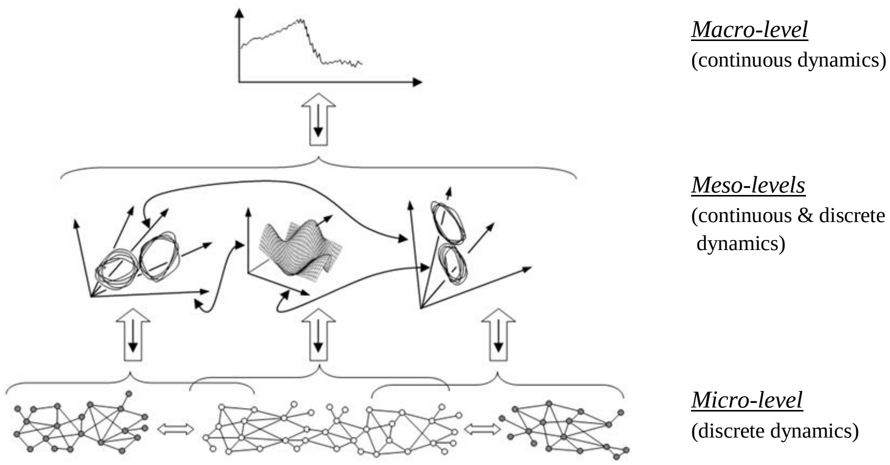


Fig. 6. The challenge of modelling multilevel systems of systems of systems

Figure 6 illustrates the challenge of creating coherent multilevel theories to support policy. At the microlevel the dynamics emerge from the interactions of individuals. Currently a favoured scientific way of investigating the microlevel behaviour is agent-based simulation, which generates two-level dynamics at the level of the individual and the emergent behaviour of the community. Simulations do not give 'predictions' of future system behaviour, but give insights in to possible system behaviour and, at best, estimate the likelihood for a policy to have its desired outcome at that level.

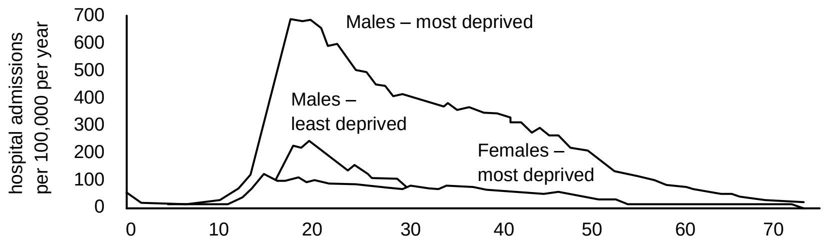


Fig. 7. Admissions to English NHS hospitals for assault involving 13 to 14 year olds. [11]

Figure 7 illustrates macro-level statistics that provide a macro-level snapshot in the terms of Figure 6. They come from a report entitled ‘Dying to Belong’ published in 2009 by the Centre for Social Justice (CSJ) which gives an in-depth analysis of street gangs in Britain [12].

Between 6th and 10th August in 2011 London and other British cities experienced violent riots, looting, arson, assault and robbery. Many thousands of lawless people took to the streets including rival street gangs acting together.

The report ‘Ending Gang & Youth Violence’, published in November 2011 by the Secretary of State for the Home Department was the basis for a policy response to the riots. “One thing that the riots in August did do was to bring home to the entire country just how serious a problem gang and youth violence has now become.” [13] This report set out detailed policy plans for the agencies to work together, including providing support to local areas; preventing young people becoming involved in serious violence; pathways out of violence and gang culture; punishment and enforcement to suppress the violence of those refusing to exit violent lifestyles, and partnership working to join up local area responses to gangs and youth violence.

The 2012 CSJ report ‘Time to Wake Up’ questions the effectiveness of the police practice of identifying and removing gang ‘elders’: “it seems that an unintended consequence of the arrest of senior gang members has been to heighten tensions and violence. … There was a consensus that the current gangs neither have [no] cohesive leadership, which is resulting in increased chaos, violence and anarchy. [14]

These reports discuss the problem of gangs at the macro level of state policy, at meso level of local authority policy, and at the microlevel of dealing with individuals. For example, ‘Ending Gang and Youth Violence’ documents the life of “Boy X” from his birth to his seventeen year old crack cocaine addicted mother to life imprisonment for murder at age twenty one, and his many contact points with the social and emergency services (Fig. 8). [13]

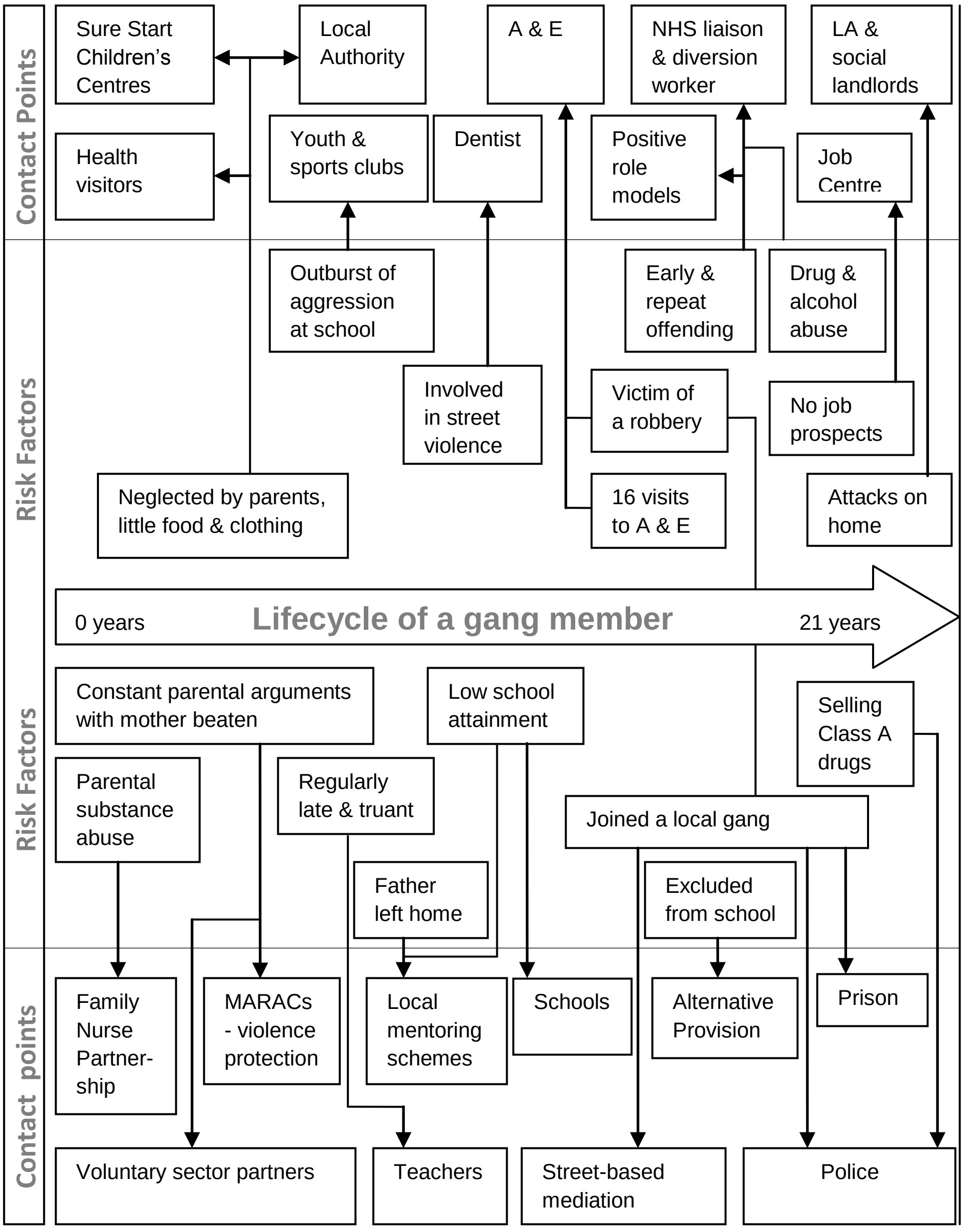


Fig. 8. The lifecycle of a gang member (Source: HMG: Ending Gang Youth Violence [13]).

If this were the story of just one person it would be regrettable but not an issue for policy. Figure 8 is a kind of model of the life of this individual at the microlevel and his interactions with the mesolevel social and emergency services. Implicitly it is intended to generalise to *classes* of individuals at higher mesolevels of aggregation. Policy cannot target individuals at the microlevel but requires *models* of the behaviour of individuals within classes or higher level aggregate entities, *i.e.* hypernetworks.

The various reports on gangs cited above contain propositions about the behaviour of gang members. These include postulating classes of 'Elders' and 'Youngers' and their modes of interaction at the microlevel, and classes of rival gangs and their modes of interaction at higher mesolevels. Much of this is empirically based and forms a theory of gang behaviour on which to base policy. What can formal modelling with hypernetworks add to this?

Social policy is inevitably expressed in natural language within a legal framework for implementation. In comparison, technical or formal models of systems are stated in their own language which may include mathematics and computation, but they are always embedded in a metalanguage such as vernacular English. This is as true in engineering as it is in social administration.

The problem with the theories in the reports cited above is that their vernacular models are *untestable* before implementation. For example, the failure of the 2011 police policy to remove Elders from gang was *predicted* in 2009 [12] but this was ignored or overlooked by the police. Why? Perhaps due to predictions presented in natural language carrying little more weight than opinions because the outcome of their premises and logic cannot be demonstrated before empirical testing on the street.

Technical models can translate vernacular models into formal theories that can be implemented in computers to generate their logical and empirical consequences. Given the many possible initial conditions such models must be run many times to characterise the space of possible policy outcomes. For policy purposes the output of the computational model will again be evaluated in a vernacular metalanguage, but the intermediate step of *computation* can add a lot in terms of understanding the real social system and its dynamics.

Many hypersimplices can be abstracted from Figure 8, *e.g.* $\langle$neglected by parents, parental substance abuse, parental violence; $R_{\text{experienced\_0-5}}\rangle$ suggests a child at high risk. The hypersimplex $\langle$outbursts of aggression as school, involved in street violence, many visits to A&E; $R_{\text{experienced\_10-15}}\rangle$ indicates a child becoming increasingly violent and dangerous, while $\langle$regularly late and truant, school low attainment, excluded from school;; $R_{\text{experienced\_10-15}}\rangle$ is a likely precursor to $\langle$joined a local gang, selling Class A drugs, early and repeat offending, drug and alcohol abuse; $R_{\text{experienced\_16-21}}\rangle$.

In many cases subsequent enquires discover that neighbours, welfare agencies, schools, the police and even the postman had evidence of tragic events to come, but this evidence was not ‘joined up’ as a hypersimplex. This is an easy conclusion but it is difficult to rectify the lack of structure to prevent further incidents. How can micro-level individuals join up the information spread across their mesolevel organisations? How can institutional structures such as $\langle$welfare agencies, schools, police, postman, milkman; $R_{\text{share\_and \_synthesie\_information}}\rangle$ be formed and run at politically bearable costs?

5. Hypernetworks, Big Data and Policy Informatics

Recently it has become clear that the dynamics of society are carried by an unprecedented flux of data. *Big Data* includes billions of phone calls, text messages, emails, financial transactions, personal data, and so on. Data mining has well established techniques for abstracting useful information from these data, and many involve discovering hypersimplices and relationships between them in hypernetworks.

Policy informatics is the process of building computer models of social systems with the explicit intention of using them as policy tools. Increasingly these models have the structure shown in Figure 6, with massive microlevel simulations creating more aggregate information at higher levels. For example, the TRANSIMS system developed at Los Alamos National Laboratory in the nineteen nineties simulates the trip making of millions of travellers in US cities, including explicit representations of each individual traveller, their family structure, their activity patterns and so on.

Policy informatics is built on *social informatics*, the process of building computer models of social systems to investigate their dynamics. Social informatics is not necessarily policy-driven and can be pure research. Today big data plays an important part in social informatics and the development of new models of social processes.

Policy informatics may answer the question of forming institutional structures able to synthesise atomic social data for policy purposes. In future it is likely that computers will shift from having their constructs supplied by humans to them abstracting ‘relevant’ constructs for themselves, where a *construct* is a hypersimplex, of subordinate constructs assembled by a relation. To illustrate this consider the faces shown in Figure 3. What is the correct way of grouping these faces? For example, the faces could be aggregated on the basis of round versus square, or they could be aggregated on the basis of smiles versus frowns. Of course there is no ‘correct’ way of aggregating them, only more or less *useful* way of aggregating them.

Hypernetworks provide essential structure for computer systems to build vocabularies automatically for searching big data in policy informatics. [9]

6. Conclusions

Network theory is fundamental in both social informatics and policy informatics. Hypernetwork generalise networks and hypergraphs, allowing relations between many things to be modelled by hypersimplices which have much richer structure than edges in hypergraphs. Hypersimplices provide a coherent way of representing multilevel systems to integrate their bottom-up and top-down micro-, meso- and macrolevel dynamics. They provide a natural way of representing structures in a policy context, and this enables polices to be tested by computation before they are implemented with possibly unexpected consequences. Hypernetworks have immediate interpretation as data structures and can bridge the gap between vernacular models and computation of social processes. Hypernetworks will play an important role in the use of big data in social informatics and policy informatics, and will play a central role in future policy formulation and implementation.

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