Mach and Froude Numbers on Mars

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Abstract

Mars atmospheric global circulation models exhibit transonic jet streaks during northern winter, which motivates this study of the Mach number, Ma (the ratio of flow speed to the speed of sound), and Froude number, Fr (the ratio of flow speed to the speed of buoyancy waves), as a function of season and location. Two global reanalyses spanning Mars Years (MY) 24 to 32 are used as input, EMARS and OpenMARS. The study’s vertical coordinate is potential temperature, θ, ranging from θ = 400 to 1100 K (from z ≈ 32 to 66 km); the floor is set to the lowest level that avoids intersecting mountains. Area-weighted global means and standard deviations and 5 yr temporal means using the complete years MY 25, 26, 29, 30, and 31 are compared. EMARS and OpenMARS show general agreement below θ = 700 K (z ≈ 53 km), where the observational constraints are strongest, but can vary significantly at higher levels. Both reanalyses contain transonic jet streaks in every northern winter sampled. The Fr signal is roughly twice the Ma number signal, as expected where the temperature lapse rate is small compared to the dry adiabatic lapse rate. Mach numbers are similar in both reanalyses but show larger year-to-year variability in OpenMARS. Maps of standard deviations indicate a depression between the main peaks in Tharsis and higher variability in Mare Boreum than Mare Australe. The main conclusion is that the atmosphere of Mars routinely operates in the compressible regime (0.3 < Ma < 0.8), unlike Earth. Aspects of all flow-speed meteorology are discussed.

Unified Astronomy Thesaurus concepts: Mars (1007); Planetary atmospheres (1244); Jets (870)

1. Introduction

This is an empirical study of the atmosphere of Mars that establishes basic climate statistics on how the Mach and Froude number fields, Ma and Fr, vary as a function of season, altitude, and region. The main goal is to determine when and where the flow is incompressible (Ma < 0.3), compressible (0.3 < Ma < 0.8), and transonic (0.8 < Ma < 1.2). Meteorologists working with Earth’s atmosphere need not consider this step: on Earth, the speed of sound near the surface is 340 m s\(^{-1}\), with only a 10% degradation near the tropopause, so that the fastest jets barely reach the upper limit of the incompressible regime (0.3 × 340 m s\(^{-1}\) ≈ 100 m s\(^{-1}\) ≈ 370 km hr\(^{-1}\) ≈ 230 mi hr\(^{-1}\) ≈ 200 kn). This is not the case on Mars, where the northern winter polar jet exhibits transonic jet streaks (Dowling et al. 2016). Given that this is the first systematic assessment of compressible regimes on Mars, the results are interpreted in terms of their broad implications for model development.

1.1. EMARS and OpenMARS

The observations that enable this study were obtained in Mars Years (MY) 24 to 27 by the Thermal Emission Spectrometer (TES), aboard the NASA Mars Global Surveyor (MGS) and in MY 28 to 32 by the Mars Climate Sounder (MCS) aboard the NASA Mars Reconnaissance Orbiter (MRO; Conrath et al. 2000; Smith et al. 2001; McCleese et al. 2008, 2010). The data span 9 Mars years and include five complete or nearly complete years (MY 25, 26, 29, 30, and 31; one MY equals 1.88 Earth years).

The utility of these observations has been expanded by producing global reanalyses, whereby global circulation models (GCMs) have been used to calculate wind fields via optimal assimilations of thermal and opacity profiles. Two independent groups have each recently made public a reanalysis of the TES and MCS data: the Ensemble Mars Atmosphere Reanalysis System (EMARS; Greybush et al. 2019) and the Open-access to Mars Assimilated Remote Soundings (OpenMARS; Holmes et al. 2020); the latter is an update to the Mars Analysis Correction Data Assimilation (MACDA; Montabone et al. 2014), which was used in the precursor to this study (Dowling et al. 2017). These reanalyses employ different models and data assimilation schemes. EMARS data assimilation estimates errors via an ensemble average of models run under different conditions, considering both temperature profiles and dust column opacities (but not dust profiles), and reports both that average and the output of a representative model (member 008, which assumes median values for dust and water ice cloud forcing); to avoid inconsistencies that may arise from averaging (the “2.5 children” issue), this study works with the latter (EMARS files labeled back_memb). The OpenMARS group has demonstrated that incorporating the total column dust opacity improves accuracy when dust loading becomes high (Lewis et al. 2007). It is important to note that in both reanalyses the wind fields are not observed directly but are calculated by the models.

The zonal and meridional wind fields from EMARS and OpenMARS are used to calculate specific kinetic energy, and the temperature and pressure fields are used to calculate potential temperature, sound speed, scale height, buoyancy frequency, and geopotential. The fields “co2ice” and “dustcol” from OpenMARS are not used. The calculation of buoyancy frequency is described in Kono & Arakawa (2006, Appendix A), and the calculation of...
geopotential is described in Konor & Arakawa (1997, Equation (3.3)). The calculations of potential temperature, sound speed, and scale height are described below.

Since this study is using output from GCMs in part to discern where the output is not accurate, there is a chicken-and-egg problem that needs to be addressed. The specific question is, how does inaccurate modeling of compressibility affect the determination of where compressibility is important? In general, one expects poorly modeled compressibility to produce relatively larger errors in phase than amplitude because compressibility affects a field variable’s contrast more than its range. This issue has been addressed for Earth models by Davies et al. (2003), who systematically compared a hierarchy of compressibility approximations. They found that each type of wave mode suffers in a different manner depending on the numerical scheme, and they concluded that only fully compressible schemes are acceptable in the long run. To get a definitive answer for Mars will require a similar systematic comparison of models, which is outside the scope of this study.

While the behavior of the Mach and Froude number fields on Mars is the focus, a useful by-product is that EMARS and OpenMARS are interpolated onto the same vertical and horizontal grids (they come on different grids) and are subject to the same statistical analyses, making direct comparisons straightforward. Discerning which is more correct is not possible by this means, since essentially all available data are already assimilated into both, but it is instructive to note when, where, and by how much they differ.

### 1.2. Spatial and Temporal Grids

In terms of longitude, the EMARS grid spacing is 6°, and that of OpenMARS is 5° (60 and 72 longitude grid points). It was elected to keep these native, i.e., to use direct insertion instead of interpolation, which results in the only grid difference between the two reanalyses in this study. The EPIC atmospheric model is used as a convenient platform for interpolations, since it employs an isentropic vertical coordinate and was set up to handle Mars reanalysis input in the precursor study (Dowling et al. 2017); note that no EPIC atmospheric simulations are part of this or the precursor study. OpenMARS, as well as the EPIC platform, works with east longitude running from −180° to 180°, whereas EMARS works with east longitude running from 0° to 360°, so the EMARS [180°, 360°] interval is mapped to [−180°, 0°] to match the others.

In terms of latitude, both reanalyses use 36 grid points, but with slightly different grid spacings. The EMARS spacing is 5°.14 everywhere, except at the polar edges, where it is 3°.86, whereas OpenMARS is 5° everywhere. Both EMARS and OpenMARS are interpolated onto the same EPIC model latitude grid, with 4°.94 spacing everywhere as described in Dowling et al. (2017). All interpolations in this study use PCHIP splines (piecewise cubic Hermite interpolating polynomials; Fritsch & Butland 1984).

In terms of time, the EMARS grid spacing is 1 Mars hour, or 1/24 sol (day), while OpenMARS is 2 hr, so in this study EMARS is sampled on the same 2 hr interval as OpenMARS (by skipping every other time). EMARS includes the time variable macOSa_sol, which when added to the EMARS variable Mars_hour/24 yields the same time variable used in OpenMARS and MACDA, making synchronization straightforward. For solar longitude, Ls, Montabone et al. (2014) suggest subtracting 0°.12 from the MACDA Ls to correct for a near-constant bias (midnight vs. actual start of spring). Since Ls matches between MACDA and OpenMARS, following Dowling et al. (2017), this correction is adopted for OpenMARS as well. A spot check at time 4351.083 sol (measured since the start of MY 24) shows that Ls in EMARS is nearly the same as the corrected value in OpenMARS, 161°.660 versus (161°.795 − 0°.12) = 161°.675; hence, no correction is applied to the EMARS Ls values.

Following Hoskins et al. (1985) and Dowling et al. (2017), the vertical coordinate in this study is an isentropic variable, the potential temperature, θ. This is calculated from the temperature, T (K), and pressure, p (Pa), via the standard adiabatic-process power law,

\[ \theta = T \left( \frac{p_0}{p} \right)^{\kappa}; \quad \kappa = \frac{R}{c_p \rho_0}, \]

where \( p_0 = 610 \text{Pa} \) and \( R = 192 \text{J kg}^{-1} \text{K}^{-1} \) are the standard reference pressure and specific gas constant for Mars, respectively (Table 1, p. 855 of Zurek et al. 1992), and \( c_p = 735 \text{J kg}^{-1} \text{K}^{-1} \) is a reference specific heat capacity at constant pressure (corresponding to \( T \approx 200 \text{K} \)), such that the representative Poisson exponent is \( \kappa = 0.261 \) (Urata & Toon 2013). See Appendix A for mention of an alternate “potential enthalpy” approach used in oceanography.

Since altitude and/or pressure have traditionally been employed instead of θ for Mars work, as an aid to interpreting the results, in plots with a θ axis the geopotential height, i.e., the geopotential, \( \Phi \), divided by the reference gravity, \( g_0 = 3.71 \text{m s}^{-2} \), is included as an alternate vertical axis (with \( \Phi \) averaged in the same manner as the variable being displayed). Dowling et al. (2017, Figures 2 and 3) show typical positions of θ surfaces versus latitude over the altitude and pressure ranges under study for the four canonical season points, \( L_s = 0°, 90°, 180°, \) and 270° (it is helpful to note that \( L_s = 270° \) corresponds to “December 21°”). The geopotential as a function of longitude, latitude, and potential temperature is calculated by integrating the hydrostatic balance equation upward from the surface geopotential (Konor & Arakawa 1997, Equation (3.3)). EMARS supplies the surface geopotential, which is used directly; OpenMARS does not supply it, so it is reconstructed as in Dowling et al. (2017, p. 41).

It was decided for this study to avoid outcroppings, that is, to pick the largest vertical span of θ values that never dips below the planet’s surface, or rises above the data ceilings, across both reanalyses. The lowest θ values that always clear the mountains in EMARS and OpenMARS are 355 and 371 K, respectively (see Figure E1 for a reference plot of θ vs. altitude). Interestingly, as θ is lowered below these values, it is the peak of Arsia Mons—the second tallest after Olympus Mons—that first pokes through. This outcropping occurs in MY 25, at \( L_s = 179°.6 \) and 186°.1, at (longitude, latitude) positions (−117°0, −7°7) and (−120°0, −7°25) for EMARS and OpenMARS, respectively. The study’s floor value is rounded up to \( \theta = 400 \text{K} \) (on average, \( z \approx 32 \text{km} \)). The highest θ values that always stay below the data ceilings for EMARS and OpenMARS are 1296 and 1168 K. As θ is raised above these values, the data ceiling is first intersected in MY 26, at \( L_s = 299°7 \) and 237°0, at positions (159°0, 69°4) and (−55°0, 72°5). These positions appear to be random with respect to longitude, but not latitude or season: they coincide with the
strong warp in the geopotential associated with the northern winter polar jet (Dowling et al. 2017, Figure 2(d)). The study’s ceiling value is rounded down to $\theta = 1100 \text{ K}$ ($\approx 66 \text{ km}$). The vertical sampling increment is set to $\Delta \theta = 20 \text{ K}$, which provides ample resolution for the purposes of this study. The reanalyses are then interpolated onto these potential temperature surfaces ($\theta = 400, 420, \ldots, 1080, 1100 \text{ K}$). See Dowling et al. (2017) for additional details regarding the vertical interpolations.

1.3. Mach Number

In fluid mechanics, the portfolio of Mach-type numbers is the set of dimensionless ratios of the form $u/(-v)$, where $u$ is the fluid velocity in a given reference frame and $v$ is the wave speed relative to the flow. The negative sign ensures that the ratio is positive for upstream waves, which is an important detail when dealing with unidirectional waves like Rossby waves (Dowling 2014). Generally $v$ does not depend on the reference frame, i.e., it is Galilean invariant, whereas $u$ and hence the Mach number are not. The full wave velocity is $c = u + v$, such that an alternate expression of the ratio is $u/(u - c)$. This ratio equals unity when the upstream-directed waves are stationary, $c = 0$, which corresponds to the “sonic” condition. For dispersive waves, $v$ is set to the fastest upstream group (envelope) velocity because information and energy travel at the group velocity and keeping track of upstream information propagation is one of the primary uses of a Mach number. The analog for Rossby waves, denoted “Ma” (with the quotation marks included) or $M_R$, has recently been employed as a field diagnostic for the atmospheres of Earth (Du et al. 2015) and Mars (Dowling et al. 2017) and for Earth’s Southern Ocean (Stanley et al. 2020). This article returns to Mars and to the original Mach and Froude numbers.

For a given altitude in kilometers, the atmosphere of Mars has much lower pressures than on Earth—an intuitive way to view the situation on Mars is to conceptually start with Earth’s atmosphere, remove its troposphere and stratosphere, and lower its mesosphere (and above) down to the planetary boundary layer. The pressures then line up with Mars, and so too do vorticity—stream function correlations line up (Dowling et al. 2017, Figures 1 and 12). The ideal-gas limit is well satisfied on Mars, implying that the speed of sound is not a function of pressure, but it does vary appreciably with temperature. The denominator of the Mach number is set to the adiabatic speed of sound,

$$
\hat{c}_s(T) = \sqrt{\frac{c_p R^\kappa T}{\mu}} = \sqrt{\frac{R T}{1 - R/c_p(T)}},
$$

where $c_p(T)$ and $c_v(T)$ are the specific heat capacities at constant pressure and constant volume (J kg$^{-1}$ K$^{-1}$), respectively; $R = 8314.46$ J kmol$^{-1}$ K$^{-1}$ is the universal gas constant; and $\mu = 43.1$ kg kmol$^{-1}$ is the mean molar mass, such that $R = R^\kappa/\mu = 192$ J kg$^{-1}$ K$^{-1}$ is the aforementioned specific gas constant. The rightmost expression in Equation (2) follows from $c_p = c_v + R$ for an ideal gas. Considering that the atmosphere is 95.1% carbon dioxide (CO$_2$) by volume, $c_p(T)$ is found by interpolating the ideal-gas CO$_2$ data in Woolley (1954); it varies by about 12% over the domain of interest. For reference, $\hat{c}_s \approx 230$ m s$^{-1}$ at $T \approx 200 \text{ K}$, which is slower than the sound speed in Earth’s atmosphere and tends to make the Mach number larger. The numerator of the Mach number is set to the local magnitude of the velocity vector expressed as $|v| = \sqrt{2 \text{KE}}$, where KE is the specific kinetic energy (J kg$^{-1}$ m$^{-2}$); thus,

$$
\text{Ma}_1 = \frac{\sqrt{2 \text{KE}}}{\hat{c}_s(T)}; \quad \text{Ma}_2 = 2 \text{KE} \frac{1 - \kappa(T)}{RT},
$$

where $\kappa(T) = R/c_p(T)$. Shapiro (1953, p.54) underscores the importance of the Mach number for compressible fluid systems: “We have found the Mach Number to be a criterion of the type of flow pattern. Later it will be shown that it is also a parameter which almost always appears in the equations of motion.” Over the domain of interest, the typical fastest sound speed is 250–260 m s$^{-1}$ for both EMARS and OpenMARS, occurring in mid- to upper altitudes at $L_s \approx 240^\circ$–$270^\circ$ (Mars is closest to the Sun at $L_s = 251^\circ$). In MY 28, which is a major dust storm year, the sound speed rises to 270 m s$^{-1}$. The slowest sound speeds are consistently in the range 162–168 m s$^{-1}$ in both reanalyses and occur at the top of the domain of interest.

1.4. Froude Number

A second member of the Mach number portfolio is the Froude number, Fr, the ratio of flow speed to the speed of buoyancy waves (gravity waves), with hydraulic jumps and controlled flow being the analogs of acoustic shocks and choked flow, respectively. William Froude is credited with establishing, in the mid-1800s, the practice of using scale models and scaling laws to obtain accurate results in engineering fluid mechanics. Darrigol (2005, pp.277–283) reviews Froude’s empirical research on ship drag, and Hager & Castro-Orgaz (2017) describe the meandering path that led to Froude’s name being attached to the analog of the Mach number for buoyancy waves.

As with any shock, hydraulic jumps occur when the Froude number descends from Fr > 1 to Fr < 1, which is described in words as transitioning from shooting to tranquil flow, or from supercritical to subcritical flow. If the jump is moving in a given reference frame, then it is called a bore. At the scale of downslopes and katabatic winds (the mesoscale), both Earth and Mars have atmospheric hydraulic jumps and bores that have been well studied. In Earth’s atmosphere, they are often associated with topography (Peña & Santos 2021), and even form over the dynamic topography of anvil clouds (O’Neill et al. 2021). On Mars, hydraulic jumps associated with katabatic winds and crater rim walls have been observed and modeled (Smith et al. 2013; Tyler & Barnes 2013; Smith & Spiga 2018), as have atmospheric bores (Kahn & Gierasch 1982; Sta. Maria et al. 2006).

There are two issues to keep in mind regarding how the Froude number is handled in this article. The first is the mild confusion caused by Fr being variously defined, over the course of its history, as the square, the reciprocal, or the squared reciprocal of the Mach-number-style definition used here. The second issue stems from the fact that, in general, buoyancy waves are dispersive (their speed depends on their wavelength), which makes working with them more complicated than working with sound waves (which are nondispersive in the small-amplitude limit). To serve as the bookkeeper for buoyancy wave information propagation, the denominator of Fr should be the fastest upstream group velocity. However, the determination of this quantity is sensitive to assumed boundary
conditions and is generally more difficult to compute than the speed of sound. In this study, the local value of \( NH \) is used to estimate the denominator of \( Fr \) (for a WKB viewpoint regarding the employment of \( NH \) in this manner, see Chelton et al. 1998),

\[
Fr = \frac{\sqrt{2KE}}{NH},
\]

where \( N \) and \( H \) are the local buoyancy wave frequency and pressure scale height, respectively, computed via

\[
N = \left( \frac{g \partial \Theta}{\theta dz} \right)^{1/2}; \quad H = \frac{RT}{g},
\]

and \( R \) is the specific gas constant, \( g \) is the acceleration of gravity, and \( T \) is the absolute temperature. Because Equation (4) is an approximation, plots of \( Fr \) in this article do not have the value \( Fr = 1 \) highlighted. Nevertheless, there are seasons and locations on Mars during which \( Fr \), defined by Equation (4), exceeds unity by enough to bring planetary-scale hydraulic jumps into consideration (Dowling et al. 2017).

2. Three Flow Regimes

To gain perspective on the Mach number fields on Mars versus Earth, consider three examples of severe winds on Earth. First, the threshold for the top hurricane designation, Category 5, is \( Ma \gtrsim 0.2 \), specifically \(|v| > 70.5 \text{ m s}^{-1} \), or 158 miles per hour. Second, the notorious gusts on the top of Mount Washington, New Hampshire, reach \( Ma \approx 0.3 \), which is \( 103 \text{ m s}^{-1} \), or 231 miles per hour. Third, the strongest tornadoes top out around \( Ma = 0.38 \), which is \( 130 \text{ m s}^{-1} \), or 300 miles per hour. This third example is the exception that proves the rule that Earth’s atmosphere generally constitutes a \( Ma \lesssim 0.3 \) fluid system. The significance to terrestrial meteorology is that 0.3 is the engineer’s rule of thumb for the start of compressibility effects. The reason the compressible regime starts at 0.3 is reviewed next, along with why modeling transonic flow is challenging.

2.1. Incompressible Flow: \( Ma < 0.3 \)

Shapiro (1953, Section 3.2) provides a lucid description of the physical differences between flow regimes as a function of Mach number. For adiabatic, compressible flow, the dimensionless change in pressure between points (a) and (b) following the flow, where the latter is a stagnation point such as at the end of a pitot tube, is given by

\[
\begin{align*}
\text{compressible: } & \quad \frac{p_b - p_a}{p_a} = \left( 1 + \frac{\kappa}{2} Ma^2 \right)^{1/\kappa} - 1, \quad \text{where } Ma \text{ is the Mach number at point (a), i.e., the upstream value. See Appendix A for a derivation of Equation (6). To obtain the incompressible approximation, valid for } |Ma^2| \ll 1, \text{ apply to Equation (6) the approximation } (1 + \alpha)^n = 1 + n\alpha + O(\alpha^2) \text{ for } |\alpha| \ll 1, \text{ plus the expression for } Ma^2 \text{ in Equation (3) and the ideal-gas equation of state in the form}\end{align*}
\]

\[
RT = \frac{p}{\rho};
\]

\[
\text{incompressible: } \quad \frac{p_b - p_a}{p_a} \approx \frac{1}{1 - \frac{\kappa}{2} Ma^2},
\]

\[
\text{with the rightmost expression states that the difference between the incompressible pressure and static pressure is } \rho_a|v|^2/2, \text{ called the dynamic pressure. The value } Ma = 0.3 \text{ with } \kappa = 0.261 \text{ yields 0.0623 via Equation (6) and 0.0609 via Equation (7), meaning that the latter underestimates the pressure difference by 2.2%. At } Ma = 0.8, \text{ the corresponding values rise to 0.5072 and 0.4330, and the incompressible error rises to 14.6%. The value } Ma = 0.3 \text{ is chosen as the boundary between the incompressible and compressible regimes because it corresponds to 2% error tolerance for pressure.}
\]

2.2. Compressible Flow: \( 0.3 < Ma < 0.8 \)

The compressible regime is set to the range \( 0.3 < Ma < 0.8 \), where the upper limit marks the start of transonic flow. It is shown below that large segments of the atmosphere of Mars are compressible, and in this respect Mars and Earth differ markedly. One effect of compressibility is a delay in the onset of turbulence. For example, the critical Reynolds number needed to initiate turbulence in the viscous boundary layer of a sphere rises by a factor of 2 as the flow speed goes from \( Ma = 0.3 \) to 0.7 (Landau & Lifshitz 1987, p. 182); in other words, at higher speeds less viscosity is needed to maintain stability. Such suppression of turbulence may be relevant to aeolian processes on Mars, including dust lifting. In general, pressure is not symmetric inside parcels in the compressible regime, which is why, to echo Shapiro (1953), the Mach number appears explicitly in the equations of motion. The emerging field of all flow-speed meteorology is discussed in Section 6, with an eye toward what is needed for Mars.

2.3. Transonic Flow: \( 0.8 < Ma < 1.2 \)

A supersonic jet streak that is impinging on a stationary tide can be expected to exhibit features more closely associated with high-performance aeronautics than terrestrial meteorology. Thus, it is significant that supersonic jet streaks occur in every Mars year in both EMARS and OpenMARS, as explored below. Shapiro (1953, p. 773) identifies the transonic regime as \( 0.75 \lesssim Ma \lesssim 1.3 \), which today is usually rounded to \( 0.8 < Ma < 1.2 \). The idea is that such a system can contain juxtaposed subsonic and supersonic flow. The precise value of the upper limit is not particularly important for Mars and so does not play a role in this study, other than to note that the planet does not have jets that stay exclusively supersonic (and none are hypersonic, with \( Ma > 2 \)).

Numerical instability at a grid point often arises if an algorithm’s stencil (i.e., its local sampling of grid points) extends beyond the point’s domain of dependence, which changes depending on the classification of the underlying partial differential equation (PDE) system. An illustration of the role of the Mach number in numerical stability is the reaction of an inviscid, adiabatic flow that is made to deviate slightly from straight-line motion. The original problem is from the study of thin airfoils, but the compressibility effects at play are also
relevant to jet streaks impinging on stationary tides. As derived in Appendix A, for small deflections away from flow in the x-direction, the steady-state case is governed by the linear PDE,

\[(1 - \text{Ma}^2) \phi_{xx} + \phi_{yy} = 0, \tag{8}\]

where \(\phi\) is the velocity potential, defined such that the velocity vector is \((u, v) = (\phi_x, \phi_y)\), and \(\text{Ma}\) is the free-stream Mach number (measured away from the deflection). Where the flow is subsonic, the coefficient \((1 - \text{Ma}^2)\) is positive and Equation (8) is like Laplace’s equation; it can be made exactly so by rescaling the coordinates. Laplace’s equation is the prototypical elliptic PDE, meaning that the domain of dependence of a given point completely surrounds the point. An example of an elliptic problem is the shape of a trampoline’s surface with a bowling ball resting in the middle. The original numerical approach to solving elliptic PDEs is via finite differences, using a centered stencil.

Now, consider that where the flow is supersonic, the coefficient \((1 - \text{Ma}^2)\) in Equation (8) is negative and the PDE is classified as hyperbolic rather than elliptic. In other words, it becomes a second-order wave equation, albeit one that describes steady flow, with the flow-direction coordinate playing the role of time (Versteeg & Malalasekera 2007, p. 35). For hyperbolic PDEs, a centered stencil can easily extend out past the domain of dependence, leading to numerical instability. Consequently, upwind ( upstream) differencing is generally used instead of centered differencing. Murman & Cole (1971) published the first accurate finite-difference solution for transonic flow by adopting a mixed approach that switched from centered differencing in subsonic regions to upwind differencing in supersonic regions (see Cebeci et al. 2005, p. 301). In computational fluid dynamics, such finite-difference schemes have now largely been replaced by finite-volume schemes, and several upwind algorithms have been developed that offer trade-offs between accuracy and computational efficiency (Niculescu & Dănilăi 2013).

The rest of the article is organized as follows. Section 3 examines representative map and meridional-plane snapshots of transonic jet streaks from both EMARS and OpenMARS. Section 4 analyzes the climate statistics of the Mach and Froude number fields from a global-mean perspective, including the area-weighted incompressible fraction \((\text{Ma} < 0.3)\) and transonic fraction \((\text{Ma} > 0.8)\), as a function of altitude and season. Section 5 analyzes regional statistics via global maps and vertical profiles from selected Mars quadrangles, namely, Tharsis (large mountains) versus Arcadia (adjacent to Tharsis) versus Hellas (large basin), and Mare Boreum (north pole) versus Mare Australe (south pole). EMARS and OpenMARS results are directly compared, which provides a convenient indication of uncertainties. Section 6 discusses the current state of affairs regarding all flow-speed meteorology, and Section 7 summarizes and makes conclusions. Appendix B includes derivations of the two compressibility results used above to distinguish the three flow regimes.

3. Transonic Jet Streaks

It is interesting to view snapshots of the Mach number field on Mars, especially when they show phenomena not seen in Earth’s atmosphere. Probably the most intriguing are the transonic jet streaks, which were discovered serendipitously as part of a vorticity—stream function correlation study using the MACDA reanalysis (Dowling et al. 2016). OpenMARS has superseded MACDA, and after interpolating OpenMARS fields onto potential temperature surfaces, a comprehensive bank of animations was made using Panoply (a free netCDF reader available from NASA/GISS). This revealed fast jet streaks, particularly during northern winter. The project was then expanded to include EMARS, and it was determined that transonic jet streaks, with supersonic cores, occur in every Mars year in both reanalyses. Map and meridional-plane views representative of northern winter are shown in Figures 1 and 2, respectively. As mentioned above, plots with a vertical axis include a reference altitude scale that is computed by averaging the geopotential height in the same manner as the variable being displayed. The EMARS and OpenMARS reference altitudes thus obtained are nearly identical on a given \(\theta\) level, so the average between them is shown. See Dowling et al. (2017, Figure 2) for seasonal variations of \(\theta\) versus geopotential height.

In both EMARS and OpenMARS, transonic jet streaks are consistently located in the north polar jet, but they sometimes also show up in the southern hemisphere, for example, in the bottom right panel of Figure 1. The general morphology and location of the winter polar jet are consistent between EMARS and OpenMARS. However, there are notable differences in wind intensity and phase (position) of eddies when comparing the same times and locations. The means and standard deviations examined in the next sections make it clear that differences between EMARS and OpenMARS become significant for \(\theta > 700\) K (\(z > 53\) km), which corresponds to where the GCMs run with little constraint from observations.

In southern winter, the Mach number values are generally not as large as in northern winter (see Figures C1 and C2). This asymmetry between winters is well known on Mars and is largely the result of the planet having strong “distance” seasons (in addition to the familiar “tilt” seasons), which in the current epoch cause the solar insolation in southern summer/northern winter to be much stronger than in northern summer/southern winter. The close proximity of the Sun during the former results in stronger hemispherical contrasts and hence faster winter polar jets, whereas the more distant Sun during the latter results in weaker hemispherical contrasts and slower jets. Transonic jet streaks are less common in southern winter but do occasionally appear (Figure C2, bottom left panel). Around both solstices there is a significant fraction of the atmosphere that exceeds the compressible threshold (\(\text{Ma} > 0.3\)). Since most Mars atmospheric models in operation today have heritage from Earth models that were designed to operate in the incompressible regime, the fraction of the atmosphere that is incompressible, \(f_{\text{in}}\), as a function of season and \(\theta\) (altitude) is of interest and is analyzed below.

During early northern winter, transonic jet streaks often appear near the equator at high altitudes, as shown in Figures 3 and 4. Their location and strength vary from year to year, similar to the variation between MY 33 and 34 shown in Rajendran et al. (2021, Figure 2). They are a regular feature in OpenMARS but are somewhat less prominent in EMARS. For example, they do not appear in EMARS in MY 31, which is why MY 28 is shown instead to represent the second spacecraft, MCS/MRO: the year MY 28 is also interesting because it has a major dust storm during this season. These northern winter, quasi-equatorial, transonic jet streaks have not been the focus of much attention to date, but similar features
can be seen in Forget et al. (1999, Figure 7(c)) and Barnes et al. (2017, Figure 9.9).

At least four factors suggest that the transonic nature of the fastest jet streaks on Mars should be taken seriously. First, transonic jet streaks are a common feature in every Mars year in both EMARS and OpenMARS. Second, one may expect that the real atmosphere has greater contrasts than can be captured in models with 5° horizontal resolution. Third, Doppler shifts in millimeter spectra of carbon monoxide give a broad measure of Martian wind speeds at 40−70 km altitude, which are in general agreement with model output (e.g., Lellouch et al. 1991; Cavalie et al. 2008). Fourth, tenuous atmospheres tend to harbor supersonic flows, with Io being a well-known example (Ingersoll et al. 1985). All told, these provide sufficient motivation to pursue advances in modeling compressibility effects on Mars.

4. Global Statistics

This section focuses on the “when” of the compressibility question, by studying global averages of the Mach and Froude number fields as a function of season and altitude. Horizontal averages are computed on potential temperature (θ) surfaces, which are unbroken by mountain outcroppings via the restriction θ ≥ 400 K, as explained above. The averages are area weighted, which in practice means weighting with the cosine of the latitude (i.e., the sine of the colatitude, which is the familiar spherical-coordinate map factor in mathematics textbooks).

4.1. Global Mach Number

To start off the statistical views, consider first the globally averaged Mach number. Given that winds are related to
temperature gradients, and temperature gradients are accentuated by heating associated with airborne dust, it is to be expected that planet encircling dust storms will strongly affect the planet encircling average of the Mach number. Atmospheric dust can increase temperatures by nearly 50 K and can be well integrated into the atmosphere up to heights of 30–40 km (Greeley et al. 1977). How such a signal manifests in terms of compressibility regimes and how different models portray it are both of interest.

The average Mach number as a function of $L_s$, color-coded by year, is shown for three different $\theta$ levels in Figure 5, with EMARS on the left and OpenMARS on the right. Overall, these time series trace a similar seasonal pattern, albeit with a good amount of interannual variability. Northern winter ($L_s \sim 270^\circ$) stands out as being stronger than southern winter ($L_s \sim 90^\circ$), as expected. The two largest dust storm events that occur in the time span under study, during MY 25 (medium orange) and 28 (medium blue), make themselves known in the average Mach number. Interestingly, both are southern-born exceptions to the rule that dust storms originating in the southern hemisphere (southern sequences) are usually smaller than those originating in the northern hemisphere (northern sequences) (Wang & Richardson 2015). One can plainly see the connection between global-mean Mach number and dust by comparing Figure 5 to the time series of dust optical depth versus $L_s$ in Montabone et al. (2015, Figure 16). Generally, EMARS and OpenMARS qualitatively agree, but there are notable differences, particularly at higher altitudes. The MY 28 dust storm, which occurred during northern winter, shows a similar average Mach number at $\theta = 1000$ K, but EMARS rises higher at $\theta = 700$ K, and OpenMARS drops lower at $\theta = 400$ K. In a few years, specifically MY 25, 26, and 27, southern winter is significantly stronger in

Figure 2. Meridional-plane view corresponding to Figure 1. These are at longitudes 3°0 and 2°5 (not zonally averaged) for EMARS and OpenMARS, respectively, which are the closest native grid points to 0°. The vertical coordinate is potential temperature, $\theta$; the corresponding reference altitude is indicated on the right.
EMARS at $\theta = 700$ and 1000 K. Meanwhile, during northern winter at $\theta = 1000$ K, OpenMARS often crosses the transonic threshold, whereas EMARS does not, except during the MY 28 dust storm. Both reanalyses exhibit a similar incompressible period ($Ma < 0.3$) that spans about 60º in solar longitude at all three $\theta$ levels. However, the phase differs: EMARS is shifted earlier and OpenMARS later, such that their incompressible periods only overlap during $L_s \approx 20^\circ - 40^\circ$. The lack of a tiebreaker between the two reanalyses suggests that a simple probabilistic interpretation is warranted: incompressibility is most likely when both exhibit it and least likely when neither does.

To see more clearly at what altitudes and seasons the incompressible, compressible, and transonic regimes come into play in the global sense, the averaged Mach number is plotted versus $\theta$ for the four canonical season points, $L_s = 0^\circ$, 90$^\circ$, 180$^\circ$, and 270$^\circ$, for EMARS and OpenMARS in Figures 6 and 7, respectively. The profiles at the equinoxes (left panels) are nearly the same and for the most part remain just inside the incompressible regime, with OpenMars tending to lean into the compressible regime at the higher altitudes. Significantly, Mars often moves into the compressible regime during the solstices (right panels), and decisively so during northern winter (bottom right panels), which is the season that exhibits the largest variations in the average Mach number across years, as well as the largest variations between EMARS and OpenMARS. In this season at the upper altitudes, OpenMARS actually goes transonic about half the time. This is a good way to showcase the distance–season effect on Mars.

4.2. Global Froude Number

Maps of the Froude number field qualitatively mimic the Mach number field (and hence are not shown). Recall from above that in a nearly isothermal region, given $\kappa = 0.261$, the Froude number values are approximately 2.28 times the Mach number values. Small temperature lapse rates relative to the
5.05 K km$^{-1}$ dry adiabatic lapse rate do typically hold away from the poles in the $z \approx 32–66$ km altitude range under study. As previously mentioned, no thresholds are highlighted in the Froude number figures in this article, since the local value of $NH$ is used to characterize the buoyancy wave speed, rather than a modal (eigenvalue) analysis.

Vertical profiles of the globally averaged Froude number are shown in Figures 8 and 9 for EMARS and OpenMARS, respectively. Overall, EMARS tends to be more muted than OpenMARS in terms of amplitudes of both Fr and Ma. The OpenMARS profiles are quite similar to their corresponding Mach number profiles in Figure 7. Apart from subtle details at the higher altitudes, for example, between MY 24 (medium red) and 28 (medium blue) in northern winter (bottom panels), the Froude number values are roughly twice the corresponding Mach number values, as anticipated. The EMARS profiles exhibit the same approximate factor of two, but above about $\theta = 600$ K, the Froude number profiles in Figure 9 do not follow the same shape as their corresponding Mach number profiles in Figure 6; instead, they include kinks. The vertical resolution in EMARS at the higher altitudes is less than that in OpenMARS, and the calculation of $N^2 = (g/\theta)(d\theta/dz)$ involves a vertical derivative, which may explain much of this discrepancy. In Appendix D, it is pointed out that EMARS occasionally yields $N^2 < 0$ values (these are removed before proceeding; otherwise, imaginary Froude numbers ensue), which are not seen in OpenMARS, and OpenMARS occasionally exhibits filamentary structures in $N^2$ that descend with time, which are not seen in EMARS.

During northern winter, both reanalyses show globally averaged Froude numbers well in excess of unity, which is an indicator that planetary-scale hydraulic jumps may be occurring.

Figure 4. Meridional-plane view corresponding to Figure 3. Longitudes are as in Figure 2.
at high altitude. Mars has well-documented mesoscale hydraulic jumps near the surface, which are associated with katabatic winds as discussed in Section 1.4. A conundrum is that EMARS and OpenMARS both have 5° horizontal resolution (and their underlying models have only slightly higher resolutions), which is orders of magnitude too coarse to resolve hydraulic jumps. Once this computational hurdle is cleared, perhaps with nested grids, there seems to be sufficient evidence to warrant a Mars modeling study of supercritical hydraulics aloft and on the planetary scale during northern winter.

4.3. Incompressible and Transonic Fractions

With an eye toward model development, consider the global fraction of the atmosphere that is incompressible (Ma < 0.3), $f_{in}$, and the fraction that is transonic (Ma > 0.8), $f_{tr}$, as a function of season and altitude. For convenience, the complement is not displayed (0.3 ≤ Ma ≤ 0.8) but can be inferred from $1 - (f_{in} + f_{tr})$. Recall that on Earth, $f_{in} \approx 1$ and $f_{tr} = 0$ on a year-round basis.

Vertical profiles of the global incompressible fraction for EMARS and OpenMARS, for the four canonical season points
(snapshots), are shown in Figures 10 and 11. Large values are good news for Mars atmospheric models that rely on incompressible algorithms inherited from Earth models, and small values are cause for concern. The equinoxes (left panels in the figures) are noticeably different between EMARS and OpenMARS but quite similar for a given model from year to year. OpenMARS continues to present as more variable than EMARS. The vertical averages of each profile (averaging from 400 to 1100 K, or from $z \approx 32$ to $66$ km) are listed in Tables 1 and 2, including a bottom row that provides best-available season-point climate means. Overall, the trend in fall and spring for EMARS is about 70% incompressible, regardless of altitude. For OpenMARS, this 70% value holds below about $\theta = 500$ K, but $f_{in}$ leans back with height and shows significant interannual variability. For southern winter ($L_s = 90^\circ$) the vertical average drops to 50%, and for northern winter ($L_s = 270^\circ$) it plummets to 28% (Table 1). OpenMARS is in agreement with EMARS for northern winter, with a similarly shaped envelope of profiles, albeit showing significant differences in some years. The vertical average of 29% (Table 2) is nearly identical to EMARS. On the other hand, at $L_s = 90^\circ$, OpenMARS in MY 25, MY 26, and to some extent MY 29 seems to be not settled into the profile typical of the other years, which also match the shape of the EMARS profiles at $L_s = 90^\circ$. Figure 12 shows the evolution of $f_{in}$ throughout the seasons for each Mars year for EMARS and OpenMARS. The noticeable dip in $f_{in}$ during northern winter at 700 and 1000 K goes hand in hand with the higher Mach numbers observed during this season in Figure 5. From the engineering point of view, the atmosphere of Mars clearly presents as a compressible fluid system.

The main result of this article is therefore that in the vertical range under consideration there is no season on Mars that exhibits comfortably large values of $f_{in}$, approaching 100% like on Earth, which would justify advection and flux algorithms designed for the incompressible regime, and that this becomes acute during northern winter. But there is a limit—Mars does not have prevailing supersonic jets. Figure 13 shows the transonic fraction profiles for EMARS and OpenMARS, side by side at $L_s = 270^\circ$ (the other season points all have negligible values and are not shown). Both show negligible signal for $\theta < 600$ K, with a large flare-up for $\theta > 600$ K in some years. OpenMARS again exhibits a systematically larger signal than EMARS. The vertical averages and climate means for $f_{tr}$ at $L_s = 270^\circ$ are listed in the rightmost column in Tables 1 and 2.

From the meteorological perspective, the fact that both reanalyses show any $f_{tr}$ signal at all is remarkable.

5. Regional Statistics

Moving to the question of “where” compressibility becomes an issue, consider the following temporal averages as a function of position. Figures 14 and 15 show average and
standard deviation maps of the Mach number field, for $\theta = 400$, 700, and 1000 K. To avoid uneven sampling of seasons, these are restricted to include the five years that have complete coverage (or nearly so) in $L_s$ ($0^\circ$–$360^\circ$), which are MY 25 and 26 from TES input and MY 29, 30, and 31 from MCS input. The sampling interval is 2 Martian hours, equal to $1/12$ sol (such that time is sampled at the constant rate of the mean anomaly, rather than at the variable rate of the true anomaly or solar longitude).

EMARS and OpenMARS have quite similar 5 yr mean Mach number fields at all three levels, especially at $\theta = 400$ K. The polar jets are obvious throughout the vertical range in view. There is no transonic threshold in the mean ($Ma = 0.8$), which suggests that improvements to the modeling of transonic effects are not likely to increase the accuracy of long-term climate simulations, just the local conditions associated with transonic jet streaks. The stationary wavenumber 2 character—a well-known feature of Mars GCMs—shows up well in these isentropic-coordinate averages. This wavenumber 2 structure is linked to large-scale topography: strong eastward winds in the winter hemisphere interact with zonally varying topography to form stationary planetary waves (e.g., Hinson et al. 2003). For orientation, the locations of the peaks of the four main volcanoes in the Tharsis region are indicated in the left panels of Figure 14. It is clear that they align with one of the maxima in the wavenumber 2 pattern. This planetary-scale pattern has been found to predominate in the northern extratropics with periods of 2–4 days, and these waves influence much of the background mean seasonal variations in temperature, momentum, and transport (e.g., Lewis et al. 2008). Apparent in the 5 yr means are Earth-like, incompressible Mach numbers situated equatorward of the polar regions, at $\theta = 400$ K (left panels). Elsewhere, Mars is decidedly un-Earth-like. Specifically, in the polar jet regions at all levels, as well as in the equatorial regions around $\theta = 700$ K ($z \approx 53$ km), both EMARS and OpenMARS have prevailing Mach numbers that are in the compressible regime.

Maps of the standard deviation about the 5 yr mean are shown in Figure 15. Qualitatively, EMARS and OpenMARS exhibit similar spatial patterns, with OpenMARS tending to have larger amplitudes in the north polar region. In the Tharsis region, especially at $\theta = 400$ K (left panels, on the left side of the equatorial region, around longitude $-120^\circ$), both reanalyses show an intriguing horseshoe- or partial-doughnut-shaped ridge of signal. This pattern approximately lines up with the peaks of the four main volcanoes (Smith et al. 2003), almost like a connect-the-dots game, which signifies that the area in between is acting as a planetary wave node. The Tharsis bulge is known to play a significant role in the topographical structure of both hemispheres as a main, large-scale, high-relief region (e.g., Hollingsworth & Barnes 1996). The standard deviation in the north polar region is larger in OpenMARS at all three levels, consistent with the interannual variability in the global signal discussed above. The southern hemisphere at $\theta = 1000$ K in OpenMARS is anomalous in the sense that the variability is not small anywhere (Figure 15, bottom right panel). OpenMARS exhibits some prevailing meridional
5.1. Comparing and Contrasting Quadrangles

The surface of Mars is divided cartographically into 30 quadrangles (see Figure F1), and it is instructive to compare and contrast temporal averages of the Mach and Froude number fields in representative examples; five are viewed here. These include the northern and southern polar “quadrangles,” which are polar stereographic projections covering latitudes 90° (pole) to 65°, named Mare Boreum (MC-1, north) and Mare Autrale (MC-30, south). Comparing these helps to quantify the distance–season effect between northern and southern winter. Moving equatorward, six conical orthomorphic (Lambert conformal) projections in each hemisphere (northern and southern) span latitudes 65°–30°. Two are examined here, Arcadia (MC-3, northern hemisphere) and Hellas (MC-28, southern hemisphere). The former is adjacent to Tharsis and has its southern region dominated by Alba Mons, the broadest shield volcano in the solar system, and a relatively smooth northern region. The latter, Hellas, contains the largest impact basin on Mars. Next, eight Mercator projections in each hemisphere span latitudes 30°–0° (equator). Examined here is Tharsis (MC-9, northern equatorial), which contains the tallest volcano in the solar system, Olympus Mons, plus three more in a line, Arsia Mons (previously mentioned in the context of outcropping), Pavonis Mons, and Asaphus Mons. A sixth quadrangle was studied, Coprates (MC-18, southern equatorial), which is kitty-corner to Tharsis and contains the better part of Valles Marineris, the longest canyon system in the solar system. For brevity, it is not shown since its signal is nearly identical to Hellas.

The vertical profiles for Tharsis, Arcadia, and Hellas are shown in Figure 16. These are 5 yr means, as in the maps above. Since the individual years are not plotted separately, it is convenient to put the EMARS and OpenMARS profiles on the same plot, in red and blue, respectively. The means and standard deviations also fit together. The Mach and Froude number fields are shown in the left and right panels, respectively. EMARS and OpenMARS give qualitatively similar results. The Froude numbers in Hellas are essentially identical for the two reanalyses, for $\theta = 400–700$ K (bottom right panel). In Hellas and Arcadia, the EMARS (red) values tend to be systematically lower than OpenMARS (blue) values. This switches in Tharsis, where EMARS is systematically higher than OpenMARS, over the range from $\theta = 400$ to about 850 K, above which the two are similar.

The agreement between EMARS and OpenMars is sufficient to deduce that the isentropic-coordinate, time-mean Mach numbers in Hellas and Arcadia are alike, as are the Froude numbers (the same holds for Coprates). At $\theta = 600$ K and above, the mean Mach numbers follow along the compressible threshold, $Ma = 0.3$. The standard deviations are similar between the reanalyses, both broad with a modest flare-out with altitude, and a bit broader in Arcadia than Hellas. These profiles establish that Mars is in the compressible regime on a regional basis, even outside the polar regions. In Hellas, below
θ = 600 K, the mean Mach numbers swing down into the incompressible regime, and the standard deviation tightens slightly. The mean Mach numbers in Tharsis are lower than in Hellas and Arcadia. Although they are below the compressible threshold, in the 1σ sense the standard deviations still place Tharsis in the compressible regime everywhere, except at the bottom of the range. The fact that the standard deviations in Tharsis are smaller than in Hellas and Arcadia is consistent with the partial doughnut in the maps. It will be interesting to see how these trends play out going below θ = 400 K, which is planned for a future study.

In the two polar quadrangles, the EMARS and OpenMARS means are in good agreement with each other over the full vertical range (Figure C3). The largest disagreement is for the northern Froude number in which EMARS is systemically larger than OpenMARS, but not appreciably so. As can be expected from the results to this point, the mean Mach number profiles are well into the compressible regime at both poles in both reanalyses, for θ > 500 K. A surprising result of this study is that the north polar versus south polar 5 yr mean Mach numbers are practically the same. The only outlier is the northern OpenMARS profile below θ = 700 K, which is systematically lower than the others. That is, even though the north polar jet is stronger than the south polar jet in a sustained manner around the solstices, as shown in Sections 3 and 4, the vertical profiles of the 5 yr mean Mach numbers are the same, north versus south. The difference comes through in the standard deviations, which are appreciably larger in the north than in the south, everywhere except at the bottom, where they come together.

6. Discussion

The main takeaway from the global and regional results above is that, in general, Mars meteorology and climate operate in the compressible regime. The planet proves to not go so far as to have a transonic climate, just occasional transonic jet streaks. In contrast, Earth’s fluid system operates in the incompressible regime. Yet even so, compressible meteorology is emerging as an important subdiscipline in terrestrial operational weather and climate forecasting (e.g., Read 1988; Davies et al. 2003; Benacchio & Klein 2019). The umbrella phrase “all flow-speed meteorology” may be used to include Earth, Mars, and exoplanets, and the following is a summary of the issues involved.

6.1. Incompressible Meteorology

In dynamic meteorology, one traditionally uses scale analysis or asymptotics to judge the relative sizes of the terms in the differential equations describing conservation of mass, momentum, and energy and then trims the equations accordingly. Various versions of this procedure have produced the quasigeostrophic, primitive hydrostatic, anelastic, and pseudo-incompressible models, which together have been the workhorses for numerical weather forecasting and process studies for the better part of a century (see Benacchio & Klein 2019). A practical benefit of trading away some measure of compressibility is that
sound waves can be filtered out. Sound waves restrict the size of a model’s time step: in the case of an explicit scheme, the time step must be small enough to maintain numerical stability, and in the case of an implicit scheme, the time step must be small enough to maintain phase accuracy.

Fortunately for terrestrial meteorologists, Earth’s fastest atmospheric jets stay just within the incompressible regime. But an unintended consequence of this for planetary applications is that incompressible thinking has infiltrated much of the theory and algorithms developed for Earth’s atmosphere, yet incompressible warning labels have for the most part not been affixed or propagated. In the dynamic meteorology curriculum the Mach number itself has been all but expunged from the equations of motion. There are exceptions, for example, Vallis (2017, p. 42) demonstrates that $|\text{Ma}|^2 \ll 1$ is the condition for incompressible flow, and Pierrehumbert (2010, Section 8.7.4) features $\text{Ma}$ in a full account of atmospheric escape to space. But these exceptions prove the rule because the list of dynamic meteorology textbooks that make no mention of “Mach” in their indices is long and impressive (e.g., Gill 1982; Holton & Hakim 2013; Wallace & Hobbs 2006).

A basic example of an incompressible construction is the traditional parcel method, which is used among other applications to calculate the buoyancy wave frequency (e.g., Holton & Hakim 2013; Vallis 2017). In that theory, air parcels are assumed to move both not too slowly and not too quickly.

The not-too-slowly restriction assures that the flow is nearly adiabatic, since air is a decent insulator when there is not enough time for appreciable heat exchange between the parcel and its environment. The not-too-quickly restriction, specifically $\text{Ma} < 0.3$, assures that the flow is nearly incompressible, which allows enough time for sound waves to communicate pressure changes throughout the parcel’s interior. In other words, in the parcel method the interaction between a parcel and its environment is all or nothing: for pressure it is all, the parcel takes on the environment’s value instantaneously, with no hint of Mach cones or wakes, whereas for entropy (i.e., potential temperature) it is nothing, the parcel ignores the environment’s value and preserves its own. While a derivation based on the parcel method is valid and useful within the limits of its assumptions, when it comes to long-term weather and climate simulations, expunging the Mach number turns out to not be a harmless abridgment.

6.2. Operational Compressible Meteorology

For the quasi-geostrophic and primitive hydrostatic models, a limitation is that convection (moist or dry) is inherently a nonhydrostatic process. Thus, one of the early drivers for moving from incompressible to compressible meteorology was the need to move from hydrostatic to nonhydrostatic models, a transition that is now nearly complete in operational meteorology. But it is now appreciated that, more generally,
suppressing compressibility artificially reduces the pressure asymmetry along the direction of motion, which leads to accumulating errors in advection and fluxes. A study by Davies et al. (2003) used a normal-mode approach (instead of scale analysis) to assess accuracy of Earth atmospheric models and concluded that “for global nonhydrostatic modeling, only the fully compressible equations are suitable.”

There are today several atmospheric models in operation that terrestrial meteorologists consider to be “fully compressible” (a representative list is given in Benacchio et al. 2014). These are regarded to be so because they are valid in both the large-scale limit, which is the hydrostatic limit where the vertical pressure gradient and gravity are in balance, and the small-scale limit, the domain of anelastic and pseudo-incompressible models, collectively called “soundproof” models because they use an elliptic pressure equation to filter out sound waves (Benacchio & Klein 2019). Most Mars atmospheric models to date are adaptations of Earth models that predate this move to fully compressible equations. The above Mach number results

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**Table 1**

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**Table 2**

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Note. The bottom row serves as a best-available climate mean for each season point. For comparison, Earth has $f_m \approx 100\%$ and $f_v = 0\%$ year-round.
support a renewed effort to adapt the newest compressible Earth models to Mars. A systematic study applied to Mars models along the lines of Davies et al. (2003) would help to guide this endeavor.

Looking ahead to when compressible meteorology is included in the planetary science curriculum, one detail that would be useful for applications to Mars and Mars-like planets is if such textbooks signaled which equations are only accurate at low Mach number. In fact, precisely this has been done to alleviate an analogous problem in physical chemistry: in Atkins (1994, p. 27), each equation that assumes the ideal-gas equation of state is signaled with a superscript ° on its equation number.

6.3. All Flow-speed Meteorology

Beyond compressible operational meteorology, there is a further step that the results above motivate, which broadens the field to include all flow-speed meteorology. In practical terms, this is the intersection of meteorological concerns with aeronautical engineering concerns and has already been a focus in certain corners of astrophysics. Taking a step back from meteorology and surveying fluid dynamics as a whole, coverage of transonic effects varies between subdisciplines in more or less the pattern one might expect. Intermediate material is found in engineering fluid mechanics texts (e.g., Shapiro 1953; Munson et al. 2006; Kundu et al. 2016) and in computational fluid dynamics texts (e.g., Cebeci et al. 2005;
Introductory material is found in astrophysics texts (e.g., Zirin 1988; Shu 1992), in geophysics texts that cover magnetospheres (e.g., Priest 1985; Bertotti & Farinella 1990; Bagenal et al. 2004), and in broad fluid dynamics texts (e.g., Batchelor 1967; Whitham 1974). But circling back to planetary atmospheres, “Mach” again vanishes from the indices of textbooks (e.g., Morrison & Owen 1996; Sánchez-Lavega 2011; Mackwell et al. 2013), even in comprehensive treatments of Mars (e.g., Kieffer et al. 1992). A breakthrough exception is research on hot Jupiters, where supersonic jets are now being simulated with shock-capturing models (e.g., Fromang et al. 2016).

The challenge for modeling Mars and Mars-like atmospheres is to develop flux and advection algorithms that satisfy the three desirable traits of modeling, accuracy, numerical stability,
and computational efficiency, while simultaneously covering the three Martian flow regimes, incompressible, compressible, and transonic. Solving this all flow-speed challenge is a goal currently shared by independent groups working in astrophysics, aeronautical engineering, and meteorology. It has not proven to be easy—fixing one end of the Mach number line tends to break the other. Perhaps surprisingly, researchers who focus on accurately modeling transonic flow consider the low end of the Mach number line to be their trouble area (e.g., Klein et al. 2001; Liou 2006). Their compressible computational fluid dynamics codes tend to have two problems in the limit Ma → 0: stalled convergence and grossly inaccurate solutions. The latter often manifests as spurious pressure oscillations and is tied to the algorithms used to model fluxes. In other words, what terrestrial meteorologists take for granted strikes fear in the hearts of aeronautical engineers, who think nothing of precisely capturing transonic flows. Nevertheless, given the needs across these disciplines, there are now concerted (if not coordinated) efforts to develop all flow-speed algorithms (e.g., Klein et al. 2001; Liou 2006; Benacchio et al. 2014; Benacchio & Klein 2019). These are timely efforts, especially considering that on Mars a helicopter drone has already been flown, and has even suffered the first unexpected weather-related flight delay on another planet (NASA’s Ingenuity Helicopter), and boots-on-the-ground astronauts are eminent. To summarize, the emerging discipline of Mars operational forecasting needs to be supported by all flow-speed model development.

7. Conclusions

Two recent global reanalyses of the atmosphere of Mars, EMARS and OpenMARS, each constrained by the same orbiting spacecraft observations, TES/MGS for MY 24 to 27 and MCS/MRO for MY 28 to 32, have been used to calculate the Mach number field in the potential temperature range θ = 400–1100 K, corresponding to the altitude range z ≈ 32–66 km. This has yielded an assessment of the compressibility of the Martian atmosphere as a function of season, year, map position, and isentropic vertical level. The Froude number field, with NH used to estimate the buoyancy (gravity) wave speed, has likewise been analyzed, which has characterized the coarse-grained atmospheric hydraulics. It is clear that having two such reanalyses to intercompare increases the value of studies such as this.

Figure 15. 5 yr standard deviations of the Mach number field, corresponding to the means in Figure 14.

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Figure 16. Vertical profiles of 5 yr means of the Mach number (left panels) and Froude number (right panels) for EMARS (red solid curves) and OpenMARS (blue solid curves), in the quadrangles Tharsis, Arcadia, and Hellas. The 5 yr are as in Figure 14. The corresponding standard deviations are indicated by the dotted curves, plotted as the mean plus or minus one standard deviation.
The spatial and temporal averages of the Mach number generated in this study establish the “when” and “where” of Mars compressibility. The stationary planetary wavenumber 2 structure of the Martian atmosphere, which is a feature of all Mars GCMs, shows up clearly in the isentropic-coordinate maps. Five-year means and standard deviations as a function of altitude have been computed in five quadrangles: Mare Boreum, Mare Australe, Tharsis, Arcadia, and Hellas. These Mach and Froude number means generally increase with altitude and are highest in the polar quadrangles (Mare Boreum and Mare Australe), as expected. All five quadrangles exhibit supersonic winds at some point, not just the polar regions. Regarding the poles, an unexpected result is that the Mare Boreum (north) and Mare Australe (south) Mach number 5 yr means are essentially identical. The statistical difference is evident in the standard deviation, which is larger in Mare Boreum than in Mare Australe. Tharsis stands out as having lower mean Mach number than the other quadrangles examined—it is the only one with mean $M_a < 0.3$—and it also has an interesting partial-doughnut standard deviation structure associated with its volcanoes.

The area-weighted, global incompressible fraction ($M_a < 0.3$) stays in the range 50%–70% for most of the year, dropping to about 30% during northern winter. On Earth, this fraction stays at 100% in all seasons and all locations; hence, the two planets are fundamentally different in terms of compressibility. This has major ramifications in terms of atmospheric model development (and model legacy). The Froude number signal is roughly twice the Mach number signal, as expected where the temperature lapse rate is small compared to the dry adiabatic lapse rate. Thus, when Mars is well into the compressible regime in terms of Mach number, it is encroaching on, and possibly exceeding, the supercritical regime in terms of Froude number.

A natural by-product of this analysis is the direct comparison of EMARS and OpenMARS as a function of season and location. This is helpful because, although both are constrained by the same spacecraft observations, each employs an independent GCM and data assimilation scheme. While a study like this cannot distinguish which model is more like Mars (essentially all available data are already folded into each reanalysis), it is encouraging that maps of the climate mean of the Mach number are nearly identical between the two reanalyses, at all levels examined. Point-for-point agreement between the reanalyses tends to be best at the lower altitudes examined, $\theta < 700$ K ($\sim$50 km), where the spacecraft observations best constrain the GCMs. At higher altitudes, disagreements between EMARS and OpenMARS are not uncommon and can become large at times.

The differences seen between EMARS and OpenMARS naturally lead to the question of what is causing them. As pointed out above, even though both reanalyses are constrained by the same observations, the underlying atmospheric models and data assimilation schemes differ. To begin to address the differences, note that it is likely that the models have different numerical dissipation. Forget et al. (1999) showed two models (one the ancestor of OpenMARS) with identical physical schemes but different numerical dissipation, which produced significantly different winds in the equatorial upper atmosphere. To further pinpoint what are the most important algorithmic differences will require systematic comparisons. As an aid to that endeavor, the variables in this study, dimensionless fields on isentropic surfaces, are proving to be sensitive diagnostics.

Regarding future model development, it is time to simulate the atmosphere of Mars with fully compressible models, in both the nontransonic sense of the latest Earth operational models and the shock-capturing sense of the latest astrophysical models. On the empirical front, work is in progress to extend this study from $\theta = 400$ K down to the surface. Since there are mountain outcroppings at least some of the time at these lower levels, the focus is on regional statistics, including an examination of links between topographic features and variations in compressibility.

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### Appendix A

#### Derivations Associated with Regime Thresholds

Derivations of two fundamental results in compressible fluid mechanics referenced in the main text, but rarely encountered in the atmospheric dynamics curriculum, are given here. Consider first the physics that leads to the identification of $M_a = 0.3$ as the threshold between incompressible and compressible flow. The following is adapted from Munson et al. (2006, pp. 128–130). One begins by integrating Newton’s second law, in the form $a = f/m$, along a streamline to obtain the Bernoulli energy equation. The forces per mass on the right-hand side are the pressure gradient force and gravity, with friction assumed to be negligible. Integrating $a = f/m$ yields

$$KE = -\int \rho^{-1} dp - \Phi + C,$$  \hspace{1cm} (A1)

where $KE$ is the integral of the acceleration; $-\int \rho^{-1} dp$ is the integral of the pressure gradient force, where $\rho$ and $p$ are the density and pressure, respectively; $-\Phi$ is the integral of the gravity, which is the negative of the geopotential ($\Phi$ is also known as the potential energy per mass); and $C$ is the usual constant of integration for an indefinite integral, which here plays the role of total energy per mass following the flow.

The next step is to specify the process governing density as a function of pressure, so that $-\int \rho^{-1} dp$ can be evaluated. Isentropic flow is usually assumed (i.e., adiabatic and reversible flow), which relates $\rho$, $p$, and $T$ via pairwise power laws; the one needed here relates the reciprocal of density to pressure: $\rho^{-1} = \rho_0^{-1} (p/p_0)^{\kappa-1}$, where $\rho_0$ and $p_0$ are reference values. As discussed in Section 1, the dimensionless ratio $\kappa = R/c_p$ varies with $T$, but for this illustration the constant value $\kappa = 0.261$ is assumed. In passing, it is worth noting that oceanographers deal with similar issues caused by variable $c_p$, but instead of treating $c_p$ and $\kappa$ as constants, they achieve accurate results by defining a potential enthalpy and a corresponding “conservative temperature” (McDougall 2003), an approach that would seem to be worth pursuing for Mars (and Venus). Sticking with constant $\kappa$ to see where $M_a = 0.3$ comes from, integration of the pressure gradient force in (A1) between two points (a) and (b) along a streamline yields the Bernoulli energy equation for
isentropic flow:

\[
\left( KE + \Phi + \frac{1}{\kappa} \rho \right)_{a} = \left( KE + \Phi + \frac{1}{\kappa} \rho \right)_{b}.
\] (A2)

To highlight the difference between compressible and incompressible flow, simplify Equation (A2) by restricting to quasi-horizontal motion such that \((\Phi)_{a} - \Phi_{b}\) is negligible, and take the downstream point \((b)\) to be a stagnation point, \(KE_{b} = 0\), for example, the end of a pitot tube (e.g., Perry et al. 1997, p. 10-8). With these restrictions, Equation (A2) reduces to

\[
KE_{a} + \frac{1}{\kappa} \rho \left( \frac{p_{b}}{\rho_{b}} \right)^{\kappa} = \frac{1}{\kappa} \rho \left( \frac{p_{a}}{\rho_{a}} \right)^{\kappa}.
\] (A3)

Working on Equation (A3) from left to right, Equation (3) brings in the upstream Mach number,

\[
KE_{a} = c_{s}^{2} \frac{Ma^{2}}{2}.
\] (A4)

For the second term in Equation (A3), the ratio \(c_{p}/c_{v}\) in Equation (2) equals \(c_{p}/(c_{p} - R) = 1/(1 - \kappa)\), which, when combined with the ideal-gas law, \(p = \rho RT\), yields

\[
\left( \frac{1}{\kappa} \rho \right)_{a} = c_{s}^{2} \left( \frac{1 - \kappa}{\kappa} \right).
\] (A5)

For the third term in Equation (A3), Equation (A5) and the isentropic density–pressure power law in the form \((\rho_{a}/\rho_{b}) = (p_{a}/p_{b})^{\kappa-1}\) yield

\[
\left( \frac{1}{\kappa} \rho \right)_{a} = \left( \frac{p_{b}}{\rho_{b}} \right)^{\kappa} \left( \frac{p_{a}}{\rho_{a}} \right)^{\kappa}.
\] (A6)

The sound-speed factor cancels out of all three terms, and then Equation (A3) with Equations (A4)–(A6) rearranges into a dimensionless change in pressure between points \((a)\) and \((b)\),

compressible: \[\frac{p_{b} - p_{a}}{p_{a}} = \left(1 + \frac{\kappa}{1 - \kappa} \frac{Ma^{2}}{2} \right)^{1/\kappa} - 1,\] (A7)

where the subscript has been dropped from the upstream Mach number for convenience. Equation (A7) corresponds to Equation (6) in the main text.

Consider second the technical difficulties associated with transonic flow. Following Shapiro (1953, pp. 286–307), it is sufficient for inviscid (frictionless) flow to assume the irrotational (zero-vorticity) property, which allows the velocity to be expressed as the gradient of a velocity potential, \((u, v, w) = (\phi_{x}, \phi_{y}, \phi_{z})\), where the subscripts denote partial derivatives, in which case conservation of mass in conservative form in Cartesian coordinates is

\[
\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} (\rho \phi_{x}) + \frac{\partial}{\partial y} (\rho \phi_{y}) + \frac{\partial}{\partial z} (\rho \phi_{z}) = 0.
\] (A8)

Under compressible conditions, the density, \(\rho\), varies strongly with position in Equation (A8). For the purposes of this example, restrict Equation (A8) to steady flow \((\partial/\partial t = 0)\) and to two dimensions in the \((x, y)\)-plane, and then expand to get

\[
\rho(\phi_{xx} + \phi_{yy}) + \rho_{x} \phi_{x} + \rho_{y} \phi_{y} = 0.
\]

Put this into standard form by dividing by density:

\[
\phi_{xx} + \phi_{yy} + (\ln \rho)_{x} \phi_{x} + (\ln \rho)_{y} \phi_{y} = 0.
\] (A9)

So far, only conservation of mass has been used. To see the role of the Mach number, recall that the adiabatic speed of sound is \(c_{s}^{2} = \partial p/\partial \rho|_{\text{adiab}}\), which by the chain rule gives the relations

\[
\text{adiabatic: } (\ln \rho)_{x} = \frac{1}{\rho} \frac{\partial p}{\partial c_{s}^{2}}; \text{ } (\ln \rho)_{y} = \frac{1}{\rho} \frac{\partial p}{\partial c_{s}^{2}}.
\] (A10)

Fold these into Equation (A9) to obtain

\[
\phi_{xx} + \phi_{yy} + \frac{1}{c_{s}^{2}} \left( \frac{\rho_{x}}{\rho} \phi_{x} + \frac{\rho_{y}}{\rho} \phi_{y} \right) = 0.
\] (A11)

To relate the pressure gradient factors in Equation (A11) back to the velocities, use conservation of momentum, which for this steady, quasi-horizontal configuration may be written as

\[
\begin{align*}
uu_{x} + vv_{y} &= \phi_{x} \phi_{xx} + \phi_{y} \phi_{yy} = -\frac{p_{x}}{\rho}, \\
uu_{x} + vv_{y} &= \phi_{x} \phi_{xx} + \phi_{y} \phi_{yy} = -\frac{p_{y}}{\rho}.
\end{align*}
\] (A12)

Fold these into Equation (A11), and thereby obtain the governing nonlinear PDE for steady, two-dimensional, inviscid, irrotational, adiabatic flow:

\[
\left(1 - \frac{\phi_{x}^{2}}{c_{s}^{2}}\right) \phi_{xx} + \left(1 - \frac{\phi_{y}^{2}}{c_{s}^{2}}\right) \phi_{yy} - 2 \frac{\phi_{x} \phi_{y}}{c_{s}^{2}} \phi_{xy} = 0.
\] (A13)

Notice the preponderance of \((1 - Ma^{2})\) factors, one for each velocity component (this also holds in three dimensions). Shapiro (1953) analyzes a comprehensive hierarchy of systems like Equation (A13). For this example, it is sufficient to restrict to uniform x-direction flow that is forced to make small deflections, which reduces Equation (A13) to the linear PDE,

\[
(1 - Ma^{2}) \phi_{xx} + \phi_{yy} = 0,
\] (A14)

where Ma is the free-stream value away from the deflections. Equation (A14) corresponds to Equation (8) in the main body.

Appendix B
Derivations of the Froude Number

In a hydrostatic atmosphere the speed of sound and \(NH\) are coupled via the same physics that makes the column-integrated internal energy and gravitational potential energy proportional to each other. Meteorologists discuss this proportionality in connection with available potential energy in a hydrostatic column (e.g., Holton & Hakim 2013, pp. 227–228), and astrophysicists discuss it in terms of the virial theorem applied
to hydrostatically balanced stars (e.g., Schwarzschild 1958; Shu 1982). There is a related coupling between the sound and buoyancy wave speeds that is helpful to recall when comparing Mach and Froude number fields. Consider the ratio of the square of the Mach number \( \text{Ma}^2 \) to the square of the Froude number \( \text{Fr}^2 \) as defined by Equation (4):

\[
\frac{\text{Ma}^2}{\text{Fr}^2} = \frac{1}{c_s^2 N^2 H^4} = \left( \frac{1 - \kappa}{RT} \right) \left( \frac{g}{\theta dz} \right) \left( \frac{R^2 T^2}{g^2} \right) = (1 - \kappa) \left( \frac{d}{dz} \ln \theta \right) \left( \frac{RT}{g} \right).
\]

Apply Equation (1) and write the local temperature lapse rate as a factor \( \alpha \) times the dry adiabatic lapse rate, \( -dT/dz = \alpha g/c_p \), and then apply hydrostatic balance, \( dp/dz = -\rho g \), and the ideal-gas equation of state, \( p = \rho RT \), to obtain

\[
\frac{\text{Ma}^2}{\text{Fr}^2} = (1 - \kappa) \left( \frac{dT}{T dz} - \kappa \frac{dp}{p dz} \right) \left( \frac{RT}{g} \right) = \kappa (1 - \kappa) (1 - \alpha).
\]

Using \( \kappa = 0.261 \) for Mars and taking the square root of Equation (B2) yields \( \text{Ma}/\text{Fr} = 0.439 \sqrt{1 - \alpha} \), where \( \alpha \) is the ratio of the local temperature lapse rate to the dry adiabatic lapse rate, the latter for Mars being \( g_0/c_{p0} = 5.05 \text{ K km}^{-1} \). To give a relevant example, in a nearly isothermal region where \( \alpha \approx 0 \), the Froude number field will mimic the Mach number field with values about 2.28 times as large.

Appendix C
Additional Figures

Figures C1–C3 support the discussion in the main text, but are not critical to the flow.

Figure C1. Map view as in Figure 1, except for southern winter. The \( L_s \) values are picked to center a south polar jet streak near the zero of longitude.
Figure C2. Meridional-plane view corresponding to Figure C1. Longitudes are as in Figure 2.
Appendix D

Issues Associated with Calculating $N^2$

D.1. EMARS: $N^2 < 0$ Values

After interpolating EMARS variables onto potential temperature surfaces, $\theta$, occasionally the value of $N^2$ came out negative (slightly superadiabatic). This did not occur when performing the same analysis on OpenMARS variables. These negative $N^2$ values resulted in imaginary Froude number values, which is how they were discovered. There were also a small number of positive $N^2$ values that were so close to zero that they resulted in unphysically large values of $Fr$. It was decided to set the rejection threshold to $Fr > 30$. Those points associated with $N^2 < 0$ or $Fr > 30$ were extracted from the Froude number fields and were not used in this study. The corresponding Mach number values were left intact, since this issue only affected the calculation of the denominator of the Froude number. Figures D1 and D2 show the locations of the rejected points, color-coded in terms of season and potential temperature, respectively. Generally, such values arose in the northern hemisphere after the northern winter solstice ($L_s \approx 270^\circ$–$360^\circ$) and were more prevalent at higher altitudes (800–1100 K). In terms of number, there were 4417 points with $N^2 < 0$ and 329 points with $Fr > 30$, out of a total of $4.89 \times 10^9$ points, such that only ~0.0001% were rejected. Because so few

Figure C3. As in Figure 16, but for the northern (Mare Boreum) and southern (Mare Australe) polar regions plotted together. The southern curves are distinguished by having crosses for mean $Ma$ and plus signs for the standard deviation intervals. The northern standard deviations are the dotted curves.

Figure D1. Locations of $N^2 < 0$ points in EMARS, color-coded by solar longitude intervals, $L_s$.

Figure D2. As in Figure D1, but color-coded by potential temperature, $\theta$ (K), which is the vertical coordinate used in this study.
points were involved, this issue had a negligible effect on the
statistics in the study.

**D.2. OpenMARS: Descending \( N^2 \) Filaments**

After interpolating OpenMARS variables onto \( \theta \) surfaces
and calculating \( N^2 \), it was noticed, first in animations of the Fr
fields and then tracing back to the \( N^2 \) fields, that filamentary
structures would occasionally form. This did not occur when
performing the same analysis on EMARS variables. Considering that (i) it was not guaranteed that these were spurious,
(ii) there was no readily available way to remove them, and (iii)
they did not dominate the signal (they were not easily seen in
the primitive variables, only in \( N^2 \), which was sensitive because
it involved a vertical derivative), these structures were left
intact. Figures D3 and D4 show examples of such \( N^2 \)
filamentary structures, one for each spacecraft, TES/MGS for

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Figure D3. Meridional-plane snapshots of Froude number, Fr (left panels), and squared buoyancy frequency, \( N^2 \) (right panels), at lon = 2°5 (not zonally averaged), for MY 26 in OpenMARS. Local time is indicated by LT in the right panels.
MY 26, and MCS/MRO for MY 31. Because such filaments were transient and sparse and only the Froude number field was directly affected, this issue had a negligible effect on the conclusions in the study.

Appendix E

Potential Temperature and Reference Altitude

Figure E1 shows vertical coordinate potential temperature ($\theta$) as a function of altitude (km).

**Figure D4.** As in Figure D3, but for MY 31.

**Figure E1.** Potential temperature (K), $\theta$, as a function of height (km).
Appendix F
Mars Quadrangles

Figure F1 shows a map of the Mars quadrangles for reference.

Figure F1. Map of the Mars quadrangles (Mars 1:5 million scale), from the Gazetteer of Planetary Nomenclature, by the International Astronomical Union (IAU) Working Group for Planetary System Nomenclature (WGPSN). https://planetarynames.wr.usgs.gov/Page/mars1to5mTHEMIS.