ABSTRACT

Modelling and observations of the Leonids have shown that maxima in the meteor storm activity can be identified as due to particles released from the comet during certain perihelion passages. If the particles originating from a certain perihelion passage can be identified, the next obvious question to ask is what information can be gained about the ejection process of particles from a cometary nucleus. We have developed a method to calculate the set of all possible dust trajectories that reach the Earth at some given time. The method involves numerical integration of a few dust particles only and is applied to the Leonid activity in the year 2000. We show that particles of different sizes entering the Earth’s atmosphere at the same time were released from the comet at different heliocentric distances. Therefore one has to make assumptions about the activity of the comet with heliocentric distance in order to derive the cometary mass distribution from an observed meteor mass distribution. However, we outline how lower limits on the ejection velocity of the observed particles can be derived.

1. INTRODUCTION

Cometary dust trails consist of large dust particles which travel on trajectories similar to that of the comet. Modelling has shown that particles in the trail which were ejected during different comet apparitions can lie separated in space such that they can be identified in the meteor flux rates [1]. To give an example, the ecliptic intersection of dust trails was calculated and is shown in Fig. 1. Section 3 describes in detail how Fig. 1 was produced. Fig. 2 show the meteor flux during the Leonids in the year 2000 [1]. By comparing Fig. 1 with Fig. 2 it can nicely be seen that maxima in the meteor flux are due to close approaches of the Earth with trails of particles ejected during different comet apparitions.

Theoretical models were developed which are able to predict time and intensity of meteor flux maxima [6,8]. The model in [8] focuses on the prediction of meteor flux rates. Therefore it is justified to make certain simplifications. For example, only dust particles which were ejected at perihelion need to be considered to find the place of the ecliptic intersection of dust particles ejected during a certain apparition. Furthermore, the model was calibrated empirically in order to predict the absolute meteor flux rates of the Leonids and no...
flux can be identified to be due to dust particles ejected during different apparitions. An attempt was made to constrain the ejection process around the source comet 55P/Temple-Tuttle. An attempt to constrain cometary ejection models from Leonid observations was made in [6]. This was done by calculating the Leonid flux consistent with a dust ejection model, varying some of the model parameters and comparing the results with observations. Reference [6] reports that the best agreement of the model with the observations was found for high particle densities of 4 g cm\(^{-3}\). As [6] used a Monte Carlo model, there is very little insight in how the results depend on the model parameters, which were not varied in the calculation. These parameters are, for example, the activity distribution for different ejection directions and the dependence of the comet activity on the heliocentric distance. It cannot be said, without re-running the model how the results in [6] are affected by the assumptions about these parameters.

In this work we try to overcome the shortcomings of the use of Monte Carlo models by defining a procedure which allows, independent of an ejection model, the calculation of the set of all dust particles which can reach the Earth at a given time. To do this we introduce a model which allows calculation of the intersection point of particle trajectories with the ecliptic in a semi-analytical way. We show that for particles of given radiation pressure coefficient, which are ejected at the same time, there is only one unique ejection velocity vector such that the Earth is reached at a given time. On the basis of this, velocity constraints on the ejection velocity can be found without invoking any ejection model.

Meteor observers not only measure the meteor rates but also measure a magnitude distribution index [5]. The magnitude distribution index is a measure for the mass distribution of the dust particles in the meteor stream [10]. It is an obvious question to ask how the mass distribution of meteoroids entering the Earth’s atmosphere is related to the mass distribution of particles ejected from the comet. We show that particles which enter the Earth’s atmosphere at the same time might have been ejected at different heliocentric distances. Therefore the relationship between the mass distribution of ejected particles from the comet and the meteor mass distribution must be expected to be complex.

2. EVOLUTION OF A DUST SHELL

The term “dust shell” was introduced in [2] and denotes the set of all places which is inhabited by dust particles which have the same radiation pressure coefficient \(\beta\) and were released from a comet at the same time \(t_0\) with the same velocity \(V\). A dust shell initially expands spherically around a comet. Due to radiation pressure, the centre of the dust shell becomes offset from the comet position and the position of a dust particle on the dust shell at time \(t\) can be approximated by (see Fig. 3 A)

\[
r = r_c(t_0, \beta) + V(t - t_0),
\]

where \(r_c\) is the position of the centre of the shell (bold variables represent vectors). This approximation was used in [2] to model dust tails formed by small particles which only spend a small time in the tail. For long times of flight the dust shell gets distorted to an ellipsoid by tidal forces of the Sun (see Fig. 3 A). In this regime Eq. 1 can be generalised [3]

\[
r = r_c(t_0, \beta) + \phi(t, t_0)V,
\]

where \(\phi\) is a 3x3-matrix. This approximation was first used in [4] to predict the position of large dust particles around 1P/Halley during the encounter of the Giotto spacecraft. Analytical equations for the matrix elements of \(\phi\) can be found in [3], where Eq. 2 was used to efficiently produce artificial images of dust comae.

One can ask whether these kinds of model are still applicable to dust trails. However, due to different orbital periods of the particles in the shell, the shell stretches out and bends around the comet orbit (Fig. 3 B). Therefore a linear approximation as in Eq. 2 is no longer valid for describing the entire dust shell. However with regard to analysing meteor fluxes on Earth, one is not interested in the shape of the entire dust shell but only in its intersection with the ecliptic plane. In the following section we show how these intersections look like and it is shown that the position of the intersection of a dust particle can be approximated by an equation analogous to Eq. 2. This is because the ejection velocity of dust particles is small compared with the orbital velocity of the comet. Therefore the dust particles move on trajectories which are very similar to the comet orbit. Assuming Keplerian trajectories, for both comet and dust particle, the dust particle reaches a maximum distance from the comet orbit during its first revolution. Still assuming purely Keplerian trajectories, this maximum distance will not increase in following revolutions around the Sun. However, if the comet and the dust particle have different orbital periods, the distance between the comet and particle along the orbit increases with every revolution. Therefore dust shells can have a typical width in the order of 100000 km, but can stretch
several astronomical units in length around the comet orbit.

To understand some of the results later in this work, another property of dust shells is worth mentioning. Assuming again that the dust travels on Keplerian trajectories, all dust particles have to come back to the point of ejection. Hence, when a dust shell crosses its point of ejection, the particles with the same orbital period, which usually form a ring on the dust shell are collapsed to a point (see Fig. 3 C). Furthermore, a ring on a shell collapses to a one-dimensional line, when it crosses its node with respect to the comet orbit plane (180° true anomaly away from the point of ejection; see Fig. 3 C).

3. ECLITIC INTERSECTION OF A SHELL

In this section we describe how the set of all dust particles which intersect the ecliptic at the same time can be calculated and how the position of the ecliptic intersection depends on the initial particle velocity.

The position of a dust particle along a dust shell, which can have a length of several astronomical units, is controlled by the orbital period of the particle. Because the width of the shell is tiny compared with its length the intersection of the shell with any plane (which has at least a moderate inclination with respect to the comet orbit plane) only contains particles which have almost the same orbital periods. For a Keplerian trajectory the orbital period $T$ is related to the orbital energy by

$$\frac{1}{2} \left( \frac{2\pi\mu}{T} \right)^{3/2} = \frac{1}{2} \left( V_{\text{comet}} + V \right)^2 - \frac{\mu (1 - \beta)}{r},$$

where $\mu$ is the gravitational constant multiplied with the solar mass, $V$ is the ejection velocity of the particle. $V_{\text{comet}}$ and $r$ are the comet velocity and heliocentric distance at the ejection time, respectively.

Fig. 3. Evolution of a dust shell. A: dust shell during revolution of ejection.
By expressing the ejection velocity $V$ by its component $V_3$ in the direction of the comet orbital velocity vector, $V_{\text{comet}}$, and a component perpendicular to this direction, $V_\perp$, the orbital energy can be written

$$E = V_{\text{comet}} \cdot V_3 + \frac{1}{2}(V_3^2 + V_\perp^2) + \frac{\beta \mu}{r} + E_{\text{comet}} \cdot (4)$$

where $E_{\text{comet}}$ is the orbital energy of the comet. From Eq. 4 it can be seen that the orbital energy, and with this the orbital period, depends to first order only on the component of the ejection velocity in the direction of the orbital velocity of the comet. With this knowledge it is easy to write a numerical procedure which searches $V_3$ such that the dust particle which started at the given time with a given $V_\perp$ and $\beta$ intersects the ecliptic at a given time. Even though the motivation for the procedure is based on Keplerian dynamics, our numerical procedure, which takes the perturbations of all planets and Poynting Robertson drag into account, robustly provides the required solution. The ecliptic intersection point of a particle with radiation pressure coefficient $\beta$, which started with an initial velocity $V_\perp$, is denoted $r(v, \beta, V_\perp)$, where we refer to the ejection time by the true anomaly $\nu$ in the following. Tab. 1 lists, for some true anomalies, the corresponding heliocentric distances.

Tab. 1. Heliocentric distances for different true anomalies of 55P/ Tempel-Tuttle during the 1866 apparition.

<table>
<thead>
<tr>
<th>true anomaly [deg]</th>
<th>heliocentric distance [AU]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.977</td>
</tr>
<tr>
<td>10</td>
<td>0.984</td>
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<tr>
<td>20</td>
<td>1.006</td>
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<tr>
<td>30</td>
<td>1.043</td>
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<tr>
<td>40</td>
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<tr>
<td>50</td>
<td>1.177</td>
</tr>
<tr>
<td>60</td>
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</tr>
<tr>
<td>70</td>
<td>1.422</td>
</tr>
<tr>
<td>80</td>
<td>1.609</td>
</tr>
<tr>
<td>90</td>
<td>1.862</td>
</tr>
<tr>
<td>100</td>
<td>2.210</td>
</tr>
<tr>
<td>110</td>
<td>2.698</td>
</tr>
<tr>
<td>120</td>
<td>3.404</td>
</tr>
</tbody>
</table>

In this work we only consider dust particles which were ejected during the perihelion passage of comet 55P/Tempel-Tuttle in 1866 and which passed through the ecliptic on the 18th November 2000 (from North to South). In Fig. 4 the ecliptic intersections of dust particles with $\beta=0$ are shown, which were released during perihelion passage ($\nu=0$). The relative position of particles with different $\beta$ or different ejection times look qualitatively the same. The ecliptic intersections are shown for trajectories which started with different directions of $V_\perp$. For each direction four values for $V_\perp$ are shown (25, 50, 75 and 100m s$^{-1}$). It can be seen that ecliptic intersections of particles, which started with a $V_\perp$ in the same direction, lie on a straight line (dotted in Fig. 4) and that the distances between two points on this line is proportional to the velocity difference. Therefore the position of the ecliptic intersection can well be approximated [9] by

$$r(v, \beta, V_\perp) = r_\perp(v, \beta) + A(v, \beta)V_\perp \cdot (5)$$

where $r_\perp(v, \beta) = r(v, \beta, 0)$ and $A(v, \beta)$ is a 2x2 matrix (note that all vectors lie in a plane; the spatial vectors in the ecliptic and the velocity vector in the plane perpendicular to the orbital velocity vector of the comet at the time of ejection). Since the elements of the matrix can be calculated on the basis of a few trajectories only, the intersection point for any initial velocity $V_\perp$ can be calculated efficiently on the basis of the numerical integration of a few trajectories only. The intersection of a dust shell with the ecliptic has elliptical shape (shown solid in Fig. 4). Such ellipses are shown in Fig. 4 and Fig. 1. The parameters which were used to produce Fig. 1 are radiation pressure coefficient $\beta=0$, ejection true anomaly $\nu=0$, $\pm 40^\circ$ and $V_\perp=20$m s$^{-1}$.

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Apart from the intersection point with the ecliptic the numerical procedure also provides the ejection velocity $V_3(\nu, \beta, V_{\perp})$ which is required to intersect the ecliptic at the specified time. Because the particles intersecting the ecliptic at the same time have almost the same orbital periods the required velocity in the direction of the orbital motion of the comet is almost independent of $V_{\perp}$. Fig. 5 shows $V_3$ for three different values of $\beta$.

![Fig. 5. Particle velocity in direction of comet orbital motion, $V_3$, which led to ecliptic crossings on November 18\textsuperscript{th} 2000. The different lines correspond to different $\beta$-values (solid: $10^{-3}$; dotted: $10^{-4}$ and dashed: $10^{-5}$).](image)

4. VELOCITY REQUIRED TO REACH EARTH

Using Eq. 5 it is easy to find the trajectories which intersect the Earth. If we assume the Earth to be a point at position $r_e$, then the ejection velocity $V_{\perp}$ required to reach Earth can be found by

$$V_{\perp}(\nu, \beta) = A^{-1}(\nu, \beta) \left\{ r_e - r_c(\nu, \beta) \right\}. \quad (6)$$

Eq. 6 only gives the ejection direction in the direction perpendicular to the orbital motion of the comet. As the velocity in the direction of the orbital motion hardly changes for different $V_{\perp}$, we can define the velocity vector required to reach the Earth at the specified time by

$$V_{\text{req}}(\nu, \beta) = V_{\perp}(\nu, \beta) + V_3(\nu, \beta, 0) \frac{V_{\text{comet}}}{V_{\text{comet}}}. \quad (7)$$

The absolute value of the ejection velocity required to reach the Earth position on 18\textsuperscript{th} November 7am UT is shown in Fig. 6 for the $\beta$-values $10^{-3}$, $10^{-4}$ and $10^{-5}$. For all $\beta$-values a maximum in the required velocity can be seen for the particles which were released from the comet shortly after perihelion. This is due to the mentioned collapse of the dust shells. Comet 55P/Tempel-Tuttle crosses the ecliptic shortly after perihelion. The dust shell which was ejected during the ecliptic crossing is therefore collapsed when it passes through the ecliptic. Similarly, the width of the ecliptic intersections of dust shells which were emitted shortly before and after the ecliptic crossing are small and high ejection velocities are required in order to reach the position of the Earth. The required velocity reaches a minimum for true anomalies $\pm 100^\circ$. The minimum is much lower for particles with $\beta$-value of $10^{-3}$ than for the other shown $\beta$-values. This is because the centre of the dust shell (position of the particle with $V_{\perp}$ and $V_3=0$) with a $\beta$-value of $10^{-3}$ is just crossing the ecliptic and only small values of $V_3$ are required to make a dust particle intersect the ecliptic at the right time. The centres of the dust shells with lower $\beta$ have already passed the ecliptic a long time ago and a $V_3$ of 14m s$^{-1}$ is required such that there are still particles which intersect the ecliptic at the given time. From this, a constraint on the ejection velocity can be derived. The length of the dust shells is controlled by the absolute value of the ejection velocity. If the centre of all dust shells for some given $\beta$-value has long passed the ecliptic then a minimum ejection velocity is required such that there are still particles which lag behind far enough such that they still intersect the ecliptic at the given time. In the chosen example this means that if there was an 1866-Leonid observed on 18\textsuperscript{th} November 2000, for which there is reason to assume that its $\beta$-value is not larger than $10^{-4}$, then it must have been ejected with an velocity of at least 14m s$^{-1}$.

![Fig. 6. Velocity required by particles released in the 1866 apparition to reach Earth on 18\textsuperscript{th} November 2000.](image)
7h UT for different values of $\beta$ (solid: $10^{-3}$; dotted: $10^{-4}$ and dashed: $10^{-5}$).

5. EJECTION DIRECTION

As the comet activity is driven by the Sun, the sunlit side of the comet is much more active than the night side and most dust particles are ejected towards the Sun. Therefore it is interesting to derive the angle between the required ejection velocity vector and the Sun direction. This angle is shown in Fig. 7 for three different values of $\beta$. For all $\beta$-values there is a rapid change of this angle shortly after perihelion. This is again because of the collapse of the dust shell. Because ecliptic intersections of dust shells shortly before the collapse appear point-mirrored with respect to the ecliptic intersection of dust shells after collapse, a rapid change in the required ejection direction occurs around this point. At true anomalies of about $\pm 100^\circ$ another rapid change in the angle can be seen for the $\beta$-value $10^{-3}$. This is because of the change of the sign of $V_3$ for these particles (see Fig. 5). As $V_3$ does not change sign for particles with $\beta$ not larger than $10^{-4}$, this rapid change is only seen for the highest $\beta$-value.

![Fig. 7](image)

Fig. 7. For particles which were released from 55P/Tempel-Tuttle in the 1866 apparition and reach the Earth on 18th November 2000 7am UT, the angle between the required ejection velocity and the Sun direction is shown versus ejection true anomaly for three different $\beta$-values (solid: $10^{-3}$; dotted: $10^{-4}$ and dashed: $10^{-5}$).

As the comet activity increases with decreasing heliocentric distance it seems valid to assume that most of the observed meteors were released from the comet near perihelion. From Fig. 7 it can be seen that shortly before perihelion particles need to be ejected towards the anti-solar direction in order to reach the Earth at the specified time. As comets eject most particles towards the Sun direction, Fig. 7 suggests that most of the observed meteors were ejected after perihelion. However, from Fig. 6 it can be seen that the particles which were ejected close to perihelion require high ejection velocities in order to reach the Earth. Therefore particles ejected close to perihelion might not have reached the Earth and most of the meteors are particles which were ejected at large heliocentric distances. The fact that particles ejected at large heliocentric distance can dominate a meteor storm was shown in [7]. However, even if particles ejected at large heliocentric distances dominate the flux, then there is still information which can be gained from Fig. 7.

6. MASS DISTRIBUTION

Due to the different dynamical behaviour of particles of different masses, one might already expect that the mass distribution of particles which enter the Earth’s atmosphere is different from the mass distribution of particles ejected from the comet per unit time. In this section we want to give an example for which this expectation is confirmed. For this we assume a very simple model for the ejection velocity of particles. We assume that the comet emits dust particles radial symmetrically. We assume the ejection velocity is independent of the heliocentric distance and only dependent on the $\beta$-value of the particle

$$V_{ej}(\beta) = 160 \frac{m}{s} \sqrt{\frac{\beta}{10^{-3}}}.$$  

(8)

For a dust particle to reach the Earth’s atmosphere the ejection velocity must equal the required velocity

$$V_{ej}(\beta) - V_{req}(\nu, \beta) = 0.$$  

(9)

Fig. 8 shows the difference of the ejection velocity and the required velocity for the three $\beta$-values $10^{-3}$, $10^{-4}$ and $10^{-5}$. Eq. 9 is fulfilled when one of the lines intersects the horizontal axis. It can be seen in Fig. 8 that for different $\beta$-values, Eq. 9 is fulfilled for different true anomalies. Hence this simple example shows that particles with different $\beta$ i.e. different masses, which reach the Earth’s atmosphere at the same time originate from different heliocentric distances. Therefore one could not derive a mass distribution of ejected particles from the mass distribution of observed meteors without making
assumptions about the comet activity in dependence on heliocentric distance.

Fig. 8. Difference between ejection velocity and required velocity for different $\beta$-values (solid: $10^{-3}$, dotted: $10^{-4}$ and dashed: $10^{-5}$).

7. DISCUSSION

The result of the previous section does not necessarily mean that the mass distribution of observed meteors can never be used to constrain the mass distribution of ejected particles of a comet. The magnitude distribution indices in [5] show significant variations if different meteor streams are compared. However, the magnitude distribution indices given for each meteor stream observed in different years are comparable. Therefore, there must be some information in the observed mass distribution otherwise the scatter of the magnitude distribution index for the meteor outbursts of a single meteor stream could be expected to be larger. From the example in the previous section one could expect that the heliocentric distances which contribute to the observed magnitude distribution index, change for observation of the same trail in different years. However, the fact that a typical range of magnitude distribution indices for each meteor stream seems to exists, suggests that there is some repeating pattern in how the mass distributions of observed meteors is related to the mass distribution of particles ejected from the comet. From the fact that different magnitude distribution indices are observed for meteor outbursts of different streams, it cannot necessarily be concluded that this difference is due to different mass distribution of the source comets. Effects due to different orbital configurations need to be ruled out before this conclusion can be drawn.

In section 4 it was shown that a lower limit on the ejection velocity of the dust particles can be derived, if the corresponding trail can be identified in the observed meteor rates. If the comet orbit was perfectly known, the value of the minimum velocity could be estimated with great accuracy, because of the high accuracy with which celestial mechanics can be modelled. However due to non-gravitational forces exerted on comets, the initial conditions of the released dust particles always remain uncertain to some extent. To understand how the uncertainty of the comet orbit affects the result, one has to recall that the required velocity of particles to reach Earth can be dominated by two different components of the ejection velocity: (1) The ejection velocity perpendicular to the comet’s orbital velocity at the time of ejection. In the example in section 4, this is the case for true anomalies, for which the width of the dust shell ecliptic intersection is small. Therefore large ejection velocities are required to reach the Earth. (2) The component in the direction of the orbital velocity. In the example, this is the case for particles with $\beta$-values not larger than $10^{-4}$ at true anomalies $\pm 100^\circ$. Here the required velocity is dominated by $V_3$, because the centres of the corresponding dust shells have long passed through the ecliptic and a minimum ejection velocity is required such that the dust shell is long enough that there are still particles intersecting the ecliptic. In the latter case, the required velocity is hardly influenced by the uncertainty of the comet orbit. However, in the previous case small uncertainties in the position where the dust particles intersect the ecliptic can significantly change the results.

8. CONCLUSION

We have defined a procedure that allows the calculation of the set of all possible particle trajectories that can reach the Earth at the same time. On the basis of this it is possible to place constraints on the ejection velocity without invoking any ejection model of the dust particles from the comet nucleus. An example was given on how a lower bound of the ejection velocity can be found. The accuracy of the estimate is only dependent on the accuracy of the comet orbit during the time of ejection. For a very simple ejection model it was shown that dust particles of different sizes, which reach the Earth at the same time, originate from different heliocentric distances. Therefore, assumptions about the dependence of the comet activity on heliocentric distance have to be made in order to constrain the mass distribution of ejected particles from the mass distribution of observed meteors. Since the magnitude distribution indices vary only slightly for a given meteor stream [5], it is suggested that there might be a repeating pattern on how different heliocentric distances contribute to a meteor mass distribution. However, as there are bigger variations of the magnitude distribution indices for different meteor streams, the different mass distributions of meteors of different streams is not necessarily due to different mass distributions of the
source comets, but could also be due to different orbital configurations.

ACKNOWLEDGEMENT

The encouraging discussions with David Asher and Rüdiger Jehn which led to the present work are gratefully acknowledged. This work was funded by the UK Particle Physics and Astronomy Research Council.

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