From Andronow to Zypkin: an outline of the history of non-linear dynamics in the USSR

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Abstract—**By employing the conventional German, rather than English or international transliterations – particularly of the name written 'Tsypkin' in English – I artfully suggest that this talk is an 'A to Z' of Russian non-linear dynamics. However, the period under consideration is not delineated by these two names. Rather, I begin with the Poincaré-Lyapunov heritage – but then concentrate on the way that this thinking was transformed through Russian mathematicians, scientists and engineers such as Mandelshtam, Andronov, Lure and others, over a large part of the twentieth century. (Tsypkin, in fact, receives only a minor mention, as his major achievements were in other areas.) As a historian of automatic control, I draw primarily on applications of non-linear theory in feedback systems.**

I. INTRODUCTION

This conference celebrates the 100th anniversary of the death of Henri Poincaré; the 120th anniversary of the publication of both his impressive *Méthodes nouvelles de la mécanique céleste* and Aleksandr Lyapunov's dissertation on the stability of motion; as well as the 60th anniversary of the untimely death of Aleksandr Andronov, a major figure in Russian non-linear dynamics of the 1930s and 1940s. My paper is drawn essentially from my researches over the last two decades into the history of control engineering in the USSR. The interested reader will find details of the primary and secondary sources for this talk in those earlier papers in the list of references, particularly [1, 2, 3]. Apart from these early papers of mine I have cited just a few of the more important and / or more recent historiographical publications in this area. For one of the few articles to set Russian work on non-linear dynamics in an international context, see [4].

II. POINCARÉ AND LYAPUNOV

Lyapunov's gravestone in Odessa – re-erected, as it happens, in 1952, after a period of loss – reads:

The creator of the theory of the stability of motion, of techniques for studying equilibrium figures of rotating fluids, and of methods in the theory of differential equations; the author of the central limit theorem in probability theory, and of other fundamental investigations in the fields of mechanics and mathematical analysis.

Until Lyapunov's work, the only rigorous approach to the stability of dynamic systems was that derived by Routh in England in 1877 (Hurwitz came to the same conclusions in 1895 in Switzerland, but using a different proof). The Routh-Hurwitz stability criterion, as it came to be known, requires a linear differential equation model, however – although the test itself is a comparatively simple matter of performing arithmetical operations on the coefficients of the differential equation.

Lyapunov was aware of the work of Routh, but his starting point was Poincaré. In fact his 'first' stability criterion is essentially equivalent to other work by Poincaré, and involves assessing the stability of a non-linear system by finding series solutions of the non-linear differential equation and then using the stability of the linear first approximation to estimate stability for small initial disturbances.

Lyapunov's 'second' criterion, however, was more general, and of much greater influence, particularly in the field of control engineering. Instead of concentrating on time-domain solutions of differential equations, notoriously difficult to obtain for non-linear systems, he introduced the concept now known as a Lyapunov function, an energy-like function of the system state variables, satisfying a particular set of conditions. The stability criterion can then be stated:

"If there exists a Lyapunov function, then the origin is a stable equilibrium point. If the derivative of the Lyapunov function is negative, then the system is asymptotically stable." Unfortunately, however, finding an appropriate Lyapunov function can be very difficult, and failure to find one gives no information at all about the stability or otherwise of the system!

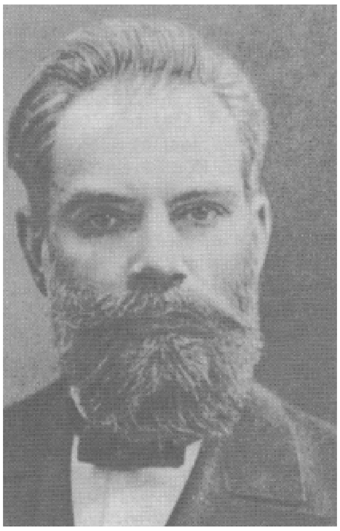

Fig. 1. A. M. Lyapunov

III. THE MANDELSHTAM SCHOOL

Leonid Isaakovich Mandelshtam was a most distinguished Soviet physicist of the first half of the twentieth century. He studied in Odessa and Strasbourg, and was ultimately appointed to the Chair of Theoretical Physics in 1925. He suggested what was later to be called the Briouillin effect, and was a co-discoverer of Raman scattering (in crystals). For the purposes of this talk, however, it is his work on oscillations in non-linear systems in which we are interested. Together with his collaborator Papaleksi and a number of talented young researchers, his group developed a range of important results in non-linear dynamics, including the theory of multivibrators [4].

One of Mandelshtam's most gifted students was Aleksandr Aleksandrovich Andronov. One of Andronov's first major papers appeared in French in 1929, in the *Comptes Rendus*, based on a paper in Russian the previous year. In this short paper he demonstrated the link between Poincaré's limit cycles and a general theory of oscillations. In a remarkably prescient observation, he remarked that oscillatory phenomena in chemistry, biology and engineering would be amenable to phase-plane techniques. Soon after the publication of this work, however, Andronov made a major career change, which led to him establishing his own 'school' as the Russians tend to call such major scientific groups, in Nizhnii Novgorod, known as Gorkii in the Soviet period.

IV. THE ANDRONOV SCHOOL

In 1931, for reasons that are still not quite clear, Andronov and his wife moved to Gorkii, 600 km east of the capital, although he appears to have retained a part-time paid position in Moscow. Whatever the reasons for the move, Andronov built up an extremely successful research group in Gorkii, while continuing to maintain and develop further his close links with Moscow.

Fig. 2. A. A. Andronov

The 1930s saw Andronov and his colleagues elaborating the theory of what the Russians call self-oscillations. As Amy Dahan-Dalmedico has put it [5]:

During the 1930s, nonlinear oscillations constituted the central and unifying topic of the group's researches, which focused on: 1) the search for a general mathematical theory; 2) the elaboration of a physics of oscillations which might exploit these mathematical results, in particular for radiotechnics; and 3) the development of a family of instruments for high-frequency and radio-communication purposes (magnetronic generators, continuous-current *synchronic machines). This work was characterized from the outset by constant interaction between fundamental and applied research and sharing sophisticated theoretical resources and diverse skills New apparatus and engineering devices were created to fulfil the technical needs of the military-industrial powers – for stabilizing nonlinear vibrations and resonance phenomena, or for manipulating and controlling waves.*

This work in the 1930s culminated in the remarkable 1937 book "Theory of Oscillations" authored by Andronov, S. E. Khaikin and A. A. Vitt (although Vitt's name never appeared in the book as he was arrested, deported and died in Siberia soon afterwards). This work, which became a classic in the USSR, and which made available to a much broader readership the Poincaré / Lyapunov techniques as further developed by Andronov and others, was not translated into English until 1949 (for further details see [1, 3, 6]). The advanced nature of this text for the 1930s is illustrated by Fig. 3, which shows a typology of phase space trajectories taken from that volume.

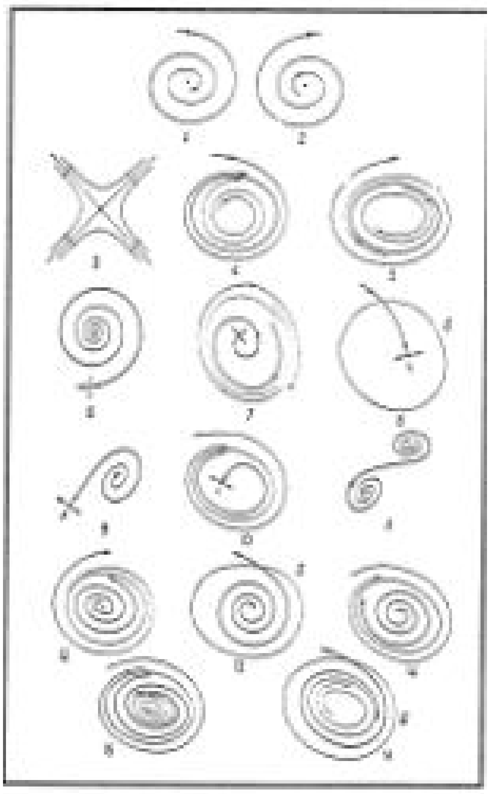

Fig. 3. A typology of trajectories from "Theory of Oscillations"

V. THE AUTOMATIC CONTROL CONNECTION

The USSR was one of the earliest nations to realize that automation and control was going to be a huge part of manufacturing and process engineering, and as early as 1939 set up the Institute of Automation and Remote Control (IAT) in Moscow. By this time the Gorkii group was beginning to take an interest in control-theoretical problems, particularly where non-linearities were involved. A specialist conference on automatic control was held in Moscow in 1940 (probably the first in the world), attended by researchers from a number of centres in the Soviet Union. Some of the recent work in the United States was reported, as was the native Soviet research.

In 1944 Andronov established a specialist seminar at the IAT. One of the young attendees, M.V. Meerov, recalled in an interview with the author in 1996: *The work of the seminar was inaugurated by A. A. Andronov on non-linear friction in the theory of the direct-acting governor, and the theory of the point transformation of surfaces. This meeting was no mere seminar session – it was a major scientific event, attended by a number of full Academicians and Corresponding Members of the Academy of Sciences, as well as by various other professors and engineers. After the lecture there was no need to worry about attendance at the seminar, it had a momentum of its own.*

The seminar naturally covered a whole range of topics in control engineering, but many of the researchers were, naturally, interested in non-linear systems, including: M. A. Aizerman who looked at non-linearities in feedback systems; L. S. Goldfarb, who developed a version of the describing function approach; and Yakov Tsypkin who, in addition to major early work on sampled-data (discrete) systems, also extended the describing function approach and wrote a *magnum opus* on relay control systems – which, of course, require non-linear analysis techniques. Between 1946 and 1960 at least six members of Andronov's group had obtained the highest Russian academic degree, *doctor nauk* [doctor of sciences] – which required a second dissertation after the PhD approximate equivalent (*kandidat*).

The links between Moscow and Gorkii were particularly close, owing to Andronov's joint role, but there were also links with other centres, particularly Leningrad, where there was a group of non-linear theorists, including A. I. Lure. Lure was a specialist in mathematics and mechanics, but also studied non-linear elasticity, flight control systems and optimal control. Lure and Aizerman in particular looked at the stability of a range of non-linearities confined within two linear regimes. Aizerman, for example, queried the stability consequences within a feedback loop of a non-linear element whose variable gain was confined between limiting systems with two different constant gains (Fig. 4). His conjecture proved to be incorrect, but it led to subsequent correct, and significant, results.

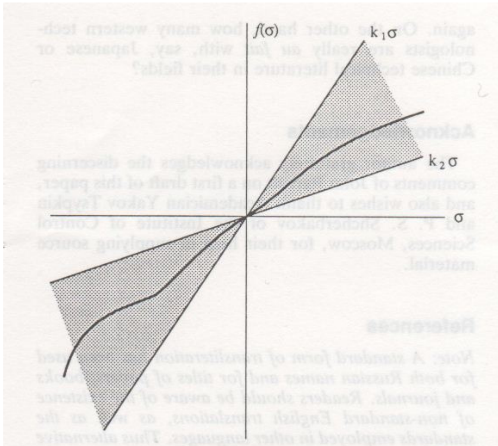

Fig. 4. Aizerman's conjecture

VI. OTHER SOVIET CENTRES OF NON-LINEAR STUDIES

This brief paper has concentrated on the two major centres of Gorkii and Moscow, with a brief mention of Leningrad. To conclude, however, a few other locations should be mentioned:

- Kiev (Ukraine), where N. M. Krylov and N. N. Bogolyubov had been making detailed investigations into the harmonic linearization approach to non-linear systems during the 1930s
- Kazan, where N. G. Chetaev and others had been applying Lyapunov methods to engineering problems
- Kharkov (Ukraine), where M. V. Meerov derived a condition for aperiodic stability in 1942 – although it was not published until 1945

VII. CONCLUSION

I hope to have demonstrated the enormous range of Soviet work on non-linear dynamics over the first half of the twentieth century. The work in this field in the Soviet Union is all the more remarkable since it took place within an often extremely repressive political system. Apart from Vitt, the individuals whose work is outlined briefly here escaped the various purges, but many of their colleagues did not. Some further information is to be found in [6] and [7].

The Soviet work permeated only slowly to the West. Lyapunov's dissertation was translated into French in 1907 and reprinted again in 1947, but an English version appeared only in 1992, to mark the centenary – and then as a translation from the French rather than directly from the Russian. Andronov, Khaikin [and Vitt] appeared in an English version in 1949, twelve years after the original. Krylov and Bogolyubov's book "Introduction to Non-Linear Mechanics", again published in Russian in 1939, reached English a little more quickly, in 1943. These translations were very much the result of activity by Americans of Russian descent, who could read Russian. The most eminent lobbyists were probably Solomon Lefschetz and Nicholas Minorsky: the former translated both Andronov, Khaikin [and Vitt], and Krylov and Bogolyubov; and the latter wrote a confidential wartime report on Soviet non-linear dynamics for the US Navy. From the mid 1950s, however, cover-to-cover English translations of the open Russian academic literature were appearing, and inter-communication was, to the extent allowed on both sides by confidentiality and sensitivity of military applications, greatly improved.

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