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Condition for tripartite entanglement

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Abstract. We propose a scheme for classifying the entanglement of a tripartite pure qubit state. This classification scheme consists of an ordered list of seven elements. These elements are the Cayley hyper-determinant, and its six associated 2×2 subdeterminants. In particular we show that this classification provides a necessary and sufficient condition for separability.

1. Introduction

It is well known that for bipartite pure qubit states a single determinental condition is enough to discriminate between separability and entanglement. It is a straightforward matter to determine whether a vector $v \in V = V_1 \otimes V_2$ is entangled or not. Here V_1 and V_2 are two-dimensional (qubit) vector spaces with basis $\{e_1 \equiv |0\rangle, e_2 \equiv |1\rangle\}$.

In general we may write $v \in V$ as

$$v \in V_1 \otimes V_2 = \sum_{i,j=1}^{2} c_{ij} \;\; e_i \otimes e_j \tag{1}$$

If v is non-entangled, i.e. separable, then

$$v = (x_1 e_1 + x_2 e_2) \otimes (y_1 e_1 + y_2 e_2) \tag{2}$$

so

$$c_{ij} = x_i y_j \quad \{i, j = 1, 2\} \tag{3}$$

from which we deduce that the matrix c of coefficients c_{ij} has determinant zero, $\det c = 0$. And this condition is clearly necessary and sufficient.

In fact, by suitably normalizing, we may use this determinant to provide a *measure* of entanglement for pure states called the concurrence $\mathcal{C}$, with

$$\mathcal{C} = 2|\det c|. \tag{4}$$

This measure of entanglement varies between 0 (separable) and 1 (maximally entangled) and may be conveniently extended to mixed states [1].

For *tripartite* states the situation is somewhat more complicated. One three-dimensional analogue of the two-dimensional determinant is the *Cayley hyperdeterminant* [2, 3] (denoted as Det and defined in section (2.2)). In [4] the Cayley hyperdeterminant (which was termed $3-$ *tangle* there) is employed as a type of hyper-concurrence to distinguish two tripartite states: the GHZ-state $|GHZ\rangle = 1/\sqrt{2}(|000\rangle + |111\rangle)$ [5] and the W-state $|W\rangle = 1/\sqrt{3}(|001\rangle + |100\rangle + |010\rangle)$ [6]. However the Cayley hyperdeterminant Det does not truly reflect the nature of entanglement of these states.

For instance, the Cayley hyperdeterminant for the GHZ-state and the W-state are one and zero, respectively. However, the W-state *is* entangled; so Det $= 0$ does not provide a criterion for separability as the simple 2×2 determinant $\mathcal{C}$ does in the bipartite case. Further, one knows that the W-state is in fact more robust under measurement-collapse than the GHZ-state. For example, if Alice measures the (first) qubit of the GHZ-state to be 0, then this leaves the separable state $|00\rangle$. And similarly for any measurement of any qubit in any of the three subspaces for the tripartite GHZ-state. On the other hand, the determination of the value "0" of any qubit in any space for the W-state still leaves the state (maximally) entangled, and only if the value "1" is measured will the collapsed state be separable. Again this difference is not reflected in the values of Det for these two states. So one needs additional indicators to reflect this difference in entanglement properties.

In this note we supply these indicators which distinguish these and other tripartite states, and - more significantly - provide a necessary and sufficient criterion for the separability of a tripartite pure state.

2. Local Unitary Transformations

We initially reconsider the bipartite case. Since every (normed) vector $v \in V$ can be transformed to the (non-entangled) state $|00\rangle$ by a unitary transformation, it is clear that entanglement is *not* invariant under unitary transformations. However, under a *local unitary transformation* U, defined by $U = U_1 \otimes U_1$, one can see that the bipartite concurrence $\mathcal{C}$ as defined in Eq.(4), for example, is invariant:

Theorem 1 *The concurrence $\mathcal{C}$ is invariant under local unitary transformations.*
Let $v = \sum_{i,j=1\ldots 2} a_{ij} e_i \otimes e_j \in V = V_1 \otimes V_2$, and the unitary matrix $U = U_1 \otimes U_2$ be a local unitary matrix; then

$$
\begin{aligned}
Uv &= \sum c_{ij} U_1 e_i \otimes U_2 e_j \\
&= \sum c_{ij} (U_1)_{ik} e_k \otimes (U_2)_{jr} e_r \\
&= \sum c'_{kr} e_k \otimes e_r
\end{aligned}
$$

where $c'_{kr} = \sum_{ij} c_{ij} (U_1)_{ik} (U_2)_{jr}$ so that $c' = \tilde{U_1} c U_2$ whence

$$
\begin{aligned}
|\det c'| &= |\det(\tilde{U_1} c U_2)| \\
&= |\det \tilde{U_1}|\, |\det c|\, |\det U_2| \\
&= |\det c| \quad \text{since} \quad |\det U_i| = 1.
\end{aligned}
$$

For a bipartite general state ρ (mixed state, density matrix) the definition of separability is:

Definition 1 (Separable bipartite state) *The state ρ acting on $V_1 \otimes V_2$ is said to be* separable *if it is given by a convex sum $\sum_i \lambda_i \rho_i^1 \otimes \rho_i^2$ $(\lambda_i \geq 0, \quad \sum_i \lambda_i = 1)$ where ρ_i^α acts on V_α.*

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When $\rho = \rho^1 \otimes \rho^2$ it is said to be *simply separable.* The above Definition 1 extends immediately to *multipartite* states.

Definition 2 (Separable multipartite state) *The state ρ acting on $V_1 \otimes V_2 \otimes \ldots \otimes V_n$ is said to be* separable *if it is given by a convex sum $\rho = \Sigma_k \lambda_k \rho_1^k \otimes \rho_2^k \otimes \ldots \otimes \rho_n^k$ $(\lambda_i \geq 0,\ \sum_i \lambda_i = 1)$ where ρ_i^α acts on V_α .*

We may see rather immediately from Definition 1 that the property of *being* separable is invariant under local unitary transformations; and from Definition 2 this extends to the multipartite case. However, an extension of the *implication* from Theorem 1 that the *measure* of entanglement is preserved by local unitary transformations does not necessarily apply to multipartite systems, since such an extension would depend on possessing a definition of *entanglement measure* for such systems, which is currently unavailable. Indeed, for general multipartite states, local unitary equivalence does *not* preserve all the relevant (state and substate) entanglement properties [7, 8], as we shall now exemplify.

2.1. Entanglement properties of two tripartite qubit states

We consider two specific examples of entangled states: the GHZ state [5]

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{5}$$

and

$$|\psi\rangle = \frac{1}{2}(|100\rangle + |010\rangle + |001\rangle + |111\rangle). \tag{6}$$

In [9] it was shown that these states are equivalent under the local unitary transformation $U \otimes U \otimes U$ where

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \tag{7}$$

That is,

$$|\psi\rangle = U \otimes U \otimes U |GHZ\rangle. \tag{8}$$

However, the physical properties of these states are *not* equivalent. In particular, as noted in the introduction, after qubit measurement in any subspace, the GHZ-state becomes separable; while under a similar action the state $|\psi\rangle$ gives a maximally entangled state. Therefore any tripartite description should distinguish between these two states.

2.2. Cayley Hyperdeterminant

For the tripartite qubit case we write

$$v \in V_1 \otimes V_2 \otimes V_3 = \sum_{i,j,k=1}^{2} a_{ijk}\ \ e_i \otimes e_j \otimes e_k \tag{9}$$

or, as a state Ψ,

$$|\Psi\rangle = \sum_{i,j,k=0}^{1} a_{ijk} |ijk\rangle \quad (i,j,k = 0,1). \tag{10}$$

For $|\Psi\rangle$ the Cayley Hyperdeterminant Det of the coefficient hypermatrix $A = (a_{ijk})$ is defined by

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Definition 3 *Cayley Hyperdeterminant* Det

$$\begin{aligned}\text{Det}\,A &= a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{100}^2 a_{011}^2 \\ &- 2\,[a_{000}a_{001}a_{110}a_{111} + a_{000}a_{010}a_{101}a_{111} + a_{000}a_{011}a_{100}a_{111} \\ &+ a_{001}a_{010}a_{101}a_{110} + a_{001}a_{011}a_{101}a_{100} + a_{010}a_{011}a_{101}a_{100}] \\ &+ 4\,[a_{000}a_{011}a_{101}a_{110} + a_{001}a_{010}a_{100}a_{111}]\,.\end{aligned} \tag{11}$$

In Table 1 we give some examples of tripartite qubit states.

Using Definition 3 one may confirm by direct calculation that Det $= 0$ for a general separable tripartite state as given in Table 1. However, Det $= 0$ for the W-state also, which is an entangled state. Therefore the numerical value of Det alone does not discriminate between separable and non-separable states.

Further the state $|\psi\rangle$ of Eq. (6) has Det $= 1$, as does the GHZ-state. But, as previously noted, the properties of retaining entanglement after qubit measurement are completely different in the two cases.

It is clear that at the very least we need supplementary indicators beyond the hyperdeterminant Det to specify the entanglement properties of tripartite states, even completely separable states.

In the following Section we propose a classification scheme.

3. Classification

From the foregoing argument it would appear that one needs to consider the subconcurrences of a tripartite state in order to distinguish their entanglement properties, and ultimately define a measure.

This amounts to listing the six submatrices of the hypermatrix A of Eq.(11). We define

$$\begin{aligned}A_{x_0} &= (a_{0ij})\,, \quad A_{x_1} = (a_{1ij}) \\ A_{y_0} &= (a_{i0j})\,, \quad A_{y_1} = (a_{i1j}) \\ A_{z_0} &= (a_{ij0})\,, \quad A_{z_1} = (a_{ij1})\,.\end{aligned}$$

The corresponding sub-concurrences are given by the moduli of the subdeterminants[1]:

$$[C_{x_0}, C_{x_1}, C_{y_0}, C_{y_1}, C_{z_0}, C_{z_1}] \tag{12}$$

where we have written

$$C_{\alpha\, i} \equiv |\text{det}A_{\alpha_i}| \qquad (\alpha = x, y, z; i = 0, 1) \tag{13}$$

which may be regarded as an *ordered list* that distinguishes the bipartite substate entanglements of the given tripartite state.

This list is by itself not capable of discriminating between tripartite states. For example, it has the value $[0, 0, 0, 0, 0, 0]$ for both a separable state and the GHZ state. We thus supplement this list by the Cayley hyperdeterminant Det, giving a classification defined by the ordered list of 7 elements

$$[|\text{Det}\,A|; C_{x0}, C_{x1}, C_{y0}, C_{y1}, C_{z0}, C_{z1}]. \tag{14}$$

As we prove in the Appendix, the vanishing of this ordered list provides a necessary and sufficient condition for separability, and thus possibly paves the way to providing a useful measure for tripartite pure qubit entanglement.

In Table 1 we describe their classification under the scheme herein proposed.

[1] The normalization factor used here is 1. The values in the Table are obtained by applying a normalization factor $1/|\text{Det}A|$ for non-vanishing Det$A$ to all the terms. For the examples given this factor is 4. For the W-state, where Det$A = 0$, we use the factor 3.

Table 1. Classification of some tripartite qubit states.

	State	Classification
General Separable State	$\Sigma a_i e_i \otimes \Sigma b_j e_j \otimes \Sigma c_k e_k$	[0;0,0,0,0,0,0]
W-state $\|W\rangle$	$1/\sqrt{3}(\|001\rangle + \|100\rangle + \|010\rangle)$	[0;1,1,1,0,0,0]
GHZ-state $\|GHZ\rangle$	$(1/\sqrt{2})(\|000\rangle + \|111\rangle)$	[1;0,0,0,0,0,0]
Cluster state	$(1/\sqrt{8})(\|000\rangle + \|001\rangle + \|100\rangle + \|101\rangle$ $+\|010\rangle - \|011\rangle - \|110\rangle + \|111\rangle)$	[1;1,0,1,1,0,1]
ψ-state	$(1/2)(\|100\rangle + \|001\rangle + \|010\rangle + \|111\rangle)$	[1;1,1,1,1,1,1]
ϕ-State [7, 8]	$(1/2)(\|000\rangle + \|011\rangle + \|101\rangle + \|110\rangle$	[1;1,1,1,1,1,1]

4. Discussion

In this note we discussed the robustness of tripartite pure qubit states under projective measurement, and devised a classification scheme, which consists of an ordered list of seven elements displaying this aspect. These elements are the Cayley hyper-determinant, and the six 2×2 subdeterminants. In particular we showed that this classification provides a necessary and sufficient condition for separability. In so far as we may extend the definition of *rank* to the Cayley hyper-matrix, as being the order of the largest non-vanishing minor, the necessary and sufficient condition for separability may be simply stated as that the Cayley hyper-matrix be of Rank 1. Further work in progress is the extension of this definition to multipartite systems.

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Appendix A. Necessary and Sufficient Condition for Tripartite Qubit Separability

As we argued above, to get a better picture of the nature of entanglement of a tripartite state it is also necessary to look at the entanglement properties of the 2-qubits obtained when one of the three qubits is measured.

We map the tripartite state $|\psi\rangle = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle$ $(i,j,k = 0,1)$ into the multilinear form

$$F(x,y,z;A) = \sum_{i,j,k=0}^{1} a_{ijk}\, x_i y_j z_k, \quad A = (a_{ijk}). \tag{A.1}$$

Thus the problem of factorization of $|\psi\rangle$ is reduced to that of $F(x,y,z;A)$. Analyzing the entanglement of the 2-qubit state after one qubit is measured is equivalent to determining the factorizability of the derivatives of $F(x,y,z;A)$, namely,

$$\frac{\partial F}{\partial z_0} = a_{000}x_0y_0 + a_{010}x_0y_1 + a_{100}x_1y_0 + a_{110}x_1y_1, \tag{A.2}$$

$$\frac{\partial F}{\partial z_1} = a_{001}x_0y_0 + a_{011}x_0y_1 + a_{101}x_1y_0 + a_{111}x_1y_1, \tag{A.3}$$

$$\frac{\partial F}{\partial y_0} = a_{000}x_0z_0 + a_{001}x_0z_1 + a_{100}x_1z_0 + a_{101}x_1z_1, \tag{A.4}$$

$$\frac{\partial F}{\partial y_1} = a_{010}x_0z_0 + a_{011}x_0z_1 + a_{110}x_1z_0 + a_{111}x_1z_1, \tag{A.5}$$
$$\frac{\partial F}{\partial x_0} = a_{000}y_0z_0 + a_{001}y_0z_1 + a_{010}y_1z_0 + a_{011}y_1z_1, \tag{A.6}$$
$$\frac{\partial F}{\partial x_1} = a_{100}y_0z_0 + a_{101}y_0z_1 + a_{110}y_1z_0 + a_{111}y_1z_1. \tag{A.7}$$

For the $2 \times 2 \times 2$ hypermatrix A in $F(x, y, z; A)$ with components a_{ijk} $(i, j, k = 0, 1)$, the Cayley hyperdeterminant Det$A$ is given as in Definition 3. Corresponding to the six equations (A.2)-(A.7), one defines the six determinants as in Eq. (13):

$$C_{z_0} = |\det(a_{ij0})|, \quad C_{z_1} = |\det(a_{ij1})|, \tag{A.8}$$
$$C_{y_0} = |\det(a_{i0j})|, \quad C_{y_1} = |\det(a_{i1j})|, \tag{A.9}$$
$$C_{x_0} = |\det(a_{0ij})|, \quad C_{x_1} = |\det(a_{1ij})|. \tag{A.10}$$

We now assert that the multilinear form $F(x, y, z; A)$, and thus the tripartite state $|\psi\rangle$, is completely factorized, i.e.

$$F(x, y, z; A) = (a_0x_0 + a_1x_1)(b_0y_0 + b_1y_1)(c_0z_0 + c_1z_1) \tag{A.11}$$

for some constants a_i, b_i and c_i, if and only if the hyperdeterminant and all six sub-determinants are identically zero, i.e.,

$$\text{Det}\, A = 0, \quad C_{x0} = C_{x1} = C_{y0} = C_{y1} = C_{z0} = C_{z1} = 0. \tag{A.12}$$

The necessary condition is easy to prove by direct substitution. Below we prove the sufficient condition.

Suppose all the six sub-determinants are zero. Let us start with $C_{z0} = C_{z1} = 0$. These conditions imply that the l.h.s. of eqs.(A.2) and (A.3) are factorized, i.e.,

$$\begin{aligned} & a_{000}x_0y_0 + a_{010}x_0y_1 + a_{100}x_1y_0 + a_{110}x_1y_1 \\ = \; & (A_0x_0 + A_1x_1)(B_0y_0 + B_1y_1), \end{aligned} \tag{A.13}$$
$$\begin{aligned} & a_{001}x_0y_0 + a_{011}x_0y_1 + a_{101}x_1y_0 + a_{111}x_1y_1 \\ = \; & (A'_0x_0 + A'_1x_1)(B'_0y_0 + B'_1y_1) \end{aligned} \tag{A.14}$$

for some non-zero constants A, B, A' and B'. Then from eqs.(A.2)-(A.7) we have

$$\begin{aligned} F(x, y, z; A) &= (A_0x_0 + A_1x_1)(B_0y_0 + B_1y_1)z_0 \\ &+ (A'_0x_0 + A'_1x_1)(B'_0y_0 + B'_1y_1)z_1 \end{aligned} \tag{A.15}$$

and

$$a_{000} = A_0B_0, \quad a_{001} = A'_0B'_0, \; a_{010} = A_0B_1, \quad a_{011} = A'_0B'_1, \tag{A.16}$$
$$a_{100} = A_1B_0, \quad a_{101} = A'_1B'_0, \; a_{110} = A_1B_1, \quad a_{111} = A'_1B'_1, \tag{A.17}$$

For $C_{y0} = C_{y1} = C_{x0} = C_{x0} = 0$, we have, respectively,

$$\begin{vmatrix} A_0B_0 & A'_0B'_0 \\ A_1B_0 & A'_1B'_0 \end{vmatrix} = 0, \quad \begin{vmatrix} A_0B_1 & A'_0B'_1 \\ A_1B_1 & A'_1B'_1 \end{vmatrix} = 0, \tag{A.18}$$
$$\begin{vmatrix} A_0B_0 & A'_0B'_0 \\ A_0B_1 & A'_0B'_1 \end{vmatrix} = 0, \quad \begin{vmatrix} A_1B_0 & A'_1B'_0 \\ A_1B_1 & A'_1B'_1 \end{vmatrix} = 0. \tag{A.19}$$

For tripartite states, we have the following situations for the solutions of eqs.(A.18) and (A.19):

1. All A_i, A', B_i and $B_i' \neq 0$ $(i = 0, 1)$:

   In this case one can factor out the common factors in each of the four determinants in (A.18) and (A.19), giving
   $$\begin{vmatrix} A_0 & A_0' \\ A_1 & A_1' \end{vmatrix} = 0, \quad \begin{vmatrix} B_0 & B_0' \\ B_1 & B_1' \end{vmatrix} = 0.$$
   Then we have
   $$\frac{A_0'}{A_0} = \frac{A_1'}{A_1} = p, \quad \frac{B_0'}{B_0} = \frac{B_1'}{B_1} = q$$
   for some constants p, q, say. This implies $F(x, y, z; A)$ is completely factorized
   $$F(x, y, z; A) = (A_0 x_0 + A_1 x_1)(B_0 y_0 + B_1 y_1)(z_0 + pqz_1),$$
   and hence $\mathrm{Det}A = 0$.

2. $X_i,\ X_i' \neq 0$, $X_{\bar{i}} = X_{\bar{i}}' = 0$ ($X = A$ or B, $\bar{i} = i + 1 (\mathrm{mod}\ 2)$):

   In this case , $F(x, y, z; A)$ is also factorized. To show this, let us take $A_0,\ A_0' \neq 0$, and $A_1 = A_1' = 0$. Then $C_{y0} = C_{y1} = C_{x_1} = 0$, and $C_{x0} = 0$ implies $B_0'/B_0 = B_1'/B_1 = p$ for some constant p. This implies $F(x, y, z; A)$ is factorized as
   $$F(x, y, z; A) = x_0(B_0 y_0 + B_1 y_1)(A_0 z_0 + pA_1 z_1),$$
   and $\mathrm{Det}A = 0$,.

3. $X_i + X_i' = 1$, $X_0 + X_1 = X_0' + X_1' = 1$ ($X = A$ or B):

   There are only four cases for X and X', namely, (i) $A_1 = B_1 = 0$, (ii) $A_1 = B_0 = 0$, (iii) $A_0 = B_0 = 0$ and (iv) $A_0 = B_1 = 0$. For these choices of X and X', the six sub-determinants are zero. This is because one of the rows or columns of the sub-determinant is zero. Interesting cases include: $x_0y_0z_0 + x_1y_1z_1$ (GHZ-state), $x_0y_1z_0 + x_1y_0z_1$, $x_1y_1z_0 + x_0y_0z_1$, and $x_1y_0z_0 + x_0y_1z_1$. They correspond to $\{A_0, A_1, B_0, B_1\} = \{1, 0, 1, 0\}, \{1, 0, 0, 1\}, \{0, 1, 0, 1\}$ and $\{0, 1, 1, 0\}$, respectively. The corresponding A_i' and B_i' are obtained by making the transformations $0 \to 1,\ 1 \to 0$.
   As is obvious, these states are not separable. Thus the vanishing of the six sub-determinants alone do not distinguish separability of the state. Therefore the Cayley hyperdeterminant is required. It is easy to show that in this case, $\mathrm{Det}A$ is given by
   $$\mathrm{Det}\, A = (A_0 B_0 A_1' B_1')^2 + (A_0' B_0' A_1 B_1)^2 + (A_0 B_1 A_1' B_0')^2 + (A_0' B_1' A_1 B_0)^2 \neq 0. \qquad \text{(A.20)}$$
   Other factors in (11) are zero, as they all involve $X_i X_i' = 0$. The conditions $X_0 + X_1 \neq 0$ and $X_0' + X_1' \neq 0$ guarantee that one of the four factors in $\mathrm{Det}A$ is non-vanishing.

Putting all these together, we see that a tripartite state is separable iff
$$[|\mathrm{Det}\, A|; C_{x0}, C_{x1}, C_{y0}, C_{y1}, C_{z0}, C_{z1}] = [0; 0, 0, 0, 0, 0, 0]$$

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