Deduction of static surface roughness from complex excess attenuation

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Abstract: Data for complex excess attenuation have been used to determine the effective surface admittance and hence characteristic roughness size of a surface comprising a random distribution of semi-cylindrical rods on an acoustically hard plane. The inversion for roughness size is based on a simplified boss model. The technique is shown to be effective to within 4%, up to a threshold roughness packing density of 32%, above which the interaction between scattering elements appears to exceed that allowed by the model.

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1. Introduction

This paper discusses the problem of deduction of surface roughness from a measured excess attenuation. There has been considerable work concerned with the prediction of excess attenuation of sound, i.e., forward scatter, from details of (hard) surface roughness and source–receiver geometries.[1–4] Although these predictions have agreed well with experimental data, the inverse problem of deducing surface roughness information from the measured excess attenuation has been addressed only for agricultural soils.[5]

Acoustic measurements of roughness have been made by use of other methods such as ultrasonic backscatter,[6] but since the proportion of backscattered energy is very low, the technique requires high power and is only viable over short distances and for small surface areas. Excess attenuation provides the opportunity to deduce statistical information about static hard rough surfaces that span a large area and will function over large distances. The excess attenuation (EA) spectrum represents the ratio of the frequency-dependent signal received from a point source over a solid boundary to that in the absence of the boundary. Its magnitude indicates a series of maxima and minima resulting from constructive and destructive interference between the direct and reflected acoustic signals.[1] It has been shown that the presence of roughness on the surface causes a change in shape of the EA spectra which may be interpreted as a consequence of a change in the effective impedance. The effective impedance may be deduced from complex EA data.[7] Although semi-analytical theory has been shown to be reliable at calculating the real and imaginary parts of excess attenuation for a known source–receiver geometry, the inverse problem has to be solved numerically.[7]

The multiple scattering theory developed by Boulanger *et al.*[8] could be used to give more accurate results for semi-cylinder configurations, but it would be restricted to these shapes of roughness whereas the intention here is to explore a potentially more general method. Although similar work has been performed in the context of deducing the roughness of cultivated ground surfaces,[5] in the work reported here

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roughness parameter fits are made via the deduced complex surface impedance rather than by fitting only the magnitude of EA spectra. Also of note is that lasers are often used to measure surface roughness, however the method generally requires moving parts, whereas the broadband acoustic technique proposed here has the potential to categorize surface roughness over a large area using fixed equipment.

2. Theory

This work is based on theories by Tolstoy[2] and Twersky,[3] which use a "boss" model. Hence a rough surface is considered as a flat hard surface, upon which, for example, the two-dimensional semi-cylindrical rods or scatterers are distributed (see Fig. 1). Although this represents an idealization and simplification of a randomly rough surface, according to Tolstoy it will be more accurate than theories based on perturbation methods if the roughness includes any steep slopes. The roughness is typically categorized by the roughness (semi-cylinder) volume per unit area, and the mean center-to-center spacing.[4] The model assumes that the roughness size and mean spacing are small compared with the smallest wavelength of interest. In this work the simulated roughness is described by the percentage of the surface that contains scattering elements.

Theory for propagation from a point source over an impedance boundary can be used to calculate a complex EA spectrum from a given surface admittance.[8] This requires accurate measurements of source and receiver separation, and their respective heights above the surface. Expressions have also been derived to calculate the effective surface admittance from the statistical shape of a given surface,[4] by the relationship $\beta = \eta - i\xi$, where β is the complex admittance, and,

$$\eta \approx \frac{k^3 b V^2}{2}(1 - W^2)\left\{(1 - \sin^2(\alpha)\sin^2(\varphi) \times \left[1 + \left(\frac{\delta^2}{2}\cos^2(\varphi) - \sin^2(\varphi)\right)\sin^2(\alpha)\right]\right\} + O(k^5), \quad (1)$$

$$\xi \approx kV[(\delta - 1)\cos^2(\varphi) - \cos^2(\alpha)(1 + (\delta - 1)\cos^2(\varphi)], \quad (2)$$

where k is the wave number, b is the mean center-to-center spacing of scatterers (which is related to the correlation length on a randomly rough surface), V is the volume of scatterers per unit area (which is related to the mean height of a randomly rough surface), $W = nb^* = b^*/b$ is a measure of the randomness of the distribution, b^* is the minimum (center-to-center) separation between two cylinders, α is the angle of incidence with respect to the normal, ϕ is the azimuthal angle between the wave vector and the roughness axes, and $\delta = 2/(1 + I)$ is a measure of the dipole coupling between the semi-cylinders, where $I = (a^2/b^2)I_2$, a is the radius of the semi-cylinders and I_2 is a function of W as given by Boulanger *et al.*[7] In the experiments reported in Sec. III, the incident wave vector is assumed to be normal to the semi-cylinder axes ($\phi = 0$) and the angle of incidence is near grazing ($\alpha \approx \pi/2$).

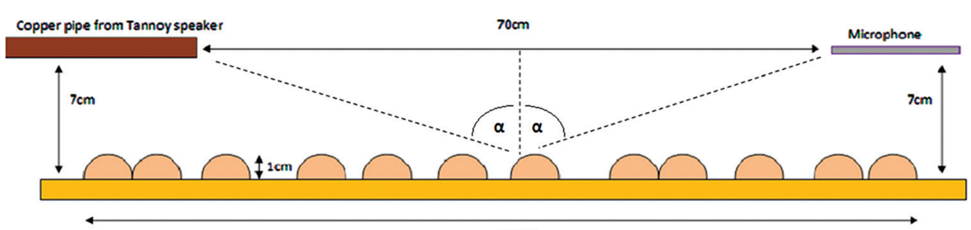

Fig. 1. (Color online) Experimental arrangement based on the boss model. Array of 1 cm radius rods arranged on a flat hard surface.

Since wave number $k = \frac{2\pi f}{c_0}$ is proportional to frequency f (c_0 is the speed of sound in air), the frequency dependence can be represented in the following simplified form by grouping the other physical quantities into (assumed) constants α_1 and α_2,

$$\beta = \alpha_1 f^3 + i\alpha_2 f, \tag{3a}$$

where $\alpha_1 = \frac{4\pi^3 bV^2}{c_0^3}$, and $\alpha_2 = \frac{2\pi V(\delta - 1)}{c_0}$.

It has been found that allowing the exponent of the frequency term to be adjustable results in more accurate data fitting hence,

$$\beta = \alpha_1 f^{\gamma_1} + i\alpha_2 f^{\gamma_2}, \tag{3b}$$

where γ_1 and γ_2 are adjustable frequency exponents.

3. Experiments

The experimental setup can be seen in Fig. 1. Up to fifty 1 cm radius semi-cylindrical rods were placed at random intervals on a 1 × 1 m plywood surface acting as an acoustically hard base. A Tannoy speaker attached to a 1 m long, 2 cm diameter copper pipe was used as a point source and generated white noise in a frequency band from 0 to 20 kHz. The resulting sound fields were measured at a Bruel and Kjaer Type 4954 quarter-inch microphone positioned at 70 cm away from the source. Both source and receiver were at a height of 7 cm above the surface. The free-field response of the source was recorded without the hard base present to provide a reference spectrum from which to calculate EA spectra. Rod arrangements were characterized by the percentage of surface covered by rods. Readings of EA were taken for ten random arrangements corresponding to coverage percentages from 5% to 95% in 10% increments. The amplitudes of the ten EA spectra were then averaged. A curve fitting routine was used to optimize the values of variables α_1, γ_1, α_2, and γ_2 in Eq. (3b) to give the complex frequency-dependent admittance which would give rise to a complex EA spectrum whose magnitude matched with that of the measured EA.[8] Correlations between these four variables and the known surface parameters of volume of rod per unit area, and the center-to-center spacing between rods were then investigated. An empirically derived relationship was then tested against data from other rod arrangements at coverage percentages from 0% to 100% in 10% increments.

4. Results and discussion

Table 1 shows typical ranges of the fitted values of variables α_1, γ_1, α_2, and γ_2. Deduced values of variable α_1 are consistently very small, O(10^{-6}), since the imaginary part of deduced effective admittance is much larger than that of the real part which is in agreement with previous results.[1] As suggested by Eqs. (2) and (3b) the volume of scatterers per unit area of surface (a measure of mean roughness height, H) is found to be linearly related to variable α_2, for surface coverage percentages below 32% by the relationship,

$$V = 0.076\,\alpha_2 - 0.006. \tag{5}$$

Table 1. Typical values of fitted variables.

Variable	Typical range
α_1	10^{-8}–10^{-6}
γ_1	2.4–3.2
α_2	0.08–0.25
γ_2	0.8–1.3

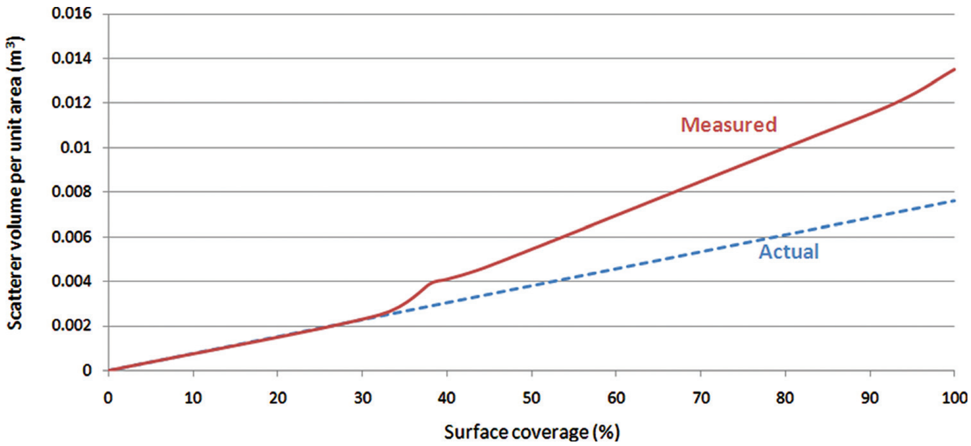

Fig. 2. (Color online) Actual and acoustically measured scatterer volume per unit area (m^3) vs surface scatterer coverage (%), showing an accurate measurement to within 4% error up to a coverage of 32%.

Figure 2 shows the scatterer volume estimation using Eq. (5) against the percentage of surface covered. It can be seen that up to around 32% coverage the method enables determination of the surface roughness volume per unit area quite accurately (maximum error of 4%). It is thought that above 32% coverage the interaction between scattering elements becomes larger than that permitted by the model and thus adversely affect the results. In Eq. (3), δ is a measure of interaction between adjacent elements, which is a function of a, b, and W. This would imply an effective admittance that would be different for each random arrangement. Moreover the Twersky model relies on the assumption that $b \gg a$. When the surface coverage (roughness density) is low, this condition is upheld, and δ is approximately constant. For such cases the imaginary part of effective admittance varies linearly with V, the scatterer volume per unit area. As the roughness density increases, and the ratio between b and a decreases; δ values begin to dominate the imaginary part of effective admittance, and the simplification of Twersky's model in Eq. (3) no longer applies. This technique also suggests a method for establishing the limitations of Twersky's model. For surface coverage greater than 32% it appears that the following empirical relationship could be used,

$$V = 1.0895\,\alpha_2 - 0.0071. \qquad (6)$$

In principle, the mean spacing, b, could be calculated from the deduced value of scatterer volume per unit area V and the assumed constant α_1, by using the first term of Eq. (3). It was found, however, that for the scattering configurations investigated here the values of α_1 are too small to allow this.

5. Concluding remarks

A method of characterizing hard surface roughness based on forward scatter has been investigated and has been found to be useful up to 32% roughness area coverage. Potentially the technique is useful at greater ranges than ultrasonic backscatter while involving lower transducer cost and less maintenance than current ultrasonic devices. It also facilitates the roughness characterization of larger areas than ultrasonic transducers. Further investigations will consider modification of $\mathrm{Im}(\beta)$ to include dependence on the standard deviation in height and roughness correlation length, as well as on mean height.[5,9]

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