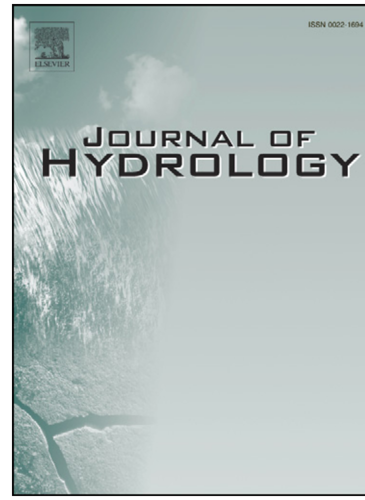
Accepted Manuscript



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PII: S0022-1694(09)00538-1
DOI: 10.1016/j.jhydrol.2009.08.034
Reference: HYDROL 16773

To appear in: *Journal of Hydrology*

Received Date: 17 March 2009
Revised Date: 24 July 2009
Accepted Date: 30 August 2009

Please cite this article as: Youngs, E.G., Rushton, K.R., Steady-state ditch-drainage of two-layered soil regions overlying an inverted v-shaped impermeable bed with examples of the drainage of ballast beneath railway tracks, *Journal of Hydrology* (2009), doi: 10.1016/j.jhydrol.2009.08.034

STEADY-STATE DITCH-DRAINAGE OF TWO-LAYERED SOIL REGIONS OVERLYING AN INVERTED V-SHAPED IMPERMEABLE BED WITH EXAMPLES OF THE DRAINAGE OF BALLAST BENEATH RAILWAY TRACKS

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Abstract

Water-table heights due to steady surface accretion in drained two-layered soil regions overlying an inverted V -shaped impermeable bed are obtained using both the Dupuit-Forchheimer approximate analysis with flow assumed parallel to the bed and also from numerical solutions of Laplace's equation for the head distribution. For illustration, water-table profiles obtained by the two procedures are compared for surface accretion draining to ditches in a typical two-layered ballast foundation for a railway track where a very permeable ballast material overlies a less permeable sub-grade on top of an inverted V-shaped impermeable bed that slopes away both sides from a central line to drainage ditches. These results are found to be in good agreement except very near the drainage ditches where the Laplace numerical solution takes into consideration a surface of seepage that is ignored in the Dupuit-Forchheimer analysis. The Dupuit-Forchheimer analysis is also in good agreement

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with results of a laboratory model experiment. It is concluded that the approximate Dupuit-Forchheimer analysis can be used with confidence in these situations. It is used to investigate the effect on the water-table elevation caused by the reduction of hydraulic conductivity of the porous materials due to clogging.

Keywords: Drainage, Layered soils, Sloping bed, Dupuit-Forchheimer analysis, Laplace numerical solutions, Railway ballast beds

1. Introduction

Water flow due to surface accretion to drains in two-layered porous bodies overlying an undulating impermeable base is a problem occurring both in agricultural lands and engineering structures. In ridge and furrow drained lands a permeable structured surface soil overlies less permeable soil that lies above an impermeable base that rises and falls, with the furrows acting as drainage ditches for rainfall infiltrating through the soils. Similarly, ballast beds, that provide a foundation for railway tracks, often consist of a layer of very permeable material overlying a layer of finer less permeable material laid on top of an impermeable sub-grade whose surface slopes away from a peak midway between drainage channels.

The Dupuit-Forchheimer approximation is conventionally used in investigations of the two-dimensional groundwater problem presented by flow to transverse drains due to steady accretion on the surface of lands overlying a moderately sloping impermeable bed, either assuming horizontal flow (Werner, 1957; Schmid and Luthin, 1964; Yates et al., 1985) or more realistically assuming flow parallel to the sloping bed (Wooding and Chapman, 1966; Childs, 1971; Towner, 1975; Lesaffre, 1987; Chapman, 1980). Towner (1975) showed that the Hele-Shaw viscous flow analogue results of Guitjens and Luthin (1965) agreed with

Dupuit-Forchheimer calculations for even large slopes when the flow was assumed parallel to the bed although the agreement was poor when the flow was assumed horizontal.

These studies all considered transverse drains on a continually rising sloping bed. For steady-state drainage of lands overlying an impermeable bed that rises to a peak midway between uniformly spaced parallel drainage ditches, the water-table height midway between drains is a maximum. The Dupuit-Forchheimer analysis is then simpler than that for the problem of interception of rainfall over sloping lands by parallel ditch drains along the contours where the location of the maximum water-table height is part of the solution. As shown by Towner (1975) for drains along contours and by Youngs and Rushton (2009) for the present drainage situation for a uniform soil, there is little difference in the results assuming horizontal flow and those assuming flow parallel to the sloping bed when the slope is less than 10%. However, the difference becomes significant for larger slopes when the Dupuit-Forchheimer analysis assuming horizontal flow gives poor agreement with numerical computations of the water-table profile solutions based on Laplace’s equation describing the head, while the analysis assuming flow parallel to the slope gives good agreement.

In all these studies the soil over the sloping bed was assumed to be uniform. For soils whose hydraulic conductivity varies with height, Guirinsky’s (1946) extension of the Dupuit-Forchheimer analysis can be used for soils overlying a horizontal impermeable base, while Youngs’ (1965, 1966) seepage analysis, founded on Charny’s (1951) work on flow through earth dams, gives an exact formulation of the problem of groundwater flow in layered soils, leading to estimates of bounds for the water-table profiles. This exact analysis was extended to groundwater flow in layered sloping lands (Youngs, 1971), but does not provide solutions for the water-table profiles.

In this paper the drainage of a two-layered soil region overlying an inverted V-shaped impermeable bed has been addressed analytically by assuming Dupuit-Forchheimer conditions with flow parallel to the bed and with the water table drawn down to the water level in ditch drains. We assume the impermeable bed slopes away at a uniform angle from a peak midway between the ditches. We compare and discuss our results with those obtained from numerical solutions of Laplace's equation for the head assuming boundary conditions that include the existence of a seepage face at the drainage outfall. Comparisons are also made between the Dupuit-Forchheimer calculations of the water-table profile and the laboratory model results for steady surface accretion on drained railway ballast foundations published by the Department of Civil and Environmental Engineering, University of Massachusetts, Amherst, USA (Heyns, 2000). We also demonstrate the use of the Dupuit-Forchheimer analysis by examining the effect on the water-table elevation due to clogging of the ballast.

2. The physical problem

We consider the two-dimensional flow region through a cross section of the two-layered soil that is sketched in Fig.1. The soil overlies an impermeable bed that slopes downwards at an angle to the horizontal from a central plane at $x = 0$. The soil consists of a lower layer of depth t with hydraulic conductivity K_0 overlain by a more permeable layer of conductivity K_1. There is uniform steady accretion q over the surface which drains at $x = \pm D$ to a head H_D. The accretion maintains the water table at a height $H(x)$ above the floor at position x over the area. Above the outfall water level H_D at $x = D$ a seepage surface exists to a height h_f.

The location of the water table in the two layers varies depending on the slope of the bed, the accretion rate, the ratio of the hydraulic conductivities and the outfall head. When the outfall head is below the boundary of the two soil regions, for small values of q, $q < q_1$, the water table is wholly contained in the lower soil layer of conductivity K_0. This situation is shown in Fig. 2a for a sufficiently small value of $q < q_1$ (Case 1) that does not allow the water table to rise above the impermeable sloping base at the centre $x = 0$. If $q_1 < q < q_1$ (Case 2) the water table is above the base at $x = 0$ and the situation becomes that shown in

Fig. 2b. For a range of values of q, $q_1 < q < q_2$ (Case 3), the water table is in the more permeable soil region with conductivity K_1 over a section $x_1 < x < x_2$ of the region but is in the lower soil region near the centre and near the drainage ditch (Fig.2c). Again, the water table might drop to the impermeable base at $x = 0$ when the water table configuration in the lower layer in the central region becomes similar to that of Fig. 2a. For large values of q, $q > q_2$ (Case 4a), the water table can be in the lower layer in a region near the outfall but in the upper layer over the rest of the region near the centre. This is shown in Fig. 2d. However, the height of the seepage surface h_f can be above the boundary between the two layers (Case 4b). In this case the water table lies wholly in the upper soil layer as shown in Fig.2e. When the outfall ditch level is above the boundary between the two layers (Case 5), for smaller values of q and larger slopes, the water table could drop into the lower ballast layer as shown in Fig. 2f, but for large values of q and smaller slopes the water table is wholly in the upper layer. Conditions giving rise to these situations can occur with ridge and furrow lands and also with railway ballast foundations with a very permeable ballast overlying a less permeable sub-ballast. When significant rainfall occurs, the water table rises progressively through the lower layer into the very permeable layer above as shown in Cases 1, 2 ,3 and possibly 4(a) and/or 4(b); when the rainfall stops the water table falls progressively through the situations described by these conditions. Case 5 occurs when there is drainage surcharge and the water head builds up in the drainage channel.

The flow in each soil layer can be obtained by solving Laplace's equation $\nabla^2 h = 0$ for the hydraulic head h at (x,z) in the groundwater region. The boundary conditions of the problem are shown in Fig.3a. These are that there is no flow through the base of the lower layer and through the plane of symmetry at $x = 0$ and there is continuity of flow and hydraulic head between layers with the vertical flux, assumed equal to the accretion rate q, through the water table where $h = z = H$. Thus we assume that flow is vertical in the unsaturated soil above the water table where it is refracted on entry. It was argued by Childs (1945) that this was a reasonable assumption for uniform soils in considering water-table heights in drained lands, but Kacimov (2003) has demonstrated that with soils overlying a V-shaped impermeable bed, the flow diverges from the vertical in the unsaturated region, leading to non-uniform flux through the water table. However, for small angles of slope such divergence may be assumed to be small. At $x = D$ water drains out of the soil into the ditch where there is a surface of seepage, so that $h = H_D$, $0 < z < H_D$, and $h = z$, $H_D < z < h_f$, where H_D is the ditch-water level and h_f is the height of the top of the seepage face. Analytical solutions of Laplace's equation with these boundary conditions have not been possible so that numerical methods of solution are needed.

3. Dupuit-Forchheimer solutions

The boundary conditions to be applied with the approximate Dupuit-Forchheimer analysis to obtain the water–table profiles in the two-layered drainage situation shown in Fig.1 when flow is assumed parallel to the impermeable base, are shown in Fig.3b. The water-table height is a maximum at the centre of the soil region at $x = 0$. With a uniform accretion rate q, assumed to be the vertical flux through the water table, the flow per unit width down the slope is qx, discharging qD into the ditch. However, as discussed by Youngs and Rushton (2009) the assumption of flow parallel to the slope requires the ditch face to be normal to the sloping bed and the inclusion of fictitious flow regions upslope from the central plane and another overhanging the ditch as shown in Fig.3b. When is small, as in the examples given later in this paper, the overhang becomes unimportant, increasing the total inflow by less than 0.5 % for a 5% slope.

3.1: Water table in upper layer

When the water table is in the upper soil layer, the flow per unit width assumed parallel to the sloping bed is

$$qx = -\{K_0 t + K_1[H - t - (D - x)\tan(\alpha)]\}\cos(\alpha)\frac{\mathrm{d}H}{\mathrm{d}s} \qquad (1)$$

where s is the coordinate measured down-slope with $s = 0$ corresponding to the water-table height at $x = 0$ and $s = s_D$ (a function of the slope of the bed, accretion rate and hydraulic conductivities of the layers) at $x = D$ (see Fig.3b), so that

$$s_D - s = \frac{D - x}{\cos(\alpha)} + [H - (D - x)\tan(\alpha)]\sin(\alpha)$$

giving

$$\frac{\mathrm{d}s}{\mathrm{d}x} = \cos(\alpha) - \frac{\mathrm{d}H}{\mathrm{d}x}\sin(\alpha) \qquad (2)$$

as deduced by Childs (1971) and Youngs and Rushton (2009).

Thus, in terms of the horizontal coordinate x

$$qx = -\{K_0 t + K_1[H - t - (D - x)\tan(\alpha)] - qx\tan(\alpha)\}\frac{\mathrm{d}H}{\mathrm{d}x} \qquad (3)$$

An analytical solution of eq,(3) is obtained using the substitution

$$w = [H - D\tan(\alpha) - (1 - K_0/K_1)t + (1 - q/K_1)x\tan(\alpha)]/x \qquad (4)$$

Equation (3) then becomes

$$\frac{\mathrm{d}x}{\mathrm{d}w} = \frac{-\ xw}{w^2 - (1 - q/K_1)\tan(\alpha)w + q/K_1} \tag{5}$$

so that after integration, *x* as a function of *w* is given by

$$x(w) = x(w_i)\exp\left[-\int_{w_i}^{w} \frac{w\,\mathrm{d}w}{w^2 - (1 - q/K_1)\tan(\alpha)w + q/K_1}\right] \tag{6}$$

where the lower integration limit of *w* is w_i at $x = x(w_\mathrm{i})$. Noting that

$$\int \frac{v\mathrm{d}v}{\mathrm{a}v^2 + bv + c} = \frac{1}{2}\ln(av^2 + bv + c) - \frac{b}{a\sqrt{4ac - b^2}}\tan^{-1}\left(\frac{2av + b}{\sqrt{4ac - b^2}}\right)$$
$$= \mathrm{f}(v) \text{ say} \tag{7}$$

we can write the solution of eq.(6) in the form

$$x(w) = x(w_i)\exp[-\ \{\mathrm{f}(w) -\ \mathrm{f}(w_i)\}] \tag{8}$$

with a =1, $b = -\ (1 -\ q/K_1)\tan(\ )$ and $c = q/K_1$ in f(v) of eq.(7). In eq.(8) w_i is obtained from eq.(4) for the given value of H at $x(w_\mathrm{i})$. With eq.(8) giving the coordinate x as a function of w, the water-table height H at a given x is obtained from eq.(4) so that

$$H = wx(w) + D\tan(\alpha) + (1 -\ K_0/K_1)t -\ (1 -\ q/K_1)x(w)\tan(\alpha) \tag{9}$$

3.2: Water table in lower layer

When the water table is located in the lower layer, the flow q is given by (Youngs and Rushton, 2009)

$$qx = -\{K_0[H-(D-x)\tan(\alpha)] - qx\tan(\alpha)\}\frac{\mathrm{d}H}{\mathrm{d}x} \qquad (10)$$

With u defined by

$$u = [H - D\tan(\alpha) + (1 - q/K_0)x\tan(\alpha)]/x \qquad (11)$$

$$\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{-xu}{u^2 - (1-q/K_0)\tan(\alpha)u + q/K_0} \qquad (12)$$

so that

$$\begin{aligned} x(u) &= x(u_i)\exp\left[-\int_{u_i}^{u}\frac{u\,\mathrm{d}u}{u^2-(1-q/K_0)\tan(\alpha)u+q/K_0}\right] \\ &= x(u_i)\exp[-\{\mathrm{f}(u)-\mathrm{f}(u_i)\}] \end{aligned} \qquad (13)$$

with a =1, b = - (1 - q/K_0)tan( ) and c = q/K_0 in f(v) of eq.(7). The lower limit of integration u_i in eq.(13) is obtained from eq.(11) for the known value of H at $x(u_i)$. With eq.(13) giving x as a function of u, the water-table height H at a given x is then found from eq.(11) as

$$H = ux(u) + D\tan(\alpha) - (1 - q/K_0)x(u)\tan(\alpha) \qquad (14)$$

The Dupuit-Forchheimer analysis of the drainage problem assumes the water-table height is drawn down to the ditch-water level. Thus at the ditch from eqs.(11) and (4)

$$u = \frac{H_D - D\tan(\alpha)}{D + H_D \tan(\alpha)} + (1 - \frac{q}{K_0})\tan(\alpha), \quad x = D + H_D \tan(\alpha), \quad H_D < t \qquad (15)$$

$$w = \frac{H_D - t(1 - K_0/K_1) - D\tan(\alpha)}{D + H_D \tan(\alpha)} + (1 - \frac{q}{K_1})\tan(\alpha), \quad x = D + H_D \tan(\alpha), \quad H_D > t \qquad (16)$$

At the positions x_1 and x_2 where the water table crosses over from one layer to the other, $u = t/x - q/K_0 \tan( )$ and $w = K_0 t/K_1 x - q/K_1 \tan( )$. The water-table profile is obtained by finding x as a function of u from eq.(13) when the water table is in the lower layer or as a function of w from eq.(8) when it is in the upper layer; H at a given x is obtained from eq.(14) or eq.(9), using the appropriate value of u_i or w_i in eq(13) or (8). Values of u and w where the water table crosses the interface between layers are found by trial and error, hence determining x_1 and x_2. The calculation of the water-table profile in a practical example is given in Table 1.

3.3: Application to individual cases

Case 1: Water table wholly in lower layer, $H_D < t$ at $x = D$, $H = D \tan( )$ *at* $x = 0$. Fig.2a sketches this situation. The water table lies wholly in the lower layer with the accretion rate $q < q_1'$ insufficient to raise the water table to meet the boundary between the two layers at any distance from the outfall. The situation is thus that discussed by Youngs and Rushton (2009). Also the accretion rate in this case is insufficient to raise the water table above the impermeable base at the centre. $x(u)$ is calculated from eq.(13) with u_i given by eq.(15) , , and the water-table height H found from eq.(14). In this case the parameter u is finite at $x = 0$ and is found by trial and error. The limiting value of q/K_0 below which the water table meets the impermeable bed at $x = 0$ is $(\tan\ )^2/4$ (Youngs and Rushton, 2009). It is to be noted that the water table meeting the impermeable bed at $x = 0$ results from the assumption of the uniform surface accretion travelling to meet the water table without diverging from the vertical. If the divergence (as would be the case at large slope angles) were taken into consideration, then the water table would meet the impermeable bed at some distance down slope.

Case 2: Water table wholly in lower layer, $H_D < t$ *at* $x = D$, $H > D \tan(\ )$ *at* $x = 0$. Fig.2b illustrates this case when the accretion rate $q_1' < q < q_1$ is sufficient to raise the water table above the impermeable floor at the centre but insufficient for the water table to penetrate into the upper layer. Again this corresponds to the situation considered in Youngs and Rushton (2009). The calculations proceed in the same way as for Case 1, but in this case $q/K_0 >$ $(\tan\ )^2/4$ at x = 0 and $u \rightarrow\ $ as $x \rightarrow 0$.

Case 3: Water table in lower layer $x < x_1$ *and* $x > x_2$, *in upper layer* $x_1 < x < x_2$, $H_D < t$ *at* $x = D$. This is the situation sketched in Fig.2c. The accretion rate $q_1 < q < q_2$ is such that the water table is in the lower layer in the vicinity of the central area but crosses into the upper layer at $x = x_1$ before descending into the lower layer at $x = x_2$ to drain at $x = D$ to an outfall at height $H_D < t$. The range of values of accretion rate $q_1 < q < q_2$ when this occurs is determined by the hydraulic conductivities of the two layers, the thickness t of the lower layer and the slope of the bed. In the region $x_2 < x < D$, H is obtained from eq.(14) as for Case 1 with u_i given by eq.(15) and $x(u)$ calculated from eq.(13) with the calculation proceeding until $u = t/x_2 - q/K_0 \tan(\ )$. For $x_2 > x > x_1$ the water table height H is given by eq.(9) with $w_i =$ $K_0t/K_1x_2 - q/K_1 \tan(\ )$ and $x(w)$ given by eq.(8), the calculation proceeding until $w = K_0t/K_1x_1$ $- q/K_1 \tan(\ )$. Between $x_1 > x > 0$ the water table is obtained from eqs.(13) and (14) with $u_i =$

t/x_1 - q/K_0 tan( ). q_1, the lower value of q, is when $H – D$ tan( ) → t as x_2 → 0. q_2, the upper value, is when $H – D$ tan( ) → t as x_1 → 0.

Case 4: Water table in lower layer $x > x_2$, in upper layer $x < x_2$, $H_D < t$ at $x = D$. This situation, sketched in Fig.2d for the case of a seepage surface with h_f < t and in Fig.2e for a seepage surface with h_f > t, occurs when the accretion rate is increased beyond q_2. The procedure for the Dupuit-Forchheimer calculation of the water table is the same as for Case 3 except only the first two calculations are performed. Since the seepage surface at the outfall is neglected in the Dupuit-Forchheimer calculations and the water table in the soil is assumed to be drawn down to the ditch water level, Case 4b shown in Fig.2e would give the same result as for Case 4a shown in Fig.2d with the water table wrongly calculated to cross into the lower layer.

Case 5: Outfall above layer boundary. With the water level at the outfall above the boundary between the two layers, the water table can penetrate into the lower layer for small accretion rates and large slopes, as illustrated in Fig.2f. The calculation of the water-table profile then follows the last two parts of the procedure given for Case 3 with w_i given by eq.(16). For large accretion rates and small slopes the water table is located always in the upper layer when the outfall is above the boundary between layers and is calculated from eq.(9).

4. **Numerical solutions of Laplace's equation**

The reliability of the application of the Dupuit-Forchheimer analysis can be checked by obtaining numerical solutions of Laplace's equation with the physical boundary conditions given in Fig.3a. Both the water-table profile and the height of the seepage surface are unknown and emerge as part of the solution in the numerical investigation.

Numerical solutions for specific problems can be obtained using the finite difference approximation method (Rushton and Redshaw, 1979). Due to the unknown location of the water table and hence the top of the seepage face, an iterative technique is required with a series of trial solutions for the water-table location. When the water table is entirely in the lower layer, Cases 1 and 2 (Figs.2a and 2b), or entirely in the upper layer, Case 4b (Fig.2e), a systematic series of trial solutions leads to the water-table elevations and the height of the top of the seepage face, h_f. However, when the water table crosses the interface between the two regions, many trials are required before a satisfactory approximation is obtained for the crossover points and the water-table elevation. For most problems, a regular rectangular finite difference mesh is used. On the other hand, for Case 4(a) the steep fall in the water table from the interface towards the downstream boundary, requires closer vertical grid lines towards the downstream face. Due to the sudden change in the hydraulic gradients at the interface between the two layers, solutions were obtained for a ratio of the hydraulic conductivities in the upper and lower layers of $K_1/K_0 = 10$. For larger values of K_1/K_0, it is difficult to obtain reliable finite difference solutions. For all the numerical solutions, results are presented for the water table profile, the equipotentials and the height of the seepage face h_f.

5. **Dupuit-Forchheimer calculations and Laplace numerical results**

The Dupuit-Forchheimer calculations of the water-table profiles for different cases that can arise for drainage of a two-layered permeable region overlying an inverted V-shaped impermeable bed are compared with Laplace numerical solutions for the particular example presented by the drainage of ballast foundations beneath railway tracks. The results are given in Figs 4-9. In these we considered a typical railway ballast geometry with a half-width D = 2.25 m of depth t = 0.125 m; the slope of the base above the horizontal is $= \tan^{-1} 0.05$. The upper layer is assumed to have a hydraulic conductivity $K_1 = 10\ K_0$. (Note that in these figures the vertical coordinate is five times that of the horizontal.) In most cases, there is good agreement between water-table elevations deduced from the two approaches except in the vicinity of the drainage ditch where the Dupuit-Forchheimer analysis ignores the existence of a seepage face which is included in the Laplace solution. For Case 4(b) shown in Fig.8 the downstream water level is at the base of the aquifer so that the water table, according to the Dupuit-Forchheimer approximation, falls to this level. However, in the Laplace solution the seepage face is found to be above the interface. Consequently there is a significant difference between the water-table elevations for large x near the drainage ditch. Due to the discrete mesh used in the numerical solution of the Laplace equation, the accuracy of the water-table elevation is about 0.005 m. When the outfall is above the boundary of the two layers, the seepage surface in the more permeable layer included in the Laplace solution is a small distance above the interface; when the outfall level is below the interface it is a prominent feature. The equipotentials obtained are very nearly normal to the impermeable base except near the outfall ditch and midway between ditches.

The area over which the upper layer contains the water table when the accretion rate becomes large depends on the slope of the bed, the hydraulic conductivities of the two layers, the elevation of the downstream water level and the accretion rate. Fig.10 plots the values of x_1 and x_2, the distances at which the water table crosses the boundary between layers (see Fig. 1), against the ratio of the accretion rate to the hydraulic conductivity of the lower layer when the ditch-water level is zero for the examples given in Figs 4 to 8. It is seen that x_1, the cross-over distance nearest to the watershed, occurs over a limited range 0.0069 q/K_0 0.016. The horizontal line on Fig. 10 refers to $q/K_0 = 0.01$ for which $x_1 = 0.317$ m and $x_2 = 1.75$ m as in the example shown in Fig. 6. For Cases 1 and 2, $q/K_0 < 0.0069$; consequently the water table is always below the interface. For Case 4(a) shown in Fig. 7 with $q/K_0 = 0.02$, $x_2 = 2.05$ m, but there is no value for x_1 because the water table does not fall below the interface towards the watershed.

6. Dupuit-Forchheimer results and laboratory experiments

Heyns (2000) reported an extensive study of the drainage of railway ballast using a tilting tank containing a sub-ballast overlain by a ballast; water was sprayed from nozzles to simulate rainfall. His main interest was in non-steady state conditions, especially the recession of the water table after the rainfall ceased. However, the simulated rainfall continued for a sufficient time for a steady-state to be reached.

In Fig.11 we compare one of his steady-state results with our Dupuit-Forchheimer calculations; water-table heights deduced from five piezometers are shown in the figure by the symbol +. Due to the experimental technique, in which the piezometers are connected to the base of the tank, the accuracy of the estimates of water table elevation is unlikely to be better than 0.005 m. A two-layered railway ballast bed was modelled with a lower less permeable layer of porous material having a hydraulic conductivity equal to 65 md^{-1} (K_0) to a depth t = 0.14 m overlain by a very permeable material of conductivity 3250 md^{-1} (K_1). In the experiments water was sprayed on to the surface for three hours at a rate 2.7 md^{-1}. Outflows occurred from both the upper and lower layers at a distance 1.88 m from the mid-plane; this is assumed to be the value of D used in the calculations. Two Dupuit-Forchheimer calculations are included on the figure. The full line corresponds to H_D = 0.0 with the water table in the lower layer in the region midway between drains, crossing into the upper layer before entering again the lower layer (Case 3 of Section 3); for the broken line H_D = 0.14 m where the water table does not enter again the lower layer (Case 5), chosen to represent conditions actually observed in the experiment of outflow both from the upper and lower layers at this downstream boundary. It is seen that good agreement is obtained between the experimental and Dupuit-Forchheimer estimates.

Table 1 contains some results of calculations that give the Dupuit-Forchheimer plots in Fig.11, following the procedure given for Case 3 in Section 3. These calculations were performed using computer algebra with the mathematical package Mathcad (Mathsoft Inc., 201 Broadway, Cambridge, Mass., U.S.A.)

Parsons (1990) discusses the reduction of hydraulic conductivity of the ballast [upper layer] due to clogging of pores. From experiments using a falling head permeameter, Parsons found that the hydraulic conductivity for moderately clean ballast is typically one tenth of the value for clean ballast; for moderately fouled ballast the hydraulic conductivity is about one-twenty fifth of the value for clean ballast. In Fig.12 we also show the calculated water-table profiles when K_0 and K_1 are reduced to 0.1 and 0.04 of their original values to represent clogging. With the hydraulic conductivities at 0.1 of the original values, the maximum water- table elevation is 0.24 m above the base (0.1 m above the interface). For moderately fouled ballast, with hydraulic conductivities set at 0.04 of the original values, the maximum water-table elevation is 0.34 m above the base (0.2 m above the interface). This means that the water table approaches the bottom of the track sub-structure a condition that needs to be avoided. For hydraulic conductivities 1% of their original values, the maximum water table elevation is 0.61 m above the impermeable base.

7. Discussion and conclusion

This paper has particular relevance to the problem of drainage of the ballast beneath railway tracks that, if not attended to, risks the water table reaching the level of the sleepers when severe operational speed restrictions have to be imposed due to reduced strength of the track foundations. The problem considered here also occurs in agricultural ridge and furrow drainage where the water table is controlled to provide suitable conditions for root development and livestock grazing. In these situations there is an underlying impermeable undulating base, overlain by a permeable surface layer on top of a less permeable layer. We have used the Dupuit-Forchheimer analysis to consider the steady-state drainage of two-layered soil regions overlying an inverted V-shaped impermeable bed that approximates the situation. Our results have been compared with numerical solutions of Laplace's equation for the head distribution and also the results of laboratory model experiments of the drainage of a railway ballast foundation.

These steady-state results provide a theoretical background for more general time-variant studies of the problem (Rushton and Ghataora, 2009). Cases 1 to 4 discussed here can occur with the drainage of the railway ballast where a very permeable ballast overlies sub-ballast with a hydraulic conductivity at least an order of magnitude less. Case 5 is relevant to the situation when there is surcharge in the drain and a low accretion rate.

The upper layer clearly plays a major role in preventing the water table rising to the surface when the accretion rate is large. Our results show that the high conductivity of the surface layer insures the water table to follow close to the boundary between the layers when the accretion rate is sufficient for the water table to rise above the less permeable layer.

The agreement obtained between the Dupuit-Forchheimer results and the Laplace numerical calculations of the water-table profile in drained two-layered soils overlying a sloping bed gives confidence in using the approximate analysis in these situations. This is important since two-dimensional numerical computations involving a water table crossing between layers are time consuming and particularly difficult when there is a large difference in conductivity in the two layers, while Dupuit-Forchheimer calculations are easily performed with computer algebra. Further confirmation of the efficacy of the application of the Dupuit-Forchheimer is given with the agreement between the calculations of the water-table profile and the experimental results of Heyns (2000) laboratory experiment of a two-layered railway track ballast foundation. Application of the Dupuit-Forchheimer analysis shows the effect on the water-table heights in such situations due to the fouling of the ballast and sub-ballast with the consequent reduction in hydraulic conductivities.

Notation

D = drainage ditch half-spacing (L);

H = water-table elevation (L);

H_0 = water-table elevation at watershed (L);

H_D = ditch-water level (L);

h = hydraulic head (potential) (L);

K_0 = hydraulic conductivity of lower layer (LT^{-1});

K_1 = hydraulic conductivity of upper layer (LT^{-1});

q = accretion rate (LT^{-1});

q_1 = limiting accretion rate for the water table to be wholly in the lower layer (LT^{-1});

q_1' = limiting accretion rate for the water table to meet the impermeable bed at the watershed and be wholly contained in the lower layer (LT^{-1});

q_2 = limiting accretion rate for the water table to be in the upper layer over a section of the region but in the lower layer near the watershed (LT^{-1});

s = coordinate along sloping bed (L)

s_D = value of s at $x = D$ (L);

t = thickness of lower layer (L);

u,w = parameters used in calculating the water–table profile when the water table is located below and above the boundary between layers, respectively;

u_i,w_i = lower integration limits of u,w;

x = horizontal coordinate (L);

x_1,x_2 = coordinates where the water table crosses the boundary between the layers (L);

z = vertical coordinate (L);

= slope angle.

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Legend to Figures

Fig. 1. Drainage to ditches in two-layered soil regions overlying an inverted V-shaped impermeable base.

Fig. 2. The water-table profiles in a drained two-layered soil overlying an inverted V-shaped impermeable base for the different cases discussed in the text.

Fig.3. (a) The boundary conditions of the two-dimensional physical problem; (b) the flow conditions assumed in the Dupuit-Forchheimer analysis.

Fig. 4. Calculated water-table profiles for Case 1 sketched in Fig.2a.

Fig. 5. Calculated water-table profiles for Case 2 sketched in Fig.2b.

Fig. 6. Calculated water-table profiles for Case 3 sketched in Fig.2c.

Fig. 7. Calculated water-table profiles for Case 4a sketched in Fig.2d.

Fig. 8. Calculated water-table profiles for Case 4b sketched in Fig.2e.

Fig. 9. Calculated water-table profiles for Case 5 sketched in Fig.2f.

Fig. 10. Locations x_1 and x_2 where the water table crosses the interface as a function of the accretion rate q.

Fig. 11. Steady-state water-table profiles calculated by the Dupuit-Forchheimer analysis compared with Heyns' (2000) laboratory experiment: (K_1 = 3250 md^{-1}; K_0 = 65 md^{-1}; q = 2.7 md^{-1}).

Fig. 12. Effect of clogging of ballast on water-table elevations.

Table

Table 1. Calculation of the steady-state water-table profile shown in Fig.11 for zero ditch-water level and for a ditch-water level at the top of the sub-ballast.

Figure1

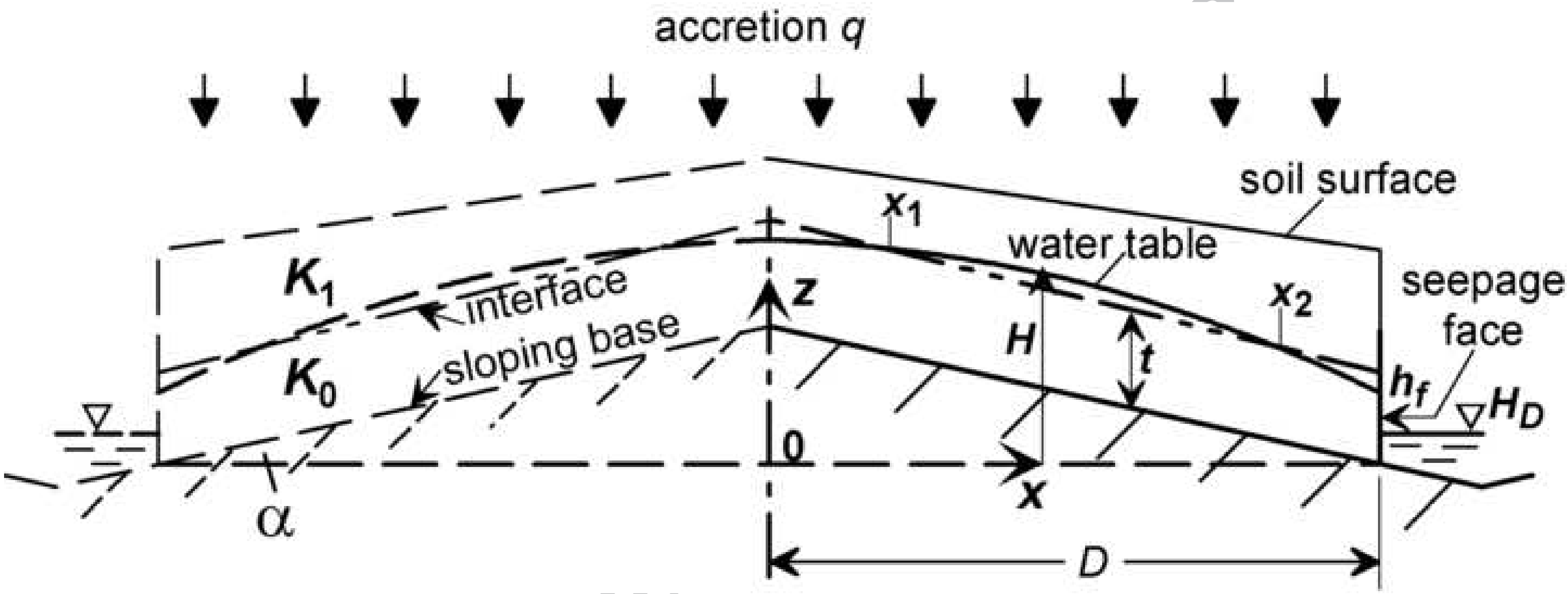

accretion q
soil surface
x_1
water table
K_1
interface
K_0
sloping base
z
H
t
x_2
seepage
face
h_f
H_D
0
x
α
D

Figure2

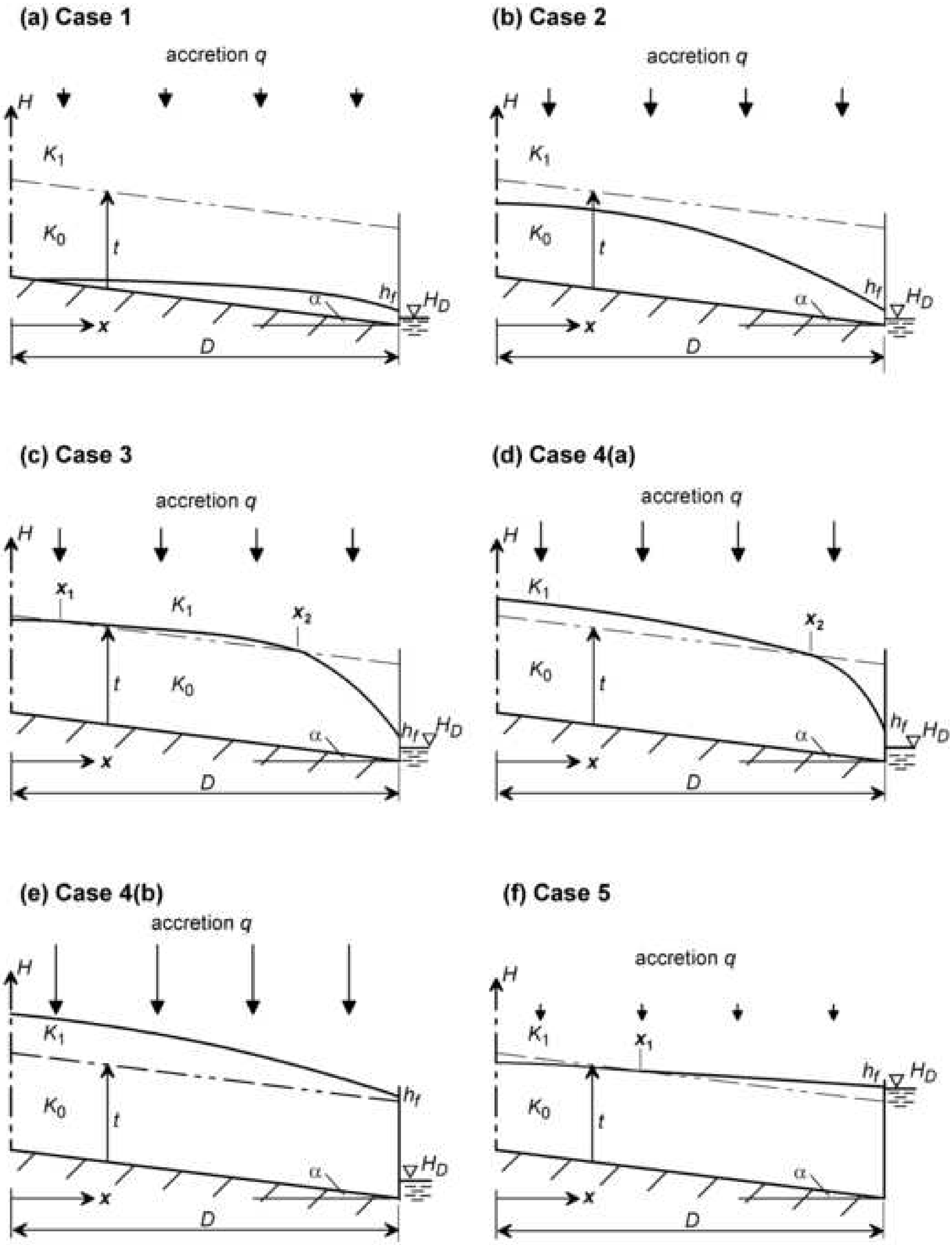
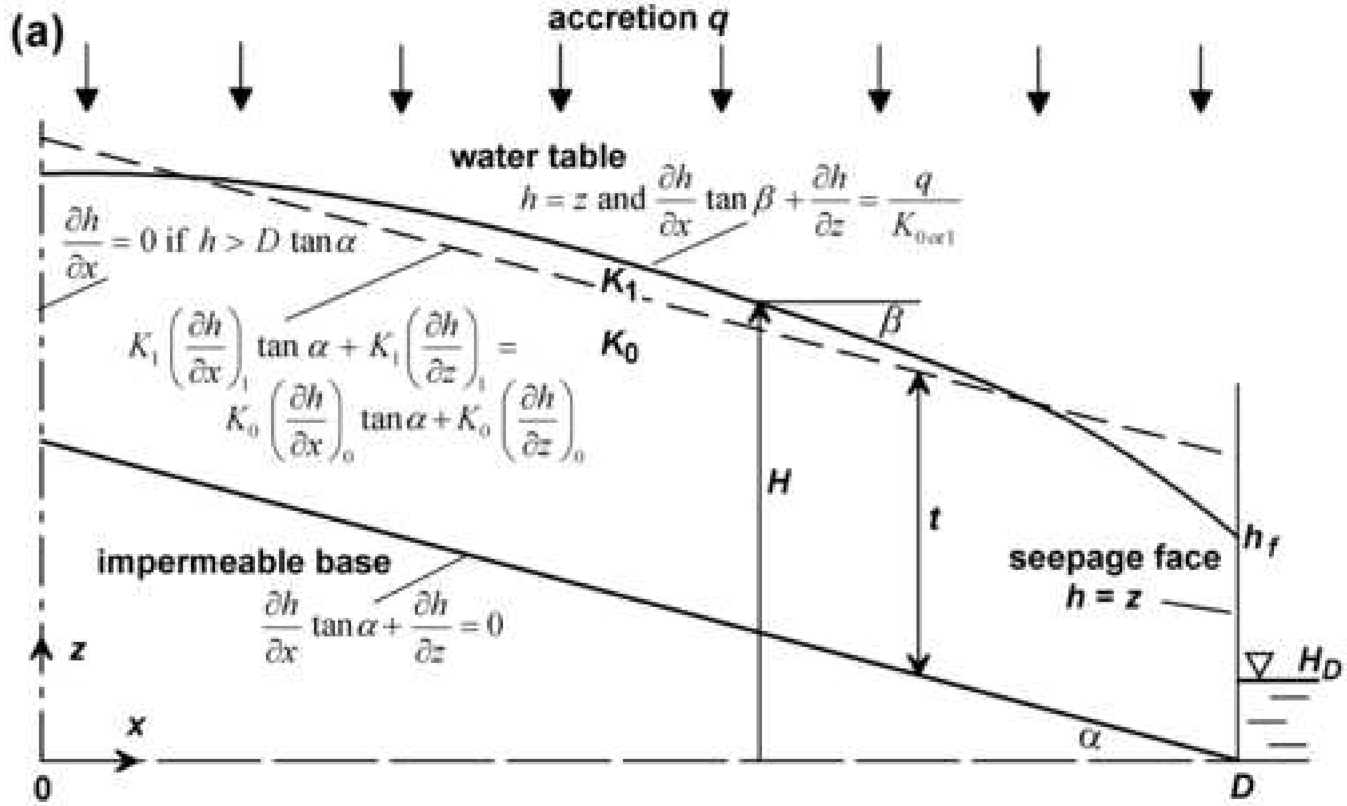
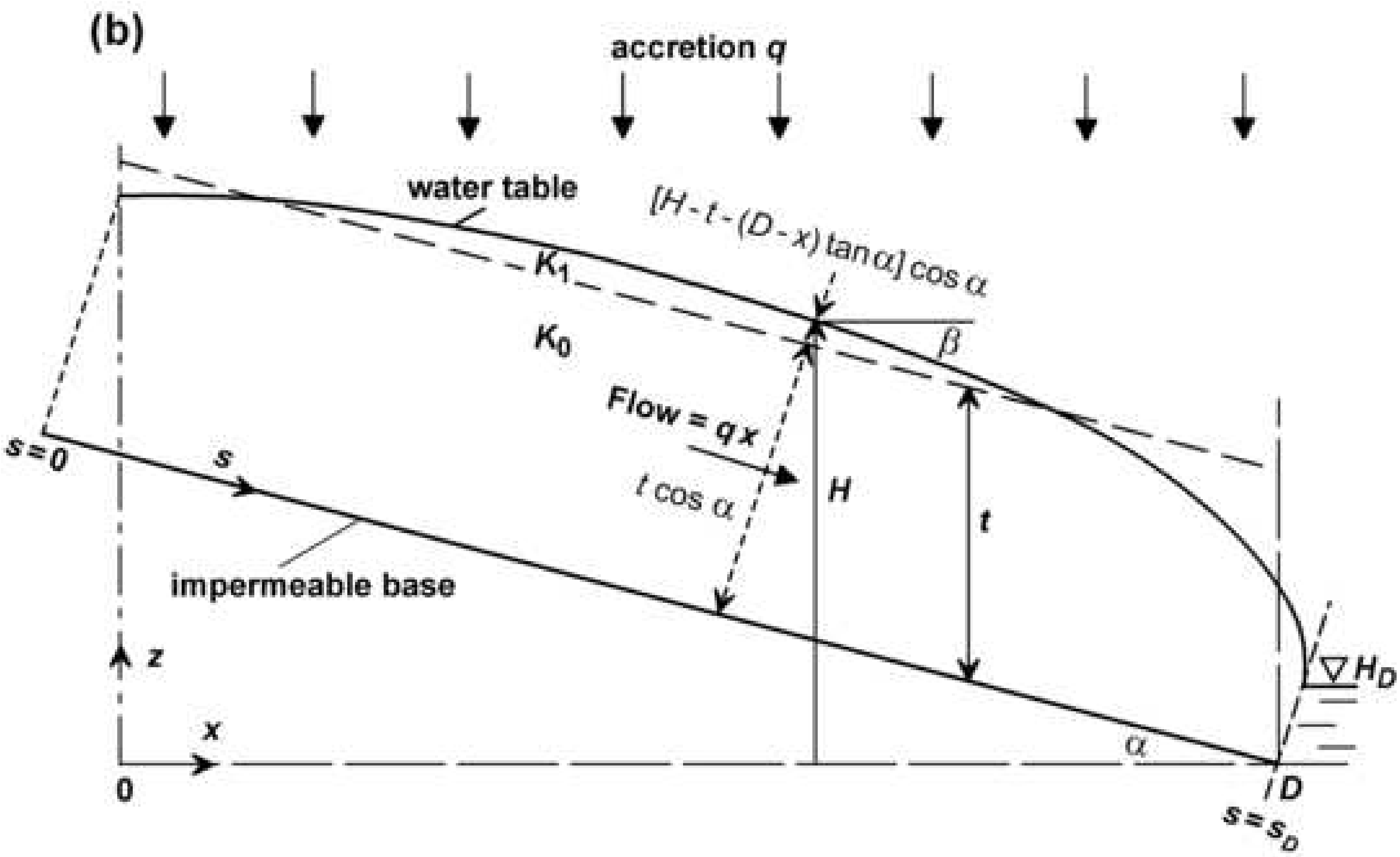

(a) Case 1
accretion q
H
K1
K0
t
hf
HD
α
x
D
(b) Case 2
accretion q
H
K1
K0
t
hf
HD
α
x
D
(c) Case 3
accretion q
H
x1
K1
x2
t
K0
hf
HD
α
x
D
(d) Case 4(a)
accretion q
H
K1
x2
K0
t
hf
HD
α
x
D
(e) Case 4(b)
accretion q
H
K1
K0
t
hf
HD
α
x
D
(f) Case 5
accretion q
H
K1
x1
hf
HD
K0
t
α
x
D

Figure3

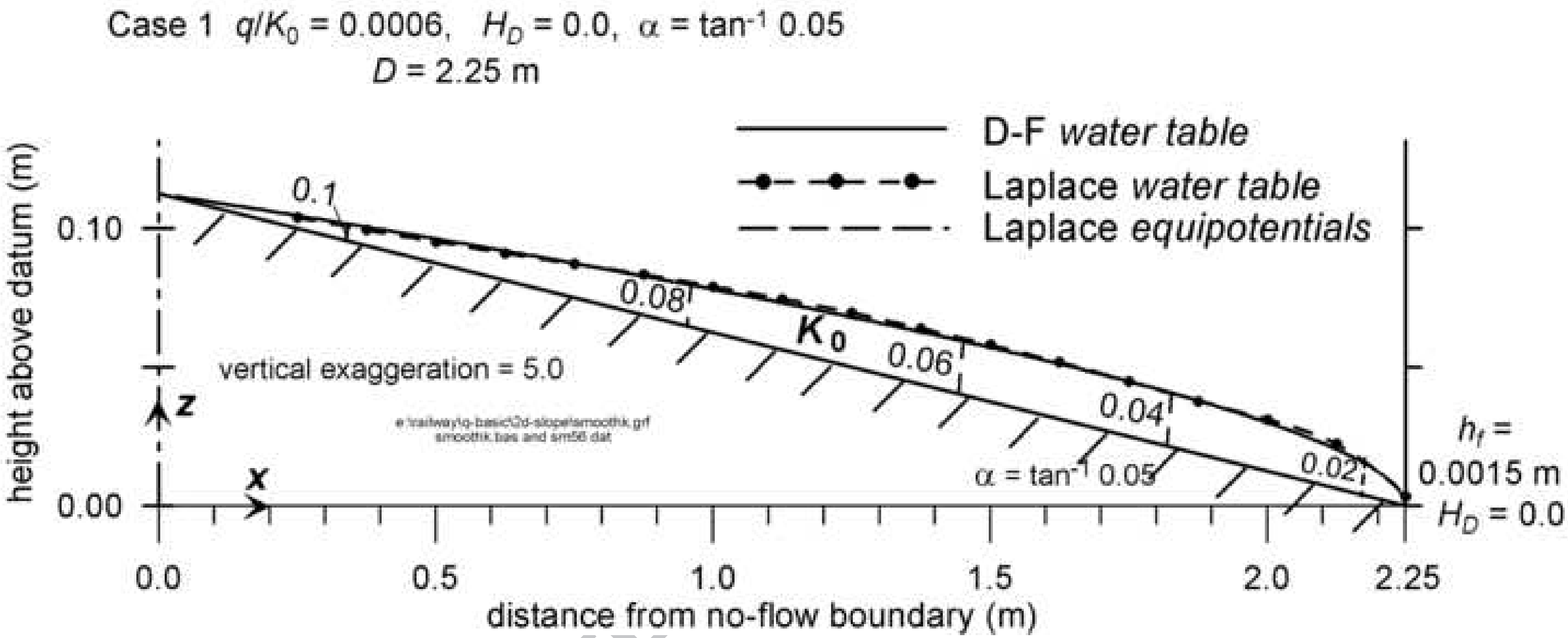


(a)
accretion q
water table
h = z and ∂h/∂x tan β + ∂h/∂z = q/K0or1
∂h/∂x = 0 if h > D tan α
K1
K0
β
K1 (∂h/∂x)1 tan α + K1 (∂h/∂z)1 = K0 (∂h/∂x)0 tan α + K0 (∂h/∂z)0
H
t
hf
seepage face
h = z
impermeable base
∂h/∂x tan α + ∂h/∂z = 0
z
x
HD
α
0
D



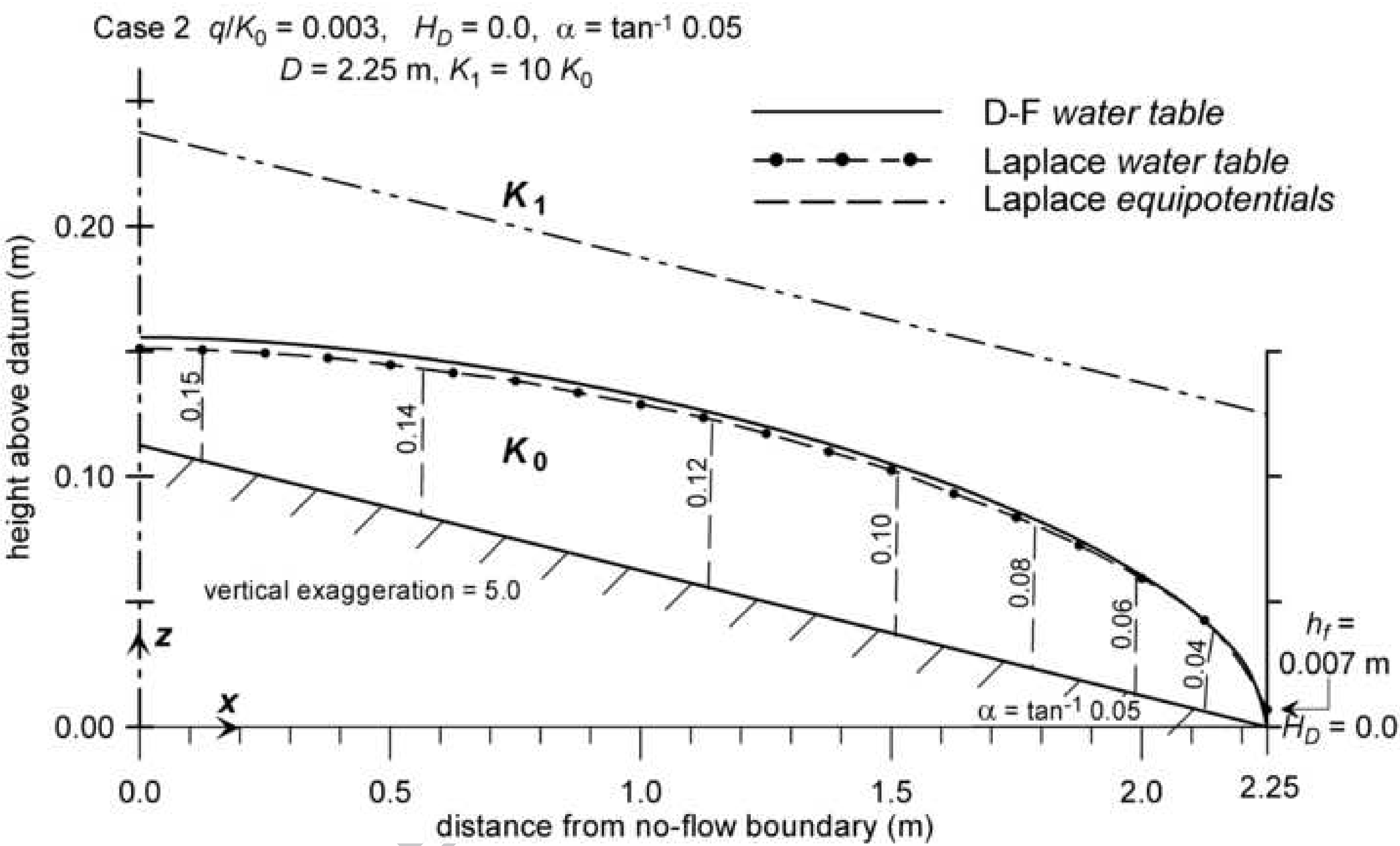
(b)
accretion q
water table
[H - t - (D - x) tan α] cos α
K1
K0
β
Flow = q x
t cos α
H
t
s = 0
s
impermeable base
z
x
HD
α
0
D
s = sD

Figure4



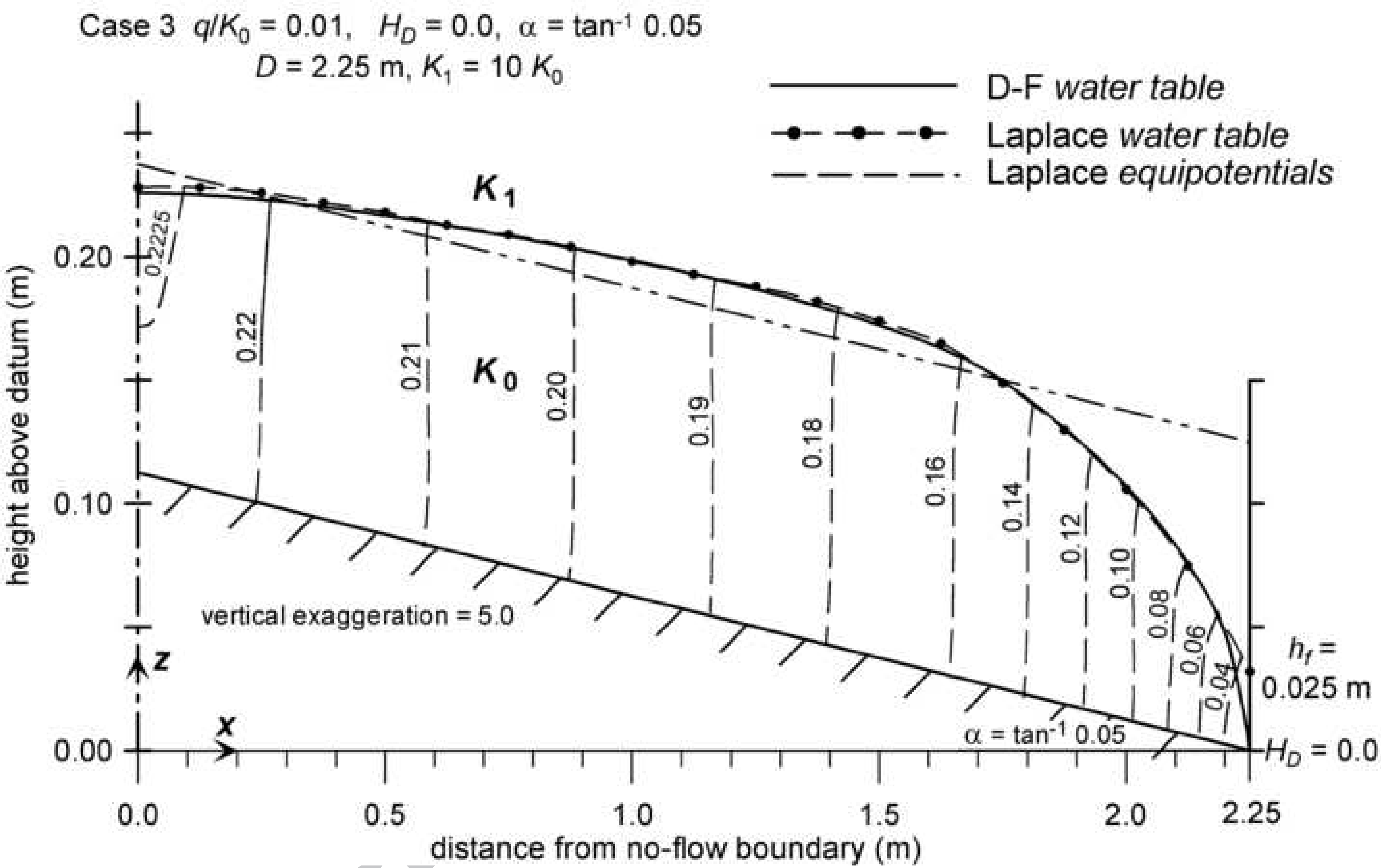
Case 1 q/K0 = 0.0006, HD = 0.0, α = tan-1 0.05
D = 2.25 m
D-F water table
Laplace water table
Laplace equipotentials
height above datum (m)
0.10
0.00
0.1
0.08
K0
0.06
0.04
0.02
vertical exaggeration = 5.0
z
x
α = tan-1 0.05
hf =
0.0015 m
HD = 0.0
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure 5



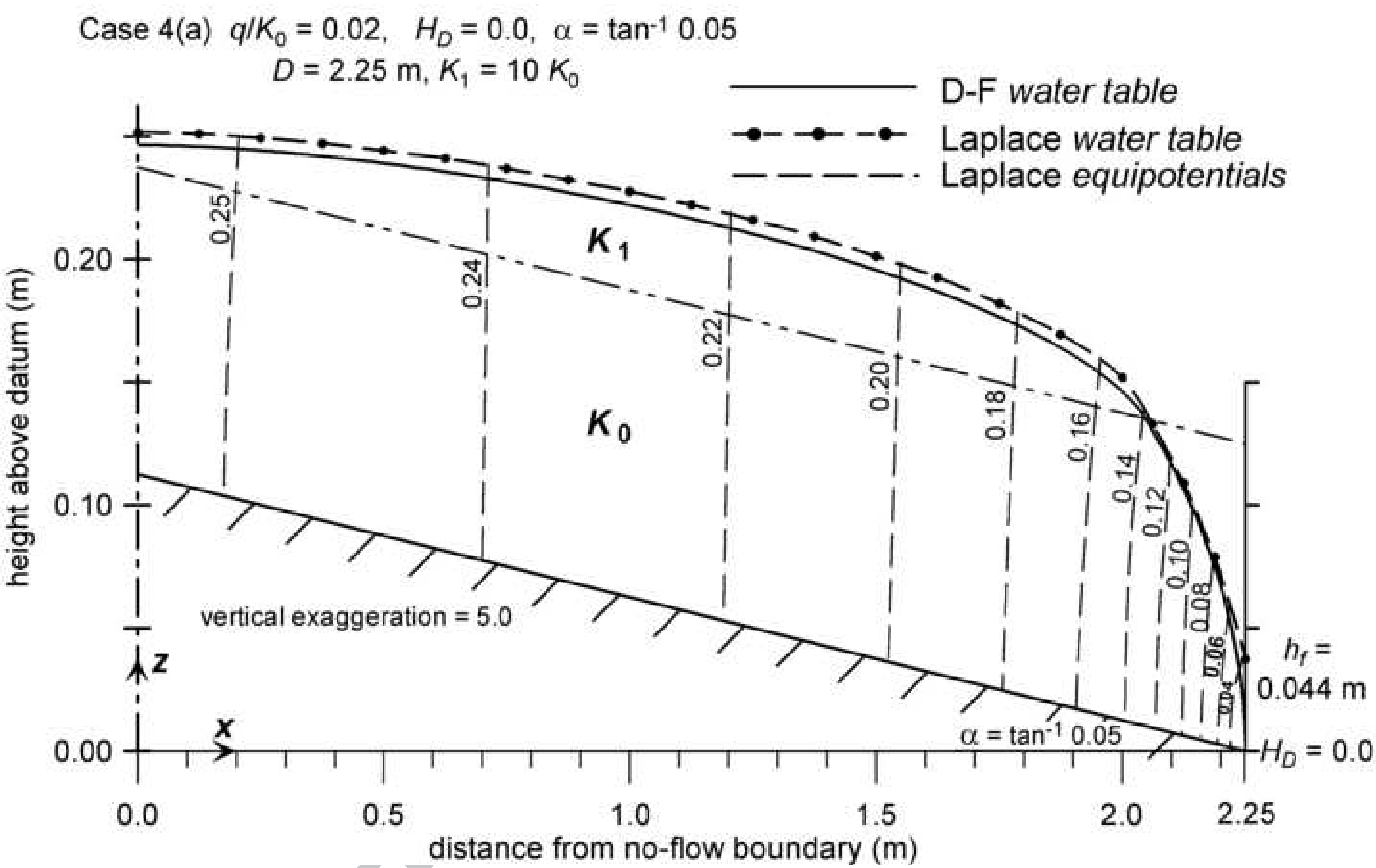
Case 2 q/K_0 = 0.003, H_D = 0.0, $\alpha = \tan^{-1} 0.05$
D = 2.25 m, K_1 = 10 K_0
D-F water table
Laplace water table
Laplace equipotentials
K_1
K_0
0.15
0.14
0.12
0.10
0.08
0.06
0.04
vertical exaggeration = 5.0
z
x
$\alpha = \tan^{-1} 0.05$
h_f = 0.007 m
H_D = 0.0
height above datum (m)
0.00
0.10
0.20
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure6



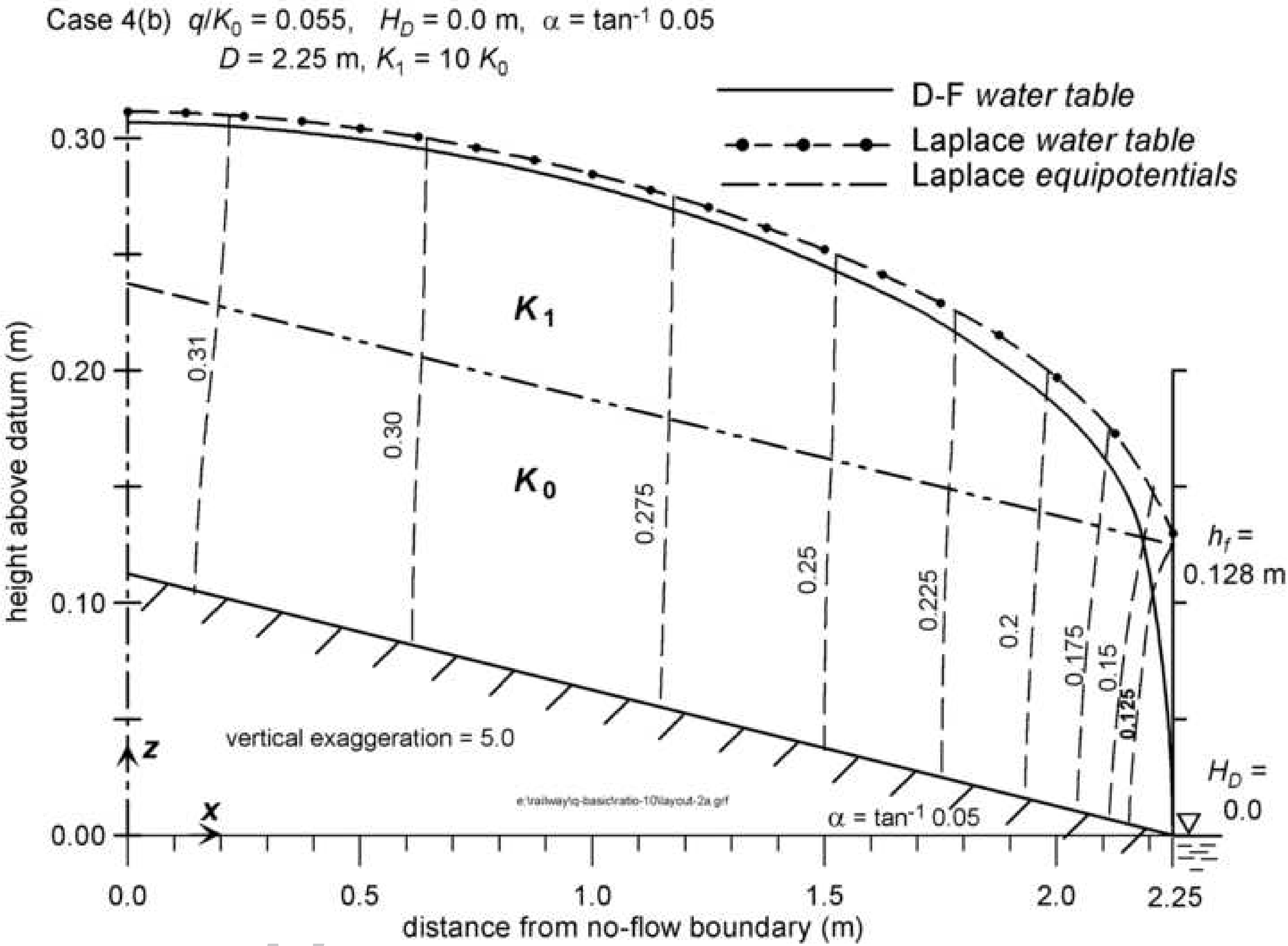
Case 3 q/K0 = 0.01, HD = 0.0, α = tan-1 0.05
D = 2.25 m, K1 = 10 K0
D-F water table
Laplace water table
Laplace equipotentials
K1
K0
0.2225
0.22
0.21
0.20
0.19
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
hf =
0.025 m
HD = 0.0
vertical exaggeration = 5.0
α = tan-1 0.05
z
x
height above datum (m)
0.00
0.10
0.20
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure7



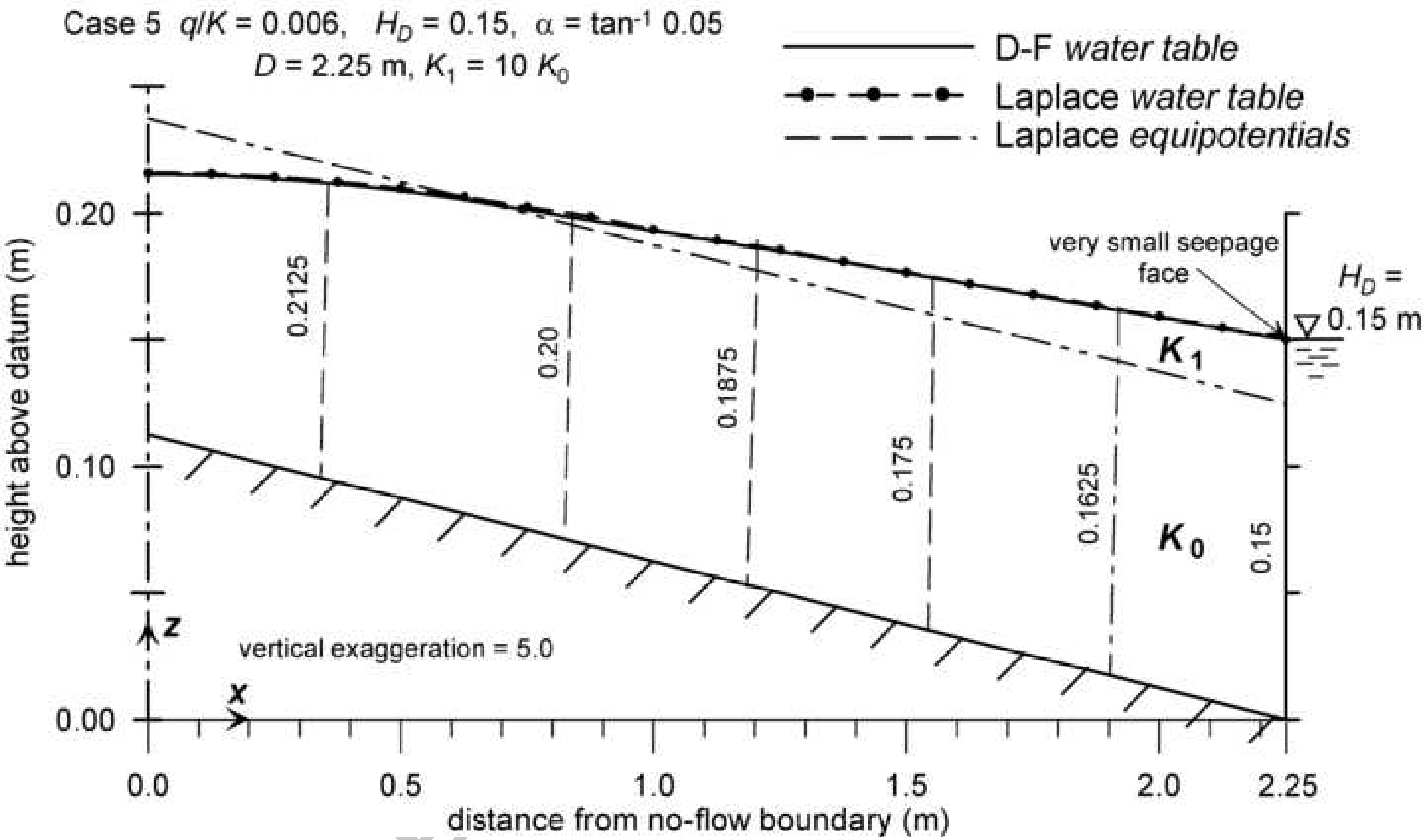
Case 4(a) q/K_0 = 0.02, H_D = 0.0, α = tan⁻¹ 0.05
D = 2.25 m, K_1 = 10 K_0
D-F water table
Laplace water table
Laplace equipotentials
K_1
K_0
0.25
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
height above datum (m)
0.20
0.10
0.00
z
x
vertical exaggeration = 5.0
α = tan⁻¹ 0.05
h_f =
0.044 m
H_D = 0.0
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure8



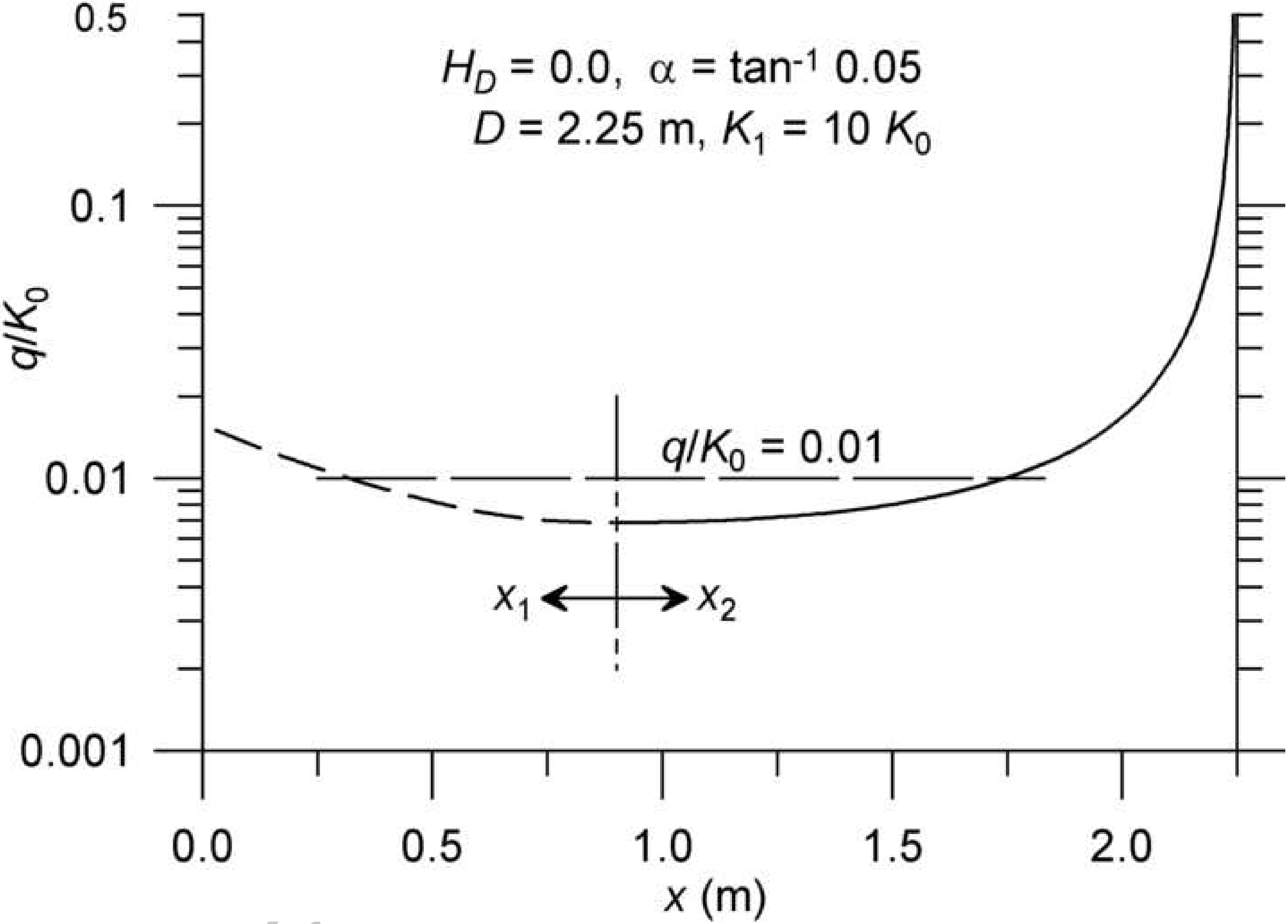
Case 4(b) q/K0 = 0.055, HD = 0.0 m, α = tan-1 0.05
D = 2.25 m, K1 = 10 K0
D-F water table
Laplace water table
Laplace equipotentials
K1
K0
0.31
0.30
0.275
0.25
0.225
0.2
0.175
0.15
0.125
hf = 0.128 m
HD = 0.0
vertical exaggeration = 5.0
α = tan-1 0.05
z
x
height above datum (m)
0.30
0.20
0.10
0.00
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure9



Case 5 q/K = 0.006, H_D = 0.15, α = tan-1 0.05
D = 2.25 m, K_1 = 10 K_0
D-F water table
Laplace water table
Laplace equipotentials
very small seepage face
H_D = 0.15 m
K_1
K_0
0.2125
0.20
0.1875
0.175
0.1625
0.15
z
x
vertical exaggeration = 5.0
height above datum (m)
0.00
0.10
0.20
0.0
0.5
1.0
1.5
2.0
2.25
distance from no-flow boundary (m)

Figure10



H_D = 0.0, α = tan⁻¹ 0.05
D = 2.25 m, K_1 = 10 K_0
q/K_0 = 0.01
x_1
x_2
0.5
0.1
0.01
0.001
q/K_0
0.0
0.5
1.0
1.5
2.0
x (m)

Figure11

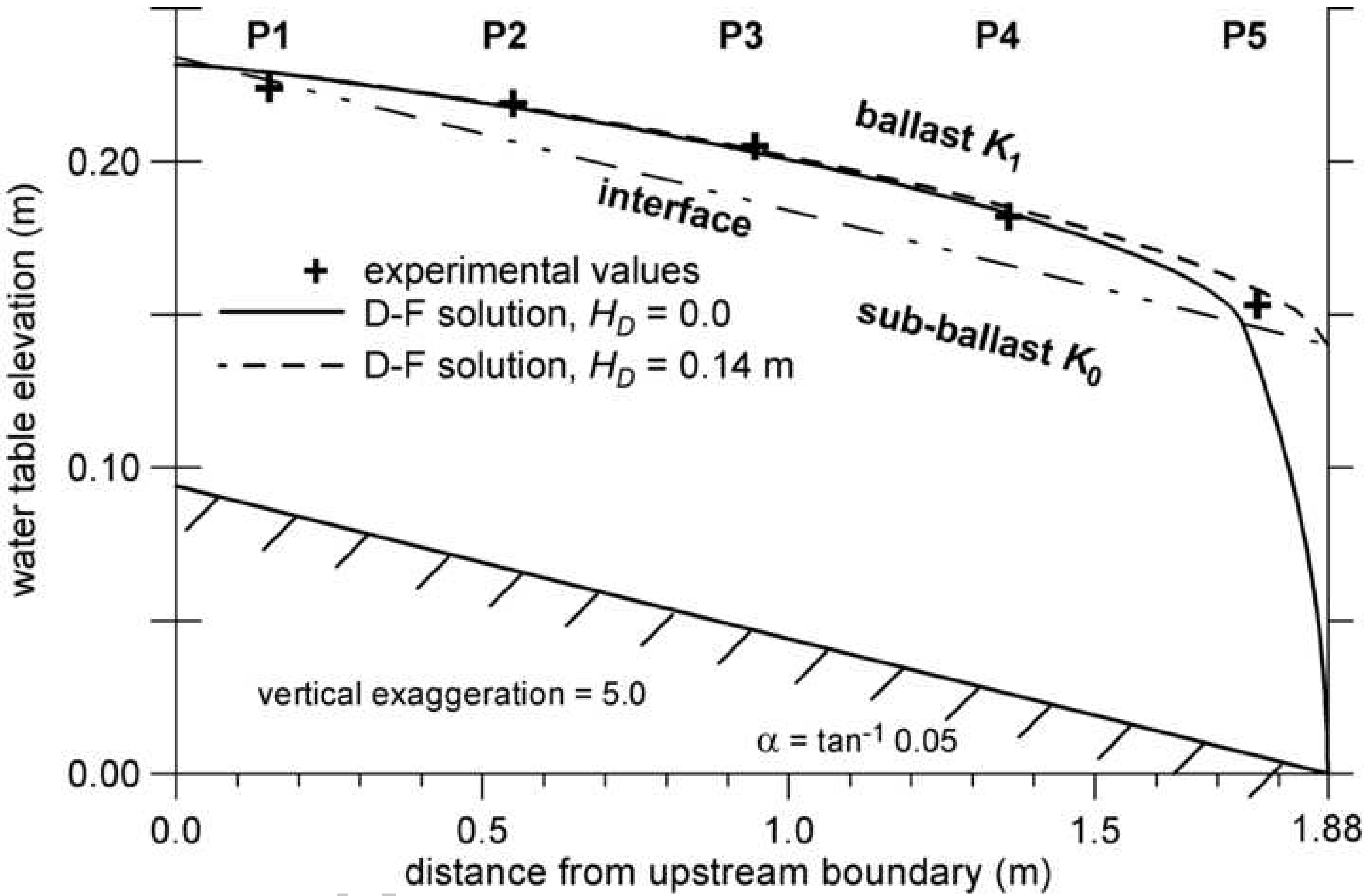
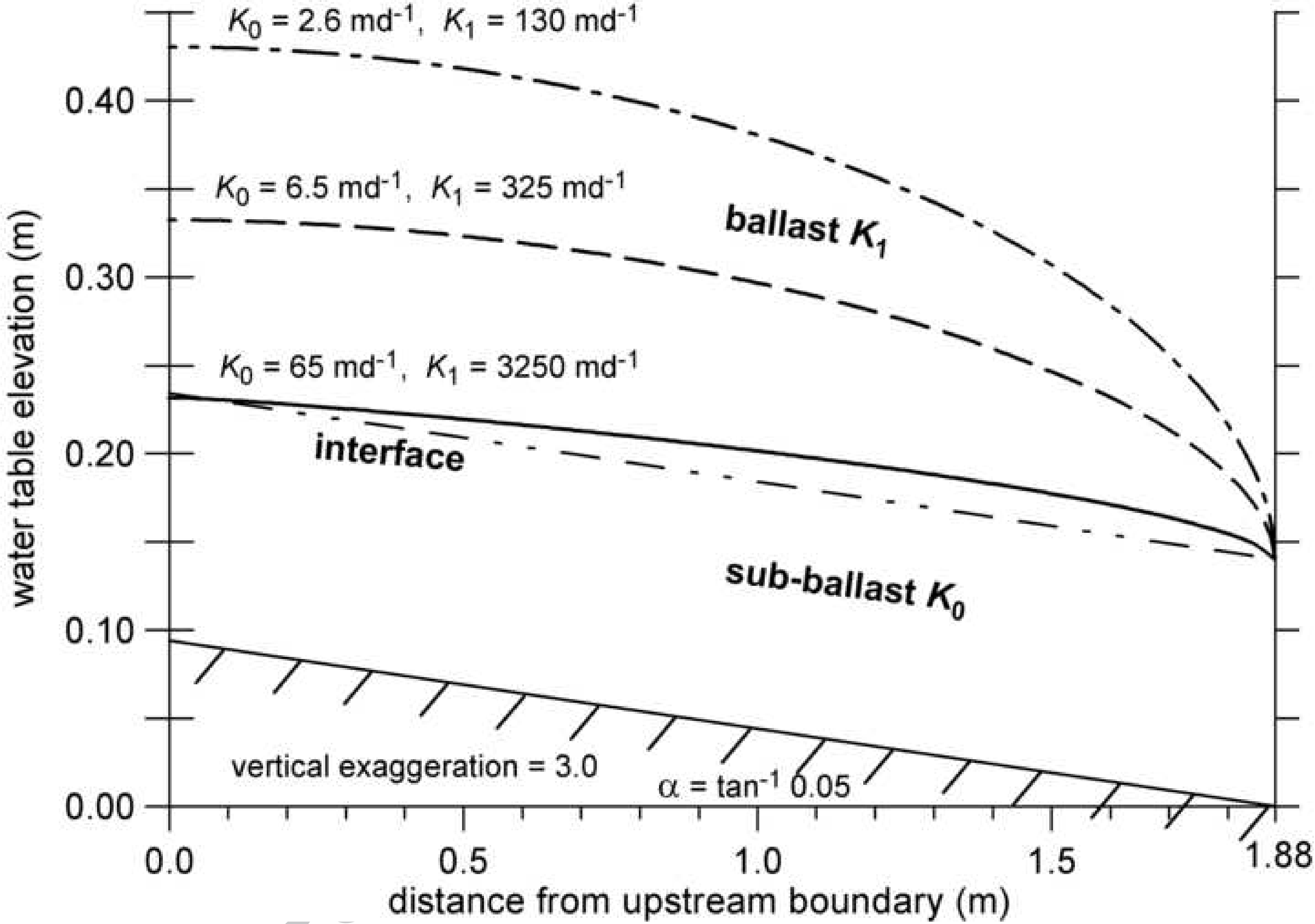

P1
P2
P3
P4
P5
ballast K_1
interface
sub-ballast K_0
experimental values
D-F solution, H_D = 0.0
D-F solution, H_D = 0.14 m
vertical exaggeration = 5.0
α = tan^{-1} 0.05
water table elevation (m)
0.20
0.10
0.00
0.0
0.5
1.0
1.5
1.88
distance from upstream boundary (m)

Figure12



K0 = 2.6 md-1, K1 = 130 md-1
K0 = 6.5 md-1, K1 = 325 md-1
K0 = 65 md-1, K1 = 3250 md-1
ballast K1
interface
sub-ballast K0
vertical exaggeration = 3.0
α = tan-1 0.05
water table elevation (m)
0.40
0.30
0.20
0.10
0.00
0.0
0.5
1.0
1.5
1.88
distance from upstream boundary (m)

Table 1. Calculation of water-table profiles shown in Fig. 11.

u	w	x (m) eq.(8) or (13)]		H (m) eq.(9) or (14)
		$\mathbf{H_D = 0}$		
- 0.00208		1.880	(D)	0
0		1.880		0.0039
0.002		1.880		0.0077
0.005		1.879		0.0133
0.01		1.878		0.0228
0.02		1.871		0.0418
0.04		1.844		0.0794
0.06		1.800		0.1157
0.07797	0.001559	1.749	(x_2)	0.1466
	0.005	1.734		0.1532
	0.01	1.645		0.1655
	0.015	1.440		0.1809
	0.02	1.111		0.1979
	0.025	0.7328		0.2130
	0.03	0.4308		0.2227
	0.035	0.2509		0.2275
	0.04	0.1592		0.2297
	0.05	0.07615		0.2313
2.99780	0.059956	0.04669	(x_1)	0.2317
5		0.02784		0.2319
10		0.01386		0.2320
100		0.001380		0.2321
1000		0.0001380		0.2321
		$\mathbf{H_D = 0.14}$ **m**		
	0.001442	1.880	(x_2,D)	0.14
	0.01	1.776		0.1603
	0.015	1.555		0.1768
	0.02	1.200		0.1953
	0.025	0.7909		0.2115
	0.03	0.4650		0.2220
	0.035	0.2708		0.2272
	0.04	0.1683		0.2296
	0.06	0.05028		0.2318
3.16237	0.063250	0.04424	(x_1)	0.2319
5		0.02786		0.2320
10		0.01387		0.2321
100		0.001381		0.2322
1000		0.0001381		0.2322