Dynamic Symmetrical Topology Models for Pervasive Sensor Networks

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Abstract

The success of pervasive computing environments using ubiquitous loco-dynamic sensing devices is very dependent upon the sensor deployment topology (DT) employed. This paper presents a systematic mathematical model for efficient sensor deployment and provides a comparison with other popular topologies. The model focuses upon blanket coverage of a surveillance area using a minimum number of sensing devices, with minimal intra-sensor overlapping to reduce collisions and co-existence problems. Simulation results are presented for the Hexagonal, Triangular and Square grid topologies for various dimensions of surveillance area. The results confirm that the hexagonal model gives optimal performance in terms of requiring the minimal number of sensors. The paper also highlights the improved performance of ubiquitous wireless sensor networks when a hexagonal topology (HT) is used.

1. Introduction

A key advantage of symmetrical topological architectures for pervasive sensor network is that they reduce operational costs significantly while concomitantly improving operational efficiency. A pervasive environment of seamlessly integrated, loco-dynamic wireless sensing devices is regarded as one of the most important technologies of the future [1]. Each device is usually capable of collecting, storing and processing information, and also communicating with neighboring nodes [2,3,4]. Weiser [5] identified the rapid growth of micro-electromechanical systems and low powered wireless communications as facilitating the deployment of very dense, fully distributed sensor/actuator networks for a wide range of monitoring applications. These include perimeter surveillance, structural health monitoring (using non-invasive bio-signals like EEG and ECG sensors), tracking of accidental chemical leaks, environment

Ubiquitous wireless sensor technology is embedding processing capability, storage, localization capability via the global positioning (GPS) or local positioning systems (LPS) and wireless links to the neighboring nodes into devices operating in acoustic, seismic, infrared (IR) and electromagnetic modes [1]. Data fusion strategies are evolving in order to correlate sensor outputs from multiple nodes since no system component is independent for achieving the global objective [2].

The deployment topology of a network is crucial to the effectiveness of a wireless network of densely distributed ubiquitous sensing devices. A DT includes the description of network dimensions, location and density of sensing and control devices, coverage estimation, surrogate localization and ownership resolution mechanisms. Since pervasiveness requires context-awareness, localization is indispensable for systems [8] which depend upon sensor placement. Chong et al [1] has regarded the problem of sensors density and sensor deployment as one of the most challenging from a technical perspective.

Most techniques being developed target application development on top of an assumed sensors deployment topology and rarely discuss the infrastructural level issues relating to pervasive topological models. Both Krishnendu [9] and Sameera [10] have realized the importance of topologies in sensor networks. Krishnendu [9] presented different sensor placement techniques in a square grid with different levels of complexities for various sensor ranges because they do not define the mechanism for grid formation. The approach used in [10] evolved different topologies for different density requirements, posing serious limitations on generalized application level development. Their techniques result in uncovered areas for different neighboring requirements. VTriangle (sensors placed on the vertices of triangles) topology has been considered by Tseng [2] for agent-

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based location tracking, while Nirupama [8, 11] assumed a square-grid topology for extension of existing sensor network, but did not define the original topology. This paper considers the importance of network topology and presents a systematic mathematical HT model and compares its performance with other modeling techniques, especially those based on square and triangular grids. The HT model considers the following key parameters to determine its effectiveness: Topology Design Dimensioning, Sensor Placement and Sensor Density.

For open space monitoring a unique subset of sensors with minimum density must be used so that the problems of coexistence, interference, and resource ownership resolution should not impose a computational overhead on the network. The sensor nodes with low memory and low computational power should be less dependent on Parent Nodes (PN), so that if a PN fails the network should keep working in a safe mode. This implies that a network topology should be mathematically simple.

The rest of this paper is organized as follows: Section 2 presents the proposed HT model, while Section 3 mathematically compares the four models. Section 4 discusses the simulation results, to highlight the comparative performance of each model, with some conclusions presented in Section 5.

2. Proposed Model

The proposed HT approach is based upon coding theory principles [12] and low-order polynomial time algorithms that will reduce the overall computational, power and memory requirements. It is assumed that all sensors have isotropic radial coverage and use radio frequency to communicate with neighboring sensor nodes.

2.1. Hexagonal Grid Formation

Let X,Y be the dimensions of a rectangular field that is to be monitored such that, if R is the side length of each regular hexagon in the resulting hexagonal grid and also the radius of the sensor coverage then:

$$v:\ R/2,\ \Delta x:\ R+v,\ \Delta y:\ (R/2)\sqrt{3}$$

Where x and y are differences in the horizontal and vertical coordinates respectively. The rectangular grid is formed in such a way that each x and y grid line passes though the centres of hexagons in the resulting hexagonal grid. In Figure 1, symbols 'o' and 'X' indicate the intersection of even (x, y) and odd (x, y) coordinate pairs respectively. To now generate the hexagonal grid from the rectangular gird, let:

$$M=\{0y,1y,2y,3y,\ldots,my\}$$
$$N=\{0x,1x,2x,3x,\ldots,nx\}$$

where M & N are respectively the sets of vertical and horizontal coordinates on the rectangular grid. The pair-

-elements from these sets are selected such that the vertices of a hexagon can be generated. To select a pair a

"Pairing Function (PF)" is used:

$$f(p)=\{(j,i)\,|\,\forall i\ M, j\ N\ \exists\ (j,i)=(0,0)\vee(1,1)\}$$

Based on PF, for any pair *(j,i)* there always exists a "Mod Function" (j,i) such that:

$$(j,i)=(nMOD2, mMOD2)$$

To determine if given pair *(j,i)* will produce vertices of a hexagon, (j,i) must produce a "Mod Vector" *(a,b)* such that *a=b*. Once a set of all such pairs have been generated by PF then each pair is used to generate a pair of hexagonal vertices H_{v1} and H_{v2} such that:

$$H_{v1}=(j+v,i)\ \text{And}\ H_{v2}=(j-v,i) \qquad (1)$$

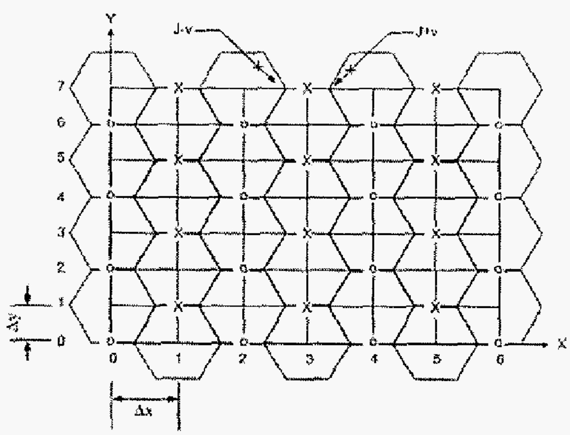

Figure 1, Hexagonal Grid

This procedure produces the vertices of all the hexagons in the resulting grid. To combine the vertices to form the hexagons, consider the vertex u in Figure 2.

$$u=\{(j_k+v,i_p)\vee(j_k-v,i_p)\,|\,1\le k\le n,1\le p\le m\}$$

Three other adjacent vertices u_1, u_2, u_3 are:

$u_1 = \{(j_k - v, i_p) \vee (j_k + v, i_p) \mid 1 \le k \le n, 1 \le p \le m\}$

$u_2 = \{(j_{k+1} - v, i_{p\ 1}) \vee (j_{k-1} + v, i_{p\ 1}) \mid 1 \le k \le n, 1 \le p \le m\}$

$u_3 = \{(j_{k+1} - v, i_{p\ 1}) \vee (j_{k-1} + v, i_{p\ 1}) \mid 1 \le k \le n, 1 \le p \le m\}$

Combining every u with each corresponding u_1, u_2, u_3 generates the complete hexagonal grid shown in Figure 2.

2.2. Sensor Placement

For a given field, it is firstly assumed that the HT model is already implemented and that each hexagon in the

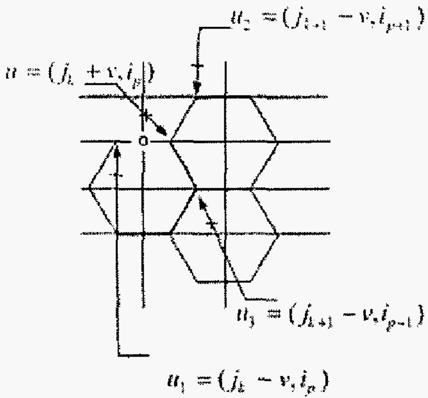

Figure 2 -Adjacent vertices for $u \quad \{(j_k \quad v, i_p)\}$

model is a vertex of an undirected graph G. Next, the location of sensors is found based on the following rule: Find a best covering of the vertices of graph G by a set of sensors placed at certain vertices in G so that whole field is covered. The covering of a vertex should be associated with a unique subset of sensors.

Sensor placement has been solved using the theory of Identifying Codes [9] by placing sensors at the centre of each hexagon using a "Non-Pairing Function (NPF)":

$$f(p) = \{(j,i) \mid \forall i \ M, j \ N \exists \ (j,i) \quad (0,1) \quad (1,0)\}$$

Based on NPF, for any pair *(j,i)*, there exists a "Mod Function" (j,i) which results in a "Mod Vector" *(a,b)* such that $a \neq b$. This means that sensors are always placed at the points generated by NPF which are the intersections of odd x and even y coordinates and vice versa.

Lemma 1: The NPF Condition. Sensors must always be placed at the centre of each hexagon cell.

Proof: According to hexagonal grid formation technique, both the horizontal and vertical grid lines pass through the centre of each hexagon and so intersect each other at three points, two opposite sides and at the centre of hexagon, (Figure 2). The PF used two intersections at the opposite parallel sides of a given hexagon to produce four vertices, leaving the intersection which is at the centre of the hexagon (Figure 2). This pair is used by the NPF for sensor placement

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Sensor Placement Algorithm (SPA):

1. Let the radial coverage of sensors be:
   $v = \alpha R$ Where: R is side length of hexagon and is an integer.
2. Let the centre of a hexagon be denoted by:
   $\delta_{pq} = \{(c_p, r_q) \mid 0 \le p \le \overline{C}, 0 \le q \le \overline{R}\}$ where $\overline{C}$ and $\overline{R}$ are the total number of hexagonal grid columns and rows respectively.
3. Initialize $\delta_{(1)(0)} = (c_1, r_0)$ where $c_1 = 1, r_0 = 0$
4. Position of the first sensor in each row is defined by: $\delta_{pq} = (c_p, r_q)$ where $c_p = s + k$
   $k \quad NOT(tMOD2)$
   where s and t are the sensor column and row numbers respectively.
5. Next sensor *position* in same row:
   $\delta_{(p+1)q} = (c_{p+1}, r_q)$ where $c_{p+1} = c_p + 2\alpha$
6. Next sensor *position* in same column:
   $\delta_{p(q+1)} = (c_p, r_{q+1})$ where $r_{q+1} = r_q + \alpha$

■

Both Table 2 and Figure 4 show sample sensor positions for 3 sensor rows and columns for α =2.

Table 2 (= 2)

3	(1,4)	(5,4)	(9,4)
2	(3,2)	(7,1)	(11,1)
1	(1,0)	(5,0)	(9,0)
t / s	**1**	**2**	**3**

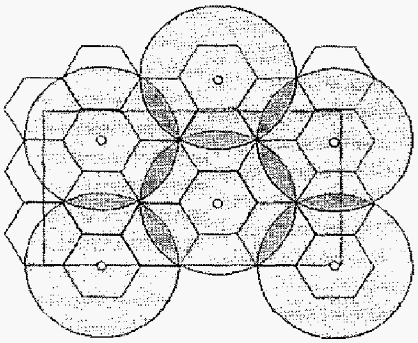

Figure 4, Sensor Placement for = 2

2.3. Sensor Density

The main aim is to minimize the number of sensors required to blanket cover the entire field. Lemma 2 shows that the number of sensors required to cover the whole area has an upper limit governed by the number of hexagons in the grid.

Lemma 2: If sensor of coverage radius $\gamma = \alpha R$, where $\alpha = 1$, are placed at the centre of adjacent hexagons of circumradius R, the minimum number of sensors S_n required to cover all the hexagons is always equal to the number of hexagons H_n in the grid. i.e. $S_n \geq H_n$

Proof:

1. If A_c is the sensor coverage circle area and A_h is the hexagon area, then the sensor radial coverage $\gamma = \alpha R = R$ for $\alpha = 1$, and the coverage will be a circumcircle R of the hexagon and :

$$A_c > A_h \qquad (2)$$

2. Now consider two adjacent hexagons with centers $\delta_{pq} = (c_p, r_q)$ and $\delta_{(p+1)q} = (c_{p+1}, r_q)$ as shown in Figure 5. If $c_p = 2$ and $r_q = 1$, then from the SPA algorithm, $c_{p+1} = c_p + 2\alpha = 4$. Therefore the two points are $\delta_{pq} = (2,1)$ and $\delta_{(p+1)q} = (4,1)$ which fulfill Lemma 1 (NPF condition), and so are candidates for sensor placement and by implication must be the centre of two adjacent hexagons.
3. From (2), the area covered by a sensor will always be greater than that of a hexagon. In order to prove that sensors have to be placed in both hexagons to cover the whole area, we consider the case where a hexagon is surrounded by six other adjacent hexagons.
4. Assume that six surrounding sensors would also cover the central hexagon so there is no need to place a sensor in the central hexagon.
5. Let: $A_c = A_h + \beta_h$

   Where $_h$ is the excess area covered by a sensor outside the hexagon. This excess area will be for six surrounding hexagons, so the excess area covered for the central hexagon by one surrounding hexagon is:

$$A_E = \beta_h / 6$$

   Area of central hexagon covered by six surrounding hexagons:

$$A_E = 6\beta_h / 6 = \beta_h$$

   From the hexagon geometry:

$$A_h = R^2 3\sqrt{3}/2 \text{ and } \beta_h \quad R^2(2\pi - 3\sqrt{3})/2$$

$$\Rightarrow \beta_h << A_h$$

   (3)

   (3) shows that even in the best case, an area equivalent to $A_h \quad \beta_h$ of central hexagon is left uncovered. Therefore the assumption made in 4 above is wrong and it is required to place a sensor in every hexagon in order to cover the whole field.
6. If $\overline{A}$ is the area to be monitored and area of one hexagon is A_h, then number of hexagons required is given by: $H_n = \overline{A} / A_h$

   This is equal to the number of sensors required.

Corollary-1: Given a field of dimensions X, Y, there exists a hexagonal pattern for which the number of sensors required is equal to the number of hexagons. i.e.:

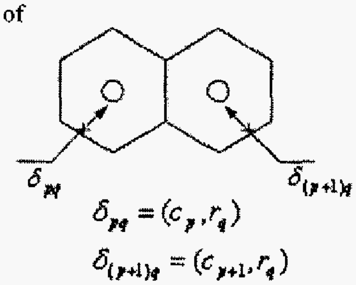

Figure 5, Sensors in Adjacent Hexagons

$$\{\forall A(X,Y) \exists H(\overline{C}, \overline{R}) \mid S_n = H_n\}$$

2.4. Hexagon Dimensions

The greater the dimensional disparity between the radial coverage of sensors and underlying topology, the higher will be the computational overhead on the network. In this Section, this disparity factor is shown to be a minimum for the HT model.

Lemma 3: If a sensor of coverage radius $\gamma=\alpha R$, where $\alpha=1$ is placed at the centre of a regular hexagon of circumradius R, the relationship between sensor coverage radius and hexagon side length that covers the whole hexagon and minimizes the extra area covered is given by: $\gamma=R$

Proof: From hexagon geometry: $\beta_h=\pi\alpha^2-3\sqrt{3}/2$
(4)

1. It is also known that a circle with area $A_c = A_h + \beta_h$, centered about a hexagon covers the whole hexagon, touching the hexagon at its vertices (Figure 6a).
2. If $\alpha>1$:
   (5)
   In this case, area in (5) is greater than the area in (4). This results in a circle that covers larger extra area than that of the hexagon as shown in Figure 6b.
3. If $\alpha<1$:
   (6)
   In this case, area in (6) is less than the area in (4). In fact $_h$ is negative since the circle does not cover the complete hexagon. This results in circle that covers a smaller area than the hexagon (Figure 6c).
4. From the above, it is clear that only when =1, the sensor radial coverage covers the whole hexagon with minimal extra area coverage given by (4).

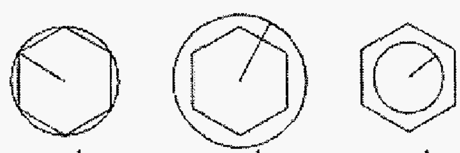

Figure 6a **Figure 6b** **Figure 6c**

3. Comparison with Other Models

The performance of HT model presented in Section 2 has been compared with equilateral triangles and squares models [2,9,10] on the basis of grid formation, dimensional complexity and sensor density.

The dimensional disparity of different topologies with the radial coverage is compared by considering the complexity of dimensional relationship. If R, E and F are the side lengths of a hexagon, square and triangle respectively, then the relationships between the radial coverage of the sensor and R, E and F are:

$$\gamma=R,\quad \gamma=\frac{E\sqrt{2}}{2},\quad \gamma=\frac{F\sqrt{3}}{2}$$

From this it can be seen that the hexagonal dimensional relationship is simpler than other topologies and HT model incurs the lowest computational overhead.

The dimensional difference implies that a larger number of squares will be required in the square grid topology which increases sensor density. For VTriangle topology as shown in Figure 7, the number of sensors to monitor a hexagonal area is seven which is a high number compared to HT which requires only one sensor.

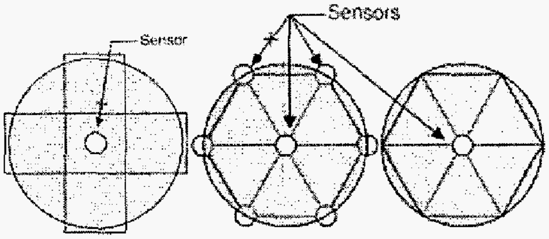

Figure. 7 Square, VTriangle and HT Models

4. Simulation Results

In the various simulation studies, sensor networks were designed for an open rectangular area of dimensions varying from 100u² to 25000u², where u is any general unit. The sensors were placed according to the HT, square [9,10] and two triangular models; CTriangle (sensors at the center of triangle) & VTriangle (sensors at the vertices of triangles) [2]. In the studies, sensors for two different ranges (5u and 10u) were analyzed.

4.1. Sensor Density

This Section presents comparative simulation results of sensor density for the four models in Figure 8. It was observed that the density was a maximum for the VTriangle topology, while the HT and CTriangle models required significantly fewer sensors. Figure 8 also confirms the rise in the density of sensors required with.

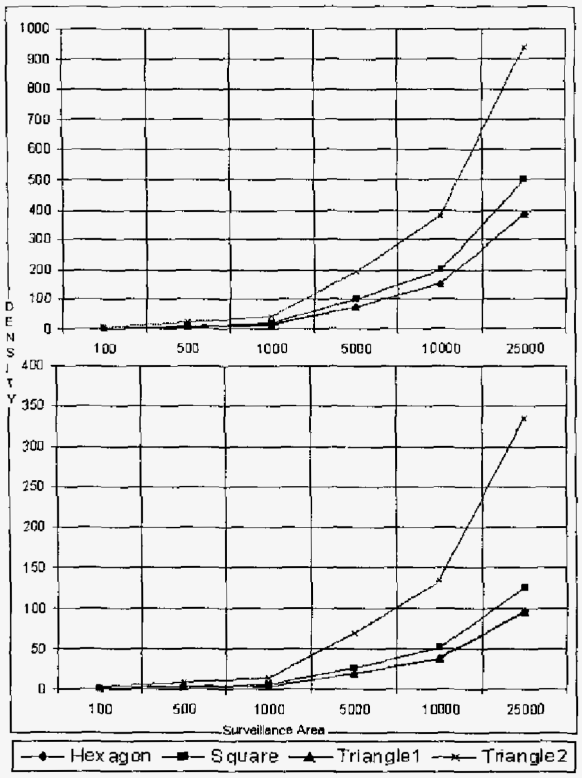

Figure 8 (Sensors Density)
Above: Sensor Range 5u, Below 10u

increasing surveillance area dimensions, with the rise less steep for the Hexagonal and CTriangle topologies

4.2. Blanket Coverage

Blanket Coverage (BC), measures the total area coverage of isotropic radial sensors. Table 3 shows the excess area covered by different topologies excluding the area to be monitored. Figure 9 shows the total coverage area by sensors deployed in each topology. The HT and CTriangle models covered less excess area implying these models will have minimal effects upon their surroundings. Figure 9 also illustrates that doubling the sensor range does not increase the excess area coverage with the same ratio for the HT and CTriangle models, while it increased significantly for the other two topologies.

Table 3, Excess Area Covered-(Sensor Range 5u)

Area (u^2)	Hexagon	Square	CTriangle	VTriangle
1000	200	575	125	2075
5000	775	2575	775	9325
10000	1550	5075	1550	18275
25000	3875	12575	3875	45575

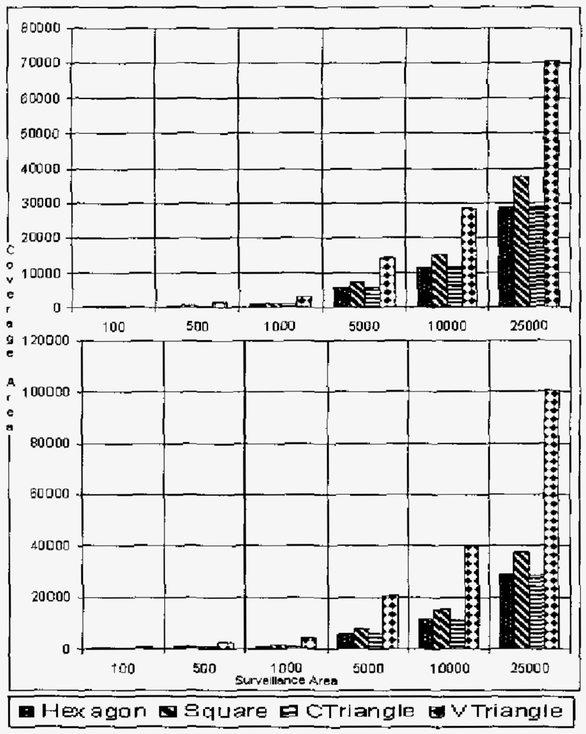

Figure 9 (Sensors Coverage)
Above: Sensor Range 5u, Below 10u

5. Conclusions & Future Work

This paper has analyzed and evaluated the importance of sensors placement in a highly interconnected pervasive environment of communicating devices by emphasizing the importance of DT of sensing devices. The Hexagonal model has been presented and compared with Square and Triangular topologies on the basis of topology design dimensions, placement of sensors and sensors density. The results have proven that HT and CTriangle models required fewer sensors and both performed better in providing efficient blanket coverage. However CTriangle topology evolves into hexagonal topology and is overridden by the HT model. VTriangle topology performed worst in all scenarios whereas Square topology failed to show its edge on the HT and CTriangle models at all levels.

Since the HT model has proved to be comparatively optimal at infrastructure level, we intend to extend this model at application level for tasks including location tracking, ad-hoc sensor placement, network self-configuration, disaster recovery and recasting of randomly distributed sensors with different sensing ranges.

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