A computer-assisted proof of dynamo growth in the stretch-fold-shear map

How to cite:

For guidance on citations see FAQs.

© 2021 Farhana Pramy

https://creativecommons.org/licenses/by/4.0/

Version: Poster

oro.open.ac.uk
A computer-assisted proof of dynamo growth in the stretch-fold-shear map

Farhana Akond Pramy (farhana.pramy@open.ac.uk)
Supervisors: Dr. Ben Mestel, Dr. Katrine Rogers And Dr. Robert Hasson
School of Mathematics and Statistics.

### Introduction
- Our work is on a functional linear operator called the Stretch-Fold-Shear (SFS) operator [1] which arises from a model of dynamo growth.
- The existence of an eigenvalue of this SFS operator S of magnitude greater than one ensures dynamo growth.
- Aim of this research is to prove such existence using a computer-assisted proof.

### The SFS Operator
- \( F = \{ c(x) \text{ complex-valued function of a real variable } x \in [-1,1] \} \)
- \( S: F \to F \)
- \( Sc(x) = e^{\frac{i\alpha(x-1)}{2}} c \left( \frac{x-1}{2} \right) - e^{\frac{i\alpha(1-x)}{2}} c \left( \frac{1-x}{2} \right) \)
- \( \alpha \geq 0\) (real parameter) is the shear parameter.

### Mathematical Tools
- Computer-assisted proof
- Interval arithmetic
- Function ball
- Julia Programming Language

### Our Work
**Theorem (Computer assisted):**
Let \( \alpha \in [0,5] \). Then there exists an eigenvalue-eigenfunction pair \((\lambda, c(x))\) of the operator \( S \) satisfying the following:
1. For \( \alpha \in [0,1.5705] \), \( |\lambda| < 1 \)
2. For \( \alpha \in [1.571,5] \), \( |\lambda| > 1 \)
3. For \( \alpha \in [1.5705,1.571] \), \( |\lambda| \in [0.99624,1.00374] \).

### Graphical Investigations:

**Figure 1:** Modulus of Eigenvalues of \( S \).

**Figure 2:** Investigating the first family of eigenvalues of \( S \).

### Conclusion
- To prove the theorem a computer-assisted proof is used to find rigorous bounds around a function ball of each approximately calculated eigenvalue-eigenfunction pair by implementing interval arithmetic in Julia.
- In future we will make \( \alpha_{max} \) as large as possible, hopefully \( \alpha_{max} = 10 \).
- We are also investigating the eigenvalues (carefully around \( \alpha = \frac{\pi}{2} \approx 1.5705 \)) graphically up to \( \alpha_{max} = 15 \).

### Literature Cited