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REVIEW ON FUZZY CLUSTERING ALGORITHMS

M. Ameer Ali
Dept. of Electronics & Communication Engineering, East West University, Dhaka, Bangladesh.

Gour C. Karmakar
Gippsland School of Information Technology, Monash University, Churchill, Australia

Laurence S. Dooley
Gippsland School of Information Technology, Monash University, Churchill, Australia

ABSTRACT

Image segmentation especially fuzzy based image segmentation techniques are widely used due to effective segmentation performance. For this reason, a huge number of algorithms are proposed in the literature. This paper presents a survey report of different types of classical fuzzy clustering techniques which available in the literature.

Keyword: *fuzzy clustering, image segmentation, fuzzy c-means, PCM.*

1. INTRODUCTION

The application of digital images is rapidly expanding due to the ever-increasing demand of computer, Internet and multimedia technologies in all aspect of human lives, which makes digital image processing a most important research area. Digital image processing encompasses a wide and varied field of applications from medical science to document processing and generally refers to the manipulation and analysis of pictorial information. Image processing is mainly divided into six distinct classes: i) Representation and modelling, ii) Enhancement, iii) Restoration, iv) Analysis, v) Reconstruction, and vi) Compression. Image analysis embraces feature extraction, segmentation and object classification [1-5], with segmentation for instance, being applied to separate desired objects in an image so that measurements can subsequently be made upon them.

Segmentation is particularly important as it is often the pre-processing step in many image processing algorithms. In general, image segmentation refers to the practice of separating mutually exclusive homogeneous regions (objects) of interest in an image. The objects are partitioned into a number of non-intersecting regions in such a way that each region is homogeneous and the union of two adjacent regions is always non-homogeneous. Most natural objects are non-homogeneous however, and the definition of what exactly constitutes an object depends very much on the application and the user, which contradicts the above generic image segmentation definition [3, 6-9].

Segmentation has been used in a wide range of applications, with some of the most popular being, though not limited to: automatic car assembling in robotic vision, airport identification from aerial photographs, security systems, object-based image identification and retrieval, object recognition, second generation image coding, criminal investigation, computer graphic, pattern recognition, and diverse applications in medical science such as cancerous cell detection, segmentation of brain images, skin treatment, intrathoracic airway trees, and abnormality detection of heart ventricles [6, 10-12].

Different applications require different types of digital image. The most commonly used images are *light intensity* (LI), *range* (depth) image (RI), *computerized tomography* (CT), thermal and *magnetic resonance images* (MRI). The research published to date on image segmentation is highly dependent on the image type, its dimensions and application domain and so for this reason, there is no single generalized technique that is suitable for all images [12, 13].

There are numerous image segmentation techniques in the literature, which can be broadly classified into two categories [12] namely: i) *classical* and ii) *fuzzy mathematical*. The former [14- 18] comprises the five main classes [12] shown in Figure 1: i) Gray level thresholding [16, 19, 20], ii) Iterative pixel classification (e.g. relaxation, *Markov random fields* and neural network based techniques) [21-23], iii) Surface-based segmentation [24], iv) Colour segmentation [25], and v) Edge detection [14, 26]. Fuzzy mathematical techniques are widely used in multifarious computer vision applications as they are far better able to handle and segment images, particularly noisy images, by using fuzzy membership values. The various fuzzy mathematical techniques identified in Figure 1 will be examined in greater detail in Section 2. There are also other image segmentation techniques which are not classified in either category, including those based upon *Markov random models*, Bayesian principles and the Gibbs distribution, with further details being given in [23, 27-30].

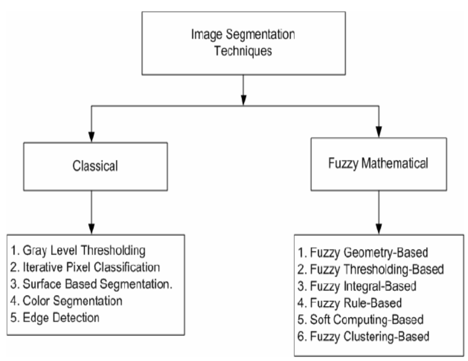


Fig. 1 General classification of image segmentation techniques.

Segmentation is certainly one of the most challenging tasks in image processing and computer vision for many reasons, some of which are [6-8, 12]:

- Image types such as MRI, CT or *Single Photon Emission Computed Tomography* (SPECT) contain inherent constraints that make the resulting image noisy and may include or introduce some visual artefacts.
- Image data can be ambiguous and susceptible to noise and high frequency distortion as in SPECT imaging for instance, where object edges become fuzzy and ill-defined.
- The shape of the same object can differ from image to image due to having different domain and capturing techniques as well as various orientations. An object's structure may not be well defined in many natural images and can also be very hard to accurately locate the contour of an object.
- The distributions of gray scale pixel values of the same object are not the same for all images and even in the same image, pixels belonging to the same class may have different intensities and distributions.
- Objects to be segmented are highly domain and application dependent-for example, in order to automatically estimate the myocardial wall thickness from a captured X-ray image of the human heart region, the inner and outer contours of the heart's left ventricle may be the two objects required to be segmented, while for another application, the entire heart may need to be segmented.
- The properties of an object can differ in their representation depending upon the type of image and its domain, so there needs to be a trade-off between the desired properties that are to be employed for segmentation. For example, some gray scale images have a Poisson distribution, though this would not be true for either an RI or MRI image, so the segmentation strategy requires both semantic and *a priori* information concerning the image type and with other relevant object information such as the number of objects in the image.

Thus, it can be concluded that most images contain some form of ambiguity. Pal and Pal [12] showed that gray tones (LI) images possess ambiguities because of possible multi-valued brightness levels. This ambiguity may be defined in terms of grayness and/or spatiality. The gray ambiguity represents *indefiniteness* in deciding whether a pixel is either black or white, while spatial ambiguity means *indefiniteness* in the shape and geometry of a region contained in the image. Classical techniques produce a crisp (hard) decision, though such decisions are unsuitable for ambiguous and ill-defined data. For this reason, it is crucially important to have a segmentation strategy for image processing systems that is able to handle all types of uncertainty at any processing stage. Prewitt first suggested that image segmentation yielded fuzzy regions [13, 31], which was the catalyst for the development of various fuzzy-based techniques, which have since proven to be very effective in efficiently handling such ill-defined image data, by assigning a membership value to every pixel (datum), which denotes the possibility of belongingness of that pixel to a region (cluster). This is main discriminating feature between fuzzy and hard decision-making and is one of the main motivations for using fuzzy-based image segmentation techniques.

In a fuzzy system, every image contains a number of regions $R_1, R_2,..........R_c$, where c is the number of regions (objects) [32-35], with a number of pixels forming a region and each pixel in a region assigned a membership value which measures the probability of that pixel belonging to that particular region. Each datum X(x, y) of image *I* having coordinates (x, y) is assigned a membership value μ by mapping the gray levels into the close interval ranging from 0 to 1, so the membership function μ for *I* can be defined as follows:-

$$\mu(X): \Omega \rightarrow [0, 1] \quad (1)$$

where Ω denotes a universal reference set of all values for all the data in image *I* .

In using a fuzzy technique, the particular characteristics of an image including brightness, contrast, edges, regions, connectivity and complexity can be represented by *linguistic variables* such as VERY COMPLEX, COMPLEX and SIMPLE [36]. Medasani *et al* [32] used both fuzzy and crisp methods to measure geometric properties such as area, perimeter, height, extrinsic and intrinsic diameter and elongatedness, together with nongeometric properties like average pixel intensity, entropy, and homogeneity for both real and synthetic images. They showed that fuzzy techniques consistently provided better results than crisp techniques for all images due to using fuzzy membership functions and also tested both approaches upon noisy data, with experimental results confirming their superiority for both geometric and non-geometric properties. It was also proven that if fuzzy techniques are applied in noisy conditions, it is not necessary to apply noise removal techniques to the image, even in textured regions where noise removal is often very difficult.

Any segmented image therefore will inherently produce fuzzy regions (objects) [13], so fuzzy-based image segmentation techniques do afford an attractive and effective approach for handling imprecise image information by employing fuzzy membership functions for each datum. This was the overriding reason for making literature on fuzzy image segmentation.

The organization of the paper: Section 2 describes the different types of fuzzy image segmentation techniques while existing fuzzy clustering algorithms are detailed in Section 3. Different types of existing classical fuzzy clustering techniques are presented in Section 4 with some concluding remarks in Section 5.

2. FUZZY IMAGE SEGMENTATION TECHNIQUES

Fuzzy image segmentation techniques have become very popular [6] due to the rapid development of fuzzy set theory based on mathematical models, genetic algorithms and neural networks, and are widely used in diverse applications including image processing, pattern recognition, robotic vision, engineering tools, security and computer vision systems. Fuzzy image segmentation techniques as shown in Figure 1, are broadly classified into six categories [37] :- i) Fuzzy geometric, ii) Fuzzy thresholding, iii) Fuzzy integral-based, iv) Fuzzy rule-based, v) Soft computing-based, and vi) Fuzzy clustering. A detailed description of existing fuzzy clustering techniques is now provided.

3. EXISTING FUZZY CLUSTERING Algorithms

Clustering is the process of separating or grouping a given set of unlabeled patterns into a number of clusters such that the patterns drawn from the same cluster are *similar* to each other in some sense, while those are assigned to different clusters are *dissimilar* [38-42]. Most of the time, objects are defined by a set of features and so those with similar features are classified into one cluster [42]. For a physical interpretation of the clustering process, the example shown in Figure 2 contains four separate clusters.

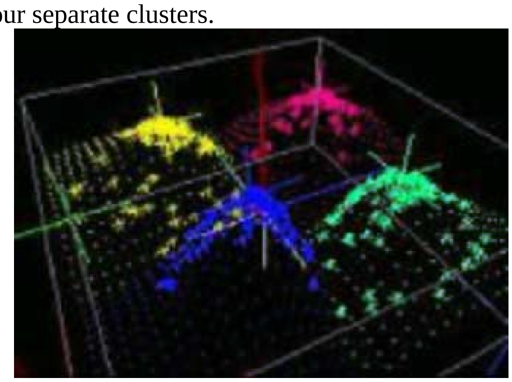


Fig. 2 Example showing four clusters

As highlighted in Section 1, there are mainly two types of clustering, namely *hard* (crisp) (HC) and *fuzzy*-based [6]. In a HC algorithm [43-44], the decision boundary is fully defined and one pattern is classified into one and only one cluster, i.e. the clusters are mutually exclusive [41, 42]. However in the real world, the boundaries between clusters are not clearly defined. Some patterns may belong to more than one cluster and so in this case, fuzzybased clustering techniques [39, 40, 45, 46] provide a better and more efficient approach to classifying these patterns by assigning a membership value to each individual pattern. As mentioned in Section 1, among fuzzy-based techniques, fuzzy clustering is considered in this paper as the basis for the literature review due to their effective segmentation performance. Fuzzy clustering algorithms are broadly classified into two groups: i) Classical and ii) Shape-based [38]. There exist many classical fuzzy clustering algorithms in the literature, among the most popular and widely used being: i) *Fuzzy cmeans* (FCM) [40], ii) *Suppressed fuzzy c-means* (SFCM) [46], iii) *Possibilistic c-means* (PCM) [45], and (iv) *Gustafson-Kessel* (GK) [47], while from a shape-based fuzzy clustering viewpoint, well established and popular algorithms include: i) Circular shape-based [48], ii) Elliptical shape-based [49], and (iii) *Generic shape-based* techniques [50].

A detailed review of the above mentioned classical fuzzy clustering algorithms is now provided.

4. CLASSICAL FUZZY CLUSTERING ALGORITHMS

Clustering algorithms that use general feature sets such as PL, PI or CIL are generally treated as classical fuzzy clustering techniques. These are dependent on both the features used and the type of objects in an image. A review of the three main classical fuzzy clustering techniques mentioned above is now detailed.

4.1.1 Fuzzy c-Means Algorithm

The FCM algorithm [40] was developed by Bezdek in 1981 and is still the most popular classical fuzzy clustering technique, widely used directly or indirectly in image processing. It performs classification based on the iterative minimization of the following objective function and constraints [39, 40, 51-57]:

$$J_q(\mu, V, X) = \sum_{i=1}^{c}\sum_{j=1}^{n}(\mu_{ij})^q D_{ij}^2 \tag{1}$$

subject to

$$0 \le \mu_{ij} \le 1;$$
$$i \in \{1,\ldots,c\} \text{ and } j \in \{1,\ldots,c\} \tag{2}$$

$$\sum_{i=1}^{c}\mu_{ij} = 1;\ j \in \{1,\ldots,c\} \tag{3}$$

$$0 < \sum_{j=1}^{n}\mu_{ij} < n;\ i \in \{1,\ldots,c\} \tag{4}$$

where n and c are the number of data and clusters respectively. μ is the fuzzy partition matrix containing membership values $[\mu_{ij}]$, q is the fuzzifier where $1 < q \le \infty$, V is cluster centre vector $[v_i]$, X is a data vector $[x_j]$ and $D_y = d(x_j, v_i)$ is the distance between datum x_j and v_i . Using a Lagrangian multiplier, the following can be derived by optimizing the objective function in (1) with respect to μ and V .

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{D_{ij}}{D_{kj}}\right)^{2/(q-1)}} \tag{5}$$

$$v_i = \frac{\sum_{j=1}^{n}(\mu_{ij})^q x_j}{\sum_{j=1}^{n}(\mu_{ij})^q} \tag{6}$$

The membership values are initialized randomly and both these and the cluster centres are iteratively updated until the maximum change in μ_{ij} becomes less than or equal to a specified threshold ξ. q is normally set to 2 as this is the best value for the fuzzifier (Step 1) while the membership μ_{ij} is randomly initialized in Step 2. The cluster centre *iv* and membership values μ_{ij} are then iteratively updated using (6) and (5) respectively (Steps 3.1-3.2) until either the maximum number of iterations (max_Iteration) or threshold ξ is reached (Step 3.3). The complete FCM algorithm is given in Algorithm 1, which for n data points incurs $O(\,)\,n$ computational time complexity [13, 58].

Algorithm 1: *Fuzzy c-means* (FCM) algorithm.

Pre condition: Objects to be segmented, number of clusters c , threshold ξ and the maximum number of iterations max_Iteration.

Post condition: Final segmented regions $\Re$.

1. Fix $2 = q$.
2. Initialize μ_{ij}..
3. FOR 1 = 1, 2, 3, ..., max_Iteration
   3.1 Update cluster centres v_i using (6).
   3.2 Update membership values μ $\mu_{ij}^{(i)}$ using Equation (5)
   3.3 IF $\left\| \mu_{ij}^{(l)} - \mu_{ij}^{(l-1)} \right\| \le \xi$ THEN STOP.

The number of clusters c , fuzzifier q and threshold ξ all need to be set manually. The selection of q is especially important because if $1 = q$ then FCM produces crisp (HC) instead of fuzzy regions. Also **(5)** and (6) are not sufficient to achieve the local minimum of (1) [55, 59], since if any of the distance value $D_{ij}=0$ **(5)** will be undefined. FCM strongly supports probability, but not the degree of *typicality* because it has the constraints in (2)- **(4)** which preclude the trivial solution $\mu_{ij}=0$. The relative membership values in **(5)** are calculated using these constraints which can be interpreted as the degree of sharing, but not the degree of *typicality* as required in many fuzzy set theory applications [60]. Antonio *at el* [52] tried to solve this problem by considering the Euclidean distance, the Mahalnobis distance and the covariance matrix in [47], and proposed the following two new objective functions:

$$J_{Ec}(\mu, V, X) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij} D_{ij}^2 \qquad (7)$$

$$J_{Mh}(\mu, V, X) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^{3/2} D_{ij}^2 \sqrt{\left| G_i^{-1} \right|} \qquad (8)$$

where the J_{Ec}, (μ, V, X) and J_{Mh} (μ, V, X) functions use the Euclidean and Mahalnobis distances respectively and G_1^{-1} is the covariance matrix for all data in the i^{th} cluster. According to [52], if the *membership function density* is defined as $\mu_{ij} \sqrt{\mu_{ij}} \sqrt{\left| G_i^{-1} \right|}$, then the membership values are respectively updated for the Euclidean and Mahalnobis distances by:

$$\mu_{ij}(Ec) = \frac{\left(\mu_{ij} \sqrt{\left| G_i^{-1} \right|} \sum_{k=1}^{n} \mu_{ik} \right)^{1/(q-1)}}{\sum_{i=1}^{c} \left(\mu_{ij} \sqrt{\left| G_i^{-1} \right|} \sum_{k=1}^{n} \mu_{ik} \right)^{1/(q-1)}} \qquad (9)$$

$$\mu_{ij}(Mh) = \frac{\left(\mu_{ij}^{3/2} \sqrt{\left| G_i^{-1} \right|} \right)^{1/(q-1)}}{\sum_{i=1}^{c} \left(\mu_{ij}^{3/2} \sqrt{\left| G_i^{-1} \right|} \right)^{1/(q-1)}} \qquad (10)$$

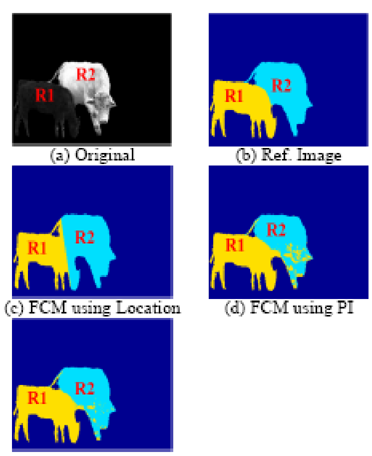
And the cluster centres are correspondingly updated by:

$$v_i = \frac{\sum_{j=1}^{n} (\mu_{ij})^2 x_j}{\sum_{j=1}^{n} (\mu_{ij})^2} \qquad (11)$$

where the fuzzy covariance matrix for the k^{th} cluster denoted by k G is defined as:

$$G_k = \frac{\sum_{j=1}^{n} \mu_{kj}^2 (x_j - v_k)(x_j - v_k)^T}{\sum_{j=1}^{n} \mu_{kj}^2} \qquad (12)$$

As mentioned in Section 1, the popularity of FCM is firmly based upon its flexible mathematical foundations and being an analytical solution for constraint optimized functions. This means it is possible to incorporate image feature information, such as *pixel location* (PL), *pixel intensity* (PI), and shape within its theoretical framework for segmentation purposes, and furthermore it is able to both effectively handle noisy and large datasets. FCM does arbitrarily divide objects into a given number regions (objects) whenever PL, PI, and *combination of pixel intensity and location* (CIL) are used as the selected features in the image segmentation process [61]. The experimental results of FCM separately using PL, PI, and CIL are given below in **Figure 3**.



(e) FCM using CIL

Fig. 3 (a) Original *cow* image, (b) Manually segmented reference of (a). (c)-(e) Segmented results of (a).

4.1.2 Suppressed Fuzzy c-Means Algorithm

By using a fuzzifier q and membership value μ_{ij} , the performance of FCM is better than any HC technique [43], though the convergence speed is much lower. Moreover, if the fuzzifier is large (q>2), it increases the gap between the membership values which may lead to a decrease the overall segmentation performance of FCM [38]. To address these issues, the *rival checked fuzzy c-means* (RCFCM) algorithm [62] was introduced on the basis of competitive learning, by magnifying the largest membership value and suppressing the second largest membership value. The main step in the RCFCM algorithm is to modify μ_{ij} in the FCM algorithm as follows.

Assume the largest membership value of datum x_j for the p[th] cluster is μ_{pj} and its second largest membership value in the s^{th} cluster is μ_{sj}. After modification, the membership value of x_j belonging to each cluster is then:

$$\mu_{pj} = \mu_{pj} + (1-\alpha)\mu_{sj} \tag{13}$$

$$\mu_{sj} = \alpha\mu_{sj} \tag{14}$$

where $1\ 0 \leq \leq \alpha$. The main problem with RCFCM is that it only pays attention to the largest and second largest membership values, so if the choice of α is unsuitable, it can lead to the second largest membership value to be modified being actually less than some others, which causes a disturbance in the original order [46]. For this reason, the convergence of RCFCM is not assured and so to solve this, the *suppressed fuzzy c-means* (SFCM) algorithm was introduced to magnify only the largest membership value and to suppress the rest [46]. If μ_{pj} is the largest membership value for datum $j\ x$, the modified values are:

$$\mu_{pj} = 1-\alpha\sum_{i\neq p}^{c}\mu_{ij} = 1-\alpha+\alpha\mu_{pj} \tag{15}$$

$$\mu_{ij} = \alpha\mu_{ij}\,; \qquad i \neq p \tag{16}$$

where the various parameters are as defined above. Since SFCM prizes the largest and suppresses all other membership values, it does not disturb the original order and so eliminates the drawback of RCFCM. When $\alpha = 0$, SFCM produces the same results as HC, while for $1 = \alpha$ it becomes the FCM algorithm, so this establishes a more natural and realistic relationship between the HC and FCM algorithms, so that for a suitable α value, SFCM can compromise the advantages of faster convergence speed of HC techniques, with the better clustering performance of FCM without impacting on the time complexity which remains the same as FCM, i.e., $O(n)$.

Since SFCM reduces the sensitivity of the fuzzifier q it actually improves the segmentation performance of FCM. A sample experimental result of SFCM is shown in **Figure 4**.

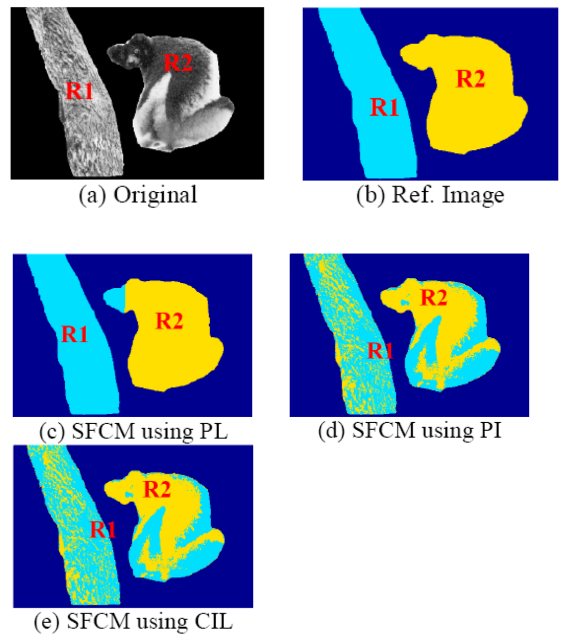


Fig. 4 (a) Original *babacoot* image, (b) Manually segmented reference of (a). (c)-(e) Segmented results of (a).

4.1.3 Possibilistic c-Means Algorithm

FCM uses a probabilistic constraint **(3)** so that the sum of the membership values of a datum across all clusters is 1. The membership values generated by FCM using constraint **(3)** represent the degree of sharing, but not the degree of *typicality* or compatibility with an elastic constraint. *Typicality* here means the actual degree of belongingness of a datum to a cluster rather than an arbitrary division of data [45, 60, 63]. Krishnapuram *et al* addressed these issues by proposing the *possibilistic c-means* (PCM) algorithm whose membership values represent the degree of *typicality* rather than the degree of sharing and as consequence constraint **(3)** is eliminated [45, 64]. Every cluster is independent of the other clusters in PCM and the FCM objective function is modified as follows:

$$J_q(\mu, V, X) = \sum_{i=1}^{c}\sum_{j=1}^{n}(\mu_{ij})^q D_{ij}^2 + \sum_{i=1}^{c}\eta_i\sum_{j=1}^{n}(1-\mu_{ij})^q \tag{17}$$

subject to:-

$$0 \le \mu_{ij} \le 1;\ i \in \{1,\ldots,c\} \text{ and } j \in \{1,\ldots,c\} \tag{18}$$

$$0 < \sum_{j=1}^{n} \mu_{ij} < n; \qquad i \in \{1,\ldots,c\} \tag{19}$$

$$\max \mu_{ij} > 0; \qquad j \in \{1,\ldots,c\} \tag{20}$$

where η_i is the *scale* (resolution) parameter that determines the zone of influence of a particular cluster. The PCM algorithm is applied twice, using the scale *i* η the first time by setting:

$$\eta_i = \frac{\sum_{j=1}^{n} (\mu_{ij})^q D_{ij}^2}{\sum_{j=1}^{n} (\mu_{ij})^q} \tag{21}$$

and the second time:

$$\eta_i = \frac{\sum_{x_j \in (\pi_i)_\alpha} D_{ij}^2}{\left|(\pi_i)_\alpha\right|} \tag{22}$$

where $(\pi i)_\alpha$ is an appropriate α-cut of π_i. By minimizing the objective function $J_q(\mu, V X)$ in **(17)**, the membership value μ_{ij} and cluster centre v_i can be calculated using the following two equations that are iteratively updated:

$$\mu_{ij} = \frac{1}{1 + \left(\frac{D_{ij}}{\eta_i}\right)^{2/(q-1)}} \tag{23}$$

$$v_i = \frac{\sum_{j=1}^{n} (\mu_{ij})^q x_j}{\sum_{j=1}^{n} (\mu_{ij})^q} \tag{24}$$

If the fuzzifier q=1, PCM produces crisp (HC) regions. PCM provides good-segmented results for oisy data, but it is highly dependent on the initialization and the estimation of scale parameter *i*η, for which FCM can be effectively used for both purposes. The computational time required for PCM is O(*n*) [13, 58]. It should be noted that PCM can generate trivial solutions since the solution spaces are not constant over all clusters, moreover it only achieves a local minimum and so is unable to minimize the objective function (17) in a global sense [58, 64-66]. The performance of PCM for noisy data can be improved using the modifications proposed in [67, 68], though again only a local not global minimum can be reached. The improvement in [67] may increase the possible number of local minima which produce a number of *bad* minimizers that are likely to trap PCM iterations into poor classification.

Algorithm 2: *Possibilistic c-means algorithm* (PCM)
Precondition: Objects to be segmented, number of clusters c .
Post condition: The final segmented regions $\Re$.

1. Fix c , and $2 = q$.
2. Initialize μ_{ij}^{0} .
3. Estimate η_i using (21).
4. FOR l = 3 , 2 , 1.........,
5. Update prototypes using **(24)**
6. Compute $\mu_{ij}^{(l+1)}$ using (23)
7. IF $\left(\left\|\mu_{ij}^{(l+1)} - \mu_{ij}^{l}\right\| < \xi\right)$ THEN STOP.

ENDFOR

8. Estimate η_i applying (22).
9. FOR $l = 1,2,3,\ldots,$
10. Update cluster centre v_i using **(24)**.
11. Update $\mu_{ij}^{(l+1)}$ using (23).
12. IF $\left(\left\|\mu_{ij}^{(l+1)} - \mu_{ij}^{l}\right\| < \xi\right)$ THEN STOP.

ENDFOR

13. Return $\Re$.

In summary, PCM gives more emphasis to *typicality*, that means it is able to separate visually distinctive objects well, but conversely produces poorer segmentation performance when objects are not visually different. **Figure 5** highlights the experimental results of PCM separately using PL, PI, and CIL.

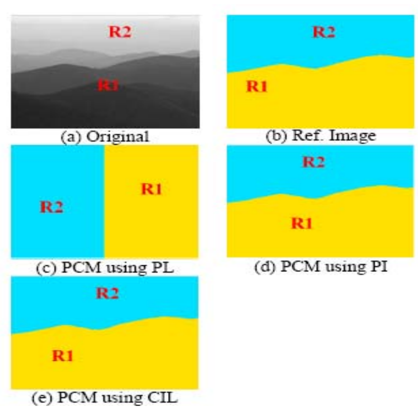


Fig. 5 (a) Original *scene* image, (b) Manually segmented reference of (a). (c)-(e) Segmented results of (a).

4.1.4 Gustafson-Kessel Algorithm

The *Gustafson-Kessel* (GK) algorithm [47] is a powerful clustering technique that has been used in various image processing, classification and system identification applications [40, 57]. It is characterised by adapting automatically the local data distance metric to the shape of the cluster using a covariance matrix and adapting the distance inducing matrix correspondingly [47, 69-71]. The GK algorithm is based on the iterative optimization of the following FCM-type objective function [39, 40]:

$$J_q(\mu, V, X) = \sum_{j=1}^{n}\sum_{i=1}^{c}(\mu_{ij})^{*} D_{ij}^{'2} \tag{25}$$

$$0 \le \mu_{ij} \le 1 \; ; \; i \in \{1,\dots,c\} \text{ and } j \in \{1,\dots,n\} \tag{26}$$

$$\sum_{i=1}^{c}\mu_{ij} = 1 \; ; \qquad j \in \{1,\dots,n\} \tag{27}$$

where D is the data distance norm calculated for clusters of different shapes in one dataset that is given by:

$$D_{ij}^{'2} = (x_j - v_i)^T A_i (x_j - v_i) \tag{28}$$

where i A is the *norm inducing* matrix, which allows the distance to adapt to the local topological structure of the data [69, 70]. Using the Lagrangian multiplier in **(25)**, the membership value μ_{ij} can be calculated as follows:

$$\text{IF } (D_{ij}^{'} = 0)$$

$$\text{THEN } \mu_{ij} = 1 \text{ maintaining } \sum_{i=1}^{c}\mu_{ij} = 1 \tag{29}$$

$$\text{ELSE } \mu_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{D_{ij}^{'}}{D_{kj}^{'}}\right)^{\frac{2}{q-1}}} \tag{30}$$

The cluster centre v_i is updated as:

$$v_i = \frac{\sum_{j=1}^{n}(\mu_{ij})^q x_j}{\sum_{j=1}^{n}(\mu_{ij})^q} \tag{31}$$

To adapt to the structure of the cluster shape, the distance norm inducing matrix A_i is used which increases the distance of the furthest data points while decreasing those data points close to the cluster centre. A_i is defined as:

$$S_{fi} = \frac{\sum_{j=1}^{n}(\mu_{ij})^q (x_j - v_i)^T (x_j - v_i)}{\sum_{j=1}^{n}(\mu_{ij})^q} \tag{32}$$

$$A_i = \left|\rho_i \, det(S_{fi})^{1/P'} (S_{fi})^{-1}\right| \tag{33}$$

where S_{fi} is the *fuzzy covariance* matrix, $'$ P is the dimension of hyper-spherical cluster, and ρ_i is the cluster volume, which is usually set to 1. In the GK algorithm, the parameters values are set to $2 = q$ and $1 = \rho_i$ (Step 1) followed by the initialization of membership values μ_{ij} (Step 2). The cluster centre i v is updated using **(31)** in Step 3.1, while the data distance norm is calculated (Steps 3.2 and 3.3) to iteratively update the membership value μ_{ij} using (29) and (30) (Step 3.4) until either fulfilling the specified threshold ξ or the maximum number of iterations is exceeded (Step 3.5). The detailed steps of the GK algorithm are given in Algorithm 3.

Algorithm 3: *Gustafson-Kessel* (GK) algorithm
Precondition: Objects to be segmented, number of clusters c **,** threshold ξ and ion max_Iterat .
Post condition: The final segmented regions $\Re$.
1. Fix $2 = q$ and set $1 = i$ ρ .
2. Initialize μ_{ij} .
3. FOR 1 = 1, 2, 3, ..., max_Iteration
    3.1 Update cluster centre v_i using **(31)**.
    3.2 Compute cluster covariance matrix using **(32)** and **(33)**.
    3.3 Calculate data distance norm by **(28)** .
    3.4 Update $\mu_{ij}^{(l)}$ using (29) and (30).
    3.5 IF $\left(\left|\mu_{ij}^{(l)} - \mu_{ij}^{(l-1)}\right| \le \xi\right)$ THEN STOP.

The performance of the GK algorithm is not very good for either small datasets or when data within a cluster are (approximately) linearly correlated, because in such cases the covariance matrix becomes singular. Babuska *et al.* (2002) overcame these drawbacks by considering the ratio of the maximum and minimum eigenvalues [70] in calculating the *fuzzy covariance matrix*. In summarising, the GK algorithm adapts the local structure of the cluster shape using a *distance norm inducing matrix* A_i , with the *modified GK* algorithm [70] able to effectively handle both large and small datasets. These characteristics are exploited by using the GK algorithm as key part of the shape-based algorithm [50] for integrating generic shape information into the clustering framework. To clarify the performance of GK, a sample experimental result is provided in **Figure 6**.

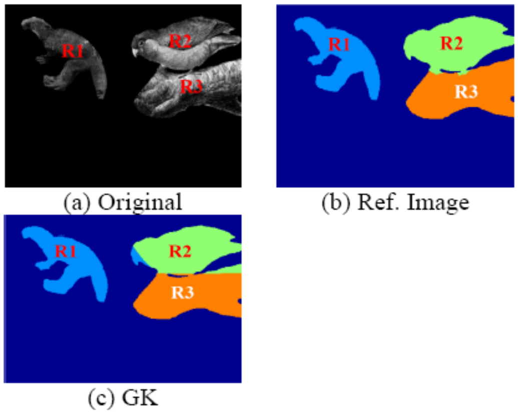


Fig. 6: (a) Original *crocodile* image, (b) Manually segmented reference of (a). (c) Segmented results of (a).

4.1.5 MISR Algorithm

Based on the analysis, the fuzzy clustering algorithms including FCM, SFCM and PCM are highly dependent on the features used. For example, FCM using PI is suitable feature for one type image for segmenting objects while using PL produces better results for other. In some cases, FCM using CIL shows good segmentation performance [61, 72-76]. This raises an open question which feature set produces best segmentation results for which type of image [61]. Addressing this issue, Ameer e*t al* proposed a new algorithm namely *merging initially segmented regions* (MISR) [61] which merges initially segmented similar regions produced by clustering algorithm separately using a pair of feature set from PI, PL, and CIL. The detailed description of the MISR algorithm is given in Algorithm 4 with the full details in below.

It is shown in [61], FCM using either CIL or PI is unable to properly segment the objects having *similar surface variations* (SSV) which requires to apply PL feature for segmentation process. For this reason, the foreground (objects) of an image (*f*) is segmented by FCM using CIL (Step 1) to separate the objects having SSV from those having *dissimilar surface variations* (DSV) (Step 2). To complete the segmentation process, objects with SSV are segmented by SFCM using PL (Step 3) as SFCM outperforms FCM mentioned in Section 4.1.2. For the case of objects having DSV, if there is more than one such object then it requires several processes to complete the segmentation process. In this regard, the feature sets for initial segmentation are selected based on the overlapping regions. To select the best feature set, two cases are considered, namely

$\theta_1^{'} > \pi/4$ and $\theta_1^{'} \le \pi/4$.

(i) When $\theta_1^{'} > \pi/4$, CIL dominates PL in the segmentation process and there is a high pixel misclassification risk when merging, because of the existence of two objects with vastly differing brightness values so PI will outweigh PL and the feature set combination of CIL and PI will generate a lower degree of overlap.

(ii) When $\theta_1^{'} > \pi/4$, in order to decrease misclassification, the feature sets are selected based on the minimum value of the angle between the corresponding decision boundaries as follows:

$$feature\ sets = \begin{cases} CIL, PL & if\ \theta_1^{'}\ is\ minimum \\ PL, PI & if\ \theta_2^{'}\ is\ minimum \\ CIL, PI & if\ \theta_3^{'}\ is\ minimum \end{cases} \quad (34)$$

where $\theta_1^{'}$ = angle between the decision boundaries for FCM using only CIL and PL; $\theta_2^{'}$ = angle for FCM using only PL and PI; $\theta_3^{'}$ = angle for FCM using only CIL and PI.

To apply the merging technique, two cases need to be considered namely if: i) there are more than two objects having DSV(D>2) and ii) two objects have DSV(D=2).

For the former, PI and CIL are used together with the *connectivity_Flag* being set because using the connectivity property will correctly classify those pixels that are misclassified by PI. For the case of *D=2* , in applying the *connectivity_Flag* is set to **FALSE** for $\theta_1^{'} > \pi/4$, (Step 4), otherwise it is set **TRUE**. The reason for this is that if $\theta_1^{'} > \pi/4$, it can be intuitively argued that connectivity should not be applied because while each region consists of having objects of distinct pixel intensities, either one or more pixels of a region may possess a similar intensity to another region that is actually connected to it. In such circumstances to reduce the possibility of pixel misclassification, connectivity is not applied. The complete processing steps for the MISR algorithm are given in **Algorithm 4**.

Algorithm 4: Fuzzy image segmentation considering surface characteristics and feature set selection (FISFS) algorithm **Precondition:** Objects $(\,)f$ to be segmented, number of clusters c, *connectivity_Flag*, θ'_1, θ'_2 and θ'_3,

Post condition: The final segmented regions $\Re$.

1. Segment f by FCM using CIL into regions represented by R^C .

2. Find R_k^M and R^D *for* R^C.

3. IF ($K \geq 1$) THEN FOR i-1,....., k

Segment R_i^M into M regions by SFCM using PL.

END IF

4. IF ($D \geq 2$) THEN

connectivity_Flag=TRUE

IF D = 2 THEN

IF $\theta'_1 > \pi/4$, THEN

connectivity_Flag=FALSE

Segment R^D into D regions

for R^I and R^C .

ELSE

Select feature sets considering overlapping.

Segment R^D into D regions.

END IF

ELSE

Segment D R using I R and C R .

END IF

END IF

The experimental result of the MISR algorithm is given below. In assimilating the overall segmentation performance, out of the 186 test images, MISR produced superior results for 90, with for the remainder of images, FCM, SFCM and PCM provided better results for only 35, 30 and 18 images respectively.

5. Summary

This paper has reviewed various classical fuzzy clustering algorithms, with FCM being chosen as the design platform for the new clustering framework as it is able to incorporate object specific information like pixel location, intensity and shape within its generic structure.

For object-based image segmentation, classical fuzzy clustering algorithms like FCM are unable to segment objects satisfactorily using only low-level features such as pixel location, intensity and their combination. Different objects can be segmented well using different features in FCM, though no single algorithm is suitable for segmenting all objects within a general framework using a particular feature. To address this, Ameer *at el* introduced a new algorithm *merging initially segmented regions* (MISR) that aims to generalise the FCM clustering framework. From the critical analysis of the experimental results for MISR, it has been shown that its segmentation performance for objects having DSV is highly dependent on the initially segmented results. Moreover, in some cases MISR produced poor segmentation performance for objects having SSV due to applying the PL feature, so to address these matters a strategy is mandated that incorporates shape information into the clustering framework for segmentation.

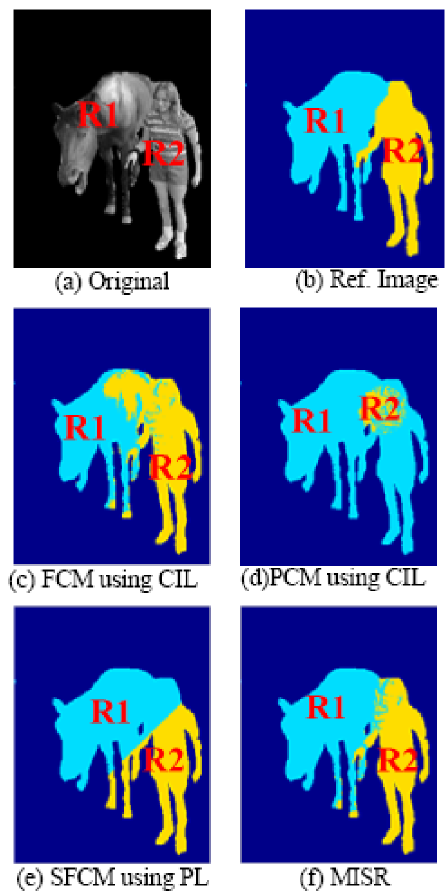


Fig. 7 (a) Original *horse* image, (b) Manually segmented reference. (c)-(f) Various segmented results of (a).

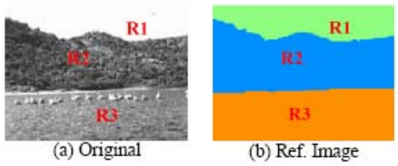


Fig. 8 (a) Original *scene* image, (b) Manually segmented reference of (a). (c)-(f) Segmented results of (a).

Corresponding Author
M. Ameer Ali
Asst. Professor
Dept. of Electronics & Communication Engineering
East West University
Dhaka, Bangladesh

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