

## DECOMPOSITION OF STOCHASTIC PROPERTIES WITHIN IMAGES USING NON-PARAMETRIC METHODS

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### Abstract

This paper discusses the application of three different non-parametric methods for decomposing images into regions which exhibit special stochastic properties, together with the constituent components of the stochastic. These are:- 1) order statistics in connection with steps in the empirical estimated distribution functions; 2) detection of stochastic information within an image by hypothesis testing; 3) rank order statistics to decompose the different types of stochastic within an image. The decomposition is used to isolate different image regions and to estimate the processes which are the constituent stochastic components. In order to achieve this, decisions based upon membership relations are employed and adapted thresholds used. The thresholds are obtained by the ordering of terms calculated by stochastic estimation methods together with one of the aforementioned techniques.

### 1 Estimation of selected parts of distribution functions

The decomposition of images into separate parts with different stochastic properties is obtained by calculating selected portions of the distribution function, using ordering of statistics. The ordering uses a specific set of grey values of pixel elements in the image. For such an ordering set an empirical integral function of the distribution [1] is calculated, by summation of step functions with selected thresholds within a stochastic region. These overlaid stochastic processes are described by using the difference and the height of the levels of the pre-processed integral distribution function. These levels are estimated by analysis of the stochastic empirical properties. The non-parametric method of ordering allows a soft decision to be made in classifying the membership set of pixel values, to a selected stochastic property.

A-priori knowledge concerning possible stochastic properties within regions of the image, can be identified by testing assumptions. The test provides a value that represents the similarity between the stochastic property in a selected region with one of the assumed a priori properties. The set of elements of the selected region is given by

$$\mathbf{x} = \{x_{n,m}, x_{n,m+1}, \dots, x_{n,m+k}, x_{n+1,m}, x_{n+2,m}, \dots, x_{n+r,m+k}\}$$

Where  $l = k + r$  and the distribution function (DF) of the selected region with  $l$  other neighbouring elements written as:-

$$F_l(\mathbf{x}, x_{rel}) = \frac{1}{p} \sum_{r=1}^p u(\mathbf{x} - x_{grey}^{(r)}) \quad (1)$$

Where  $p$  is the number of grey levels  $x_{grey}$  that occurs in the  $l$  elements and  $u$  is the unit step function. A threshold can then be calculated to decide upon the degree of similarity.

Ordering of the DF does not afford the possibility to detect shifting areas, while the correlation between regions does. Another description of the stochastic may be obtained by the ordering of a special sub-region  $s$  with elements  $x_s$ . Such ordering in  $v$  sub-regions can be expressed as:-

$$F_s(x, \mathbf{x}_{area}^{(v)}) = \frac{1}{v} \sum_{k=1}^v \{u(x_s - x^{(k)}) - u(x - x^{(k)})\}$$

A simple example of which would be where the values  $x_s$  are obtained by setting a minimal threshold, and then summing over a special area. This is identical to the bit-image of the least significant bits.

The number of the inversions is also a measure of DF-characterisation of stochastic information. For different levels and distances, with the threshold  $c$  the following relationship holds.

$$I_{diff}(c) = \frac{1}{n} \sum_{i=1}^n u(x_i + c - x_j) - \frac{1}{n} \sum_{i=1}^n u(y_i + c - y_j)$$

Here  $x_i$  are the element values of one area and  $y_j$  the values of the other.

All three of the above discussed DFs, obtain different stochastic information for an image and the fusion of this information can be obtained by applying the fuzzy integral.

## 2 Testing of hypothesis of assumed types of stochastic

This method is based upon a special kind of multi-dimensional testing of hypotheses, where the thresholds are estimated by stochastic properties connected by the  $\alpha$ -quantile. The image will be tested for the presence of special stochastic parts, where it is assumed that one part has a special stochastic and the other part does not. A test image is created with similar non-stochastic contents as the original, but without the stochastic process to be detected. This can be obtained by non-linear filtering of the original image. The grey values for the bitwise pixels in the original and the test images are used to obtain an estimation of the contributions of different stochastic in the original image. For the testing of the hypotheses, not only the stochastic properties of the grey values are used, but also those of the regions where the same kind of stochastic is present. The cross-correlation of the two images or parts of these are used for the ratio of the likelihood [1]. By applying the Neuman and Pearson criterion, the condition for the detection of the related stochastic is obtained, which provides for the possibility to calculate the threshold  $C$  using the value of the likelihood. This procedure is realised using pixels from both the original and test images. The algorithm is based upon an optimisation of the different sums of the values of the hypotheses, where a special selected stochastic is present and where it is not. This is then applied to a collection of different kinds of stochastic. The decision for the stochastic is given if the sum is greater than the estimated constant  $C$ .

The next issue is to solve the decision problem as to whether an image region  $B_A$  contains the same stochastic contents as a region  $B_D$ . It is assumed that the grey value of the image region  $A$  consists the noise component  $n_{ij}$  and the stochastic information

component  $x_{ij}$ . A similar condition for the region  $B_D$  exists, so that  $w'_{ij} = x'_{ij} + n'_{ij}$ ; where the terms relevant to region  $B_D$  are identified by an apostrophe. For the moments in region  $B_A$  the following conditions are assumed:-

$$\begin{aligned} \langle w(x_{ij}) \rangle &= 0, \langle n_{ij} \rangle = 0, \langle w(x_{ij})w'(x_{kl}) \rangle = \delta_{i,k} \delta_{j,l} S(w(x_{ij})), \\ \langle n_{ij}n_{kl} \rangle &= \delta_{i,k} \delta_{j,l} Q(n_{ij}), \langle w_{ij}w_{kl} \rangle = \delta_{i,k} \delta_{j,l} [S(x_{ij}) + Q(n_{ij})] \end{aligned}$$

These same conditions apply to region  $B_D$ . To find the equivalence of both regions, the hypothesis's test (HT) is used. If the requisite stochastic information is present in the image, then:-

$$\frac{\langle w_{ij}w'_{kl} \rangle}{\langle w_{ij}w_{kl} \rangle} = \frac{S}{S+Q} \quad \text{otherwise}$$

$$\frac{\langle w_{ij}w'_{kl} \rangle}{S+Q} = 0 \quad \text{If it is not. For the variation of } w_{ij}w'_{kl}$$

where the special stochastic information is present, using the above conditions, gives the following relationships:-

$$\frac{(w_{ij} + w'_{kl})^2}{\langle w_{ij}w_{kl} \rangle - \frac{\langle w_{ij}w'_{kl} \rangle \langle w_{ij}w'_{kl} \rangle}{\langle w_{ij}w_{kl} \rangle}} = \frac{(w_{ij} + w'_{kl})^2}{(S+Q) \left( 1 - \frac{S^2}{(S+Q)^2} \right)}$$

If the information is absent, with  $S=0$  this becomes,

$$\frac{(w_{ij} + w'_{kl})^2}{Q}$$

If the logarithm of the likelihood  $l$  is used for the HT, then

$$\begin{aligned} \ln l(w_{ij}, w'_{ij}) &= \frac{S}{2[(S+Q)^2 - S^2]} \sum_{i,j=1}^n (w_{ij} + w'_{ij})^2 - \\ &\frac{n}{2} \ln \left[ \left( 1 - \frac{S^2}{(S+Q)^2} \right) \frac{S+Q}{Q} \right] \end{aligned}$$

This gives the value for the HT of stochastics:-

$$\sum_{i,j=1}^n (w_{ij} + w'_{ij})^2 \geq C \quad (2)$$

Where  $C$  is calculated by the logarithmic term of the upper equation.

C is obtained by the maximum of the likelihood:

$$C = \frac{n[(S+Q)^2 - S^2]}{S} \ln \left[ \left( 1 - \frac{S^2}{(S+Q)^2} \right) \frac{S+Q}{Q} \right]$$

This method may be extended to search for more stochastic processes, by iteratively applying this technique, and thus enabling more complicated stochastic properties to be discovered.

### 3 Rank order for decomposing different kinds of stochastic processes

Of considerable important in analysing the results from ordering in single regions, is the selection of individual regions related to the information in the image. To achieve this, the relationships between the rank orders of the original and specific parts of the image are pre-processed. As for the ordering function, the distance of grey values, a local distance, a distance between derivated values or absolute values about a threshold can all be used. For the ordering process, an adapted area is selected, which can be described by a construction of ensembles [2]. There exist many different possibilities to construct such an ensemble, including partial histograms, signum ordering or spatial relationships of nearest neighbourhood pixels. The rank given in [1] for instance, can be written for images in the usual form as in [3], while in [4] the rank algorithm is used principally for noise smoothing and contrast enhancement. The rank algorithm proposed here however, separates different structures within an image, by decomposing the different stochastic properties. As in the above technique, adapted thresholds are used for the decision making. The single area on which elements the rank relations are applied can be found with help of the difference of the pixel values and the value of the central pixel  $x_{ij}$ . The set of values of the difference in an array of N neighbours  $x_{r,s}$  in the quadrant  $(N\Delta)^2$  related to the inner point  $x_{ij}$  of the region  $e_N$  is then :-

$$B_D(x_{ij}, C, e_N) = \{ w(l+k), |w(x_{i\pm l\Delta, j\pm k\Delta}) - w(x_{ij})| \leq C, 0 < l, k \leq N, x_{i\pm l\Delta, j\pm k\Delta} \in e_N \}$$

All elements  $x_{r,s} \in e_N$  for which this condition is fulfilled are members of the region of values of distance ( $B_D$ ). C is an adapted constant or it can be defined as:-

$$C = \min_{0 \leq l, k \leq N} \{ w(x_{i\pm l\Delta, j\pm k\Delta}), x_{i\pm l\Delta, j\pm k\Delta} \in e_N \}$$

For ordering the elements in  $B_D$  the rank relationships are used:-

$$R_{ij} = \sum_{l, k = -N}^N u(w(x_{ij}) - w(x_{i\pm l\Delta, j\pm k\Delta}))$$

If a priori it is known that the image contains stochastic and determined parts and we are interested in elimination of the real stochastic part, then this problem can be solved by extension of the sign-rank algorithm [2]. For a better representation of the fluctuation in the image, positive and negative values about a mean value are used. Such values are obtained for the region  $e_{NM}$  if we subtract the maximum from the minimal value of  $\{r(m)\}$ , that is

$$d = [\max(w(e_{NM})) - \min(w(e_{NM}))] / 2$$

and then solve the equation:-

$$S_e(\{w(x_{ij})\}) = \sum_{i, j = -N}^N \text{rank}(|w(x_{ij}) - d|) u(w(x_{ij}) - d) \geq D \quad (3)$$

Here D is a constant, which is estimated. With the help of the expectation value in eqn. 3, the constant D can be estimated [3].

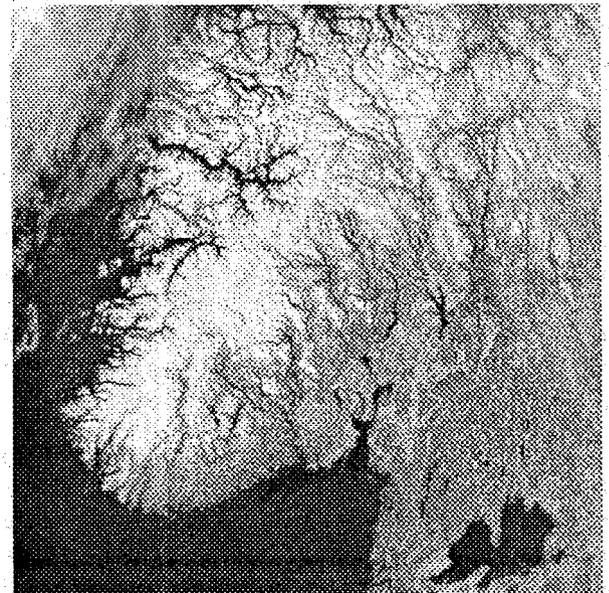


Fig. 1; Norway, original image with clouds and snow

## Discussion of Results

As an example of applying the theory which was developed above, a satellite picture (Figure 1) containing many different kinds of stochastic information is used.



Fig. 2 . Stochastic structures of clouds and snow on the mountains, separated by rank ordering

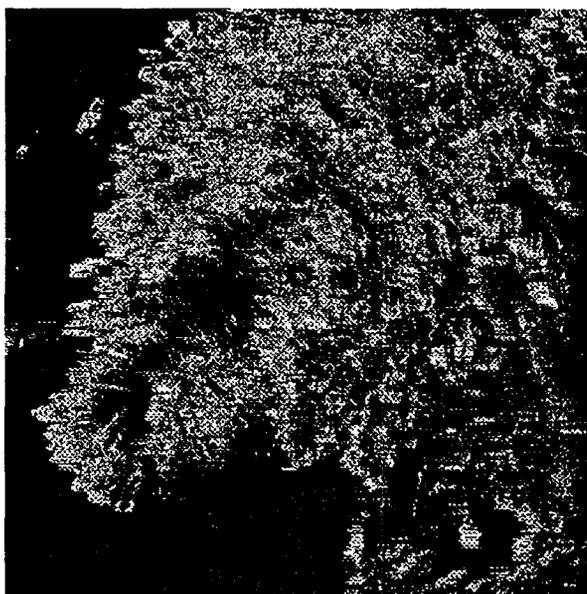


Fig. 3. Contours of the coastline of Norway split from the clouds.

By only utilising the brightness, it is very difficult to distinguish between snowy areas, the clouds and the coastline of Norway. By using rank ordering however, the stochastic in the clouds and the snow can be separated as shown in Figure 2. Using the distribution function in combination with hypotheses testing, the coastline can also be detected by spitting the cloudy regions from the snowy areas as illustrated in Figure 3. It is therefore possible to detect the coastline from under the clouds as well as splitting the snow-capped mountains in the picture.

## Conclusions

This paper has reviewed three diverse methods which facilitate the separation of different stochastic properties within an image. By combining the stochastic features derived from these different techniques, improved results can be obtained. These methods have been successfully applied to a number of different images, an illustration of which is presented in the paper.

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