Main Belt Asteroid Shapes from SuperWASP Photometry

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Abstract

The SuperWASP telescope was used to search for exoplanetary transits. It also serendipitously observed Solar System objects, with a large number of main belt asteroids being observed for at least one apparition by the telescope. Data are available for the year 2004, and then from 2006 to 2012, but data collected after 2006 has remained largely unused. The large field of view of the telescope (974 square degrees) as well as the length of time the telescope has been observing has resulted in large amounts of photometry for asteroids with an apparent brightness above $V = 15$. In many cases, the amount of photometry for an asteroid in the SuperWASP dataset is comparable to the other photometry currently in existence for that asteroid.

This thesis describes the pipeline created to take existing SuperWASP photometry, reject observations of high photometric uncertainty and produce lightcurves suitable for shape modelling. A novel method of fitting double-sinusoids to lightcurves to quickly determine the rotational period of an asteroid is also described, and uses and limitations discussed. A total of 89 models were created for 50 asteroids, with 6 of these asteroids not previously having models. A significant difference between the existing model and model created in this work was found for 6 asteroids.

The properties of models produced in this work and previously published models are studied. It is shown that the anisotropy of asteroid spin axis longitudes is present in the sample of asteroids that have models. The relationship between spin axis longitude and other dynamical properties such as semi-major axis and argument of perihelion are explored, and it is demonstrated that the anisotropy is more prevalent for asteroids with an estimated diameter greater than 20 km.
Acknowledgements

I would like to thank my supervisor Simon Green, whose tireless support, input and advice from the first day of this project to the last have been invaluable. The expertise and guidance of co-supervisors Colin Snodgrass and Ben Rozitis are deeply appreciated, as well as their good humour and willingness to sit through many, many meetings for my benefit. I would also like to thank my external supervisor Benoit Carry, who not only made my visits to Nice some of the best times for me in the last four years, but has also provided much of the software and know-how that has made this project possible. You have all been first-class supervisors and I am truly grateful to have had you all supervise me.

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Nomenclature

CAI    Calcium Aluminium Inclusion
CCD    Charge-Coupled Device
DaFEED DAMIT Feeder
DAMIT  Database of Asteroid Models from Inversion Techniques
ESO    European Southern Observatory
FWHM   Full Width Half Maximum
HED    Howardite–Eucrite–Diogenite
IMCCE  Institute of Celestial Mechanics and Computation of Ephemerides
KBO    Kuiper Belt Object
KOALA  Knitted Occultation, Adaptive Optics, and Lightcurve Analysis
KS Test Kolmogorov–Smirnov Test
LHB    Late Heavy Bombardment
NEA    Near Earth Asteroid
NEATM  Near Earth Asteroid Thermal Model
<table>
<thead>
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<th>Abbr.</th>
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<td>NEO</td>
<td>Near Earth Object</td>
</tr>
<tr>
<td>NGTS</td>
<td>Next Generation Transit Survey</td>
</tr>
<tr>
<td>PDS</td>
<td>Planetary Data System</td>
</tr>
<tr>
<td>PHA</td>
<td>Potentially Hazardous Asteroid</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
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<tr>
<td>SAGE</td>
<td>Shaping Asteroids with Genetic Evolution</td>
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<tr>
<td>STM</td>
<td>Standard Thermal Model</td>
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<td>SuperWASP</td>
<td>Super Wide Angle Search for Planets</td>
</tr>
<tr>
<td>TESS</td>
<td>Transiting Exoplanet Survey Satellite</td>
</tr>
<tr>
<td>TNO</td>
<td>Trans Neptunian Object</td>
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<td>YORP</td>
<td>Yarkovsky–O’Keefe–Radzievskii–Paddack Effect</td>
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Chapter 1

Introduction

This thesis presents the motivations, methods and findings of work on creating shape models of main belt asteroids using data from the SuperWASP telescope. Asteroid data obtained from SuperWASP has remained largely unexploited in recent years, but contains large amounts of photometry for asteroids that were serendipitously observed during the course of its exoplanet survey. This photometry is ideal for shape modelling. Before the photometry can be used, the uncertainties in each observation must be estimated, as observations with large uncertainties will introduce these uncertainties into the shape model. This work describes the creation of shape models from a combination of photometry from SuperWASP and other sources, and discusses the implications of them. Possible future work is also discussed. A summary of each chapter is given below.

Chapter 2 contains an introduction to Solar System Objects, with a focus on Main Belt asteroids. It outlines their formation, evolution, physical and dynamical properties as well as a brief history of observing techniques.

Chapter 3 is a more in depth explanation of techniques used to observe asteroids, including optical photometry, infrared and radar observations as well as (spacecraft and meteorites), with a particular focus on the SuperWASP telescope. There is also an explanation of how various properties of asteroids are calculated.

Chapter 4 describes the pipeline created to prepare SuperWASP data for use in a
shape modelling algorithm. This includes lightcurve cleaning, period determination and validation.

Chapter 5 outlines the techniques used in this thesis to create shape models, and also describes other methods of creating shape models. The stability and degeneracy of shape models are discussed, as well as an ESO large programme to image large asteroids with adaptive optics, as well as my contribution to it.

Chapter 6 contains the results of using SuperWASP data to create shape models. It highlights where the new models disagree with the models in the literature, as well as the models for asteroids which previously did not have a model.

Chapter 7 analyses these models in more detail, as well as models already in the literature. Models are validated against models made using adaptive optics, and includes more in depth looks at unusual models. Minimum densities are derived for these models, and there is a discussion on the evidence for and possibly cause of anisotropy in spin direction longitudes.

Chapter 8 reviews the findings in this thesis, and suggests areas where future work would be most fruitful.

There are supplementary tables of information at the end of the thesis.
Chapter 2

An Overview of Asteroid Science

This chapter is an overview of the science and history relevant to creating and analysing asteroid shape models. It starts with a summary of the history of asteroid discoveries and the techniques used to discover them up to the present day. It then describes various theories of asteroid formation models and the evidence for them. Next is a section describing the dynamical, physical and spectral properties of asteroids, and what they tell us about the asteroids. Finally, some of the current outstanding questions of asteroid science are discussed.

2.1 The Solar System

The Solar System can be described as a collection of bodies orbiting the Sun, and the satellites of these bodies. How we classify these bodies depends on their properties. The Sun is a star as it produces energy via fusion\(^1\). The planets are currently defined as being in orbit around the Sun, in hydrostatic equilibrium and having ’cleared their local neighbourhoods’ according to the IAU definition\(^2\). Previously, the largest asteroids and Pluto have been considered planets at various times, but by the current definition there are only eight planets. The other bodies in the Solar System fail to meet one or more

\(^1\)IAU definition of a star is that it must have sufficient mass to fuse deuterium (13 M\(_\odot\))

\(^2\)The definition can be found at https://www.iau.org/static/resolutions/Resolution_GA26-5-6.pdf.
of these definitions.

Smaller than the planets are dwarf planets, which orbit the Sun directly, are in hydrostatic equilibrium but have not cleared their own neighbourhoods of other small bodies, and hence are not planets. Examples are Ceres and Pluto, which are clearly rounded and therefore in hydrostatic equilibrium, orbit the Sun but are situated within the asteroid belt and the Kuiper belt respectively (see subsection 2.1.1 for an overview of the locations of small bodies in the Solar System), and thus have not cleared their local neighbourhoods. This definition is not without controversy, and it has been argued that the phrase clearing the neighbourhood is vague enough such that depending on interpretation some of the eight planets have not cleared their neighbourhood, such as Jupiter sharing its orbit with many Trojans). What is clear is that the eight planets are far more dominant in their orbits than any other smaller objects. The other dwarf planets are the Kuiper Belt objects Eris, Pluto, Haumea and Makemake, with the possibility of hundreds more existing in the TNO (Trans Neptunian Object) population (Muñoz-Gutiérrez et al., 2018). There are no dwarf planets other than Ceres in the asteroid belt under the current definition. Pallas, Vesta and Hygiea may be round enough to meet the criterion for hydrostatic equilibrium and therefore be considered dwarf planets in the future subsection 5.5.1.

The rest of the small bodies that are not in hydrostatic equilibrium can be subdivided depending on their dynamical properties and composition. Generally, small bodies closer to the Sun are rocky, while those further away are composed of ices and dust. This is because within the ‘snow line’ volatiles are unable to condense into ices so objects that formed within the snowline will be rocky and depleted in volatiles. Asteroids are objects primarily made of rock, but some may contain large amounts of ice, especially in the outer belt and some are possibly comets that have lost their surface volatiles. Icy bodies that are not satellites can be categorised into Trans Neptunian Objects, Centaurs or Comets depending on their dynamical properties.

The final group of objects are those that do not orbit the Sun directly, but rather orbit another body. Satellites have been found all across the size spectrum of objects,
both in absolute size and comparative to the object they orbit. There are satellites of NEOs that are 10 m across (these small moons are sometimes known as moonlets) (Chapman et al., 1995), and the largest planetary satellites Ganymede and Titan are larger than the planet Mercury, although neither is as massive. Binary systems are where the secondary body is of comparable size to the primary, such as the asteroid 90 Antiope (Michałowski et al., 2004). Regular moons are moons which are thought to have formed in situ around the planet, while irregular moons are moons which did not form around the planet and are thought to be captured objects. These are chiefly differentiated by their orbits, with regular moons having orbits that are closer to the planet, and with smaller inclinations and eccentricities compared to irregular moons.

2.1.1 Orbits

The dynamical properties describe the orbit of an asteroid. Asteroids orbiting the Sun can be modelled as a two body system. The position of the asteroid can be described by six orbital elements (see Figure 2.1 for an explanation). Asteroids are classified into groups depending on their orbital parameters.

The vast majority of asteroids are in the belt between Mars and Jupiter, usually known as the main belt (Figure 2.2). The main belt can be further subdivided into the inner, middle and outer main belt, as well as several other groups including the Hungarias and Hildas. The inner, middle and outer belts are divided by the Kirkwood gaps at 2.5 and 2.82 AU, which are unpopulated areas due to Jupiter’s 3:1 and 5:2 mean motion resonances overlapping with the \( \nu_5 \) and \( \nu_6 \) secular resonances of Saturn. Mean motion resonances are where objects have ratios of orbital periods that are integers. This overlapping of resonances makes asteroid orbits in this region unstable, even though mean motion resonances by themselves are usually stable orbits, such as the Jupiter Trojans in a 1:1 resonance and the Hildas in a 2:3 resonance with Jupiter, which locates them between the main asteroid belt and Jupiter.

Near Earth asteroids have orbits that bring them near to the vicinity of Earth for at
Figure 2.1: A diagram of the orbital elements of an asteroid. Not shown are the semi major axis and eccentricity, which are half the largest diameter of the ellipse, and a measure of how circular the orbit is. The symbol \( \Upsilon \) points towards the vernal equinox, this is the reference point by which the other angles are measured. \( \Omega \) is the angle between the vernal equinox and the longitude of ascending node. The ascending node is where the object moves up through the ecliptic plane (the rectangular plane). \( \omega \) is the argument of perihelion, the angle between the longitude of ascending node and where the asteroid is at perihelion. The true anomaly (\( \nu \)) is the angle between the argument of perihelion and where the object currently is in its orbit. The inclination (\( i \)) is the angle between the ecliptic plane and the orbital plane.

least part of their orbits, with perihelia less than 1.3 AU. As these objects are generally small they do not concern this work, but are of great importance due to the possibility of one impacting Earth. Asteroids with orbits that bring them close to a planet generally have unstable orbits. The expected orbital lifetime of an asteroid in near Earth space is short compared to the age of the Solar System. NEOs will eventually either impact into one of the terrestrial planets or the Sun, be ejected from the inner Solar System or disintegrate from thermal fracturing or spin up (Gladman et al., 2000).

Asteroids being ejected from the main belt feed the NEA population. While some comets come close enough to Earth for them to be observed with the naked eye, they only spend a short time in the vicinity of Earth they cannot be said to be Near Earth
2.1. The Solar System

Figure 2.2: Figure (a) shows the distribution of asteroids and other groups of small bodies within the solar system (Lisse et al., 2007). Figure (b) shows the main belt in more detail. Note the vertical axis is different in each figure.
Objects. The transitional objects between NEAs and the main belt are known as Mars crossers, with perihelia within Mars’ orbit and aphelia beyond the orbit of Mars, although it should be noted that while Mars crossers can become NEAs, not all Mars crossers do (Michel et al., 2000).

Any planet is capable of trapping asteroids in a 1:1 mean motion resonance known as Trojan asteroids. Jupiter has by far the most Trojans in the Solar System. These orbit at the L4 and L5 Lagrangian points, which are stable points 60° in front and behind the object respectively. This means that if a Trojan asteroid’s orbit is slightly perturbed, it will continue to orbit around the L4 or L5 point. It is thought that Jupiter’s Trojans did not form in situ, and instead have been transported there from the Kuiper belt as there are spectral similarities between KBOs (Kuiper Belt Objects) and Trojans suggesting a common origin (Morbidelli et al., 2005). Jovian Trojans also show similarities with Neptunian Trojans, suggesting a common origin with these objects. The presence of the Trojan populations is one of the problems that must be solved with any model of giant planet migration (subsection 2.3.2).

Outside the orbit of Jupiter are Centaurs and the Kuiper Belt. Centaurs have semi major axes greater than that of Jupiter’s but less than Neptune’s. This makes interactions with the giant planets relatively common, meaning their orbits are unstable over millions of years (Tiscareno & Malhotra, 2003). Most of these objects are likely to have come from the Kuiper Belt. The Kuiper Belt is beyond the orbit of Neptune, and the vast majority of the objects here are expected to be cometary in nature. It is still possible that some are asteroids scattered by the giant planets.

It is expected that an asteroid that formed from the protoplanetary disc to have a low inclination, low eccentricity orbit, as this is the typical orbit for matter in an accretion disk. These types of orbits are referred to as dynamically ’cold’. If the orbit has a high eccentricity and/or inclination, this type of orbit is referred to as dynamically ’hot’, and is the product of the asteroid having an interaction with another body. These properties can tell us other things about asteroids as well. If two asteroids have similar orbital properties, there is a possibility that they share a common origin.
2.2 Discoveries and Surveys

2.2.1 Initial Discoveries

Humanity has had at least some knowledge of the Solar System since ancient times. The earliest records of the planets Mercury, Venus, Mars, Jupiter and Saturn were from Mesopotamian astronomers, and these planets share their names in English and several other languages with the Roman Gods. The word planet comes from the ancient Greek 'wanderer'. The idea that the planets orbit the Sun has been around since at least the 3rd century BC, but the geocentric model was widely accepted until the 15th century when Copernicus reintroduced the idea of Heliocentrism in Europe. In the 17th century Kepler inferred that planets move on elliptical orbits, and Newton later showed that this was a consequence of a gravitational force that is of the form of an inverse square relation. Newton also showed that the then known comets orbit the Sun on elliptical orbits, and described them as compact objects.

In the 2nd half of the 18th century, it was noted that the semi major axes of the known planets followed the pattern of a geometric series plus a constant, known as the Titius-Bode law. William Herschel discovered Uranus in 1781, and it was found that the new planet fitted into this series, seemingly confirming its validity. The reason for the law is not known, and Neptune does not fit the Titius Bode law, seemingly proving it to be a coincidence. Recent analysis has shown that some exoplanetary systems show similar behaviour to our own Solar System in that they show geometric progression, and that there may be some validity to the law (Chang, 2010). After the discovery of Uranus the Titius Bode law was considered valid, but there was a gap in the series. The series predicted a planet at 2.8 AU between the orbits of Mars and Jupiter, but none was known. Astronomers began searching for this missing planet in the region

\[ \text{many of the discoveries of the workings of the Solar System will have been independently made by other cultures outside of the 'western sphere'. For example the Indian astronomer Aryabhata may have known that the planets moved in elliptical orbits (Ramasubramanian et al., 1994), and later Indian astronomers knew that comets were periodic phenomena centuries before astronomers in Western Europe (Sharma, 1987). However, as the initial discoveries of asteroids were made by astronomers in Western Europe, I will describe the discoveries known to those astronomers.} \]
occupied by the asteroid belt.

The first discovery of an asteroid was Ceres in 1801 by Giuseppe Piazzi at Palermo observatory, although rather than being motivated by the search for the missing planet, he was mapping stars and came across Ceres quite accidentally. Before the advent of photographic plates moving objects were discovered by making star maps every night and looking for objects that moved from night to night. Piazzi was making a new star map of then unparalleled precision when he discovered Ceres as an object moving against the other stars. Initially he believed it to be a comet as these were what all small moving objects had been up to that point, but when he realised that the orbit was not highly elliptical like that of a typical comet, he classified it as a planet. It was clear from its apparent brightness and the fact that it could not be resolved into a disc that this object was smaller than the other planets. Searching for Ceres, Olbers discovered a second asteroid, Pallas, from his home in Bremen in 1802. Olbers proposed that Ceres and Pallas may be fragments of the hypothetical planet between Mars and Jupiter as a way of explaining their small size, so the search for more fragments continued. Juno was discovered in 1804, and Vesta in 1807, but both of these were far smaller than the other planets as well. The search continued for a few years, but no more objects were found and most astronomers assumed there were only four of these objects between Mars and Jupiter.

This changed in 1845 with the discovery of Astraea by Hencke, still working in Bremen. He had persisted in the search for more asteroids, and proceeded to discover Hebe in 1847. With proof that there were more than 4 asteroids, interest in these objects was renewed. The word asteroid was coined by William Herschel, meaning 'star-like' in Ancient Greek, because asteroids were unresolved point sources like stars but orbited the Sun like planets. They were known as planets until enough of them were found for them to merit their own name, and the term asteroid caught on.

4The fragmentation theory of asteroid formation lost popularity in the 20th century in favour of the asteroids being remnant planetesimals.
2.2.2 Photographic and Digital

Initially asteroids were discovered by making a star map, returning to the area of sky a few nights later to see if any of the objects on the map had moved. A breakthrough occurred when photographic plates were employed, allowing for asteroids to show up as streaks across the plate, and not requiring star maps to be made by hand. The photographic plate was first successfully used to discover asteroids by Max Wolf (Wolfschmidt, 1998), with his discovery of 323 Brucia. The blink comparator was invented at the turn of the 20th century allowing images taken on different nights to be compared, and asteroids would stand out as moving against the background sky. This method of asteroid discovery continued for nearly a century, until the advent of surveys, and is famous for being the method used to discover Pluto in 1930 (Hoyt, 1976).

As well as being useful for discovering asteroids, photographic plates have the added advantage that they record the brightness of objects, rather than relying on error-prone human estimates of brightness.

The CCD (Charge-Couple Device) is a device that can convert photons incident on its surface into a charge, then count these charges thereby counting photons and digitising the image. This has several advantages over photographic plates. One of these is that the translation of the image to a digital object is immediate. While photographic plates can be scanned and converted into a digital image, it is not practical to do on a large scale. The use of digital images for automation of data reduction and analysis is almost imperative for large amounts of data. This increased ease of dealing with large amounts of data allowed for increasingly large surveys of the sky to be done. Asteroids can be seen moving between sequential images (Figure 2.3).

2.2.3 Surveys

Surveys are important to asteroid science. Some surveys are designed for astrometry, which can discover new asteroids, and help refine the orbits of previously discovered

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5Images taken from [https://upload.wikimedia.org/wikipedia/commons/1/16/704_interamnia_oate.gif](https://upload.wikimedia.org/wikipedia/commons/1/16/704_interamnia_oate.gif).
Figure 2.3: Images of 704 Interamnia moving between images on successive nights as seen from the TNT telescope at Collurianio-Teramo Observatory. The asteroid in the top left can clearly seen to have changed the positions between the images.

The vast majority of asteroids have been discovered by surveys, as their ability to observe large areas of the sky and use of dedicated pipelines for detecting moving objects in images can greatly increase the number of objects discovered. This is particularly useful for characterising the orbits of NEOs, as this includes the population of PHAs (Potentially Hazardous Asteroids) that pose a risk to our society.

There have been many asteroid surveys, but the ones listed below are the some of the most notable for advancing the field.

**Yerkes McDonald**

The first systematic survey looking for asteroids was the Yerkes McDonald Survey started in 1950, which surveyed the ecliptic down to magnitude 16 and found 1550 asteroids (Kuiper et al., 1958). This survey was designed to determine the absolute magnitudes of asteroids and used a 0.25m aperture telescope with a field of view of 53 square degrees.
2.2. Discoveries and Surveys

**Palomar Leiden**

The Palomar Leiden Survey went down to magnitude 20, but rather than viewing the entire ecliptic, focussed on one patch of sky. This was to study fainter asteroids than those in the Yerkes McDonald survey. Approximately 2000 asteroids were discovered (Van Houten et al., 1984). The survey was intended to be an extension to the Yerkes McDonald survey, and used a telescope of .

**Spacewatch**

Spacewatch was set up in 1980. The goal was to discover small bodies from NEOs (Near Earth Objects) all the way out to the Kuiper Belt. It uses two telescopes on Kitt Peak observatory (apertures of diameter 0.9 and 1.8 m) to detect moving objects and the Bok (2.3 m diameter) and Mayall (4 m diameter) telescopes for follow up. Spacewatch was novel in that it was the first survey to use CCDs to look for small bodies, and the development of running software on live data to detect moving objects (McMillan, 2006).

**IRAS**

*IRAS* (Infrared Astronomical Satellite) was a satellite launched in 1983 (Neugebauer et al., 1984; Davies et al., 1984). It was the first all-sky infrared survey and would remain the largest dataset for asteroid diameters for over 20 years. As an all-sky survey, the asteroids were observed serendipitously. It discovered 2750 asteroids, including four Near Earth Asteroids (NEAs) including 3200 Phaethon, the parent body of the Geminids meteor shower (Green et al., 1985). The aperture of the telescope was 0.57m, and a combined Field of View over all detectors of approximately 0.1 square degrees.

**LINEAR**

Lincoln Near Earth Asteroid Research (*LINEAR*) is a survey to detect and track NEOs based at the Lincoln Laboratory’s Experimental Test Site in Nevada, started in 1998.
Observations started with a 1 m aperture telescope, and a second 1 m telescope was added to double the search capacity in 2002. Combined the field of view of these telescopes is It has discovered many asteroids in the main belt in addition to NEOs, and has also discovered several comets (Stokes et al., 2000).

**Catalina**

Catalina Sky Survey (Larson et al., 1998) is a survey to find NEOs begun in 1998 as a result of NASA being directed to find 90% of NEOs 1 km or larger. It consists of a 1.5 m survey and 1 m follow up telescope at Mt Lemmon and a 1 m survey telescope on Mt Bigelow. In 2005 it surpassed LINEAR for the number of NEO detections in a single year.

**AKARI**

AKARI (Murakami et al., 2007) is a Japanese satellite that surveyed the sky in the infrared between 2006 and 2011. There are currently 5185 asteroids in the asteroid flux catalogue. The satellite orbited in a Sun synchronous orbit such that the direction of orbit was always perpendicular to the Sun-Earth line, and looks directly away from the centre of the Earth. This allows the sky to be scanned in six months, but due to the Moon obscuring some parts of the sky, and observations not being taken in the South Atlantic anomaly, some parts were not seen. This affected later observations, with the parts of the sky missed having to be observed later. This led to parts of the sky being mapped more often than others, and so the number of observations varies between asteroids.

**Pan STARRS**

Pan STARRS is a 1.8 m telescope at the Haleakala Observatory that started observing in 2008, and is an all-sky survey that is designed to detect variable objects, including Solar System objects. Its current primary mission is to detect NEOs, but has discovered
thousands of small bodies throughout the Solar System during the survey (Kaiser et al., 2002; Denneau et al., 2013).

**WISE**

*NEOWISE* (Near Earth Object Wide Field Infrared Survey Explorer) is the most complete of the infrared surveys, having observed 36,000 objects, an order of magnitude more objects than previous observations. The satellite itself is called WISE and launched in 2009, but after the coolant ran out in 2010 it continued to be used to look for Near Earth Objects and other Solar System objects in a mission extension known as *NEOWISE*, and was put into hibernation after it had surveyed the entire asteroid belt. The telescope was taken out of hibernation in 2013 to look for potentially hazardous asteroids. It does have issues with some bright objects due to saturation of the sensors, although most of these objects have recently had excellent diameter characterisations from adaptive optics imaging. *NEOWISE* has had two observing modes, one with all four bands at 3.4, 4.6, 12 and 22 \( \mu m \), and then a second observing period with only 2 bands after its cryogenic coolant ran out. From these data asteroid diameters can be determined (Masiero et al., 2011).

**Gaia**

Gaia is a satellite that observes the positions of celestial objects with extreme precision. This excellent astrometry is primarily meant to map stars and their proper motions within the Milky Way, as well as detecting quasars and Solar System objects. As well as detecting asteroids, by monitoring their motions Gaia will be able to provide accurate mass estimates for large asteroids (Bancelin et al., 2012). It is currently observing, but there have not been any data releases yet for asteroids.

**Rubin Observatory Legacy Survey of Space and Time**

The Vera C. Rubin Observatory is an 8.4 m optical telescope with a 12 square degree field of view that is currently under construction in Chile that will survey the entire
night sky every few nights. It is expected to increase the number of known small bodies by an order of magnitude. It is expected to get first light in 2020 (Jones et al., 2016).

2.3 Formation and Evolution

2.3.1 Formation

Asteroid science is advanced both by theory and observations. Observations tell us the current state of the asteroid belt. This state includes knowing the dynamical, physical and chemical properties of the asteroids. Any theory that does not recreate the current state is either incorrect or incomplete. We can increase our understanding of asteroids by creating and refining theories such that they predict the current Solar System with increasing accuracy, and by observations which allow the current state to be observed more accurately.

The currently favoured hypothesis for the formation of the Solar System begins with a nebula consisting of molecular hydrogen and some helium, with small amounts of heavier elements from past supernovae. This nebula either collapsed under its own gravity, or collapsed after being shocked by a nearby supernova (Boss & Keiser, 2010). The presence of short half life elements suggests they either formed with or shortly before the Solar System. The cloud will have some intrinsic angular momentum, which will inhibit collapse normal to the direction of the angular momentum vector, but allows collapse along that direction resulting in a disc of material, known as a protoplanetary disc (Figure 2.4). These discs have been observed around other stars, with evidence of large bodies forming, giving credence to the nebula hypothesis theory (Andrews et al., 2018). The disc consists of dust and gas, with most of the gas being hydrogen and helium, with the dust being the remnant material from supernovae or late type stars, and the exact dust composition varying depending on the origin and evolution of the particular dust grains. Some meteorites contain Calcium Aluminium
Inclusions (CAIs), which are dated by lead-lead isotope dating as being 4.567 billion years old, and are the oldest materials found in the Solar System so far (Bouvier & Wadhwa, 2010). This date is taken to represent the start of the Solar System.

The traditional coagulation formation theory of asteroids and planets is that of hierarchical growth. Dust grains in the disk stick to each other via electrostatic forces to form larger particles, and by further collisions these particles stick to each other to grow larger and are compacted by impacts and their own gravity into consolidated objects. The issues with this model are firstly that smaller particles have sufficiently high

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6Figure taken from https://www.eso.org/public/images/eso1436a/.
mutual velocities and small enough self gravity that collisions are more often destruc-
tive than constructive, and secondly that for metre-sized objects, drag from interactions
between the particle and the disc will cause them to fall into the Sun faster than they can
grow. These are known as the 'bouncing barrier' and 'metre sized barrier' respectively
(see Johansen & Lacerda (2010) for a summary). They can be overcome somewhat if
ice is the primary material of the particles rather than rock, as ice will stick together
more easily than rock, but this can only explain the formation of objects beyond the
snow line where ices can exist.

As the hierarchical growth model has problems that can’t obviously be resolved,
other theories have gained popularity. The basic premise of these theories is that rather
than hierarchical growth, large planetesimals formed directly from the gravitational
collapse of material in the disc. Gravitational collapse requires that the gravitational
potential of the material be greater than the kinetic energy. The easiest way to explain
this occurring is if the material is densely concentrated, increasing its gravitational
potential. Various methods have been put forward for concentrating material in the
disc. The streaming instability theory (Youdin & Goodman, 2005) states that a small
overdensity of particles in the disc will exert a drag force on the local gas which heats
it. This causes both the particles and the gas to drift inwardly more slowly than the rest
of the disc. This leads to particles from further out in the disc drifting radially in to the
overdensity, which will then grow and further slow its inward drift. This overdensity
can then collapse at once to form a planetesimal, or planetesimals (Figure 2.5).

An alternative to streaming instability is turbulent concentration (Cuzzi et al., 2001).
The result comes from hydrodynamical simulations, but in simple terms the process
starts with a turbulent region of the disc composed of eddies. These eddies will break
up into smaller scales, but the energy and particles must be conserved when the ed-
dies split. If the eddies do not split equally, then some of these eddies will have an
enhanced concentration of particles. As this process is repeated as part of a turbulent
cascade, the concentrations can be increased, or decreased. Some of these increased
concentrations will be high enough to collapse into planetesimals, or make the small
Figure 2.5: Diagrams demonstrating the formation of planetesimals via the streaming instability process. The protoplanetary disk consists of gas and dust. The heat of the Sun causes a higher pressure nearer to it, and creates a pressure gradient in the disk. The pressure gradient reduces the effective gravity on the gas, causing it to orbit more slowly than the Keplerian velocity. This does not apply to the dust, which experiences drag due to the slower orbiting gas. This causes dust to drift radially inward. The drag force on the dust has a reaction affect on the gas accelerating it. In regions with higher densities, the radial drift is slower for the same reaction force. This slower drifting dust will be joined by dust drifting more quickly from further out, causing the density to grow and drift more slowly. Once the overdensity has grown sufficiently, then it can collapse directly into a planetesimal.

overdensities required for streaming instabilities.

The final step of planetesimal formation is pebble accretion, and is where planetesimals are large enough such that their gravitational influence will cause them to accumulate further material (Lambrechts & Johansen, 2012). This is only applicable for the growth of large bodies the size of planetesimals, and does not answer questions on the early stages of planetesimal formation.

It is uncertain exactly when the planetesimals formed, and how long they took to form. It can be said with certainty that they must have formed after CAIs, and
before the protoplanetary disc dissipated (10-25 Myr), but there is room for them to have formed in between these times, and in different parts of the protoplanetary disc. This may explain why different asteroid types exist, as a result of compositional heterogeneities in the disc. Additionally, the different times of asteroid formation may have made different asteroid types. As the streaming instability theory requires that all asteroid parent bodies formed within the protoplanetary disc, all asteroids that are currently observed are either fragments of these parent bodies or objects of extrasolar origin (Namouni & Morais, 2018).

Recently, analysis of (486958) Arrokoth which is a Kuiper Belt Object observed by the New Horizons mission seems to have confirmed that planetesimals form via cloud collapse, rather than hierarchical growth (McKinnon et al., 2020). Arrokoth is a ‘cold classical’ object, which means that it has not dynamically evolved due to the influence of Neptune, unlike ‘dynamically hot’ objects. Being in the Kuiper belt where orbital speeds are slower and the surface density of objects is lower, it is also an environment with far fewer collisions than the asteroid belt increasing the likelihood that it has not been significantly altered since it formed. The round lobes show that there were no collisions in the object’s history significant enough to alter the shape of the lobes (Figure 2.6). As Arrokoth is a contact binary, the lobes must have been separated before spiralling in. They have similar flattening, and the planes along which they are flattened are closely aligned. This along with being compositionally indistinguishable means they must have formed as a binary and slowly spiralled in. The roundness of the individual lobes suggests that they are not the result of violent collisions, so the only possible explanation is a cloud collapse mechanism.

2.3.2 Evolution

The majority of Solar System dynamical evolution is thought to have been driven by migrations of the planets. In some exoplanet systems, there are planets that are dis-
The image is a composite of images from the MVIC and LORRI instruments.

integrating due to being too close to the parent star (Sanchis-Ojeda et al., 2015). It is more likely that they formed further out and migrated in to this position than formed in a place that is too hot for them to form by known methods. There are other systems where the planets are in mean motion resonances, which are unlikely to form with all of the planets forming in situ, but likely to happen if the planets are allowed to migrate (Luger et al., 2017). As the giant planets are thought to have formed first and have the largest mass, it is their migrations that have dominated the evolution of the Solar System. It is safe to assume that migrations could have occurred in our Solar System, and the currently favoured models indicate that they did.

The first of these processes thought to have occurred is known as the grand tack (Walsh et al., 2011). As it is required to have occurred when there was a significant disc, it would have taken place a few million years after the formation of the Solar System. In this model, Jupiter and Saturn form from the disc and proceed to accrete matter. This causes gaps in the disc to form where all of the matter has been accreted, such that Jupiter and Saturn are orbiting within their respective gaps. They then migrate inwards due to interactions with the disc either side of the gaps, scattering asteroids
interior to Jupiter outwards past Saturn. Their motion brings them into a mean motion resonance which causes their movements to be coupled. After Saturn has migrated in towards Jupiter, the gaps each planet has formed in the disc merge. This causes the direction of migration to reverse and the planets start moving outwards. The migration scatters objects exterior to Saturn, including some of the objects scattered earlier, either back towards or away from the Sun. The migration stops when the disc dissipates.

The second potential process is the Nice model (Tsiganis et al., 2005). Here, it is assumed that the four giant planets were closer to the Sun and each other than they are currently, and a disc of small bodies extended outwards from the orbit of Neptune. Neptune would have encounters with objects from this disc, and to conserve angular momentum the small bodies would be scattered inwards, and Neptune outwards which would then scatter more of the bodies in this disc inwards. The bodies scattered inwards would do so again with Uranus and Saturn, causing the planets to migrate outwards. Conversely, Jupiter would have scattered the bodies back out again and itself migrated inwards. Eventually Jupiter and Saturn would have crossed their 1:2 mean motion resonance, moving rapidly in semi major axis and eccentricity. This destabilised the orbits of Uranus and Neptune which were driven into the disc by Saturn, scattering the bodies towards the inner Solar System. This is thought to be the cause of the Late Heavy Bombardment (LHB), and left the giant planets at their current orbits.

Recently, the Nice model has been called into question (Hartmann, 2019). One of the major motivations for this model is material from all of the Apollo landing sites dated to approximately 4 Gyr ago, which would require impacts or resurfacing of the entire lunar surface, and a simultaneous injection of asteroids into the inner Solar System. The reason for it being called into question is that the material ages may be caused by a biasing in the collection of material, and represent the formation of Imbrium basin, rather than being representative of the Moon as a whole. The Nice model has been modified to occur earlier in the Solar System to better explain the observations, but the idea that impacts in the early Solar System declined from formation monotonically rather than spiking nearly a billion years after formation is gaining popularity,
and some theories do not require any planet migration at all (Mojzsis et al., 2019). The problem of this abundance of possible migration scenarios is partly due to the fact that computational modelling has become more sophisticated with increased availability of computing power. So many parameters can be adjusted in the model, that many initial conditions and evolution scenarios can be fitted to produce the current Solar System. In other words, the problem is under constrained, and more observations will be required to make real progress with our understanding of Solar System evolution. However, even if the Nice model is no longer required to explain the LHB, then the constraint on the timing of the migration is no longer there, but this does not necessarily mean that the Nice model is incorrect.

These theories are not mutually exclusive, and all theories agree that after approximately 4 Ga ago, there was no large scale changes to the architecture of the Solar System, and any evolution between asteroids was driven by interactions with other asteroids, or when an asteroid is driven into an orbit that brings it close to a planet. Many asteroids are collisionally evolved, as evidenced by the existence of asteroid families (subsection 2.3.3) that are more recent than the last giant planet migration. Overall, it is estimated that more than 99% of the original mass of the asteroid belt has been removed (Clement et al., 2019).

As well as the dynamical evolution of the asteroid population as a whole, there has been evolution for individual asteroids. This is mainly due to collisions and internal heating. Magnetic induction heating and heating from gravitational collapse have both been proposed as internal heating mechanisms, but neither can produce enough energy to explain the amount of heat required to create minerals in certain meteorites (see Ghosh & Weidenschilling (2006) for a summary of proposed mechanisms). Instead, the heating is thought to have come from the decay of short lived radionuclides, specifically Aluminium 26 ($^{26}\text{Al}$) (Lee et al., 1977). The evidence for this is an excess of $^{27}\text{Al}$ in meteoritic samples. As $^{26}\text{Al}$ has a short half life of $7.17\times10^5$ years, it must have arrived at the nebula or protoplanetary disc not long before the asteroids formed.

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Footnote:

$^{8}$It is actually an excess of Mg-26 that is measured as a proxy for $^{27}\text{Al}$. 
or formed in the nebula itself. After the asteroids formed, these radioactive elements will have heated the interiors of asteroids, causing them to melt and chemical fractionation to occur. The amount of melting would be proportional to the amount of $^{26}$Al in the body, as well as the surface area to volume ratio which determines the rate of heat loss to space. This in turn will depend on whether the protoplanetary disc had a heterogeneous distribution of $^{26}$Al, as well as when the asteroids formed. The later they formed, the less $^{26}$Al would be present in the object.

The semi major axis of an asteroid's orbit can evolve over long time scales due to the Yarkovsky effect (Vokrouhlický et al., 2015). This only effects small asteroids on timescales relevant to the Solar System, but the mechanism is that asteroids absorb radiation from the Sun on the solar facing side, then radiate energy from the same surface over time. As there is a lag between the absorption and emission, there is a preferential direction of emission depending on the rotation. For a prograde rotator, this is in the same direction of the direction of travel, and so very slightly accelerates the asteroid, raising its orbit and increasing the semi major axis. The opposite is true for a retrograde rotator, decreasing the semi major axis over time. If the asteroid is non spherical, the vector of the force resulting from the emission will not act through the centre of mass of the asteroid, producing a torque on the asteroid. This is the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect (Nesvorn & Vokrouhlick, 2008). This will cause the asteroid to change its rotation period, the magnitude and sign of the change depending on the shape and initial rotation of the asteroid. The spin axis direction will also change such that the direction moves to higher ecliptic latitudes, which in turn changes the magnitude of the Yarkovsky effect (Figure 2.7) for that asteroid. Both of these effects are dependent on the size of the asteroid. For asteroids of several tens of kilometres in diameter or greater, the effects are small enough that the time scale for the period or semi major axis to change is greater than the age of the Solar System, so this effect can be neglected entirely for large asteroids.
2.3.3 Families

Asteroid families are groups of asteroids that share a common origin. They are the result of the disruption of a parent body, with the family consisting of any remaining parent body plus the fragments of the parent and the impactor (an asteroid is considered destroyed if none of the fragments from a collision contain more than half of the mass of the original body). After a collision, the fragments will have very similar spectral and physical properties as they come from the same parent body, although this will depend on the heterogeneity of the composition of the parent. They will share similar dynamical properties as they come from the same point of origin, with slightly different initial velocity vectors. The Yarkovsky effect (Vokrouhlický et al., 2000) will cause the fragments to drift apart in semi major axis over time, spreading the family out (Figure 2.7). The affect is inversely proportional to the diameter, so smaller members disperse more quickly than larger members.

2.4 Properties

The properties of an asteroid can be assigned into one of three main types: Dynamical properties that describe the passage of the asteroid through the Solar System, chemical properties that come from the composition of the asteroid, and physical properties that describe the dimensions and orientation of the asteroid.

2.4.1 Physical Properties

Mass, Size and Density

Three of the main physical properties of an asteroid are the mass, density and shape (which includes the volume and diameter). These three properties are intrinsically linked. The density is determined by the materials the asteroid is made from, as well as its structure. This bulk density (the mean density of the entire asteroid) is complicated if the asteroid is a mixture of materials or if it contains voids, which is the case for a
re-aggregation of fragments after a catastrophic collision. This will lower the average density of the asteroid compared to the average density of the constituent materials, and asteroids like this are known as rubble piles. The bulk density is calculated from the volume and mass of the asteroid, but except for those objects that have shape models from flybys, or large asteroids with adaptive optics shape models, the volume is not well constrained.

Objects that have been visited by spacecraft have very accurate mass estimates, but the mass of small bodies is most commonly calculated by observing their mutual
perturbations. If the orbits of the asteroids are known before any mutual encounter and known after, then the deflection of the orbits can be measured and the masses calculated. The accuracy of the mass determination is proportional to the accuracy of the astrometry. The Gaia mission can track objects to within 10 microarcseconds, which will allow for excellent astrometry and better mass estimates for $10^5$ to $10^6$ objects (Mouret et al., 2007). This method works best for large asteroids, as the orbital deflections will be larger and easier to measure. Currently, the mass estimates of asteroids have large errors associated with them. Recently, work has been done to calculate the masses from mutual perturbations with a different method. Previously, the mass was calculated with a least squares error, while the new method uses a Monte Carlo method that does not have the same assumption about the parameter uncertainties being Gaussian. This leads to the typical uncertainty being larger but more realistic than those previously reported, but with broadly similar mass estimates for most asteroids (Siltala & Granvik, 2020).

Masses can similarly be calculated from planetary ephemerides, rather than asteroid ephemerides. For the orbits of planets to be properly described, the gravitational influence of the largest asteroids has to be considered. The masses can be calculated from modelling the gravitational effects of the asteroids on the planets, but as with mutual asteroid interactions the uncertainty of the masses are of order 10% with some as high as 100%. The majority of the mass of all asteroids is contained within a few objects.

The volume of an asteroid is determined from the shape and the diameter. The shape can often be approximated by a triaxial ellipsoid, and can be more accurately determined from shape modelling. The volume is obtained by scaling the model, usually from diameter estimates. The diameter is usually determined by the brightness of the object or infrared observations, which is explained in subsection 3.3.3. The sizes of asteroids follow a power law (Equation 2.1),(Figure 2.8).

$$N(> D) \propto D^q$$  \hspace{1cm} (2.1)
The value of $q$ is not constant, but is about $-2.5$ for asteroids larger than 100 km (H 8) and about $-1.8$ for asteroids smaller (Tsirvoulis et al., 2018). It is thought that this 'knee' in the distribution at 100km/H=8 represents a change from asteroids that are primordial, and those that are collisional fragments. However, while there are few large asteroids, they contain the vast majority of the mass in the asteroid belt.
Rotation and Spin Axes

The rotation period is how long it takes a small body to spin on its axis. The spin axis of an asteroid is the direction of the axis around which it rotates. This is usually described with reference to the ecliptic plane. Asteroids formed directly from the protoplanetary disc, so it is expected for them to have formed with prograde rotations. After this, the scattering and collisions that have occurred in the intervening years will have randomised the spin orientations, but there is expected to be an excess of prograde rotators among the larger asteroids that have not been disrupted. Additionally, in a completely collisionally evolved system the distribution of rotation frequencies will be Maxwellian.

It is also possible for an asteroid to be in a non-principal-axis rotation state, where the asteroid rotates around more than one axis. Asteroids in this state are referred to as ‘tumblers’, and it is more difficult to uniquely determine both rotation periods for these objects. As it is harder to excite the rotation of increasingly large asteroids, there are not many large tumbling asteroids, and those that do exist are almost all exceptionally slow rotators.

There is a cut-off in rotation periods at 2.2 hours, with the typical rotation period of an asteroid being 8-12 hours, but this is dependent on size (ref). If an asteroid spins any faster than this, it will disintegrate. This period is known as the spin barrier (Figure 2.9). The barrier is a result of the balance of internal forces against the centripetal force of the asteroid rotation. For small objects, internal strength dominates over gravity in terms of forces keeping the object together. This means that only monolithic objects can exceed the spin barrier, as rubble piles lack the internal cohesion to rotate faster than the spin barrier. As large asteroids are large enough that gravity dominates the cohesional forces, there are no objects in excess of the spin barrier. It is also very hard to spin up a large asteroid, so very few objects even approach the spin barrier.

Large asteroids should be spherical if they are in hydrostatic equilibrium, as is the case for Ceres. With the exception of Hygiea none of the other objects are as round.
Figure 2.9: Diameter against rotation frequency. The dotted line is the spin barrier of 2.2 hours. Non-monolithic asteroids that are bound by gravity cannot exceed this spin limit without breaking up. The objects above this line are monolithic objects. Figure taken from Carbognani (2017).

section 5.5.1, and have been shaped by collisions, disruptions and re-accumulations over the history of the Solar System. A common approximation is to describe an asteroid as a triaxial ellipsoid. This is a reasonable approximation for larger asteroids, which while not in hydrostatic equilibrium are rounded by their own gravity.

Some asteroid systems have been confirmed to be binaries, such as 90 Antiope. In most cases, the primary is much larger than the secondary, but there are a few double asteroids where the two parts are of approximately equal size. Binaries are useful because the orbit of the secondary body allows for the mass of the primary to
be calculated precisely from the orbital period and separation of the secondary, rather than relying on measurements of orbital perturbations.

2.4.2 Spectral Properties and Composition

The spectral type of an asteroid is determined by its reflectance spectrum, with different types representing different surface compositions, formation mechanisms and parent bodies. There are various taxonomies that have been developed that group asteroids according to their spectra, and in some taxonomies their albedos. Each taxonomy splits asteroids into groups/complexes, and then subdivides these groups into types. Most taxonomies are based on cluster analysis, with several parameters measured for each asteroid, and then all of these asteroids are plotted into the n-dimensional parameter spaces, and the cluster analysis returns groupings of asteroids within this n-dimensional space.

There have been many asteroid taxonomies developed, with the two most widely used being the Tholen and Bus-DeMeo taxonomies. In the Tholen system, the features used to distinguish the asteroids are the colour and the albedo (Tholen, 1984). The advantage of this method is that these properties are easy to determine from poor quality data, meaning that types are easy to find from survey data. The colour of a spectrum describes the slope, with red spectra having increasing reflectance with increasing wavelength and blue spectra having decreasing reflectance with increasing wavelength, and a neutral spectrum having unchanging reflection with wavelength.

The main groups are the C group, which are carbon rich asteroids, the S groups which are silicate rich, and the X group being a mixed group for metallic, moderate albedo M type asteroids, and asteroids with similar spectra but smaller (P type) or large (E type) albedos. Each of these groups contain sub-types, and there are some types separate from these groups. For example, the majority of Trojans are D type which is not part of any of the C, S or X groups.

The Bus-DeMeo classification is a newer system of classification, and uses only
spectral features and does not use albedo (Figure 2.10) (DeMeo et al., 2009a). The main complexes here are again the C, S and X complexes but with more spectral types used. Not all asteroids that were placed in to the C, S and X groups in the Tholen classification are put into the same complexes in the Bus-DeMeo classification. In addition, there are various end members that consist of ungrouped spectral types such as A and V types. The Bus-DeMeo taxonomy uses data from the SMASS survey so more asteroids have types in this classification than the Tholen classification. The Bus-DeMeo taxonomy builds on the SMASS classification, which also used spectral features rather than albedo.
The physical meaning of the types is that the type is dependent on the formation conditions of the parent body, as well as the evolution of the parent body. For example, C types are so named because of their similarities with carbonaceous chondrites, and the S types were named for their stony/silicate composition, and are similar to stony iron meteorites.

The types are not uniformly distributed in the Solar System (Figure 2.11). C types appear more frequently with increasing heliocentric distance, and are the most common type in the outer main belt, while S types are generally found closer to the Sun. This is probably indicative of different formation conditions, although the fact that taxonomies are driven by colours and spectra rather than the physical interpretation of these prevents us from conclusively finding the reason for their differences in spectra and distributions. Definitively linking asteroid types to meteorite types would allow us to link compositions to asteroid types, and would be a major step in understanding the evolution of the Solar System.

Some meteorites have been associated with parent bodies. For example, the Howardite Euchrite Diogenite (HEDs) are thought to come from Vesta, due to spectral similarities as well as the proximity of Vesta family asteroids, which are mostly V types, to resonances that can change their orbits to approach Earth (Buratti et al., 2013). HEDs are also from a differentiated body due to homogeneity of oxygen isotope ratios, and as a large object with a basaltic crust Vesta definitely satisfies this criterion.

The NEA (25143) Itokawa visited by the Hayabusa mission, and although only 1500 grains were collected from the asteroid, it was enough to determine that the material matched that of an LL chondrite (Nakamura et al., 2011). Itokawa is an S type asteroid, and the grains were of ordinary chondrites. As both of these were thought to be stony objects, Hayabusa confirmed the link between stony asteroids and stony meteorites. This also resolved the issue that although ordinary chondrites and S types were both stony and thought to be the same objects, their spectra did not match. This was confirmed by Itokawa to be because of the particle size at the surface altering the spectrum of S types asteroids from that of a Q type, which have larger particles
Another example of an asteroid with both a sample on Earth, and a spectrum of the asteroid is the Almahata Sitta meteorites (Bischoff et al., 2010). These come from asteroid 2008TC3, which landed in Sudan in 2008. This was special because the asteroid was detected before it entered the atmosphere of the Earth. Spectra taken showed that the asteroid was an F type, which is part of the C complex, implying a primitive and carbon rich composition. Carbon rich grains were found among most of the samples, but were achondritic in nature, which is caused by differentiation, which makes the parent body non primitive. The remainder of the samples were from at least 2 types of
chondrites, making the asteroid an agglomeration of different meteoritic types.

There are also statistical arguments that predict the source region of L and LL chondrites from orbits of NEOs, which can then be matched to the Gefion and Flora families respectively, but there is no definitive link from a meteorite with a known spectral type and an asteroid with a known spectral type. Both of these families are mainly S type asteroids in the inner asteroid belt (Nesvorný et al., 2009).

Asteroids can be categorised as containing processed or unprocessed material, or a mixture of both. The least processed objects contain very primitive material. This is what D type asteroids are thought to be made from, and is material from early stages of the formation of the Solar System that has not been significantly altered by heating or differentiation. We find increasingly processed material in other types. Some have been subject to shock, pressure and heating, while others have undergone differentiation.

Asteroids have a property known as the phase function. This function is the brightness as a function of phase, which is the angle subtended at the asteroid by the Sun and observer. This is a non-linear function due to scattering at the surface of the asteroid. This is especially true at small phase angles, where there is a sharp increase in brightness near opposition.

An asteroid family (page 25) is a group of asteroids with a common origin. If an undifferentiated asteroid is impacted, then the collisional fragments will all start off with similar orbital parameters and compositions. Over time the fragments drift due to the Yarkovsky effect, and may be scattered through interactions with other asteroids. This spreads the family out within orbital space in a characteristic manner, with smaller objects moving away more quickly than larger ones. By grouping objects with common orbital and compositional characteristics, we can try and group objects to families and identify the parent body, the most massive of the family. This is important because a large asteroid not associated with a family may have never been collisionally fragmented, meaning it is as it was when it formed, and can be defined as a primordial object. Alternatively, the family may have been spread out sufficiently that it is no
longer possible to detect, and if members drift into an unstable resonance, then they will have their orbital parameters altered to the point where it is impossible to establish a relationship with the family.

In this work *primitive* is a property of a material, and refers to material that is unaltered since it was incorporated into a small body. If the small body is broken apart in a collision but the material in the body is unaltered, it is still primitive. *Primordial* is a property of an individual object. The object can suffer impacts and have internal processes altering it, but as long as it is not significantly disrupted, it is still primordial.

### 2.5 Questions to be answered

There are many unanswered questions in asteroid science. Below are some of the ones most pertinent to a project on shape modelling.

#### 2.5.1 Rubble Piles and Consolidated Objects

The amount of empty space within an asteroid is referred to as its macroporosity. As planetesimals formed large directly from the protoplanetary disc, then they should have zero macroporosity. Although the initial grains that stick together to form the pebbles will have space between them, and the pebbles that make the self gravity of a large planetesimal will not be packed together with zero porosity unless they are squashed together by a large objects self gravity. Most planetesimals are expected to then broken apart in collisions. If the energy of the collision is sufficient to break apart the object, but insufficient to break the mutual gravity of the fragments, these fragments will eventually re-coalesce into a group of objects (an asteroid family). Like breaking apart a jigsaw and trying to put the pieces together randomly, there will be spaces between the fragments. This will leave the asteroids as rubble piles. An asteroid that hasn’t been broken apart will be a consolidated object, and will not have any macroporosity.

Understanding which objects are rubble piles and which are not will allow us to distinguish between primordial objects and those which are re-accumulations of ma-
terial. It is worth noting however, that just because an object is primordial does not make it primitive. Ceres and Vesta are both consolidated objects, but both have internal processes that have altered their composition so their constituent material is no longer primitive.

2.5.2 Physical properties of families

As asteroid families are remnants of collisions, they are an ideal way to study these events. It has been shown that asteroids that are members of family have different axial ratios from asteroids not associated with any families, suggesting that collisional fragments have different shapes from primordial objects (Cibulková et al., 2016).

An asteroid that belongs to a family is by definition not a primordial object, as it has been broken apart. It is reasonable to assume that an asteroid that has never been part of a family is a primordial object, but it is difficult to prove that an asteroid has never been part of a family, as it may have moved far from the rest of the family over the history of the Solar System. Delbo et al. (2017) claim to have identified some objects that do not belong to a family, and are of sufficient size to have been a planetesimal, assuming planetesimals did indeed ‘form big’.

2.5.3 Period and Spin Pole distribution

The periods of collisionally evolved asteroids is expected to be Maxwellian, and for large asteroids this has been observed. For small asteroids that are spun up or down due to the YORP effect, the periods should diverge to either zero or infinity. Due to the finite age of the Solar System, this is not true but there is an excess of both slow and fast rotators compared to the Maxwellian.

As it is expected that most of the asteroid belt is collisionally evolved, the distribution of periods should be Maxwellian and the distribution of spins should be uniform. As asteroids formed from the protoplanetary disc, they would have formed with prograde rotations to conserve angular momentum. Primordial asteroids should retain
this preference for prograde rotation, as they will not have suffered enough impacts to significantly alter their spin direction, and are too large for the YORP effect to have altered their spin direction. In addition, the YORP effect will cause small asteroids spins to be normal the their orbital plane. The YORP effect will also effect the spin rate of asteroids, causing them to either spin up to the point they break apart, or slow down in rotation speed. The distribution of the longitudes of the spins is expected to be isotropic, as they should be random with no known physical mechanism from their formation or collisions to have a bias in a particular direction. It has been found that there is an excess of asteroids clustered around 30-110 degrees ecliptic longitude, with a deficit around 120-160 degrees (Cibulková et al., 2016). A possible mechanism that has been proposed is the Cassini resonance, where the procession of the asteroid spin and orbit are aligned, causing the spin axis to be at a specific value both in latitude and longitude (Vokrouhlický et al., 2003).

2.6 Goals

As SuperWASP small bodies data are best used for relative magnitudes rather than absolute, it is ideally suited for shape modelling. Due to the large number of asteroids observed from many geometries, it should be possible to create many shape models from the dataset of large main belt asteroids. The sizes of these asteroids would range from some of the largest asteroids such as 6 Hebe, down to a few tens of kilometres in diameter. This straddles the range of sizes that have been suggested as the boundary between primordial and collisionally evolved asteroids. As collisionally evolved asteroids would be rubble piles, they would have a lower density than the primordial objects of the same composition. A shape model would allow improved estimates of the volume of the asteroid over the volume from the assumed diameter. While estimates of asteroid density are currently limited by the uncertainty in the mass, once Gaia data are available to use for asteroid masses, volume will become the larger uncertainty in the data calculation. By creating shape models now, once the Gaia data are released it
2.6. Goals

should be possible to calculate the density of individual asteroids, and see if there is any suggestion of a boundary between the densities of primordial asteroids and rubble piles.

Other studies have also used survey data for shape modelling, but these studies have used data from 'sparse’ sources with observations of each asteroid hours or days apart (Durech et al., 2020), (Cibulková et al., 2018). The output is often not a complete shape model due to the small number of observations, but rather an axial ratio and a spin direction. The large number of objects observed means that these properties can be determined for a very large number of objects. The small success rate of shape model creation means that there is little overlap with the models in this work which focuses on the largest asteroids, with a notable exception being 33 Polyhymnia which is a new model.

The original objective of this project was to create new and improve existing shape models. These would then be combined with masses from Gaia to derive improved densities. This was not possible as Gaia data release 4 will become available after the end of this project. This would also have required thermophysical modelling to scale the shape models. Therefore, over the course of the project, the science goal has shifted somewhat to accommodate these factors. By having a large sample of shape models, it is possible to do population statistics on the main belt. (A recent example of this was to analyse the shapes of asteroid family members to get an idea of the shape distribution of collisional fragments). This sort of analysis does not require the exact volume or mass to be calculated, although the diameter is still useful when looking to see if any of the other physical properties are correlated with size. In addition to using models using purely SuperWASP data, I will later use models publicly available on DAMIT to aid in the ensemble properties of asteroids. This will be especially useful for smaller asteroids, as these do not appear frequently in the SuperWASP database. There will also be larger asteroids that happen to have not been observed with SuperWASP that will be useful additions to the dataset.

One of the properties that are useful for ensemble investigations are the axial ratios.
Lightcurve analysis provides a lower limit of the axial ratio of the longest and shortest axes, but a shape model is needed to get the true axial ratio. As the axial ratios have been linked to properties such as whether the object is a collisional fragment, they are useful quantities to know. The longitude of the spin axis is also useful to know, as some modelling methods that have been used in the DAMIT (subsection 3.1.5 database) uniquely determine the longitude, rather than giving two values 180 degrees out of phase from each other. As some unexpected non-uniformity has already been found for this, it is a useful quantity to measure. These values that are well known for shape models can be compared with other well known properties such as types and sizes to see if there is any relation.

The secondary objective of the thesis is to make the SuperWASP data usable to the community at large. A significant proportion of the SuperWASP dataset has data with a signal to noise ratio too poor to be useful, and for many projects potential users have decided it is not worth the time and effort to extract the useful data. It is therefore useful to come up with an automated pipeline for cleaning the SuperWASP data to extract the maximum amount of useful data, as even after discarding poor quality data the amount of photometry is comparable to the entire historical dataset for some asteroids. Such a large amount of data will be useful to many planetary scientists, and the fact that it is historical data means it will be useful for applications that require a long baseline of observations in the future.
Chapter 3

Data Used in this Thesis

This chapter outlines the data used in this thesis as well as other relevant data. The acquisition of optical photometry is outlined, and the history pipeline and previous work done with SuperWASP asteroid data is explained. The other methods of observing asteroids including adaptive optics, occultations, infrared, radar, spacecraft and meteorites are explained.

3.1 Optical Photometry and SuperWASP

3.1.1 Optical Photometry

The primary method of observing asteroids is with photometry acquired from telescopes. Optical telescopes are most commonly used, and these can be used to determine the orbit of the asteroid by observing its position on the sky and both the mean and variations in brightness can be used to calculate physical properties. The absolute magnitude (H) is a measure of the brightness of the asteroid\(^1\) and the variations in brightness are recorded as a lightcurve, which is is the brightness of the asteroid against time. These variations are caused by the rotation of the asteroid which changes the amount of reflected light reaching the observer. By looking for repetition in the

\(^1\)The definition of the absolute magnitude is what the brightness of the asteroid would be at a distance of 1AU from the Earth, at 1AU from the Sun and at zero phase angle.
lightcurve, the period of the asteroid can be found. Although the asteroid will have a nearly constant rotation speed, the constantly changing geometry between the asteroid, Earth and Sun will cause the lightcurve to change gradually with time. In addition, albedo variations across the surface will also change the shape of the lightcurve. For example, Ceres is extremely round so its lightcurve is albedo driven, primarily by the salt deposit in Occator Crater which is very reflective (Nathues et al., 2015). Optical data are produced by SuperWASP, which is the data used in this work.

### 3.1.2 SuperWASP History and Specification

The WASP (Wide Angle Search for Planets) consortium is a group of UK Universities with an interest in exoplanet surveys, and includes the Instituto de Astrofísica de Canarias, the Isaac Newton Group of telescopes, Cambridge University, Keele University, Leicester University, the Open University, Queen’s University Belfast and St. Andrew’s University (Pollacco et al., 2006).

SuperWASP is designed as an exoplanet survey, the main goal of the survey being to discover ‘Hot Jupiters’, which are Jupiter sized exoplanets orbiting close to their host star. Due to the large size of the planet, when it transits the star it creates a relatively large dip in brightness of the star of about one percent (Deeg & Alonso, 2018). This is visible on a lightcurve as a characteristic ’U’ shape, with the depth and width of the ’U’ being indicative of the size of the planet. To maximise the number of detections, the survey favours field of view over photometric depth, as this maximises the number of stars that can be viewed at once. The trade off is that only transits across bright stars can be detected, and only large gas giants comparable in size to Jupiter can be found as the planet has to be sufficiently large compared to the host star to cause a detectable decrease in brightness. Exoplanets on close in orbits are easier to detect as they transit the host star more frequently, making it more likely for them to be detected. The shape of the transit lightcurve is determined by the size of the planet relative to the star. Combining the size and the mass gives the bulk density which can be used to
classify the planet as either rocky or gaseous. SuperWASP was very successful, finding nearly two hundred exoplanets to date. Besides asteroids, the SuperWASP survey has also provided insights into other areas of time domain astronomy such as variable stars (Norton et al., 2007), with periodicity on the scales from hours to months available. The telescope is currently used to track space debris around the Earth.

SuperWASP north started observing in 2004 in manual mode as a successor to the WASP0 telescope, which was designed as a test for the SuperWASP hardware. This was followed by a break from observing in 2005 to upgrade the number of cameras and install SuperWASP south. Both telescopes have observed continuously since 2006. In 2012 the observing strategy was changed from a cadence of a pair of observations 40 seconds apart every 240 seconds to a single observation every 40 seconds.

The successor to SuperWASP is the Next Generation Transit Survey (NGTS), designed to find ‘Super Earths’ and Neptune sized planets Wheatley et al. (2018). The survey will go deeper to 18 magnitude and will cover 100 square degrees of sky at any given time. For asteroid science, this will allow it to survey objects down to a smaller size, but offer less completeness at any given size. It will also mean that Solar System Objects will move out of the field of view more quickly, meaning fewer serendipitous observations for any given apparition.

SuperWASP consists of two telescopes, one at La Palma in the Isaac Newton Group of telescopes (located at the Roque de los Muchachos Observatory) in the northern hemisphere, and one at the South African Astronomical Observatory, just outside Sutherland in South Africa in the southern hemisphere. Each telescope consists of 8 cameras (Figure 3.1), with each camera having a 7.8 by 7.8 degree field of view. The cameras have 2048 x 2048 pixels giving each pixel an angular size of 13.7 arcseconds, and the total field of view across all cameras of 973 square degrees. For comparison, Kepler (a space-based exoplanet survey) has 21 detectors with a combined field of view of 115 square degrees Borucki et al. (2010) and the full moon subtends a solid angle of 0.2 square degrees. This makes the field of view extremely large, able to view many asteroids, but at the cost of large pixel size, which causes increased blending.
(section 4.1). It has a limiting magnitude of $V=15$, but in practise to get good signal to
noise the object needs to be brighter than this.

As a traditional dome would limit the amount of sky visible to the telescope, the
enclosure uses a rolling roof design that can cover the structure within a minute in
case of inclement weather. The SuperWASP telescopes use commercially produced
Nikon lenses, which had excellent chromatic properties and the fastest shutter speeds
for commercial lenses at the time. These cameras were mounted in a 2x4 arrange-
ment (with slight offsets in viewing angle to cover different parts of the sky). Both
telescopes were fitted with broad-band filters in 2006, and the lenses of SuperWASP
South were changed in 2012. Each telescope has its own computer for data acquisition,
and a weather monitoring system to determine if the weather is suitable for observing,
and is capable of automatically stopping observations in bad conditions. The original
SuperWASP images are stored on an archive server at the Rutherford Appleton Lab-
oratory, but all of the photometry is stored as fits files at Warwick university NGTS
cluster.
3.1.3 SuperWASP Pipeline

The SuperWASP data reduction pipeline was created to process all SuperWASP images (Pollacco et al., 2006), and starts by applying image corrections; the bias, dark, flat and shutter correction maps. Then for each image, a subset of the Tycho-2 catalogue is extracted containing all of the stars expected to be within the field (Høg et al., 1998). Then, all stars detected in the field are matched with the catalogue, and any objects that either do not match both the position and brightness of a source from the Tycho-2 catalogue are recorded as orphans, which have their own separate pipeline. Outliers, cosmic rays and gradients are then removed from the image. If the amount of the image removed exceeds 50 percent, the entire image is rejected as it is indicative of heavy cloud cover. The brightnesses are calculated using aperture photometry, which is performed with radii of 2.5, 3.5 and 4.5 pixels. The sky background annulus has an inner radius of 13 pixels and an outer radius of 17 pixels. At this point the blend index is extracted from the flux ratios of the apertures, as a nearby contaminating star will change the flux ratios from the expected ratios for a single source (near 1:1:1). If there is no blending present, the only brightness comes from the central source. If there is a significant increase in brightness for larger apertures, it is indicative of one or more secondary sources. Then, the calibration from SuperWASP to Tycho-2 magnitudes is done, and a linear fit is used across each camera using a series of predefined standard non variable stars.

3.1.4 Previous Work with SuperWASP and Potential Uses

SuperWASP is a useful tool for asteroid science. The extremely large field of view and long run time of over a decade means that it has serendipitously observed many asteroids over its operational life. It has observed 12460 asteroids as of the time of writing, with a total of 7692660 individual observations. As the pipeline matches unknown objects in the SuperWASP catalogue to known asteroids it has not discovered any new asteroids, but due to the limiting magnitude it is unlikely it would have made any new
discoveries other than bright NEOs. Due to the limiting magnitude of SuperWASP, most of the asteroids will be large main belt asteroids with a diameter greater than 10s of km. There are also some NEOs in the dataset, but the ones with usable data will generally be large such as Eros, and at a close approach to Earth. As seen in Figure 3.2, NEOs and outer main belt asteroids are under-represented in SuperWASP. Some of the largest Jupiter Trojans are also present, but generally have insufficient lightcurve quality for creating shape models, but the photometry could have other uses. There are no Centaurs or Kuiper Belt objects in the dataset as they are too dim to be seen.

SuperWASP does not suffer from many of the biases that targeted observations have (Marciniak et al., 2015)(Figure 3.2). It is beneficial for astronomers creating shape models to view asteroids with a large lightcurve amplitude and fast rotation, as a full rotation period can be viewed with minimal telescope time, and with a smaller telescope than a slow rotating low amplitude object. The result is that when doing targeted observations of individual asteroids, slow rotators and low amplitude objects are left out because the photometry required to characterise them is harder to obtain. SuperWASP gathers data on all asteroids within the field of view, so does not suffer from this bias, although the uncertainty in the night-to-night magnitude calibration for moving objects prevents SuperWASP from collecting photometry suitable for shape modelling on slow rotators (P>24 hours). It also only views relatively bright asteroids, so is not useful for main belt objects smaller than a few 10s of km in diameter. Although the lack of a targeted survey is a bonus in that it help prevents observational biases, it has the downside that many asteroids have little data as they have, by chance, rarely or never been in the field of SuperWASP.

The observing strategy of SuperWASP is to look at the same area of sky for several weeks before moving on to another area. This strategy is done to optimise the exoplanet search, but it is also beneficial for characterisation of a asteroid lightcurves, especially for asteroids with periods longer than a single night’s observation time. The large field of view increases the mean time for an asteroid’s apparent motion to take it out of the frame, typically taking weeks. Many asteroid surveys (subsection 2.2.3)
Figure 3.2: Figure (a) is a histogram of the semi major axes of numbered asteroids, and the figure (b) is a histogram of the semi major axes of asteroids observed by SuperWASP.
(a) All Numbered Asteroids

Figure 3.3: A histogram of the H magnitudes of all of the asteroids observed by SuperWASP. The number of objects peaks at 13\textsuperscript{th} magnitude. This is in contrast to all asteroids, where the number of objects increases with an increase in absolute magnitude Figure 2.8.

rapidly change the area of observation, not allowing for uninterrupted lightcurves. While sparse photometry is increasingly being used for asteroid science other than NEO discovery and classification, it is much harder to use for shape modelling than continuous (also referred to as dense) photometry as the phase folding of the photometry is much more difficult, as well as the need for very precise night-to-night magnitude calibration. As SuperWASP has several lightcurves from consecutive nights, finding the period is easier and the continuous data are helpful in cases where a single noisy lightcurve may not provide a conclusive period, but by folding many lightcurves for better signal to noise a period may be determined. The cadence of SuperWASP observations are well suited to asteroid shape modelling. The mean cadence of observations is about 4 minutes (section 4.2.2), while the minimum asteroid rotational period for non monolithic asteroids is greater than the spin barrier (2.2 hours). This allows sufficient sampling of the lightcurve for all asteroids.

Unfortunately, SuperWASP suffers from a drift in the calibration magnitude between cameras, and across individual sensors due to temperature gradients. The pipeline does attempt to standardise photometry by comparisons of stars with their catalogue brightness, but as the temperature gradients across the detectors are not easy to model,
this does not completely eliminate the problem. This makes SuperWASP data self consistent, but brightnesses may not be consistent with other sources of asteroid photometry due to the calibration issues. For asteroids with long rotations where only a partial lightcurve is retrieved in a single night, it is difficult to fold these lightcurves due to the issues with the calibration (see subsection 4.2.5 for how lightcurve folding is done in this work).

SuperWASP has been running for many years and has observed in both hemispheres, so many asteroids have been seen from multiple geometries. Viewing an asteroid from multiple geometries is important because it is a requirement for the production of a shape model for that asteroid. Views from multiple geometries will require multiple observation sessions months or years apart depending on the orbit of the asteroid, so a survey is useful to get several different viewing geometries of an asteroid, known as apparitions. Even if it is not possible to create a shape model from the data, the variety of geometries will help to get the maximum possible range of the lightcurve amplitude, which makes period and axial ratio determination easier. The large variety of geometries is especially useful when combining SuperWASP data with data from historical lightcurves. Many historical datasets have large amounts of data, but lack the specific geometries needed to create a model, for example if an asteroid has only been observed near perihelion. The SuperWASP dataset can provide supplementary information necessary to create a model, as shown in the results of this work.

The amount of time SuperWASP has been observing for also makes it useful for serendipitous asteroid discoveries. In addition to my project, there is a separate project to look for transient events in the SuperWASP catalogue, such as an asteroid undergoing an outburst or experiencing an impact. As well as the potential to obtain observations at the time of the event itself, SuperWASP will likely have photometry from before and after an event, which means that a period change or a long term brightening can be observed. As the data are historical, SuperWASP is a unique source, having many observations that potentially contain events not seen by any other telescope.

There are several reasons why SuperWASP data has not been widely used before
now. The largest barrier is that the archive is not public. The automated observing of the telescope led to it sometimes observing when it should not have been such as under excessive cloud cover, leading to poor quality data. In addition, if the asteroid was close to the detection limit, the signal to noise would be very poor, producing lightcurves that were not of sufficient quality to use for studies but are not flagged as such. I have done significant processing of these data to try and remove bad nights and sections of lightcurves from the SuperWASP dataset, while retaining as much usable data as possible. Another difficulty of using SuperWASP data is a result the large pixels. The pixels are 13.7” each side, and as the cameras are defocussed the asteroid point spread functions (PSFs) are larger than that. The result of this is that if a star is within about 60” of the asteroid then it will contaminate the signal from the asteroid, a phenomenon known as blending. By calculating the separation of the star, the contamination can be estimated, but sometimes will not be removed entirely.

Particularly bright asteroids can saturate the detectors of the SuperWASP telescopes, but only Ceres and Vesta seem to suffer from this with none of the other asteroids having high enough apparent brightness. As they have both been visited by the DAWN spacecraft, the only use for the data would be looking for outbursts or validating SuperWASP. SuperWASP data will continue to be useful because deeper surveys will be saturated by not only Ceres and Vesta but other large asteroids, preventing them for obtaining photometry.

Although SuperWASP data has not been fully exploited, it has been used previously. The main user of the SuperWASP small bodies data has been Neil Parley who developed the asteroids pipeline to extract the asteroid data from the main SuperWASP pipeline (Parley et al., 2005). The main pipeline was designed for detecting exoplanets, and so only dealt with stellar objects. It did identify non stellar objects, but did not do anything with these orphans other than catalogue their location and brightness. While Parley finished extracting the asteroid data and continued to do so for some years, he did not have time to extract much scientific value from the data before moving on to other projects). The work done was mainly demonstrating the potential uses of the
data, such as finding rotation periods and phase curve coefficients for various asteroids, but no conclusions were obtained from these results.

There has also been interest from others in the planetary science community, but limited use has been made of the data due to lack of availability and unreliability of some of the data, with most deeming the return of a few more lightcurves for an object of interest not worth the effort of cleaning the available data to get it into a usable state (Simon Green, personal communication). This has led to the asteroid data from SuperWASP remaining largely unexploited.

### 3.1.5 DaFEED

DaFEED is short for DAMIT Feeder. DAMIT is the Database for Asteroid Models from Inversion Techniques Ďurech et al. (2010). Both DAMIT and DaFEED are run by Josef Hanuš and Josef Ďurech. This is a database for asteroid models made from lightcurve inversion, and contains 4213 models from 2408 asteroids at the time of writing. The difference is that DAMIT is publicly available and shows the best model for each object. DaFEED shows all models for each object, as well as the lightcurve data for each model. Each object has a series of lightcurve files, with new files being added with new data, either recently acquired or made available. The data comes from many observers, and from several decades. Each lightcurve file contains the name of the observer that recorded each lightcurve, as well as the night the data was taken to make it more human readable. The lightcurves are in chronological order, with the exception of sparse data which is always at the end of the file. The data goes back up to 70 years for some objects, and is taken from many observers’ data. For each model the best period, spins and shape are given, as well as a plot of the synthetic lightcurve against the data. The data are in the format of the light time corrected time of the observation, then the Cartesian coordinates of the asteroid and Sun in AU is added.

There are a few differences between SuperWASP and historical data. The first is that historical data covers a much longer period of time, up to decades while Super-
WASP only covers a single decade. This is fine for period determinations, but usually the historical data has more apparitions than SuperWASP for the same asteroid. On the other hand, SuperWASP will typically have more observations per asteroid (Figure 3.4), but these observations are grouped in time, while the data from DaFEED is more spread out. Finally, DaFEED has good quality data, but there are a few that need to be removed as they are not of sufficient quality for shape modelling. On the other hand, SuperWASP data needs cleaning before much of it can be used for shape modelling.

Figure 3.4: Comparison of numbers of photometric points for asteroids in both SuperWASP and DaFEED for the first 1000 numbered asteroids. Points below the line are asteroids with more data from DaFEED, while points above it have more data from SuperWASP.
3.2 Other Observing Methods

3.2.1 Adaptive Optics

Observations from Earth of asteroids will be disc integrated unless the telescope has resolution greater than the apparent size of the object. Telescopes with Adaptive Optics (AO) are capable of resolving asteroids that are sufficiently large as viewed from Earth (Figure 3.5), as for telescopes without adaptive optics the resolution is limited by atmospheric seeing rather than diffraction. For an object such as Vesta that is both large and relatively close, many features can be distinguished such as individual craters (section 5.5.1). For a more distant and smaller object such as Iris, only broad features can be seen such as larger impact basins (section 5.5.1). These observations can be combined with shape models from lightcurve inversion to create more detailed and stable models than those made from lightcurve inversion alone (section 5.3). The diameter can also be estimated from resolved images as the resolution of the telescope and distance to the asteroid provide the size of each pixel at the distance of the asteroid. This can only be done to the nearest pixel, so the uncertainty can be improved by obtaining higher resolution images or viewing the asteroid at an apparition where it is closer to Earth. AO derived shape models can be combined with occultation data of an asteroid to give improved diameter estimates.

3.2.2 Occultations

An occultation is where a small body passes in front of a star as seen by an observer, which blocks out the light of the star. Occultations are a useful way of determining the size of asteroids, as the measured diameter is independent of albedo. To observe an occultation, a moving body has to pass in front of a star as viewed from Earth. The object will cast a shadow on the Earth. As the star is effectively an infinite distance away, the shadow will be equal to the size of the occulting object. By positioning telescopes in the expected path of the occulting object’s shadow across Earth, the star will appear
Figure 3.5: An adaptive optics image of 16 Psyche taken using the SPHERE instrument on the Very Large Telescope using visible light (600-900 nm). Visible are some of the large craters on the surface, with some concavities visible in the outline (Dohlen et al., 2006)

from the observer’s perspective to disappear for a short time. The duration of the time the star is gone for combined with the known orbital velocity of the asteroid gives the size of a single chord of the asteroid’s two dimensional cross section. By placing observers in a line across the surface of the Earth (Figure 3.6), several of these chords can be obtained to give the two dimensional cross section, which gives an approximation of the asteroid outline from that single geometry. An example is the occultation of Arrokoth Figure 2.6 (Buie et al., 2020). As the timings of the occultation are accurate to within a second, the asteroid diameter can be calculated to within a few kilometres.
3.2. Other Observing Methods

Figure 3.6: Map showing the path of an asteroid’s shadow across the Earth. Telescopes must be placed at regular intervals along the expected path of the shadow in order to get the most chords for the occultation, allowing for the best estimate of the asteroid’s outline. The second image shows the results of these chords, clearly outlining the asteroid. This is a particularly good way to search for close or contact binaries, as the outline(s) will show large concavities. The size, shape, density and ring of the dwarf planet Haumea from a stellar occultation (Ortiz et al., 2017).

The occultation could resolve binary objects as long as the two components are not obscuring each other.

Occultations are predictable when the apparent motion of the Earth, small bodies and stars are known with sufficient precision. The occultation has to occur somewhere on Earth that a line of telescopes can easily be set up, which in practice limits them to locations with an active amateur astronomy community with mobile telescopes, or where observatories already exist (Figure 3.6). The occultation data can be incorporated at the shape modelling stage to help produce the shape, and constrain the diameter. This is not possible for all small bodies, as not all small bodies have occulted stars, have had an occultation that would be practical to observe.

3.2.3 Infrared

Asteroids are some of the brightest infrared sources in the sky, making them ideal targets for infrared observations. The infrared radiation from asteroids comes from
the re-emission of the energy from solar radiation absorbed at the asteroid’s surface. Infrared observations are useful for asteroids as they allow for a more accurate calculation of the diameter than those obtained from optical data. Two properties that are important for calculation of the diameter are the albedo and thermal inertia. The albedo determines how much radiation the asteroid absorbs. Asteroids with higher albedo will absorb less radiation, so will need to emit less radiation in order to maintain thermal equilibrium. The thermal inertia is how quickly the asteroid will react to a change in thermal conditions, and depends on other properties including the heat capacity and thermal conductivity. These properties are required for thermal modelling.

Infrared observations are difficult because Earth’s atmosphere emits and absorbs infrared radiation, making the atmosphere opaque. The best infrared observations are obtained from high altitude or space based telescopes, reducing the number of facilities available for observing asteroids. The telescopes themselves can also emit in the infrared so they have to be constantly cooled in order to continue observing. For space based telescopes where the coolant cannot be replaced this limits the operational lifetime of the telescope. The majority of asteroid infrared data has historically come from three sources; IRAS, AKARI and NEOWISE (subsection 2.2.3). Other telescopes such as Spitzer have also had many asteroid diameter calculations (Trilling et al., 2020), but have not been the leaps in number of asteroids detected that the former three have provided.

3.2.4 Radar

Radar observations of asteroids are obtained by transmitting radio waves towards an asteroid, then receiving the reflected waves (Ostro et al., 2002). By measuring the Doppler shifts in the returned waves as well as polarisation changes, the shape of the asteroid can be reconstructed. This is because the rotation of the asteroid causes a doppler broadening in the return signal. This only works for nearby or exceptionally large asteroids, as the power of the received signal drops as $D^4$, where $D$ is the distance
to the asteroid from Earth. As there are few radio telescopes on Earth powerful enough
to be able to detect the return pulse, telescope time is a practical limit as to how many
asteroids can be studied this way. The radar albedo of the asteroid also affects how
strong the returned signal is. The spectral type will give some indication of the radar
albedo, with metallic objects typically having higher radar albedos. Using the radar
albedo and return signal strength, size can be estimated.

3.2.5 Spacecraft

The most direct way to study asteroids is to visit them with a spacecraft (Figure 3.7).
This can tell us the shape, spin state, rotation period, mass and density extremely
precisely. In addition, the chemical and thermophysical properties across the entire
surface of the asteroid can be known, rather than the disk integrated amount at the time
of a single observation. More recently with the Hayabusa, Hayabusa2 and OSIRIS-
REX missions, sample extraction and return has been attempted (Fujiwara et al., 2006;
ichiro Watanabe et al., 2017; Lauretta et al., 2017). This allows for the context of the
samples to be known, connecting analysis in the laboratory to asteroids seen through
telescopes. Only a few asteroids have been visited by spacecraft, and as of the time
of writing the only samples returned are from Itokawa and Ryugu by Hayabusa and
Hayabusa 2.

3.2.6 Meteorites

With the exception of the Hayabusa samples, meteorites are the only samples we have
of asteroids to study up close. They can tell us things such as when the asteroids formed
via isotopic ageing, what they are made of as well as whether they have undergone any
significant processing since their formation. At the moment, it is difficult to definitively
associate any meteorite with a single parent body, although several relations have been
proposed. Meteorites tell us the dates the asteroids were formed, as well as any un-

\[\text{\footnotesize Vesta image taken from the website } \text{https://www.nasa.gov/mission_pages/dawn/multimedia/pia15678.html.}\]
altered constituents. The composition, specifically the amount of volatiles reveal the approximate location of the asteroid within the protoplanetary disk. This can be helpful for showing the distribution of materials in the protoplanetary disk, which can help with determining the material that the planets formed out of (see subsection 2.4.2 for a summary of links between meteorites and asteroids).
3.3 Calculating Properties

3.3.1 Orbits

Orbits can be calculated from as few as three observations of an asteroid, or two if the velocity at the time of each observation is known. More observations will improve the accuracy of the orbital parameters (Figure 2.1). Determination of orbits is particularly important for PHAs, as this determines the risk they pose to Earth.

3.3.2 Spectral Classification

Infrared and optical observations are also used for spectral classifications of asteroids (subsection 2.4.2). In the Tholen classification, this is based on spectra from the UV through to IR data from the eight colour asteroid survey, as well as albedos to separate some of the types that are spectrally similar, such as the E, M and P types. SMASS uses higher resolution spectral data, so has used absorption bands to separate types. However, it does still use the C, S and X complexes from the Tholen classification for continuity, as X and C types are not well separated in this classification as separate complexes, although the types within these complexes are well defined.

3.3.3 Diameters

The diameters in this work are taken from Benoit Carry’s compilation of asteroid diameter estimates (Benoit Carry, personal communication). Most asteroid diameters are from thermal infrared modelling, and similar to asteroid mass estimates have errors typically of the order of 10 to 100 percent. The error in the volume is approximately 3 times the error in the diameter, which can lead to large volume errors. When combined with the mass error, the density error can be extremely large. Part of the reason for this is that infrared diameter modelling uses the assumption that the object is spherical, and so the accuracy of the estimate is dependent on how spherical the object is.

Asteroid diameters can be calculated from one or more sources. As most asteroids
have an absolute magnitude estimate, a diameter can be calculated. By assuming an albedo and using the absolute magnitude the diameter can be calculated from Equation 3.1.

\[ D = \frac{K}{\sqrt{p_v}} 10^{-\frac{H}{5}} \]  

(3.1)

In this equation \( D \) is the diameter in kilometers, \( H \) is the absolute magnitude, \( p_v \) is the bond albedo and \( K \) is a constant, usually taken to be 1329. This makes the assumption that the shape is spherical, does not take account of illumination or any other phase effects and the albedo is known and uniform across the surface of the asteroid. The albedo is assumed if it is not known, based on the type and location of the asteroid (Warner et al., 2009b). While some asteroids can be close to being spherical such as Hygeia and Bennu, for smaller objects the shape can deviate significantly from a sphere, such as Kleopatra and Itokawa. If the shape is known, then the illumination can be corrected for. Most asteroids will not have a shape, phase curve or albedo, so diameters obtained by this method have large uncertainties.

Infrared data can be used in place of optical with smaller uncertainties as albedo has a much smaller error associated with it, but has the added complication that the infrared is both emitted and reflected. The Sun emits most of its light at wavelengths shorter than infrared which the asteroid absorbs and then reemits at infrared wavelengths. As most of the radiation is emitted and not reflected the emissivity rather than albedo is required to calculate the brightness. The emissivity \( \varepsilon \) is related to the albedo \( p \) by Equation 3.2.

\[ \varepsilon = 1 - p \]  

(3.2)

As the albedo is small for most asteroids, the emissivity is large. When the albedo is close to zero a large change in albedo causes a small change in emissivity. This makes the uncertainty in diameter from the albedo larger than the uncertainty in the diameter from the emissivity. As most of the radiation is emitted some sort of thermal
model is needed that takes account of effects such as the rotation of the asteroid and its current distance to the Sun that will affect its thermal equilibrium. One such model is the Standard Thermal Model (STM) (Lebofsky et al., 1986), while a more recent model is the Near Earth Asteroid Thermal Model (NEATM) (Harris, 1998), which was motivated by the need to incorporate high phase angle data, and the difference in the size of regolith on near Earth asteroids compared to main belt asteroids which were not accounted for in the STM.

These models start with a shape model for the asteroid which can be derived from optical data (section 5.2), but can be assumed spherical with the pole orientation as a free parameter if there is no model available. Although a spherical model can be used, shape models give a better fit for the thermal infrared data, allowing for better diameter estimates with smaller uncertainties. Using a shape model also produces added context for the temperature variations across the surface. For example, water ice has been detected on 24 Themis via spectral measurements, but in order to find where on the surface it might exist it is necessary to use a thermal model to find where water ice can be stable for many orbits Campins et al. (2010)(Figure 3.8). Once a model has been chosen the incident sunlight, reflected light and emitted light are calculated for each polygon of the shape model using a scattering law. The change in temperature can be calculated for each polygon. The transfer of heat between polygons by conduction and radiation is also calculated. Time can then be stepped forward and the new temperature across the asteroid can be calculated. It is important to initialise the modelling well before the observations to be fitted in order for the asteroid to reach thermal equilibrium. From the total emitted and reflected light, the brightness in various wavebands can be calculated. This can be compared with the actual data to produce a goodness of fit. The model parameters can be adjusted for the next iteration model, until the best set of parameters has been found.

An alternative way of measuring the diameter of an asteroid is via stellar occultations (subsection 3.2.2) (see Lucas & A. (2004) for an overview). The length of the chords can be used to scale the outline of the asteroid obtained from the occultation.
3.3.4 Masses

Asteroid masses are derived mostly from their orbital perturbations. Each asteroid has a number of mass estimations associated with it. For this work I will use the list of masses produced by Carry (Carry, 2012). As the accuracy of asteroid masses is determined by astrometric accuracy, Gaia data will be useful for refining asteroid masses in the future. It is worth noting that many of the derived asteroid diameters have errors that are underestimated, with probable errors in the order of tens of percent (Siltala & Granvik, 2020).
Chapter 4

Data Pipeline

This chapter outlines the pipeline for the SuperWASP data, which includes the observations of the telescopes, detection of asteroids in images, removal of bad data, creation and formatting of individual lightcurves, period determination and finally insertion into the shape modelling. The first step is running the existing findast and blendmag codes to extract the asteroid photometry from the SuperWASP orphan files and inputting it into an SQL database. This matches known asteroids to unknown objects in the SuperWASP database and then removes contamination from stars near the asteroids. These data are then moved to a local machine, and the needed data are extracted from the SQL database, filtering out extremely poor data points. The pipeline that I have create takes this data which is then cleaned, with bad nights, groups of points and individual points being removed, and the remaining data formatted into nightly lightcurves for shape modelling. The lightcurves are then combined with other historical data, and then a periodogram is created for each asteroid. The lightcurves are then folded and the period can be uniquely determined. Using either this period, or in the case of there being no period determined, the period from the PDS (Planetary Data System) archive\(^1\) (Warner et al., 2009a), the shape model can then be produced.

\(^1\)The rotation periods were taken from the lightcurve derived properties which can be found from the following link: https://sbn.psi.edu/pds/resource/lc.html
4.1 Post 2010 Extraction

SuperWASP has its own pipeline, as described in Pollacco et al. (2006). As an exoplanet search, the pipeline was not designed to have a dedicated process to detect asteroids. Objects that don’t match up with stars in the US Naval Observatory Catalogue are recorded as ‘orphans’ to be examined later. The primary objects present in the orphans table are transients, small solar system objects and catalogue objects with discrepant brightness.

The pipeline to recover asteroids from the SuperWASP orphans file was developed by Neil Parley as part of his PhD work and maintained to process new data until 2010. He wrote several codes for processing the orphans data, but the main two I used are the findast and blendmag codes. Both codes are detailed in Neil Parley’s thesis Parley et al. (2005), but I shall describe them briefly here.

The findast code starts with the list of orphans, with each unidentified object in each image being recorded as a separate orphan detection. It then takes an astorb file containing the ephemerides of all numbered asteroids correct for the month the image was taken, then for each asteroid calculates its apparent brightness and location in the sky as viewed from the SuperWASP telescopes. These potential asteroid sightings are then compared with the orphans database to match asteroids to orphans. If the orphan is within 60 arcseconds of the predicted asteroid position, then it is deemed to be a match. The orphans matched to asteroids are then entered into an SQL table called asteroids which contains the photometric data for the asteroid. In the asteroids table a single row is a single photometric observation. Table 4.1 shows the columns in the asteroids table and a short description of each.

The blendmag code attempts to correct for the blending present in the SuperWASP dataset. The maximum aperture size used for photometry is 4.5 pixels, or about 60 arcseconds. This large aperture leads to contamination of the signal from the asteroid by nearby stars that are either in or near the aperture, also known as blending. The contribution of this extra light causes an apparent brightening of the asteroid propor-
<table>
<thead>
<tr>
<th>Column Header</th>
<th>Column Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>jd</td>
<td>Julian date</td>
<td>Julian date of the observation</td>
</tr>
<tr>
<td>jd_lc</td>
<td>Julian Date Light-time Corrected</td>
<td>Julian date corrected for light travel time</td>
</tr>
<tr>
<td>ra</td>
<td>Right Ascension</td>
<td>Right ascension of the asteroid</td>
</tr>
<tr>
<td>declination</td>
<td>Declination</td>
<td>Declination of the asteroid</td>
</tr>
<tr>
<td>number</td>
<td>Asteroid number</td>
<td>Catalogue number of the asteroid</td>
</tr>
<tr>
<td>pre_RA</td>
<td>Predicted Right Ascension</td>
<td>RA predicted from ephemeris</td>
</tr>
<tr>
<td>pre_dec</td>
<td>Predicted Declination</td>
<td>Declination predicted from ephemeris</td>
</tr>
<tr>
<td>error_ang</td>
<td>Error Angle</td>
<td>Separation of observed and predicted position</td>
</tr>
<tr>
<td>image</td>
<td>Image</td>
<td>Image ID of observation</td>
</tr>
<tr>
<td>night</td>
<td>Night</td>
<td>Night of observation (given as a date)</td>
</tr>
<tr>
<td>flux</td>
<td>Flux</td>
<td>Measured flux</td>
</tr>
<tr>
<td>flux_err</td>
<td>Flux Error</td>
<td>Error in measured flux</td>
</tr>
<tr>
<td>mag</td>
<td>Magnitude</td>
<td>Apparent magnitude of the asteroid</td>
</tr>
<tr>
<td>mag_err</td>
<td>Magnitude Error</td>
<td>Error in the apparent magnitude</td>
</tr>
<tr>
<td>phase</td>
<td>Phase</td>
<td>The phase angle of the asteroid</td>
</tr>
<tr>
<td>as_earth</td>
<td>Asteroid to Earth distance</td>
<td>Asteroid-Earth distance in AU</td>
</tr>
<tr>
<td>as_sun</td>
<td>Asteroid to Sun distance</td>
<td>Asteroid-Sun distance in AU</td>
</tr>
<tr>
<td>lon_pab_ecl</td>
<td>Longitude of the PAB</td>
<td>The longitude of the PAB</td>
</tr>
<tr>
<td>lat_ast_ecl</td>
<td>Latitude of the PAB</td>
<td>The latitude of the PAB</td>
</tr>
<tr>
<td>lon_ast_ecl</td>
<td>Ecliptic longitude</td>
<td>Ecliptic longitude of the asteroid</td>
</tr>
<tr>
<td>lat_ast_ecl</td>
<td>Ecliptic latitude</td>
<td>Ecliptic latitude of the asteroid</td>
</tr>
<tr>
<td>offset1</td>
<td>SuperWASP offset</td>
<td>Mean SuperWASP magnitude offset</td>
</tr>
<tr>
<td>offset1_error</td>
<td>Error in offset 1</td>
<td>Error in offset 1</td>
</tr>
<tr>
<td>offset2</td>
<td>NOMAD offset</td>
<td>Mean NOMAD magnitude offset</td>
</tr>
<tr>
<td>offset2_error</td>
<td>Error in offset 2</td>
<td>Error in offset 2</td>
</tr>
<tr>
<td>offset3</td>
<td>TYCHO offset</td>
<td>Mean TYCHO magnitude offset</td>
</tr>
<tr>
<td>offset3_error</td>
<td>Error in offset 3</td>
<td>Error in offset 3</td>
</tr>
<tr>
<td>pre_mag</td>
<td>Predicted magnitude</td>
<td>Magnitude predicted from ephemeris</td>
</tr>
<tr>
<td>c_mag</td>
<td>Corrected Magnitude</td>
<td>Magnitude corrected for distance and blending</td>
</tr>
<tr>
<td>c_mag_error</td>
<td>Corrected magnitude error</td>
<td>Error in corrected magnitude</td>
</tr>
<tr>
<td>blend_index</td>
<td>Blend Index</td>
<td>Blend Index is a measure of the fluxes in different sized apertures around the asteroid which can indicate blending</td>
</tr>
<tr>
<td>blend_mag</td>
<td>Blend Magnitude</td>
<td>Estimated magnitude of blending</td>
</tr>
<tr>
<td>blend_mag_error</td>
<td>Blend Magnitude Error</td>
<td>The error in blend magnitude</td>
</tr>
<tr>
<td>blend_fraction</td>
<td>Blend Fraction</td>
<td>Fraction of light from blending</td>
</tr>
<tr>
<td>blend_dist</td>
<td>Blend Distance</td>
<td>Separation of object and blending star</td>
</tr>
<tr>
<td>blend_star</td>
<td>Blend star</td>
<td>Magnitude of blending star</td>
</tr>
<tr>
<td>camera</td>
<td>Camera identifier</td>
<td>ID of camera</td>
</tr>
</tbody>
</table>

Table 4.1: Column names and descriptions in the *asteroids* table which stores asteroid observations from SuperWASP, and a short description of each. In the *asteroids* table a single row is a single photometric observation. PAB stands for Phase Angle Bisector, which is the line that bisects the Sun-Asteroid-Earth angle.
tional to the closeness and brightness of the star, and needs to be removed. This is done by estimating the contribution of the star to the flux inside the aperture, by calculating the proportion of the stellar PSF (Point Spread Function) inside the aperture in each image, and removing it.

Neil had been running these codes for all SuperWASP data up until the end of 2010, while the SuperWASP computing cluster was hosted at Leicester University. The cluster was then moved to Warwick and migrated onto the NGTS (Next Generation Transit Survey) cluster, which meant that Neil was no longer able to access the orphans table and continue the processing of the asteroids. It also meant that in order for the codes to be run again, they had to be fixed to work in the new environment. The orphans are saved on the cluster as \textit{fits} files, with each month being a separate file. The post 2010 data had files up until the end of 2013, with the rest of the orphan files yet to be made.

The main issues with the \textit{findast} code were that no images were initially identified due to the orphans file being moved, and then no asteroids were found in these images as I was lacking SQL table privileges. In addition, automatic downloading of the \textit{astorb} files every month ceased when the cluster was migrated, so the astorb file created nearest to the time of the image was used. As all of the objects in the SuperWASP catalogue are large main belt objects, their orbital elements had not changed enough such that they were no longer detected from the orphans. Finally, the SQL connections used as temporary storage and to output the results were broken. For the \textit{blendmag} code, the major problems were that the yearly SuperWASP stellar catalogues that were used for the \textit{blendmag} code were only updated to 2011, so an average of the previous catalogues had to be used for data after this. Both codes shared some common problems, with old C++ libraries no longer being available on the cluster to recompile the codes, and SQL databases that were needed not having been created. Various other files were missing or had been moved.
4.2 Pipeline

My own pipeline for SuperWASP data started with data in the form of an SQL table with the asteroid photometry in the format required for shape modelling, specifically the light-time corrected Julian date and the corrected magnitude. The output is a separate file for each asteroid containing the light-time corrected Julian date, relative flux and Cartesian positions of the Earth and Sun relative to the asteroid. There is then a code to take this file and create a periodogram based on trying to fit a double sinusoid to the photometric data. The steps are outlined in detail below.

4.2.1 Formatting

The first step to cleaning the data is to examine the database and determine which points can be discarded either due to being unphysical or with too large of an uncertainty to ever be useful. By examining a histogram of each column of the database, certain results stand out as being impossible. First is that some data have negative fluxes (Figure 4.1), which, due to the flux directly correlating to the number of photons arriving at the detector, must be positive.

Next is the apparent magnitude. This had the issue of magnitudes being far too...
bright or far too dim. Only main belt asteroids are considered in this work, and the brightest main belt asteroid as viewed from Earth is Vesta, with an maximum apparent magnitude of 5.2. This does not take into account possible blending of the asteroid with a nearby star, which would make the asteroid have a higher apparent brightness. To calculate an upper limit on brightness, the value of Vesta must be adjusted according to how bright it would be with an acceptable level of stellar contamination. To be conservative, it was decided that 50 percent contamination would be the maximum permitted amount. An object twice as bright as a magnitude 5.2 object is 0.7 mags brighter, so 4.5 was the maximum brightness cut-off. In reality, there were very few observations (∼300 out of 6 million) between 0 and 7 magnitude, with the majority of bright objects having zero magnitude, which was probably due to an error somewhere in the data reduction pipeline, with 0 returned as an error value. The blending pipeline also has a function that discounts objects on pixels that are saturated, so there should be no saturated objects in the dataset.

At the other end of the scale, there are objects as dim as 27 mag. The nominal limiting magnitude of SuperWASP is 15. The likely reason for brightnesses this low is some error in the orphans pipeline, while detected objects around magnitude 16 are probably However, this is not an exact value, so simply putting the cut-off here could exclude real data, although any real data at this magnitude would likely have a small signal to noise ratio. As the number of objects with magnitude dimmer than 16 was small (∼1000) compared to the size of the sample and comparable to the number of data points excluded for being too bright, this seemed like a reasonable compromise between excluding erroneous data and good data.

There are data with negative magnitude uncertainties, similarly to the negative fluxes these were excluded. Objects with large uncertainties (> 2 mag) were also excluded, as there were few points (∼200) and would have an uncertainty in excess of the realistic maximum amplitude of an asteroid lightcurve, and therefore have an extremely low SNR. The only asteroids where the signal to noise ratio is higher will have to have an extreme axis ratio, especially for a large asteroid. Assuming that the signal
is dominated by the cross sectional area of the asteroid reflecting light to the observer, and that an asteroid can be modelled by a triaxial ellipsoid, the ratio of the cross sectional areas can be shown to be the ratio of the semimajor axes, for two ellipses with the same semiminor axes. For the amplitude of the lightcurve to be in excess of 2 mag, the brightness ratio from brightest to dimmest will vary by a factor of 5. Therefore the axis ratio must be in excess of 5. For large asteroids approaching hydrostatic equilibrium, the only objects with this axis ratio will be contact binaries. Indeed, 216 Kleopatra is a probable contact binary Ostro et al. (2000) with a reported lightcurve amplitude of 1.2 mag. Therefore, keeping all points with an uncertainty less than 2 mags will not throw out any usable real data. The reason for the conservative approach to cut at 2 mags rather than 1.5 is that the SuperWASP error estimation is not always reliable, as discussed below.

Similarly to other quantities that should always have a positive value, objects with a negative corrected magnitude uncertainty were excluded. It is important to note that there was a relatively large population of objects with a corrected magnitude of -99, which are those for which the blending program had failed in some way, usually because of non-numerical values in the input data. Again, objects with a corrected magnitude uncertainty larger than 2 were also excluded from the sample. The blend fraction has to be between 0 (no stellar contamination) and 1 (only stellar contamination). Any data outside this range were excluded, as it demonstrates a mistake somewhere in the pipeline. Anything with a negative blend distance (Table 4.1) was discarded for the same reason. There was also a population of ~5000 points that had a right ascension between 719.7 and 720, while the rest are limited between 0 and 360 degrees. After closely inspecting the points, they had similar properties to the rest of the population with no evidence that they are otherwise erroneous, and were therefore not excluded, and had the right ascension corrected to be less than 360.

It is important to consider why the data were not rejected on the quoted uncertainties alone, which means having to create a new cleaning method. SuperWASP was an early generation of robotic telescope, and the detection for poor observing conditions
Figure 4.2: A folded lightcurve for 24 Themis. The size of the error bars is not well correlated to whether the photometric point is close to the fitted lightcurve. While not being close to the fit does not necessarily make the observation erroneous, it should not be used for shape modelling.

was not perfect. This means that the quoted uncertainties are assuming photometric conditions, even though the cloud cover significantly impacted the photometry Figure 4.2. When cleaning a SuperWASP lightcurve with uncertainties, while having a restrictive uncertainty threshold will result in data that is not consistent with the rest of the lightcurve being removed, it comes at the cost of the majority of good data being removed simultaneously. Therefore it was decided to develop a method of removing poor quality data based solely on the lightcurve, without using the quoted uncertainties.

Once a database of points that meet the criteria for all of the fields described above has been assembled, the extraction could begin. The first step was to write the photometric data to a text file, and then converted to a CSV file which can be easily read as an input to further parts of the pipeline. The text file was made by querying the SQL asteroids table, and then piping the output to a text file. It is also important to remember that the same asteroid could be observed simultaneously in multiple cameras, and that due to the problem of fields having non-negligible absolute offsets, each lightcurve has to be treated separately. To ensure this, data are sorted by camera and then by Julian date.
4.2. Pipeline

The pipeline to format and clean the data was written in python, as was the code to create the periodograms by fitting double sinusoids. The programs to find the inertias and collate the physical properties was written in bash. The program to run the K-S tests (subsection 7.2.1) was written in python. The code to clean lightcurves has been through several iterations (5 at the time of writing), so for the sake of clarity rather than explaining why each function was added or removed I will explain only the final version of the code. In this section when referring to a single observation, the meaning is a single photometric measurement of an asteroid (i.e one point on a lightcurve). The code is split into several parts; reading the SuperWASP file for a particular object and acquiring the fluxes and positions for each observation, removing bad groups of observations within a single night, removing single bad observations, removing bad nights, formatting the output and writing to file.

Before the lightcurve could be cleaned, first it was necessary to convert magnitudes into fluxes, as the shape modelling code uses normalised fluxes such that the mean flux for each lightcurve is 1. The SuperWASP catalogue does have asteroid fluxes, but these can’t be used because the blending correction is only applied to the corrected magnitude. To obtain fluxes, the conversion from magnitudes to flux was used (Equation 4.1).

\[
M_1 - M_2 = -2.5 \log(f_1 / f_2)
\]  

(4.1)

For each data point, it would be possible to treat \(f_2\) as the mean flux and \(M_2\) as the corresponding magnitude of the asteroid, and \(M_1\) as the corrected magnitude and \(f_1\) the unknown normalised flux. This leaves the equation with 3 unknowns, \(f_1\), \(m_2\) and \(f_2\). This can be solved by imagining that there is an observation of the asteroid with a known and constant magnitude \(M_2\) and flux \(f_2\). These can be combined to one unknown constant, which leaves us with 2 unknowns, one of which is to be found (\(f_1\)). By using the constraint that only relative fluxes are needed, the equation can be solved with two unknowns and two constraints. Rearranging to get \(m_2\) and \(f_2\) on one side of
the equation gives Equation 4.3.

\[ M_1 - M_2 = -2.5 \log(f_1) + 2.5 \log(f_2) \]  \hspace{1cm} (4.2)

\[ M_1 = -2.5 \log(f_1) + (2.5 \log(f_2) + M_2) \]  \hspace{1cm} (4.3)

Here the term in brackets dealing with the imagined reference asteroid can be combined into one constant \( K_2 \) (Equation 4.4), and the equation can be rearranged to make \( f_1 \) the subject (Equation 4.5).

\[ K_2 = (2.5 \log(f_2) + M_2) \]  \hspace{1cm} (4.4)

\[ f_1 = 10^{-K_2/2.5} \cdot 10^{-M_1/2.5} \]  \hspace{1cm} (4.5)

\( K_2 \) can be changed into a constant \( C_2 \) Equation 4.6.

\[ \frac{1}{C_2} = 10^{-K_2/2.5} \]  \hspace{1cm} (4.6)

\[ f_1 = \frac{10^{-M_1/2.5}}{C_2} \]  \hspace{1cm} (4.7)

The constraint that the fluxes must be normalised gives Equation 4.8.

\[ \sum_{i=1}^{n} f_i = n \]  \hspace{1cm} (4.8)

Setting \( f_1 = f_i \) and rearranging solves for \( C_2 \).

\[ C_2 = \frac{1}{n} \sum_{i=1}^{n} 10^{M_i/2.5} \]  \hspace{1cm} (4.9)

This produces the normalised flux (Equation 4.10).
\[ f_j = 10^{M_j/2.5} \cdot \frac{n}{\sum_{i=1}^{n} 10^{M_i/2.5}} \] (4.10)

\( C_2 \) is a constant for each light curve.

Once the lightcurves had the fluxes calculated, the Cartesian positions of the Sun and Earth relative to the asteroid in astronomical units were added. This was done by querying the IMCCE’s (Institute of Celestial Mechanics and Ephemeris Calculations) Miriade service for Solar System objects ephemerides. The query was a text file with the non time corrected times of the photometric observations as Julian dates, along with the asteroid number. This returned a text file with a table of the Sun and Earth coordinates relative to the asteroid for each time. The light corrected Julian dates, fluxes and positions were then used as the basis for the cleaning algorithm.

### 4.2.2 Cleaning

Cleaning a lightcurve is removing noise from the lightcurve, while retaining as much of the signal as possible (Figure 4.3). A difficulty with cleaning SuperWASP data is that the quoted SuperWASP uncertainties are only weakly correlated with how well an observation fits the trend. While a threshold on uncertainty can be set such that all observations that do not fit the trend of the lightcurve are removed, depending on the lightcurve up to half of the usable data is also rejected when using quoted uncertainties alone. Therefore the task of the cleaning code is twofold; to determine a better method of calculating the uncertainty for each observation as well as establishing a threshold for these uncertainties above which the observation should be discarded.

**Synthetic Test**

If a new way of measuring the uncertainty on each observation was to be devised, then there had to be a way of testing the method to see if the calculated uncertainty is correlated with the actual error. To test the cleaning method, synthetic lightcurves

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2This work has made use of Miriade’s VO tool.
Figure 4.3: A partial lightcurve of 16 Psyche having the cleaning algorithm applied, with potentially erroneous points identified in blue and confirmed erroneous points in red. The potentially erroneous points are near the definitely erroneous points as expected.

were developed. The initial synthetic lightcurve was made using a sum of sines and cosines up to second order. Then, each synthetic point had a random error added to simulate low level noise. Then, some points were selected at random to have another error added, to simulate situations such as a cosmic ray hit. Finally, groups of points were selected at random to have flux removed to recreate the effect of passing cloud. Each lightcurve had the actual error and the estimated errors plotted against each other. A linear fit would mean that the errors were being estimated correctly, and as the actual error is not needed for shape modelling, the slope did not need to be unity.

Several methods of cleaning were tried. Removing individual points or groups of points was done before testing for bad nights because while removing the worst points makes the mean uncertainty of a good night much better, removing the worst points from a bad night does little to improve the night of data as a whole.

For a point on a lightcurve \( n \), the scatter was defined as the difference between the flux of point \( n \) and the flux of point \( n - 1 \). The scatter was unreliable as it was biased against lightcurves with large amplitudes. Use of the gradients between \( n - 1 \) and \( n \), and \( n \) and \( n + 1 \), suffered from similar flaws. The best method was fitting a straight line between \( n - 1 \) and \( n + 1 \), and stating the error as the distance between point \( n \) and this line. Other fits including quadratic and linear fits using \( n - 2 \), \( n - 1 \), \( n + 1 \) and \( n + 2 \) did
yield slightly better results, but were far more computationally expensive. This has the added bonus that it gets around the complication that the cadence of the observations is non-uniform. The cadence of observations alternates between 38 and 144 seconds. This means that the expected variation between points varies, as points farther apart in time are expected to be farther apart in flux.

All methods had the disadvantage that if estimating the error for point \( n \), and the magnitude of the point \( n + 1 \) is far from the trend, the linear fit between \( n - 1 \) and \( n + 1 \) will be wrong, causing the error estimate of \( n \) to be incorrect (Figure 4.4, Figure 4.5). To solve this, a fit between \( n - 2 \) and \( n + 2 \) was also used to calculate a second uncertainty, and the smaller of the two uncertainties was taken to be the overall uncertainty. In addition, several passes were performed on a lightcurve. On the first pass, all points were eligible to be used as \( n \pm 1 \) or \( n \pm 2 \). If a point is determined to be erroneous (i.e. with a calculated error significantly larger than the typical scatter), it would not be used to estimate the uncertainty of other points in the future. However, if on subsequent passes it was not found to be erroneous it would be included again. The data would be repeatedly passed over until there was no change on which points are deemed to be erroneous.
Figure 4.5: Synthetic lightcurves and the associated estimated error (vertical axis) against actual error (horizontal axis). The points identified as erroneous are highlighted in red. Note how there is a significant population of points below the linear trend, which are caused by groups of bad points making it difficult for the cleaning algorithm to determine the probable flux at a given time.

Second was the removal of groups of bad points. The general premise was to identify clusters of points that had been removed in the previous step, and to make sure all points within this cluster have been removed. The method was to find a point that was considered bad compared to the rest of the night. Then points after this were tested to see if they were also considered bad. If two successive points were considered good, or if successive bad points were sufficiently far apart, then the last bad point was chosen as the end of the group. For a group to be removed, it must be sufficiently long (a value of 15 points was chosen) and at least half of the points must have been bad. The reason for not requiring all of the points to be erroneous can be explained with an example; if five consecutive points all had an increase of 1 mag, the middle point would appear to be perfectly acceptable as it had a small change in flux relative to the adjacent points. This cleaning method has the advantage that the threshold for
a bad point was set by the data from the rest of the night, rather than the SuperWASP catalogue as a whole. This stops over-aggressive cleaning of lightcurves with a lower than average signal-to-noise ratio that are otherwise still usable.

Finally, the nights with poor signal to noise ratio were removed. The way the 'goodness' of the night was determined was by the average uncertainty on every point. Many nights had to be cleaned in order to generate a list of nights and their 'goodnesses', so that a representative distribution of uncertainties could be created. Whether a night is useable or not is subjective (since there is no formal definition of a useable lightcurve), so there will be some nights that are unusable that are still included. However, as there is a visual check for each night later in the pipeline it is better to include these possibly unusable lightcurves and discard them later rather than discard useable lightcurves.

In addition to using SuperWASP data, I was able to use DaFEED (DAMIT Feeder), a database containing the lightcurves used to create the models on DAMIT (Database of Asteroid Models from Inversion Techniques) Říček et al. (2010). This facility is curated and maintained by Josef Hanus and Josef Durček at Charles University Prague, and contains lightcurves compiled from various historical sources. These data are already in the correct format for shape modelling. In addition, some of the lightcurves are from the SuperWASP archive as data were provided for the large program. These data have to be combined with SuperWASP, and so I wrote a program to combine the two datasets automatically. First, the SuperWASP lightcurves were written to a new compilation file, then the DaFEED data were appended with any sparse data removed. Sparse data are any lightcurves that span more than one night of data, with observations typically on a timescale of hours rather than minutes. Finally, the number of combined lightcurves was computed and written as the header.

The DaFEED data were not subject to the same cleaning procedure as SuperWASP. The data are normally of a superior quality compared to the SuperWASP data, as they are not collected automatically and any obviously erroneous data had been removed by the observer. Some of the nights had noisy data but were still usable, and there was one
Figure 4.6: The apparent brightness, blending distance, blending magnitude and corrected magnitude for a single night of observations of 71 Niobe. The observed magnitude shows an increase in brightness, but the brightness increase is misaligned with the blend distance. This leads to a spike in the corrected magnitude.

case where instead of having a lightcurve the file had what appeared to be elevation curves for the object.

After running the cleaning algorithm, it was decided that while the cleaning pipeline was efficient at removing especially poor nights and individual points, the data were not always of sufficient quality for shape modelling. They were however good enough to get an estimate of the period accurate to \( \sim 1 \) percent. It was also discovered that occasionally there were nights that upon folding the data with the correct period, the resulting lightcurve was not a single lightcurve as would be expected, but one or more differently shaped lightcurves on top of the well folded lightcurves. This was probably due to an incorrect estimate of the position or brightness of a nearby star, causing an incorrect estimate of the blending contribution which would substantially alter the lightcurve shape (Figure 4.6).

One possible solution was to fold the lightcurves, then remove any lightcurves that did not fit well to the average shape. This method has the difficulty that if the true
4.2. Pipeline

lightcurve is a minority of the folded data, the true lightcurve would not conform to the average shape and would be wrongly rejected. The alternative was to examine the lightcurves by eye. This had previously been rejected for a cleaning algorithm as being too time consuming. However, as only objects with sufficient data and observation arcs are to be shape modelled, and entire nights rather than individual data points need to be removed, the number of individual ‘entities’ being looked at is reduced by at least 2 orders of magnitude, making this a much more manageable task. This has the added bonus of being a visual check to make sure the automated cleaning procedure has removed all of the nights. Finally, as the SuperWASP observing strategy means that the same object will be observed for many nights in a row, there are often redundant data. Multiple lightcurves from the same geometry add additional complexity and therefore run time to the shape modelling algorithm, but do not improve the shape, and can actually make the shape worse by biasing in favour of a certain geometry. Sometimes it is favourable in the case of an object where all of the available data are good to remove some of the data to speed up the process of shape inversion. The final version of the code displays all of the lightcurves over a 30 night period, and the user can then reject a single night, all nights or no nights. In the case of a single night being rejected, the user is given the opportunity to reject another night before moving on.

4.2.3 Validation

In order to validate the pipeline, it is useful to compare the output with other sources. The way this was done was to check for objects which had been observed simultaneously by SuperWASP and another telescope. Data from DaFEED is the easiest to compare with as it has the same output format. After manually checking for objects that were observed simultaneously, a night of data was found for 39 Laetitia \(^3\) (Figure 4.7). The data from SuperWASP matches that from DaFEED almost perfectly, with no offset in normalised flux and does not lag or lead the other dataset in time. In addition the coordinates of the Earth and asteroid match. This demonstrates that the

\(^3\)The results did not initially match, this was later found to be a bug in the pipeline and was corrected.
pipeline works as intended, correctly calculating the normalised flux and positions of
the asteroid and Earth for each lightcurve. Some of the lightcurves were also sepa-
ately checked to make sure that the metadata was correct (i.e. the number of nights of
observations, the number of observations in each night and the date of each night), and
these were also found to be correct.

4.2.4 Period Search

Once the lightcurves have been cleaned, they are ready for shape modelling. At the
beginning of the project, the lightcurves were put straight on to the Nice cluster and
combined with DaFEED lightcurves to then be used for shape modelling. As all of
the lightcurves use relative fluxes, this is straightforward as phase effects do not need
to be considered. This presented a couple of problems. First the DaFEED data had to
have the sparse data removed by hand, and the WASP data added manually. This is
time consuming, and it is easy to make a mistake, miscalculating the total number of
lightcurves or forgetting to remove the sparse data. Therefore a script was written to
combine the DaFEED and WASP data before it was submitted to the cluster. First it re-
moved the sparse data, removed any SuperWASP data already present in the DaFEED file and then combined the two datasets into a single file. Another problem is that not all asteroids have accurate period predictions, and a large amount of period space has to be searched if the period of the object is not well known. While shape modelling gives the best period estimate, it is slow. It can take 48 hours of cluster time to run a complete period search from 1 to 100 hours for a single asteroid. It was suggested (by Marcus Lohr, personal communication) that there could be a quicker method of determining the period than a full shape solution, as his work on variable stars had used methods of period determination that were far quicker. Apart from speed, another advantage of an alternative method is that for asteroids with an existing period determination, an extra estimation can provide an idea of the accuracy of the period.

Previous work has been done by Harris et al. (1989) on asteroid period searches, and his method fits a polynomial curve to a lightcurve, with the order of the polynomial and the initial period estimate being defined by the user. The problem with this is that while it is less computationally intensive, it is more time intensive for the user as it requires lots of manual input to define the parameters, as well as there being different parameters for each lightcurve. I therefore decided to write my own code, which would be less time consuming for the user to run. Most large asteroids can be modelled as triaxial ellipsoids because they are close to hydrostatic equilibrium, and a rotating ellipsoid has a characteristic double peaked lightcurve. As the body rotates, the cross section of the illuminated area goes from a maximum to a minimum twice per rotation period. The exception to this is when the asteroid is being viewed pole on, where the lightcurve will be approximately flat. The starting point for the period search algorithm was to try and fit a double peaked lightcurve to a series of lightcurves folded onto a single period. As the asteroid is moving relative to the Sun and the Earth continuously, the observing geometry and therefore the shape of the lightcurve for each night is changing continuously. For short lengths of time of about a month the viewing geometry does not change significantly for an MBA and hence the change in the lightcurve shape is small enough to be successfully folded. The number of nights
that can be folded is left as a variable, but by running the code with various folding
lengths, and defining the best run to have the largest contrast between the correct value
and the worst, the ideal folding length was found to be 15 nights of data.

The lightcurves were then sorted into groups of length no more than 15 days, and
then phase folded onto a single period. An issue that quickly became apparent is that
the normalisation of the lightcurve would cause issues with the amplitude. For exam-
ple, take a lightcurve with complete coverage for one period. The normalisation will be
correct for this asteroid. Now let us consider that this lightcurve is actually obtained on
two separate nights, with the parts with a normalised flux >1 being obtained on night
A, and the parts with a normalised flux <1 being obtained on night B. As can be seen
in Figure 4.8, the lightcurves have been shifted as the mean of each segment has now
changed from the mean of the full lightcurve. Clearly the mean of the entire lightcurve
must be calculated before the normalisation can be considered correct. However, there
is no way of knowing the mean of the entire lightcurve from a partial lightcurve.

To compensate for this, it is possible to treat the amplitude and shift in brightness
as free parameters. However, leaving more parameters free allows the fitting algorithm
to fit erroneous periods too easily. If the amplitude is left totally free a situation could
arise where the amplitude of each section is reduced to 0 and shifted to a brightness
of 1, such that the lightcurve is perfectly flat. The solution used to avoid this situation
is to take the lightcurve section with the highest amplitude for that object, and take
the largest and lowest amplitudes from this section, which will become the global

Figure 4.8: A phase folded lightcurve of 16 Psyche before and after vertical shifting. Each colour
represents a different night’s data. The non shifted sections do not contain any minima or maxima.
maximum and minimum. For every section, if it was found to contain a maximum and no minimum, it was shifted up such that the maximum amplitude of that section was now equal to the global maximum, and for a minimum shifted down such that it is equal to the global minimum. If the section contained a combination of maxima and minima or neither, it was left as it was. There is the additional constraint that any maxima or minima must be sufficiently far away from the end of the lightcurve to avoid shifting any lightcurves that had an artificial maximum introduced by noise at the end of the lightcurve.

Then a double sinusoidal function was fitted (Equation 4.11).

\[
Flux = A + B \sin(2\pi \omega t) + C \cos(2\pi \omega t) + D \sin(4\pi \omega t) + E \cos(4\pi \omega t)
\] (4.11)

The code used the optimize function in scipy to determine the best constants for each period. This was done 2000 times, with \(\omega\) varying such that periods between 2 and 100 hours were tested, with sampling density decreasing with increasing period. For each fit, a chi-square was calculated based on the difference between the fit and the observations, which could then be plotted as a periodogram (Figure 4.9).

Once the periodogram has been created, the goodness of fit for each period tested is written to a text file. However, for asteroids with noisy or small amounts of data, there are often multiple minima, with the true period not always being the best in the periodogram. Often it is the case that there is also a good solution at half the true period (i.e. the single peaked lightcurve solution) as the constants for the double peaked lightcurve can be picked such that \(D\) and \(E\) are small. In this case, it is usually the larger period that is the correct one. There will not be a value at double the period, as the second order sinusoids cannot be made to fit a quadruple peaked lightcurve. It is important to remember that there are some single peaked lightcurves which are driven by albedo variation rather than shape, and that some discriminant between the two cases is needed. Usually this is when a double peaked lightcurve has the same shape.
Figure 4.9: A periodogram produced by the period search for 51 Nemausa. There are several minima, with the real value being at 7.78 hours. Note how there are solutions often double the time of other solutions, such as at 4.6 and 9.3 hours. These represent the single peak and double peak solutions.

for both maxima and minima, although this can also be caused by a highly symmetric object.

4.2.5 Folding

For the cases with multiple possible periods it is necessary to analyse each solution by eye, so a folding code was written. Like the period search it starts by grouping the lightcurves. The total number of nights to be folded is again user defined. The number of nights to be folded can be slightly larger than the number for the period search, as the emphasis is covering the entire period of the lightcurve rather than conserving the shape. Generally up to 30 nights can be folded. Increasing the number of nights folded is especially useful for asteroids with a period close to a multiple of 12 hours. This is because these objects tend to have a significant overlap between nights, meaning that only a part of the lightcurve is visible, and there is a degeneracy for the multiples of 12 (i.e the period could be any of 12, 24, 36 hours).

The folding program takes a period as a user input, finds the closest minimum on the periodogram, and plots the resulting lightcurves for verification of the period by eye (Figure 4.10). After some testing, it was clear that some lightcurves did not
have a good period found by the fitting code. This was true when the shape of the lightcurve is not a double sinusoid, which can be caused by an irregular asteroid shape or an albedo driven lightcurve. An example of this is the lightcurve of 10 Hygiea, which is largely driven by the albedo of the asteroid rather than its shape, as it is a highly spherical object. The true period was found in the periodogram as a potential period, but was not the best period. After visually inspecting the folded lightcurves for each likely period, the true period was found, demonstrating the importance of seeing the folded lightcurves after the automated search. The period search also struggled with asteroids that have long periods, as often within the 15 days there would be no overlapping lightcurve sections, so the program can essentially fit any plot to the data. To help with this an override option was added, where rather than finding the nearest minimum in the periodogram, a user defined value, such as one taken from the JPL small body browser, could be used. None of the periods produced in this work use this functionality, as the uncertainties introduced in the model are likely larger than those of the periods quoted in the literature. If the resulting lightcurves looked correct, then the period could be written to file. This was avoided whenever possible, as one of the goals of the period search tool was to correct any incorrect periods in the literature.

The final step of the folding was to write the period to a file that contains the periods of all of the currently tested asteroids. It is also important to note that often this method would not give an unambiguous period, especially for those objects with a period close to 24 hours. However, the periodogram clearly indicates that the period must be at one of several minima, which helps decrease the search time for the shape modelling code.

The advantage of this method is that ~200 periodograms could be created a day, and ~100 inspected using the folding code. Each period produced this way is correct to ~2 decimal places. This uncertainty is derived from the Full Width Half Maximum. The other advantage is that by being able to see the folded lightcurves, the user can judge how likely the period is to be correct, or whether the period fitting procedure has identified an alias. The disadvantage is that the code does not work well for asteroids with a lightcurve that does not approximate a double sinusoid.
Figure 4.10: Several consecutive nights of data for 16 Psyche folded onto a single period, with each night represented by a different colour. The black line is the best fit for a second order sinusoidal fit. Although it is a crude fit, it is adequate to determine the period to within a few percent. Extra terms could increase the accuracy of the fit, but would make it easier to fit erroneous periods.

4.2.6 Automation

Once all of the codes had been written, they were converted to take a list of targets rather than the user entering an asteroid number when running a program. The extraction of data from the MySQL table, conversion of extracted data to a CSV file and the cleaning and formatting of these data were done in a single script for multiple objects with no supervision required. If there is no SuperWASP data available, the output is an empty file. This also had the alteration from previous versions that the cleaned lightcurves were not displayed on screen, speeding up the process but not allowing manual examination. Next was the script to combine the DaFEED and SuperWASP datasets, with the addition of a warning if the file of either dataset did not exist or was empty. Finally was the script to run the period search. The advantage of running all of the targets with a single script is that it drastically reduces the amount of time required by the user to process many target objects. If each lightcurve is examined for a typical time of two seconds, and if an object has 100 lightcurves in the SuperWASP archive, it could take 3 minutes to do a single object. While seeing all of the lightcurves acts an assurance that the code is working correctly, it does significantly slow progress, and
for exceptional asteroids (e.g. with a large amplitude) this can always be run later as a check.

The downside of the automation is that it is hard to tell if there has been an issue with the lightcurve cleaning. This will not lead to erroneous shape models, because bad data will result in a shape model not being made, as the modelling code will not be able to converge on a good solution. It may however result in no shape model being made when there are sufficient data to create one. The solution to this is to use the folding script on every object, even if there is already a good period in the literature. By visually checking the lightcurves, the quality of the data and the period can be checked.

Once all of the cleaning scripts have been run, the lightcurves were uploaded to the Nice cluster, ready to be used as the input for the shape modelling code. The code is designed to take multiple lightcurve files, but for the sake of speed these are combined locally into a single file before being uploaded. Once uploaded, the KOALA (subsection 3.2.1) algorithm was used to find the shape model. For the periods and spins, the jobs are run in batches, with each run as hundreds of separate jobs to optimise the number of cluster cores that could be used at once to speed up the process. Using the period estimates from the periodogram, a period range of the estimate ±0.1 hours was searched for each possible period. If one of these periods produced a positive result (i.e. a single period not within 5 percent of the rest of the periodogram) then a spin pole detection was attempted. The spin grid used was 3 degrees latitude and 6 degrees longitude spacing between chi square calculations. Similarly, if a spin pole detection was found with sufficient significance, then a final inversion was done for each possible spin for that object. The period results file, spin results file and models are then copied from the cluster. The inertia calculation code is then run on each model. Once all of the models were collated, further codes were run to collate the physical properties from all of the models, in table 4.1.
Chapter 5

Shape Modelling Methods and the Large Programme

This chapter will discuss shape modelling in practise, with a focus on lightcurve inversion which is the method used in this work. I describe other disc-integrated methods of shape modelling, the properties of shape models created using disc-integrated methods and briefly discuss models from spacecraft and radar observations. I discuss the use of adaptive optics and occultations in asteroid modelling, and finally discuss the large programme at ESO (European Southern Observatory) to make detailed shape models using adaptive optics, its findings, and the part I had in it.

5.1 Shape Models

Shape models are useful as they give detailed information about the physical properties of an asteroid. The simplest model of an asteroid is a sphere with fixed diameter. This is not an accurate representation of the vast majority of asteroids, but in the absence of other information it is the only model to use, and is often used in thermal models to calculate asteroid diameters from infrared data (subsection 3.2.3). The model can be improved, where the asteroid is modelled as a triaxial ellipsoid with 3 different axes. This gives an idea of how elongated an asteroid is as well as the spin axis.
orientation, but gives no indication of any surface features. It is also not suitable for asteroids that have significant concavities such as contact binaries. For many asteroids a triaxial ellipsoid is a reasonable approximation, as rotating objects in hydrostatic equilibrium will have a triaxial shape, and many large asteroids are close to hydrostatic equilibrium, with dwarf planets such as Ceres being in hydrostatic equilibrium (Thomas et al., 2005).

The most complex type of shape model is a polyhedron of arbitrary complexity. Here the model is described as a series of vertices, each with an x,y,z coordinate. The surface is constructed from polygons, and in this work triangles, each of which is described by the vertices that bound it. The number of vertices is a parameter the user can set when the model is created, but the typical number of vertices in this work is about 2000. The maximum number of vertices that is useful is determined by the signal to noise ratio of the photometric data, as well as the coverage of the asteroid. Areas of the asteroid surface that are less visible from Earth, or at the edge of the disc as seen from Earth will be less well characterised and therefore need fewer polygons in the shape model. The scattering law used in the construction of the asteroid is only an approximation, so there is a limit to the resolution that can be achieved.

The method to create shape models used in this work is lightcurve inversion, which uses disc-integrated photometry (subsection 3.1.1), but there are some other sources of shape models worth mentioning. The first is from spacecraft visits. As only fourteen asteroids have been visited by spacecraft, these models are only available for a tiny fraction of asteroids. These visits can be divided into two types; flybys and orbital rendezvous. Orbiting an asteroid is more difficult but will allow for full surface coverage, and depending on the orbit and the instruments can map the asteroid on the scale of metres. Flybys are easier to achieve as they do not require the craft to slow down enough to be captured by the object’s gravity, but will only be able to view one hemisphere at the best resolution. Because small bodies rotate, the other hemisphere can be viewed before or after closest approach, but the resolution will not be as good as that at closest approach. This is the case with 21 Lutetia and 253 Mathilde, which were both
viewed as the Rosetta (Pätzold et al., 2011) and NEAR Shoemaker (Yeomans et al., 1997) craft respectively passed them, but only one hemisphere of each was accurately mapped. The other hemisphere can be created using data from Earth-based observations, but is of lower resolution (Veverka et al., 1999; Sierks et al., 2011). Asteroid models derived from spacecraft data are characterised very well in terms of mass, volume and composition. The composition allows for a good estimate of the density of the material, while the mass and volume produces the bulk density. The discrepancy between the material density and bulk density can be interpreted as heterogeneity, usually in the form of differentiation or macroporosity. The second source of models is from radar (see subsection 3.2.4 for a summary). Radar models are advantageous as they provide a snapshot of the surface, meaning that a shape model can be constructed in comparatively little observing time.

5.2 Convex Models

5.2.1 Lightcurve Inversion

As radar and spacecraft models of asteroids are only available for a few asteroids, some other method is needed if shape models are to be found for large numbers of asteroids. The most common form of data for asteroids is photometry, and as the shape of the asteroid dictates the brightness, then this is a natural place to start. It is relatively straightforward to start with an asteroid shape and calculate the lightcurve as seen from Earth using a scattering model, but the inverse problem of starting with the lightcurve and calculating the shape is more difficult. The method of lightcurve inversion was developed in the early 2000s by Kaasalainen et al. (2001), and takes lightcurves as an input and generates a shape model. The shapes are convex, which means that there are no depressions on the surface of the models such as impact craters. Instead, there is a large flat area where the crater is. The reason for this is that the photometry used is disc-integrated, and changing the depth of the crater in the model does not have a
significant affect on on the fit between the model and the observations. The depth of
the impact crater is poorly constrained, and so the convex solution is used.

For this method relative brightnesses are used. Astronomical brightnesses are mea-
sured as magnitudes, and a typical asteroid lightcurve will change in brightness by a
fraction of a magnitude around its mean brightness. For relative brightnesses, the
magnitudes are converted to fluxes and then adjusted such that the mean flux of the
lightcurve is one. Relative brightness is used because the scattering laws used in the
model do not adequately account for the opposition effect, which is a rapid increase in
brightness of an asteroid as it approaches opposition (Belskaya & Shevchenko, 2000).
The scattering models are usually basic models, such as Lambert’s scattering law. By
using relative brightnesses, this problem is not an issue as the extra brightness at op-
position is effectively removed. The scattering law used in shape modelling cannot be
too complex, as adding parameters causes the model to be fine tuned and unstable. The
need for relative brightness in shape modelling is a convenience for this project, as the
SuperWASP brightnesses are affected by the temperature dependence of the detector
(subsection 3.1.2).

The lightcurve inversion method works as follows (see Kaasalainen & Ðurech
(2020) for a summary). For each stage, the goodness of fit is defined as the differ-
ence between the observed and synthetic lightcurves made using a model (Figure 5.1).
First, the period is determined. The user defines a minimum and maximum period.
The steps between period searches is defined by Equation 5.1.

\[ \Delta P \approx \frac{1}{2} \frac{P^2}{T} \tag{5.1} \]
Here $P$ is the period, $\Delta P$ is the step between successive period changes and $T$ is the baseline of observations (Kaasalainen, 2004). The reason for this step spacing is that the period can only take certain values, where the maxima and minima in a phase folded lightcurve align. For each period, the spin direction is calculated for a low resolution shape model. The spin axis direction is somewhere on the celestial sphere. To find the spin direction, six starting points are placed evenly across the celestial sphere. From each point, the shape model is made and the goodness of fit calculated. Gradient descent is used for each starting point to find the best spin direction. The
reason to use multiple points is to ensure a global minimum is found, rather than a local
one. The six starting spins will converge on one or more minima, and the minimum
with the best goodness of fit is deemed the global minimum. The period with the best
goodness of fit is then used for the full spin search. Due to the large number of models
that have to be constructed, this is by far the most time consuming part of light curve
inversion methods, especially if a large range of periods is being searched.

Once the period has been found it is held as a fixed parameter and the spin is de-
determined. The user defines the size of the latitude and longitude grid spaces on the ce-
estrial sphere to be searched, and the goodness of fit is calculated for each grid square.
This will often result in multiple possible solutions for the spin direction. These will be
pairs of solutions, with the pairs having the same latitude, but longitudes close to 180°
apart. The shapes from these pairs will be close to mirror images of each other, and
will rotate in opposite senses. The only way to break this degeneracy is to have either
an occultation or an AO image to get an outline of the asteroid, which will match one
of the two mirror images. There may be multiple pairs that give usable solutions, in
which case more photometric data can be used to determine which pair is correct. The
reason for these pairs is that all observations of main belt asteroids are made on the
ecliptic plane or very close to it, depending on the inclination of the asteroid’s orbit. It
is a feature of the inversion technique that there are two indistinguishable solutions of
the spin and shape if all of the observations are made within a single plane.

The final inversion is then done. Both the period and spin are left as free parame-
ters, but the starting point is determined by the best period and spin found previously.
Then the final period, spin and shape are found by gradient descent. The best period
and spin must be sufficiently deep minima for the shape model to be considered reli-
able. The convention used is that there is a single minimum point, and that there is no
other point within 5% of the minimum. For many objects, there is a dip in the peri-
odogram indicating that the minimum is in that region, but the data are not sufficient
for the period to be adequately constrained.

After the shape model has been produced, the moments of inertia are calculated as-
suming a uniform density. The axis of the object with the minimum rotational energy should be close to the current rotational axis. Due to tidal stresses from the rotation, the asteroid will convert rotational energy to heat energy, which is then radiated and lost to space. This loss of rotational energy will cause the asteroid to change its rotational direction typically over $\text{Ma}$ timescales (Pravec et al., 2005). The timescales for excitations of the spin state are expected to be longer than the time scale for dampening, so most asteroids have been observed in a low energy state.

### 5.2.2 Sparse Photometry

There has been an increase in the use of sparse data for shape modelling. Traditional shape modelling techniques use dense data, where lightcurves are acquired in a single night, and multiple nights of data are used to create the model. By contrast, techniques using sparse data use as little as a single photometric point in a given night. This introduces several complications. First, as there is no way of calculating the normalised flux, absolute magnitudes must be used, which requires photometric precision better than the lightcurve amplitude. There is the additional complication that an accurate scattering law must be used to take account of phenomena such as the opposition effect to correct for the phase angle. These limits on the models mean that often only a spin direction and a triaxial ellipsoid can be produced from sparse data and a convex hull in only a few cases (Hanuš & Ďurech, 2012), but given the huge amount of sparse data that exists and will come from future asteroid surveys, this could increase the number of shape models substantially.

### 5.3 Concave Models

As synthetic lightcurves from convex models are a good fit to the real lightcurves used to create the models, there is little need for a concave model as it will not be able to appreciably improve upon the convex solution unless there is disc resolved data. As most observations are disc-integrated photometry, this is the case for most asteroid models.
5.3. Concave Models

Concave models are still desirable as they provide additional information about an asteroid, such as depressions and craters in the model. The advantages of concave models are that the volume is determined more accurately, as well as large scale depressions that will have had a significant impact on the object’s history will have the dimensions determined rather than being represented by a flat facet on a convex model. This can be used to investigate the collisional history of the object.

Concave models can also be formed with methods other than lightcurve conversion. The SAGE (Shaping Asteroid Models through Genetic Evolution) (Bartczak & Dudziński, 2018) algorithm works differently to lightcurve inversion. Rather than taking the lightcurves and working backwards towards the shape model as an inverse method, it tries to guess the shape model to find the best solutions. The algorithm starts with a sphere as the model for the asteroid, and adds a small mutation such as a bump on the surface. In a single generation there are several models created, each with a different mutation from the previous generation. The models then have synthetic lightcurves created, which are then compared with the observations. The best period and spin orientation is determined for each model. The model which best matches the lightcurves is chosen as the seed for the next generation of models. This process is then repeated, with the best model of each generation serving as the seed for the next generation. When the next generation does not appreciably improve the fit between the synthetic lightcurves and the data, the process stops. The whole process is repeated several times with slightly different starting models, and the end result will be different for each starting model. This will create an ensemble of models, which can then be layered on top of each other allowing for identification of well defined regions where the models agree well, and regions that are not well constrained. The advantage of this is that it will quantify the uncertainties as opposed to the convex hull which doesn’t produce any uncertainties. Whereas a convex model will have a flat surface for a depression, the concave model will have a crater with a range of uncertainties. In addition, any areas that are not well observed will be picked out as not being well constrained.
Adaptive optics can also be used to identify concavities in asteroids. Most telescopes are limited by atmospheric turbulence, known as seeing. The motion of the atmosphere blurs images and limits their resolution. Adaptive optics works by having a deformable mirror in the telescope which can be adjusted constantly to compensate for the distortion of the incident light by the atmosphere. This means that telescopes will be limited by diffraction rather than atmospheric turbulence, and the resolution is a function of the size of the aperture, so a large enough telescope with adaptive optics will be able to resolve an asteroid. Disc resolved images show how much light is reflected off of a region of the asteroid surface, but from the image alone it is not possible to tell whether a dark patch on the object is a region of low albedo or whether it is due to shadowing. By combining the images with a shape model from lightcurve inversion, the shadowed areas can be matched with suspected depressions to tell which low reflectance areas are due to topography. As adaptive optics can be used on large asteroids to resolve them, it is possible to get very accurate volumes for the asteroids observed as well as their shapes. This is the purpose of the large programme, as discussed in section (section 5.5). The package used in this work is known as KOALA (Knitted Occultation and Adaptive Lightcurve Analysis) (Carry et al., 2012). While this can accept disc resolved data such as AO images and occultations as an input, in this work only the photometric data are used. KOALA has the capability of combining all of the data together, but as there is no disc-resolved data it is essentially lightcurve inversion.

5.4 Stability and Degeneracy

The advantage of only using the convex solution from lightcurve inversion is that the resultant models are stable, meaning that the model will not change significantly if new data are added, as opposed to the concave solutions that can change significantly for small changes in the input data. In addition, lightcurves are needed for creating shape models from adaptive optics images to stabilise the solution. There are some
cases where shape modelling will not work at all. One of these is excited rotation, where there is non-principal axis rotation. The rotation can be modelled by using more parameters, but unless the rotation periods are identified before modelling, it is difficult to search the entire parameter space and find a unique solution. This was the case with 'Oumuamua (Meech et al., 2017), an interstellar object in an excited rotation state which has never had a definitive rotation period or shape found (Fraser et al., 2018), mostly due to its tumbling. Another case is the YORP effect, where the rotational speed and direction are constantly changing by a very small amount. For large asteroids, these can be neglected on the timescale of their observation history, but for smaller asteroids this needs to be taken into account when making a shape model. A small period change can lead to a large difference in phase across decades of observations.

The method used in this work is lightcurve inversion, which uses lightcurves and solves an inverse problem to create the shape model. Like a lot of inverse problems, the solutions are not unique. For lightcurve inversion, this manifests itself as the problem of concavities having undefined depth. The convex hull is unique, however. In addition, solutions typically have two spin solutions, with some sort of disc resolved data required to find the correct solution. The quantity that is not uniquely determined is the spin longitude for which two approximately symmetric shapes are derived (Figure 5.2).
5.4.1 Properties Derived from Models

The inertia calculation is based on the algorithm by Dobrovolskis (1996), which can calculate the moments of inertia of any polyhedron. The program is written in C, and returns the vertices, facets, area, volume, equivalent diameter, centre of mass, inertia tensor, principal moments, as well as the ratio and angle phi of the model. For every asteroid, the principal moment of inertia is not exactly aligned with the axis of rotation. The ’ratio’ quantity is the ratio of the rotational moment of inertia to the principal moment of inertia, and the angle ’phi’ is the angle between the principal moment of inertia and the current rotation axis. If either of these is large, it suggests a non relaxed rotation state. As for most asteroids the damping timescale for excited rotation is short compared to the mean time between collisions, it is not expected that many asteroids will have an excited rotation (Henych & Pravec, 2013). Any shape model that does have an excited rotation is subject to much scrutiny because of this, and is usually rejected as erroneous.

A measure known as the sphericity can be used to define how round an asteroid
is. It is defined as the ratio of the volume to a surface area, as shown in Equation 5.2.

In this work, ‘phi’ will be used to refer to the angle between the principal moment of inertia and the current rotation axis, and sphericity will not be referred to as its commonly used symbol, $\phi$

$$\Phi = \pi \frac{(6V)^{\frac{2}{3}}}{A} \quad (5.2)$$

A sphere is defined to have a sphericity of 1 which is the maximum possible sphericity for any object, with zero being the minimum achieved by an object of zero volume but finite area (e.g. a plane). Dwarf planets have a sphericity very close to 1, with irregular asteroids having smaller values (Vernazza et al., 2020). The axial ratios of the asteroid can also show how round the asteroid is, but do not account for surface features that may affect sphericity.

The volume of a convex model is always an upper limit on the volume, as when concavities are taken into account the volume will be smaller. There is a linear relationship between flat areas on a convex model and the volume difference between the convex and concave model, although as can be seen in Figure 5.3 there is considerable deviation from this relationship for individual objects. This scatter is tens of percent, which is added to the volume uncertainty. This is a relatively small uncertainty for the volume compared to the uncertainty from the diameter of the asteroid, but it still needs to be considered. The physical motivation is that more heavily cratered asteroids will have more flat facets on a convex shape model, and the scatter is due to the variation in the depth of these concavities, with shallow concavities resulting in a good match between concave and convex volumes, and deep concavities resulting in a significant drop in volume for the concave shape.
5.5 Large Programme

5.5.1 Programme Summary

The ESO Large Programme (P.I P. Vernazza) is a project to image approximately forty large main belt asteroids with adaptive optics. It is sometimes referred to as the HARISSA survey, but in this work is referred to as the 'large programme'. The main telescope used is the Very Large Telescope (VLT) with the SPHERE instrument at
Paranal observatory in Chile. Using AO both the size and the shape can be determined accurately, allowing for an accurate estimate of the volume. Using only lightcurve inversion, large basins on the surface of the object are apparent as flat areas, but small scale features are not apparent at this resolution, and there is no way of determining the albedo variation across the surface of the object. Adaptive optics helps with both of these issues, with the increased resolution helping differentiate between illumination and albedo. This is also helpful for modelling objects with high albedo variations, such as 10 Hygiea which is known to have an albedo driven lightcurve. Lightcurve inversion techniques rely on the assumption that the albedo is approximately uniform. If this is not true, then there is a degeneracy between albedo and shape, and the model produced will not match the true shape, as it has attributed albedo variations to the shape.

Using the asteroid mass the density can be calculated, allowing for possible compositions to be determined based on spectral type. As these asteroids are large, they are not expected to have significant macroporosity, so the bulk density should be representative of the mean material density of the asteroid. The definition of a dwarf planet is that it must be in hydrostatic equilibrium, but it needs to be empirically verified what size this occurs at. By measuring the sphericity of the large asteroids, the radius at which hydrostatic equilibrium occurs can be found.

The surface of asteroids, in particular the number of large craters, reveals the collisional history of the object. This is useful for knowing things such as the total material excavated from the object. Primordial asteroids have particularly useful cratering history, as they have experienced the entire evolution of the solar system, and therefore have the most useful constraints.

Some of the main findings for individual asteroids to come out of the programme are:

**Hebe**

6 Hebe is the probable parent body of H chondrites, with comparable density to meteoritic samples (Marsset et al., 2017).
Vesta

The model of 4 Vesta matches up extremely well with the shape model from the Dawn mission, which validates the method of shape modelling. As Vesta is close and large, it has the best resolution model of all of the asteroids in the study, with medium sized craters and the mountain Rheasilvia easily visible on the shape model produced from the adaptive optics (Fétick et al., 2019).

Hygiea

10 Hygiea is an extremely spherical object, with sphericity comparable to that of Ceres. As there is a large family associated with Hygiea, we would expect there to be a large impact basin, as is the case with Vesta. The lack of a basin or any other large craters implies that the object was completely disrupted by a giant impact that formed the family and re-accumulated from the fragments that did not meet escape velocity to form Hygiea. The sphericity of Hygiea also means that it is in hydrostatic equilibrium and therefore a candidate for a dwarf planet (Vernazza et al., 2020).

Interamnia

Interamnia is the fifth largest asteroid, and yet very little was known about it because of its spherical shape, dark albedo and distance from Earth making high signal to noise lightcurves of it difficult to acquire. The large programme has shown that its sphericity is similar to that of Vesta, but not in hydrostatic equilibrium, so is not thought to be a candidate dwarf planet (Hanuš et al., 2020).

Psyche

16 Psyche, once thought to have been the iron core of a protoplanet, was found to have a density more in common with the mesosiderite meteorites than iron meteorites (Viikinkoski et al., 2018). However, since the publication of (Siltala & Granvik, 2020) the new estimate of 16 Psyche’s mass is a significant reduction from the previous mass,
further lowering its bulk density. However, the unusually high radar albedo of 16 Psyche still suggests a high metallic content. The *Psyche* mission should shed light on the conundrum (Oh et al., 2019).

**Iris**

Iris is a large asteroid with a shape that is consistent with an oblate spheroid with an equatorial excavation. Its density is consistent with LL chondrites, but the lack of an associated family suggests that the impact must have happened at least 3Gyr ago, although it could be that no family was produced due to a hit and run collision, or that this is the original shape of Iris (Hanuš et al., 2019).

### 5.5.2 Contribution to Project

AO is not enough by itself to generate a shape model. It must be combined with lightcurve data in order for the solution to be stable, as well as supplement areas of the object that are not well imaged. My contribution was to supply SuperWASP data to the project, and ensure the data was accurate and usable. Due to the large volume of data, SuperWASP contributed a significant number of lightcurves to the total. Of the above asteroids where the number of lightcurves from each source has been explicitly stated (3 Hebe, 704 Interamnia, 16 Psyche and 7 Iris), 377 lightcurves were from SuperWASP and 296 were from other sources. For many of the asteroids SuperWASP data was critical for creating the models. Approximately 2000 lightcurves were used in the large programme.
Chapter 6

Results

This chapter outlines the results of the shape modelling, including the periods, spins and shapes. There are examples of periodograms, spin maps and shape models for successful and unsuccessful attempts at determining these properties. I then go on to discuss the properties of the ensemble population, including periods, spins and inertias. Next I discuss the uncertainties involved with the shape modelling process, and attempt to calculate them where relevant. Finally, I discuss the selection biases that have influenced which models have been produced.

6.1 Shape Modelling Requirements

For shape modelling, it is generally necessary that there are at least 1000 photometric observations for a shape model to be produced. This is not always a sufficient condition, and also requires multiple observing geometries and also depends on the quality of the data and the rotation period of the object. Even with good data over a number of apparitions 1000 observations is still needed. 1000 observations was taken to be the minimum amount of data for a shape model to be attempted to be constructed, and no shape models were attempted to be created from less data. A photometric file was created for every object in the SuperWASP database, and then the DAFEED file was merged to create a complete photometric catalogue. Modelling was attempted for the
objects with at least 1000 data points, while no models were attempted for objects with less data than this. There were 240 objects that met this requirement.

### 6.1.1 Initial Rotation Periods

Obtaining an estimate of the rotation period before shape modelling will greatly reduce computational time for shape modelling. Before starting the shape modelling, rotational periods were attempted to be found for the first thousand numbered asteroids as all asteroids with sufficient SuperWASP data for period determination were in this group, using the double sinusoid fitting method described in (Equation 4.11). There are no asteroids numbered larger than this that SuperWASP has photometry suitable for shape modelling at the time of writing with more than one thousand points of photometry from more than two apparitions. Asteroids 1015, 1041, 1048, 1114, 1303, 1469 had more than one thousand points from only a single apparition. Asteroids 1021, 1086, 1107, 1171, 1323 had more than one thousand points from two apparitions. After a visual inspection of the lightcurves, none of these objects were found to have enough lightcurves of sufficient quality for shape modelling, but this data may be useful if more observations of these objects are made in the future. Of the first thousand numbered asteroids 512 periods were definitively determined. For the objects that did not have a definitive period returned, some had low signal to noise that did not allow for any estimate of a period to be obtained, either because of a flat lightcurve or excessive noise in the data, while some had no data and some had ambiguous periods. Many of these ambiguous objects had periods close to a multiple of 12 hours, leading to incomplete lightcurve coverage for any single apparition leading to ambiguous period determinations. Most of the periods found agreed with the literature (Figure 6.1, Table A.2), but there were a few objects for which the literature did not agree with the results from this work. Of these 12 differed by a factor of 2 or 0.5, and 9 were not an integer ratio of the literature period. Many of these objects have ambiguous spin periods, with either a very small amplitude lightcurve or a very symmetric one, making the
The most obvious object to have a different value was that of Hygiea which is the fourth largest asteroid, although the correct period was discovered independently of this work section 5.5.1, but the PDS archive has not yet been updated.

Figure 6.1: Asteroid periods from this work and the literature. The dotted lines represent where the periods from this work are half and double the periods from the literature. The vast majority of objects show good agreement between this work and the literature. Objects where only one solution is available are not included.

These periods are all synodic (the rotation period with reference to the Earth, not referencing the Sun), and will therefore be slightly incorrect compared to the true sidereal period. Using the shape modelling code produces sidereal periods, as well as giving a much smaller uncertainty, although both the uncertainty in the period due to
6.1. Shape Modelling Requirements

<table>
<thead>
<tr>
<th>Asteroid</th>
<th>This Work</th>
<th>Existing Period</th>
</tr>
</thead>
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<tr>
<td>10 Hygiea</td>
<td>13.823</td>
<td>27.63</td>
</tr>
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<td>74 Galatea</td>
<td>8.653</td>
<td>17.268</td>
</tr>
<tr>
<td>81 Terpischore</td>
<td>21.861</td>
<td>10.943</td>
</tr>
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<td>101 Helena</td>
<td>15.48</td>
<td>23.08</td>
</tr>
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<td>115 Thyra</td>
<td>14.491</td>
<td>7.241</td>
</tr>
<tr>
<td>207 Hedda</td>
<td>15.024</td>
<td>30.098</td>
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<td>241 Germania</td>
<td>11.498</td>
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</tr>
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<td>307 Nike</td>
<td>7.900</td>
<td>11.857</td>
</tr>
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<td>333 Badenia</td>
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<td>338 Budrosa</td>
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<td>425 Cornelia</td>
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<td>14.667</td>
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<td>702 Alauda</td>
<td>16.713</td>
<td>8.353</td>
</tr>
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</table>

Table 6.1: A comparison of the rotation periods derived in this work that differ from those in the literature.

The systematic error (< 1%) and the error due to measuring the synodic period are still small (< 0.1%). 512 objects had a period derived in this way. All of these periods were based on a period either derived earlier in this work or using a period in the PDS archive. Due to the way the shape modelling works, there were several objects that had a clear detection of the period, but with an uncertainty too large to use it for shape modelling Table A.4.

Overall, the periods found agreed very well with those from the PDS archive. Of those that disagreed, most differed by a factor of two (Table 6.1). This is due to either one or more of the lightcurve maxima being very small or very similar. If they are very small, identifying all of the maxima is difficult. If a maximum is missed and a double peaked lightcurve is assumed, then the lightcurve estimate will be double the true value. If the shape is very similar, then it is impossible to know if it is an albedo driven single peaked lightcurve, or a very symmetric shape driven lightcurve.

The large number of successful period determinations shows that double sinusoid
fitting is viable as a quick way of searching a large period space to find the correct period. It also works well for the cases of single period lightcurves, for example correctly finding the 13 hour period for Hygiea as the best period (Figure 6.2). However, asteroids with more complex shapes and not having double peaked lightcurves are not as well identified. There are several asteroids with triple peaked solutions, and these will not be as well determined as double peaked asteroids. A solution for finding non-double-sinusoidal lightcurves could be to add third order terms to the fitting equation, but this will run the risk of fitting triple peaked solutions to double peaked lightcurves. Due to the fact that successive nights of lightcurves will subtly differ from each other, it is preferable to fit the longest period possible. As SuperWASP tends to see bright asteroids, it is not surprising that the periods agreed well, as these objects have the longest observation arc and will have the most data to derive a period.
6.1. Shape Modelling Requirements

Figure 6.2: These periodograms are from fitting the double sinusoid to the lightcurves. All periodograms using the sinusoid fitting method are from 2 to 100 hours, as it is assumed that there are no faster rotating asteroids due to the spin barrier, and slower rotating asteroids have lightcurves that are too flat to fit the data. Figure (a) is a periodogram that fits the data extremely well, as shown by the deep and narrow minima. Note the second solution at double the period. In these periodograms, it is hard to fit a lightcurve with double the period as this would be a quadruple sinusoid, which is impossible to fit well, meaning that the smallest good solution is usually the correct one. Figure (b) is a periodogram of Hygiea, the correct period at about 13.7 hours is correct, but not the best solution on the periodogram. This is because the lightcurve is single peaked, so the solution at double the period appears superior. Upon inspection of the folded lightcurves, the correct period becomes apparent. In this periodogram the longer periods are not shown. Figure (c) is an image of an inconclusive periodogram, where there is no good candidate for a period. These are caused by either insufficient amount of data, or insufficient coverage, such as the period being close to a multiple of 12 hours.
Of the objects that had periods found using the shape modelling code (Figure 6.3), fifty successfully had spins found Table A.3. Thirty eight of these had two spins determined while the remaining twelve have one solution. Those within 10 degrees of the ecliptic pole, do not have well constrained longitudes, and the longitude for these objects is not quoted. The objects were determined to have a spin successfully determined if there were two well localised minima on the spin axis map (Figure 6.4), which were also sufficiently deep. For the minima to be sufficiently deep, any pixels with a chi squared within 5% of the minimum value must be within 10 degrees of the minimum. If there are chi square values within 5 percent of the minimum and located within 10 degrees of it, then the spin solution had too large an uncertainty for it to be considered secure. For the minima to be well localised, there must be no more than 2 minima on the plot. This prevents the cases with lots of minima due to a poor solution, some of which happen to satisfy the deepness criterion. Six of these models are for objects that did not previously have models.
6.1. Shape Modelling Requirements

(a) Psyche

Figure 6.3: Periodograms for 16 Psyche and 57 Mnemosyne created using the shape modelling method. The periodogram for 16 Psyche is made of a main, smoothly varying trend. At the minimum, the trend is not smoothly varying, with one point below the main trend. The dotted line is a 5 percent increase on the chi squared from the minimum value. If there are more than 2 points below the dotted line, the solution is empirically rejected as not accurate enough. If there are no points below the main trend, then the period is not sufficiently well constrained to attempt a spin solution. The next is a periodogram with insufficient evidence for shape modelling, with the minimum value less than 5 percent away from the main body of the periodogram. There are many possible periods within the 5 percent criterion, making the solution unsuitable for shape modelling.
Figure 6.4: Spin maps for 16 Psyche and 8 Flora created using the shape modelling method. Each pixel is 6 degrees longitude and 3 degrees latitude. The most likely areas for a spin to exist are in white. The map for 16 Psyche is a typical spin map for an object with a well constrained spin with the minima being relatively small and deep. The spin map for 8 Flora is more typical of a poorly constrained spin map, with the minima being wide and relatively shallow. There are also multiple minima.

6.2 New Models

Six asteroids had models made in this work that did not have a model in the DAMIT database at the time of writing. They each have two spin solutions with the exception of 33 Polyhymnia, which only has one model. A similarity between these objects is their complex lightcurves. Three have lightcurves that are not the typical double sinusoid, and two have incorrect periods listed in some databases, possibly contributing to the lack of models.

33 Polyhymnia

This is an S type asteroid in the outer belt, and is currently in the 22:9 resonance with Jupiter putting it on an unstable orbit (Šidlichovský, 1999). The rotation solution for Polyhymnia was found to be 18.608(1) hours. This is in agreement with the literature period of 18.608 hours. The two spin solutions have a latitude and longitude of -54°, 36° and -54°, 192°. The two solutions are similar to each other in latitude and nearly 180 degrees apart in longitude, indicating a well constrained spin solution with the expected degeneracy. It is an oblate object, with the two main features being a single protrusion and a crater on the equator.
6.2. New Models

Figure 6.5: Shape model for one of the solutions of Polyhymnia. The crater is most easily visible on the left side of the lower left image.

63 Ausonia

This object has previously had a model published, but the authors noted that the model was not a good fit to the data used to create it (Ďurech et al., 2011). Ausonia has a rotation period of 9.2975(2) hours, in excellent agreement with the existing prediction of 9.298 hours. The spin solutions are 120°, -18°, and 306°, -27°. Both solutions are elongated objects, meaning it could be a contact binary of similar sized lobes, or an elongated ellipsoid. It has a diameter of about 103 km, and as an object in the Vesta family (although it is an S type rather than a V type), it is not thought to be a primordial object and came from one or more fragments that came from an impact on Vesta. Due to the rotational period not being unusually fast, it is unlikely the elongation is from
rapid rotation. It is possibly a contact binary, with the two lobes forming from the ejecta from an impact.

Figure 6.6: Shape model projections of Ausonia. There are no noticeable surface features, with the possible exception of a small most easily seen looking up the x axis (lower left).

113 Amalthea

This is an S type asteroid with a diameter of 46 km. A previous occultation found that Amalthea probably has a satellite thought to be about 5 km in diameter ((113) Amalthea And Its Satellite S/2017 (113)1). In this work it is assumed that the moon is small compared to the asteroid and does not significantly contribute to the shape, although if the moon is found to be large this would mean that the derived models are not reliable. Current estimates put the moon at about one tenth the diameter of
the parent body, suggesting a cross sectional area of 100. The exact brightness ratio will depend on illumination fraction and albedos. Amalthea is also thought to be a fragment of a larger parent body along with 9 Metis, with which it shares a very similar composition (Kelley & Gaffey, 2000). The spin solutions are close to the ecliptic south, making the uncertainty in longitude larger. It has a rotation period of 9.9392(3) hours, and spin solutions of 348°, -81° and 192°, -60°. In both solutions there is an obvious crater on the equator.

Figure 6.7: Shape model for Amalthea. The large crater is visible looking down the y axis (centre of the top right image).
123 Brunhild

A possibility for the lack of models is an incorrect period quoted in some parts of the literature of 10.04 hours as opposed to the 9.873 hours found in this work, although the PDS archive has the more recently derived correct period listed. The lightcurve for this object is triple peaked, suggesting a more complex shape than an ellipsoid. It is an S type asteroid with a rotation period of 9.8735(1) hours, and spin solutions of 78°, 69° and 240°, 54°).

![Shape model for Brunhild.](image)

126 Velleda

Angular object with no notable surface features and a diameter of 48 km. It is also an S type. Similarly to Brunhild, there are some places in the literature providing a
rotation period of over 5 days, possibly contributing to the lack of a model. The period solution is 5.36708(4) hours, and the spins are $132^\circ$, $42^\circ$ and $312^\circ$, $36^\circ$. It also has a complex lightcurve, indicating a complex shape. There are no obvious depressions on the surface, although the angular nature of the convex hull indicates a more complex shape than an ellipsoid.

Figure 6.9: Shape model for Velleda.

337 Devosa

A rounded object, but seems to be missing part of a hemisphere, possibly from a large impact. This object has a triple peaked lightcurve, similarly to 123 Brunhild. Devosa does have large flat facets on the model, indicating large depressions, possibly causing the lightcurve shape. The period derived is 4.65277(3) hours, and the spin solutions are
210°, -9° and 36°, -12°. It is an X type in both the Tholen and SMASS classifications.

Figure 6.10: Shape model for Devosa.

6.2.1 Differing Models

The asteroids in this section already have an existing shape solution. As the models here use the data from DAMIT, it is unsurprising that they are mostly in agreement. There are a few models that have different spin solutions, but for most objects the difference from the literature in the spin solution is less than ten degrees in longitude and latitude. Some of the larger longitude differences are from the latitude of the spin axis direction being near the ecliptic pole, where changes in longitude are very hard to detect and the solution is essentially correct.

Table 6.2 contains the models which are not in agreement with the existing solu-
6.2. New Models

<table>
<thead>
<tr>
<th>Asteroid</th>
<th>DAMIT Longitude</th>
<th>DAMIT Latitude</th>
<th>SuperWASP Longitude</th>
<th>SuperWASP Latitude</th>
<th>Angular Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 Angelina</td>
<td>138 14</td>
<td>317 17</td>
<td>132 6</td>
<td>313 6</td>
<td>9 12</td>
</tr>
<tr>
<td>201 Penelope</td>
<td>84 -15</td>
<td>-</td>
<td>81 -38</td>
<td>259 -26</td>
<td>23 -</td>
</tr>
<tr>
<td>218 Bianca</td>
<td>- 126 17</td>
<td>-</td>
<td>305 17</td>
<td>321 34</td>
<td>- 22</td>
</tr>
<tr>
<td>270 Anahita</td>
<td>15 -50</td>
<td>-</td>
<td>42 -81</td>
<td>253 -77</td>
<td>23 12</td>
</tr>
<tr>
<td>665 Sabine</td>
<td>310 -77</td>
<td>-</td>
<td>167 59</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>776 Berbericia</td>
<td>347 12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.2: Comparison of spins for asteroid models taken from DAMIT and those in this work (SuperWASP), as well as their angular separations. A dash indicates that the solution is not available from that collection of models.

All have a period that agrees with the existing solution. The angular separations are all within 25° of each other with the exception of 776 Berbericia. 270 Anahita and 665 Sabine have solutions near the ecliptic pole, making the longitude difficult to constrain, and therefore only a single solution is available from SuperWASP. 776 is unusual as there is only a single SuperWASP-derived model, but the spin solution is not near an ecliptic pole. This is probably due to the high inclination of the orbit of the asteroid. The other solution is not statistically significant from the SuperWASP and DAMIT combined data, but is the best solution for the models taken from DAMIT. The latitudes are also different by 47°, an unusually large discrepancy for a pair of spins.

Table 6.2 are the models where the period and spin solutions agree. This does not necessarily mean that the model is correct, as if the added SuperWASP data are from a similar geometry to the data from DAMIT, we do not expect the solution to change significantly.
Figure 6.11: The difference in spins between the literature models and all of the models in this work. The mean offset was 9.7 degrees in longitude and 5.7 degrees latitude, and a total angular offset of 8.3 degrees. This is greatly influenced by the solution for 776 Berbericia, which has 180 degrees difference between the existing and new solution. Excluding this outlier, the mean longitude difference is 6.1 degrees and the mean offset is 6.2 degrees. 43 of the models had an equivalent model, while 6 objects had a solution that was significantly different. The second figure is just the solutions where the new model spin agrees with the existing spin within 20 degrees.

6.3 Physical Properties of All Models

6.3.1 Spin Axes

Spin axis determination was attempted for 225 asteroids, which is all asteroids that had a successful period determined via shape modelling. Of these, 50 had a spin successfully determined with a sufficient significance to claim the spins form a unique set of solutions, with a total of 89 spin solutions found. Of these spins, 40 were new spin values, and 49 matched with the literature. Some of the spins that did not match the
literature will be the alternate solutions for some asteroids that previously only had one spin estimate. The spins did not match exactly with those quoted in the literature, and as most asteroid spins do not come with uncertainty determinations it is impossible to say how significant the difference is. Spins are quoted in latitude and longitude, but near the poles the spins can be far apart in longitude, but still have a small angular separation. An example is 187 Lamberta, for which the most current solution from DaFEED is 153, -56, while the solution in this work is 40, -86. All of the objects with either a latitude or longitude variation of more than 20 degrees have at least 1 solution close to the ecliptic pole. The objects that do not match are 201 Penelope, 218 Bianca, 270 Anahita and 776 Berbericia.

6.3.2 Inertias

All of the asteroids had an angle phi (Table 4.1) of less than 2 degrees. This indicates that the models are good fits, with no significant rotation around non principal axes.

For all models, a table was made with various properties for comparison. For each model the parameters related to the unscaled shape were taken from the inertia files, and the parameters related to the other physical properties were taken from several places, as outlined in Table A.1. For each object, all of the data was pulled from the relevant files, concatenated and written to a data base.

The semi major axes $a, b$ and $c$ were calculated from the inertias using the formulae:

\[
a = \sqrt{\frac{5PM_2 + PM_3 - PM_1}{2V}}
\]  
(6.1)

\[
b = \sqrt{\frac{5PM_1 + PM_3 - PM_2}{2V}}
\]  
(6.2)

\[
c = \sqrt{\frac{5PM_1 + PM_2 - PM_3}{2V}}
\]  
(6.3)

These formulae assume that the asteroid can be modelled as a triaxial ellipsoid. The
alternative and perhaps more obvious method is to take the axes of the shape model, and measure the distance along the axis where the model intersects each axis. This method is sensitive to small variations in the shape of the asteroid in the local area of each axis such as a crater, while the axis lengths derived from the inertias is dependent on the shape as a whole, and is more representative of the axial ratios, which in the absence of scaling data is the useful quantity.

6.4 Uncertainties

6.4.1 Periods

The uncertainties for periods are relatively easy to calculate, with the uncertainty being proportional to the width of the minimum of the periodogram. A typical uncertainty is the full width half minimum (FWHM) (Figure 6.12), combined with the uncertainty due to it being the synodic period. The uncertainties for periods derived using shape modelling methods are far smaller than those from lightcurve fitting, due to the correspondingly narrower minimum for the produced periodogram. For periods derived from shape modelling, the width of the minimum is dependent on whether a solution has been found or not. In the case of a found solution (Figure 6.3), the minimum is very narrow and so the uncertainty is very small. For cases where the solution hasn’t been found the minimum is much wider, so the period has a greater uncertainty.
6.4. Uncertainties

Figure 6.12: A demonstration of the calculation of the FWHM from a periodogram.

For models where a solution is found, the uncertainty come from Equation 5.1. This is because the true period must be between successive tested periods. For a typical rotation period of 8 hours and an observing baseline of 40 years, the uncertainty is approximately 0.0001 hours, or about half a second.

6.4.2 Spin Axes

Similarly to periods, the spin uncertainties are not a product of the shape modelling algorithm. The uncertainty can be estimated from the spin map. Similarly to the period the uncertainty can be determined by the width of the minimum, with the difference that for the uncertainties will be in two dimensions. In addition, the resolution for the spin map is far lower than the resolution for the periodogram.

The spin uncertainty is estimated from the size of the minimum on the spin map. These are harder to quantify due to the shape of these minima being irregular, and not aligned along the principal axes. The uncertainty quoted in this work is the size of the area within 5 percent of the minimum. As all shape models with a spin uncertainty greater than 10 percent are rejected as not being valid spin solutions, there are no spins with a greater uncertainty than 10 percent in this work.
6.4.3 Shapes

The nature of the shape models makes it hard to quantify any uncertainties. The best way would be to use a method similar to the SAGE algorithm and create an ensemble of models, but ensembles are not practical for the code used in this work. The period, longitude and latitude would have to be varied in a small way to create an ensemble of models around the best solution to see which parts of the shape model are stable under small changes and which are sensitive to small changes in the initial conditions. There are two reasons why this wasn’t done. First, the model is very sensitive to changes in the period, as changing the rotation period by a second can cause the asteroid rotation to be out of phase by several hours across the entire observational range. The second issue is that the final inversion in the shape modelling works by gradient descent, meaning that the best solution near to the initial conditions is found. Therefore the code itself will have to be changed in order to do this kind of uncertainty analysis.

(Figure 6.13) compares the shape models for 42 Isis from three different sources.

![Three models for 42 Isis](image)

(a) DAMIT Concave  (b) SuperWASP Convex  (c) DAMIT Convex

Figure 6.13: Three models for 42 Isis. The first model is a concave model made using adaptive optics taken from DAMIT. The central image is from this work, and the final model is a convex model also from DAMIT. All models are oriented such that they are looking down the spin axis. The large scale feature that is present in all three images is a crater visible in the upper right.
6.5 Selection Bias

There are several selection biases present in shape modelling (Marciniak et al., 2015).

6.5.1 Rotation Periods

First is the period of the asteroid rotation. As asteroids that have longer rotations require more nights of observing to cover the full lightcurve, they will have fewer full rotations worth of data, making them inherently harder to converge to a solution. They have the added disadvantage that because SuperWASP does not have good calibration magnitudes (subsection 3.1.4), for objects with periods longer than about 24 hours lightcurve folding becomes very difficult as the magnitude becomes a free parameter as successive nights of data do not cover much or any of the same phases of the rotation period. There are no objects with a period longer than 24 hours successfully modelled in this work, out of a total of 50. This compares to all asteroids with a number below 1000, where 138 objects have a derived period greater than 24 hours.

Asteroids with rotation periods close to a multiple of 12 hours are also harder to model, as the same part of the lightcurve is sampled every night. Periods close to 12 hours are less of an issue because most of the lightcurve can be sampled in a single night. For 24 or 36 hour periods larger sections of the lightcurve will be missing, and will so parts of the shape model will be unconstrained, requiring observations from a different geometries. Due to the increased number of apparitions needed to successfully create a shape model, these objects are less likely to have completed shape models. Defining a period of within 5 percent of either 12 or 24 hours in PDS, 2 objects out of 74 were successfully modelled in this work (2.7%), while 50 of the rest the population of 926 were successfully modelled (5.4%). This means that an object with a period close to a multiple of 24 hours is approximately half as likely as other objects to be modelled, although given the small number of models here this is not definitive proof.
6.5.2 Orbit and Spectral Types

SuperWASP has a limiting magnitude of 15, which makes observing outer belt asteroids difficult. In addition, darker C types are much more prevalent in the outer belt, making them even harder to detect. Using the number of SuperWASP detections as a proxy for apparent brightness, in the inner belt the mean number of observations of an asteroid in the size range 20-50 km was 6457, while in the outer belt the mean number of observations was 2486 per object, meaning an inner belt object has a mean number of observations more than twice that of an object in the outer belt. The mean number of observations for S and C type objects in the inner belt was 8074 and 3023 per object respectively (a factor of 2.67), while the number of detection in the outer belt was 3582 and 1243 (a factor of 2.88). The difference between types is similar for the inner and outer belts (2.67 and 2.88), demonstrating that the effect is not due to the distribution of types dependency on heliocentric distance. Comparing the mean number of observations of inner belt S type asteroids (8074) and outer belt C types (1243), the factor of 6 difference demonstrates the bias in favour of nearby reflective asteroids. The reason for using the number of observations rather than the apparent magnitude is that shape models need approximately 1000 observations to be stable. Objects on elliptical orbits spend most of the time near to aphelion, so the number of observations better captures the amount of time an asteroid has a high enough brightness to be seen by SuperWASP.

6.5.3 Lightcurve Amplitude and Axial Ratio

The other major bias is lightcurve amplitude. Asteroids with a higher signal to noise are easier to model. This bias is harder to measure, as while most asteroids have a secure rotation period, fewer asteroids have well constrained lightcurve amplitudes; many only have a minimum lightcurve amplitude which is affected by the geometry of the observations, meaning that an asteroid could have a small lightcurve amplitude simply because it happened to be observed pole on.

The minimum and maximum recorded lightcurve amplitude is recorded for every
asteroid on PDS. These can be compared to the axial ratios of the asteroids in the sample to see how good of a proxy lightcurve amplitude is for axial ratio, and then compare the distributions for both the maximum and minimum lightcurve amplitudes for asteroids that have been modelled and asteroids that have not. The minimum lightcurve amplitude is strongly dependent on the orientation of the asteroid spin axis. The minimum lightcurve amplitude is zero when the asteroid is observed directly down the spin axis, and will be largest when the spin axis is perpendicular to the line between the observer and the asteroid. Therefore the minimum lightcurve amplitude would increase for asteroids with spin axis latitude further from the ecliptic equator. As the maximum lightcurve is observed when the spin axis is perpendicular to the observer asteroid vector, and for all spin axis directions this observing geometry is possible, then the maximum lightcurve is dependent on the axial ratio rather than the spin axis. The asteroid may not be observed under these conditions, and illumination must be taken into account when determining brightness, but it is expected that minimum magnitude is dependent on spin axis direction and then axial ratio, and maximum is dependent on axial ratio and then spin axis.

Figure 6.14 shows plots of minimum and maximum lightcurve amplitudes against axial ratios $a/b$ and $a/c$. Maximum amplitude is moderately correlated with the $a/b$ axial ratio ($r>0.7$). The other plots all have weak correlations ($r<0.5$). This is as expected, where the maximum amplitude is expected to be dependent on the axial ratio, while the minimum amplitude is not.

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1 Here $r$ is the Pearson Correlation Coefficient, a statistic that measures the linear correlation between two variables.
Figure 6.14: The comparison of the a/b and a/c axial ratios and the minimum and maximum lightcurve amplitudes. There is only a weak correlation between the minimum lightcurve amplitude and the axial ratio, while the correlation between maximum lightcurve amplitude is stronger, especially for the a/b axial ratio.

Figure 6.15 shows plots of minimum and maximum lightcurve amplitude against the sine of the ecliptic latitude of the spin axis. There is almost no correlation between ecliptic latitude and either maximum or minimum lightcurve amplitude. There could however be a small bias introduced, in that the asteroids with the largest maximum amplitude lightcurves all have a sin $\beta$ of more than 0.8, suggesting very high inclination asteroids are more likely to be modelled.
Figure 6.15: (Maximum and minimum lightcurve amplitudes plotted against the spin pole inclination.)
Chapter 7

Analysis

This chapter outlines the main findings of the thesis. These include an investigation of the non-isotropy of spin axis longitudes, an estimate of the minimum densities of modelled asteroids and the axial ratios of different spectral types. Models produced in this work are compared to those produced in the large programme for validation of the models, and finally unusual objects are examined in more detail.

7.1 Selecting a sample

To look at population statistics for the models derived in this work and available from DAMIT, it was necessary to get a representative sample of asteroids. There are the biases that cannot be completely corrected, such as the preference to model asteroids with high amplitude and short period lightcurves, but these can be quantified (section 6.5). Other biases include preference for modelling nearby asteroids, large asteroids and asteroids of certain spectral types. The completeness of detected objects in the asteroid belt is not a trivial quantification (Ryan et al., 2015), but a diameter of 6 km is a reasonable estimate for main belt completeness. Picking a representative sample according to spectral types is harder as not all asteroids have a spectral type, especially for fainter asteroids, but it can still be done for the asteroids that do have a type. Figure 7.1, Figure 7.2, Figure 7.3 and Figure 7.4 demonstrate the biases in shape
7.1. Selecting a sample

models.

Figure 7.1: The period distribution of asteroids in the main belt with a diameter greater than 20 km that do not have a shape model, and those with a shape model. While there are nearly equal numbers of objects with and without shape models at the quickest rotation speeds, as rotation period increases there are significantly fewer objects with shape models. The data has been cut off at rotational periods of 100 hours for scaling. There are a few slow rotators, the vast majority of which do not have shape models. All of these figures have the data taken from the small body browser.¹

Figure 7.2: The distribution of asteroid semi major axes for objects with and without shape models. The nearest asteroids are more likely to be modelled than not, while in the outer belt there are far fewer objects with models.

¹https://ssd.jpl.nasa.gov/sbdb.cgi
Figure 7.3: Histogram of the diameters of asteroids with and without shape models. There are relatively more objects with shape models at large diameters, and relatively fewer objects with shape models at smaller diameters. Asteroids over 250 km in diameter are not shown.

Figure 7.4: Histogram of the Data Arc, which is the number of days between the first observation of the asteroid and the most recent, for asteroids with and without shape models. Asteroids with shape models typically have a longer Data Arc than those without shape models. As the data arc is not an intrinsic property of the asteroid, it is not considered for the reduced bias sample selection. Asteroids over 250 km in diameter are not shown. All larger than 250 km have shape models although some of these such as Hygiea and Interamnia have only recently been made within the last couple of years.

To select this representative sample, the asteroids were grouped into 'bins'. First, the asteroids were separated radially into Inner (semi major axis <2.5 AU), Middle (2.5-2.82 AU), Outer (2.82-3.5 AU) and Extended (>3.5 AU) semi-major axis zones.
Trojans are not included as there aren’t any with sufficient SuperWASP data to make an improved shape model. The physical reasoning for these zones is the presence of the Kirkwood gaps, and this labelling of the belt is common. Next, the asteroids were sorted into size bins of 20-50, 50-100, 100-150 and >150 km, according to their mean estimated diameters (see subsection 3.3.3 for asteroid diameter estimates). These size ranges were chosen as there was a possible change in physical properties at 100 km (Morbidelli et al., 2009), so using these groups it is possible to say at which size the behaviour changes. The other boundaries were chosen to have approximately equal amounts of objects in each size bin. Finally, the asteroids were sorted by spectral type into C group, X group, S group, other types and no types according to their Bus-DeMeo classification. The reason for using this classification as opposed to the Tholen classification is that asteroids in the Bus-DeMeo classification cannot have multiple types, making grouping them easier.

All asteroids were then assigned to each ‘bin’ of a single location, size and type, and then the assignment was done only for asteroids with models. An example would be all asteroids greater than 150km in diameter that are also S types and in the inner belt. Approximately 60% of asteroids over 20 km have models, but for individual bins this varied between 100% and 0%. For each bin, the number of asteroids to be in the sample was half of the number of asteroids rounded up to the nearest integer. If the number of models was less than this, all of the models were used. This is not a truly representative sample as some bins did not have enough models to be proportionally represented. In particular, the outer belt is still very under-represented, especially for small C types, which have some of the lowest apparent brightnesses of the whole belt as seen from Earth. Once the subset had been chosen, a copy of the physical parameters file was made, with only models from the sample included.

Asteroid models of differing spatial resolution can have different derived physical properties, so when choosing between multiple models the model with the number of facets closest to 2000 was used, as this is the number of facets of models created in this work. This is relevant to any property derived from the surface area of the asteroid, as
the area of an asteroid is a function of the resolution of the asteroid model. To try and compensate for this, models were chosen to have the same number of facets wherever possible.

## 7.2 Group Properties

### 7.2.1 KS Test

The KS test (Kolmogorov–Smirnov test) is a statistical test to determine the probability that a sample comes from a given distribution. The two-sample KS test determines the probability that two-samples come from the same distribution, but does not make any assumptions or provide any information on what the distribution could be. In this work, the KS test is used to compare the properties of different subsets of the sample of shape models, such as the rotation periods of different asteroid types. The KS test is useful because it is easily generalised to any two distributions. It is however insensitive to differences in the tails of the distribution. This can lead to issues with data such as longitudes, where the test will be insensitive to changes near 0 degrees or 360 degrees. It is also not a useful test for small samples. As the sample size directly affects the confidence of the test, using it will not lead to erroneous conclusions but will not allow for secure conclusions to be drawn.

To run KS tests a programme was written in python to extract the data for the required asteroids, then do a two-sample test KS test using the `KS_2samp` function from the statistics package in python, and then plot the ECDF (Empirical Cumulative Distribution Function) in python for both samples on the same plot. This has the limitation that non-continuous parameters such as type cannot be compared. In the case that a one-sample test is needed to compare an empirical distribution with a known distribution (e.g. rotation periods and a Maxwellian), then a sample is generated for the known distribution (in this case the Maxwellian) and a two-sample test is done. This is easier to implement than the one-sample test, and the sample size of the test
distribution can be increased to the point where the uncertainty is dominated by the error in the test sample rather than the deviation of the comparison sample from the distribution, effectively making it a one-sample test.

### 7.2.2 Periods

The rotation periods of asteroids are expected to be Maxwellian for a purely collisionally evolved system. Groups of asteroids that are not yet collisionally evolved, or that have experienced other non-collisional effects that change the rotation period such as the YORP effect will have non-Maxwellian distributions. It is the frequency of rotation rather than the period that is Maxwellian, and all frequencies in this work are calculated as rotations per day unless otherwise stated. To investigate this effect, a Maxwellian was fitted to the rotational frequencies of a subset of asteroids, from which a representative sample was taken to be compared with the real sample in a KS test. By doing this it is possible to see with what probability the distribution is Maxwellian.

It was found that asteroids with diameters greater than 30 km have a rotation frequency distribution that is Maxwellian, with a probability of 99% (Figure 7.5. This result has been found before, with the uncertainty in the lower bound of the diameter for the Maxwellian distribution being dominated by the uncertainty in the diameter estimates. This result is useful though as it validates the use of the KS test to further probe the properties of the shape models produced in this work. This was also done with a Poisson distribution, which was not as well correlated as the maxwellian.

### 7.2.3 Spin Axes

The spin axes of an asteroid are described by the longitude and latitude. The distribution for latitudes is well described (subsection 2.5.3), with an excess of prograde rotators (spin latitude greater than 0°) expected for primordial objects, an isotropic distribution for collisionally evolved objects, and an excess of high latitudes (close to 90°) or -90°)) for objects subject to the YORP effect.
Figure 7.5: The frequency distribution of objects in the 30 to 100 km range, and the best Maxwellian fit. This is rotation frequency, so faster rotators are on the right. There is an excess of both fast and slow rotators.

The longitude distribution of asteroids is not isotropic, as discussed in section (subsection 2.5.3), and the reason for the distribution is not yet known. Looking at the distribution of all asteroid spins (not just those in the representative sample) compared to a uniform distribution (Figure 7.6), it is clear that the distribution of spin longitudes is indeed not uniform. It has previously been established that the longitude distribution is less uniform for asteroids with low inclination orbits (<1°) (Cibulková et al., 2016). These studies have calculated asteroid spins in a way that does not break the degeneracy of the spin solutions. The degeneracy means that it is not currently known what the longitude distributions for 0-180 and 180 to 360 degrees longitude are. For clarity, any solution with the pole in the 0-180 degree range will be pole A, and any in the 180-360 degree range will be referred to as pole B. Plotting the longitude distributions, it is clear that there is an excess of objects with spin axis longitudes between 40 and
80 degrees and 220 and 260 degrees. These are 180 degrees out of phase from each other, as expected from the fact that the folded longitudes of previous studies showed only a single maximum in the distribution of spin longitudes. From here on asteroids with spin axis longitudes between 40 and 80 degrees will be known as Group A, while those between 220 and 260 will be known as Group B. Group A has 110 objects, while Group B has 92.

![Spin Axis Longitude Distribution](image.png)

Figure 7.6: Comparison of spin longitudes to a uniform distribution. The longitudes have been changed such that longitudes larger than 180 degrees have 180 degrees subtracted. The bump in the distribution is indicative of an excess of objects in the range of 40 to 80 degrees and a deficit in the range of 80 to 220 degrees compared to the uniform distribution.

Using shape models from DAMIT it is possible to test whether Group A and Group B have the same properties. Figure 7.6 shows the spin longitudes for all asteroids, with spin longitudes less than 180 degrees compared to those with spin longitudes greater than 180 degrees. It shows that there is indeed a difference, with the probability that
they are different by chance is $p=0.00035$ using a KS test. Restricting the sample to only those objects in Group A and Group B, the most significant differences are dynamical (Figure 7.7, Figure 7.8), with differing distributions of statistical significance observed for semi major axis ($p=0.00037$), eccentricity ($p=0.0015$) and inclination ($p=0.0069$). The distribution of the axial ratio $a/b$ for Group A and Group B was also statistically different, with a probability of them being different by chance being $p=0.024$. However, when comparing the objects with spin longitudes between 0 and 180 degrees to those in the range 180-360, the difference is no longer statistically significant unlike the semi major axis, eccentricity and inclination. The probability is also an order of magnitude larger than the dynamical differences, and is probably not a real difference.
Figure 7.7: Comparison of the distribution of semi major axes of asteroids with spin longitudes in the range of 40 to 80 and 220 to 260, as well as the whole asteroid belt. The locations of the Kirkwood gaps are shown as dotted lines, with the stronger resonances indicated by longer dashes. The flattening of the distribution around the gaps can be seen for all objects, and is especially pronounced for the objects in the 40 to 80 degree range.

Looking at the distributions of the semi major axes (Figure 7.7), Group B has an excess of objects in the range 2.5 - 2.8 AU compared to Group A and vice versa. A qualitative difference that is not picked up by the KS test is that Group B has several ranges of semi major axis where there are no asteroids. These seem to line up with the Kirkwood gaps. This lack of objects is present for the whole asteroid belt, with the difference being that the subset has much larger gaps in the distribution than the asteroid belt as a whole. This effect is particularly apparent at the 3:1 resonance, which separates the inner and middle belts. Also, the 8:3 resonance located within the middle belt is very minor in the distribution of the entire population, but significant for the
subset. The other major resonances are the 5:2 marking the outer boundary of the middle belt and the 7:3 resonance in the outer belt, sometimes used to determine the boundary between the outer and pristine belts. These two gaps are similar in size for both the subset and the whole population.

If the increased gap size around the resonances is real, they could be indicative of some sort of migration in this population. There are three possible scenarios here for migration of asteroids; radially inwards, radially outwards or both. For radially inward migration, the population of asteroids interior to the resonances would migrate inward, but would not be replaced, as no asteroids could replace them by travelling across the resonance. The region exterior to the resonance would be filled by asteroids migrating inwards, so would not be depopulated. This has the net effect of extending the depopulation caused by the resonance in the direction of migration, either inwards or outwards but not both. If the migration is inward for some objects and outward for others, as is the case for the Yarkovsky effect, then the objects near the resonances would move away, but would only be replaced by objects migrating from a single direction, as the resonance is blocking migration from the other way. This could lead up to a depopulation on both sides of the resonance, but this would also depend on the distribution of asteroids between the resonances and the rate of migration. The fact that these objects exist must mean that they are being produced on a timescale shorter than the timescale to migrate to a resonance. Therefore whether the resonances become depopulated in this scenario is a complex problem involving the production rates in different regions of the asteroid belt, as well as the timescale for migration. In Figure 7.7 the gaps in the distribution caused by the 3:1 resonances are not symmetrical around the resonance. This asymmetry indicates that if the effect is real there is a preferential direction of migration outwards. An alternative explanation is that for some reason the strength of the resonance is larger for asteroids with these spin longitudes, causing the gaps to widen for the subset. This distribution in semi major axes is important because it is not affected by selection biases in the same way argument of perihelion and longitude of ascending node are, as the semi major axis is not influenced by things like observations
close to the galactic plane.

It also appears that objects less than 15 km in diameter do not show the same distributions for these dynamical properties, so have been omitted from many of the analyses. One of the reasons for this lack of similarity could be that the small objects are newer, resulting from recent collisions and not having time to be sufficiently affected by whatever mechanism is acting on these objects. As small objects are the most affected by Yarkovsky drift, it is possible that the Yarkovsky effect dominates for small objects.

Another difference between Group A and Group B is in the argument of perihelion (Figure 7.8). Group A has no objects with an argument of perihelion in the range of 130 to 150 degrees, while the Group B has no objects with an argument of perihelion in the range of 210 to 230 degrees. This suggests that this combination of spin and argument of perihelion is somehow forbidden. These perihelion ranges are both 20 degrees wide, which could mean that they are caused by the same underlying mechanism. What is unclear is why they are centred on 140 degrees and 220 degrees. These are not 180 degrees out of phase with each other like the longitude distributions. The argument of perihelion can assumed to be uniformly distributed, as mean motion and secular resonances are dependent on the procession of the argument of perihelion rather than the argument of perihelion itself. If it is assumed that the argument of perihelion is otherwise uniformly distributed, then the chance of an asteroid having an argument of perihelion outside the 130-150 range is 340/360, or approximately 94%. If the argument of perihelion for each object is independent of every other object, then the probability that there are no objects with an argument of perihelion in the range 130 to 150 degrees is \(0.94^n\), where \(n\) is the number of asteroids in the sample. For Group A, \(n\) is 143. Therefore the probability of this gap being there by chance is \(0.94^{143} = 0.00028\), a very unlikely occurrence. The calculation for Group B is similar, although there are four objects that are in the range 210 to 230 degrees. Using similar arguments, the probability of four or less objects in the range 210 to 230 is \(p=0.18\). This is not statistically significant. The interpretations of this are either that the gap is
a statistical chance and has no physical meaning, or that somehow the combination of spin longitude and argument of perihelion is harder to achieve for objects in Group A than it is for Group B. This can be resolved by finding more asteroids with uniquely determined spin solutions to increase the sample size.

Figure 7.8: The argument of perihelion for both subsets and the entire sample, and a uniform distribution. The whole asteroid belt is approximately uniform, as are the subsets with the exception that the subsets have a deficit between 130 and 150 degrees and 210 and 230 degrees respectively.

Cibulková et al. (2016) noted that there seems to be an increase of objects with inclinations less than a degree that have an excess of objects in Groups A and B compared
to other longitudes (subsection 2.5.3). While these data appear to show the opposite, with the objects under 3 degrees not showing a significant deviation from a uniform distribution, there are no objects with an inclination of less than a degree. This makes it impossible to come to any conclusions as to whether small inclinations are more likely to exhibit the concentrations in spin longitudes demonstrated in this work.

There seems to be some relationship between spin latitude and longitude (Figure 7.9). Compared to the entire asteroid belt, Group A displays an excess of retrograde rotators, while the second has an excess of prograde rotators. The analysis was redone, but this time splitting each longitude grouping into prograde and retrograde rotators and then plotting the distributions for semi major axis and argument of perihelion. No difference was seen for these properties between the prograde and retrograde objects. (Vokrouhlický et al., 2003) found a relationship between longitude and latitude in their work, but this was limited to asteroids under 40km in diameter, and was much more pronounced than the differences in this work.
Figure 7.9: The distributions of spin latitudes for objects in the two longitude groupings. These groupings do not have the same distribution of latitudes.
The significant differences between the two sub populations and the differences between the groups and the main belt are dynamical (Figure 7.11), including the previously found preference for low inclinations, so it is probable that the cause is dynamical and the objects are interacting with some external force. The alternative possibility is something similar to the YORP effect where the asteroids are acted upon by electromagnetic radiation, but due to the fact that the spin distribution appears to be independent of asteroid size (Figure 7.12), any effect must independent of size. This makes an effect that relies on electromagnetism such as YORP less likely as it will scale with surface area, and gravitational effects more likely. The dominant gravitational effects on the asteroid belt come from Jupiter. A way to test this theory would be to examine if these effects are also true for other small body populations. Due to the very limited observing geometries and low apparent brightnesses for outer Solar System objects, the obvious place to look will be for NEAs.
(a) Inclination

(b) Diameter

(c) Period

(d) Mean anomaly

Figure 7.11: The cumulative distribution functions for inclination, diameter, period and mean anomaly. None of them show any statistically significant difference between the two populations.

Figure 7.12: Cumulative distributions for the semi major axis and argument of perihelion for the two groups, but this time considering objects smaller than 20 km rather than larger than 20 km. The differences between the two distributions are much smaller.
The two groups are opposite each other on the celestial sphere which suggests that the Solar System has a preferential direction. The main possibilities are the sun and the planets. As the sun has an axis of rotation close to ecliptic north and emits near isotropically, especially as a time average, it is unlikely to be the source of this effect. A more likely scenario is some sort of spin orbit interaction with one of the planets, with Jupiter and Saturn the most likely. As the planets orbits are not perfectly circular, this is an obvious source of a preferential direction for any interactions. This is a related concept to 'Planet 9', whose existence has been suggested partly by the clustering of arguments of perihelia in outer Solar System objects, demonstrating the possible effects of planets on small bodies (Batygin et al., 2019), (Bromley & Kenyon, 2016).

Of the 419 objects in the sample, 71 are objects that are part of a family and are not the parent body. Of these, 32 are part of the Eos family. Without these objects, the overall conclusions were not changed, although they were less statistically significant. This could be due to a reduction in the sample size. A piece of future work could be to change the sample to only include objects from asteroid families, and see if there is any difference between asteroid families, with attention paid to the family age, and the size of the member objects.
Figure 7.13: The distribution of absolute magnitudes and orbital periods for shape models made in this work. The absolute magnitude is clearly smaller than all observed objects in the SuperWASP database (Figure 3.3), demonstrating the shape modelling bias for larger objects. The distribution of orbital periods shows the bias for shorter period objects.

### 7.2.4 Densities

Densities can be calculated directly from mass and volume estimates. The mass of asteroids is discussed in section (subsection 3.3.4), and these uncertainties will be reduced with upcoming Gaia masses, leaving the volume as the limiting uncertainty. The volume can be calculated directly from a shape model produced using multiple data sources, but for the majority of models that are produced by lightcurve inversion, so some scaling is needed after the model has been created. Therefore the volume uncertainty can be decomposed into the shape uncertainty and scaling uncertainty. As discussed in section 5.2, convex hulls are stable solutions, but have an uncertainty caused by the undetermined size of concavities of approximately 10%. The model can be scaled using thermal data, with the scaling having its own uncertainty. As the shape models in this work are made using lightcurve inversion, they are not scaled. As scaling them using a thermal data is a demanding process both in terms of time and computing power (B. Rozitis, personal communication) it has not been done for models in this work.

Most diameters derived from infrared and optical diameters assume a spherical
shape, and do not use data from multiple apparitions (Masiero et al., 2020). This could lead to erroneous diameter results because of the observing geometry. An asteroid observed along the short axis appears larger than if it is viewed along the longest axis. It has already been found that this has an effect on phase curves (Cibulková et al., 2020). To test this, both the dimensions of some asteroids and the size as estimated by thermal and optical data are required. The dimensions can be taken from asteroid models made from multiple data sources, in this case models created in the course of the large programme (section 5.5). The uncertainties on each axis of these models is typically less than 5%. To compare the sizes of the diameter estimates, the volume equivalent diameter for the large programme asteroid was calculated (Table 7.1).

The best estimate of the volume was the optically derived diameters. This is in contrast to what is expected, as the uncertainty in albedo typically makes the uncertainties in optically derived diameters large. The albedos were taken from the JPL small body browser. Many of these albedos were derived by taking a diameter obtained using a thermal model, then finding the visible albedo from the diameter and absolute magnitude. Therefore these are not really optically derived albedos at all, but rather infrared diameters. Comparing the infrared diameters to the AO diameters, both the models overestimate the size of the asteroids; STM by 3% and NEATM by 9%. There is a very weak correlation (regression coefficient) between axial ratio and error (r=0.44). This shows that while the orientation and shape of the asteroid could be a contributor to diameter uncertainties, it is not the dominant factor.

Instead of calculating the density from the mass and volume for each model separately, the collective properties can be used to probe densities. For an object that is bound together by gravity (i.e. not a monolith with internal strength), the centripetal force at the surface must balance the gravitational force at the surface. This puts a minimum limit on the density of the object. Most asteroids will not be near this limit, but putting a minimum density even for a single asteroid can be useful as asteroids that share a type (or family) can be assumed to have a similar composition, and therefore

\[ \text{https://ssd.jpl.nasa.gov/sbdb.cgi} \]
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<th>STM(km)</th>
<th>NEATM(km)</th>
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<td>679 Pax</td>
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Table 7.1: The diameters of a selection of asteroids calculated using adaptive optics (and complementary data), the Standard Thermal Model and absolute magnitudes.
material density.

To calculate the density, the forces at the surface of the asteroid must be calculated. This method relies on the assumptions that the asteroid can be modelled as a triaxial ellipsoid and that the asteroid rotates along the shortest axis, as this is a result of the assumption that the asteroid is in the minimum energy rotation state which is true for large asteroids, and can be checked from the shape model. As the other axes are perpendicular to the axis of rotation, then the largest axis can be taken as the point on the equator furthest from the centre, and therefore subject to the minimum gravitational and maximum centripetal force, giving the best constraint on the minimum density.

Holsapple (2001) showed that the sum of the gravitational and centripetal potentials at the surface of a rotating triaxial ellipsoid along the longest axis is given by Equation 7.1, here $a$, $b$ and $c$ are the axis lengths, $\rho$ is the density, $G$ is the gravitational constant and $\omega$ is the rate of rotation.

$$V = \pi G \rho abc \int_0^\infty \frac{ds}{(a^2 + s) \sqrt{(a^2 + s)(b^2 + s)(c^2 + s)}} x^2 - \frac{\omega^2}{2} x^2$$

(7.1)

This can be simplified by defining the terms involving axes as $I$ (Equation 7.2).

$$I = abc \int_0^\infty \frac{ds}{(a^2 + s) \sqrt{(a^2 + s)(b^2 + s)(c^2 + s)}}$$

(7.2)

This integral in these terms can be done numerically. This quantity is dimensionless, which means that the absolute values of the axes are not important, but rather their ratios. The advantage of this is that no accurate measurement of the size of the asteroid is required. If the asteroid is only just bound, then the force at the surface needs to be zero. The force is the derivative of the potential (Equation 7.3). For the force to be zero, then the gravitational and centripetal contributions must be equal, and can then be rearranged to give the limit on the density (Equation 7.5).

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x} \left( \pi G \rho abc I x^2 - \frac{\omega^2}{2} x^2 \right)$$

(7.3)
\[ \omega^2 = 2\pi G \rho abcI \] 

(7.4)

\[ \rho \geq \frac{2\pi}{GT^2 abcI} \] 

(7.5)

The gravitational constant is usually quoted in SI units, but using grams, centimetres and hours is it approximately 0.865 \( cm^3 g^{-1} h^{-2} \). Densities are most commonly quoted in grams per cubic centimetre, and rotation periods in hours. The term \( abcI \) is dimensionless, so only the ratios of the axes are required, rather than the absolute values. Not having to scale the asteroid is useful, as the density limit is independent of the diameter calculation, a major source of errors for the direct calculation from mass and volume. The downside is that it is a lower limit on the density, and unless the asteroid is spinning close to the spin limit, then the density will be an underestimate.

Plotted on Figure 7.14 are all of the objects from this work and DaFEED. The only large object close to the spin limit is 321 Florentina, an S type asteroid with a minimum density of 2.28 g cm\(^{-3}\), placing it close to the density commonly quoted for S type asteroids (2-3 g cm\(^{-3}\)) (Carr, 2012). This implies that it is close to the spin limit, which is to be expected from its rotation period of 2.871 hours, close to the spin barrier limit of 2.2 hours. The size of the asteroid (21 km) suggests that this asteroid has been spun up by collisions rather than the YORP effect. Other than this object, the only other large asteroid with a type that is close to the spin limit is 216 Kleopatra, which is a contact binary and the approximations of a triaxial ellipsoid is incorrect.
7.2. Group Properties

Figure 7.14: The period against the unitless quantity $abcI$, where $a$, $b$ and $c$ are the axes and $I$ is the quantity in Equation 7.2. Lines of constant density are plotted, with lines lower on the graph representing higher densities (densities of 1, 1.5, 2 and 2.5 g cm$^{-3}$). The quantity $abcI$ can be interpreted as a measure of elongation, with larger values indicating less elongated objects.

7.2.5 Spectral Types

Spectral types are determined by the surface composition of asteroids. Different compositions may have different strengths, which could manifest as a difference in the shape and axial ratios. The C types were found to have a statistically significantly different axial ratio to the other types, and there were small differences between each of the other types (Figure 7.15). The C complex contains the roundest objects, while X complex objects are slightly rounder than S complex. The X complex is different from the C and S complexes because it is thought to be less compositionally homogeneous than either the C or S complexes, with at least some M types (from the Tholen classification) being dense and metallic, while Tholen P types are very primitive. As the Tholen classification splits the X complex along albedo rather than spectral variations, it is a better proxy for density as primitive objects tend to have lower albedos, while more evolved objects have higher albedos (subsection 2.4.2. We therefore expect to see more differences between the Tholen X complex types than the SMASS types. Unfortunately there are not enough SMASS X type objects with Tholen types to draw
any strong conclusions. It is interesting that the X type is more similar in shape to the S types, as compositionally the X types are more similar to the C types.
7.2. Group Properties

(a) C Types

(b) S Types

(c) X Types

(d) No Type

Figure 7.15: The distributions of the a/b axial ratio for C, S, X and untyped asteroids. All objects are over 20 km in diameter.
Untyped objects are the least round objects, but this appears to be a product of the bias in size (Figure 7.16). Less round objects tend to be small, which are harder to obtain a spectral type for, leading to untyped objects being small. Limiting the sample to asteroids with a diameter less than 70 km, untyped objects have similar axial ratios to the rest of the asteroid belt. When constraining the size like this, the C types appeared to be of a more similar roundness to the rest of the belt, but the difference was still statistically significant when conducting a KS test. As C types are darker and generally further out in the asteroid belt, the asteroids that have been modelled are on average larger than the S types that are more common in the inner asteroid belt. There were not enough of the types that do not to belong to any of the C, S or X complexes to determine their comparative roundness.
Figure 7.16: Plots of the distributions of the a/b axial ratio for C, S, X and untyped asteroids. All objects are between 50 and 100 km in diameter.
7.3 **Comparison to Large Programme**

Out of the forty objects in the large programme that have models available, there are four that are also in this work. Using the objects from the large programme as ground truth due to the low uncertainties in the shape, it is possible to validate the convex models, with individual examples below. Overall, the convex models match the models from the large programme well in terms of overall shape, but miss small details and in certain cases have a noticeably different axial ratio. The comparison is done by visually comparing the approximate size and locations of surface features such as depressions, and the overall shape can be compared by the triaxial ratios, although as the comparison is between convex and concave shape models the exact ratios will always be different, even for models of the same object. The models labelled concave are made using AO, while the convex objects are a product of this work.

7.3.1 **16 Psyche**

The largest difference between the models produced in this work and the large program for 16 Psyche is the size of the smallest axis, which is relatively larger in the convex model. The convex model has a single flat facet ([Figure 7.17](#)) that does correspond to the deepest depression on the concave model, but it is not the only depression in the concave model.
7.3. Comparison to Large Programme

Figure 7.17: Shape models from this work and the large programme for 16 Psyche.

7.3.2 42 Isis

The overall shape for these models matches well, but similarly to 16 Psyche the shallow depressions have not been picked out as flat facets on the convex model.
Figure 7.18: Convex (left) and concave (right) models for 42 Iris. The top models demonstrate how the convex model has a large depression corresponding a relatively flat area on the concave model. The bottom images show that shallower depressions on the concave model are not seen on the convex model at all.

### 7.3.3 354 Eleonora

For Eleonora, the convex model does have a flat facet, but in this case seems to be the combination of two depressions on the concave model visible slightly above and below the location corresponding to the flat area on the convex model (Figure 7.19).
7.3. Comparison to Large Programme

7.3.4 511 Davida

This object does not have any large depressions for comparisons between the models. There are some shallow depressions on the concave model, but there are no flat areas on the convex model with which to compare.
7.4 Individually interesting objects

There are some objects found in this work that have interesting shapes, either because they have unusual surface features such as large flat areas, or an unusual shape such as an extended object.

7.4.1 27 Euterpe

This asteroid is notable for having a very large flat facet which could indicate a large crater. As it is also the parent body of the Euterpe family, whether or not this crater is evidence of an impact will depend on whether the family-forming collision was catastrophic or not. This is not a new model (Stephens et al., 2011), but the presence of the flat facet was not noted at the time. The authors noted that there was an expected albedo variation which could have been caused by illumination effects in the crater. It will approach to 1.04 AU of Earth in November 2022, making it an ideal target for AO imaging.

Figure 7.21: Shape model of 27 Euterpe. The large flat facet is at the top of the model.
7.4. Individually interesting objects

7.4.2 39 Laetitia and 43 Ariadne

These have elongated shapes that are not axisymmetric. This in contrast to convex models such as those for 216 Kleopatra (Shepard et al., 2018), which concave models show is a contact binary with lobes of approximately equal size. The lobes more resemble Itokawa (Fujiwara et al., 2006), an NEO made of two lobes of unequal size.

![Laetitia](image1)

![Ariadne](image2)

(a) Laetitia  
(b) Ariadne

Figure 7.22: Convex shape models of 39 Laetitia and 43 Ariadne.

7.4.3 55 Pandora and 532 Herculina

![Pandora](image3)

![Herculina](image4)

(a) Pandora  
(b) Herculina

Figure 7.23: Shape models for 55 Pandora and 532 Herculina. These shape models have shapes resembling hemispheres.

The model for Pandora is oblate, but has a shape approximated by a hemisphere, with one side being very flat and the other being relatively rounded. Herculina has a similar shape, previously being described as 'toaster like'\(^3\) (Kaasalainen et al., 2002). This could be indicative of a large impact.

\(^3\)Disc-resolved images will be required to determine if it has two slots or four.
7.4.4 218 Bianca

The model of Bianca appears to be complex with two large flat areas and a smaller flat area, which could mean it has multiple large concavities.

Figure 7.24: Shape model of 218 Bianca. Shown are two flat areas, which can be interpreted as depressions. Not shown is a small flat facet on the reverse side of the model.
Chapter 8

Conclusions and Future Work

8.1 Thesis Summary

8.1.1 Pipeline

I have successfully created a pipeline to clean SuperWASP photometry to make it usable for other applications, with an emphasis on making it suitable for shape modelling. This pipeline takes the SuperWASP SQL table, and produces lightcurves in a format that is suitable for shape modelling. This has been validated against data from other sources and the final lightcurves show good agreement with these other sources of photometry. The pipeline has also cleaned the lightcurves, removing outlying observations, sections of lightcurves and lightcurves that were too noisy. A novel method of determining periodograms by fitting double sinusoids has been made, and can quickly find the most probable periods for an asteroid’s rotation based in its lightcurves.

8.1.2 Shape Models

89 shape models were created in this work for 50 objects. Six of these objects did not previously have shape models. The amount of data produced by SuperWASP is a significant fraction, and in some cases the majority, of all photometry available for large main belt asteroids. This data will be useful in the future in the creation of shape
models for more objects. The large programme (section 5.5) also used lightcurves from SuperWASP, and this data was useful in refining and in some cases necessary in order to create these models. The automatic cleaning of lightcurves was a significant benefit to the large programme, which otherwise would have been unable to use this valuable resource.

8.1.3 Model Properties

Using these models and the models from DAMIT, I looked at the properties of these objects. The main findings from this were:

The spin periods and directions, as well as the shapes of asteroids that were modelled in this work.

The distribution of the longitudes of spin axes for main belt asteroids is indeed non-uniform as other works have found (subsection 6.3.1). There seem to be two distributions, and the dynamical properties are subtly different for each, suggesting a dynamical mechanism causing these non uniformities. Further shape models will help refine these results, especially models with a uniquely determined spin axis.

C complex objects are significantly rounder than objects from the S and X complexes. This could help constrain the physical properties of the different asteroid spectral types.

8.2 Future Work

Since the start of this project, there have been other sources of photometric data that could be used for shape modelling. One example is the Transiting Exoplanet Survey Satellite (TESS), that has lightcurves for many objects that do not yet have shape models. Due to the operating time (<2 years) most objects only have a single apparition, but could be enough for some objects to create a new model. It has the added advantage of good photometric quality.
8.2. Future Work

8.2.1 De-trending

Various other fields have methods for cleaning lightcurves, such as studying variable stars. The difference is that these objects have a repeating pattern, and this can be exploited via some sort of folding and moving average. Due to the constantly changing lightcurve shape, this isn’t applicable to asteroids.

Pipeline in this work was designed to take SuperWASP lightcurves and automatically detect anomalous spikes. This would have been easier if the data had been de-trended first. It is hard however to design a de-trending algorithm that works for all possible time series data. As asteroids have a constant rotation speed asteroids have a characteristic timescale for the signal that astronomers are interested in, usually on the order of hours. When removing noise from the lightcurve, it is important to only remove features that have a shorter duration than the rotation period, or else the signal that astronomers are interested in will be removed.

One possibility is to do a multi scale analysis, and wavelets could be a possible tool to do this. A wavelet transform is similar to a Fourier transform, but whereas a Fourier transform reveals the exact frequencies but does not provide any information on where in the lightcurve these frequencies occur, a wavelet transform provides approximate frequencies and their approximate locations. Asteroid lightcurves are suitable for wavelets as they are smoothly varying. This means that any high frequency components of the lightcurves will be noise. By identifying in the lightcurve where the high frequency components are and removing them the lightcurve can be cleaned. This breaks down in the case of extremely low signal to noise ratio, where the entire lightcurve consists of high frequency noise.

A wavelet transformation gives the size and location of different frequencies present in the lightcurve. The lightcurve can be decomposed into its different frequency components, and sections of lightcurve that have large high frequency components can be removed. This kind of approach would be universally applicable to any lightcurve that is smoothly varying.
8.2.2 Using the Shape Models

Thermal data can be used with a shape model of an asteroid to scale the shape model. This can be used to derive properties such as the diameter, albedo and thermal inertia. This was not done in this thesis due to time constraints, but this is an obvious use of the models made in this work.

Previous work has shown that there is a dependency on the phase curve of an asteroids on its shape. Further models will help study this effect, and may also reveal whether there is any systematic bias in asteroid diameter estimates that were not adequately constrained within this work.

More work needs to be done to determine the cause of objects having non isotropic spin longitudes. The large size of some of the objects means that the YORP effect is unlikely to be a reason for these spin states. Due to the apparent lack of any non-dynamical differences between these objects and the rest of the asteroid belt, it is hard to suggest a mechanism. If the same effect is observed outside of the main belt, it would suggest that gravitational interactions from the giant planets are not the cause, as the effect will be weaker the further the asteroids are from Jupiter and Saturn.

8.2.3 Publishing WASP data

The SuperWASP data is a valuable resource for asteroid science, and as such it is a goal to make the archive easily usable for the small body community. The greatest barrier to this is the reliability of the data, with some photometry not being usable due to poor observing conditions and large errors. The automated cleaning process goes a long way to making the data usable to the wider community.
# Appendix A

## Supplementary Tables

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Table A.1: The columns used in the inertia files. Some of the values listed are the physical parameters for the equivalent triaxial ellipsoid. The reason for creating these is that the inertially derived parameters such as triaxial volume and area are different from the volume and area of the model. This reduces the effects of local topography on the physical parameters. This is particularly important for the calculation of sphericity, as sphericity is particularly dependent on deviations away from an ellipsoidal shape. While these inertially derived values were calculated, they were not ultimately used in the analysis of the models with the exception of the axial ratios.
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Table A.2: Periods derived from the fitting system described in chapter 4. While these are the formal uncertainties associated with each period, the dominant uncertainty comes from the simplistic model, which does not allow for light travel time or account for the difference between the sidereal and synodic rotations. Anecdotally, by comparison with the existing periods in the literature, the vast majority agree to one decimal place, and approximately half agree to two decimal places or better. Literature values were taken from PDS, which sourced them from the LCDB (Warner et al., 2009a)
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<td>228</td>
<td>33</td>
<td>-</td>
</tr>
<tr>
<td>714</td>
<td>Ulula</td>
<td>6.998380 (70)</td>
<td>42</td>
<td>-15</td>
<td>228</td>
</tr>
<tr>
<td>776</td>
<td>Berbericia</td>
<td>7.667013 (84)</td>
<td>168</td>
<td>57</td>
<td>-</td>
</tr>
</tbody>
</table>

Table A.3: Periods and spin solutions derived from shape models created in this work.
<table>
<thead>
<tr>
<th>Asteroid</th>
<th>Rotation (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>13.719136(269)</td>
</tr>
<tr>
<td>25</td>
<td>9.949157(141)</td>
</tr>
<tr>
<td>35</td>
<td>31.901272(1452)</td>
</tr>
<tr>
<td>44</td>
<td>6.483716(60)</td>
</tr>
<tr>
<td>49</td>
<td>10.353997(153)</td>
</tr>
<tr>
<td>51</td>
<td>7.783061(86)</td>
</tr>
<tr>
<td>57</td>
<td>12.292609(216)</td>
</tr>
<tr>
<td>65</td>
<td>6.080638(53)</td>
</tr>
<tr>
<td>71</td>
<td>35.838938(1833)</td>
</tr>
<tr>
<td>77</td>
<td>9.000218(116)</td>
</tr>
<tr>
<td>80</td>
<td>14.03082(281)</td>
</tr>
<tr>
<td>82</td>
<td>13.000489(241)</td>
</tr>
<tr>
<td>98</td>
<td>16.48012(388)</td>
</tr>
<tr>
<td>100</td>
<td>27.068218(1046)</td>
</tr>
<tr>
<td>103</td>
<td>23.742662(804)</td>
</tr>
<tr>
<td>111</td>
<td>22.064637(695)</td>
</tr>
<tr>
<td>116</td>
<td>12.032283(207)</td>
</tr>
<tr>
<td>117</td>
<td>9.124169(119)</td>
</tr>
<tr>
<td>121</td>
<td>5.492723(43)</td>
</tr>
<tr>
<td>122</td>
<td>10.687276(163)</td>
</tr>
<tr>
<td>173</td>
<td>6.111105(53)</td>
</tr>
<tr>
<td>173</td>
<td>6.111614(53)</td>
</tr>
<tr>
<td>184</td>
<td>6.441104(59)</td>
</tr>
<tr>
<td>192</td>
<td>13.623507(265)</td>
</tr>
<tr>
<td>202</td>
<td>23.670117(799)</td>
</tr>
<tr>
<td>205</td>
<td>14.899316(317)</td>
</tr>
<tr>
<td>207</td>
<td>15.060833(324)</td>
</tr>
<tr>
<td>Period</td>
<td>Period [s]</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>233</td>
<td>19.682401(553)</td>
</tr>
<tr>
<td>236</td>
<td>12.337578(217)</td>
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<tr>
<td>245</td>
<td>14.355677(294)</td>
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<tr>
<td>284</td>
<td>8.561707(105)</td>
</tr>
<tr>
<td>304</td>
<td>18.20905(473)</td>
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<tr>
<td>331</td>
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<tr>
<td>334</td>
<td>7.360127(77)</td>
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<td>12.027196(206)</td>
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<tr>
<td>365</td>
<td>12.705414(230)</td>
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<tr>
<td>451</td>
<td>9.725975(135)</td>
</tr>
<tr>
<td>478</td>
<td>16.10334(370)</td>
</tr>
<tr>
<td>554</td>
<td>13.712063(268)</td>
</tr>
<tr>
<td>579</td>
<td>16.281622(378)</td>
</tr>
<tr>
<td>654</td>
<td>31.856887(1448)</td>
</tr>
<tr>
<td>690</td>
<td>8.617972(106)</td>
</tr>
</tbody>
</table>

Table A.4: Periods derived from objects that had a successful period determination using the shape model method, but for which a spin solution could not be found.
Appendix B

Lightcurves
Appendix B. Lightcurves

14

15

17

18

19

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21

23
Appendix B. Lightcurves

![Lightcurve plots for objects 32, 33, 34, 35, 36, 37, 38, 39]
Appendix B. Lightcurves

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Appendix B. Lightcurves

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Appendix B. Lightcurves

![Lightcurve 97](image1)

![Lightcurve 98](image2)

![Lightcurve 99](image3)
References


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