Crystallization model of magnesium primary phase in the AZ91/SiC composite

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Abstract

Metal-matrix composites are widely applied in the aerospace and automotive industries because of their low density and high rigidity. We focus on composites of magnesium-alloy (AZ91) matrix reinforced with SiC particles, with the aim of building a micro-macro model for their solidification. Heat-flow processes are modelled on the macro-scale, with latent heat release during nucleation and grain growth. Calculations are for a fixed cooling rate, consistent with the micro model for magnesium primary phase solidification. The population density of grains in the matrix is assumed to depend on the maximum supercooling and on the mass fraction of SiC particles. The present model gives reasonable agreement with measurements.

Keywords: A. Metal-matrix composites (MMCs), Particle-reinforcement; B. Microstructures; C. Computational modelling; E. Casting
1. Introduction

Metal-matrix composites (MMCs) can be made ‘ex-situ’ by addition of reinforcing particles to a liquid or semi-solid alloy, or ‘in-situ’ by forming the particles during the solidification of the matrix itself. The present work is concerned with ‘ex-situ’ composites consisting of the Mg-based AZ91 alloy (9 Al, 0.6 Zn, 0.2 Mn, 0.03 Si, 0.002 Fe, 0.003 Cu and 0.001 Ni (all wt%) [1]) reinforced with particles of silicon carbide. These composites solidify with equiaxed dendrites of a Mg-based primary phase and a non-equilibrium eutectic [2]. The eutectic is ignored in our treatment, as the primary phase has the predominant effect on the mechanical properties of the composite. Previous work has shown that the presence of reinforcement particles [3] or fibres [4] can affect the final microstructure of metal-matrix composites because of the effects of the reinforcing phase on the local solidification conditions.

Comprehensive treatments of solidification include macromodelling of fluid flow and heat transfer, and micromodelling of grain nucleation and growth [5]. The literature offers many examples of the application of so-called micro-macro models [6–10]. The most uncertain part of such models is often the nucleation law, which it is convenient to describe in terms of the nucleation rate, \( i.e. \) the rate at which the population density of grains increases as the supercooling is increased. In his experiments on cast iron, Oldfield [11] noted that the population of grains was proportional to the square of the supercooling, corresponding to the nucleation rate increasing linearly with supercooling. More recent modelling usually takes the nucleation rate to have a Gaussian form as a function of supercooling, often with superposed Gaussians for different types of nucleation [12].
In such approaches, it is implicit that the number of nucleated grains depends on the maximum supercooling reached and that if the melt were held at a constant supercooling there would be no ongoing nucleation as a function of time. The free-growth model [13] offers a possible explanation of such behaviour. The model is based on there being a size distribution of nucleant particles. Each particle can become an active nucleation centre when a critical supercooling is reached that is inversely proportional to the diameter of the particle.

The model presented in this article is modification of the kinetic-diffusion model of Liu and Elliott [7]. It builds on earlier work on the solidification of AZ91/SiC composites [1, 14−17]. The aim is to develop a method for predicting the mean grain size in the matrix of these composites from the mass fraction of SiC particles and the maximum supercooling as input parameters.

2. Mathematical model

A numerical micro-macro model of primary-phase solidification in AZ91/SiC composites must treat diffusion of heat and solute. The primary phase is α-Mg solid solution, approximated here as the binary alloy Mg-9wt%Al. The temperature field in the model can be described with the Fourier equation taking into account the heat of crystallization of the α-Mg phase:

\[
\frac{\partial T}{\partial \tau} = a \nabla^2 T + \frac{q_v}{c_v},
\]  

(1)
where \( a \) is the thermal diffusivity (\( m^2 \text{s}^{-1} \)), \( c_V \) the heat capacity (\( J \text{m}^{-3} \text{K}^{-1} \)), \( T \) the temperature (K), and \( \tau \) the time (s). The rate of release (per unit volume) of the heat of crystallization (latent heat) is given by:

\[
q_V = L \frac{df_s}{d\tau},
\]

where \( L \) is the latent heat (\( J \text{m}^{-3} \)), and \( \frac{df_s}{d\tau} \) is the rate of increase of solid fraction (\( s^{-1} \)). The solid fraction of primary phase, ignoring impingement, is given by:

\[
f_s = \sum_{i}^{n+1} \frac{4\pi}{3} R_i^3 N_i,
\]

where \( R_i \) is the radius of a spherical grain of the solid (m), \( N_i \) is the volumetric grain density (\( \text{m}^{-3} \)), and the index \( i \) denotes the class of grains that appears during the \( i^{th} \) time step since beginning of nucleation. The rate of increase of solid (\( \alpha \)-Mg) phase fraction in the \((n+1)^{th}\) time step is given by:

\[
\frac{df_s}{d\tau} = \sum_{i}^{n} 4\pi R_i^2 N_i \frac{dR_i}{d\tau} + \frac{4\pi}{3} R_{n+1}^3 \frac{dN_i}{d\tau}.
\]

The population densities \( N_i \) of grains in Eq. (4) are evaluated from an expression of Fraś et al. [9, 10] that relates the overall grain population density \( N_V \) to the maximum supercooling. This is based on (i) an exponential size distribution of nucleant particles (as also measured by Greer et al. [13]), and (ii) an inverse relationship between the diameter of a nucleant particle and the supercooling necessary for nucleation to occur on it [13].
It is expected that $N_V$, the overall population density of grains, must also depend on the mass fraction of added SiC particles. A complete expression has been derived for $N_V$ in the matrix phase of AZ91/SiC composites [1, 16, 17]. The solution of Eq. (4) requires knowledge of $N_V$ and of the local growth rate of the primary-phase grains.

The growth rate is controlled by the supercooling at the solid–liquid interface and by solute diffusion in the liquid. Two stages of solidification are considered. The first ends when the maximum supercooling is reached and from this point no more grains are nucleated. In the second stage, the growth rate of the already nucleated primary-phase grains depends on the supercooling and is controlled by solute (Al) diffusion in the liquid. Diffusion of the base metal (Mg) is not considered in this model. The diffusion of Al in the liquid around a spherical grain of Mg-based primary phase is shown schematically in Fig. 1. The solute concentrations shown in Fig. 1 are $C_c$ in the grain centre, $C_a, C_L$ on the solid and liquid sides of the moving interface, respectively, and $C_0$ in the liquid away from the computational domain.

The rates of change of aluminium concentration $C_a, C_L$ are, for primary-phase solid:

$$\frac{\partial C_a}{\partial \tau} = D_a \left( \frac{\partial^2 C_a}{\partial r^2} + \frac{2}{r} \frac{\partial C_a}{\partial r} \right) + \frac{r}{R} \frac{\partial C_a}{\partial r} \frac{\partial R}{\partial \tau}, \quad (5)$$

and for liquid:

$$\frac{\partial C_L}{\partial \tau} = D_L \left( \frac{\partial^2 C_L}{\partial r^2} + \frac{2}{r} \frac{\partial C_L}{\partial r} \right) + \frac{R_0 - r}{R_0 - R} \frac{\partial C_L}{\partial r} \frac{\partial R}{\partial \tau}, \quad (6)$$
where $D_a, D_L$ are the diffusion coefficients (m$^2$ s$^{-1}$) of aluminium in the Mg-Al primary-phase solid and in the liquid, $r$ is the distance between the centre of the $\alpha$-Mg grain and the node of the numerical grid, and $R_0$ is the maximum radius, dependent on the population density of grains in the primary phase:

$$R_0 = \sqrt[3]{\frac{3(1 - f_s)W}{4\pi}}.$$  \hspace{1cm} (7)

Here, $V$ is the standard analysed volume. The parameter $R_0$ allows grain impingement to be taken into account. The mass balance at the solid-liquid interface is described by:

$$\left(C_L^* - C_a^*\right) \frac{\partial R}{\partial \tau} = D_a \left. \frac{\partial C_a}{\partial r} \right|_{r=R^*} - D_L \left. \frac{\partial C_L}{\partial r} \right|_{r=R^*},$$  \hspace{1cm} (8)

where $C_L^*, C_a^*$ are equilibrium aluminium concentrations in the liquid and solid (wt. %).

The boundary conditions are as follows:

- at the crystal-liquid interface:
  
  $C = C_a^*, \ r = R^*, \ on \ the \ grain \ side,$
  
  $C = C_L^*, \ r = R^*, \ on \ the \ liquid \ side,$

- at the centre of a grain:

$$\left. \frac{\partial C_a}{\partial r} \right|_{r=0} = 0,$$  \hspace{1cm} (9)

and in the liquid away from the crystal-liquid interface:

$$\left. \frac{\partial C_L}{\partial r} \right|_{r=R_0} = 0.$$  \hspace{1cm} (10)
To determine the velocity of the solid-liquid interface, Eqs (5) and (6) are coupled with mass balance, Eq. (8). A similar model has been presented in ref. [10]. Numerical solution of Eqs (1) to (8) allows prediction of $N_v$, the population density of grains in the matrix phase in the MMC, as a function of the mass fraction of SiC particles.

3. Modelling results

Calculations have been performed for the composite AZ91/SiC, with the assumption that the particles of the reinforcing phase SiC are uniformly distributed in the liquid. This assumption facilitates calculation of the effects of the mass fraction and size of the SiC particles on the heterogeneous nucleation of primary magnesium grains and the cooling rate. The supercooling $\Delta T$ during solidification of the primary phase is defined as the difference between the nucleation temperature $T_N$ (the point at which the first grains of the primary phase appear) and the current temperature, $\Delta T = T_N - T(\tau)$. As the solidification occurs under non-equilibrium conditions, the measured $T_N$ is taken as the best estimate of the liquidus temperature. In the model, new grains are taken to have an initial diameter of 1 $\mu$m. Further inputs are:

a) diffusion coefficients of Al [18]: $D_a = 2.7 \times 10^{-10}$; $D_L = 2.7 \times 10^{-8}$ m$^2$ s$^{-1}$.

b) nucleation temperature calculated from the empirical equation (for selected mass fractions of SiC particles) [1, 16, 17]:

$$T_N (m_{f_{SiC}}) = 606 - 5.8 \exp(-90.4 m_{f_{SiC}}),$$

(11)
where $mf_{SiC}$ denotes the mass fraction of SiC particles.

c) the initial temperature of the liquid is taken to be 700°C, well above the liquidus temperature.

d) the SiC particle diameter $d$ is taken to have a Weibull distribution [9, 10]:

$$n(d) = \frac{1}{d_{SiC}} \exp\left(-\frac{d}{d_{SiC}}\right),$$

where the characteristic value for this distribution is taken to be $d_{SiC} = 45 \mu m$ [1, 16, 17], which is the arithmetic average diameter of all SiC particles put into composite and $n(d)$ denotes the number of particles of diameter $d$ present in melt. This function based on an exponential relationship between the population density of grains $N_v$ and $\Delta T$ [9]. This law is modified to take mass fraction into consideration, as it is expected to affect the nucleation rate. Unknown parameters in the model are obtained by fitting to the measured data [1]:

$$N_v(\Delta T, mf_{SiC}) = 1.42 \times 10^{13} \exp\left(61.9mf_{SiC} - \frac{36.25 \exp(29.3mf_{SiC})}{\Delta T}\right).$$

(13)

e) $L = 267.86 \text{ K}$ [19].

f) imposed cooling rate $\Phi = a\nabla^2 T$, depends on temperature, mass fraction of SiC particles and the foundry mould:

$$\Phi(T) = \begin{cases} 
-40 & \text{ for } T > 625^\circ C \\
200 & \text{ for } 625^\circ C \geq T > 620^\circ C \\
5T - 2900 & \text{ for } 620^\circ C \geq T > 600^\circ C \\
100 & \text{ for } T \leq 600^\circ C.
\end{cases}$$

(14)
analysed volume: \( V = 1 \text{ m}^3 \).

These calculations are compared with data from measurements on plate-shaped samples of AZ91/SiC composite. The temperature was measured with a K-type thermocouple at the geometrical centre of the sample. Results for samples with 0.1, 2, 5\% mass fraction of SiC particles, gathered in Table 1, show that \( N_v \) increases with the mass fraction of SiC particles. The same tendency for finer microstructure can be observed in experiment and in numerical simulation. The experimental results and the simulations are correlated. Selected examples are presented in the figures discussed below.

Figure 2 presents cooling curves of the AZ91/SiC composite obtained from simulation and experiment. There is good agreement, specifically within the solidification range of the magnesium primary phase, from the pouring temperature down to 560\(^\circ\)C. The cooling curves start to diverge below 560\(^\circ\)C. The differences most probably originate from the micro-model calculations, which derive the cooling rate from Eq. (12) rather than from thermophysical parameters.

The numerical calculations permit determination of the temperature at which nucleation of the \( \alpha \)-Mg phase is terminated (at a time \( \tau_r \) marking the onset of recalescence). After this time, the numerical calculation continues to model the growth of existing grains, but there is no further nucleation. This calculation can predict the population density of the grains in the as-cast primary phase (\( \alpha \)-Mg). The computed \( N_v \) in the primary phase for selected cases are in reasonable agreement with the value measured in experiment, Table 1.
The numerical simulation suggests that nucleation of the primary phase occurs over a period of about 10 s during the solidification of the AZ91/5%SiC composite (Fig. 3). Figure 4 shows the kinetics of primary-phase solidification: the rate increases rapidly for 100 seconds and then stabilizes.

4. Conclusions

Numerical calculations, based on a model for nucleation and growth of the magnesium primary phase during solidification of an AZ91/SiC composite, give predictions in reasonable agreement with experimental measurements. The differences between the simulation and measured data originate from inaccurate treatment of the cooling rate in the simulation. In the chosen case of an AZ91/SiC composite, the predicted grain densities of the matrix phase are consistent with the measured grain size, capturing the key trends in the microstructural scale. It is expected that the present model can be used to simulate the solidification of the $\alpha$-Mg matrix phase in AZ91/SiC composites generally, giving reasonable predictions for the population density of grains in the matrix phase as a function of the mass fraction of SiC particles.

Acknowledgements

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References


Table Captions

Table 1: Population density $N_V$ of grains in the matrix phase of AZ91/SiC composites: comparison of measurements and values from numerical simulation, for different mass fractions of reinforcement particles.

<table>
<thead>
<tr>
<th>SiC&lt;sub&gt;p&lt;/sub&gt; content (wt.%</th>
<th>0.1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_V$ from simulation ($10^{11}$ m$^{-3}$)</td>
<td>0.88</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>$N_V$ from experiment ($10^{11}$ m$^{-3}$)</td>
<td>0.95 ±0.33</td>
<td>1.2 ±0.4</td>
<td>1.4 ±0.3</td>
</tr>
</tbody>
</table>

[16, 20]
Fig. 1. Aluminium concentration around a spherical grain of primary phase growing into the liquid alloy, treated as binary Mg-9Al (symbols are defined in the text).
Fig. 2. Cooling curves obtained from simulation and experiment. The eutectic reaction revealed by the thermal arrest at ~425°C on the experimental cooling curve was not taken into consideration in the modelling.
**Fig. 3.** Numerical simulation of the increase in the total number of grain nucleation events as an AZ91/5%SiC sample is cooled according to the simulated (black) curve in Fig. 2. The number of nucleation events corresponds directly to the total number $N_v$ (per unit volume) of primary $\alpha$-Mg phase grains in the as-cast structure.
**Fig. 4.** Corresponding to Fig. 3, numerical simulation of the increase in solid fraction of the matrix as an AZ91/5%SiC sample is cooled according to the simulated (black) curve in Fig. 2.