Complex beyond words

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Abstract

This paper addresses the fundamental question of whether mathematics is necessary for the science of complex systems or whether a verbal approach is sufficient. The science of complex systems is presented as interdisciplinary, in which case mathematics is essential. In case this interdisciplinary approach is rejected, classes of systems are identified that are inherently mathematical for which research cannot be conducted in words alone. This includes almost all systems involving human beings! By definition it includes all social-technical systems, and it includes all aspects of systems that involve mathematics, the statistical analysis of data, time series or power laws, chaos, flows, have large numbers of heterogeneous elements, have many heterogeneous relationships, have top-down and bottom-up dynamics, have complicated entailments with positive and negative feedback loops, have stochastic dynamics, or involve infinitesimals. Thus a certain level of mathematical knowledge is essential to be a competent complex systems scientist. This is because complex systems research requires teams of scientists with complementary knowledge, and it is essential that they can communicate using the core ideas of the science. In order to make mathematics and other essential scientific concepts available to the whole complex systems community, the Complex Systems Society is coordinating the definition of a core curriculum and is providing free web-based education and certification through the Paris-based Open University for Complex Systems.

1. Introduction

This paper addresses the general questions:

- is mathematics necessary for the science of complex systems, or
- can the science of complex systems be conducted through words alone?

There is no doubt that natural language is necessary for science. A glance at any book on mathematics itself will show that at some level the meta-language is vernacular. The same is true for any scientific text. The question therefore becomes whether natural language is sufficient to represent complex systems and their dynamics. This will be investigated as follows:

- Science, strategy and survival

  A view will be presented of complex systems science as inherently interdisciplinary, where many of the disciplines involve mathematics. If this view of complex systems science is accepted, then mathematics is necessary for complex systems science. Across the domains in the natural and social sciences complex systems have the ability to coevolve with their environment. The ability to adapt is a fundamental strategy in evolution and survival, and the science of complex systems needs mathematical tools to research these dynamics rigorously.
Devils, detail and data

Complex systems generally have many heterogeneous sets whose elements interact in many heterogeneous ways with emergent dynamics. Generally complex systems have *multilevel* dynamics, with bottom-up and top-down dynamics. They are often sensitive to initial conditions, where some small change locally in some small and apparently isolated part can cause profound change globally. The devil is in the detail, which implies large complicated data sets are essential, the details of which cannot be captured in broad brush terms or generalities.

Mathematics, metaphor and methodology

The mathematics developed for complex systems science has produced some evocative term such as ‘chaos’ and ‘fractals’. Very often these are used as metaphors without understanding of their technical meaning. For example, it is easy to talk about the ‘butterfly effect’ but much more demanding to understand and implement the use of the *Lyapunov exponent* to investigate chaos in any particular system. In this case the analogical use of the word ‘chaos’ amounts to the proposition that “some systems are sensitive to initial conditions, and that any small change at one time may result in a large change at another time”. The problem with this is that it has no predictive power for any particular system. It is ‘just words’.

Vernacular, verisimilitude and vulnerability

There is an unresolved scientific problem in understanding the difference between vacuous generalities, and propositions that have a rigorous theoretical context and that are grounded in observation and data. For example, a company may analyse the proposition that “implementing policy $x$ will achieve our objective $y$”. If this proposition can be demonstrated to be true, then implementing policy $x$ can be a good business decision. However, if the proposition is not ‘true’ in the sense that it is not supported by data, computation and argument but is ‘just words’, then the company may make itself very vulnerable by implementing that policy. Verisimilitude – the quality of appearing to be true or real – is not sufficient when the stakes are high.

For those domains which are inherently mathematical the proposition that science needs mathematics holds tautologically. Therefore we focus on the domains that are not universally considered to be mathematical with dynamics that some people believe can be modelled using words alone. In particular we focus on social systems and their management.

2. The Interdisciplinary science of complex systems

Before addressing our questions, let us clarify what is meant by a science of complex systems. Almost every domain studied by scientists is ‘complex’; no scientist would accept that they are studying simple systems. Traditionally scientists have studied systems within particular domains, boring deep into their discipline and becoming great experts within them. Thus the disciplines have become relatively compartmentalised over the last two centuries, with vertical disciplines such as physics, economics, biology, and so on. In contrast to this, as illustrated in Figure 1, complex systems scientists ask what have been called *fundamental questions* that cut horizontally across the traditional vertical domains. For example, the implications of the dynamics of a system being sensitive to initial conditions is as relevant in management, economics and biology as it is the physics of weather systems.

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1 see, for example, http://hypertextbook.com/chaos/43.shtml
Complex system science seeks knowledge across the domains

Figure 1 illustrates an essential constraint on Complex Systems scientists. On the one hand it is essential to cross domain boundaries in order to understand the deep questions of Complex Systems in comparative contexts. On the other hand, in the modern world, no-one can be an expert in all vertical fields. We distinguish depth and breadth of knowledge. To be a competent scientist in any domain requires study of the concepts and literature in those domains. Post-doctoral and senior researchers with good study skills can become competent but not necessarily expert in a new domain in six months to one year. Compared to this in-depth knowledge, there is another level in which researchers can become familiar with the main ideas in a discipline and able to contrast these with other disciplines, while not being competent to do original work in that discipline.

If this interdisciplinary view of complex systems science is accepted, then many domains of science are inherently mathematical, and the necessity for mathematics in complex systems science is established.

3. Is mathematics needed to model and manage complex system?

Let us assume that the multidisciplinary approach to complex science developed in the previous section is not accepted, and that there is something about some social systems that enables them to be modelled and managed without mathematics.

Mathematics is widely used in economics and the management of complex social systems and tautologically anything involving these approaches uses mathematics. So let these be
excluded from the argument. For example Figure 2(a) shows a time series which is a common way that data are collected for complex systems. Here there is a clear seasonal cycle combined with an upward trend. Figure 2(b) shows the kind of power law that one see frequently at complex systems meetings. These are both mathematical structures and neither can be represented adequately by words alone.

Are time series and power laws relevant to a system being researched? If the scientist does not know what these things are they cannot make an informed decision on this. To this extent they are an incomplete scientist – avoiding what may be fruitful avenues of research through lack of training.

Chaos provides another example. the inability to predict the long-term weather is due to ‘sensitivity to initial conditions’. In systems with this property, even given a perfect deterministic model small errors in measuring the initial conditions can have large effects as time progresses. Since all measurement has errors, computing system states based on tiny errors can diverge greatly from the actual observed system state. Boundedness and sensitivity to initial conditions are the “essential ingredients” of chaos (Arrowsmith, 1991). The first condition ensures the system trajectory does not go off towards infinity (or real systems disintegrating), while the second underlies apparently random changes in system behaviour. To illustrate this the so-called Lorenz attractor is shown in Figure 2.

![Figure 3. The Lorenz Attractor (Source: L. Bradley).](a) 3-D trajectory (b) x-z projection (c) y-z projection

A strange attractor is “an attracting set that has zero measure in the embedding phase space and has fractal dimension. Trajectories within a strange attractor appear to skip around randomly.” (Weisstein, 2005). Putting to one side the technicalities, the last part is the key to why the theory of chaos explains apparent unpredictability in some systems. In Figure 2(b) there are clearly two areas mapped out by the system trajectory. While it remains in either of them the system seems to behave reasonably predictably. However, sometimes it jumps from one to other. Sensitivity to initial conditions may make it impossible to predict when these jumps happen. It may be impossible to discriminate two sets of initial conditions, and the ‘same’ initial observation might result in the system behaving quite differently. This is not due to the inherent dynamics being one-to-many, but because of the nature of observation.

The discovery of chaos has changed our view of what is predictable, as expressed by Laplace in his 1814 *Essai philosophique sur les probabilités*, “We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes”. Contrary to this, even if we had all the data and the right formula, inevitable limitations to measurement mean that some things will be uncertain in the future. Beyond inapplicable generalities such as the butterfly effect, chaotic dynamical systems have to be investigated using mathematics.
Cellular automata are one of the most important tools used to investigate the dynamics of complex systems. At their simplest they are formed from grids of cells with state change rules that give the state of a cell at time $t+1$ in terms of the states of neighbouring cells and itself at time $t$. These transition rules are applied iteratively and the dynamics of the system emerges from the interactions of the cells. This underlies another approach to chaos illustrated by Wolfram’s investigations in the effect of changing the initial conditions for the various classes of cellular automata. Wolfram found that classes 3 and 4 shown in Figure 4 were very sensitive to initial conditions, while classes 1 and 2 were not. Systems that show class 4 behaviour are said to be ‘at the edge of chaos’. The theory of cellular automata involves mathematics, and the investigating the behaviour of particular cellular automata requires computers.

There is a very large class of socio-technical systems in which human and physical subsystems interact. It includes, for example, those systems that assume the relevance of the Internet, mass transportation, and the weather. Certainly the technical parts of these systems involve mathematics, so let the class of socio-technical system be excluded from the our argument.

Consider a large complex organisation with, say, a thousand or more people. Generally such organisations are multilevel, with departments, divisions, and so on. At the microlevel are interactions of individual people, with departments and divisions interacting at meso and macro levels. This multilevel structure is analogous to way that cells make organs and organs make bodies, and not to be confused with top-down hierarchies of authority. Although fashionable management consultants have advised organisation to be ‘flat’, it is inconceivable that an organisation as complex as a health service could operate at a single level (although it can be argued that its current multilevel structure and dynamics are not well suited to its purpose).

Whatever its organisation structure, let us suppose that everyone working in the organisation wants to be paid. This implies a payroll system, with flows of money to the employees. Unless the employees are all the same, there is a need for data to compute the payroll, and experts to calculate the taxes and take home pay. Already this organisation seems to need an accounting department, and an IT department, so multilevel structure seems inevitable. Furthermore the IT systems of any organisation implicitly or explicitly contain models of the organisation. These models are likely to expressed as flowcharts, trying to capture relationships between parts of the organisation and its workflows. In some organisation they are modelled by Petri nets\(^2\) which are inherently mathematical structures.

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\(^2\) See for example http://en.wikipedia.org/wiki/Petri_net
In today’s world no system that involves significant flows of money, materials and information can be modelled and managed without the use of computers and the implicit or explicit mathematical models underlying data processing systems.

Taking away all the quantitative aspects of social systems leaves just the relationships between the parts and narratives of their dynamics. Relationships are widely modelled by networks, as illustrated in Figure 3 which taken from Cross and Parker’s (2004) book *The hidden power of networks: understanding how work really gets done in organizations*. In this example, Cross and Parker make a compelling case for the importance of network structure in organizations.

Figure 5. An information-seeking network in an organization (Source Cross and Parker, 2004).

The organization in Figure 5 combined expertise in strategy and organization design together with expertise in technical fields such as warehousing and architecture. By integrating these specialisms the organization hoped to provide a service not available from its competitors.

However, when one looks at this network it clearly has two major parts. The people on the left were the strategists and organizational designers, while those on the right were expert in knowledge management, architecture and databases. The picture says it all – contrary to the managements aspirations, these two groups were not integrated in terms of information flows.

The reason for this was that Alam, circled in Figure 5(a) had the potential to connect the two groups but did not do so. After he left the organisation, the two parts of the business became connected and it began to achieve its commercial objectives.

The networks shown in Figure 5 are mathematical objects. They can be used in an intuitive way, but network theory has many technical concepts used for modelling them. One of these is the notion of *connectivity* which was so fundamental in this (simple) example.
Figure 6. Interacting positive and negative feedback cycles. (Source: Newell and Wasson, 2001)

Figure 6 shows a set of entailments drawn as a diagram. These entailments could be expressed in words but, arguably, they are clearer expressed graphically. This network diagram immediately leads to the notation of positive and negative feedback cycles which are mathematical objects. Given the combinatorial nature of the cause-effect arrow combination, it would be impractical to list them all by words. Even to try to do so would be futile because this approach would not the capture the dynamics – for this one needs mathematical modelling and computer simulation.

As shown in Figure 6 the dynamics of this system depend on deterministic entailments, but in general the entailments are stochastic where the outcome of a given set of conditions depends on probabilities. In such cases mathematics and computation are essential.

This point is made even more emphatically by Figure 7, where the drawing of the food chain network is too complicated to be understood by inspection alone.

Before bringing this section to a close, consider two examples in which mathematics is able to simplify aspects of systems that otherwise can be very confusing.

Any system that involves infinitesimal needs mathematics. For example, there is the famous paradox of Zeno that a runner such as Achilles cannot overtake a slower runner such as a tortoise if the former gives the latter a heads start: The slower when running will never be overtaken by the quicker; for that which is pursuing must first reach the point from which that which is fleeing started, so that the slower must necessarily always be some distance ahead. This and many other paradoxes were only resolved when Cantor pinned down the nature of infinite sets in the nineteenth century.

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3 See for example http://en.wikipedia.org/wiki/Stochastic_processes
As another example, consider the following conundrum: Simpson’s finger is part of Simpson. Simpson’s is part of the Philosophy Department. Simpson’s finger is part of the Philosophy Department. Let $s$ be Simpson, $f$ his finger and $p$ the philosophy department. Then let us write $f R_1 s$ to mean that $f$ is related to $s$ by being one of its body parts. Let $s R_2 p$ mean that $s$ is related to $p$ by being one of its legally constituted members. Then we can write $f R_1 \circ R_2 p$ to mean $f$ is $R_1$ related to something (unspecified) that is $R_2$ related to $p$. The conundrum here lies in the ambiguous use of the term ‘part of’ to mean all of $R_1$, $R_2$ and $R_1 \circ R_2$ when they are different relationships. Here the use of words leads to confusion while the use of mathematics and symbols gives clarity and precision.
Although many details are missing, this discussion can be summarised as follows:

*Words alone may be used analyse and predict the behaviour of systems that*

- do not explicitly involve mathematics
- do not involve the statistical analysis of date
- do not involve time series or power laws
- do not involve chaos
- do not have technical components as socio-technical systems
- do not involve flows
- do not have large numbers of heterogeneous elements
- do not have many heterogeneous relationships
- do not have top-down and bottom-up dynamics
- do not have complicated entailments
- do not have positive and negative feedback loops
- do not have stochastic dynamics
- do not involve infinitesimals

Even this incomplete list precludes the great majority of complex social systems from being modelled by words alone. In all other cases the systems are better modelled using a combination of natural language as the meta-language and mathematics for the detailed multilevel dynamics.

4. Synthesising Mathematics and Natural Language for Complex Systems Science

We can conclude from the above discussion that *both* mathematics and natural language are necessary for a science of complex systems. Although it has not been stressed above, computation also has a central role to play. By hypothesis, systems that are sensitive to initial conditions become unpredictable beyond a finite time horizon and deterministic models give way to stochastic models. To explore the implications of many possible state transitions across large complex systems requires computation – it is the only way we know how to investigate possible futures from data. At its simplest, one might simulate the possible future states and trajectories of a commercial organisation to give a distribution of ‘predictions’ of possible future states. One then uses natural language to reason about the distribution at a higher meta-level, possibly to formulate policy and make decisions. But natural language itself is a complex system and the use of human cognition and intelligence are also complex systems. The hope of some in the complex systems community is that the part of the science conducted in vernacular language can be better understood to overcome its inconsistencies and problems when applied in a scientific context. Indeed current research into the semantic web is trying to find sufficient regularity in human behaviour, interaction and communication that it can be modelled within computers. The challenge then is not to discard any of mathematics, computing, or natural language but to understand how they can be synthesised to support better scientific research into complex systems.

5. Implications

The practical implications of this discussion are that any analysis of complex systems expressed in words alone will be superficial and probably misleading if the system has any of the properties listed above.

A number of management consultancies set up over the last few years conduct their work entirely in words. As seen above, words cannot capture the subtleties of wide classes of complex systems. To employ such consultants is akin to a bridge building project appointing structural engineers who conduct their analysis in vernacular language alone, with no

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4 In this paper I have deliberately not discussed drawings, images, sounds etc. as means of representing systems in scientific research. The argument can be extended to include these.
mathematical modelling or computer simulation of the structure and its dynamics. Caveat emptor!

More positively, the need for mathematics to research the great majority of complex systems is widely recognised, and is being addressed by the international professional body, the Complex Systems Society and the Paris-based Open University for Complex Systems.

The argument that complex systems science requires mathematics does not extend to an argument that only mathematicians can research complex systems, which would be absurd. As noted in the introduction, the study of any domain requires great in-depth knowledge based on many years study in that domain.

Avoiding an exclusive ‘mathematicians’ only approach, the Complex Systems Society has the view that complex systems research involves teamwork – combinations of specialisms that are essential because no-one knows everything. The important thing is that the scientists are able to communicate clearly in each others’ languages. This includes those from technical backgrounds learning the constructs and ideas developed in the social sciences.

To be a competent complex systems scientist it is essential that one understand the various mathematical structures sketched in this paper. It is not essential that everyone becomes a creative mathematician, statistician or computer scientists, but it is essential that everyone understands the basics and can use the technical terms and symbolic notation. For this reason the Complex Systems Society has launched a call for a discussion on the core curriculum for complex systems science covering mathematics, statistics, data, computation, simulation, physics and other areas.

Over the next two years the Complex Systems Society together with the Open University for Complex Systems will provide free education on the core curriculum in complex systems with associated certification. This process has already begun and will be the basis of establishing basic professional competence in complex systems science. Apart from filling in gaps in all our knowledge, the core curriculum will facilitate better communication between those researching complex systems, and should accelerate the transfer of ideas between disciplines.

We hope that everyone in the complex systems community will welcome the opportunities this offers, both to learn and to share knowledge in order to advance our science.

References


