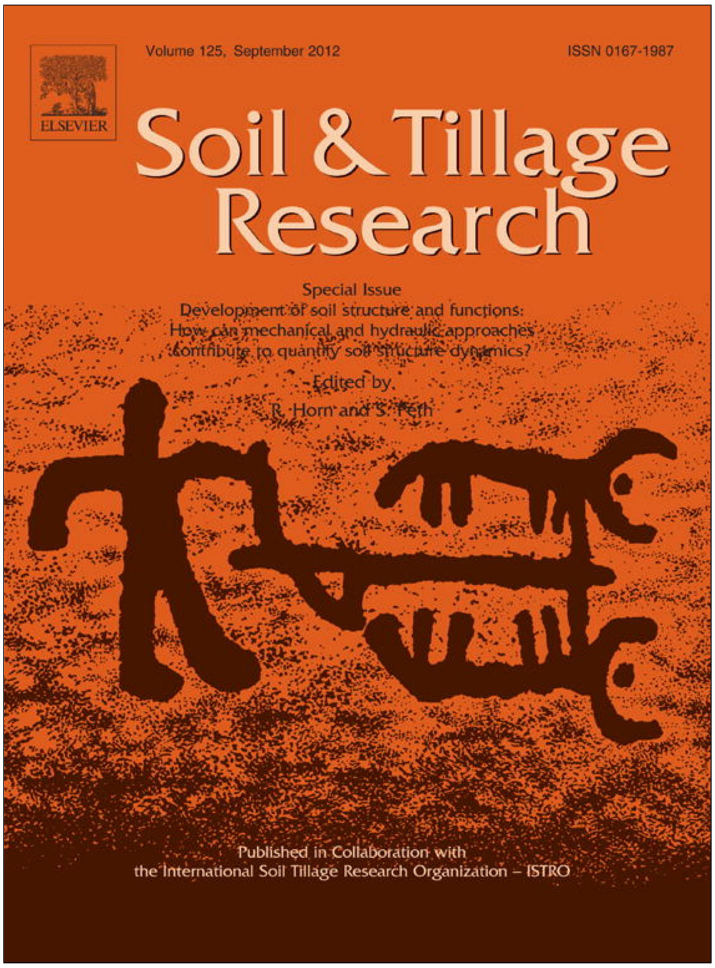
Volume 125, September 2012
ISSN 0167-1987
ELSEVIER
Soil & Tillage Research
Special Issue
Development of soil structure and functions:
How can mechanical and hydraulic approaches
contribute to quantify soil structure dynamics?
Edited by
R. Horn and S. Peth
Published in Collaboration with
the International Soil Tillage Research Organization – ISTRO

Soil & Tillage Research 125 (2012) 30–37

Contents lists available at SciVerse ScienceDirect

Soil & Tillage Research

journal homepage: www.elsevier.com/locate/still

The velocity of shear waves in unsaturated soil

W.R. Whalley [a,*], M. Jenkins [b], K. Attenborough [c]

[a] Rothamsted Research, West Common, Harpenden, St Albans AL5 2JQ, UK
[b] Delta-T Devices, 130 Low Road, Burwell, Cambridge CB25 OEJ, UK
[c] The Open University, Walton Hall, Milton Keynes MK7 6AA, UK

ARTICLE INFO

Keywords:
Soil
Shear wave velocity
Unsaturated
Consolidation
Tri-axial testing

ABSTRACT

The velocities of shear waves V_s in two soils, a loamy sand and a sandy clay loam, were measured at various matric potentials and confining pressures. We used a combination of Haines apparatus, pressure plate apparatus and a Bishop and Wesley tri-axial cell to obtain a range of saturation and consolidation states. We proposed a single effective stress variable based on a modification to Bishop's equation which could be used in a published empirical model (Santamarina et al., 2001) to relate shear wave velocity to soil physical conditions. Net stress required a nonlinear transformation. Matric potential was converted into suction stress with the function proposed by Khallili and Khabbaz (1998), thus requiring an estimate of the air entry potential. We found it was possible to fit V_s to void ratio, net stress and matric potential with a set of four parameters which were common to all soils at various states of saturation and consolidation. In addition to the data collected for this study we also used previously published data (Whalley et al., 2011). The utility of shear wave measurements to deduce soil physical properties is discussed.

1. Introduction

Previously we have explored how consolidation of saturated soil affects the velocity of shear waves (Whalley et al., 2011). We used an empirical model for shear wave velocity, V_s, described by Santamarina et al. (2001) and written as

$$V_s = AF_e\left(\frac{\sigma}{1\ \mathrm{kPa}}\right)^{\beta} \tag{1}$$

where F_e is a the void ratio normalization factor which takes account of changes in elastic modulus (shear modulus in this case) due to differences in porosity, σ is the effective stress (often σ' is used but here we use σ to be consistent with Whalley et al. (2011)), A is the value of V_s when σ = 1 kPa and β is a fitted parameter. The value of the void ratio normalization factor, F_e is given as follows

$$F_e = \frac{(2.97 - e)^2}{1 + e} \tag{2}$$

where e is the void ratio (Lo Presti, 1995). The constant in the numerator bracket can be obtained empirically by fitting to data, but a value 2.97 has been recommended for angular particles (Santamarina et al., 2001) and has been found to be suitable for a range of agricultural soils (Whalley et al., 2011). In most agricultural soils void ratio is very sensitive to the effective stress and they are related to each other by the compression characteristic which is commonly written as

$$e = n - \lambda \log_{10} \sigma \tag{3}$$

where n and λ are fitted parameters and depend on soil type. By combining these equations we were able to collapse V_s data from a range of soils and consolidation states onto a common relationship (Whalley et al., 2011). The estimated value of β was 0.39 which is midway between the expected values of 0.25 for rough angular particles and 0.75 for a porous material where particle contacts are governed by Coulomb forces (see Santamarina et al., 2001).

A limitation of the common relationship, described by Whalley et al. (2011), is that it is restricted to saturated soil. Here the stress supported by the fabric of the soil is given by

$$\sigma = \sigma_c - u_w \tag{4}$$

where σ is the effective stress, σ_c is the total stress (in this case the pressure in the tri-axial cell) and u_w is the pressure of the soil water. This is the effective stress according to Terzaghi (see Santamarina et al. (2001) or Mitchell and Soga (2005) for discussion). For *unsaturated* soil the effective stress is given by Bishop and Blight (1963) as

$$\sigma = (\sigma_c - u_a) - \chi(u_w - u_a) \tag{5}$$

where $(u_w - u_a)$ is the matric potential $(\sigma_c - u_a)$ is the net stress (excess of total stress over air pressure), χ is a factor that converts matric potential ($\psi = u_w - u_a$) into what is sometimes referred to a "suction stress" (e.g. Lu et al., 2010) and u_a is the air pressure in the

* Corresponding author. Tel.: +44 01582 763133; fax: +44 01582 760 981.
E-mail address: Richard.whalley@rothamsted.ac.uk (W.R. Whalley).

0167-1987/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.still.2012.05.013

soil sample. Alonso et al. (2010) observed that because of a lack of data, the degree of saturation, S, is often used as a candidate for χ. In the special case of tensile failure testing Mullins (2000) found that for $S > 0.5$ the simple assumption that $\chi = S$ worked well. Essentially χ weights the contribution of the matric potential according to its effect on effective stress. Although, S is widely used to estimate χ (e.g. Whitmore et al., 2011) an alternative has been suggested by Khallili and Khabbaz (1998) who proposed that

$$\chi = \left(\frac{\psi}{\psi_{ae}}\right)^{-0.55} \quad \text{for } \psi < \psi_{ae} \text{ otherwise } \chi = 1 \qquad (6)$$

where ψ_{ae} is the matric potential at which air invades a drying soil, often called the air entry potential (see Mitchell and Soga (2005) for discussion).

It is known that the velocity of elastic waves in unsaturated soil depends on both the net stress applied to the soil volume under test as well as the saturation state. This has been shown clearly by Lu and Sabatier (2009) in a two year survey of an outdoor field site exposed to the prevailing weather. They measured matric potential, water content and the velocity of compression waves (P waves), V_p. For the soil they monitored they were able to develop an empirical power law relationship between V_p and matric potential of sufficient quality ($r^2 = 0.92$) to form the basis of a calibration. They also found correlations between V_p and soil water content. The measured values of V_p were also sensitive to depth; deeper layers tended to have higher wave speeds (V_p). This observation was explained by the effect of overburden and it is consistent with the Bishop and Blight equation for effective stress. In deeper layers the higher overburden pressure would tend to increase wave speeds but these layers also tend to be wetter which reduces wave speed (see Velea et al., 2000).

This paper has two objectives. First, we wish determine the relationship between χ and soil moisture in unsaturated agricultural soils. Specifically we wanted to test the function for χ proposed by Khallili and Khabbaz (1998) which has never been applied to agricultural soils. Our second objective is to characterize the additive nature of net stress ($\sigma_c - u_a$) and suction stress ($\chi\psi$) in so far as they can be used to estimate a single effective stress variable for use in the empirical model of Santamarina et al. (2001) for the velocity a shear wave in variably consolidated and saturated soil. Here the effects of stress history will not be considered.

2. Materials and methods

2.1. Soils and other data

The properties of the agricultural soils used in this work are listed in Table 1. One of the soils (Butt Close) was previously used and described by Whalley et al. (2011). Warren Field soil has not been previously used to make V_s measurements and it was included here because it is from an important group of soils used for arable agriculture. We included the data of Whalley et al. (2011), but excluded any data collected when the soil was on the elastic rebound curve.

2.2. Measurement of V_s in unsaturated, but unconfined sands and soil

We used the Haines apparatus shown in Fig. 1 to measure the velocity of shear waves in sand at various matric potentials. To measure the shear wave velocity in two agricultural soils (see Table 1) we used a pressure plate apparatus (see Fig. 1).

2.3. Measurement of V_s in confined soil

Shear wave velocity in consolidated soils was measured using a Bishop and Wesley tri-axial cell (GDS instruments, 32 Murrel Green Business Park, Hook, Hampshire, RG27 9GR, UK) (see Bishop and Wesley (1975) and Fig. 1a). These soil samples, 100 mm long and 50 mm in diameter, were obtained by packing thin layers of soil into a split brass mould with an axial pressure of approximately 10 kPa. Before packing the water contents were adjusted so the soils were at their most compactable (see Gregory et al., 2010). Once in the triaxial cell, the soils were saturated by increasing the pressure of the water confining (σ_c) the soil sample and the pressure of the soil water u_w to 600 and 590 kPa respectively over a period of 24 h. The soils were then consolidated to a range of effective stresses between 10 (initial condition) and 400 kPa. During consolidation a record of the sample length (measured with a LVDT) was kept. The S wave was generated by a piezoelectric device in the top-cap and detected in the pedestal by a similar device. This apparatus is commercially available from GDS, who supplied the Bishop and Wesley cell. The input electrical signal was a single cycle of a sine wave with a 0.2 ms period. The time for the S wave to travel through the soil sample was determined by comparing the input and detected shear waves (Whalley et al., 2011).

For the loamy sand (Butt Close) but not on the sandy clay loam (Warren Field) measurements were made on both normally consolidated soil and also soil during elastic rebound and in both cases the applied effective stress was isotropic. On the Butt Close loamy sand the relationships between V_s and effectives stress was not greatly affected by stress history (Whalley et al., 2011) and the effects of rebound could be ignored. During the tests on Butt Close soil at an effective stress of 98 kPa, the soil was drained by simultaneously increasing the pore air pressure (u_a) and the confining pressure in the triaxial cell (σ_c). This had the effect of

Table 1
Description of soils used in this work (Butt Close and Warren Field) as well as those soils used by Whalley et al. (2011) for which data was used in the curve fitting.

Property	Units	Broadbalk FYM	Rowden	Butt Close	Warren
Location		Rothamsted Res. Hertfordshire	North Wyke Res. Devon	Woburn Farm Res. Bedfordshire	Woburn Expt. Farm Bedfordshire
Grid reference	GB national grid	TL121134	SX652994		SP968364
	Latitude	51°48′36″N	50°46′42″N	52°00′42″N	52°01′06″N
	Longitude	00°22′30″W	03°54′54″W	00°32′42″W	00°35′30″W
Soil type	SSEW group	Paleo-argillic brown earth	Stagnogley soil	Brown Earth	Brown Earth
	SSEW series	Batcombe	Hallsworth	Cottenham	Flitwick
	FAO	Chromic Luvisol	Gleyic Luvisol	Cambic Arenosol	Dystric Cambisol
	USDA	Paleudalf	Haplaquept	Udipsamment	Hapludalf
Landuse		Arable; cereals; farmyard manure	Grass; unfertilized; grazed	Arable	Arable; cereals; beans
Sand (2000–63 μm)	$g\,g^{-1}$ dry soil	0.252	0.147	0.875	0.538
Silt (63–2 μm)	$g\,g^{-1}$ dry soil	0.497	0.396	0.055	0.203
Clay (<2 μm)	$g\,g^{-1}$ dry soil	0.252	0.457	0.072	0.260
Texture	SSEW class	Clay loam	Clay	Loamy sand	Sandy clay loam
Particle density	$g\,cm^{-3}$	2.508	2.439	2.65	2.587
Organic matter	$g\,g^{-1}$ dry soil	0.060	0.138	0.01	0.038

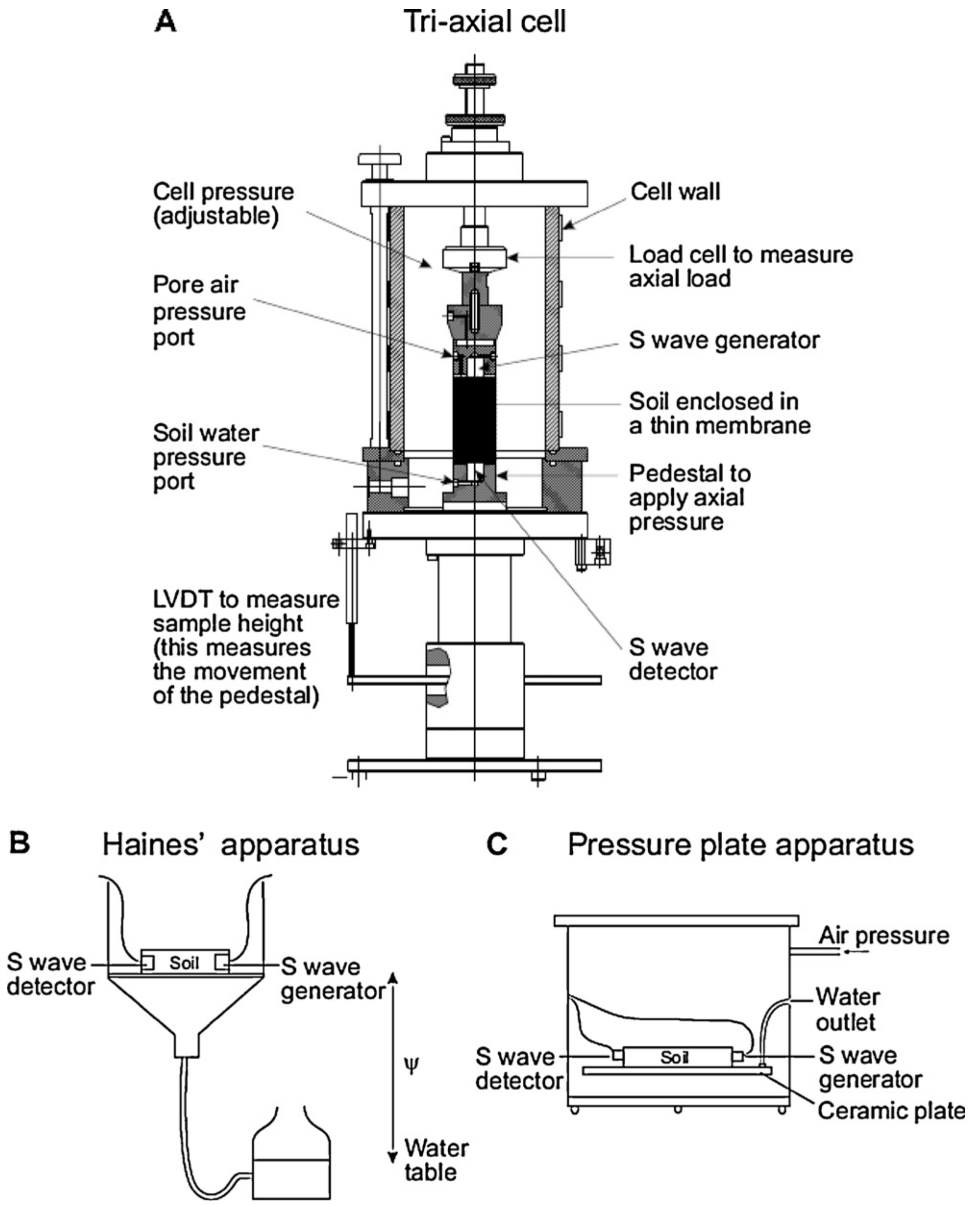

Fig. 1. Three different experimental systems which were used to make shear wave measurements as a function of consolidation and saturation.
Source: Figure A is redrawn from Whalley et al. (2011).

keeping the net stress constant ($\sigma_c - u_a$) but it applied a matric suction ($u_w - u_a$) to the soil water. Drainage of the soil sample could be monitored with changes in the volume of the pressure–volume controller which regulated the pressure of the pore water.

2.4. Extension of the Santamarina model to unsaturated soil

We propose to test the following heuristic function

$$V_s = A\frac{(2.97-e)^2}{1+e}\left((\sigma_c - u_a)^r - (u_w - u_a)\left(\frac{(u_w-u_a)}{(u_w-u_a)_{ae}}\right)^{-0.55}\right)^{\gamma} \quad (7)$$

to describe how V_s varies with matric potential ($\psi = u_w - u_a$) and net stress ($\sigma_c - u_a$). We used the expression for χ proposed by Khallili and Khabbaz (1998) which was tested by measuring V_s in loose sands and soil to determine if a linear relationship between effective stress and V_s could be obtained when $\sigma_c = 0$. When $\psi = 0$ the proposed function for unsaturated soils (Eq. (7)) will reduce to the one for saturated soils proposed by Whalley et al. (2011) if $r\gamma = 0.39$. However, values of $r\gamma$ in the range 0.25–0.75 would be consistent with published values for soil (Santamarina et al., 2001). We also used the data set of Whalley et al. (2011) to determine the values of the parameters of Eq. (7).

3. Results

3.1. Loose sand and soil on a tension plate or pressure plate $\sigma_c = 0$

The water release characteristics of the two sands are plotted in Fig. 2a and the shear wave velocity is plotted against matric potential in Fig. 2b. The relationship between V_s and matric potential can be divided into three regions each of which is linear. The boundaries between these linear regions correspond approximately to the air entry potential and the matric potential at which the residual water content is reached in drying sand. At matric potentials higher than the air entry potential the sand is tension saturated and V_s increases at a constant water content but with decreasing matric potential. When the matric potential

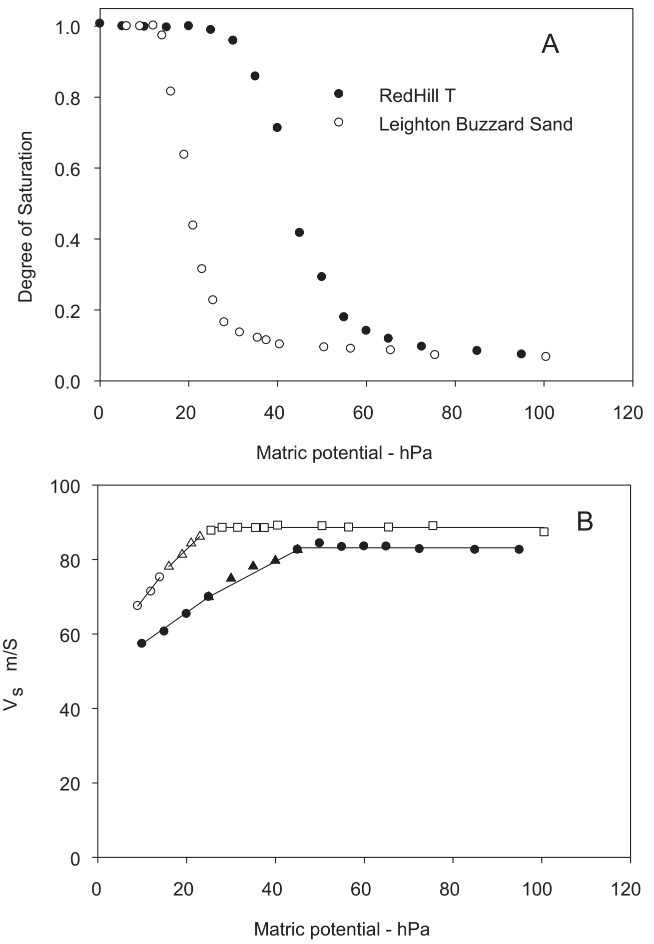

Fig. 2. Water retention characteristics for Redhill T and Leighton Buzzard sand are shown in the top panel. In the lower panel V_s is plotted against matric potential for these two sands. In the lower panel the three linear regions were identified visually and correspond to the regions of tension saturation, capillary fringe and residual water content.

was smaller than the air entry potential the sand was in the capillary fringe and water content decreases with matric potential. In the capillary fringe V_s increased with decreasing matric potential, but more slowly than in the tension saturated region. At matric potentials corresponding to residual water contents V_s remained constant. The Redhill T sand had a greater range of matric potentials in the capillary fringe compared to the Leighton Buzzard sand and the effect of the capillary fringe on V_s was more easily seen. In Fig. 3 effective stress for the Redhill T sand is plotted against V_s using three different assumptions: (A) $\chi = S$, (B) $\chi = S$, if $S > 0.5$ otherwise $\chi = 0.5$ or (C) the function Khallili and Khabbaz (1998). Only the proposed function of Khallili and Khabbaz (1998) linearized the relationship for matric potentials in both the tension saturated range and the capillary fringe (Fig. 3c).

Fig. 4a shows V_s for unconsolidated loamy sand and a silty loam soil against matric potential. For both soils the relationships are non-linear. To apply the effective stress function of Khallili and Khabbaz (1998) to these soils required an estimate of the air entry potential. In agricultural soils this can be difficult to estimate by simple inspection of the water release curve as is in the case with sand (e.g. Fig. 2). To overcome this difficultly we approximated the air entry potential from the point of inflexion on the water release characteristic, which is the point at which water drains most rapidly as the matric potential is lowered (Dexter and Bird, 2001).

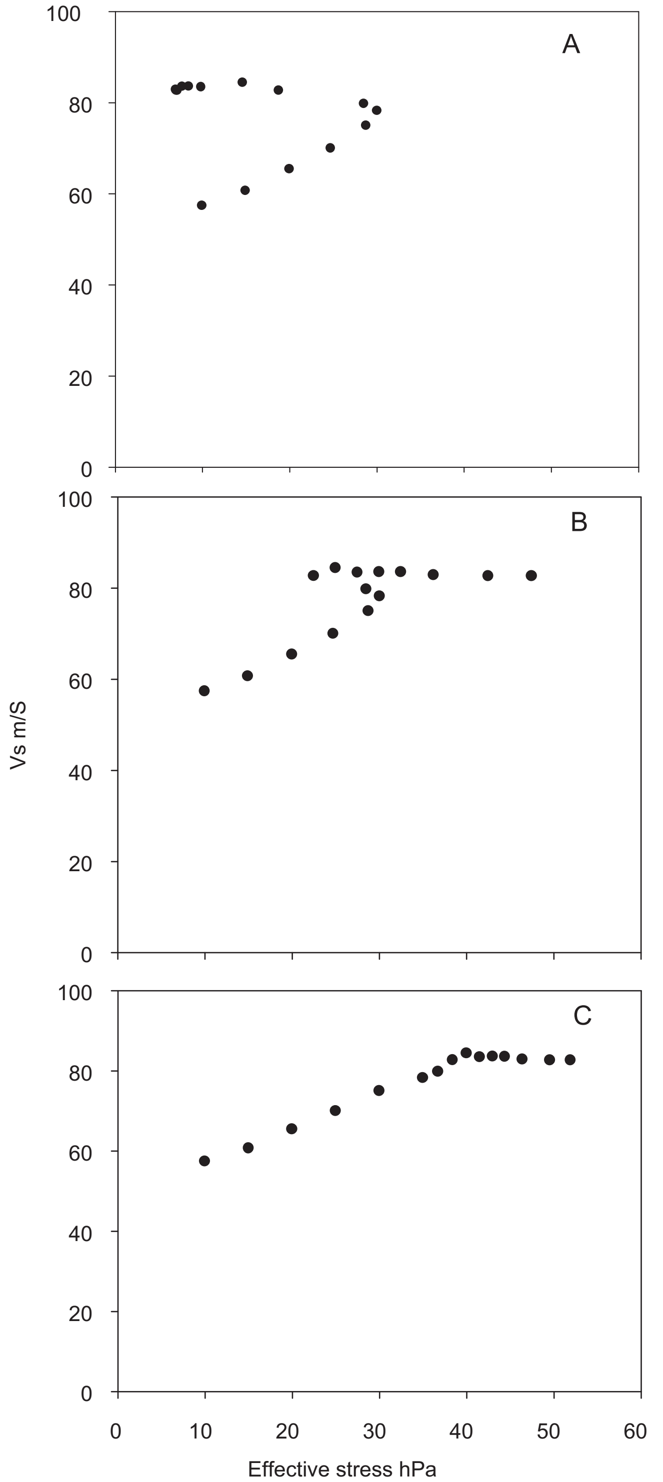

Fig. 3. These panels show V_s plotted against effective stress for Redhill T sand. Three different assumptions about the calculation of effective stress were made: (A) $\chi = S$, (B) $\chi = S$, if $S > 0.5$ otherwise $\chi = 0.5$ and (C) the function of Khallili and Khabbaz (1998).

The matric potential at the point of inflexion, ψ_{inf}, on the water release characteristic is given by

$$\psi_{inf} = \frac{1}{\alpha}\left(\frac{1}{m}\right)^{1/n} \quad (8)$$

where α, n and m are the van Genuchten parameters (see Dexter and Bird, 2001). The water release characteristics for the loose Butt

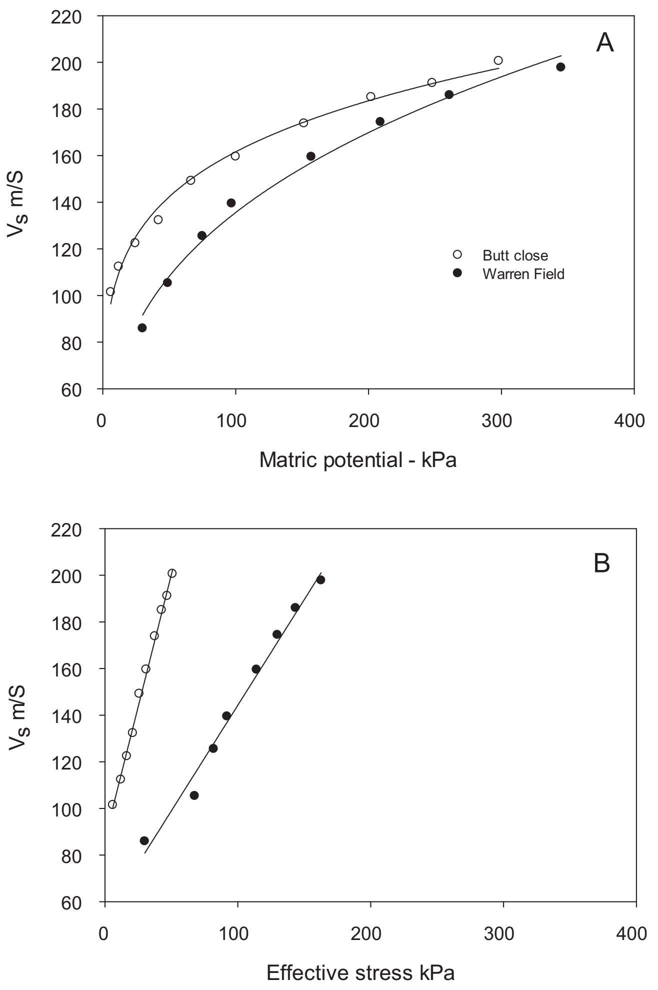

Fig. 4. Shear wave velocity, V_s, is plotted against matric potential for two agricultural soils (A) and against effective stress (B) calculated with the function of Khallili and Khabbaz (1998).

Close (loamy sand) and Warren Field (sandy clay loam) are given in Fig. 5 and they were used to calculate the air entry potentials of −12 and −88 kPa, respectively. The use of these matric potentials with the effective stress function of Khallili and Khabbaz (1998) produced a linear relationship between V_s and effective stress (Fig. 4b).

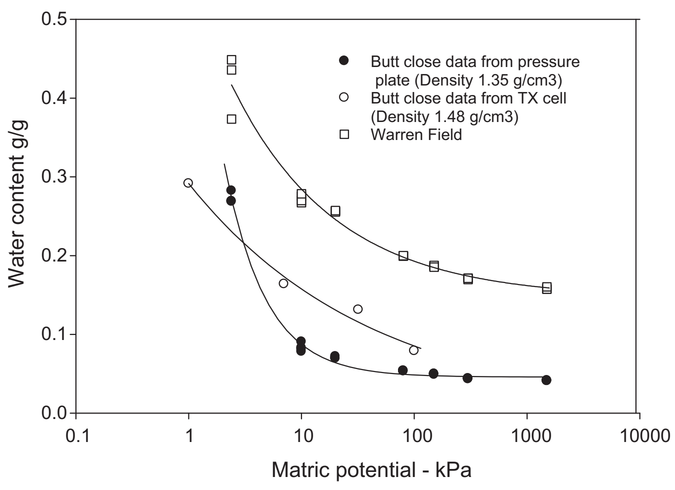

Fig. 5. Water release characteristics of the loamy sand (Butt Close) and the sandy clay loam (Warren Field). Two sets of data are plotted for the loamy sand one of them of determined in the pressure plate apparatus and the second was determined during tri-axial testing of a consolidated soil sample.

3.2. Consolidated soils

Fig. 6a and b shows shear wave velocity of soil plotted against effective stress for consolidated loamy sand and sandy clay loam. Also plotted are the data for loose soil in Fig. 4b on the assumption that $\sigma_c = 0$. For the saturated soils effective stress was calculated with Eq. (4). For the unsaturated soils effective stress was calculated with Eq. (5) and χ was estimated with Eq. (6) on the assumption that $\psi_{inf} = \psi_{ae}$ (i.e. Eq. (8)).

At an effective stress of approximately 100 kPa the loamy sand (Fig. 6a) was drained to matric potentials of −7, −32 and −100 kPa. The height of the samples was 10 cm and it took 8 weeks for the sample to equilibrate at a matric potential of −100 kPa. In Fig. 5 the water release characteristic of loose Butt Close soil is compared with that determined on a more compact soil in the tri-axial cell. The differences between the two sets of water release data are likely to be due to the effects of compaction (Gregory et al., 2010). Once the sample had been equilibrated to a matric potential of −100 kPa, the cell pressure was varied to give a range of net stress values for when $\psi = -100$ kPa and measurements of V_s were made.

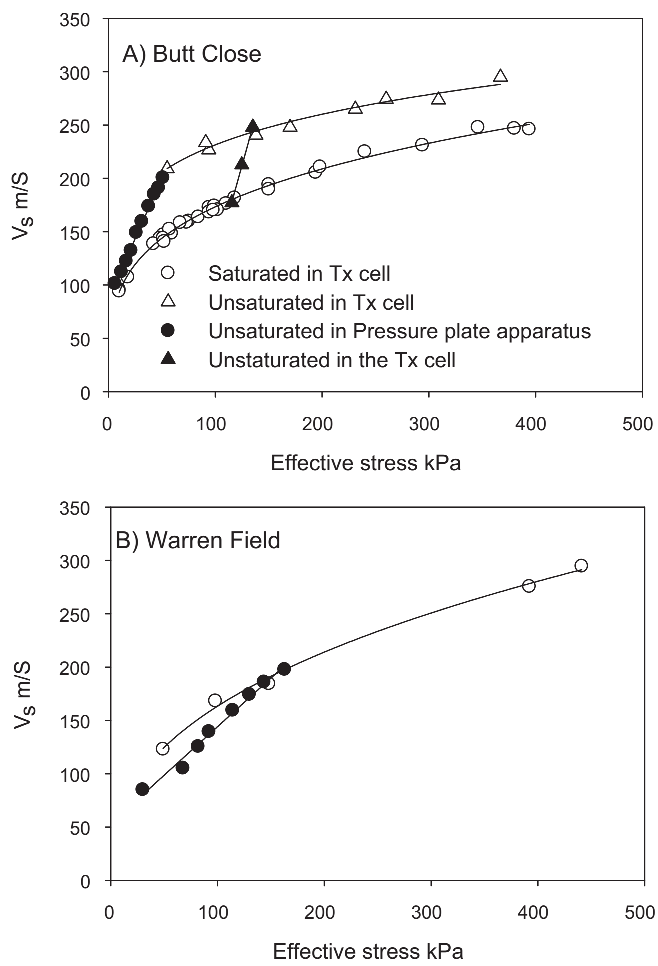

Fig. 6. Shear wave velocity, V_s, plotted against effective stress for a loamy sand (Butt Close) in panel A and for a sandy clay loam (Warren Field) in panel B. Effective stress was calculated with the Bishop equation; $\sigma = (\sigma_c - u_a) - \chi(u_w - u_a)$ where $\chi = ((u_w - u_a)/(u_w - u_a)_{ae})^{-0.55}$ for $(u_w - u_a) < (u_w - u_a)_{ae}$ otherwise $\chi = 1$. The air entry potential $(u_w - u_a)_{ae}$ was assumed to be equal to the point of inflexion on the water release characteristics (see Fig. 5). For the compacted loamy sand we assumed that the point of inflexion on the water release characteristic was insensitive to compaction.

Table 2
Values of the parameters for Eq. (7) determined from curve fitting. The standard errors of the fitted parameters are given in parenthesis.

Soil	Parameter	Separate A	Common parameters
FYM (clay loam)	*A*	26.75 (±1.63)	2.368 (±0.624)
Rowden (clay)	*A*	36.06 (±2.44)	
Warren Field (Sandy clay loam)	*A*	30.11 (±1.80)	
Butt Close (loamy sand)	*A*	23.68 (±1.20)	
	r	0.650 (±0.011)	0.746 (±0.013)
	γ	0.340 (±0.013)	0.356 (±0.013)
Constant in the void ratio normalization factor		2.97[a]	7.085 (±0.794)
	b[b]	−0.55[b]	−0.55[b]
Percentage of variance accounted for		95.7	95.6

[a] The value recommended by Santamarina et al. (2001).
[b] The value recommended by Khallili and Khabbaz (1998).

In Fig. 6b the relationship between V_s and effective stress for loose unsaturated (or tension saturated) Warren Field soil (from Fig. 4b) is compared with that for a consolidated saturated soil. Here, as a first approximation, V_s is determined by effective stress whether calculated by the Terzaghi equation in saturated soil or with the Bishop and Blight equation in unsaturated soils and using the χ value recommended by Khallili and Khabbaz (1998). However, from our Butt Close loamy sand data (Fig. 6a) is it clear that this is not always the case.

3.3. A common function to explain V_s in consolidated soil at various soil matric potentials

We fitted Eq. (7) with Genstat® (VSN International Ltd., Hemel Hempstead, UK) to the data in Fig. 6a and b and included the data of Whalley et al. (2011) in the fit (ignoring rebound data). The values of the fitted parameters are given in Table 2. We fitted Eq. (7) in two different ways. Firstly, we allowed *A* to vary with soil type, but used a common *r* and γ. The value of the exponent in the Khallili and Khabbaz (1998) function was −0.55 as they have recommended and as used in Fig. 4b. The value of the constant in the void ratio normalization function was kept at 2.97 as recommended by Santamarina et al. (2001). Secondly, we have sought a fit which would provide a common set of parameters for all the soils. To achieve this we found it necessary to adjust the constant of the void ratio normalization factor for best fit. The fitted value of 7.085 is within the range of values published by Mitchell and Soga (2005). Interestingly both approaches gave similar results from a statistical perspective (Fig. 7).

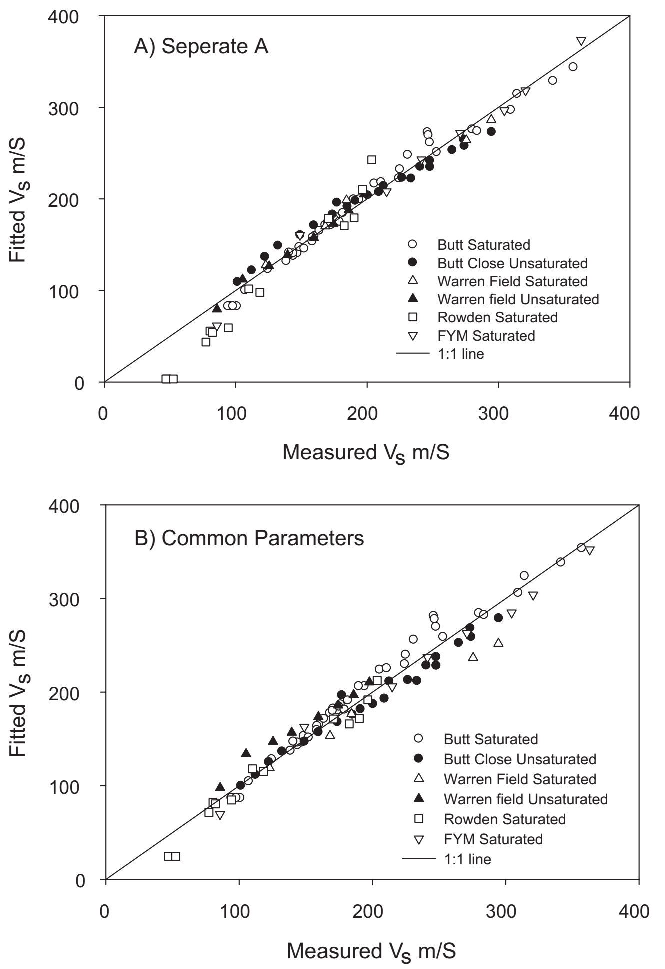

Fig. 7. Fitted values of V_s are plotted against measured values. We used Eq. (7) with either separate *A* parameters for each soil (panel A) or with common parameters (panel B) for all soils (see Table 2).

4. Discussion

4.1. The relationship between χ and soil moisture

In the pure sands the three separate regions of soil drying (tension saturated, capillary fringe and residual water content) can be identified. In each of these regions the relationship between V_s and matric potential is linear, but with different slopes (see Fig. 2). A successful estimation of χ should provide a relationship between V_s and effective stress over both the tension saturated and capillary fringe regions with no discontinuity. In the tension saturated range $\chi = 1$. For the two sands (Redhill T and Leighton Buzzard) the tension saturated regions are easily identified (Fig. 2). The only approach we have used to calculate effective stress which gives a linear relationship with V_s over both tension saturated and capillary fringe regions is that proposed by Khallili and Khabbaz (1998). This is seen most clearly in the Redhill T sand (Fig. 3c). It is less clear in Leighton Buzzard sand, probably because the capillary fringe occurs over a narrow range of matric potentials. The approximation proposed by Mullins (2000) where $\chi = S$, if $S > 0.5$ otherwise $\chi = 0.5$ is better than assuming $\chi = S$ (compare Fig. 3a and b). In the range of residual water contents V_s does not change with decreasing matric potential which suggests that capillary break down has occurred and the water content and matric potential of the soil water are no longer determined by the water table on the Haines apparatus (Fig. 1). In these circumstances the water release curve should be treated as a discontinuous function. At matric potentials smaller than those in the capillary fringe water transport in these sands can only occur by diffusion in the vapor phase.

The relationship between V_s and matric potential in agricultural soils is nonlinear and the different regions of soil drying cannot be detected in either of these data sets (Fig. 4a) or in the water release characteristics, as is the case for the sands (Fig. 2). It possible that the additive effects of the broad range of pore sizes in such soils (Gregory et al., 2010), compared to pure sands (Fig. 2), make a single air entry potential hard to detect. For the loamy sand, even in the range of matric potentials associated with the residual water content V_s increases with decreasing matric potential. This suggests that changes in air pressure in the pressure-plate apparatus (used to drain these soils) are reflected in a change in matric potential in the soil matrix. To use the effective stress function of Khallili and Khabbaz (1998) an air entry potential is needed. We have found that by approximating the air entry potential with the matric potential at the point of inflexion of the van Genuchten curve (Dexter and Bird, 2001) a linear relationship between V_s and effective stress can be obtained (Fig. 4b).

4.2. The additive nature of the net stress ($\sigma_c - u_a$) and suction stress ($\chi\psi$)

In saturated soils a nonlinear relationship between V_s and effective stress has been reported (Whalley et al., 2011; Santamarina et al., 2001; Lu et al., 2004; Nakagawa et al., 1997). In these soils effective stress is given by ($\sigma_c - u_w$) and it is a macroscopic phenomenon and nonlinearly related to the inter-particle forces that determine V_s (see Santamarina et al., 2001). For unsaturated soils when net stress = 0, effective stress is determined by capillary processes and it is a microscopic phenomenon. Effective stress calculated with the function of Khallili and Khabbaz (1998) has a linear relationship with V_s, although the relationship is not common between different soil types (Fig. 4b). Even at high levels of net stress ($\sigma_c - u_a$), the Khallili and Khabbaz (1998) function linearizes the relationship between V_s and effective stress (Fig. 6a). In proposing Eq. (7) for unsaturated soil we assume that the macroscopic stress ($\sigma_c - u_a$) is nonlinear according to the exponent, r, and that the effects of matric potential are linearized according to Khallili and Khabbaz (1998). In saturated soil ($\psi \geq 0$ kPa), u_a needs to be set to u_w for the equation to be valid and $r\gamma = \beta$, where β is the exponent used by Whalley et al. (2011) and described by Santamarina et al. (2001).

In so far as V_s is concerned, with modification the Bishop effective stress appears to provide a single stress variable that takes into account both the effects of net stress and suction stress (Fig. 7). However, since the net stress required a nonlinear transformation to collapse data from different soils into single relationship (Fig. 7), the additive function,

$$\left((\sigma_c - u_a)^r - (u_w - u_a)\left(\frac{(u_w - u_a)}{(u_w - u_a)_{ae}}\right)^{-0.55}\right) \tag{9}$$

is dimensionally inconsistent. Nevertheless, it provides a useful relationship (Eq. (7)) which can be used to relate V_s to important soil properties. The goodness of fit is similar (Table 2) whether we use common parameters for all soils (Fig. 7b) or values of A which depend on soil type (Fig. 7a). Allowing only A to vary with soil type, accounts for 95.7% of the variation in the V_s data compared to 95.6% when all parameters are common (Table 2). Given the additional complication there is little advantage to be obtained by allowing parameters to differ between soil type. The best fit obtained using a single set of parameters for all soils (Fig. 7b) is achieved by allowing all parameters including the void ratio normalization constant to be adjusted. This gives a void ratio normalization constant of 7.085, which is much greater than the value of 2.973 recommended for angular particles by Santamarina et al. (2001), although within the range of report values (Mitchell and Soga, 2005). Since we had shown that a b value of -0.55, as recommended by Khallili and Khabbaz (1998) gives a linear relationship between V_s and suction stress, irrespective of the net stress ($\sigma_c - u_a$) (Figs. 4b and 6a), it was not included in the fitting process.

4.3. Deducing soil physical characteristics from measurements of V_s

In practice the application of Eq. (7) to deduce soil properties (e, ψ, or σ_t) may be complicated by over consolidation. However, Whalley et al. (2011) have shown that this can be corrected for in saturated soils as is also the case for unsaturated soil (Mancuso et al., 2002). It should be noted that in some soils the effects of over consolation on the relationships between V_s and effectives stress are small and the Butt Close loamy sand is an example of such a soil (Whalley et al., 2011). In a field application Eq. (7) is likely to be written as

$$V_s = A\frac{(2.97 - e)^2}{1 + e}\left(\sigma_s^r - \psi\left(\frac{\psi}{\psi_{ae}}\right)^{-0.55}\right)^{\gamma} \tag{10}$$

where σ_s is the overburden pressure due to the weight of soil and ψ is the matric potential of soil water. In shallow and newly cultivated soil layers one way to use V_s would be to independently measure e or ψ and then determine the other variable. Matric potential would be the most easily measured (e.g. Whalley et al., 2009) and then void ratio could be estimated. Methods to measure void ratio are either invasive or use buried sensors combining TDR and thermal methods (Liu et al., 2008).

In a following paper (Gao et al., in this issue) we will show that the penetrometer resistance of soil is closely related to the small strain shear modulus G ($G = \rho V_s^2$, where ρ is the dry bulk density of soil). This provides an alternative use of V_s to estimate penetrometer resistance which can give an estimate of soil strength which has an important influence on root elongation and ultimately crop yield (Whalley et al., 2006, 2008).

In future work we propose to test the use of Eq. (10) in the field. The development of applications for V_s measurements in soil science may be more complex and take a longer development time than those of TDR because V_s is a function of more than one soil physical property. However, this should be treated as more of an opportunity than a limitation. In common with TDR (see Whalley, 1993), the calibration function for V_s measurements has a physical basis.

5. Conclusions

We have shown how shear wave velocity can be related to void ratio, matric potential and net stress with a function with four fitted parameters. A common set of parameters provided a function that could be fitted to a range of soils at various states of consolidation and saturation.

Acknowledgements

This work was supported by Engineering and Physical Sciences Research Council of the UK (EPSRC) grant EP/H040617/1. Rothamsted Research receives strategic funding from the Biotechnology and Biological Sciences Research Council of the UK. We thank Jerry Sutton at GDS Instruments for frequent and helpful discussions. We thank Rodger White for help with the statistical analysis.

References

Alonso, E.E., Pereira, J.-M., Vaunat, J., Olivella, S., 2010. A microstructurally based effective stress for unsaturated soils. Geotechnique 60, 913–925.

Bishop, A.W., Blight, G.E., 1963. Some aspects of effective stress in saturated and partly saturated soils. Geotechnique 13, 177–179.

Bishop, A.W., Wesley, L.D., 1975. A hydraulic triaxial apparatus for controlled stress path testing. Geotechnique 25, 657–670.

Dexter, A.R., Bird, N.R.A., 2001. Methods for predicting the optimum and the range of soil water contents for tillage based on the water retention curve. Soil & Tillage Research 57, 23–212.

Gao, W., Watts, C.W., Ren T., Whalley, W.R. Predicting the penetrometer resistance of drying compacted soil. Soil & Tillage Research, http://dx.doi.org/10.1016/j.still.2012.05.013, in this issue.

Gregory, A.S., Bird, N.R.A., Whalley, W.R., Matthews, G.P., Young, I.M., 2010. Deformation and shrinkage effects on the soil water release characteristic. Soil Science Society of America Journal 74, 1104–1112.

Khallili, N., Khabbaz, M.H., 1998. A unique relationship for χ for the determination of the shear strength of unsaturated soils. Geotechnique 48, 681–687.

Liu, X., Ren, T., Horton, R., 2008. Determination of soil bulk density with thermo-time domain reflectometry sensors. Soil Science Society of America Journal 72, 1000–1005.
Lo Presti, D.C.F., 1995. General report: measurement of shear deformation in geomaterials. In: Shibuya, S., Mitachi, T., Miura, S. (Eds.), Prefailure Deforemation of Geomaterials. Balkema Rotterdam.
Lu, Z., Hickey, C.J., Sabatier, J.M., 2004. Effects of compaction on the acoustic velocity in soil. Soil Science Society of America Journal 68, 7–16.
Lu, Z., Sabatier, J.M., 2009. Effects of soil water potential and moisture content on the sound speed. Soil Science Society of America Journal 73, 1614–1625.
Lu, N., Godt, J.W., Wu, D.T., 2010. A closed-form equation for effective stress in unsaturated soil. Water Resources Research 46, W05515.
Mancuso, C., Vassallo, R., d'Onofrio, A., 2002. Small strain behaviour of a silty sans in controlled-suction resonant column – torsional shear tests. Canadian Geotechnical Journal 39, 22–31.
Mitchell, J.K., Soga, K., 2005. Fundamentals of Soil Behaviour, 3rd ed. John Wiley & Sons, New Jersey.
Mullins, C.E., 2000. Hardsetting Soils. In: Sumner, M. (Ed.), Handbook of Soil Science. CRC Press, pp. G65–G85.
Nakagawa, K., Soga, K., Mitchell, J.K., 1997. Observation of biot compressional wave of the second kind in granular soils. Geotechnique 47, 133–147.
Santamarina, J.C., Klein, K.A., Fan, M.A., 2001. Soils and Waves. Willey & Sons, Chichester, England.
Velea, D., Shields, D.F., Sabatier, J.M., 2000. Elastic wave velocities in partially saturated Ottawa sand: experimental results and modelling. Soil Science Society of America Journal 64, 1226–1234.
Whalley, W.R., 1993. Considerations on the use of time domain reflectometry (TDR) for measuring soil water content. Journal of Soil Science 44, 1–9.
Whalley, W.R., Clark, L.J., Gowing, D.J.G., Cope, R.E., Lodge, R.J., Leeds-Harrison, P.B., 2006. Does soil strength play a role in wheat yield losses caused by soil drying? Plant and Soil 280, 279–290.
Whalley, W.R., Watts, C.W., Gregory, A.S., Mooney, S.J., Clark, L.J., Whitmore, A.P., 2008. The effect of soil strength on the yield of wheat. Plant and Soil 306, 237–247.
Whalley, W.R., Lock, G., Jenkins, M., Peloe, T., Burek, K., Balendonck, J., Take, W.A., Tuzel, İ.H., Tuzel, Y., 2009. Measurement of low matric potentials with porous matrix sensors and water-filled tensiometers. Soil Science Society of America Journal 73, 1796–1803.
Whalley, W.R., Jenkins, M., Attenborough, K., 2011. The velocity of shear waves in saturated soil. Soil Science Society of America Journal 75, 1652–1657.
Whitmore, A.P., Whalley, W.R., Bird, N.R.A., Watts, C.W., Gregory, A.S., 2011. Estimating soil strength in the rooting zone of wheat. Plant and Soil 339, 363–375.