A Novel Precolouring-Random Demodulator Architecture for Compressive Spectrum Estimation

***D. Karampoulas* [*1], *S.M.Kouadri* [*2], *L.S.Dooley* [*3]**

**Department of Computing and Communications, The Open University, UK.*

[1]dkarampoulas@gmail.com
[2]soraya.kouadri@open.ac.uk
[3]laurence.dooley@open.ac.uk

Keywords: Random Demodulator, Autoregressive Filter, Minimum Description Length, Precolouring, Spectral Concentration

Abstract

One of the main challenges of conventional spectrum estimation methods in cognitive radio applications is the very high sampling rates involved, which imposes significant operating demands upon the *analog-to-digital converter* (ADC). This has given impetus to employing *compressive sensing* (CS) techniques, such as the *random demodulator* (RD) structure to relax the input ADC specification. It has been recently shown the RD spectrum estimation performance for quadrature *phased shift keying* (PSK) modulated signals can be significantly improved in terms of *spectral concentration* and signal-to-noise ratio, when signals are precoloured by an *autoregressive* (AR) filter. This paper presents an extended AR-RD architecture, which provides enhanced CS capability for higher-order digital modulation schemes, including 16 *quadrature amplitude modulation* (16QAM), 64QAM and *binary* PSK (BPSK). Quantitative results corroborate the improved CS performance of the AR-RD structure for higher-order modulations schemes, which provides a propitious design trade-off between AR-RD complexity, latency and CS performance.

1 Introduction

A key objective in *cognitive radio* (CR) systems is efficient spectral detection and estimation techniques, to readily sense and identify available spectrum [1]. One of the challenges of conventional spectrum estimation methods is the high sampling rates involved. A consequence of this is that it is either too costly or problematic to practically construct an *analogue-to-digital converter* (ADC) to be able to operate at the requisite rates [2]. This has given impetus to the emergence of *compressive sensing* (CS) techniques, which are a new sampling paradigm for signal acquisition [3, 4, 5]. A necessary precondition for efficient CS is the signal of interest must be sparse in some domain [3]. Signals in wireless networks are often sparse in the frequency domain, due to spectral under-utilization [6], so CS can effectively be applied to both sample and recover sparse wireless signals, by taking a relatively small number of random linear, non-adaptive measurements [3,7].

One specific CS technique is the *random demodulator* (RD) [8, 9] which has been shown to be particularly effective for signals which are characterised by being bandlimited, periodic and sparse, and thereby are able to be approximated by a set number of tones [10]. For multiband signals with continuous spectra however, the RD requires a sufficiently high number of tones to obtain an acceptable approximation, which impacts upon the computational cost of both the sampling and recovery stages [11]. In [12], it was shown that the RD can be employed to efficiently recover the *power spectral density* (PSD) of *quadrature phased shift keying* (QPSK) modulated, sparse multiband signals. By firstly precolouring the signal with an *autoregressive* (AR) filter, signal sparsity is enhanced by sharpening dominant frequencies while attenuating weaker spectral components which lie outside the bands of interest. The augmented sparsity achieved in this AR-RD structure reduces the computational complexity upon both sampling and recovery stages, so providing more efficient PSD recovery.

While QPSK modulated signals were analysed in [12], this paper presents an extended AR-RD arrangement, which can be applied to other high-order digital modulation schemes, including 16 *Quadrature Amplitude Modulation* (16QAM), 64QAM and *binary* PSK (BPSK). These are all defined in the IEEE 802.22 standard for CR [13], as well as in both the IEEE802.16 standard for *Worldwide Interoperability for Microwave Access* (WiMax) [14], and the *3rd Generation Partnership Project* (3GPP) *long term evolution* standard [15].

While the AR filter order in [12] was empirically chosen, determining the most appropriate filter order is an important design objective because of the corresponding influence on both the quality of the signal estimate and computational complexity [16]. Moreover, the order of the filter affects the time delay introduced by the AR-RD structure. This paper therefore examines the CS performance sensitivity in regard to the choice of the AR filter order and proposes a new strategy for the best filter order selection, by using the *minimum description length* (MDL) criterion [16,17]. The MDL is framed as a function of the AR filter order incorporating both a modelling error and penalty term for increasing the model order [16, 17].

The results corroborate the extended AR-RD structure provides improved performance in terms of both spectral leakage concentration and *signal-to-noise ratio* (SNR) for all these higher-order modulation types compared with the original RD structure.

The remainder of the paper is organized as follows. Section 2 presents a brief overview of both the RD and AR-RD structures, before Section 3 details both the precolouring mechanism and MDL criterion used for AR order selection. Experimental results are analysed in detail in Section 4, with some concluding comments being provided in Section 5.

2 The RD and extended AR-RD structures

2.1 RD basics

The basic RD structure is shown in Figure 1 and has three constituent components: demodulation, filtering and uniform sampling [10, 18]. The input signal *x(t)* is multiplied by a square pulse train of random values {±1} generated by a pseudorandom sequence called the *chipping sequence* $p_c(t)$ which alternates between values at least as fast as the Nyquist frequency of *x(t).* The demodulation process smears the energy of the signal tones across the entire spectrum, giving each tone a unique spectral signature that can be discerned by a low-pass filter, which is implemented as an integrator. The final stage involves generating the CS output *y(n)* using an ADC which is clocked at a lower sampling frequency rate than Nyquist. The PSD of *x(t)* can then be recovered by applying any l_1 -norm minimization technique such as those in [3, 19].

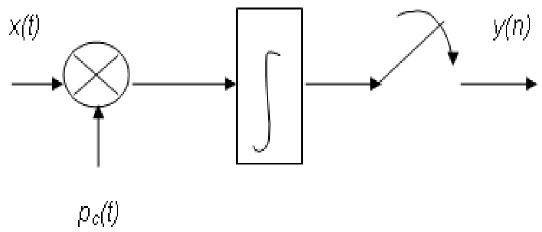


Figure 1. Block Diagram of the Random Demodulator (RD)

2.2 The AR-RD structure

The extended AR-RD includes an AR filter [16] to precolour the input signal before its sampling by the RD [12]. Precolouring has the dual effect of reducing leakage for dominant spectral components and of increasing the spectral dynamic range, so the spectrum becomes sharper and the signal sparsity increases. This is achieved because weaker frequency components which lie outside the bands of interest are either eliminated or significantly attenuated [12]. The output of a p^{th} order AR filter is given by:

$$y(n) = x(n) + \sum_{k=1}^{p} a_k y(n-k) \qquad (1)$$

where α_κ are the AR filter coefficients which are optimally estimated using the modified covariance method [17]. Subsequently, the intermediate PSD $S_y(f)$ is recovered after CS by the RD, as depicted in Figure 2.

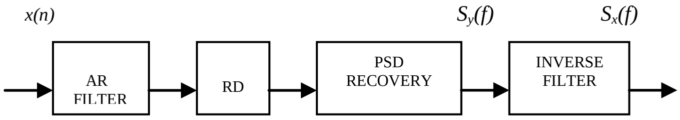


Figure 2. The AR-RD structure

The influence of the precolouring filter is removed by inverse filtering of the intermediate PSD estimate, to derive $S_x(f)$, with the relationship between $S_x(f)$ and $S_y(f)$ being expressed as:

$$S_y(f) = \frac{S_x(f)}{\left|1 + \sum_{k=1}^{p} a_k e^{-j2\pi fk}\right|^2} \qquad (2)$$

Note the precolouring stage does not necessarily have to be embedded within the Figure 2 CS architecture. It could for example, be alternatively implemented by a licensed (primary) user within a CR network, with the signal being precoloured before transmission. This avoids the requirement for a high-speed ADC to be implemented which negates the benefit derived by using the lower sampling rate of the RD.

3 Precolouring and MDL

3.1 Precolouring mechanism

An AR parametric model acts as a colouring filter because it takes a zero-mean white noise process *w(n)* which is spectrally flat and generates the input signal *x(n)* [16]. The filter coefficients are optimally calculated using the modified covariance method, so that the AR model fits the signal *x(n)* [17], in order to be able to accurately generate it. The transfer function of such an AR filter is analogous to the spectrum of *x(n)*. Therefore, it operates on *w(n)* so that it amplifies the spectral components according their dominance in the signal *x(t)*. The output *x(n)* is given by:

$$x(n) = w(n) + \sum_{k=1}^{p} a_k x(n-k) \qquad (2)$$

The frequency domain equivalent of (2) is given by:

$$S_x(f) = |H(f)|\sigma^2 \qquad (3)$$

where *H(f)* is the AR transfer function and σ^2 the variance of *w(n)*.

When *x(n)* is precoloured by the AR filter, the more dominant frequency components will become enhanced. This has the effect of reducing the spectral leakage described in Section 2.2, though in these circumstances, it is the time signal *x(t)* which is coloured rather than *w(n).* In essence, the better the AR filter approximates *x(t)*, the superior the CS performance of the RD.

3.2 MDL selection criterion for AR processes

The order of an AR model plays an important role, in the CS process, since it determines the quality of the spectral estimate [16] i.e., the number of filter coefficients and the computational complexity. If a lower order is selected then the spectrum estimate may not be sufficiently accurate, while if an unnecessarily high order is used, then the complexity and latency increase and also the estimate can become prone to over-fitting [20].

Many criteria have been proposed for the selection of the best AR model order, including the *final prediction error* (FPE) [21], the *Akaike's Information Criterion* (AIC) [22], the MDL [23] and the *criterion autocorrelation transfer* (CAT) [24]. The underlying premise of all these criteria is the introduction of a penalty term that increases with model order p [16]. MDL is an improved variant of FPE and AIC [17], and its calculation is much more straightforward than CAT, whereas the latter provides equivalent performance to both AIC and MDL as the number of samples N increases [16]. For this reason, MDL was chosen for this analysis and it can be formally defined as:

$$MDL(p) = N\ln(\sigma^2) + p\ln(N) \qquad (4)$$

where p is the AR order and σ^2 the white noise variance, which represents the modeling error. The best p is then the minimal MDL value in (4).

4 Simulation Results

A series of experiments were performed upon a MATLAB-based computing platform, to evaluate the performance of the extended AR-RD structure in Figure 2. Three 16QAM/64QAM/BPSK respectively modulated test signals were generated, with each of 300msec duration, a bit-rate of 215bps and two bands centered at carrier frequencies at 1kHz and 2.5 kHz respectively, where these frequencies take cognizance of the maximal frequency recovery capability of the RD [11]. The sampling frequency was chosen 1.67 times greater than the Nyquist rate which corresponds to a signal length of 2048 samples. The CS performance was then respectively analyzed at sub-sampled rates of 68%, 34%, 17%, 8.5% and 4.25% of the Nyquist rate which corresponds to 1024, 512, 256, 128 and 64 samples respectively. It is assumed *additive white Gaussian* (AWG) noise has been added to $x(n)$, so that input SNR was 8.1db as prescribed in the proposed IEEE 802.22 standards for CR [13,25].

4.1 AR filter design

The AR order in (4) takes into account the modelling error and computational complexity introduced. The normalised MDL value is plotted against p in Figure 3 for three different modulation types, i.e., 16QAM, 64QAM and BPSK. It can be observed that for all three, the minimum MDL is attained at p=8, though it is evident there exists an abrupt improvement in the MDL when p=4. Also apparent is the sharp decrease in the MDL value when $p=4$. Table 1 shows the MDL values for both $p=4$ and $p=8$, together with the corresponding *time delays* (TDL) values for each modulation scheme.

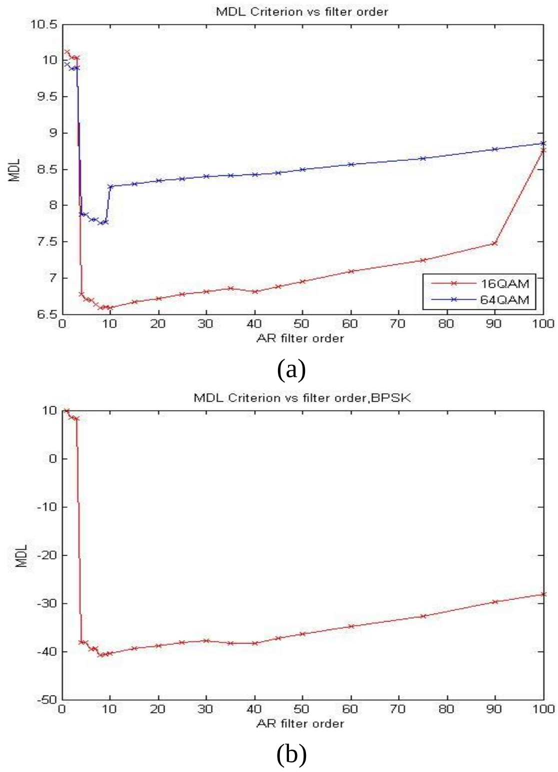


Figure 3. Effect of AR filter order on MDL in (4) for (a) 16QAM and 64QAM and (b) BPSK

p	MDL 16QAM	TDL 16QAM (msec)	MDL 64QAM	TDL 64QAM (msec)	MDL BPSK	TDL BPSK (msec)
4	6,7726	130	7,88	189	-38,2	136
8	6,585	176	6,76	240	-40,85	197

Table 1. MDL criterion values and corresponding AR filter delays for $p=4$ and $p=8$

When contrasting the respective MDL results for $p=4$ and $p=8$, the MDL is lower by 2.77%, 1.53% and 6.95% at 16QAM, 64QAM and BPSK respectively, however this comes with a corresponding increased filter latency of 36%, 27% and 45%. Designing for $p=4$ thus provides a pragmatic filter design trade-off between minimal order and latency, compared with the p=8 option.

Figure 4 shows the effect of the AR precolouring filter order p upon the extended AR-RD performance, where the percentage of the total energy content within the bands of interest is termed *PSD spectral concentration*. The corresponding spectral leakage for the bands of interest is then the difference from the ideal PSD spectral concentration (100%). The sampling rate was set to 68% of Nyquist, corresponding to 1024 signal measurements. It is observed that for all three modulation types there is a significant improvement in spectral concentration when p=4. Moreover there is no significant improvement as p increases further, and degradation is observed when p is greater than 40 due to the aforementioned AR model over-fitting [20].

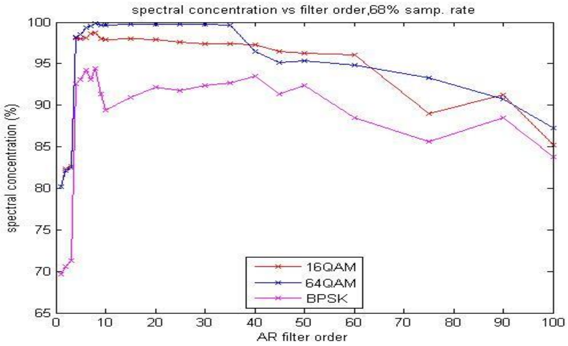


Figure 4. The effect of AR filter order on *PSD* spectral concentration

4.2 AR-RD performance results

The extended AR-RD performance for the modulation types of interest are shown at Figure 5, where the percentage of the total signal energy content within the bands of interest is termed *PSD spectral concentration*.

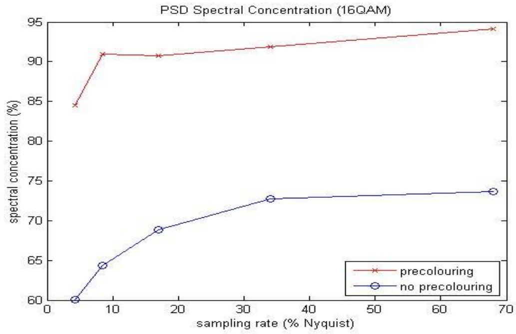
The results conclusively show that as with QPSK modulated signals [13], when precolouring is applied across the range of sub-Nyquist rates, the corresponding spectral concentrations achieved by the extended AR-RD structure are enhanced, on average by approximately 33%, 18% and 20% for 16QAM, 64QAM and BPSK respectively, compared with the original RD model.

In the precolouring case, the spectral leakage is never greater than 15% of the total signal energy at any of the sub-Nyquist sampling rates. BPSK and 64QAM in particular, provide leakage of less than 10% of the signal energy. In contrast, when no-precolouring is applied, the spectral leakage is always higher than 15% of the signal energy, with the non-precolouring case for 16QAM in particular, revealing a leakage greater than 25%.

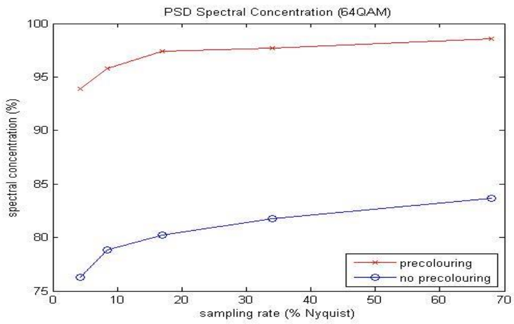
Figure 6 displays the corresponding SNR performance of the AR-RD system, for 16QAM, BPSK and 64QAM modulated signals. The SNR is defined as:

$$SNR_s = 10 * \log \frac{\|S\|_2}{\|S - S_x\|_2} \quad (5)$$

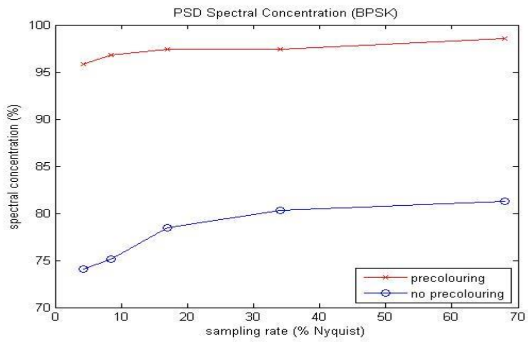
where S is the signal PSD when it is sampled either at or above the Nyquist rate, and S_x is the PSD of the under-sampled CS signal. The results confirm the superior SNR performance when precolouring is applied for all three higher-order modulation types and despite increased SNR degradation, the system SNR is always higher, even when the sampling rate falls below 10% of the Nyquist rate, and the respective graphs converge due it no longer being feasible to collect a sufficient number of samples.



(a)



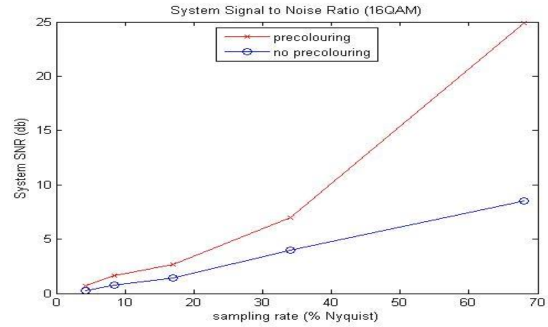
(b)



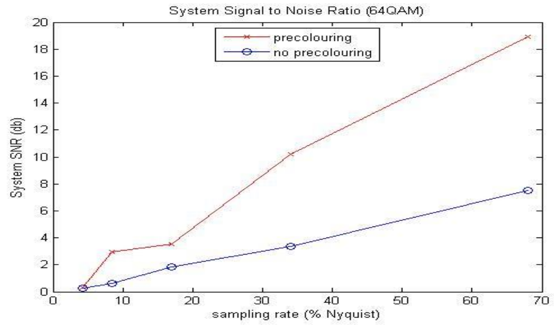
(c)

Figure 5. The effect of precolouring on spectral concentration for (a) 16QAM, (b) 64QAM and (c) BPSK

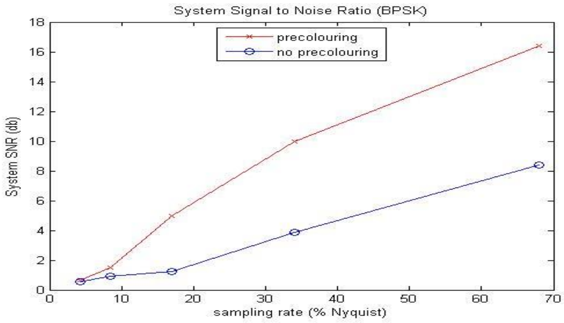
Finally, Figures 7 and 8 respectively plot the recovered PSD both without and with precolouring at sampling rate 4.25% of Nyquist rate, which corresponds to 64 samples. The spectral leakage is manifestly less pronounced when precolouring is applied, even at this very low sampling rate. In particular, for the precolouring case, the occupied bands are still able to be identified in contrast to when no signal precolouring is employed.



(a)

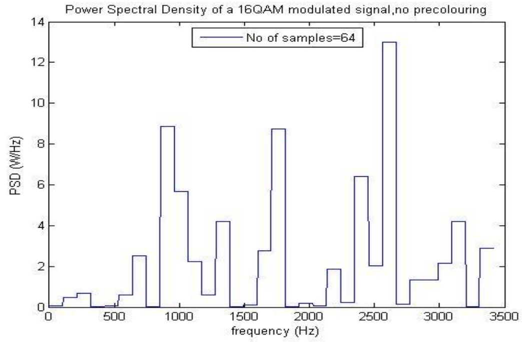


(b)

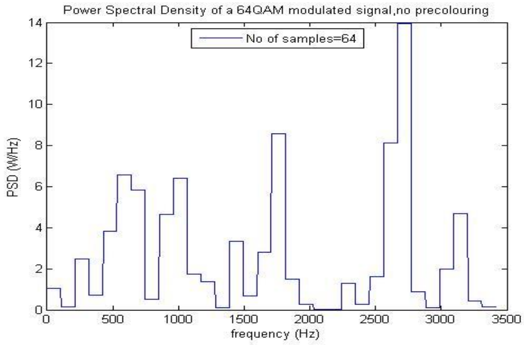


(c)

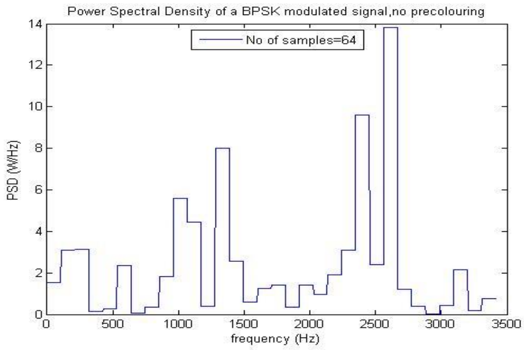
Figure 6. The effect of precolouring on SNR for (a) 16QAM, (b) 64QAM and (c) BPSK



(a)



(b)



(c)

Figure 7. Recovered normalised PSD at sampling rates 4.25% Nyquist with no precolouring for (a) 16QAM, (b) 64QAM and (c) BPSK

5. Conclusion

This paper has presented an extended AR–RD architecture which provides compressive spectrum estimation capability for various CR-related digitally modulated signals. A strategy has been developed to determine the best AR filter order to be adopted for precolouring and the corresponding impact on the performance of the AR-RD design has been analysed. Experimental results for 16QAM, 64QAM and BPSK, have shown that the proposed model both reduces PSD spectral leakage and provides superior SNR performance. Moreover, the existence of a beneficial design trade-off between filter order and AR-RD performance has been presented. Future research will focus on investigating the application of precolouring to other CS techniques particularly the compressive multiplexer and modulated wideband converter, together with analysing the impact of precolouring on the content of the transmitted signal.

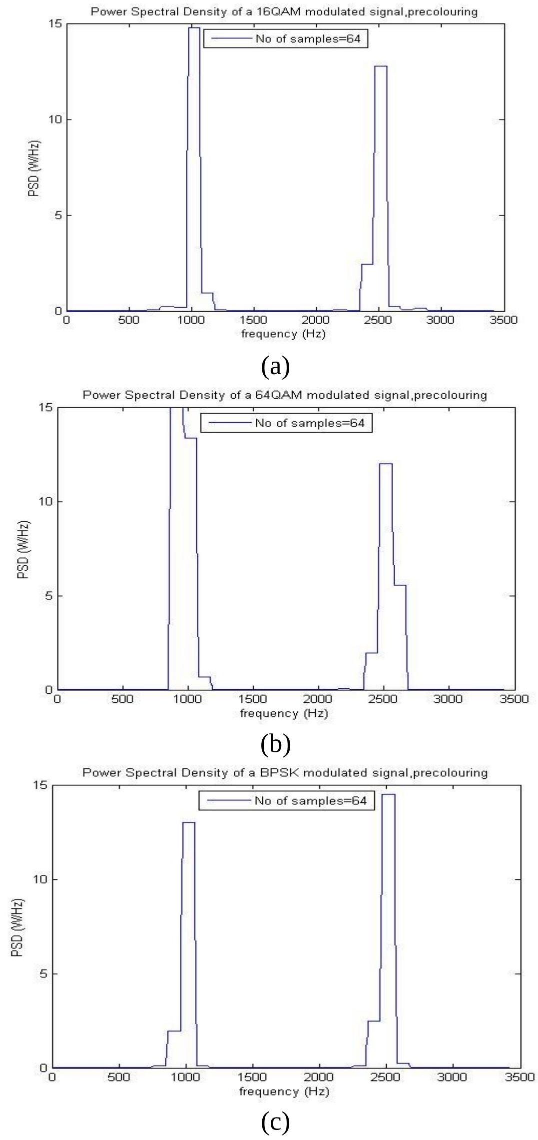


Figure 8. Recovered normalised PSD at sampling rates 4.25% Nyquist with precolouring for (a) 16QAM, (b) 64QAM and (c) BPSK

References

[1] Z.Tian, G. B. Giannakis, “Compressed sensing for wideband cognitive radios”, IEEE International Conference on Acoustics, Speech and Signal Processing, pp. IV-1357-IV-1360, Apr. 2007.

[2] Walden R.H. “Analog-to-digital converter survey and analysis”, *IEEE Journal of Selected Areas in Communications*, vol 17, no.4, pp.539550, Apr. 1999.

[3] E. Candès, E. Wakin, “An Introduction To Compressive Sampling”, IEEE Signal Processing Magazine, Vol.25, Issue 2, pp. 21-30, Mar. 2008.

[4] E. J. Candès, J. Romberg, T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information”, IEEE Trans. Information Theory, vol. 52, no. 2, pp. 489-502, Feb. 2006.

[5] D. L. Donoho, “Compressed sensing”, IEEE Transactions on Information Theory, vol. 52, pp. 1289-1306, Apr. 2006.

[6] S. Haykin, “Cognitive radio: Brain-empowered wireless communications”, IEEE Journal on Selected Areas in Communications, Vol.23, No.2, Feb. 2005.

[7] Candès E., Romberg J., “Practical signal recovery from random projections,” *SPIE 17th International Symposium on Electronic Imaging III*, vol. 5674, pp. 76–86, Jan. 2005.

[8] S. Kirolos, J. Laska, M. Wakin, M. Duarte, D. Baron, T. Ragheb, Y. Massoud and R. Baraniuk, “Analog-to-Information Conversion via Random Demodulation”, IEEE Workshop on Design, Applications, Integration and Software, pp.71-71, Oct. 2006.

[9] J. N. Laska, S. Kirolos, M. F. Duarte, T. S. Ragheb, R. G. Baraniuk and Y. Massoud, ‘‘Theory and implementation of an analog-to-information converter using random demodulation”, IEEE International Symposium on Circuits and Systems, pp. 1959-1962, May 2007.

[10] J. A. Tropp, J. N. Laska, M. F. Duarte, J. K. Romberg and R. G. Baraniuk, “Beyond Nyquist: Efficient sampling of sparse bandlimited signals’’, IEEE Transactions On Information Theory, Vol. 56, No. 1, pp.520-544, Jan. 2010.

[11] Y. Eldar & G. Kutyniok, “*Compressed Sensing, Theory and Applications*’’, Cambridge, UK, Cambridge University Press, 2012.

[12] D. Karampoulas, L.S. Dooley, S.K. Moustéfaoui, “Precolouring in Compressive spectrum estimation for cognitive radio”, IEEE EUROCON, Zagreb, Jul. 2013.

[13] A. N. Mody, G.Chouinard, “IEEE 802.22 Wireless regional area networks”, IEEE 802.22-10/0073r03, Jun. 2010.

[14] J. Rakesh, W. Vishal, U. Dalal, “A survey of mobile WiMax IEEE 802.16m standard”, International Journal of Computer and Information Security, vol.8, no. 1, pp.125-131, Apr. 2010.

[15] 3GPP TR 36.814 V9.0.0 (technical report), Mar. 2010

[16] D. Manolakis, V. Ingle, S. Kogon, “*Statistical and Adaptive Signal Processing*”, Artech House, Boston, USA, 2005.

[17] S.L. Marple, “*Digital Spectral Analysis with Applications*”, Englewood Cliffs, New Jersey, USA, Prentice-Hall, 1987.

[18] R. Baraniuk, M. A. Davenport, M. F. Duarte and C. Hegde, “An Introduction to Compressive Sensing”. [Online]. Available: http://cnx.org/content/col11133/1.5, (Accessed 14-07-2013).

[19] l1 Magic, a collection of MATLAB routines for solving the convex optimization programs central to compressive sampling. [Online]. Available: http://www.l1-magic.org/, (Accessed 24-06-2013).

[20] R.Gitlin, J.Hayes and S.Weinstein, “*Data Communications*”, Plenum Press, NewYork, 1992.

[21] Akaike H., “Fitting autoregressive models for prediction”, Annual Instit. of Statistics and Mathematics, 21: 243-247.

[22] Akaike H, “A new look at the statistical model identification”, IEEE Trans. Automatic Control, vol.19, Issue 6, pp. 716-723, 1974.

[23] Risannen, J. “Modelling by shortest data description”, Automatica, vol 14, Issue 5, pp. 465–471, Sep. 1978

[24] Multiple time series modeling: Determining the order of approximating autoregressive schemes. In P. R. Krishnaiah, Ed., *Multivariate Analysis*, vol. 4, pp. 283–295. North-Holland Publishing Company, 1977.

[25] M. Nekovee, “A survey of cognitive radio access to TV white spaces”, Hindawi Publishing Corporation, International Journal of Digital Media Broadcasting, Vol. 2010, Article ID 236568, Apr. 2010.