Flow and its Relationship with the Mathematical Classroom

Thesis

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Note: This is a redacted version in which images have been blurred to anonymise participants in the study.
Flow and its relationship with the mathematics classroom
Abstract

This thesis contributes to the field of mathematics education. Using a phenomenological approach, I investigated the multi-dimensional model of the flow phenomenon and its interdependence with the mathematics classroom. Learners of mathematics often have a great enjoyment when carrying out mathematical activities. I contend this experience can be labelled as flow. The flow experience has been described as being in the ‘zone’. Flow refers to the experience of a learner. The learner is totally absorbed in a task to the exclusion of all else with a complete connection and successful outcome. Flow experiences can improve educational experiences for students in the secondary mathematics classroom by heightening enjoyment, intrinsic motivation and creativity. Between 2015 and 2016 I carried out an 18-month longitudinal study with a group of secondary students, anticipating how flow can assist positive relationships with mathematics. I recorded, videoed and questioned students’ experiences of flow in the classroom. I argue first, that from examining how flow occurs in the mathematics classroom, I have constructed an understanding of its appearance to the classroom observer and attended to its perceived benefits. A second contribution of the research is that flow, a quality of experience, positively affects learning in the mathematics classroom, and is a feasible vector for teacher development.
Acknowledgements

I indebted to the numerous individuals and colleagues who have supported and helped me with researching and writing this thesis. In particular, I would like to thank all of the students that were involved in the project, both for their time and their permission for me to question and examine their mathematical experiences. I would also like to thank my colleagues at school, in particular the help the mathematics department and senior leadership gave me along the way. I extend my appreciation to colleagues at University College London. I would also like to thank my supervisors from the Open University; Dr Els DeGeest, Dr Clare Lee and Dr Cathy Smith for all of their time, encouragement and understanding throughout the study. And finally, I would like to thank my friends and family for putting up with me, especially Dawna for never-ending support during the process of researching and writing this thesis.
Preface

The research was an unfolding story, even the research questions themselves changing several times, as notions became apparent. There are narrative parts to the story that I have deliberately kept away from the reader; for example, the events leading up to the research are only documented briefly in the methodology chapter, and the changing nature of the questionnaires is not presented in detail. That I have left out the parts of the story not relevant to the central theme is hoped an aide rather than a hindrance. My aspiration is that I have included enough that will enable the reader to satisfy curiosity and picture my analysis of the relationship between flow and the mathematics classroom.

I attempted to guide the research lessons towards flow occurring, and it may appear (unintentionally), that some of the descriptions are presenting a perfect picture, and the reader can rest assured that the perfect picture did not exist. Because I focused on when flow occurred, there were also times when students were not engaged, and times of flow ‘non-occurrences’ did exist, in line with realities of teaching.

Throughout this thesis all names connected with my classroom and school are anonymised and pseudonyms are used for teachers and students. Permission has been given for classroom images to be shared with examiners and supervisors only, and other library copies will have the images redacted. Ethics forms have been signed by all students and teachers (examples in appendix B).

There was an active reflexivity on my part as researcher and this is explicitly described throughout with the use of extracts from my field journal, compiled from 2011-2016. For the sake of euphony, I have used some words interchangeably; (e.g., learner/student, flow/optimal experience, solitary/individual flow, group/social flow, self-report/self-assessment), although have attempted to avoid confusion and keep this to a minimum. Square [brackets] are always my additions to any direct quotes. Mihály Csíkszentmihályi is pronounced Me High E Cheeks Sent Me High.
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PART ONE -- SETTING FORTH
In this chapter I provide an early definition of flow, and an establishment of some of the key terminology used. I also outline my personal and professional motivations for the research and summarise the context of where the research was carried out and with whom. Additionally, I indicate the aim and purpose of the research, including the research questions.

An early definition

An individual experiences flow when he or she is totally absorbed in the situation and/or task to the exclusion of all else, with a complete connection (Csikszentmihalyi, 1990a). The latter author describes flow extensively: ‘The holistic sensation that people feel when they act with total enjoyment’ (Csikszentmihalyi, 1975, p. 36); ‘…a psychological state in which the person feels simultaneously cognitively efficient, motivated, and happy’ (Moneta and Csikszentmihalyi, 1996, p. 276); ‘…An almost automatic, effortless, yet highly focused state of consciousness’ (Csikszentmihalyi, 1996a, p. 110).

The flow experience has also been described as the ‘zone’. (Martinez and Scott, 2014, p. 139; Goleman, 1996, p. 91). Artists such as musicians sometimes describe when they are ‘just playing’ (Sawyer, 2007; Goleman, 1996). Authors, writers and novelists also commonly experience an awareness of flow. This phenomenon is also often associated with athletics (e.g., Jackson and Csikszentmihalyi, 1999; Spittle and Dillon, 2014). It is possible to dip in and out of the mental streams of flow states during tasks that are performed during the day, for example reading a book (McQuillan and Conde, 1996; Kirchhoff, 2013); surfing and using the web (Pace, 2004); and whilst conversing with friends, family or colleagues (Kotler, 2014).

Personal and professional relevance

I have been led to investigate the flow experience as it relates to mathematics in the secondary classroom, by certain observations made during learning and teaching mathematics.
I have an early recollection of playing a piece of flute music, aged about 15. Being so absorbed in the playing, that everything else disappeared, all outside noises, sights, feelings, smells and tastes. All that mattered was the music, I was not thinking of the music, just playing it. There was no noticing of what note to play next, no conscious thinking of how to play. I can only describe it as magical, lost in a world of luminosity and yet with such relevance. Many years later, whilst I was completing a proof of the Fibonacci sequence, this experience suddenly repeated itself and a connection and relevance to mathematics was made!

From 2009 onwards, up until present day, I have used journaling as a tool to become more aware of many of my experiences as a teacher and learner of mathematics. In particular, I have sometimes used them as a classroom practitioner, sometimes as a researcher, and also occasionally as a learner of mathematics. The journal entry below was written in 2010 and further illustrates the anecdote described in the vignettes above.

Journal entry 9/9/10
I was doing an inductive proof of Fibonacci sequence and I felt a state of mind, and time stopped still and had no meaning. All the outside influences vanished, all the worries and concerns of everyday life, disappeared. Only thinking in that moment.

Moreover, there were times, during my teaching career, when I noticed my students had a great enjoyment of the lesson, similarly lost in the moment whilst doing mathematics. (Although many may have some difficulty admitting it, perhaps due to peer pressure and other reasons.) Another journal entry below, this time about my teaching, sheds light;

Journal entry, September 9th, 2014
Year 9 class. I let the maths take the centre stage. Comments such as ‘we really learned loads today’ ‘thanks sir’ ‘wow is that the time’.

I contend that this experience can be labelled as flow (Seligman, 2011; Goleman, 1996; Csikszentmihályi, 1975). In my role as a mathematics teacher, I have found occasions of flow being demonstrated by students could be when carrying out particular mathematical tasks, questions or problems; when working either individually, with a partner or in a group doing a task. It may be that the classroom ethos, character and atmosphere is ‘conducive’ to learning.

I continue to have a fascination, curiosity and interest in this enjoyment and propitious atmosphere; and with a focus on flow, it may be possible to improve educational experiences for secondary mathematics students. The purpose of this research was to provide further understanding of the link between flow and secondary learners of mathematics.
Introduction

An outline of the context

The particular community I investigated was a single secondary inner-city school (‘Meadows’ School) in the North London area. I carried out a series of research lessons with a group of secondary students aged 11-14 between 2015 and 2016. I was their mathematics teacher, having left a leadership position in another inner-city school to carry out the research. I explored the flow phenomenon in my mathematics classroom; examining multiple data sets and collecting evidence repeatedly from a single class over a period of 18 months, a generally accepted practice to investigate teaching and learning experiences (Stone et al., 1991; Wheeler and Reis, 1991).

I used video to capture the flow occurrences, and additionally utilised responses to student’s self-assessment questionnaires. To examine and analyse the 12 hours of video, I adapted a method of analysis called multi-slice imagining (Bohnsack, 2008), used here, I believe, for the first time in an educational context. In addition, I utilised and further developed empirical indicators of flow, which I have coined ‘flow markers’ (Williams, 2002). I also included a focus on social flow; a relatively new concept (Walker, 2010).

Building on existing knowledge

There is reference to the research and literature in the general area of flow, (e.g., Engeser and Rheinberg, 2008; Csíkszentmihályi, 1990a; Csíkszentmihályi, 1975). There is also exploration in specific disciplines; for example sports (e.g., Crust and Swann, 2013; Hefferon and Ollis, 2006; Jackson and Csíkszentmihályi, 1999); music (e.g., Diaz, 2011) (Byrne et al., 2003); computers (e.g., Alina, 2012; Rha et al., 2005); spirituality (e.g., Martinez and Scott, 2014; Peterson et al., 2005) even the military (see Kotler and Wheal, 2017).

Additionally, a number of studies revealing the flow state, teaching and education were unearthed; for instance, research into North American high school children (Shernoff et al., 2014; Hektner et al., 2007), online learning and students (Esteban-Millat et al., 2014; Meyer and Jones, 2013). Seligman et al. (2009) looked at flow, happiness and education, and Egbert (2003) researched flow and the modern foreign language classroom.
Nevertheless, investigation specifically examining mathematics teaching and learning, and the flow state is in its infancy. Accordingly, there is scope for a contribution to existing knowledge (Hammersley et al., 2001); thus this thesis has the element of originality, in the sense it is building on existing knowledge and practice; such as that done by Williams (2000), researching resilience and mathematics pedagogy using flow as a measure, and Armstrong (2008) examining social flow and the mathematics classroom.

With the aim of examining the relationship with flow and the mathematics classroom, this research set out to identify when the flow experience transpires and to whom and with whom it occurs. From there I set out to examine the pedagogy present at the relevant time. Thus, my focus was;

Research question 1 (RQ1): How is the notion of flow attested, manifested and demonstrated\(^1\) by mathematical learners?

Research question 2 (RQ2): What pedagogy (including task design, and questioning techniques), elicits flow?

It is important, perhaps at this juncture, to be clear that flow leads the research, and is the initial main interest (RQ1). Although the pedagogy (and classroom environment) will play a part, this will take second place. I shall look for an unfolding and elucidation of the influences and causes of the flow experience. This resonates with the definition of an explanatory research question (Hammersley et al., 2001).

An outline of the sections

I start with a detailed review of the existing literature surrounding flow. A definition of flow, origins of the phenomenon, and its place in the mathematics classroom is explored. Next, I examine the methodology utilised, detailing the paradigm, theoretical framework, the methods of analysis, and the instruments and datasets. Included is a description of the context of the classroom and the class that was researched. In part II, Finding Flow, I present the findings of the research lessons and a discussion of the findings. Finally, I conclude with a discussion of the limitations of the study, future research opportunities and an outline of my contribution.

\(^1\) The term ‘attested’ is used here to mean ‘to be evidence or proof of, testify to, vouch for’ (see Oxford English Dictionary, 2018a). ‘Manifested’ here is taken to mean; ‘(to) display a quality, condition, feeling, etc. by action or behaviour; to give evidence of possessing, reveal the presence of, evince’ (see Oxford English Dictionary, 2018d). Demonstrated here is taken to mean ‘To establish the truth of (a proposition, theory, claim, etc.) by reasoning or deduction or (in later use) by providing practical proof or evidence’ (see Oxford English Dictionary, 2018c).
CHAPTER 2: A review of the literature

In this chapter I present existing literature concerned with flow. In addition, smaller theoretical reviews of particular notions and terms that arose during the writing are included in each subsequent chapter. These were appended on a continuing basis as the research unfolded, a notion supported by Urquhart (2013, p. 11), who reassures us; ‘[the literature review is] determined by the emergent theory’ (my parenthesis).

I start by establishing a more detailed meaning of flow, and the origins of the optimum experience. I look at how it may have contrasting effects on different individuals. I also explore some of the associated theories, and its importance and desirability in the classroom.

Section 1: What is flow?

An adequate explanation of consciousness, mental streams and awareness has a long history. One of the oldest commentaries on consciousness and associated states is from the Upaniṣads; the story of ‘The Great Forest Teaching’ (e.g., Thompson, 2017), which forms a part of a text written in 7th century BCE (Brhadāranyaka Upaniṣad). This tale describes self, interpreted as consciousness, which in turn is defined as the inner light. Flow is perhaps a more modern attempt at this rationalisation, with a focus also on the activities that generate complete absorption and engagement.

Nevertheless, describing flow as a state of consciousness, or an ‘altered state of consciousness’ does not completely explain the phenomena (Csikszentmihályi and Nakamura, 2018, p. 103). For example, ‘flow is always a positive state’ (Kotler, 2014, p. 28) and it is not possible to have a negative experience whilst in flow, in part by definition (it is always intrinsically rewarding). States of consciousness fluctuate and can be either rewarding or not (see Thompson, 2017). Even redefining flow as ‘an always positive altered state of consciousness’ difficulties may be found (see also Kotler, 2014, p. 28), and sooner or later it will be required to define consciousness, a complex task, and beyond the remit of this thesis.
Flow occurs when a student is motivated by a teleonomy of self, that is a group of freely self-chosen goals (Nakamura and Csikszentmihályi, 2002). In other words, a motivation that ‘leads to reorganisation of growth in the order and complexity of consciousness’ (Moneta and Csikszentmihályi, 1996, p. 277).

Plato appears to describe this experience in the Phaedrus (Ph 245c), using the ancient Greek word ‘ekstasis’ to describe ‘standing outside the self’ (Seeskin, 1976, p. 575); moments when ‘a door opens in your mind or soul, an expanded sense of being and intense euphoria’ (Kotler and Wheal, 2017, p. 11).

**Flow theory**

Flow has been researched in many fields, including educational settings. Thus, I next situate flow in a general academic field, later narrowing to pedagogic experiences. Mihály Csikszentmihályi, the main proponent of flow (e.g., Seligman, 2002), began his research identifying reasons for intrinsic motivation and endeavoured to assimilate ‘motivation, personality and subjective experience’ into a consolidated framework (Moneta and Csikszentmihályi, 1996, p. 276). This framework became known as ‘flow theory’

Flow theory states that, no matter the activity, flow can be characterised by nine phenomenological elements (e.g., Csikszentmihalyi, 1997; Csikszentmihalyi, 1996a; Csikszentmihalyi, 1975). These are: challenge interrelated with perceived skill; goal setting; transparent feedback; merging of action and awareness, a loss of self-consciousness; concentration on the task; being in control; temporal distortion; and an autotelic experience. Support for this characterisation of flow, particularly within educational contexts, is found in several studies (e.g., Shernoff et al., 2014; Sentürk, 2012; Armstrong, 2008; Rathunde and Csikszentmihalyi, 2005; Moneta and Csikszentmihályi, 1996).

The following text elaborates on the nine flow elements. For each, I give Csikszentmihalyi’s characterisation and relate to the context of teaching mathematics.

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2 I limit myself here to usages which define ‘theory’ as that which is modifiable, reasonable and systematic of the phenomenon (e.g., Egbert and Sanden, 2014).
Challenge-skill balance

Flow occurs when both challenge and perceived skill exceed the learners’ levels they are used to and expect. An appropriate balance between skill and challenge is necessary (Moneta and Csikszentmihályi, 1996). It is in the relatedness of these two constructs that flow may happen. Too low a challenge compared to skill or too high a challenge compared to skill and the optimal experience will not be attained.

Similar to other flow engendering activities such as rock climbing or chess, within a mathematical task the degree of challenge is the degree of uncertainty, both with method and outcome. Figure 2.1 below summarises this juxtaposition between skill and challenge.

- Flow manifests when both challenges and skill exceed the learners’ normal levels. Students feel interested and successful.
- When challenge is greater than students expect, and the skills demanded are not present then anxiety takes place. Feelings of apprehension are displayed.
- Boredom arises when the challenge is less than students expect, and the skills are high.
- Apathy occurs when both the skills and challenge demanded by the task are lower than those which the learner is regularly capable of displaying or engaging with.

![Figure 2.1 the skill/challenge relationship (adapted from Milton and Dunleavy, 2014)](image)

The account below, taken from my field journal, further illustrates this;

*Field journal entry, November 13th, 2014*

*Today doing some maths at own level, when the maths (which initially I thought to be a challenge) was in fact pretty easy, I soon lost interest. My skill was way above the challenge.*
The constitutive relationship between skill, challenge and flow suggest that flow is a structure of consciousness. Both challenge and skill, central to flow theory, are perception based (Csikszentmihályi, 1990a). An individual student experiencing flow has perceived the challenge of the activity and perceived their own skill as appropriate to take on the challenge. Within this framework, flow can be labelled as phenomenological (Moneta and Csikszentmihályi, 1996). The question of balancing challenge and skill is of daily significance for teachers in monitoring motivation and classroom environment whether that be attention-seeking responses such as disruption or simply disengagement and disinterest (see also Simonsen et al., 2010). Flow states are experienced as soon as the challenge and skill are appropriately balanced. Interest and curiosity are awakened, and mathematics is enjoyed.

Flow does not fit easily into an objective measurement of competence or task difficulty (in terms of ‘levels’ or grades). There is a perpetual pursuit of complexity within the framework of flow theory and measurement of flow will be of necessity subjective. The relationship between a learner’s skill and the challenge of the mathematical task is a constantly changing nexus, as the task is attempted, the learner’s skill changes. This is known as stimulant complexity (Jeanne, 2009), and is an integral constituent to flow and is typically associated with the phenomenon.

Mathematics can evoke both frustration and delight within the same classroom, dependent on the appropriate challenge and skill. It is suggested that almost any activity can create a flow experience, and it is not the objective challenges and skills rather the subjective ones that dominate a student flow experience (see also Nakamura and Csikszentmihályi, 2002).

The environment (in which flow manifests) has a substantial effect on a learner’s optimal experience (see Lerner, 1998). The interaction that a student has with their environment impacts their flow experience, and flow theory suggests that it is a dynamic system (e.g., Nakamura and Csikszentmihályi, 2002). The classroom environment can be conducive or otherwise to a flow experience and is a reified element a teacher has within their sphere of influence.

**Clear goals**

The next constituent towards a flow experience, according to the theoretical flow model is; there must be clear goals to the activity. The learner must know specifically what they are going to achieve. These goals will be offering an appropriate challenge for the learner’s skill and provide
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purpose and direction for flow to occur. The flow phenomenon is an emergent motivation; that is, as flow occurs new proximal goals are developed (Jeanne, 2009). The author eloquently describes;

‘What happens at any moment is responsive to what happened immediately before, rather than being dictated by a pre-existing intentional structure located within person (e.g., a trait) or environment (e.g., a role or script). Motivation is emergent in the sense that proximal goals arise out of the interaction.’ (2009, p. 3)

Teaching mathematics in a secondary classroom provides countless and excellent illustrations of the proximal goals depicted above. For example, a learner is attempting to solve a problem involving the time taken for a swimming pool to fill, she has the overall task goal of a final answer but has to work out volume of the pool initially. This leads to another proximal goal of unit conversion, and then a rate of flow of water and so on. Eventually the learner works out the final answer. She then has to check her solution using another approach; a further additional goal. All the while the goals and subset of goals provide further and further challenge, stimulating the optimum state of flow.

Before continuing with the attributes of flow I shall compare the proximal goals that are referred to and arise from flow theory with the learning goals that are detailed and shared by teachers. The educational reforms of the late 1990’s brought with it the idea that lessons should be planned with ‘objectives (that) involve understanding’ (OFSTED, 2008, p. 4). For further example, (Hodgen and Wiliam, 2006, p. 4) state; ‘…in order to learn, students must understand the learning intention, which requires understanding of what would count as a good quality work (success criteria)’. In many of the lessons I have come across (in my previous 25 years of classroom practice and observation) the learning intention or objective set will be relating to the overall lesson, not to the immediate task at hand and not necessarily the emergent motivation which is implied by flow theory.

For example, in my experience a typical objective in a secondary mathematics classroom, perhaps displayed on the board might be something along the lines of; ‘by the end of this lesson I will be able to … (e.g.) understand 2 step equations’. However, the immediate task set that will directly relate to flow may not be directly about two step equations, it might be preparatory work, or if it is about two step equations then the task that a student is trying to solve is related more so to the actual question, e.g., how I can manipulate five so it goes to the other side of the equals sign and I am left with a single x on its own. A very different objective than that suggested originally.
With careful thinking about the sequences of activities and tasks and what is a likely goal for each it may be possible for teachers to prescribe something of a match to an emergent goal typified in flow theory. Nevertheless, many variables present themselves in the representative classroom, including the size of the class and that learners are all different not only in their perceived and expected skill level but also in how they learn. Any objectives that are related to flow and mathematics will be organic, that is to say they are created and suggested as the task or problem reveals itself and are particular to the individual learner in that context of the mathematics.

Students bring their own ‘utility’ to the task; the establishment of the usefulness and construction of meaning for the mathematical concepts and understandings (Ainley et al., 2006, p. 30). Thus, if a teacher tries to ‘break down’ tasks into their emergent goals there may be a disconnect between what the teacher perceives a student will need as an immediate goal for their mathematical activity and subsequent understanding and what they actually require.

This aspect of flow theory does have a resonance to the pedagogical framework; ‘inner and outer tasks’ offered by Tahta (1981) cited in Mason and Johnston-Wilder (2004a, p. 31). This framework details the differences between the external goals set for an explicit purpose (e.g., understanding 2 step equations) and the internal goals that students may confront whilst carrying out an activity. Alongside mathematics goals students may set other ‘internal’ goals, not specific to the mathematics. These performance goals have characteristics including being influenced by previous mathematical incidents, (that occurred during the flow state); being malleable; and influenced by future mathematical experiences (that occur during the flow state) (Hannula, 2006). Performance goals are distinct from learning goals in that they are about the judgements that a learner makes about themselves, either internally or externally whereas learning goals focus on understanding something new or increasing competence (Dweck, 1986).

Goal setting encompassing flow theory requires students to have space to set goals for themselves and have control over their learning. Being in control is another element of flow theory discussed later in this section. Goals contribute to the emergence of flow, which provide and fosters a longer-term interest in the activity (Nakamura and Csikszentmihályi, 2002). Having found flow within the activity through progressively more complex challenges a learner is intrinsically motivated to return to mathematics.
Transparent feedback

Flow happens when there is clear and immediate feedback. A student must be able to make course corrections and adjust their responses dependent on their answers for a flow state to occur. The feedback will be unambiguous and transparent (e.g., Jackson and Marsh, 1996), and although it may differ in its form it will provide the student with the necessary information that they have succeeded (or not succeeded) in their goal. It will provide information on the present and allow for course correction and will point the way forward in the sense of identifying where a student could be (perhaps a change to a proximal goal). Flow does not allow for a pause and a reflection on the feedback, automatically it is taken onboard, a student moves on and the activity continues. The student is so absorbed in the situation that evaluation is instinctive, there is a limitation of awareness because of this absorption as Csikszentmihalyi (2014, p. 144) describes.

‘In the artificially reduced reality of a flow episode it is clear what is ‘‘good’’ and what is ‘‘bad.’’ Goals and means are logically ordered. A person is not expected to do incompatible things, as in real life. He or she knows what the results of various possible actions will be.’

Often classroom feedback has a purpose of deciding if a student understands, and if they do (possibly through a display of a correct solution) then they can move on in challenge complexity. It is a ‘consequence of performance’ (Hattie and Timperley, 2007, p. 81). Flow theory suggests this decision is automatic and occurs without the need for external approval, unless in a group discussion, then the course correction would be immediate so as to not take them out of the flow state.

Merging of action and awareness

Another element of flow, suggested in the theoretical model, is a merging of action and awareness. There is no awareness of doing the task, there is just doing the task. As soon as a learner is made aware of their outside surroundings, as soon as they are cognisant of what they are doing and not just doing, then no longer are they in flow. By its very definition one cannot be aware of being in flow; action and awareness are merged (Csikszentmihalyi, 1996a). From my classroom experience learners describe being ‘in the zone’ or ‘caught up in the mathematics’ when recounting their action-awareness coalescing.
A loss of self-consciousness

Because an awareness of self-ceases, there occurs total immersion in the mathematical task and when in flow there is an integration of mind and doing. The self is congruent and cannot be differentiated. There is no ‘I’ that is engaged in reflective dialogue, the ‘I’ is still ‘acting controlling, attending, observing, but not reflecting dialectically’ (Logan, 1988, p. 178). For many, this non-dualism is one of the most attractive characteristics to flow experience, as an example stress and daily problems disappear with the total absorption and loss of self-awareness. The incessant chatter between what Krishnamurti terms the ‘I and the ‘not -I’ is suspended during the flow experience (Krishnamurti, 1973).

Concentration on the task

With a complete concentration on a limited stimulus field flow occurs. The homogenous nature of flow results in outside influences having no effect; distractions and diversions are removed from the attention of the student (Csikszentmihalyi, 2014). Being in flow means being in that moment and being in that moment with a one hundred per cent attention. Consequently, food and drink, and other interruptions then are put aside for the duration of the mental state. It has also been suggested that the reverse is possible; with reduced external cues and limited stimulus field, the flow state can be anticipated. (e.g., Csikszentmihalyi, 1996a; Kotler, 2014).

Being in control

Flow will manifest when there is no worry of failure on the part of the learner and there is total control exerted by the learner. Flow theory states that more complicated, structured and organised activities such as learning mathematics enable a ‘flow experience’ manifestation; conceivably because arranged activities provide an individual with significant control over perceived skills and challenges. Structure offers the opportunity to learn new skills and select the difficulty and type of challenge. Typical situations when flow occurs are when an individual is allowed to select the type and complexity of an activity and is presented opportunities for self-determination acquiring new and novel skills. (Moneta and Csikszentmihályi, 1996). I suggest that the latter could be from a description of a successful mathematical lesson.
A review of the literature

A classroom deliberately structured where students have control over the complexity and type of task or method is likely to engender flow. The amount of control a student has over an activity has been connected with the amount of engagement the student has in the activity (see Deci et al., 1981). When learners have control over their learning the rewards become intrinsic (Csíkszentmihályi and Csíkszentmihályi, 1988). As an example, children start their learning through play; an intrinsic motivator, they play because they enjoy playing.

Temporal distortion

The passing of time is misrepresented; inferring time passes by seemingly very rapidly (e.g., Nakamura and Csíkszentmihályi, 2002). In my experience of teaching in secondary mathematics classrooms, a distinctive feature of a successful mathematics lesson is a proclamation by students of temporal distortion.

An autotelic experience

The word autotelic is derived from two Greek words, auto; meaning self and telos meaning goal (see Oxford English Dictionary, 2018b). An autotelic activity is one that has enjoyment for its own self and purpose. The task in itself is the reward; it becomes autotelic (i.e., doing the activity for sole purpose of the experience of the activity, the outcome is not the reward). The successful, advantageous and rewarding outcome is the frame of mind itself. Moneta and Csíkszentmihályi (1996, p. 278) describe; ‘...(flow) as an endless maximization process toward higher and higher levels of perceived challenges and skills, wherein the target of the optimization is experience itself, rather than a predefined result’.

The rewards are therefore of an intrinsic nature (Csíkszentmihályi and Csíkszentmihályi, 1988). Intrinsic motivation is taken to mean performing a task or activity for its innate fulfilment, as opposed to a separable outcome (Ryan and Deci, 2000b). Experiencing an intrinsic motivation is a key attribute of flow theory.

The phenomenological (and subjective) nature of flow necessitates that not all of the above aspects of the flow experience need be present for flow to manifest. These nine holistic attributes were integral to the entirety of the research. Next, I examine the notion that flow may have distinctive effects on different individuals, dependent on the context and climate of learning.
Autotelic characteristics

It is suggested that an autotelic personality encompasses an individual able to take up the flow state more readily than others (Csikszentmihalyi, 1990a), although there is some disagreement amongst scholars as to a complete definition of the label (Crust and Swann, 2013; Jackson and Eklund, 2002). The term personality, however, implies attributes fixed in nature. Instead, I will use the term characteristic to define attributes acquired over time. The autotelic individual will develop a non-self-seeking persona. Logan (1988, p. 173) illuminates this point well, stating individuals with autotelic characteristics are ‘… able to get caught up in mental and physical activities precisely because they do not dwell on themselves’.

Csikszentmihalyi (1990a, p. 88) suggests that it is a learned rather than genetic advantage. The author eloquently writes; ‘The neurological evidence does not, however, prove that some individuals have a genetic advantage in controlling attention and therefore experience flow. The findings could be explained in terms of learning rather than inheritance.’ In my experience of the mathematics classroom, this position holds true, it is possible for any and all individuals to develop autotelic characteristics. This theory is supported by Kotler (2014) who posits that autotelic characteristics can be nurtured, influenced and grown, through focus, preparation and repeated practise. It is suggested that four notions from flow theory in particular are necessary to enable the autotelic self to grow; the setting of goals; immersing in the activity; paying attention to what is happening (loss of self-consciousness for the sake of interaction as opposed to the sake of the ego) and learning to enjoy the immediate experience (Csikszentmihalyi and Csikszentmihalyi, 1988).

Students developing autotelic characteristics, and become motivated by a teleonomy of self, are anticipated to be more creative and skilful, their choices are established by their drive for the need of flow (E.g., Moneta and Csikszentmihalyi, 1996). Limiting the field of attention seems to also be an essential quality. (Holcomb, 2004). There is a correlation between dispositional flow (i.e., an autotelic personality) and characteristics such as perceived ability, measures of perceived skill and perceived success (Jackson and Eklund, 2002). An individual who perceives they are going to succeed in mathematics will be more predisposed towards flow. Mathematical tasks will be easier to surmount to a learner who is developing strong autotelic characteristics, for example the implied intrinsic motivation indicates external threats such as being stuck are more easily renegotiated. By encouraging students to develop autotelic characteristics they will have increased control over their environment and capable of changing...
the focus that they are choosing, i.e. the attending to the mathematics (see also Csíkszentmihályi and Csíkszentmihályi, 1988). The flexibility in being able to view challenge as just that, a challenge, rather than an insurmountable obstacle, is an autotelic characteristic, and one to be encouraged by teachers for all students within the classroom if flow is to flourish.

The ability to concentrate has a key association to the theoretical flow model and a shortage of control of attention may inhibit the manifestation of flow. The notion of attention deficit and whether a disorder of attention such as ADHD or similar is inherent or learned is still debated, and outside the ambit of this research.

To benefit, enjoy and appreciate a situation for a learner to acquire autotelic characteristics she must be encouraged to be adept at focusing completely, entirely and absolutely on that situation. As teachers we have a responsibility to encourage and advance flow experiences as they relate to mathematics. Csikszentmihalyi (1990, p. 132) articulates and reassures us; ‘… a teacher who understands the conditions that make people want … to do sums - is in a position to turn these activities into flow experiences and thereby set students on a course of autotelic learning.’

Social flow

The notion of social flow or group flow as distinct from individual or solitary flow is a relatively new understanding that has risen in prominence over the last 15 years (Kaye, 2015; Walker, 2010; Sawyer, 2007). The initial research, led by Mihály Csíkszentmihályi in the 1970’s, prompting flow theory, concentrated on flow as the phenomena experienced by an individual and was described a solitary activity (see Csíkszentmihályi, 1975).

In earlier writings, Csíkszentmihályi does not see differences between individual and social flow. Instead, he suggests flow occurs in groups as a ramification of the flow experienced by the respective individuals, rather than an effect that the group has. Tangible external constructions, such as a conversation, or a visual prompt or presence of another being are merely stimuli that enable the optimal experience.

In later writings however, the concept of social flow seems to be supported, (e.g., Csíkszentmihályi and Csíkszentmihályi, 1988). As an example, Csíkszentmihályi (1990a, p. 65) describes surgeons experiencing flow during an operation having;
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‘... the sensation that the entire operating team is a single organism, moved by the same purpose; they (the surgeons) describe it as a “ballet” in which the individual is subordinated to the group performance, and all involved share in a feeling of harmony and power’.

Contribution

Within the literature there is a plethora of work on individual flow, and a dearth around group or social flow. Walker confirms this, suggesting ‘basic research on the conditions and forms of social flow is limited’ (2010, p. 3). In comparison to solitary flow, there are only a handful of studies on social flow including the influential work of Keith Sawyer (who himself was a doctoral student of Mihály Csíkszentmihályi), and in the 1990’s carried out a study of flow occurring within social interactions and collaborations, in particular focusing on jazz musicians and business professionals (Sawyer, 2007). As far as I can tell, there is only one published study on with a focus on social flow in the mathematics classroom (Armstrong, 2008). Accordingly researching group or social flow within the mathematics classroom is a part of my contribution.

A definition of social flow

It is widely agreed within the field of mathematics education that students will learn differently when in group situations then when alone (e.g., Lee, 2006; Pimm, 1989; Bandura, 1971). Social flow is an optimal social experience that occurs when learners are involved and absorbed in a collective state (Sawyer, 2007). Social flow is not many experiences of individual flow, it is when all are flowing together (Walker, 2010). Social and individual flow are two different experiences with many similarities. Although individual or solitary flow is an individual experience, it can occur when other humans are around, and it does not have to happen in a completely solitary situation.

Social flow is likened to the state of worker bees, a ‘hive mentality’ and consisting of ‘improvised collaboration’ (Sawyer, 2007, p. 57). Social flow is an altered state just as individual flow is (Kotler, 2014). Social flow has the ability to generate a self-efficacy that is shared within the group (Salanova et al., 2014) as well as to engender exceptional creativity, (Gaggioli, 2013; Sawyer, 2007).
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A possible explanation for this is the group perform together. As previously stated, social flow is distinct from individual flow, in that it is the interaction between individuals affecting the flow experiences. It is a collaboration rather than just many experiences of individuals. There are not a series of individuals who are all in flow and acting individually, rather it is as if a single organism is functioning in an optimum state. The sense of ‘collective effervescence’ (Durkheim, 1915, p. 226) occurs when a group ceases to behave as separate persons and functions as one, merging amorphous mass, a distinct entity (Gaggioli, 2013).

This has a resonance with the work of Vygotsky (1978, p. 57) who places great importance on interactions between individuals, describing the link between ‘intrapsychological’ (internal) development, and ‘interpsychological’ as social development. The author writes; ‘All the higher functions originate as actual relations between human individuals.’

Co-active and interactive social flow

Walker (2010), working in the context of whether social flow is more enjoyable than individual flow proposes two distinct categories of social flow. Both categories have varying impact on the flow that students demonstrate in the classroom.

Co-active social flow

Co-active social flow occurs when individual activities appear within a group. It takes place when co-operation is required, but not necessarily co-involvement (see also Kotler, 2014). The presence of other students will impact the experience, making it distinct from individual flow. An example could be playing golf, or swimming. A classroom specific example would be a teacher led question and answer session; some students may be more passive and less participatory in their learning, yet still in a state of social flow. In addition, Walker describes low and high conversation categories of co-active flow (2010).
Interactive social flow

Interactive social flow takes place when co-operation and co-involvement is needed; ‘… a task best done by a group and impossible alone’ (Walker, 2010, p. 5). Interaction and dialogue are innate to the activity. Contribution by all of the group participants is central. Social interaction and discussion become necessary and crucial when learners rely on each other’s mathematical skills. The flow experience is shared due to the collaboration and participation of the group. Tasks and activities have an influence on the type of flow experienced along with classroom environment. Mathematical tasks that may give rise to interactive flow more readily include group solving activities, team quizzes, and mathematical games (e.g., activities involving manipulatives, mini plenaries requiring discussion with peers).

Classifications of social (co-active and interactive) and individual flow are particularly germane because the group associated dynamics of secondary classrooms and the presence of others may have different effects on a learner’s optimal experience.

Social flow elements

Sawyer posited ten requirements for social or group flow; and that when these occur social flow is more likely (e.g., Sawyer, 2007). These conditions are holistic, and it is not necessary for all to transpire for social flow to manifest. They are familiarity; equal participation; moving it forward; the group’s goal; communication; blending egos; close listening; complete concentration; being in control; and the potential for failure. Analogous to individual flow in many ways; many of these elements are comparable to corresponding solitary flow components, although there are some differences (see Table 2.1 below).
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Table 2.1 a comparison of the elements of social flow with individual flow

This table roughly equates the elements of individual and social flow. The table should be read left to right. Challenge vs perceived skill is not directly comparable to a single element of social flow, however there are similarities to familiarity, equal participation and moving it forward.

Familiarity

Interactions will be determined by an informal and unwritten collection of systems for learning, including behaving in the classroom. At a very basic level, groups must have a shared understanding of the group’s goal, and enough familiarity that ideas and conjectures within the mathematics classroom can be communicated with confidence. There is a balance between structured thinking and unconventional thinking similar to solitary flow. Common languages will be used, (natural language and mathematics) and much of the communication is based on common understandings, (see e.g., Kotler, 2014).

There must not be too much familiarity however, as when a group is too familiar with each other then all will be normal and expected, there will be no need for close listening and the prerequisite of challenge will not be present. This makes intuitive sense in the mathematics classroom; a group of students will reach a peak experience when many (but not all) of the mathematical concepts are already shared knowledge. Once groupings become overly familiar then they may split (Sawyer, 2007).

Equal participation

For there to be a social flow opportunity the perceived skill level of all who are in the group
needs to be at a similar level (Sawyer, 2007). Social flow will be obstructed if the level of understanding of one student’s mathematics is significantly different to the others in the group. This could also relate to complimentary skills such as language, such as a learner who has similar mathematical skills but able to communicate them less well. A learner who is not as advanced in their perceived mathematical skill may become frustrated, and conversely if there is one student ahead of their peers, they may be bored.

Also, social flow will be less likely to occur if there is one person who is overbearing and self-important, who believes they have nothing to learn. The terms ‘driving’, and ‘playwriting’ are apt metaphors from the acting world (Sawyer, 2003); ‘driving’ meaning when an actor takes over a scene, or ‘playwriting’ when an actor is thinking one step ahead, attempting to predict what the other actor will say. With equal participation there is a negotiation of meaning and there emerges a ‘collaborative zone of proximal development’ (Armstrong, 2008, p. 102), similar to the Vygotskian notion, but in a social sense.

Moving it forward

Students will take ideas and extend and build on them. Emergent and improvised solutions are found (Sawyer, 2007, p. 49). One of the hallmarks of social flow includes the leadership role moving around, depending on the context and the particular problem to solve in that particular time. Kotler defines this as ‘dynamic subordination’ (2017, p. 14). Although roles may be predefined, they are also flexible (e.g., Armstrong, 2008). Individuals will ‘step up to the plate’ when needs necessitate and offer input and leadership (Kotler and Wheal, 2017; Morrison, 2018).

The group’s goal

The goal for the group must be clear and unambiguous in order for social flow to take place (similar to the fundamentals of solitary flow outlined previously). There is somewhat of a paradox here that social flow has been identified within jazz groups where the goal is not as clearly defined (fuzzy) as say within a sports team or within a small team of elite army operators (all situations where social flow occurs freely) (Kotler and Wheal, 2017). A focus on creativity is one way to explain this apparent conflict; during both fuzzy and defined goal orientated activities, individuals must creatively find, determine and clarify the problems as they solve the problems (see also Sawyer, 2007).
As an example, the main goal of an improv group is that of entertaining an audience and skilfully playing, the main goal of a football team is to win (as well as skilfully playing). Both need a larger goal to guide them towards the finish line (albeit a fuzzy one) however as progress is made towards this, individuals will need to find solutions which will involve defining new goals. The creativity that social flow affords assists in this process. This is apposite to mathematical tasks, which may need the creativity provided by social flow; the aforementioned swimming pool problem as an example.

The group goal must be just complex and challenging enough for the students to know that a solution is near but unencumbered so the problem-solving creativity that social flow enables is allowed to flourish (Sawyer, 2003).

**Communication**

Genuine, unselfconscious and spontaneous communication is necessary in social flow situations (Sawyer, 2007). This element is a collective version of transparent feedback (Kotler and Wheal, 2017). Social flow involves communication in groups, contributes to a focus on social interaction within the classroom, and a motivation towards social and socio-mathematical norms and Vygotskian notions of social learning (Morrison, 2018). Verbalising mathematics and mathematical thinking are recognised as a beneficial pedagogical routine (e.g., Armstrong, 2008; Walker, 2010; Tatsis and Koleza, 2008).

The communication should be enquiring, and not closed. ‘How do we solve this quadratic equation?’ answered with x=54.2 and -14.5 will shut any group coalescence down, whereas answered more spontaneously and openly with ‘I think the answer is a decimal so why should we factorise’ may be more appropriate.

**Blending egos**

When social flow takes place action and awareness merge in the group situation. There are no individuals monopolising the attention, there is a humility necessary for the task to be the central focus and not the individual (see also Sawyer, 2007). Surrendering to the purpose of the task enables social flow to occur. Working and moving together, belonging to something that is of more significance than the individual are key traits of social flow.
A review of the literature

Close listening

The members of the group must be engaged in the here and now. Flow, whether solitary or social is always about the present moment, a parallel with many eastern ideas around meditation. Pre-ensignoned ideas and thought-out scripts put downs and sarcasm, irrelevant and off task remarks tend to dampen social flow (Sawyer, 2003). Listening should be active, and a part of this is asking pertinent questions.

Complete concentration

Having a concentration on the task at hand and the interaction with the group will increase the likelihood of social flow. A learner in social flow is aware of the group yet unaware of their surroundings. Students will focus on the activity that is being executed, losing themselves in the dialogue and interaction. They will ‘[lose] awareness of any environmental stimulus that is not related to what they are doing’ (Esteban-Millat et al., 2014, p. 112).

Being in control

Social flow increases when there is group autonomy. Autonomy from the teacher in terms of allowing learners to use their natural mathematical powers (e.g., Mason and Johnston-Wilder, 2004a) and in terms of control over their ideas. Just like solitary flow, when the group is in charge of their own learning then social flow is more likely to materialise. Nonetheless, contrasting to solitary flow, paradoxically within groups for flow to occur there must also be a flexibility where learners must listen and yield to the emergent thoughts of others (Sawyer, 2007).

The potential for failure

Similar to solitary flow, failure is not a complication. Learners are empowered so that the experience becomes the objective, the rehearsal is always now, the final performance is always next time (Sawyer, 2007).
Descriptive and prescriptive framework

It is possible to classify the elements of flow into conditions (inputs) and characteristics (outputs) (e.g., Jeanne, 2009; Quinn, 2005). This is shown in Table 2.2 below. Conditions are those which contribute more readily towards the flow state, namely challenge vs perceived skill, clear goals and transparent feedback. Characteristics are those which are indicators of a flow state; namely a merging of action-awareness, a loss of self-consciousness, being in control, time distortion and an autotelic experience.

<table>
<thead>
<tr>
<th>Conditions (predictive ‘inputs’)</th>
<th>Solitary flow elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenge vs Skill</td>
<td></td>
</tr>
<tr>
<td>Goal Setting</td>
<td></td>
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<tr>
<td>Transparent Feedback</td>
<td></td>
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<tr>
<td>Characteristics (descriptive ‘outputs’)</td>
<td>Non-duality and a loss of self-consciousness</td>
</tr>
<tr>
<td>Merging of action and awareness</td>
<td></td>
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<tr>
<td>Concentration on the task</td>
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<tr>
<td>Being in control</td>
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<tr>
<td>Temporal distortion</td>
<td></td>
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<td>Autotelism</td>
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</tbody>
</table>

Table 2.2 flow conditions and characteristics
This diagram illustrates the elements of the theoretical model of individual and group flow. There are inputs, those elements that can predict flow and influence its manifestation and outputs, those elements which are easier to observe.

There is some disagreement as to whether all of the elements can be influenced, an argument is put forward that all the elements can be flow triggers (see Kotler, 2014). For example, allowing learners control over their learning may impact an optimal flow experience rather than indicate one. Although there may be some overlapping of conditions and characteristics, categorising in such a way also allows an insight into what is within my control (Morrison, 2018).

Summary so far

To summarise this section, the theoretical model of flow has nine characteristics and conditions posited by Mihály Csikszentmihályi that are intimately interconnected and are explicable by reference to the phenomenon as a whole. These holistic elements can be extended to group flow, which can also be further categorised into interactive and co-active flow. I argue that together these elements provide a framework that can be applied to both observing and encouraging the manifestation of flow within the mathematics classroom.
Section 2: Antecedents and origins of flow

Although flow, as posited by Csíkszentmihályi (1975) has found a niche in popular science, it is not the only linguistic metaphor describing an optimal experience and has overlapping concepts (see also Moneta and Csíkszentmihályi, 1996; McAdams, 1994). For example, theoretical proposals similar to flow that examine subjective experiences include self-determination theory, suggested by Deci and Ryan (1985; 1981) which examines intrinsic motivation and relates it to the states of individuals (e.g., Ryan and Deci, 2000b; McAdams, 1994). Additionally, a framework for intrinsically motivated phenomena is outlined by Koch (1956). Adler (1981) posits a phenomenon labelled momentum; described as ‘a heightened state’, with an ‘enhanced progression toward a desired end’ (Adler and Adler, 1978, p. 154).

Next, I continue with a description of flow by comparing and contrasting two other ephemeral states; Maslow’s self-actualisation and Eastern ideas of meditation. I have focused on these because self-actualisation and meditative states are more recognised and established experientially defined events comparable to flow (Privette and Bundrick, 1991).

Self-Actualisation

The description of flow (both individual and group) has similarities to the transient moments of self-actualisation, the peak experiences described by the psychologist Abraham Maslow (1943).

The motivation of individuals who arduously labour because the activity itself (product) becomes the purpose, reward and outcome (process), not for more conventional and traditional recompense of status, kudos or wages is described by Maslow (1943) as self-actualisation. Legge and Harari (2000, p. 5) assure us;

‘Maslow’s hierarchy of needs contains elements that cannot come from outside the person. It is possible to say well done to a child and satisfy his or her self-esteem need for recognition, but the satisfaction of having successfully solved a mathematics problem comes from within.’.

Flow theory also has intrinsic motivation at its core. Maslow’s concept of ‘peak experiences’ of self-actualisation and the contrast between product and process had an important and profound influence on Csikszentmihalyi’s original notions on flow and flow theory (e.g., Csikszentmihalyi andCsikszentmihalyi, 1988).

A review of the literature
In addition, like flow, peak experiences are accompanied by a loss of fear, anxiety and doubt, and both flow and self-actualisation are characterised by extreme states of enjoyment. Reading the description of self-actualisation below it is possible to imagine a description of flow:

‘... self-actualisation means experiencing fully, vividly, selflessly, with full concentration and total absorption. It means experiencing without the self-consciousness of the adolescent. At this moment of experiencing, the person is wholly and fully human.’ (Maslow, 1964, p. 111)

Maslow focused his efforts on what he termed high achievers; looking at Albert Einstein, Eleanor Roosevelt and Fredrick Douglass amongst others whereas Csíkszentmihályi considered the population as a whole (see Kotler, 2014). Self-actualisation is at the peak of Maslow’s noted hierarchy of needs, which describes ascending levels of requirements. It is useful to note that (unlike flow), there are intensities of peak experiences of self-actualisation (Wrigley and Emmerson, 2011). Maslow (1964, p. 111) proposed a peak experience as an infinite continuum. This is a departure from flow theory which proposes no deeper flow; it is all or nothing, you are either in flow, or not in flow (Csíkszentmihályi, 1990a). Being in flow does imply an infinite search for added complexity as skill matches challenge (Moneta and Csíkszentmihályi, 1996) but not that the phenomenon itself has a gradient. Nonetheless, others disagree and suggest flow is not a binary state, rather a graduated process (see also Kotler, 2014; Benson and Proctor, 2003).

Religion and spirituality

The flow phenomenon has a comparable description to the portrayal of mindfulness, meditation and meditative states which have origins in religious and spiritual movements from the East, such as Buddhism and Taoism as well as martial arts and yoga (Diaz, 2011; Csíkszentmihályi, 1990a). Stillness of thought, unification of mind, the gap between thoughts, not thinking, nothingness are all descriptions of meditation (Hanh, 1975), that have much in common with flow theory and flow characteristics such as a loss of self-concentration, and a merging of action and awareness. Both pursue non-duality, that is ‘I’ and the ‘me’ merge. Flow and meditative states both necessitate being in the present moment (e.g., Reid, 2011) and are characterised by temporal distortion. They are both autotelic pursuits, the pleasure is derived from the activity itself, not the outcome. Strength of mind is gained from meditative states, similar to perceived benefits of the flow state.
Complete concentration is an element of both meditation and flow (Diaz, 2011). For example, it is a part of theoretical flow model and is observed in the writings of the Buddha; as the following illustrates.

‘In what is seen there must be just the seen; in what is heard there must be just the heard; in what is sensed (as smell, taste or touch) there must be just what is sensed; in what is thought there must just be thought’ (Udana, I: 10).

Nonetheless, there is a significant distinction; concentration (and a merging of action and awareness) are a part of the purpose of meditation; they are the outcome and the activity itself (Kapleau, 2000, p. 11), whereas during a flow experience they are the effect of the phenomenon. When in a state of flow, whilst aware of our actions we are not aware of our awareness. When in a state of mindfulness and working towards the higher attainments of Eastern philosophy we are aware of that loss of awareness (see also Reid, 2011). A meditation may have a mantra associated with it, to aid with the awareness of mind. Flow states on the other hand have an associated activity (i.e., mathematics) to aid the loss of awareness of mind.

To conclude at this juncture, it may be useful to characterise the description of flow state, as being useful in the moment. Further, when flow occurs, there is a ‘doing’, an ‘effort’ or ‘action’ element to the activity. A flow state is reached when an individual is cognitively efficient (Moneta and Csíkszentmihályi, 1996). It is not simply being lost and absorbed in the point in time as, for example, the experience of viewing certain television programmes may provide (Csikszentmihalyi, 1977). Csikszentmihályi (1990a, p. 213) assures us; ‘[that] optimal [flow] experience is not the result of a hedonistic, lotus eating approach to life. A relaxed, laissez-faire attitude is not a sufficient defence against chaos. … One must develop skills that stretch capacities, that makes one become more than what one is.’
Section 3: The importance and desirability of flow

Throughout this thesis there is the presumption that flow is a desirable state for students learning mathematics. This section reviews literature supporting and critiquing this perspective. Flow is important for humans to experience and has been called ‘the optimal experience’ (Csikszentmihalyi, 1996a). Flow also provides order to consciousness, described as psychic negatroph (Csikszentmihalyi, 1996a). A study by Csíkszentmihályi and LeFevre (1989) found that the more time spent in flow throughout the day, the greater the quality-of-life experiences, greater concentration, creativity, and positive emotions. Flow theory states that being in flow not only comprises an advantageous experience, but also benefits in mental health, performance and creativity (Moneta and Csíkszentmihályi, 1996). Flow leads to an increase in intrinsic motivation, happiness, resilience and creativity (Csikszentmihalyi, 1997). Next, I shall examine these each in turn.

Intrinsic motivation

Students who demonstrate flow will, according to flow theory, have an experience that is intrinsic and motivating (Shernoff et al., 2014). The self-teleonomy of flow (individual seeking meaning of self) is driven by the juxtaposition between skill and challenge (Moneta and Csíkszentmihályi, 1996).

The intrinsic motivational characteristic of flow is advantageous. By making the activity the enjoyment, as the flow state does, the emotions of hate and dislike are removed entirely from the experience and resistance is completely absent (Csikszentmihalyi, 1996a). Through an autotelic experience by entering into flow state when doing mathematics, conflicting emotions are removed from the predicament. There are many learners who will report that they do not want to be in the classroom, and yet once there choose to want to do mathematics; entering a flow state is a part of that reason. Csikszentmihalyi (1996b, p. 2) writes; ‘A teacher who understands the conditions that make people want to learn -- want to read, to write, and do sums -- is in a position to turn these activities into flow experiences. When the experience becomes intrinsically rewarding, students' motivation is engaged, and they are on their way to a lifetime of self-propelled acquisition of knowledge.’

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A study by Nakamura (1988) indicated those who did mathematics and experienced flow learned to enjoy the discipline, advanced their skills, whereas those who did not experience flow did not enjoy the subject and made little or no progress. All the participants in this Chicago study were students who had previous high mathematics scores compared to the rest of the state, did not differ in race, gender or socio economically and the study used the experience sampling method, an accepted measure of experiences (Hektner et al., 2007). Ryan and Deci (2000b, p. 55) articulate; ‘Because intrinsic motivation results in high-quality learning and creativity, it is especially important to detail the factors and forces that engender versus undermine it’. Thoughts and feelings about mathematics, will influence a student’s intrinsic motivation, towards doing mathematics (irrespective of the outcome of the task).

A plethora of research suggests students motivated intrinsically are more independent, better, creative learners (e.g., Ryan and Deci, 2000b; Lepper et al., 1997; Deci and Ryan, 1985). Behaviour within the classroom improves when students are motivated intrinsically, without the need for separable consequences such as certificates, phone calls home, and prizes. Long term, extrinsic rewards tend to dampen the incentive to learn, the likelihood of disenchantment with ensuing activity or task increases when rewards are offered for the task outcome (Ryan and Deci, 2000a; Middleton and Spanias, 1999). With intrinsic motivation the student learns to self-regulate, and she no longer has the need to seek outside help for motivation to fulfil the task.

Being in the flow state is a quality of experience. This quality of experience leads to control of consciousness, a human endeavour illustrated not only by the Yogs and the Tao and the Zen philosophy but sought after by many religions and spiritual movements of the world (see also Kapleau, 2000). Control of learning and the need for the possibility of control are a part of the theoretical model of flow. The positive emotions produced from learning where students are in control are connected to student engagement and intrinsic motivation (see Deci et al., 1981).

**Happiness**

The flow experience comprises a necessary part of happiness. Upon emerging from a state of flow, a learner is in a state of elation (Hektner et al., 2007; Csíkszentmihályi, 1975). There are copious studies indicating a positive relationship between flow and happiness (e.g., Nakamura and Csíkszentmihályi, 2002; Csíkszentmihalyi, 1997; Goleman, 1996; Seligman, 1995). Maintaining
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happiness and satisfaction represents a by-product of the flow state and flow and happiness are synergistic.

Happiness, like flow, is a fuzzy concept, and is not just the ebullience and positive emotion suggested by popular opinion (Seligman, 2011; Seligman, 2004). Seligman et al. (2009, p. 296) eloquently states; ‘Happiness is too worn and to weary a term to be of much scientific use.’

The philosopher Bertrand Russell (1930) suggests a more simplistic expounding of happiness as a discernible, meaningful engagement with the external world.

It is important of note here, in terms of flow theory, the difference between pleasure and enjoyment. Activities satisfying basic requirements such as physical well-being and the need for food can be classified as pleasurable. Whereas activities that stretch, arouse and challenge the intellect are defined as enjoyable (see Seligman and Csikszentmihályi, 2000). Flow occurs when the challenge is almost unpleasurable, for example, a student who is getting all of her mathematical questions correct, with unbroken failure is likely not experiencing flow (Seligman, 1995). Regrouping and trying again is often seen by teachers as a form of failure yet this is the very essence of the flow experience.

Positive psychology posited by Seligman (2004) suggests another explanation of happiness with a direct association to flow. This interpretation attempts to divide happiness into three main components; pleasurable, meaning and engagement\(^3\). Each of these aspects can be pursued individually or simultaneously. The first component, termed pleasurable, is often associated with happiness. For example, watching a movie in class in the last week of term. Difficulties with the pleasurable life include its heritable nature, that it habituates quickly and is not particularly malleable. We tend to quickly normalise whatever makes us happy; a process known as the hedonic adaptation (e.g., Frederick and Loewenstein, 1999).

The second, that of meaning, comprises ‘using your higher strengths for something higher than you’ (Seligman, 2002, pp. 112-121). An example in the mathematics classroom could be doing something for the good of others, for example picking up the rubbish in the class, being polite to a classmate, allowing another student (whom it would benefit more) to answer a question.

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\(^3\) Since suggesting ‘positive psychology’ Seligman (2011) has modified the original theory, adding accomplishment and positive relationships. However, flow is still very much a part of his definition of happiness.
The third aspect of happiness and apposite to this study, is engagement and is defined by Seligman (2002) as entering a state of flow. For a student, this would occur through the carrying out of a mathematical activity which would provide the necessary challenge to demonstrate an optimal experience.

Of note is that without flow (engagement) present, the pleasurable constituent of happiness is low in scale compared to the other two components; i.e., the sum of the parts is greater than the sum of the whole (Peterson et al., 2005). Also, of importance at this juncture flow experiences comprise a necessary part of happiness; maintaining happiness and satisfaction represents a by-product of the flow state, and the two are synergistic.

Flow is not happiness

Nonetheless, flow in itself does not cause enjoyment and happiness, rather, the student is concerned with the feeling generated by the mathematical activity itself, and it is only when a learner emerges from a sense of flow that happiness is experienced. The activities that engender flow provide a basis and foundation for a happy life (see Csikszentmihalyi, 1996a).

When students experience happiness participating in and carrying out mathematical tasks, activities and problems, then it seems reasonable to propose there will be more purpose, enthusiasm and learner engagement towards the subject. Student identity seems to feature as an influence in student desire to continue mathematics post 16 (Reiss et al., 2011) and connects to happiness (Smith, 2010). If mathematical learners were repeatedly exposed to flow and built a mental association between flow and mathematics, now mathematics becomes a focus for enjoyment, and a learner conceivably connects and identifies with the subject (Shernoff et al., 2014; Armstrong, 2008; Csikszentmihalyi, 1997; Csikszentmihalyi, 1990).

Resilience

Repeated exposure to flow can cause resilience in young people (Seligman, 2011). A possible suggested reason being that when a flow state is entered, the ego temporarily disappears into the experience itself, which obliges the self to become more complex. Upon emerging, the ego is included in the experience and the self is made stronger, building resilience (Csíkszentmihályi, 1990a). Contrasted with other everyday activities, where the ego is bolstered
by outside input, in flow state, unusually the ego is strengthened by virtue of embracing the ego in the activity.

When an individual enters the flow state, the actions taken and the mind are one, in other words there is an inseparability of thought and actions. To clarify, it is not that there is no thinking whilst in flow, instead there is no gap between intention and acting. A paradox arises; although the mind is fully absorbed to the exclusion of all else when in the flow state, the mind begins to grow and is stimulated when in the flow experience (Moneta and Csíkszentmihályi, 1996). Once the flow experience is over, the benefits linger. During flow, the consciousness is together, with no distractions, working towards a common goal - some describe the feeling of flow as being at one, there is no conscious thought, it flows. Therefore, when it is over, it is possible to feel more together than before, both internally and also with respect to others and the world at large (e.g., Kotler, 2014).

Engendering resilience in mathematics students by causing flow states has particular relevance to this study, in main because of the challenge vs perceived skill condition in the theoretical flow model. Mathematical resilience calls for a positive response to overcoming an adverse situation (Kooker et al., 2015); the challenge complexity that emerges from a flow state will often cause a mood of elation. Mathematically resilient students overcome mathematical challenges that are just beyond reach (Johnston-Wilder and Lee, 2010); and for a student to enter a flow state there must be a challenge just beyond reach of the perceived skill (Csíkszentmihályi, 1975).

Creativity

It is widely agreed within the field of flow that it stimulates, fosters and encourages creativity. (e.g., Schutte and Malouff, 2020; Zubair and Kamal, 2015; Byrne et al., 2003; Seligman and Csikszentmihályi, 2000; Csikszentmihalyi, 1996a; Moneta and Csikszentmihályi, 1996; Csikszentmihályi, 1975). Creativity and flow have a synergy, that is; ‘…so fascinating we are involved in it, we feel that we are living more fully than during the rest of life’. (Csikszentmihályi, 1996a, p. 2). Social flow in particular has been found to stimulate creative thought and action. (Gaggioli et al., 2015; Gaggioli, 2013; Sawyer, 2007)

The relationship and juxtaposition of challenge and skill enabling the self to grow and become more complex plays a large part in learning and understanding mathematics and the creativity seen by those who experience flow (see also Csikszentmihályi, 1990a). The author further affirms;
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‘(flow) provided a sense of discovery, a creative feeling of transporting the person into a new reality’ (1990a, p. 74). By entering the flow state, students attain a different perception from normal everyday interactions. The lack of self-awareness (suggested by flow theory) whilst in the flow state contributes to the heightened creativity. Creativity as it relates to the flow phenomenon is found in the interplay between the individual, stimulus and environment and thus rather than ‘what is creativity’, Csíkszentmihályi (1990b, p. 200) suggests ‘where is creativity’.

Learning mathematics is progressed with creativity (Holt, 1983). Teaching approaches such as the use of learning journals, along with open-ended problems, the opportunity to make mistakes and find different solutions to similar problems may foster creativity in the mathematics classroom (Grégoire, 2016; Coles and Banfield, 2012).

Of note though is the fact that creativity does not only manifest during flow states, however during flow states creativity manifests. I am not suggesting that flow is necessary for the creativity to solve mathematics, rather by accessing the flow state, the implied creativity means a mathematical problem may be more easily solved, and with intrinsic reward and other implicit benefit.

Experiencing mathematics in the classroom

Widescale reforms of teaching and learning and changes to classroom practice have been the aims of many policymakers and government commissioned reports (e.g., Smith, 2017; DfE, 2013; Vorderman et al., 2011; Smith, 2004; Cockcroft, 1982). Aims of enjoyment, learning, preparation for the world at large, preparation for jobs, encouraging problem solving, fostering creativity; the list goes on. Despite this, there has been little meaningful change in how teachers teach, and students learn. (Lepper et al., 1997) and the lofty intentions of policymakers for the mathematics classroom are often very different from the reality (Wright, 2020). Mathematics, in the secondary classroom can be seen as an acrimonious activity that has to be studied, rather than the adolescent wanting to study mathematics (Foster, 2013; Larson, 2000). In my professional teaching I have seen many demotivated and disenfranchised students in mathematics classrooms, and often overrepresented by students from disadvantaged socio-economic groups.
One possible way forward in the step towards an engaging curriculum is a focus on a manifestation of flow in the (mathematics) classroom (Shernoff and Csíkszentmihályi, 2009). When teachers enable an optimum experience and engender flow, students will benefit and progress in mathematics, experiencing the intrinsic motivation, happiness, resilience and creativity outlined previously.

Summary

This chapter has concluded that flow has a theoretical model, that has characteristics and conditions. I have raised the idea that social flow is distinct from individual flow, and that flow has many positive benefits that can be of use in the classroom. I have also shown how the theoretical model of flow can provide meaning for classroom experiences.
CHAPTER 3: Methodology - underpinnings and methods

In this chapter I describe a broader perspective of methodological approaches, alongside the instruments used, a detailed description of how and where the research was carried out, and a description of the methods.

Section 1: Methodological approaches—the broader picture

I regarded phenomenology, and interpretivism as useful lenses to approach this study. They were selected for their comprehensiveness, effectiveness and heuristic value. Flow theory (described previously) and aspects of teacher research also contributed to an emerging bricolage of methodology. Bricolage is intended here as combining notions and ideas in new ways and improvising with what is available (e.g., Kincheloe, 2001). As Hammersley (1981, p. 219) articulates, all the techniques were; ‘serving as crutches, but one still has to find one’s own way’. Phenomenology, interpretivism, teacher research and flow theory informed the methodology (how I engaged with the research questions, the design behind the methods i.e., what was known); and also elucidated the ontology (what the reality of flow is and what can be known about it). The above frames of reference contributed and defined the epistemology, namely the connection between flow and me, as a knower. Epistemology also allied the knower of flow, (i.e., the learner), with the experiences in the mathematics classroom. Figure 3.1 below illustrates the interlinking components underpinning the research including the methods.
A paradigm is a term represented in varying ways amongst academic literature (Egbert and Sanden, 2014). Within this thesis I limit ‘paradigm’ to a descriptive label for the philosophical stance behind the theoretical framework (e.g., Crotty, 1998).

Interpretivism

It is possible that within this study objectivity is achievable, but, because of the individual nature of experiences the possibility arises that it may not be (Egbert and Sanden, 2014). This agrees with an interpretivist way of thinking and resonates with Crotty (1998, p. 71), who further states; ‘… the findings of natural science are themselves social constructions and human interpretations, albeit a particular form of constructions and interpretations.’ Flow, by its structure, definition and nature is a human interpretation (as substantiated by flow theory). Control and predictability are not always possible when experiences are being examined, and the historical, cultural and environmental contexts become relevant.

By utilising interpretivism I am attempting an overarching evaluative description; a viewpoint of the buried assumptions behind the theoretical framework of phenomenology, to better understand the research questions and flow and in the mathematics classroom.

The position taken is also broadly interpretivist in the sense that an experience is interpreted within the mathematics classroom, which is a multi-layered and complex social world.
Additionally, interpretivism was germane and applicable considering social flow. We are individuals who have individual experiences; however, we operate in a social world and this world has an influence on us. Social flow is an interaction between individuals and how much we interact with this world is a choice that we make. As the psychologist Mark White describes, we are ‘individual in essence, social in orientation’ (White, 2011, p. 86).

Theoretical framework

I describe a theoretical framework as an overarching and distinct entity; essential to hold together, support and provide a foundation for the research (e.g., Egbert and Sanden, 2014). It explains the factors, concepts and variables contributing towards an explanation of the findings and results. Phenomenology and flow theory provided a structure for the flow state to be interpreted, analysed and categorised. Teacher research provided a rationale for what can be understood as worthwhile processes of knowledge creation about flow in my classroom. First, however, I shall clarify why flow is a phenomenon.

Flow is a phenomenon - How is flow demonstrated by learners in the classroom?

I shall start at the beginning. The flow state is a phenomenon, and its definition is perhaps not as clear cut as imagined, (Kienzler, 1991). Previous to Edmund Husserl in the early 20th century, whom many academics credit with the start of the phenomenological movement (Dreyfus and Hall, 1982; Smith, 2018), the designation phenomenon was understood in two ways. Firstly, an empiricist sense, such as patterns of external happenings, comparable say to the look of a particular equation, or the feel of the exercise book. Secondly, it was seen in a rationalist sense, for instance ideas such as equations balancing, or the equivalence of binary and decimal systems. Franz Brentano (who preceded Husserl) thought of phenomena as having a ‘directness’ or ‘aboutness’ (Brentano, 1995, p. 111). Phenomena, as Brentano wrote in 1889, are; ‘things as they are given to our consciousness, whether in perception or imagination or thought or volition’ (Smith, 2018, p. 3). It was Husserl however, who advanced this school of thought; describing all ideas of mind as mind transcendent. Thus, the flow state is not a part of the internal workings of the mind, it has a ‘character that is pointing beyond itself towards something (über such hinausweisen)’ (Husserl, 1970a, p. viii).
Similar to Brentano, Husserl compartmentalised phenomena into ‘psychic phenomena’ i.e., that which can be felt, seen, heard, - the feeling of elation working through a mathematical equation and solving, observing a peer in flow, participating in dialogue around that equation and manifesting social flow - and physical phenomena – (the experience of) the worksheet, the whiteboard, the classroom door. However, he categorises both as being epistemologically identical, and also uses a definition of phenomena to include mental acts as those ‘that are, and not as they are apprehended or apperceived as’ (Husserl, 1970b, p. 860). Some argue Husserl’s definition of phenomena is incomplete, and he moved quickly onto phenomenology without a full definition (e.g., Kienzler, 1991). Here I limit myself to an interpretation of the flow phenomenon in Husserl’s classical phenomenological philosophical sense as not only the psychological experience of flow that we are conscious of, aspects of which can be observed by oneself and maybe an observer, but also the description of the meaning of the flow state, a cognitive experience told to oneself and others.

Relevance

The relevance at this juncture is twofold. Ontologically flow fits the above parameters of Husserl’s notions of ‘psychic’ phenomenon and flow is a mental act that exists independent of the perception of an individual (although affected by that perception). Flow’s inherent non-dualism; namely that whilst in a flow state there is no separation from person and object confirms the phenomenon designation. It is an inner state of being and a lived experience (Csikszentmihályi, 1975) with associated feelings and related emotions. The assumed reality of a phenomenon is multiple and a construct, affected by both individual and environment (Jeanne, 2009); this description is consistent with flow theory.

Phenomenological tradition

As flow is a phenomenon, it was logical that phenomenology (the study of phenomena) was a part of the theoretical framework on which I situated the study. Phenomenology has shaped the debate around inner awareness (which encapsulates flow); another cause to champion phenomenology as an appropriate springboard. It places an emphasis on the world and the human experience as it is lived (Laverty, 2003; Creswell, 2007). The phenomenological tradition discusses emotional states of mind and the awareness of time distortion (Smith, 2014); all of which
are particular to flow and form a part of flow theory. As discussed in chapter 2, additional phenomenological elements of flow theory such as perceived control and clear goals and instruction are also pertinent to the mathematics classroom.

The meaning of flow

Although flow states will have common characteristics, the individual will attach the meaning, because each individual will have a unique experience of flow. The effect on the learner of the experience, namely phenomenography is outside the ambit of this research. Phenomenography is the study of how multiple individuals experience a given phenomenon (Larsson and Holmström, 2007). Rather, the enquiry concentrates on making meaning of the phenomenon of flow through empirical study, focusing on what can be observed about the phenomenon of flow when engaged with mathematics, a ‘subjective phenomenology of intrinsically motivated activity’ (Jeanne, 2009, p. 2). A phenomenological approach allows for different perspectives to be observed and analysed. Bennett (2014, p. 4) describes phenomenology metaphorically ‘…watching a play from the wings of the theatre at the side of the stage, rather than from the auditorium where the audience sit.’. Next, I discuss the notion of intentionality, a cornerstone of phenomenology (Husserl, 1983).

Intentionality in phenomenological research

Intentionality is a concept that Husserl himself had some difficulty explaining (Dreyfus and Hall, 1982). Perhaps it can be understood allegorically, with the following vignette from my journal:

Journal entry 3rd April 2014

In class today and started to think of flow as a tangible ‘thing’. I know it isn’t but just suppose it was. For instance, if flow were a bird, then watching the bird as it flies would be a conscious act. Could I label this watching?

What I refer to here is the intentional act of noticing the phenomenon (flow); apposite to RQ1, how does flow manifest itself within the mathematics classroom?
Crotty (1998, p. 79) sets out intentionality as; ‘… the essential relationship between subjects and their objects. Consciousness is always consciousness of something. An object is always an object for someone.’ Husserl proposed that there are elements to studying a phenomenon. There is noema, or the real character of the phenomena (Dreyfus and Hall, 1982). This is the description of flow itself; being in the zone or absorbed in the activity to the exclusion of all else. In a simplistic form, this would be the actual bird in the journal entry.

The second element is called noesis, the real content. Noesis is the conscious attending to the phenomenon, noticing the flow itself. It is that intentional step to say, ‘this is the phenomena’; or as Husserl (1983, p. 201) describes, the ‘consciousness of something’. Rassi and Shahabi (2015, p. 29) report; ‘noesis is the act of thinking and ruminating, noema is considered as belonging to thinking and thoughts.’ Referring to the vignette above, it is describing the relationship between watching and the bird itself.

To summarise, noema would be flow, and noesis would be the concept of flow. Studying noema from different points of view would be phenomenography; studying noema and noesis is phenomenology and is how I will approach this research.

Relevance - how flow is attested

An advantage of distinguishing between the act content (noema) and act character (noesis) is that a phenomenon can be repeated with the same meaning, (Husserl, 1970a); i.e., flow can be experienced on multiple occasions with a confidence in the congruency of interpretation. In other words, separating the concept of experience of flow from the actual experience of flow, by using this framework, flow can be reported on by the individual learner who is experiencing it, (after the event) and by the outside researcher (during the event).

A corollary

There is an interesting notion that is apparent and somewhat unique with flow. Namely, as soon as a participant of the study notices they are in flow, they are no longer in flow, and can only describe flow with retrospect. When someone describes flow, they cannot be in the experience itself. The non-dualistic inherent attribute of flow would signify that, whilst in a flow state, act content (noema) and act character (noesis) are one, and it is only after the learner has experienced flow can she find the intentionality of the phenomenon. A separation of act content (noema) from
act character (noesis) can only occur after the flow state occurs, at least from the point of view of the individual who experiences it. Using the analogy of the stage from above, there is no need to interview actors while on the stage, but it may be relevant after the play has finished.

**Bracketing**

Next, I explore the tension between being teacher and being researcher; and how I worked to distance myself sufficiently to offer some objectivity in my insights. Bracketing (sometimes called phenomenological reduction or epoche) comprises the researcher setting aside the existing assumptions and biases (referring to flow) and suspending belief around claims of truth (or falsity) made by the learners in the study, a preunderstanding. (Dahlberg and Dahlberg, 2019). It seeks to clarify understanding and interpretation and can be described as a process of cultivating doubt and suspending beliefs in order to ‘open oneself to the work at hand’ (Laverty, 2003, p. 23). It can be seen as recognising, sorting and an awareness of the qualities of the researcher’s experiences [of flow] (Drew, 2004).

The purpose of bracketing is to mitigate the preconceptions that arise from the researcher’s feelings and emotions, the values and beliefs, the assumptions and preconceived theories surrounding the research. Accordingly bracketing intends to provoke a more complex understanding to the research questions (e.g., Tufford and Newman, 2010). Phenomenology concerns itself with being aware of phenomena, and the interpretations we make of these. Being aware of the world as a ‘series of possibilities’ permits a focus on learner experiences as individuals (Bennett, 2014, pp. 48-49) and also allows a layer of rigour if any generalising of conjectures takes place. Figure 3.2 below illustrates the various ways bracketing connects to the study.

Bracketing is important to phenomenology because of noema. Bracketing is turning the attention away from the object of flow (noema) and instead towards the noticing of flow (noesis). Becoming aware of the thinking about flow, as opposed to flow itself enables another layer in the research’s trustworthiness.
Trustworthiness was considered an appropriate indicator of the quality of the study, in view of the interpretivist and phenomenological nature of the research (Heron and Reason, 1997; Guba, 1981). Lincoln and Guba (1985) suggest four criteria of trustworthiness: credibility, establishing the knowledge is worthwhile; transferability, clarifying the context in which the results can be transferred; dependability, that methods and judgements were consistent; confirmability, the results could be confirmed or corroborated by others.

I therefore make a concerted effort at all stages to become aware of the inevitable biases that my role as the teacher-researcher would bring to the enquiry and the environment in which the research was set. This openness is one that permeates throughout the research. The emphasis on bracketing and the reduction of bias is deliberate.

It is important to note that bracketing will not necessarily remove any personal experiences and views from the analysis and research, indeed removing my views and experiences is not necessarily prudent or even attainable, a view that substantiated by Finlay, who demonstrates being open to meaning is an important part of phenomenology (Finlay, 2013).

Thus, a certain amount of creativity is needed, during the research and analysis. For example, recognising that there may not be an existing size or particular type of method that will fit this research and that methods of analysis may have to be modified and adapted. However, this is not
suggesting that the research is any less rigorous; objectivity and subjectivity can be merged as long as there is an awareness that interpretation of my results requires attending to openness of the perceiving process; an argument resonating with Heron (1992, p. 164). The author continues, stating that within phenomenology there necessitates an attention to;

‘encourage the inquirer to sustain an intuitive grasp of what is there by ‘opening his eyes’, keeping them open’, looking and listening and ‘not getting blinded’. It [phenomenology] opposes the use of Occam’s razor at the level of phenomena because it blunts exploration of the finer structures of the experience’.

Occam’s razor is a decision theory that suggests the simplest, easiest and most obvious solution is the most correct solution (Salkind, 2010).

The data was collected in the environment where I was immersed. I played two roles within the classroom - that of the researcher and that of the teacher. Although I could (and did) count my experiences as a part of the research, I recognised from the outset that the interpretive biases presented by being their teacher will always frame discussion and dialogue with students. This is particularly apposite when analysing and understanding social flow. My presence will also affect questionnaire responses. Nevertheless, without being the teacher the research would not exist; only by it being my classroom do I have the possibilities of occasioning the context for the phenomena and the access to study it. The blurring of the roles and subsequent ramifications was inevitable to some degree and bracketing compensated. The cognisance implied by bracketing answers what matters (how flow is demonstrated; RQ1) and who matters, i.e., learners and teachers (what pedagogy engenders flow; RQ2).

Practical implications to ensure trustworthiness

Actions I took to increase the trustworthiness of the research included;

- utilising a longitudinal study, with the intention (after an introductory period) students could see the research as normal.
- introducing the concept of flow to students through dialogue also enabled learner ‘buy-in’ and exercise perceived choice.
- the degrees of freedom allowed and actively encouraged within the classroom, for example discussing and talking about mathematics.
Methodology - underpinnings and methods

- encouraging independent thinking by stepping back from the answering of questions, by ‘learning from asking’ (Mason and Johnston-Wilder, 2004a).
- engaging with students’ mathematical queries and questions only when necessary, allowing natural powers of students to shine through and turning from teaching to learning (Gattegno, 1971).
- carrying out feasibility studies of some elements of teaching with a parallel class before working with the focus class.

My own learning journal also provided reflexiveness, a notion substantiated by (Tufford and Newman, 2010, p. 87), who asserts that journaling enables ‘a raising of awareness of the topic in everyday life.’

_A review up to this point_

In summary, I conducted a phenomenological study of flow with an overarching paradigm of interpretivism. Bracketing was utilised reducing tensions such as bias, preconception and increasing trustworthiness. The methods of interpretation and analysis, such as multi-slice imagining, and situational analysis, the design and choices of questions on the questionnaire contributed to the bracketing process and are detailed later in this chapter in Section 4: Operationalising. The approach of teacher research, discussed next, brought this bricolage methodology together, ensuring a relevance to the classroom.

**Teacher research**

I am using the label teacher research to define the approach taken, and to distinguish this I invoke the words of Lawrence Stenhouse who describes it as;

>'systematic and sustained inquiry planned and self-critical, which is subjected to public criticism and to empirical tests where these are appropriate. Where empirical tests are not appropriate, critical discourse will appeal to the judgement of evidence, the text the document the observation the record.' (Stenhouse, 1981, p. 113).

This type of approach is curiosity based, characterised with order and planning. Hopkins (2002, p. 32) further substantiates this and states teacher research; ‘encourages the independence of thought and argument on the part of the pupil, and experimentation and the use of judgement on the part of the teacher’. The term teacher research is intended to allow a flexibility in my approach, and,
using the words of Hopkins (2002, p. 51); ‘[I am] aware of the problems associated with too prescriptive a framework for action and the values that are embedded with it.’ Nonetheless, the iterative and inductive nature of the study was analogous with both action research and practitioner enquiry. The term action research is taken to mean exploration that improves and changes the situation that is being researched (see also Hopkins, 2002). The meaning of practitioner enquiry is research that sits between reflective thinking and critical reflective practice (e.g., Baumfield et al., 2012; Campbell, 2013).

The research method was an emerging design, using ‘inductive logic’ (Creswell, 2007) and was rooted in reflective practice. I carried forward the learning lessons from each of the three series of enquiry lessons, and in this sense the research was cyclical, reminiscent of action research (Kemmis et al., 2014). I researched flow and its connectives with mathematics and then reflected upon the action I took, and although what I expected to find changed, my general direction remained.

In the main, designing and carrying out the research was a sole, non-collaborative effort (there was one exception towards the end of the study, discussed in chapter 7). I had few opportunities to interact with colleagues whilst carrying out the research, at the ‘coalface’ of the mathematics classroom. It is indicated that teacher research is an individualistic activity (Simms, 2013), although action research is often, but not always performed collaboratively (Burgess, 1985).

The action and the research roles were most definitely combined, intertwined and synergistic; a mainstay of action research (e.g., Elliot, 1991; Morrison and Lee, 2019), although if there was a conflict between the two then the role of teacher had to come before the role of researcher, for both ethical and personal reasons. There were improvements to my educational practice, similar to action research (Ebbutt, 1985). Nevertheless, these improvements were not always intentional, my intentions were guided by the researcher role, i.e., wanting to contribute to scholarship and the academy with broader impact than that of just my classroom. This motive resembles practitioner enquiry (Dana, 2009; Murray, 1992).

Advantages and disadvantages

Some educational theorists claim an enquiry situated in the field can only be carried out by those closest to what is being researched, based on the premise that a closeness to a phenomenon gives a knowledge of it (e.g., McNamara, 1980; Hammersley, 1981).
Methodology - underpinnings and methods

Nonetheless, within an interpretivist paradigm knowledge does not arrive from exposure to the real world, rather it is constructed and interpreted. As an example, my role as the teacher constantly altered and effected the dynamics of the interactions; when I became involved in social flow, and a part of the process, the social authority dynamic was ever present. The role of teacher can never be quite that of equal (Biesta, 2015; Oyler, 1996b).

Equally there are some very real advantages in teacher research. Being an established teacher of the class, I could gain an insight into the flow experiences of learners that an outside researcher could not. The existing relationship and rapport I have hitherto developed with my students, allows access in additional ways and in greater depth and assisted with the research. It meant that classroom and behaviour issues did not interfere with a focus on the research questions
I had access to students’ feelings and thoughts, aspirations and motives in a manner that an outside researcher would not and could not comprehend and can understand learner’s behaviour with a perspective that an outside researcher would not have (Hammersley, 1993). The research placed an emphasis on meaning and description as a way of measurement (Laverty, 2003).
I had a greater knowledge of the environment, the school itself and was already assimilated. I had access to privileged information e.g., knowledge of published and unpublished school protocols such as behaviour for learning systems. My existing relationships with staff and other school members facilitated different levels of access, something that an outsider researcher would have had difficulty with. Furthermore, this situation was self-perpetuating. By engaging in teacher research, I increased my autonomy, and my kudos and reputation enhanced by the research aura enabled me further freedoms (Morrison and Lee, 2019).

Section 2: A description of the instruments

In this section I focus on a description of each of the instruments used and how they were utilised in the research. First, I discuss how I spoke to the students about flow, so they knew what to record, and then discuss video, self-assessment questionnaires, experience sampling method, stimulated-recall interviews, and journaling. I discuss my solution to the difficulties of empirical measurements of a phenomenon, connecting to RQ1; How is the notion of flow attested by mathematical learners.
In light of my review of the literature, I applied the principle that measuring flow was possible when all of the participants shared a common understanding of the fundamental characteristics that make up the flow state, i.e., an awareness what flow is, how it can be defined and the significance of flow to the mathematics journey within the classroom. This technique, of presenting participants with an explanation of the phenomenon prior to extracting, recording and analysing data about their experiences has been used in other research about flow (Pace, 2004, p. 336). The field journal entry below illustrates my curiosity with this:

Field journal entry 6th February 2014

Class today there were differing viewpoints as to what was skill and what was challenge.

This entry relates to a feasibility study in a parallel class where I probed and established the practicality of explaining and discussing flow with pupils. The real study commenced with an introductory lesson in which I presented the notion of flow to students, and some of the established aspects of flow theory such as the balance between challenge and skill, a loss of self-consciousness, time distortion, and clear goals and feedback. The field journal entry below was written after this opening lesson:

Field journal 4th May 2015

When asked what flow was, I used the example of athletics, asking if anyone did sport in any serious way. I said they may have experience being in a place where they were totally absorbed in the situation. I also gave example of musicians often experienced it, asking if anyone in the class plays music and that they may have been totally absorbed in playing, without knowing where time went. A learner described flow as when they were watching football on TV - this opened a discussion further, and we agreed that there must be an action to it, and it should have an element of benefit. A hypnotic state of watching cartoons would not be flow and something else. A discussion about meaning of positive benefit followed.

Rather than academic phrasing or the language of the research community the information was presented by me and by the students’ language, or as Wengraf (2001, p. 64) calls it ‘idiolect’, meaning a ‘close-to unique mode of talk’, avoiding confusion as to the meaning, definition and understanding of flow.

During the introductory lesson, students were asked what they would define as flow. Figure 3.3 below shows a slide that was produced in-situ with students’ words verbatim reproduced. The phrase ‘something with a method/instructions’ (in Figure 3.3) was discussed within the framework of needing a goal. The phrase ‘comes easy’ was also discussed in terms of skill and challenge. I also shared and discussed with students some of the benefits of flow, including the perceived rise in intrinsic motivation for those who experience it and that it is a contributory part of happiness.
Methodology - underpinnings and methods

Students were also given an information sheet as a part of the consent form that was sent to parents and carers. This had a summary of flow and an example is in Appendix B. Many of the notions examined in this introductory lesson were also revisited in later classes. The longitudinal nature of the study meant understandings and misconceptions around the definition of flow could be addressed and discussed and we could arrive at a shared vision.

Nevertheless, it was possible, that later self-assessment responses would be from a different, more complex perspective and so producing different results, as there was a deepening of a student’s understanding of flow. This was an accepted limitation of the research method. By introducing and sharing my intentions of research with the students, I established rapport. I share the view with Charmaz (2006, p. 19) that it is necessary to have rapport and a positive relationship with students, even if ‘we question their perspectives and practices’ in order to obtain an honest and reliable feedback and criticism.

**Video**

I videoed all classes using a two-camera system; one camera was in a fixed position and one was connected wirelessly to a microphone and followed the microphone automatically around the class. I would typically wear the microphone, although there were times when I experimented with leaving the microphone on desks, more as pointer for the video than to capture audio. I would occasionally give the microphone to a student so the video would concentrate on that student for
a time. By leaving the microphone with a specific learner I expected to pick up on students’ discussion whilst I was not present. I also experimented with other devices to gain audio; for example, a device placed on each table, but found that only approximately 20% of every lesson could be transcribed. This would be in main any whole class discussion, either myself talking to the group or sometimes students addressing the group about a mathematical issue. Due to the large class size and despite efforts to record small groups there was a need for pragmatism around data collection.

Nonetheless, I could use the video as a gambit for picture collection, assisting successfully with identifying and corroborating flow experiences (RQ1). There were images of learners in a flow state, discussing mathematics without the influence of the content aspects of my ‘teacher authority’ (Oyler, 1996a, p. 149). Video was a simple and effective way to record students’ flow experiences whilst in the mathematics classroom, had the advantage of being able to provide an uninterrupted view of flow and learners could be observed in a natural state. Other advantages of video included the capacity for moving images to better convey direction, attitude, and intention. The emotions and feelings associated with the moments in time could only be captured and analysed using video. Body language is also more readily accessible through video, including facial expressions.

Disadvantages of using video

The video camera occasionally had an adverse presence; and particularly at the start of the study the awareness of being recorded could make the situation unreal and learners would feel shy to be a part of the process because of the external eye. To offset this, the rules around ethics were made explicit to all the learners, i.e., the data would be anonymised wherever possible and any images restricted to a limited number of people. Another limitation of camera use, marked with teenagers of the 11-16 age bracket, was a demonstrating of inappropriate behaviour, for example ‘making faces’ at the camera, although this tapered off as the novelty subsided. When I noticed poor behaviour like this, I would take action; an example of the value of being their teacher and researcher.

Camera position

I adjusted the positions of both of the cameras in the recorded lessons; sometimes at the back of the classroom facing the front and sometimes at the front facing the back, seeking locations
where it could be the most unobtrusive, and also allowing viewpoints that may have not otherwise been seen. Each of the different positions had their own benefits and drawbacks, for example by placing the camera at the rear of the classroom and therefore watching the whiteboard it was possible to reach an understanding of what the student would see in the lesson and so have a better idea of what could be perhaps influencing their flow experience in terms of the central perspective. From the front looking in was more conspicuous but would afford a view of the students’ face and facial gestures. Figure 3.4 illustrates the different camera positions.

![Figure 3.4](image-url)

Figure 3.4 seating plans showing positions of the cameras. The cameras were placed in three different positions in the classroom over the 18-month study.

**Self-report questionnaires**

I used questionnaire responses as an internal lens of self-assessment to gather data. Here was an instrument encompassing the research questions that could capture first-hand the thoughts and experiences of the individuals. It could also take into account the logistical constraints regarding the transient nature of the students. There were 12 separate time data points from the questionnaire responses data collection during the study and up to 30 different students at any one time. The students filled out a total of 184 self-assessment records across the entire study.
Part A of the questionnaire consisted of nine self-reporting questions related closely to flow characteristics suggested by flow theory. Questions concerning the challenge-skill balance, action-awareness merging, transformation of time, unambiguous feedback, clear goals, concentration of the task at hand, sense of control, loss of self-consciousness, and an autotelic experience were chosen as an adaptation of the flow state scale FSS (Jackson and Marsh, 1996). The FSS made use of the experience sampling method as a methodological approach (as with this study), although with a far larger sample than I used.

I also considered the students language when formulating these questions, for example unambiguous (internal) feedback was seen as a process where thoughts occurred without outside help. Although the original FSS had 36 questions, I felt that asking more than nine would be taking time from the mathematical learning of the lesson. The questions that were asked are shown in more detail in Table 3.1 below.

<table>
<thead>
<tr>
<th>Question (part A)</th>
<th>Theoretical basis (flow theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I felt just the right amount of challenge</td>
<td>challenge- skill balance</td>
</tr>
<tr>
<td>My thoughts ran fluidly and smoothly</td>
<td>action-awareness merging</td>
</tr>
<tr>
<td>I didn’t notice time passing</td>
<td>transformation of time</td>
</tr>
<tr>
<td>The right thoughts occurred on their own</td>
<td>unambiguous feedback</td>
</tr>
<tr>
<td>I knew what I had to do each step of the way</td>
<td>clear goals</td>
</tr>
<tr>
<td>I had (no) difficulty concentrating</td>
<td>concentration of the task at hand</td>
</tr>
<tr>
<td>I felt I had everything under control</td>
<td>sense of control</td>
</tr>
<tr>
<td>I was completely lost in thought</td>
<td>loss of self-consciousness</td>
</tr>
<tr>
<td>I was totally absorbed in what I was doing</td>
<td>an autotelic experience</td>
</tr>
</tbody>
</table>

Table 3.1 theoretical basis for self-assessment Part A
This table questions related to the theoretical flow model, resonating with the flow state scale (Jackson and Marsh, 1996)
I adapted and utilised the questions from the flow state scale in order to identify if and how flow occurred (RQ1). However, the original study by Jackson and Marsh (1996) was not to identify if flow occurred, it assumed flow had occurred and how flow occurred was examined through the characteristics. I overcame this difference by clarifying the characteristics of flow to students. I explained to learners that by answering questions centered around the flow characteristics they were establishing if they were (or were not) experiencing flow. This understanding was confirmed with responses, for example written comments of ‘I was not in flow’ would be accompanied with low scores for characteristics, and vice versa.

Likert scales were used, following method of pre-existing flow studies that these questions were based on (see also Jackson and Marsh, 1996; Esteban-Millat et al., 2014). Known limitations of using a Likert scale included central tendency bias due to social desirability bias; particularly relevant to the age group studied (Csikszentmihalyi et al., 1997). I used a six-point scale in response.

A confirmation or manipulation question; ‘When I was in flow, I had difficulty concentrating’ was included (Fowler, 1995). The purpose of this question was a confirmation that students were registering and reading the questionnaire with due consideration. Later, during analysis, this identified three completed questionnaires that had been completed erroneously, without due consideration. The decision was made to disregard these in the numerical analysis, however, to include the written comments.

**Onset and continuation of flow**

In Part B a further 10 questions that centered around the entry to flow and the continuation of flow once it had begun were also included. These were based on the ‘flow questionnaire’ (Massimini et al., 1988). This study was longitudinal, as with this study, and had a sample of students in the larger sample questioned. It used the questionnaire as the sole data collection method. These questions are shown in Table 3.2 below.
Methodology - underpinnings and methods

<table>
<thead>
<tr>
<th>How does flow start/ continue?</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>The activity itself</td>
<td>Related to the start of the mathematical activity itself</td>
</tr>
<tr>
<td>Concentrating</td>
<td>Related to focusing of attention, limiting stimulus field</td>
</tr>
<tr>
<td>The mathematical challenges</td>
<td>Related to the perceived challenge</td>
</tr>
<tr>
<td>My inside motivation</td>
<td>Related to intrinsic motivation; an autotelic experience</td>
</tr>
<tr>
<td>My positive mood</td>
<td>Related to happiness, enjoyment and excitement of the lesson</td>
</tr>
<tr>
<td>The classroom atmosphere</td>
<td>Related to environment as a stimulus for flow</td>
</tr>
<tr>
<td>Talking with my friends</td>
<td>Related to social flow</td>
</tr>
<tr>
<td>Talking with my teacher</td>
<td>Related to social flow</td>
</tr>
<tr>
<td>My mathematical skills</td>
<td>Related to the perceived skill of the respondent</td>
</tr>
<tr>
<td>Ignoring distractions</td>
<td>Related to an ongoing awareness of concentrating</td>
</tr>
</tbody>
</table>

Table 3.2 The emergence and protraction of flow
This table shows the rationale behind the questions in Part B of the self-assessment.

Further detail of the journey of the questionnaire self-report is in Appendix C.

Difficulties of empirical measurements – using the questionnaire

There are fundamental difficulties when attempting to empirically measure a phenomenological experience. Flow in particular is complex, and the variables affecting it can be difficult to measure and create meaning, (e.g., Hektner et al., 2007; Bloch, 2000; Jackson and Marsh, 1996). Recollection, social pressure and desirability, along with the environment itself, such as current mood and personalities of students and classroom teacher along with the mathematical task itself all contribute to the intricacies of assessing students manifesting flow while learning mathematics.

Retrospective distortion or recency bias refers to the effect on a student’s recollection with reference to the time taken to measure a phenomenon. Multiple experiences occur over periods of time and will have influence over another. Various studies have shown that the recall of experiences (due to their nature) become clouded as individuals travel further in time from their start. (e.g., Beal and Weiss, 2003; Mazzoni and Memon, 2003; Stone et al., 1991; Csikszentmihalyi and Larson, 1987).
Social pressure and desirability are also encountered when measuring experiences. Social pressure is particularly relevant to age group studied (Csikszentmihályi et al., 1997). Self-reporting measures will be susceptible to these bias (Crust and Swann, 2013).

The longitudinal nature of the study provided some remedy; acknowledged by many as an effective approach to investigating an experience (see also Moneta and Csikszentmihályi, 1996; Stone et al., 1991; Wheeler and Reis, 1991). The repeated self-reports over lessons meant patterns could be captured and analysed. There were some inevitable issues with absences and class changes, but generally a core group of 30 students had their flow experiences in their mathematics lessons observed and examined.

**Experience Sampling Method**

In response to the issues around recall and recency bias, I also adopted a modified version of the Experience Sampling Method (ESM), when acquiring responses to the self-assessment questionnaires. This is a sampling method with a proven track record of measuring flow experiences (Hektner et al., 2007). It attempts to enable experiences to be sampled systematically, exposing ‘regularities in a stream of consciousness’. ESM (sometimes referred to as daily diary method or ecological momentary assessment) is an intensive data collection method that enables participants to report on numerous occasions over time, in a natural environment as the phenomena occurs.

ESM originally demonstrated a methodology in longitudinal empirical studies with a focus on flow carried out by the University of Chicago amongst adolescent school children (Csikszentmihalyi, 1977). The University of Chicago study looked at the quality of interactions and task activity. It involved several thousand students who would carry pagers which went off randomly during the day. When the pagers sounded the participant responded to a short questionnaire and enter brief notes about their preceding activities related to flow. Using it contributed towards the credibility of the research (Lincoln and Guba, 1985). For example, ESM has already previously been used with documented success in several large empirical scale studies on the flow experience (Beal and Weiss, 2003). It is widely recognised as a transparent and reliable method of capturing accurate data around experiences, minimising retrospective bias and can be tailored to the advantages and purpose of the individual and specific research dependent on the issues raised and context of the study (Hektner et al., 2007). In particular, it goes some way to prevent retrospective bias by sampling moments of flow close to their happening.
During the feasibility studies I attempted using a signal contingent sampling method, as suggested in the original method, sampling three times per lesson. However, I changed this aspect in the real study for several reasons:

- Each pause to fill in a questionnaire meant significant time was taken up with the measurement of flow, to the extent that there was almost a greater time taken with measurement than students learning mathematics. In the role of classroom teacher, I took an ethical stance that their learning time took precedent over research time.
- The comparatively short class length (50 minutes) meant that three pauses could occur at a maximum, which would not have a significant advantage over just one. As a comparative example, a high school study on flow led by Mihály Csikszentmihályi was over seven days and consisted of 7800 entries (Csikszentmihályi et al., 1997).
- The nature of teaching within a large group of students led to multiple tasks and mathematical events occurring during 20 minutes of a class, and different students were commenting on different parts of their flow experience in their responses.

Instead, I used an event contingent sampling method still consistent with ESM (Hektner et al., 2007). I sampled only once at the end of the lesson in order to capture a student flow experience. A beeper would randomly occur in the last 10 minutes of the lesson to signify the time to complete. Nonetheless, using ESM in this way meant several different tasks could take place during the lesson and the self-assessment responses only issued once were looking at the lesson as a whole. This was a known and accepted limitation; and one of the reasons videos were sought as an additional form of data collection.

**Stimulated-recall interviews**

Although during the planning stage stimulated recall was intended to be used more, unfortunately events transpired (a set of data was misplaced) and circumstances dictated (students were not able to be released often). Thus, only a single stimulated recall interview is included as evidence and the context and description of the actual interview is in Appendix A.
I used stimulated recall as a qualitative instrument to provide both measurement of flow and a way of probing deeper into what engendered flow and students experiences around it. To ‘stimulate’ the recall of learners, videos of themselves in flow were shown, and questions were then discussed. The subsequent audio transcript was then examined and analysed. Adaptations were made to future lessons in response to students’ comments, flow markers were obtained from the transcript and other findings are utilised and presented as evidence.

A random sample of students was chosen, the only criteria included that they were available on the day of the interviews. I used a prepared set of questions, although was prepared for a flexibility of approach depending on student answers. The initial questions focused on methods students used to get into flow, the effect of the environment and the influence of the teacher. In the interview the relationship and influence that the mathematical task had on the students flow experience was examined. Differences between social and individual flow were also investigated.

Advantages of using this as a data collection method included students were not pigeonholed into the existing categories of the self-reports may have suggested and it required students to describe what was salient to them regarding the research questions (Patton, 1988).

**Journaling**

**Field journal**

My own reflexive journal is present throughout; included is the journaling of my teaching, my research and developmental plans and also my observations of students and the complexities of the classroom presented. The field diary or journal was used as a tool to reinforce and not undermine my position as an insider (Burgess, 1989). It was done through electronic and written means.

When writing the field journal, consistent with the notion of bracketing it was important that the observations had as little bias and speculation to them as possible. I attempted (although not always successfully) to write any entries as descriptions, with as much objectivity as possible. The entries in the journals were completed within the shortest time frame possible, the influence of time being negative on the recollection of experiences (recency bias). Often, notes were taken in the lesson itself, and then entries were completed in full after the lesson, always within the 24 hours period.
There was also a separate mathematics flow journal in which I would write about my own experiences when carrying out mathematics at my own level. I examined, recorded and analysed my own experiences of flow when doing mathematics tasks. When journaling about my own mathematics, I would write notes in the margin, indicating where and when in flow. This contributed to ideas and conjectures in an indirect way, as it did not provide direct evidence related to the research questions, instead intending to add another perspective. I would also test, mirror and reproduce techniques on myself that were used with students and vice versa. Examples of this journal are in the discussion chapters 6 and 9 and also in Appendix A.

**Student journals**

Early on in the research students informally journaled their flow experiences, writing short sentences about their flow experience in their exercise books. For example;

*I’m pretty sure I was in flow when I was making a generalised formula for quadrilaterals to have the same proportion. I notice I had made calculations very quickly when making a specialised example.*

*(Caleb, flow journal entry, April 14th, 2016)*

In late 2016, a more formal student flow journal was introduced, that became an important pedagogical tool (Moon, 2006). For a period of six weeks student journals were used consistently at the end of every lesson. They provided many of the written comments around flow.

**Section 3: Backdrop and context**

The following section discusses the backdrop to the research as a whole, relevant to the methodology. I provide a thick description of the research; describing my own context as a researcher and teacher. I also discuss the ‘norms of acceptability in my professional setting’ (Mason, 2002a, p. 43); including the social context, intentions, circumstances and history.
Behind the scenes

An awareness of my own political and educational values and beliefs is important when considering the methodological choices, and by attending to these my findings and conclusions would become less value laden (Greenbank, 2003). The risk is that by following and placing emphasis on this line of enquiry and thought, the researcher tends, perhaps somewhat ironically, to confirm a (positivist) viewpoint that qualitative research is subjective and value laden (Halpin and Troyna, 1994). Greenbank (2003, p. 795) suggests it would ‘diminish the status of more interpretivist research’. With this end in mind, I include a minimum of biographical details here.

My previous experiences

At the start of the main study, I had over 12 years employment within the secondary mathematics classroom, including the role of advanced skills teacher (AST), various leadership roles and had successfully completed several smaller scale research projects with different foci. My previous experience implied that I would be able to concentrate on the research (more so than say a newly qualified teacher) and that I would be able to relate literature to the focus more easily. Nevertheless, there were several times when the practicalities of school life and the ‘coalface of teaching’ became the priority and the research had to take second place. As an example, it was only possible to interview students once and an intended focus group was not able to be formed due to timetabling restrictions and my existing teaching load. I had to fit around the existing timetabling of the school. An outsider researcher, perhaps from an institution would have more control because teaching would not be a priority.

Nevertheless, my professional context afforded a certain amount of autonomy; for example, I was assigned to the researched class as their mathematics teacher for three consecutive years (not the department normal practice). I could examine critically and if necessary, I could deviate from the published curriculum. Although these additional freedoms afforded were in part due to my teaching experience and expertise, it was also due in part to my research role (Stenhouse, 1983).
The context of the core setting

All of the research (including the feasibility studies) took place in a natural classroom setting and was carried out while the researcher (myself) was employed as a mathematics teacher at three separate inner-city secondary schools. Two of the schools were used in the feasibility studies, and the third school, Meadows School (a pseudonym) was used for the real study. All three schools had similar demographics and intakes. The fieldwork was carried out between 2013-2016, the feasibility studies between 2013 and 2014 and the full-scale real study commenced in January 2015 and was completed in November 2016. The backdrop of a secondary inner-city school had both scope and limitations.

Feasibility studies

Pretesting examined and established the viability of the research questions (Teijlingen and Hundley, 2002). Figure 3.5 below shows a summary of the feasibility studies. Ambiguities in questionnaire design were resolved before the real study took place. Audio and video were also introduced before the real study, and several ‘practice runs’ of the logistics were carried out. One of the main issues identified in this time period was the difficulty of recording audio in a classroom where there were approximately 30 learners talking about mathematics. The first of the pilot studies involved a small key stage 4 class with many challenging students, using questionnaires and a researcher field journal as instruments. Individual students were also questioned, testing the notion of stimulated recall and then, after I changed employment a larger key stage 4 class was used as a testing ground for video and audio. A key stage 3 class was used in the final pilot study at Meadows School (where the real study took place). The aspects of practical measurement and of pedagogy that I used in my feasibility study were close to my normal practice as an Advanced Skills Teacher.
Evidence from the feasibility studies cannot be directly included as an ethics number had not been issued at the time of collection. Nonetheless this period of time contributed to the thinking behind the research questions and the development of the questionnaires.

The pilot studies demonstrated using several classes at a time across schools tended to widen the number of variables that could affect flow, and in response the real study, in the core setting, focused entirely on one class within one secondary school.

Real study

The real study was longitudinal (over 18 months) and started when the students were in year 7 (aged 11-12) and completed when they were in year 9 (aged 13-14). Over the 2 years, 40 different students were used altogether in the study. The actual number of students in each class would fluctuate ranging from 12 to 32. Short term student absences were an inherent attribute of
school, and there were changes in the class at the end of each academic year. Figure 3.6 below illustrates the details of the real study.

May 2015
Series 1
One double lesson with the selected class, who are year 7 key stage 3.
Students were introduced to concept of flow in a previous lesson in order for a common understanding of the concept.
Instruments used were video, questionnaire and field journal.

Sept - Oct 2015
Series 2
The class are now in year 8. Many unrecorded incidences of flow had taken place in lessons and the only evidence was my field journal and so a series of consecutive lessons were planned and carried out over a 6 week period. Instruments as before.

Feb 2016
Student interview dataset
A stimulated recall interview was planned. Instruments included audio and field journal.

July 2016
Series 3
A double lesson was planned and taught with two colleagues using the lesson study model. Instruments used were video, questionnaire, field journal. Student work was sampled. There was also a stimulated recall interview after the lesson.

Sept - Nov 2016
Journal dataset
The class are now year 9. Over 8 weeks I used a student 'flow' journal as the main instrument to record and measure students thoughts and feelings around flow and the mathematics in detail. I also used my field journal and student work.

Figure 3.6 timeline of the real study
This diagram shows a timeline of the real study in Meadows school.

Socio-cultural demographics

The students (n=40) had varying ethnicities and gender. 72% of the class were male, and 28% female. 60% classed themselves as BME and 40% as non-BME 40%. Pupil premium (a measurement of social mobility) was less than the school average (24% for the class vs 57% for the school). I have also met most (80%) of parents/carers at various parents’ events. 95% of parents/carers agreed to their children taking part in the study. Nevertheless, I decided to use socio-cultural demographics as descriptive notions and not as factors. The research question could become too broad with the inclusion of these as factors and take the study further from the mathematics focus.
Methodology - underpinnings and methods

Attainment

The class is a group of ‘high attaining’ students, and there are some particularly gifted mathematicians within the class. The class had been ‘set’ in terms of attainment although there was still a wide range of mathematical learning needs and typically individuals would do well in some tasks and not others.

Where possible, many of the tasks would be purposely open ended, and low-threshold high-ceiling, although I had some leeway there were restrictions were placed by the department scheme of work and this was not always possible. Students had been introduced to investigative work previously in their first year with me as their teacher and had some exposure to this type of ‘discovery’ teaching approach. In addition, I attempted where possible to match the needs of all students; tasks could be different for specific learners, with scaffolding and subsequent fading (Mason and Johnston-Wilder, 2004b).

Therefore, it was very possible that learners would be reporting back on different tasks as to their flow experiences, and this had implications for measuring the individual experiences of learners, with a particular effect on the second research question (what tasks cause flow to manifest). I was also cognisant of my possible preconceptions about learner abilities to do tasks, consistent with the phenomenological perspective of bracketing.

I taught the group for all of their time at secondary school and have a good rapport and relationship with the class. The class are used to answering probing questions and enquiries about the process and method. They are also used to my ‘ways of working’, where I attempt not to answer questions they ask, rather reframing their questions and entering into dialogue. The group are accustomed to talking about mathematics and mathematical processes.

The school is one of a minority of schools within London that has not become an academy and is still under local authority control. This had an effect on the ethos of the school, in particular the mathematics department commitment to social justice is evident in a prevailing tendency towards inclusion. The mathematics department is supportive and collegiate, and my research was fully supported by the department and senior management.

Ethical protocol

I carried out the research in compliance with the six (Open University) Human Research Ethics Committee (HREC) Principles for Research Involving Human Participants; compliance
with protocol; informed consent; openness and integrity; protection from harm; confidentiality; professional codes of practice and ethics.

Compliance with protocol

I received a favourable opinion and formal approval for the school-based research on 30th January 2015 by the HREC Review Panel of the Open University. A copy is in Appendix B.

Informed consent

Informed consent, in line with the HREC panel approval decision was gained from both the school senior leadership and students’ parents. 38 of the 40 students examined over the 18 months agreed to take part in the study, also giving explicit permission through parental/carer contact. Students were encouraged to discuss the study with their parents/carers, and to assist with this an information sheet was included with the forms sent home. I additionally gained consent from any adults that were included in the study. Appendix B includes sample copies of consent and information forms.

Openness and integrity

All video and audio recordings were overtly done, cameras were clearly on display. I discussed the research design with students. The two students who had refused approval were still given all of the resources the rest of the class were given and included with all of the flow activities (questionnaires and flow journals), however the data was not used and destroyed at the time. I was also careful to not include either in any of the video and audio that is presented here.

Protection from harm

I continued to act as a teacher during all dealings with pupils and as such there was no risk of harm foreseen by conducting this research. I had every reason to consider from my reading of the literature that the notion of flow was advantageous to my students. I had enhanced DBS clearance and had completed safeguarding and PREVENT training. Student interviews always took place with others present. I complied with the legal frameworks that were stipulated in the HREC ethical protocol. All data protection requirements were followed. Recordings have been kept securely on a password protected machine.

Confidentiality

I understood that the school and therefore students could be identified by a determined investigator and this was kept in mind when decisions were made about what to publish.
Throughout this thesis all of the names have been changed, including students, colleagues and school names. Feasibility studies are discussed in general terms, and no evidence is included from them. Where necessary, dates are generalised, to further avoid identification of sources.

**Professional codes of practice and ethics**

The research also was in line with British Educational Research Association (BERA) code and guidelines (BERA, 2018).

**Ethical dimensions of being a teacher researcher**

The dual roles that I took on had implications on the reliability of the data collection. My role as a teacher had the potential to influence students’ self-reports and their willingness to describe flow accurately. As a teacher I was an authority, there was a power dynamic that would have always been present, irrespective to attempts at impartiality (Oyler, 1996b).

My descriptions of flow, and the language that I used may have influenced students completing the self-reports. Learners may also have deliberately not shared their honest thoughts, perhaps wanting to exaggerate their experiences; thinking it would garner favour, or conversely have negatively skewed their answers in response to process authority (Oyler, 1996a).

Nevertheless, a common understanding of flow was essential for meaningful results, and although my descriptions may have influenced pupil responses, my descriptions were, as far as possible, taken verbatim from the theoretical flow model. The role of teacher would always take precedent over any research role. I was bound, ethically, to put students learning mathematics first; for example, it would not be ethical to set a control group in which some students did not purposely experience flow. The time that was necessary to devote to research was often not available; this may have impacted the timeliness of subsequent analysis.

**Section 4: Operationalising**

In this section I discuss the types of data that were collected, and the methods and procedures used for analysis and collection of data and evaluate the reasons, impact and implications of these choices.
I attempted a reflexive and adaptive conceptualisation, and although the analysis was based on accepted qualitative methods, during the process it grew organically. This concurs with Mason (2002a, p. 86), who states ‘you (the researcher) must be… self-critical about your own ability to transcend the partiality of any perspective of a setting.’

**Types of data**

The different types of data collected and available for analysis (video, derived images, learner comments from questionnaires and journals, researcher narrative and evaluative journal entries, quantitative questionnaire results, images of student work, images of resources and tasks) are all individual and provide different perspectives each contributing to the overall discussion. They are all sources of data relevant to the same situation. Glaser and Strauss (1967, p. 65) substantiates this idea, and assures us; ‘no one kind of data on a category nor technique for data collection is necessarily appropriate’ (Glaser and Strauss, 1967, p. 65).

Each dataset played a role in supporting or contradicting any conjectures. Both visual and textual data was used to not only categorise, but to track various actions, inactions and interactions by the participants. The analysis assisted in building a new theory and the data gleaned from this evidence directed not only the direction and speed of the analysis but also to some extent the direction of the research.

The many different ways of collecting data was purposeful triangulation from the outset; I always intended to provide video evidence and self-assessments using experience samplings method (E.S.M.). There were also aspects of these methods evolving in time as I created new opportunities for flow; for example, audio was dropped as a main measure of flow after it became evident that the noise level in the class prohibited accurate transcription, and instead the self-assessments had more space for formative comments added (see Appendix C), and the student flow journals were increased in frequency after the rich student comments written during series 1.

**Mixed Methods**

Quantitative and qualitative methods have evolved from different paradigms that may be seen in some quarters are seen as contradictory (Egbert and Sanden, 2014; University, 2003). However, they can be used together – with care – to enable substance and transparency (Maxcy, 2002) and trustworthiness (Forero et al., 2018).
The majority of the data collected was qualitative. The unstructured data such as the written comments in the questionnaires and various journals, the images from students work, and the videos and audio transcription were all qualitative data and treated as such in the analysis. Unstructured data is denoted here as data not coded at the point of collection (Hammersley et al., 2001). Qualitative methods should be transparent and trustworthy, with particular reference to flexibility and context, a position that is substantiated by Mason (2002a, p. 17) who says; ‘… [qualitative methods] should not attempt to position itself beyond judgement and should provide the audience with material upon which they can judge.’ Qualitative methods can be utilised within an interpretivist approach, as less emphasis on causality can be associated with quantitative methods. Interpretivism is also associated with ‘human sciences’ (employing qualitative methods) as opposed to physical sciences (using quantitative methods). Teacher research also lends itself towards qualitative methods. Additionally, my inside role as class teacher gestured towards a qualitative method of collection. The questionnaire responses also included a series of discrete counts using Likert scales from two sections of questions based on the flow short scale, a self-reporting instrument designed to measure flow (see also Jackson and Eklund, 2002). These numerical responses were subsequently analysed using basic and straightforward quantitative methods, and thus the designation of mixed methods is appropriate. The use of quantitative methods of analysis is not intended to subtract, devalue or distract from the importance of qualitative data collection and analysis, rather is an attempt at bringing added value, rigour and a richness to the study by contributing to triangulation. Not in the sense of observational data that bears claim to the same knowledge, but rather that these accounts reflect the mathematics classroom as it is and may not necessarily all point to one ‘true statement about relevant features of the situation to which various accounts relate’ (Hammersley, 2008, p. 25). The purpose is to shine light on what is flow through the mixed methods chosen. I experienced some hesitancy in using quantitative analysis because of the typically small sample size (n=12-30), with minimal number data points (a =12) (e.g., Siegel, 1957). As an example, the original study led by Csikszentmihalyi (1977), with a focus on activities that engendered flow amongst high school students, had 753 self-reports.
Notwithstanding, I decided that fundamental numerical analysis such as averages expressed as percentages would add richness and transparency to the discussion; the sample size accepted as a limitation of sorts. Any numerical analysis would be accompanied with relevant contextual, qualitative analysis. In addition, by focusing my research on these particular students and not necessarily taking the class as a representation of the larger population, the sample size was not relevant. There was an awareness that each different instrument and method contributed differently, and that they may not point to the same conclusion nonetheless would still add to the study (Hammersley, 2008).

The choice of method was dictated in part by the research questions and purpose of the investigation, and not by methodological approach (Crotty, 1998). By ensuring the category of methods, whether quantitative, qualitative or mixed methods, were distinct from methodological approach the former became a less restrictive label and the research remained flexible.

Theoretical sampling

Throughout the research and particularly during analysis I looked for flow experiences happening; taking care to identify and separate flow experiences from non-flow experiences, based on my developing analytic framework, a notion consistent with aspects of theoretical sampling (Glaser and Strauss, 1967). Theoretical sampling is distinct in that the samples (in this case the flow experiences) are chosen not to embody a population, but rather to progress an emerging theory coherent with my research questions (see also Hammersley, 1981; Glaser and Strauss, 1967). During theoretical sampling the number of students involved in the study does not need to be preconceived, (Taylor, 1998) something that suited this study well because of the transient nature of classroom groups.

Data collection activities were informed by on-going analysis. I concentrated on my emerging theories of flow and the relationship with the mathematics classroom, trying to find environments and contexts where the phenomenon occurs, rather than attempting to replicate and generalise the ideas generated in other situations. For example, I did not select data that showed flow not occurring. Moreover, the focus was an enquiry into an experience and not an investigation of comparison i.e., not testing hypothesis with control groups. Theoretical sampling suggests how data may be sought but not how it may be analysed (Hammersley, 1981).
This understanding was consistent with teacher research, which rejects comparisons of random samples as difficult and often unethical in classroom educational contexts, partly due to the multitude of variables in schools, e.g., socio-cultural, personalities of both learners and teachers, ethos and so on (see Hopkins, 2002). The notion is also evocative of interpretivism which would rebuff isolating variables in the social world; the context of the classroom being paramount (Baumfield et al., 2012).

It would have been difficult to compare like for like with rigorous application and ethical integrity, (as a psycho-statistical investigation would imply). There could not be lessons that were deliberately geared towards students not making adequate progress. For example, providing a ‘control’ such as denying a learner a full experience within the classroom, i.e. deliberately trying to create a state of ‘anti-flow’ would be disadvantageous and could be perceived as unethical. In my role as the students’ mathematics teacher, I am attempting to address individual needs and progress, powerfully described by Lawrence Stenhouse (1979) cited in Hopkins (2002, p. 37); ‘The teacher is like a gardener who treats different plants differently, and not like a large-scale farmer who administers standardised treatments to as near as possible standardised plants.’

**Visual methods of analysis**

Within qualitative fields, text is frequently both the form of data and the suggested method upon which to base analysis. This may be due to the ease of use, for example reality is generally more easily defined through statements of words. This notion resonates with Popper (2002, p. 88) who states ‘… if we wish to build up a science, we have first to collect [basic or] protocol sentences.’ [My parenthesis.] Text can be seen as a self-referential system, i.e., the context where the data is drawn from itself supplies the rationale (Bohnsack, 2008).

However, there is a recent profusion of video and image related activity within society; for example, the rapid rise of visual social media, and therefore it is easier to introduce and establish visual methods of data collection. Here lies an inconsistency of sorts. When video (with audio) is utilised, there will be a transcription of the examined event and the accompanying visual media will be discarded after this. Perhaps ironically, regardless of the technological advances over the last years in images and video, there is an insubstantially small amount of research using qualitative visual data as a collection method with subsequent analysis (Bohnsack, 2008). Visual data (here meaning pictures and video) are not conventionally used within research projects. Methods of
qualitative visual analysis, specifically images, are yet to be established in academic and scholarly writings (Mey and Dietrich, 2016). Bohnsack (2008, p. 2) goes further suggesting; ‘the growing sophistication and systematisation of qualitative methods has been accompanied by the marginalisation of the picture’.

Analysis of visual aspect of the dataset was an important aspect. A considerable amount of visual data was generated over the study (13.5 hours of video over 10 lessons), and a much smaller amount of audio (<20%) was derived from this that could be successfully transcribed, due to practical considerations of the classroom (e.g., background noise and other interferences). Images are interpreted very much by the researcher and a picture either still or moving needs to be explained by someone who is possibly not a feature of the picture. Visual methods of analysis imply that the data is visual and also that the way I analyse it is visual as well.

**Multi-slice imagining**

I used an adaptation of Multi-slice imagining (Konecki, 2011) to examine the video dataset, a method of qualitative visual analysis that was a best-fit in the light of the research questions and also taking into account the other instruments used. This was consistent with aspects of the broader perspective such as the theoretical framework.

As an example, phenomenology typically involves a predominantly descriptive framework. The implied slowing down; noticing, amplifying and attending to learners’ experiences and thus generating thick descriptions of the video lessons would be consistent with bracketing (see also Finlay, 2013). Multi-slice imagining was first proposed by Krzysztof Konecki (2011). It is a newly established form of visual analysis, specifically focusing on images (Mey and Dietrich, 2016). Multi-slice imagining is a form of visual grounded theory method that is abductive. The term abductive is used in line with the current literature and refers to existing research having a bearing in the researcher’s mind and so an influence on the research and subsequent analysis (Konecki, 2011). Grounded theory methodology has differing interpretations. Originally it was seen as an inductive methodology (see also Glaser and Strauss, 1967) although more recently it is also interpreted as an abductive technique (Strauss, 1987). I have taken an abductive approach in line with the theoretical framework and paradigm modifying the emerging theory using theoretical sampling and multi-slice imagining.
The imagining process

Images are not texts, and for them to become a self-referential system, it is necessary to attempt to describe an understanding through pictures (and moving images) rather than using text to describe an understanding about pictures and images (Bohnsack, 2008). Accordingly, in addition to the narrative of the data itself there is the separate consideration of the background of the creation and production of data. The context of creation, how it was created is a part of the inner section of the image, and the narrative of the data, how it will be perceived is the outer frame of reference (Konecki, 2011, p. 140).

This is contrasted to the framework ‘accounting of’, the ‘what happened’, and ‘accounting for’; meaning possible perceptions of why the situation happened (Mason, 2002b). During the feasibility studies I initially looked through this lens; nonetheless this framework does not provide any specific frame of reference for images, distinct from text. Multi-slice imagining adds this visual dimension, alongside the narrative and evaluation of text.

Slices of data are taken; a slice being a rich, broad and thick description of aspects of the excerpt to include the images alongside my own recall, journals, any relevant self-report questionnaire. I selected still images from the video to tell the unfolding story, my selection of images dictated in part by the events of the lesson and in part by when I perceived flow as occurring. The use of still images to illustrate a video clip implied there was more to be left out than to be included, and text descriptions are also included.

I could now look through the visual data as well as about the visual data. The self-referential notional property of text as described earlier is transferred to video. I gained an imagining (note the distinction from imaging) of these different slices through a conceptual elaboration. I assume each image is multi-layered, as flow is a multi-layered phenomenon. The purpose to identify and detail the contextual and conceptual meaning of the image and thus create a greater understanding and depth of meaning.

Rubric

I used a rubric to provide a reliable, transparent and evaluative framework for each excerpt so that within each data slice (video excerpt) was included:

- The socio-cultural aspect of the images.
- An outline of the background to the creation of the lesson and what each of the images represent.
Methodology - underpinnings and methods

- What happened and the impact of what happened regarding the students.
- The connection between the images that were taken from the video, making allowances that they are taken from a series of moving images. I also contemplated how the images combine with others and change their form.
- A highlighting of themes threading through the findings (conjectures).
- The way the video was recorded and the manner and procedure that was used for creating the video, particularly regarding what played a part in the deciphering of the information.
- The gestures and body language of students. For example, how they were sitting, were they slouched in a chair, were they upright, did they have their head in their hands, and also their facial expressions; these were all pointers towards the flow state (flow markers).
- The relevance of the central perspective and what stood out.
- The ‘planimetric’, meaning a two-dimensional plane representation of the environment. These were seating plans with the addition of classroom furniture and the impact of seating on the data itself. Topographical properties were ignored. For each set of images, much of the context remains the same. The same seating plan for example was often used, and, unless there was a movement of tables and chairs the same environmental features.

Through utilising multislice imagining at least a rudimentary knowledge of teaching is necessary in order to appreciate the images in their entirety. There may well be several nuances of the classroom situation and aspects of teaching left from the description. Although it may be that non-teachers command a brand-new perspective, what I could call fresh-eyes; non-teachers may not have a total understanding of the situation.

A methodological contribution

I have extended, adapted and adjusted the technique proposed by Konecki (2011) to incorporate video, by perceiving it as a series of moving images and sampling (theoretically) the videos to produce excerpts. By applying multi-slice imagining to video there is a broadening of knowledge and thus a methodological contribution.
Situational analysis

Much of the data was collected in-situ; see Table 3.3 below. I used situational analysis as an accepted method of interpretation of the datasets (Clarke, 2005). Situational analysis at its simplest it is a way of being able to look at qualitative data gathered in situ; to communicate perspectives that may not be answered elsewhere in the prose, and to draw positional relations and surveys of all the players (Clarke, 2005). Situational analysis is steeped in describing perspective and ties in with the phenomenological and interpretivist stance. The situation is central, describing the human and non-human components first, then looking at the interdependencies and connections of these factors and the technique enables analysis of relationships between the data and datasets.

<table>
<thead>
<tr>
<th>Form of data (instrument utilised)</th>
<th>How it was collected</th>
<th>Internal Lens</th>
<th>External Lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-assessment questionnaire</td>
<td>Post hoc</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Video</td>
<td>In-situ</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Audio</td>
<td>In-situ</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Flow Journal (learner)</td>
<td>In-situ and post hoc</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Field journal (researcher)</td>
<td>In-situ and post hoc</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Learners’ work</td>
<td>In-situ</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Lesson resources (e.g., slides, task worksheets, question papers etc)</td>
<td>In-situ</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3 internal and external lenses and how data was collected

The descriptions generated allow me to research more deeply, galvanising thinking and awareness. Because of this, concurrent interpretations and perspectives are possible. The method can be used with coded data (although does not necessarily need to).
Methodology - underpinnings and methods

There is an argument that the researcher is an instrument (Cassell, 2005; Rubin and Rubin, 2011) and since the interpretivist framework does not have provision for tabula rasa; situational analysis was a way of getting my presumptions and ‘baggage’ out in the open, a notion also resonating with (Clarke, 2005) who asks (regarding situational analysis); ‘What does it mean to recognise the limits of exactitude and certainty, but still have respect for the empirical world and its relation to how we formulate and assess theory?’. Situational analysis enabled an articulation of the bracketing and the connection between the elements involved.

Internal and external lenses

I felt it necessary to collect data that helped me understand how the experience of flow manifested to an observer – the external lens – and to a participant in flow – the internal lens. Nonetheless, to have and report the internal lens the participant needs to observe themselves in flow and this is impossible.

There are few phenomena that cannot be observed whilst someone is immersed in them. Sleep (non-lucid) is one instance, and the flow experience is another. These events no longer exist as soon as they are observed by the individual experiencing them, and it is only possible to record self-observations of the phenomenon after the phenomenon ends. It is a part of the non-duality of flow, that awareness is meshed with the doing. These type of self-observations, remarks and responses I have labelled internal lenses, and are best captured soon (if not immediately) after the flow experience (Beal and Weiss, 2003; Csikszentmihalyi and Larson, 1987). Although there will have to be a retrospective element to the witnessing because of the nature of the phenomenon, this notion has resonance with ‘intraspective observations’ (Mason, 2002b), who describes an inner observer perceiving the self, engaged in the mathematics.

Another category proposed is that of external lenses; an observation made by another person, observing the individual who is in flow during the mathematics lesson. This notion highlights perspectives of an outside view and the distinctiveness between subject and object (Mason, 2002b). To assist with these external considerations, I utilised flow markers; an evolving set of physiological behavioural indicators to signify if a learner was in a state of flow. Flow markers are described in the following text.
What it means to know flow

Next, I introduce the notion of ‘flow markers’, empirical indicators of students manifesting flow, and a contribution to knowledge from this research. I also introduce the concept of ‘flow score’ and ‘flow circles’.

Flow markers - pointing towards the flow state

As part of my early analysis, I developed physiological behavioural markers, specifically to help identify when flow manifested in the class, either social flow or solitary flow. To do this, I started from the flow characteristics and conditions, and used these to identify threads in the video evidence. These were tested and refined to become analytic categories that provided structure for examination of data.

The markers grew organically as I analysed the visual evidence, the text comments in the self-assessment questionnaires, student flow journals and also the transcript of the stimulated recall interview. This follows notions of teacher research.

A clear structure was utilised in order to produce the flow markers, which had some similarities to coding of qualitative data (see Glaser and Strauss, 1967; Charmaz, 2006). There were ‘passes’; I initially produced rough categories and an initial list of markers using flow characteristics and conditions, these were refined during and after each series. I eventually settled with a full list of flow markers, shown in chapter 9.

The sampling of the videos had an impact on these markers. Some miscellanea would be missed by not taking the detailed analysis of the entire 12 hours of video footage, this was an accepted limitation dictated by time resource. I mitigated this by selecting samples that illustrated flow; the initial samples were provisional in nature and became less interim as the markers developed. The samples were revised and checked contemporaneously after a preliminary triage. In other words, the process was iterative; by utilising each successive list when extracting incidences of demonstrated flow from the video entirety, I was able to produce finer categories when writing the thick description of the context and classroom activities using multi-slice imagining method of analysis. There were three iterations of markers in total.

For example, an initial marker was ‘concentration on the task’. After the analysis of the first lesson, this then became; ‘working very quietly, not talking of anything else, sometimes with head in hands.’ This links closely to the characteristics of; concentration on the task, merging of action
and awareness, non-duality and a loss of self. The development of flow markers was instrumental in heightening my understanding of the flow experiences within my classroom. The emerging perspectives and relationships generated in this activity assisted in the discussion and conjectures. Both Williams (2000), and Custodero (1998) evolved a limited series of affective indicators, similarly based on the theoretical flow model. Nonetheless, there is partial and incomplete research on this aspect (Jeanne, 2009), and thus a contribution to knowledge.

Flow score

In addition to the numerical analysis of student responses to the self-assessment questionnaires, expressed as percentages, I developed an experimental method of summarising a measure of flow. This summarised responses to the nine flow characteristics, finding an average of these characteristics. The attributes measured may contribute unequally (Jackson and Eklund, 2002) thus I used a geometric mean rather than an arithmetic mean. The geometric mean highlights the differences between the characteristics.

Flow circles

During my construction of each data slice, I noticed there were certain times in which there was an obvious mathematical collaboration between students. This may have been the entire class, or several students involved. To indicate this, in several of the images that are presented I circled those who were involved in this togetherness.

Summary

To summarise at this point, I collected data during three series of lesson observations, a stimulated recall interview and a subsequent series of student flow journals. The data took the form of video (13 hours), self-assessment questionnaires (184), student and researcher journals and an interview transcript. Theoretical sampling was used as a framework to progress the emerging theory. Multi-slice imagining drew together the visual data, considering it as moving images. Situational analysis supported the analysis and contributed in answering the research questions, yielding ideas of flow markers, flow circles and flow score. These methods connected with the paradigm and theoretical framework and are shown in below in Figure 3.7.
Next, I present the findings and a subsequent discussion in Part II Finding Flow.
PART TWO -- FINDING FLOW
CHAPTER 4: Introduction to Part Two

This short chapter is intended to give direction to the findings, analysis and discussion that follows. In this part, finding flow, I present what happened in the mathematics classroom in order to answer the research questions. I use the existing theoretical work from chapter 2 and the underpinning methodology in chapter 3 to guide. I present the findings chronologically, as illustrated in Figure 4.1 below. By doing this I hope to shine a light on the nature of flow and its manifestation amongst learners; what flow is and looks like within my mathematics classroom.

There is a further discussion about the conjectures and findings in chapters 6, 8 and 9; chapter 6 being an interlude, covering a discussion of the first series. Chapter 6 is presented separately to assist the reader in understanding the chronology of the study and illustrating the iterative component of teacher research.

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**Series 1 (Chapter 5 and 6)**

Series 1 is an account of a single lesson that took place in May 2015, followed by analysis that gave rise to development of the flow markers, empirical behavioural indicators using flow
theory. I had previously noticed during the feasibility studies that a silent teaching approach had a positive impact on student flow experiences. The focus for this series was therefore silent teaching and the subsequent impact on the relationship between flow and the mathematics. The analysis from series 1 is presented in chapter 6.

Series 2 (Chapter 7)

Series 2 is a succession of eleven lessons, seven with video evidence and self-assessment questionnaires and four with no video and student self-assessment responses only. The series took place over the Autumn half term in September and October of 2015. The sequence has a focus on social flow; and during the accompanying analysis I also progressed and evolved previous ideas from series 1 such as silent teaching and the notion of flow markers. Throughout the entire study I carried out analytical work as and after I taught in the classroom.

Series 3 (Chapter 7)

Series 3 is centred around one lesson that took place late in the summer term of 2016. I noticed in series 2 there appeared to be a significance to the relationship between flow and the breaks that occurred within lessons. This series focused on those breaks. I also continued to iterate previous notions. The analysis from the entire study including both series 2 and 3 is presented in chapter 8 and 9.

There were two other evidence sources of note, a stimulated recall dataset, which consisted of a single transcript interview taking in the first half of the Spring term in 2016. There was also a sequence of lessons that took place over a six-week period in the Autumn term of 2016 in which I used student flow journals as a sole data collection method. The settings for these are described in Appendix A. Student quotes from these datasets are utilised in the subsequent chapters where they provide evidence that enhances findings from the three main series, and I did not feel they merited their own chapter.

The memos

All three series in chapters 5 and 7 are presented predominantly as rich descriptions; utilising multi-slice imagining method and my desire is for the reader to follow an evaluative line of thought. The featured blue and red memos are an attempt to separate the narrative from the evaluative aspects of my analysis. Contemporaneous partial analysis written during the respective series are the memos in red boxes and analysis written later are the memos in blue boxes. The
ideas, themes and emerging conjectures in the memos lay the foundation for later discussion in chapters 6 and 8 and 9.

Over the 3 series there were 13 observed lessons and via the analysis I have created 15 slices; each excerpt is a slice and includes images where available. There are headings at the start of each slice corresponding to the framework described in chapter 3.
CHAPTER 5: Findings (1) – series 1

Lesson 1

Setting the scene and context

The intention was to silently teach the first part of the lesson (10-15 minutes). This was one of the first occasions this teaching approach was utilised with this group. Silence was a way of focusing students’ attention to the mathematics and my other (than oral) teaching actions (see Morris, 2011). I communicated verbally on the board and articulated through my gestures and body language; nevertheless, I did not communicate through sound. Below is a rubric that I utilised when teaching without talking.

1. I do not talk.
2. I write a question/problem/task on the board.
3. I hand a board pen to student. I still do not talk.
4. Student gets up, goes to the board and answers question, talking with the class.
5. I give feedback through happy smiley face if correct, or sad face if incorrect, still not talking.

The cycle repeats until a significant number of students have reached the end of their learning attention, are off task, or it naturally needs some expert input that cannot be given at the time silently.

The necessity to adapt to the students, task and context was essential. For example, if a correct solution is provided by students, I may give another example (using suitable progression such as ‘fading’ any scaffolding). If an incorrect solution is provided, I may choose another student or modify a task/problem perhaps adding scaffolding or changing the question.

The ‘rules’ were tacit and never discussed; I did not explain to students the very different and unique classroom atmosphere experienced. Rather they learnt through non-verbal cues and from each other. The only explicit rule was [the] teacher must not talk. As an example, I never answered the question from students; ‘why aren’t you talking.’
The journal entry below illustrates my curiosity about silence and teaching silently.

*Journal entry Wednesday 29 April 2015*

The experiments with the silent way have been working, at least in terms of the progress that the students have made. Is this a way to engender flow? It seems as if this is one of the things having a positive bearing on the students that also engenders flow. I think that a video lesson with a longish silent part to it would be good. I could then do some analysis and see what is actually happening. And I can also use the questionnaire and see what the students say about this.

**Task description**

The task was an open-ended question, centered around unknowns, shown in figure 4.2. This was a part of a series of lessons introducing students to algebra as unknowns through generalising. The questions were deliberately phrased as open. The open-ended nature with more than one solution purposely allowed students the freedom to lead this part of the task.

![Figure 5.1 slide showing task for series 1](image)

This was an open-ended task, with a theme of generalising.

I planned a lesson goal of students gaining an understanding of generalising (and specialising) (Mason et al., 2010; Mason and Johnston-Wilder, 2004b). I placed a constraint on myself to not to use the word algebra. This was deliberate as I wanted to utilise an approach that minimised any anxiety around the topic, which I considered could be generated by using the term algebra (see also Carey et al., 2016).

By not using the term algebra there was little effect on the proximal goal of the task; the intended effect was rather on the perception of the student (and to not adversely affect the flow experience). It also encouraged students to view algebra through conceptual understanding, rather than a set of
rules and procedures (e.g., Skemp, 1976), which was intended to have a positive effect on the challenge of the activity, engendering flow.

I did not share the intended goal with learners. Students were deliberately focused only on the immediate task on the interactive whiteboard (Figure 5.1) rather than a broader lesson ‘objective’. This dovetails with the flow characteristic of setting clear objectives; with clear objectives the activity gives a vehicle to the flow experience. There was little mathematical help from me, and the learners had to rely on one another and their ‘natural powers’ (e.g., Mason and Johnston-Wilder, 2004b).

The memos signposts below (memos) are intended to present the context of what happened and how (which constitutes my inner understanding about the images) but also a description of how the data is perceived (an outer understanding through the images). The red memos were written during the series and the blue memos were written after.

The grammar (how the images, text and memos connect)

The video and audio were approximately 10 minutes long and captured the ‘silent teaching approach’ part of the lesson. 14 images were taken from the video and are shown in the following section. The images were chosen when there is an event and/or occurrence perceived relevant to flow in some way, consistent with theoretical sampling. Images (Figures 5.3 and 5.9) were also chosen to show the story of what happened and to illuminate, facilitate and supplement discussion. The questionnaire referred to was completed by students at the end of this section of the lesson. Much in the images remains the same. As an example, the camera angle of the right-hand image was constant, the angle of the other perspective altered in some images as I moved around the room. The members of the class are also a constant that connects the images together. Several students take turns to publicly share their knowledge, and although the mathematical knowledge shared is different, the way they share their knowledge is similar.

The task is a constant on the interactive whiteboard, it does not change although there were distinct approaches to the task tried by students in different images. I feature in most of the images (13 out of the 14), and camera one (the left-hand image) followed the microphone that I wore. Unfortunately, this microphone did not pick up the sound very well.
The phenomenon of flow is also a connection between the images. Attention is drawn to the many learners who exhibit flow characteristics and conditions. This is done with student name labels and by listing the observed ‘flow markers’ below each image. Where markers appear in the text they are in italic. Social and individual flow through the activity of learning mathematics connects the students, and in the process of completing the multi-layered analysis I drew circles on several of the images to encapsulate and emphasise this (see chapter 3).

Visual content and style

The images are stills from video recordings. The video is recorded on two tablets set up on tripods at different angles. The microphone is around my neck, and video one automatically tracks it. The impact on the class is negligible as I had talked with them about the equipment in the previous lesson, describing what to expect. The aim was to be as little intrusive as possible. Many students were concentrating on the board and the task throughout this part of the class. There is a ‘serious’ face from myself in most of the images. No smiling. I had a hand in one pocket in image 1 to 5.

Memo 5.2

Relevance
Without talking non-verbal cues are more prominent in students’ visual sphere (e.g., Calero, 2005). Gestures and body language from myself both play a large part of teaching silently.
Planimetric

I purposely placed students in specific seats as in this lesson I intended students of similar attainment levels to sit together. A silent teaching approach meant an expectation for students to be working together with those who were in close vicinity, in pairs or small groups. I had hoped that if students were working others of similar perceived mathematical attainment then there would be less boredom and frustration, and it could encourage an ethos of equal participation (a social flow condition). Grouping, nevertheless, was not a part of the remit of the research and is an area that could be looked at in a further study of flow and the mathematics classroom.

Figure 5.2 seating plan 5th May 2015 (series 1)

Lesson narrative - excerpt Lesson 1

It is very close in time to the start of the lesson. I am leading the lesson at this juncture. I am silent. I am using my right hand to signal to the class that the lesson is to start. There are a number of learners focused on me (e.g., Nicky, Caitlin) and there are several talking amongst each
other (e.g., Lili, Paige, Liam). There are also some who are looking at their books (e.g., Kyle, Ike). At this point audio although not able to be transcribed indicates the majority of discussion from students is not about mathematics.

Memo 5.3

The ‘off task’ talking stops soon after I raise my hand. This may be in part due to the rapport that I have with the class. Collaboration and talking about mathematics have been encouraged in this group since I started teaching them 18 months previous, and a few minutes of non-mathematical talk at times like this may be the trade-off for the interaction and dialogue that are innate to social flow.

The lesson is now three minutes in. I have the pen in an outstretched hand offering it to students. I have not said anything to students at all.

All students are focused either on their work (e.g., Caitlin, Kyle), or the task on the whiteboard (e.g., Hassan, Paige, Fae, Ike, Nicky). The task is asking students to consider how many eggs there are in each basket (see Figure 5.1 above). The audio at this point confirms a ‘hush’ in the class.
Several students appear to be in individual flow at this point, focused on the whiteboard. The lack of noise (and discussion) also indicates individual flow.

Ike has his hands under his chin resting his head. Ike appears to be in flow. His self-report responses for the lesson as a whole showed a high score for all of the attributes of flow queried (Table 5.1).

<table>
<thead>
<tr>
<th></th>
<th>Challenge-skill balance</th>
<th>Clear goals</th>
<th>Transparent feedback</th>
<th>Merging of action-awareness</th>
<th>A loss of self-consciousness</th>
<th>Concentration on the task</th>
<th>Being in control</th>
<th>Temporal distortion</th>
<th>An autotelic experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ike</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.1 self-assessment responses for Ike
A ‘1’ corresponds to ‘very much’.

The term focus(ed) is used here meaning involvement. In particular it connected with the characteristic of concentration on the task, a part of flow theory (Esteban-Millat et al., 2014). I used focus as a point of departure in identifying flow markers. Flow markers are observational remarks that assisted in identifying flow and validating conjectures.
Ike has volunteered to answer the questions on the interactive whiteboard. He is looking at me. I am pointing toward the board. I do not say anything. The rest of the students are focused on Ike and the interaction between him and I (e.g., Caitlin, Paige, Liam, Hassan, Fae, Benji).

**Memo 5.6**

At this point there may be the start of interactive social flow. There is a 'group goal', i.e., answering the task on the board. There is the start of 'blending of egos'; No one person is dominating the situation and my silence is fostering this. Students have a clear direction that they are in control of their learning. I am there to 'move it forward' if the students do not. All of these are social flow characteristics (Morrison, 2018; Kotler and Wheal, 2017; Armstrong, 2008, p. 102).

Ike is now facing the class and, after giving a specific value to question 1 is asking the class for confirmation/ feedback referring to the question (how many eggs in the brown basket). The pen is in his right hand. He is not silent. Unfortunately, audio transcription is not possible at this point.
I have not talked. I am standing in the corner out of the class’s eyeline, observing only. Paige in the front row has her hand under her chin, resting her chin on her hand. She appears to be in flow, focusing and oblivious to any noise and distraction. Fae and Josh have their hand up. Caitlin is writing in her book.

Kyle is leaning back in his chair and is engaged in a dialogue with Ike. It is not possible to see Caleb in Figure 5.6 (above) but he is also engaged in the dialogue, as the transcript below elucidates.
Audio (4:10-4:34)
Kyle: Eight (called out)
Caleb: Thirteen (called out)
Paige: 1,2,3,4,5,6, 7
Kyle: Seven eggs
(Student not identified): Seven
(Student not identified): Oooh

Memo 5.9
The 'oooh' was a student suddenly understanding an aspect of the mathematics (see Mason et al., 2010). This could be seen as evidence of progress of mathematics, and flow was occurring at this point.

Memo 5.10
Interactive flow seems accessible when students are collaborating with each other.

There are several students with their hands up, and also students are calling out without their hands up. This is where some sort of leadership could be called for, to ensure all were able to contribute and for the flow to continue. It is usually the type of role that I would play as teacher. I did not, however, step in verbally at this point, and the social flow seemingly continued.

There is an interaction between Ike and various other members of the class. It appears students are in control of their learning; in particular they are deciding what happens next. The social flow appears interactive.

Figure 5.7 image Lesson 1 at (4:21)
Interactive flow markers; a heightened mood (from the class) and talking animatedly and excitedly about the mathematical work/task/problem/worksheet (ike).
Caitlin is now looking at the front. Kyle is looking towards Ike. Ike is asking me if he is correct. He has substituted in numbers, specialising. I initially (intentionally) ignore and then shrug my shoulders (there are multiple correct answers to the task). The point of the task was not for a student to ‘guess the right answer’ but rather to arrive at a process and understanding that there could be several answers and that a general solution with a representation of letters may work better.

Memo 5.11

Ignoring intentionally appears to engender flow, dependent nevertheless on context. The journal entry below illustrates this.

Journal entry 5th May 2015

I deliberately did not give an answer to many of the questions today. This is slowly becoming a way of teaching for me. It seems to allow students more independence. I am finding that the less answers I give, the more the students give. The gap that occurs sometimes when students ask a question is awkward sometimes. But if I leave a silence often it will be the student who will fill the gap with talking. When to intervene and when not to...

Memo 5.12

Many of the students are involved in a quite heated discussion around the task, and at this stage without any verbal input from me (either written or spoken). There is a blending of egos, a social flow characteristic analogous to a merging of action and awareness (see also Sawyer, 2007). It is very possible that it is the task complexity and type (open ended with multiple answers) having a bearing on the flow experienced by students. It would be less likely to be my involvement as the silence dictates that I am very much less involved in this class. Possibly with a task that is open ended with more than one correct solution the challenge is personalised and matched the varying skill(s) of the students more readily.

Figure 5.8 image Lesson 1 at (4:35)

Flow marker: learner is in control, no worry of failure. A heightened mood.
My hand is up. Students have begun talking to a level that means the communication has stopped and there is no longer close listening. I take this as a sign that I need to move it forward, and signal with my right hand up that the students should stop talking. I have not said anything. My intervention has the desired effect and there is quiet. The audio indicates a calm in the class once I provide direction.

I was able to do this in part due to the familiarity and rapport I had with the class. Caleb has his hand up to answer. Ike is pointing towards the ‘baskets’ on the task on the interactive whiteboard, describing why he has decided on the number 7 as a solution.

Ike goes back to his seat with direction from me. My hand is outstretched, rigid with the pen in it. I now hand the pen to Caleb. There is no audio at this point, all class are quiet.
Findings (1) – series 1

**Memo 5.14**

I choose Caleb deliberately as I believe he has some preconceived knowledge around the task and topic. Also, the previous ‘disintegration of interactive social flow’ could have been due to challenge vs skill being unbalanced and choosing Caleb to lead will provide suitable challenge input that can reignite the debate and discussion around generalising.

**Figure 5.10 image Lesson 1 at (5:04)**

Flow markers: learner is in control, hands under chin, concentration, heightened mood.

I remove myself to the back of classroom. The intention is to be less intrusive and allow the students control over their learning (a flow characteristic). By changing the central perspective so I was not included, the students are encouraged to take more self-direction.

I have circled Caleb and Paige as the video shows the involvement between them in terms of flow. Caleb is writing on the board, *absorbed in the situation* Paige has her *hand under her chin*. The circles denote the way that all are *absorbed in the situation*.

The circle in the right-hand image was drawn as these students appear to be in a form of social flow. They are *looking intently at the board* at this point (Figure 5.10 above) and are absorbed in what Caleb is writing on the board but are not obviously participating.

**Memo 5.15**

Increasing the challenge through independence for example providing less instruction for students by not talking, and now being less intrusive without body language necessitated a careful consideration of the challenge of the task, and constant ‘organic planning’ (see Young and Muller, 2010).
Caleb is at the front of the class. He is leading the class and is animatedly describing his conjecture. His response was ‘very much’ to the question centred around being absorbed in the situation. Caleb is attempting to use letters and generalise but runs into difficulties and discusses with class. (audio 5:25, below). The questionnaire shows Caleb felt the complexity of the task caused his flow experience. This also is confirmed in his written comment ‘The challenge got harder causing my concentrating to get stronger’.

Audio 5:25

Caleb: Wait how would I write this down. I will just write b. That would be like, we don't know the number of how many eggs there's going to be in the brown basket. So, write down b for brown, and say you want to know the number of r, right? So, say like r, is the number.

Caleb’s explanation is not entirely correct, and he appears to have made a mistake when relating the eggs in the red basket to the brown basket. I do not intervene. There were three students ‘calling out’ trying to assist Caleb including Fae, who has her hand up. Caleb scored lower (2/3) on the question centred around ‘right thoughts occurring on their own’, a possible explanation being his peers assisted with the ‘right thoughts’ occurring.
Memo 5.16

Despite a conceptual mistake when trying to relate the variables, or perhaps it was because of the mistake there was a group discussion, and many appeared to be interactive social flow. For example, Caleb was talking animatedly and excitedly about the task (flow marker). Others in the class were attempting to assist Caleb with the solution, pointing at the work or indicating it in some way (e.g., Fae) (flow marker); and the class were ‘flowing together’. The mathematics (providing the activity) was an integral factor in creating this flow, alongside the ‘silent teaching’. There was a blending of egos; a social flow characteristic, students were working collaboratively, and there was concentration from much of the class focused on the task (flow marker).

Memo 5.17

Although I am not talking, my hand movements and gestures, e.g., pointing at the board, and writing prompts are leading and teaching the class. Without my voice, I feel the students are able to voice their ideas in more safety. A flow characteristic is that the student has no worry of failure, and the absence of my voice contributes to less potential of a judgmental outcome.

At this juncture, I make the decision to move the experience on and write a prompt on the board. I give a numerical value of 5 to one of the baskets. I also point several times to each of the baskets. I do not talk and am silent. Caleb walked away from the board to sit down.

Some students are talking to their neighbours (it appears Hassan and Liam are talking about the task. Hassan is looking at the board whilst he is talking.)
Caleb goes back to write on the board. He corrects his understanding and correctly relates the ‘b’ and ‘r’. Several members of the class are calling out (e.g., Kyle, Josh, Fae, Paige) and there is a class discussion about the use of letters. I am not involved in the discussion and stand towards the rear of the class. There are also some members of the class who are not verbally participating, and some are talking with their neighbours (e.g., Liam and Jack).

Nevertheless, all appear to be concentrating (flow marker), and are engaged in the task even the learners who are silent (e.g., Caitlin). There is still confusion as to the third question, the total number of eggs and a part of the discussion is hinged on that.
A different student, Paige is at the board, explaining her ideas. Again, audio is unable to transcribe at this point. I have not yet talked. There is still an active engagement in the class.

The questionnaire confirms that for Paige talking about the mathematics kept flow going (scoring the highest 1/3) although interestingly she scored lower (2/3) for the question revolving around whether talking about mathematics causes the flow experience. Was it her involvement at the board that assisted her flow? Paige (along with over 50% the class) answered that the activity causes the flow experience.

This section of the class ends when the buzzer sounds and the questionnaire is given out (consistent with my adaptation of the experience sampling method outlined in chapter 3).

**Summary of chapter 5**

This chapter has provided a narrative of an excerpt of a lesson that took place in May 2015. The excerpt was chosen because it featured a silent teaching approach. It raises questions about how social flow manifests, how a silent teaching approach impacts a student flow experience, and how markers can be utilised to identify occurrences of flow. These questions are dealt with in chapters 6 and 9.
Chapter 6: Discussion and Interpretation of Series 1

In this chapter, I include an initial discussion on flow markers, silent teaching and other aspects relevant to the research questions. I extend the discussion included in the memos (blue and red boxes) and journal entries from the preceding chapter. The evidence referred to in this chapter (the self-assessment results, video images etc.) is from series 1 unless otherwise stated.

The notions described, are developed as the study continued, in line with the cyclical and iterative aspects of teacher research. Table 6.1 shows a summary of the added contribution thus far.

<table>
<thead>
<tr>
<th>Conjecture</th>
<th>Research Question</th>
<th>Relevant datasets</th>
<th>Memos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow markers - the manifestation of the flow experience was demonstrated by learners, displaying distinctive physiological behavioural features. Many of the learners were in flow in this lesson excerpt.</td>
<td>1</td>
<td>Video images; Self-assessment responses</td>
<td>5.4; 5.16;</td>
</tr>
<tr>
<td>Silent teaching approach - many characteristics of flow were encouraged and fostered by teaching silently.</td>
<td>1</td>
<td>Video images; Self-assessment responses</td>
<td>5.7; 5.9; 5.18;</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Video images; Self-assessment responses</td>
<td>5.1; 5.2; 5.11; 5.14; 5.15; 5.17; 5.18;</td>
</tr>
<tr>
<td>Task type- the type of mathematical task contributed towards the flow that occurred within the lesson</td>
<td>2</td>
<td>Video images; Self-assessment responses</td>
<td>5.12; 5.16; 5.19;</td>
</tr>
</tbody>
</table>

Table 6.1 outcomes from series 1

The table above shows the conjectures resulting from series 1, the relevant research question, associated datasets and a cross reference to the related memos; found in the previous chapter.

In the following text I consider each of the conjectures in more detail.

Flow markers

Conjecture: the manifestation of the flow experience in the classroom was demonstrated by learners displaying distinctive physiological behavioural features, which I have termed ‘flow markers.’
Flow markers are empirical indicators of flow; a means of gauging and determining RQ1. It is through these markers that flow is made more visible (Williams, 2002). They are external visual observations of the characteristics (distinct from the conditions) of both solitary and social flow. More context and detail can be found in chapter 3, section 4.

**Development**

The markers were initially developed using the flow theoretical framework. Later, I used the evidence, in particular the self-report questionnaires to add more substance and triangulate the markers. For example, in lesson 1 Ike appeared to be demonstrating individual flow near the start of the excerpt; the marker that I utilised was *hands under his chin resting his head*. His self-report responses indicated he answered ‘very much’ to experiencing all nine of the flow characteristics (Memo 5.4). The term ‘triangulate’, meaning adding to the findings rather than necessarily confirming (e.g., Hammersley, 2008) (see chapter 3).

An initial list was generated, relevant to the theoretical flow model (Table 6.2 below). This table shows the list of markers that were originally compiled before the start of series 1.

<table>
<thead>
<tr>
<th>Markers (1) (initial list)</th>
<th>Indicative examples of evidence</th>
<th>Relevant flow characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus or focused</td>
<td>Figures 5.4; 5.5; 5.6. Self-report responses. Table 5.3</td>
<td>Concentration on the task</td>
</tr>
<tr>
<td>Concentrate or concentration</td>
<td>Self-report responses. Table 5.3. Figure 5.12; 5.13</td>
<td>Concentration on the task</td>
</tr>
<tr>
<td>Hands under the chin, resting head</td>
<td>Figure 5.4; 5.6; 5.10</td>
<td>Merging of action and awareness</td>
</tr>
<tr>
<td>Lack of noise</td>
<td>Figure 5.4</td>
<td>Concentration on the task, Merging of action and awareness</td>
</tr>
<tr>
<td>Self-consciousness disappears.</td>
<td>Figure 5.11</td>
<td>A loss of self-consciousness</td>
</tr>
<tr>
<td>Sole awareness of the task.</td>
<td>Figure 5.10</td>
<td>Merging of action and awareness</td>
</tr>
<tr>
<td>Outside influences have no effect.</td>
<td>Figure 5.10; 5.4</td>
<td>Concentration on the task, Merging of action and awareness</td>
</tr>
<tr>
<td>Oblivious to any noise and distraction around them.</td>
<td>Figure 5.6; 5.10</td>
<td>A loss of self-consciousness</td>
</tr>
<tr>
<td>Learner is in control, no worry of failure.</td>
<td>Figure 5.10; 5.7; 5.10; 5.13</td>
<td>Being in control</td>
</tr>
<tr>
<td>Excitement is evident.</td>
<td>Figure 5.7; 5.11</td>
<td>A loss of self-consciousness</td>
</tr>
<tr>
<td>A heightened mood.</td>
<td>Figure 5.7; 5.8; 5.10</td>
<td>A loss of self-consciousness</td>
</tr>
</tbody>
</table>

*Table 6.2 initial list of flow markers (pre series 1)*

This illustrates the list of markers that was compiled at the start of series 1.

This list was used as an initial springboard to determine when flow occurred, and then updated from my observations during series 1. Also included are examples of where these markers can be
found in the lesson excerpt. The theoretical flow model was used to compile this list and the applicable characteristics are shown in the final column. Particular characteristics took prevalence because they were easier to identify through visual means; for example, merging of action and awareness, a loss of self-consciousness and concentration on the task.

The responses in the self-assessment from the observed lesson presented in the previous chapter are shown in Table 6.3 below. They corroborate the initial markers; students also recognised concentration and focus as key characteristics of flow manifestation. Additionally, this added transparency, showing a common understanding of flow within the classroom, in particular the concentration characteristic. I introduced a definition of flow to students in an introductory lesson (see chapter 3) within the previous two weeks to the observed lesson for a common understanding.

<table>
<thead>
<tr>
<th>Name</th>
<th>To get into flow I will:</th>
<th>To stay in flow, I will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ike</td>
<td>Get focused and stick on the flow</td>
<td>Ignore distractions and concentrate</td>
</tr>
<tr>
<td>Lili</td>
<td>Concentrating even if I didn’t enjoy it</td>
<td>Distracting people around me and concentrating</td>
</tr>
<tr>
<td>Paige</td>
<td>Make sure I am enjoying the task</td>
<td>Ignore distractions</td>
</tr>
<tr>
<td>Kit</td>
<td>Focusing in the problems</td>
<td>I was concentrating</td>
</tr>
<tr>
<td>Dru</td>
<td>To relax</td>
<td>To concentrate</td>
</tr>
<tr>
<td>Caleb</td>
<td>Concentration and thinking</td>
<td>Concentration on the work</td>
</tr>
<tr>
<td>Ollie</td>
<td>To concentrate on the math[s]</td>
<td>To concentrate on the math[s]</td>
</tr>
<tr>
<td>Caitlin</td>
<td>When I get interested</td>
<td>To keep focus</td>
</tr>
<tr>
<td>Kyle</td>
<td>Concentrate and talking about the maths</td>
<td>The challenge got harder causing my concentrating to get stronger</td>
</tr>
<tr>
<td>Reuben</td>
<td>To concentrate and think carefully about the topic</td>
<td>To not give up and keep trying to find a way around and solve the problem</td>
</tr>
<tr>
<td>Jack</td>
<td>Ignoring distractions</td>
<td>Keeping my focus consistent</td>
</tr>
<tr>
<td>Katya</td>
<td>Concentration</td>
<td></td>
</tr>
<tr>
<td>Omar</td>
<td>Not giving up</td>
<td>Concentrating</td>
</tr>
<tr>
<td>Nicky</td>
<td>To focus on what I was doing</td>
<td>To focus on what we are doing</td>
</tr>
<tr>
<td>Amber</td>
<td>To work hard</td>
<td></td>
</tr>
<tr>
<td>Sadie</td>
<td>I carried on concentrating</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3 written comments from self-assessment series 1. This table shows the written comments from the self-assessment issued in series 1. The words ‘concentrate’, ‘concentrating’ and ‘focus’ were raised a significant number of times and were included in the flow markers (underlined).

Updated markers

Conforming to the cyclical and iterative nature of teacher research, the initial list of markers was critiqued and updated using the findings of series 1. The updated list is shown below in Table 6.4 below.
Discussion and Interpretation of Series 1

<table>
<thead>
<tr>
<th>Updated Markers</th>
<th>Examples of Evidence</th>
<th>Relevant flow characteristic</th>
<th>Category of flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head bent down, not sitting upright instead moving their head closer to the work.</td>
<td>Figure 5.4; 5.9</td>
<td>Concentration on the task, Merging of action and awareness</td>
<td>Solitary flow</td>
</tr>
<tr>
<td>Working very quietly, not talking of anything else, sometimes with head in hands.</td>
<td>Memo 5.4; Figure 5.4; 5.8; 5.10; 5.11</td>
<td>Concentration on the task</td>
<td>Solitary flow</td>
</tr>
<tr>
<td>Looking at the work/task/problem/worksheet; either alone or with another</td>
<td>Figure 5.6; 5.11; 5.12</td>
<td>Concentration on the task, Merging of action and awareness</td>
<td>Solitary flow/ Social flow (interactive)</td>
</tr>
<tr>
<td>Pointing at the work or indicating at it in some way.</td>
<td>Figure 5.6; 5.10; 5.13; 5.14</td>
<td>Non-duality and a loss of self-consciousness</td>
<td>Social flow (interactive)</td>
</tr>
<tr>
<td>Talking animatedly and excitedly about the mathematical work/task/problem/ worksheet.</td>
<td>Memo 5.9; 5.12; 5.16; Figure 5.6; 5.11; 5.12</td>
<td>Merging of action and awareness</td>
<td>Social flow (interactive)</td>
</tr>
</tbody>
</table>

Table 6.4 updated flow markers (after series 1)
This table shows the revised markers after the initial videoed lesson. Included are indicative examples of where these markers can be found, and also the significance to the theoretical flow model.

The analysis process of the video images (multi-slice imagining adaptation) was utilised in extending the descriptive language of the markers. Additionally, markers of social flow were included in this iteration. Similarities were mapped, merged and integrated, moving away from the direct language of flow theory in line with the analysis methods situational analysis and multi-slice imagining. For example, ‘Working very quietly, not talking of anything else, sometimes with head in hands’, would have more transparency than concentrating or focused. This marker was added alongside ‘Head bent down, not sitting upright instead moving their head closer to the work’ as I wanted to include students who may have been thinking, and not necessarily demonstrating this through a reference to work. The interpretation was an external lens; it was my perception of the students flow experience.

Were students in flow?

Conjecture: in this excerpt of this lesson a majority of the learners were in flow. This is evidenced from the flow markers demonstrated in series 1 (see Table 6.4) and the self-assessment dataset presented below. The memos in the previous chapter also discuss and provide evidence of this conjecture (for example 5.7; 5.9; 5.18).

One benefit of the positive affect of flow is enjoyment. For example, a student called out an exclamation when there was a progress in the understanding of the mathematics, when discussing the values of the variables (see Memo 5.9). The self-reports also suggested an enjoyment of the
lesson; comments such as ‘the activity was joyful’ [Ollie]; and ‘it (was) good’ [Omar]. 80% of those questioned (n=20 for this lesson) experienced time distortion, a flow characteristic.

90% described themselves as ‘very much’ totally absorbed in what they were doing, which was the question centered around an autotelic experience. 63% did not have a difficulty concentrating and 65% had thoughts that ran fluidly and smoothly. Shurik commented; ‘I managed to stay in flow.’ Table 6.5 below shows more detail.

<table>
<thead>
<tr>
<th>Flow Characteristic</th>
<th>Very</th>
<th>Partly</th>
<th>Never</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>55%</td>
<td>45%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>(11)</td>
<td>(9)</td>
<td>(0)</td>
<td>(20)</td>
</tr>
<tr>
<td>Challenge-skill balance</td>
<td>45%</td>
<td>45%</td>
<td>10%</td>
<td>100%</td>
</tr>
<tr>
<td>Clear goals</td>
<td></td>
<td>(11)</td>
<td>(1)</td>
<td>(20)</td>
</tr>
<tr>
<td>Transparent feedback</td>
<td></td>
<td>(13)</td>
<td>(0)</td>
<td>(20)</td>
</tr>
<tr>
<td>Temporal distortion</td>
<td></td>
<td>(11)</td>
<td>(6)</td>
<td>(20)</td>
</tr>
<tr>
<td>A loss of self-consciousness</td>
<td>65%</td>
<td>35%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>Merging of action-awareness</td>
<td>55%</td>
<td>15%</td>
<td>30%</td>
<td>100%</td>
</tr>
<tr>
<td>Concentration on the task</td>
<td>63%</td>
<td>26%</td>
<td>11%</td>
<td>100%</td>
</tr>
<tr>
<td>Being in control</td>
<td>63%</td>
<td>37%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>I didn’t notice time passing</td>
<td>80%</td>
<td>20%</td>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>I was totally absorbed in what I was doing</td>
<td>90%</td>
<td>10%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 6.5 student responses to part A of the questionnaire
This table shows (in percentages) the responses learners gave to the questions centered around the flow characteristics and conditions. The actual numbers of students who answered are shown in brackets. The concentration question was a confirmation question, and the scores are reversed here (more detail in chapter 3).
Table 6.6 below shows an overall indicative ‘flow score’ for each student. To find the flow score the geometric mean was taken of each of the flow attributes measured above. The lower the score, the more likely a student was in flow. The perfect score would be a 1 (a response to all nine characteristics as 1; ‘very much’). The least possible score would be 3 (a response to all nine characteristics as 3; ‘never’). Non-returns were not included. These scores should be taken with the caveat that turning a phenomenon into a comparative number could be seen as reductionist and should therefore be taken as indicative at best (Csíkszentmihályi, 1992). See Chapter 3 for more detail on the calculations for this indicative and suggestive figure. All, apart from Liam were below the score of 2, which would also strengthen the conjecture that most students were in flow for this lesson.

<table>
<thead>
<tr>
<th>Name</th>
<th>Flow score</th>
<th>Name</th>
<th>Flow score</th>
<th>Name</th>
<th>Flow score</th>
<th>Name</th>
<th>Flow score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ike</td>
<td>1.00</td>
<td>Caleb</td>
<td>1.26</td>
<td>Caitlin</td>
<td>1.36</td>
<td>Omar</td>
<td>1.49</td>
</tr>
<tr>
<td>Lili</td>
<td>1.08</td>
<td>Ollie</td>
<td>1.32</td>
<td>Katya</td>
<td>1.42</td>
<td>Liam</td>
<td>2.39</td>
</tr>
<tr>
<td>Paige</td>
<td>1.08</td>
<td>Kyle</td>
<td>1.32</td>
<td>Shurik</td>
<td>1.42</td>
<td>Benji</td>
<td>n/a</td>
</tr>
<tr>
<td>Fae</td>
<td>1.17</td>
<td>Brandon</td>
<td>1.32</td>
<td>Kit</td>
<td>1.43</td>
<td>Nicky</td>
<td>n/a</td>
</tr>
<tr>
<td>Dru</td>
<td>1.17</td>
<td>Reuben</td>
<td>1.32</td>
<td>Teo</td>
<td>1.47</td>
<td>Amber</td>
<td>n/a</td>
</tr>
<tr>
<td>Josh</td>
<td>1.26</td>
<td>Jack</td>
<td>1.32</td>
<td>Hassan</td>
<td>1.47</td>
<td>Sadie</td>
<td>n/a</td>
</tr>
</tbody>
</table>

This table shows the ‘flow scores’ for each student. The flow scores were based each student answers to the nine flow characteristics; the lower the score the more ‘1’s a student marked. A ‘1’ corresponds to ‘very much’ for experiencing the flow characteristic.

The single student who scored 2.39, Liam, may have not experienced flow, at least not for a majority of this specific lesson. In particular he scored low in the question centered around a loss of self-consciousness. Unfortunately, this was not something that I had an opportunity to ask further, and analysis of this lesson took place several months later.

Ike, Lili and Paige had perfect or near perfect flow scores (1 and 1.08). Both Ike and Paige were involved centrally in the lesson, explaining their ideas and concepts to the rest of the class. They were standing at the front, I was out of the line of sight, and other students were contributing to the discussion that they were leading (e.g., Figure 5.13 in the previous chapter). A possible reason for these high flow scores could be the collaborative working with peers and the ‘holding of the pen’; nevertheless, this should be contextualised, i.e. I am not suggesting that students who go to the front of the class are the only ones who mark themselves as experiencing flow. Indeed, going to the front may not engender flow for all, for example, Caleb, who was also at the front peer teaching experienced had a higher flow score of 1.26. Additionally, Lili was not at the front and had a near perfect flow score.
Nearly all students who completed the self-report claimed to have experienced at least some of the flow characteristics ‘very much’ (a score less than 2), during the lesson. This could have been influenced by the focus given to flow in this lesson; this was the initial lesson in the study and students were looking out for flow (and so reported it). The dominant role I played as their teacher may have influenced their self-assessment responses. Nonetheless, from the student self-reports (above) and flow markers observed, it appeared that many students were experiencing flow, and enjoying the lesson and the mathematical activity.

Social flow

So far, I have considered individual flow characteristics, but these characteristics can be compared and developed to include social flow. To assist the reader, Table 6.7 below is a reproduction from chapter two summarising these elements.

<table>
<thead>
<tr>
<th>Individual flow elements</th>
<th>Social flow elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenge vs Skill</td>
<td>Familiarity</td>
</tr>
<tr>
<td>Goal Setting</td>
<td>Equal Participation</td>
</tr>
<tr>
<td>Transparent Feedback</td>
<td>Moving it forward (closest to challenge vs skill)</td>
</tr>
<tr>
<td>Merging of action and awareness</td>
<td>The group goal</td>
</tr>
<tr>
<td>A loss of self-consciousness</td>
<td>Communication</td>
</tr>
<tr>
<td>Concentration on the task</td>
<td>Blending of egos</td>
</tr>
<tr>
<td>Being in control</td>
<td>Close listening</td>
</tr>
<tr>
<td>Temporal distortion</td>
<td>Complete concentration</td>
</tr>
<tr>
<td>An autotelic experience</td>
<td>Being in control</td>
</tr>
<tr>
<td></td>
<td>Potential for failure</td>
</tr>
<tr>
<td></td>
<td>Both of these are elements of social flow but are</td>
</tr>
<tr>
<td></td>
<td>not named as such by (Sawyer, 2007).</td>
</tr>
</tbody>
</table>

*Table 6.7 elements of social flow and solitary flow (copy of Table 2.1)*

This table surmises and roughly equates the elements of individual and social flow. The table should be read left to right.

There are nine distinguishing elements to the (individual) theoretical flow model posited by Csíkszentmihályi (1975). I have grouped them analogous to social flow elements suggested by (Sawyer, 2007). Next, I discuss the silent teaching approach, that encouraged a manifestation of social flow characteristics.
Silent teaching approach

Conjecture: many characteristics of flow were encouraged and fostered by teaching silently, in particular social flow characteristics. Individual flow occurred in the lesson, nonetheless much of the flow that occurred was social flow. This was in main due to the emphasis on conversation about the mathematics and mathematical interaction amongst students, which was in turn due to the silent teaching approach. A focus of this series was silent teaching. I had previously noticed during the pilot studies student flow states appeared to manifest during a silent teaching approach and wanted to investigate further. Silent teaching is an approach in which a teacher does not use oral communication in their teaching, instead relying on body language, the mathematical activity and student participation. The thoughts provoked by the focus on a silent teaching approach helped elaborate ideas about social flow which are discussed below and additionally in more depth in chapter 9. Next, I attend to the characteristics of social flow that were manifested in the lesson and begin to explore why a silent teaching approach engendered social flow.

Being in control

The silent teaching approach offers choice to students, who were given more control over their learning, a characteristic of flow theory. For example, the removal of my voice meant students were often teaching and questioning their peers throughout this excerpt. For example, when Ike is at the board and turns to me to ask me a question I do not answer, and he refers instead to the class, who assist him, and my silence focuses attention away from me and onto the class (see Memo 5.12). Students challenged each other readily, for example when Caleb makes a mistake, and Fae calls out, challenging his conceptual understanding when relating the variables (Memo 5.16). Withdrawing my voice assists in the removal of judgment (by an ‘expert’ teacher). Continuing the vignette above, I do not intervene with a judgement when Caleb makes his mistake, moving out of students eyeline and allowing space. This also contributes to less worry of failure, another part of the theoretical model of flow (Moneta and Csíkszentmihályi, 1996).
I intervened less when students were stuck; engendering independence. This could have been because my silence permitted me a longer time to process my reactions; it provided a longer space to think. For example, when Caleb made a mistake in his solution, and became stuck I did not intervene initially, other than indicating silently that he was wrong (Memo 5.16). This led to other students discussing the mistake with him, and although I eventually had to clarify the question, by initially standing-off I focused attention on the properties of the task, rather than a focus on me explaining the task. It was this stand-off that encouraged an independence, which although not a direct characteristic of flow, could suggest more control over learning, and is an element of an autotelic personality (Csíkszentmihályi, 1990a).

Giving a student less instruction can enable more understanding, in part by influencing a student’s skill perception in relation to the challenge. This has a resonance with the notion of ‘didactic tension’; suggesting the more obvious instruction given around a concept, the less likely a student gains an understanding of the concept (Coles and Brown, 2016; Mason, 2002b; Jaworski, 1994). For example, in the self-assessment for this lesson excerpt (Lesson 1) 63% of students reported they felt they had everything under control ‘very much’, and 55% indicated they had the right challenge ‘very much’. (See Table 6.5 above).

**Challenge and close listening**

The approach of silent teaching implies a removal of an aspect of help and assistance students were used to. It ‘faded’ a previous consistent scaffold (oral communication) which encouraged a curiosity and challenge, elements of flow. Students embraced this, becoming engaged in the moment, and engaged with the mathematics task, consistent with close listening (another flow characteristic). For instance, I deliberately did not give an answer to Ike when he was at the dry wipe board, and he quickly moved on, instead consulting with other members of the class (Memo 5.11).

**Blending of egos**

A silent teaching approach encouraged student involvement and discussion. (e.g., memos 5.6; 5.10; 5.12). The empty space created by my lack of noise was filled by students talking and verbalising mathematics, allowing students to flow together and fostering social flow through a blending of egos (Memo 5.17). Being absorbed in a collective state as suggested is a requirement of social flow (Sawyer, 2007).
Complete concentration/ concentration on task

There were also less distractions for students by not using my voice, congruent with the flow characteristic, ‘concentration on the task’. By not speaking, it enabled a learner’s voice to be expressed and heard more fluently (Gattegno, 1971). A student’s headspace was not invaded with teachers’ voice. For example, at the start of the lesson as I did not speak to start the lesson, I did not get involved in any exchange with students orally, using hand gestures to bring the learners together and to initiate the mathematical activity (Memo 5.3). 90% of students reported all of the time they were totally absorbed in what they were doing when in flow and 55% of students reported they were completely lost in thought all of the time when in flow.

Moving it forward

If the situation became stuck, and interactive flow stagnated (see Memo 5.14); I could then step in and change the task direction, ‘moving it forward’; another social flow element that bears a close resemblance and connection to the condition of challenge juxtaposed with skill. Equally, at times, students offered input and leadership, moving it forward. For instance, after I had written a brief prompt on the board, Caleb returned, corrected his mistake and there is a class discussion about the use of letters. I move out of students eyeline and purposely do not have an input (see Figure 5.13). Silence on my part did not mean that I was deficient or absent in the responsibility of class teacher and the lack of noise was utilised as a way to encourage natural powers of students. Through the added independence that my silence provided learners were influenced and encouraged to relearn to use their ‘natural powers’ (Gattegno, 1971). Learners were given more independence, although I facilitated and led at times, particularly with regard to behaviour for learning and ‘process authority’ (Oyler, 1996b). An illustration occurred when I asked Caleb to take over whilst Ike was having difficulty and several in the class were off-task. It was necessary for me to be able to think fast and adapt to a fluid situation (see Memo 5.13). The previous relationships I had established with the students meant I had knowledge and understanding as to their perceived skill level and this knowledge assisted me when needing to move forward the learning. Choosing the right student to answer questions when necessary helped move the class on and engender flow. Continuing the previous vignette, I deliberately chose Caleb to take over from Ike, as I knew he had preconceived knowledge of the topic, and so could lead at that juncture.
Flow states were reached because students innate mathematical ‘powers’ were allowed to flourish through both the absence of my oral direction and an acknowledgement of my presence enabling a ‘moving forward’.

**Disadvantages**

It could be argued several shortcomings to a silent teaching approach. For example, although the approach was successful in engendering flow characteristics, in particular social flow it took more planning and preparation than an ordinary lesson. Additionally, without the medium of speech, it became necessary to contrive ways to assist learners’ natural powers. For example, the task was a main feature, replacing my talking, and the questions needed careful thought with regards to suitable progression, challenge and perceived skill level. In this lesson there were students who preferred the emphasis on their talk rather than mine. Evidence includes written comments from the self-assessment: [Lilli], ‘Because I got to work with my best friend it made it more enjoyable and comfortable and we managed to do the task extremely quickly’; [Paige] ‘It was fun working on the whiteboard and the maths made it easy. When I'm in flow which I experienced the right thoughts just came and I didn't know I was doing it. And because I got to do it with a best friend it made it more enjoyable.’ Nevertheless, in this excerpt there were some for whom a silent teaching approach did not engender flow. For example, the self-assessment responses of Caitlin suggested the atmosphere in the class had the greatest effect on the cause of her flow experience and talking about the mathematics did not help her flow experience (see Memo 5.7).

**Task type**

Conjecture: the type of mathematical task contributed towards the flow that occurred within the lesson. To assist the reader at this point, the slide from the observed lesson showing the task is once again shown below in Figure 6.1.
The mathematical task played a necessary role, flow requires a suitable activity with appropriate challenge in order to occur.

The task (Figure 6.1 above) was open ended and allowed some ‘discovery’ which assisted and encouraged discussion (Davidson and Kroll, 1991). It was the discussion that encouraged social flow. The task structure with multiple correct answers focusing on the concept of placeholders provided stimulant complexity; students had access to the mathematics, irrespective of their perceived skill and, once accessed had some flexibility in the challenge of the mathematics, being able, for example to generalise; catering for the pursuit of complexity suggested by flow theory (Moneta and Csikszentmihályi, 1996). This is illustrated in Figure 6.2 below.
The self-assessment responses suggested the task played a part in engendering flow. 70% of those questioned suggested that the activity itself was a contributory factor to flow. The salience of the activity was additionally made clear from learner commentary in the self-assessments; ‘the challenge got harder causing my concentrating to get stronger’ (Kyle) and ‘Every task we had we got more in flow’ (Reuben).

The high score attributed here may be an indicator of the usefulness of tasks of this type, particularly in engendering flow. By providing students with an open-ended solution, it provides challenge that in this case seemingly matched the skill level of many in the class, thus providing an opportunity for flow.

For example, the task was deliberately structured to have multiple answers, with an appropriate entry level for all students in the class. Some wanted to put numerical values to the eggs in the baskets, (Ike in Figure 5.7) some wanted to write letters to represent the eggs. Some moved from specialising to generalising, increasing the challenge to match their perceived skill, for example part way through the excerpt Caleb was at the dry wipe board, at the front of the class, sharing his mathematical ideas around variables (Figure 5.11).
Nonetheless, there could have been more clarity on what students saw as the activity. For example, was it the silent teaching and extraordinary student involvement students reported on, or was it the mathematics question focused on generalising and specialising? The self-assessment having only a single completion point in the lesson, because of time constraints, was a recognised limitation and factor in this ambiguity.

To summarise, the task although not exclusive to a silent teaching approach was (necessarily) chosen and structured in order to teach silently. The ‘low-ceiling high threshold’ structure effected the balance of challenge vs perceived skill aspect essential for a student to experience flow. The discussion that the task encouraged also engendered flow.

**Summary of chapter 6**

This chapter has brought together and discussed the main ideas from series 1. I considered and examined how students demonstrated flow in this lesson. In particular I developed the notion of flow markers and started to consider the notion of social flow. I also looked at possible pedagogical ideas that could influence the manifestation of flow experience, including silence as an approach to my teaching. The ideas considered in this chapter evolved over the next 16 months and are addressed and discussed further in the following chapters.
CHAPTER 7: Findings (2) – series 2 and 3

Introduction

In this chapter I present the video datasets from series 2 and 3 and a timeline is once again shown below Figure 7.1 (a duplication of Figure 4.1 from chapter 4).

Both series are presented in a similar manner to series 1. The appearance of Series 3 is slightly different as it presents more narrative than images; this was a result of the evolving method of Multi-slice imagining, in line with an iterative teacher research approach. Series 2 focuses on a number of lessons. Series 3 is a single lesson.

In line with the description of the previous series in chapter 4, the boxes are commentaries on the narrative, where possible these are dated if they were written at or close to the time of the recording. I introduce ideas such as students taking breaks and independent learning. Furthermore, I continue to examine the notions and ideas from prior findings for example flow markers, frequency and numbers of students experiencing flow, task design, silent teaching and social flow.
Section 1: Series 2

This section focuses on a sequence of eleven consecutive lessons (series 2), during the Autumn 1 half term in September and October 2015. There were four other lessons that did not have video due to technical reasons but did have student self-assessment responses. I have included, in sequential order, a short description of these lessons to provide context to the story. The grammar (how the images, text and memos connect) and the visual content and style are a constant for all of the excerpts in series 2 and therefore are described once at the start.

Setting the scene and context

The series of lessons was planned in response to a growing awareness that a large number of students experienced flow in lessons and these experiences were not recorded other than in my field journal. The observation in the field journal illustrates this.

Field journal entry 13th July 2015

*Murphy’s law seems to be happening! Nearly every time there is a really good example of flow, the video is not on - and I don’t get any good evidence. So, it will end up being shown only in a journal (like this). Must think about something so that this can be captured on video/audio.*

In the course of series 2, rather than in the previous practice in series 1 of picking individual prepared lessons, I captured a string of lessons over a set period (one or two every week stretched over the half term). I felt that this data collection approach would be more likely to observe those flow moments that were being missed (addressing RQ1) as well as the broader context of the interdependence of the pedagogical choices made and the flow experience (addressing RQ2). I was also very aware of the possibilities of flow transpiring as a common, shared experience. The field journal entry below shows this:

Field journal entry 7th July 2015

*Today for example I was in flow along with the students - I wonder is flow possible to share maybe, or at least maybe I was in flow because of the students and my interaction with them?*

Student self-assessment responses, shown below, from series 1 also illustrated that working together seems to engender flow:
Findings (2) – series 2 and 3

**Question: Describe flow**

(Leo): ‘Talk about the work with my friends’

**Question: Describe what went well in the lesson**

(Lilli): ‘Because I got to work with my best friend it made it more enjoyable and comfortable and we managed to do the task extremely quickly’

(Paige): ‘It was fun working on the whiteboard and the maths made it easy. When I'm in flow which I experienced the right thoughts just came and I didn't know I was doing it. And because I got to do it with a best friend it made it more enjoyable.’

As a response, I chose social flow within the classroom context as a theme for this series. Improving and experimenting to find new awareness and theories as the research unfolded was germane to the inductive logic utilised and an example of the iterative and cyclical nature of the teacher research.

A part of the rubric used for analysis was the sociocultural aspect of the images. Because this did not significantly change throughout the academic year, this aspect is presented once in this chapter. Generally, the class demographical balance is similar to that described in earlier in the methodology; individual lessons also had slightly different composition because of absences.

**Task description**

The medium-term plan for the half term for this group was to focus on number. I utilised a sequence of lessons that attempted to heighten students’ awareness and confidence with number and that I felt would progress their number skills. Alongside a clear focus on number, the topics of representing numbers in other number systems and order of operations was included. The lessons were adapted from Martin (2011).

I was working on arithmetic skills throughout the term with all of my classes; all the members of the department feeling that a majority of students were grasping concepts, and a majority of the mistakes were in the arithmetic.

**The grammar (how the images, text and memos connect)**

There were nine excerpts (slices) from the video datasets taken from seven lessons and these are presented chronologically. There were four additional lessons not videoed, nevertheless, contributed to the context and did have self-reports completed. A timeline summary of these is shown in Figure 7.2 below. The excerpts (slices) were chosen to show examples of flow manifesting in the mathematics classroom.
Findings (2) – series 2 and 3

Figure 7.2 summary of the lessons in series 2
This diagram shows the timeline of series 2, illustrating the lessons detailed following this introduction. There are 11 lessons in this dataset, seven of which were videoed and 10 had self-assessment questionnaires completed by students.
Findings (2) – series 2 and 3

The images displayed from the excerpts are sequential with reference to the timeline of the lesson. The time shown refers to the moment from when the excerpt started not the start of the lesson. Each image features a commentary (a memo) to illustrate different aspects of the impact of mathematics learning and teaching on students’ flow experience. As in the previous series, audio is only included if coherent. Where the audio has been transcribed it is also an important ‘slice’ of the imagining, as well as shining a light on the research questions it provides a context and a way in for the reader to access to the lesson. Any text contained in [square brackets] are my additions to the transcripts.

Visual content and style

Similar to series 1, the images are all stills from video recordings. A majority of the students who are discussed are labelled in the images, however it may be useful to refer to the relevant seating plan, at the start of each section. The self-assessment questionnaire results, both numerical and commentary are incorporated in the commentary. Additional results of the students’ self-assessment can be found in Appendix D.

Planimetric (Seating plan)

There were a few ongoing alterations in the seating of the students, often lesson to lesson in response to inappropriate behaviour, absences and sometimes attainment groupings. Each of the excerpts has a seating plan that is specific to that lesson. I was aware of the influence of the seating arrangements on discussion and collaborative working, but this was not a focus of my data collection although this is an area that could be expanded in any future work.
Lesson 2 (1)

Note: Audio transcription was not available for either of these extracts, and there were no self-assessment questionnaire responses to report for this particular lesson.

Setting the scene, task description and planimetric

The door is open. There are two guest teachers from another school observing, described here because they also contributed to the atmosphere of the class. This may have a positive influence on the student’s behaviour, there are often other teachers who may look in, and there is a current of air going through the room. I handed out the task (on a worksheet) at the door as learners entered. This served as entrance routine for students to become easily and calmly involved in the mathematics, with a minimum fuss as they were admitted to the classroom and started the lesson. Students would have an opportunity to become engaged immediately in the mathematics and enter a flow state.

The task is shown in Figure 7.3. It is a short numerical task around division, practising arithmetic skills.

I modified the task to take into account the varying levels of attainment within the class. The question complexity varied. All of the questions could be solved using knowledge of short division. Some of the answers were whole numbers and some had decimals. Reverse operations were also used. I modified questions by writing each question with minimal difference to the prior one so students attend to the common structure between the questions (see Watson and Mason, 2006; Mason and Johnston-Wilder, 2004a). My aim was to ensure as many students as possible had an appropriate challenge in comparison to their individual skill.

In this excerpt the images connect approximately eight minutes of video. The interactive whiteboard which has the task displayed on it is the central perspective, although it may also be possible that the worksheet with the task is many students’ central perspective, particularly if they are working individually.

The seating plan is shown in Figure 7.4.
Findings (2) – series 2 and 3

Lesson narrative - excerpt Lesson 2 (1)

Flow markers signifying solitary flow include head bent down (e.g., Kyle, Caleb, Teo), oblivious to noise and distraction, working very quietly not talking of anything else. (e.g., Kit, Layla, Ava).

The task (see Figure 7.3) is being completed individually, there is no talking about mathematics at this stage of the lesson. I had already previously established a routine of no talking during the first few minutes of this ‘entrance task’. Many students are engaged in the work with focus.
This routine provides a calm mathematical start to the lesson. Establishing routines using my ‘process authority’ role along with the existing rapport and relationship I have with the class, facilitates a period of silence. This provides an opportunity in which students experience solitary flow whilst carrying out mathematics, indicated by flow markers.

After a few minutes, students are starting to talk to one another about the task (e.g., Reuben and Ollie; Josh and Hassan). Some are still working silently (e.g., Layla).

Students may be talking because they require feedback and discuss with a peer. Appropriate feedback is a condition of flow suggested by flow theory.

This is an example of solitary flow occurring alongside interactive social flow in close proximity. (see Walker, 2010) Some students want to talk about the work, and some find more enjoyment from working silently.
I am in a conversation with Liam, Jack and Sadie. I am explaining dividing 7 into 1 using money using a question; my question is ‘what is left over when 7 people share a £1 coin (the coin cannot be broken up).

**Memo 7.4**

The explanation I used here in the dialogue gave students accessible feedback and was important in the manifestation of interactive flow. Just as task design has an influence on flow, what is communicated is important for social flow to manifest. The field journal entry below illustrates this.

*Field journal entry 8th September 2015*

Money seems to work well in explanations – when I was talking [with Liam and Jack] today there was a really nice exchange it felt like we were in flow – was it the explanation? It definitely helped.
I am now talking with Nicky. Paige is also interested in the discussion. Several students are still becoming accustomed to the presence of the video cameras.

**Memo 7.5**

Both students who took the break at this point (Fae and Amber) appeared to be in flow previous to this juncture. They also then went back into flow after. The break may not necessarily be of detriment to the learning and the flow experience. Nonetheless, in another context there could be behaviour issues and the breaks could have a structure agreed.

Two students break out of flow state (Amber, Fae) because the tablet moves on its stand, following the microphone as I move around the room. The field journal entry below comments further on this:

**Field journal entry 8 September 2015**

Noticing today’s lesson that students were making silly faces into the camera - is this something to worry about or maybe it should be expected - it’s the first lesson in a while that they have been videoed, hopefully they will get used to it. It definitely moved them out of flow, or they could have been out of flow already and this provided the ‘break’ activity?
Findings (2) – series 2 and 3

Memo 7.6

It is not obvious whether Reuben and Ollie are in social flow (interactive or co-active) or solitary flow. Both students appear to be concentrating on the work and are also conversing about the work at intervals. Ollie appears to be listening to the conversation in the following image (Fig 1.8) and possibly in co-active social flow. There seems to possibly be a flux between the categories.

Figure 7.9 image Lesson 2 (1) at (6:46)
Solitary flow markers include oblivious to noise and distraction around them (Liam, Sadie, Jack, Omar, Hassan, Katya).

The first task has ended. There is a purposeful break, and students hand out the books. Some students are still wanting to continue with the task.

Memo 7.7

An example of the complexity of structuring breaks. Some students wanted to continue with the task at hand. This was after 7 minutes, some had just got into the task, some had not. To be able to predict when a break works best for students, and structure the break appropriately could be difficult, given the intricacy of teaching different learners, with different wants and needs, at the same time.
Interactive social flow markers include pointing at the work or indicating at it in some way, talking about the work (Reuben and Ollie).

I have put the answers up on the electronic whiteboard. Learners are assessing each other’s work.

Peer assessment is a way of getting students to communicate about mathematics, and so engendering interactive social flow.

Solitary flow markers include student looking at their work (Reuben, Ollie), co-active looking at the speaker, listening and not contributing (e.g., Layla, Kit).
I go over the ‘dividing into 1’ problem talking to whole class.

**Memo 7.9**

An example of possible co-active social flow, several students looking at the board (Paige, Hassan, Kit, Layla) and I was talking at this point, not engaging in dialogue.

**Lesson 2 (2)**

**Setting the scene and task description**

This extract is taken from later in the September 8th lesson, after the midpoint of the lesson (from 62:00 onwards). The door is now closed, and the guest teachers have left. Students are working in groups, and the group sizes vary, from 2 to 4.

![Binary Monsters Image](image_url)

*Figure 7.12 first slide introducing binary*

Students are starting a unit on binary. The central perspective is the electronic whiteboard and the slide in Figure 7.12 above. Several colleagues and I have adapted and modified a series of slides, taken from Martin (2011, pp. 54-55). One of the ideas we used was to introduce a question and then another question to extend the meaning of the question, evocative of the theme ‘extending meaning’, which consists of each new stage of the topic including new ideas (Mason and Johnston-Wilder, 2004a).
The word binary is on the electronic whiteboard in the title (‘binary monsters’). I deliberately intended to leave an element of curiosity and possible challenge by not explaining or describing binary. Students are given numbers and using addition only have to make the biggest number (25) the smallest and then all of the numbers in between. The numbers have been chosen deliberately as they are related to binary in differing ways.

As the slides progress, the questions become more complex. The task is ‘investigative’.

Lesson narrative - excerpt Lesson 2 (2)

The central perspective is Slide 1 from the task (Figure 7.12 above). Students have been asked to complete the task with little or no instruction. A conversation is taking place between Teo and Kit, but without audio it is difficult to know if it is about the mathematics.

Hassan and I are in dialogue about his work. He is seeking feedback.

Memo 7.10

Unambiguous, transparent and immediate feedback, a necessary part of flow theory. At times this feedback is internal, i.e., a student checks by approaching the problem in a different way or looks over their working and for other times an external reassurance is sought, sometimes from peers and sometimes from me.
Investigative and open-ended tasks similar to the one presented in this lesson may lend themselves to social flow more easily because of the interaction required between learners. They encourage the social flow ‘worker bee’ mentality (Kotler, 2014).

A dialogue transpires with a student (Layla) and I about the work, she is asking for feedback. At this point it is not possible to transcribe audio. For some, the feedback loop (a suggested flow condition) is internal and for others, an external reassurance is sought, sometimes from peers and often from a teacher.

When in interactive flow, what is always common is the dialogue that is between the individuals (Kotler, 2014).
Memo 7.13

Students often seek validation to their tasks, and an answer is not always necessary. They can often find the answers themselves and merely need to voice their concerns outside of themselves, often solving questions from this voicing. The journal entry below is evidence of this.

Field journal entry 28th June 2020

Talking with Layla today in the lesson, she was asking if she could use the same number twice. I asked her why is she asking this? She paused and then suddenly said if she did (use the same number more than once) then she could make every number. I didn’t reply and she said I’m going to use them once. Sometimes students don’t seem to want an answer, they actually just want a sounding board.

I talk with the class as a whole, engaging in dialogue. I pick up on two of the common queries from students. The transcript below illustrates some of this dialogue further.

Transcript (image 8/9 2.4):

ME: Two questions that came out. Can you make up numbers? You can’t make up or join 123 these are definitely numbers 1, 2, 3 distinct individual numbers.
The phrase ‘make up’ (meaning concatenate) was verbatim from a student comment. Using the language of students was important not only to build rapport but also to create and ensure common meaning (Wengraf, 2001).

If the markers were taken on their own without any other of the datasets, then it could be that interactive flow was taking place – e.g., talking excitedly and animatedly about the task. However, the field journal entries indicate that there may have been less interactive social flow, as I was talking, and students were listening – indicating co-active flow as opposed to interactive flow.

When I am talking too much then it may sometimes be moving the students from their flow experiences.

The two field journal entries below illustrate this further:

**Field journal entry 6th September 2015**
I'm going to try next lesson to not talk too much. One of the comments Paige made to me in the last lesson was ‘when you talk, I get out of flow’.

**Field journal 8th September 2015**
When I was explaining the idea of binary it was definitely me explaining and students listening.

It felt like when it was just me talking to the class it was an explanation by me and I wasn’t sure if they were getting it. But when I was talking later with the class (i.e. they were actively contributing) it was as if the class were understanding.
Findings (2) – series 2 and 3

Markers to show interactive flow include looking at the task (on the electronic whiteboard) while talking (e.g., Shurik); animated, pointing at the work or indicating at it (e.g., Me); focused on the speaker but also engagement and participating in the conversation (Reuben, Shurik, Hassan, Me).

I am now talking with several students (Reuben, Shurik and Hassan) illustrated in Figure 7.17.

The transcript below shows a part of the conversation.

*Transcript (image 8/9 2:31)*

Shurik: ‘You can’t you repeat the numbers?’

Me: ‘How do you know this?’ [Meaning how do you know that you can’t repeat the numbers].

The audio drops at this point and does not catch his answer.
By not providing an immediate answer and allowing students to explain their thinking I am trying to keep their curiosity and the challenge of the task is kept appropriate to their level of skill.

I move the class on (a social flow characteristic) changing the central perspective (Figure 7.19 below) and talk to the class and give an instruction asking for students to peer assess (see transcript below).

Transcript (image 8/9 3:05)

Me ‘Can you check with the person next to you, who has got more, who has got less. Have you got the same amount? How do you know this?’
Memo 7.18

Utilising my teacher’s content authority (Oyler, 1996a) to ask a ‘probing or open question’, e.g., as in figure 7.18 and yet withholding any further mathematical information allowing the class to talk about it may be a possible way to engender flow.

Memo 7.19

I felt it necessary to intervene as a way of facilitating further social flow; that there needed to be an input into the mathematics. There were several learners who were stuck, and the same questions were being asked by a number of students. I am ‘moving it on’ (social flow theory). This may however be forcing a break in flow state for a number of students, and this is in the middle of a task that many are already collaborating on.

Lesson 3

Setting the scene, task description and planimetric

Figure 7.20 seating plan for Lesson 3

The seating plan is shown in Figure 7.20 above. The students and any background out of shot are not included in the plan. The images shown in this excerpt are from the front only (camera A), the second camera was not available due to a technical issue.
At the start of the previous lesson, learners completed an exercise on division, and I noticed there were misconceptions, particularly when the divisor is larger than the dividend. I wanted to try and explore these misconceptions further, and I started this section of the lesson by writing several questions on the whiteboard. These were:

\[ 16 \div 6 = \]
\[ 42 \div 6 = \]
\[ 44 \div 7 = \]
\[ 178 \div 6 = \]
\[ (\text{Binary}) \]
\[ 1010 \div 10 \]
\[ 100111 \div 11 \]

Lesson narrative - excerpt Lesson 3

I start a dialogue and mathematical conversation with the group. I ‘model’ an example with assistance from the class \((16 \div 6)\). I used the ‘bus stop’ method as the ‘model’.
I was hoping that students would be able to offer alternative methods of long division, which would then mean that they would not need to convert the binary examples into decimals in order to solve. Nevertheless, this did not happen in this part of the lesson, the challenge of the dividend larger than the divisor was enough for the student’s skill at this moment. I adjusted the challenge of the complexity of the task to match the perceived skill of the student, deciding to keep with a short division method. The challenge complexity vs the perceived skill of a student is integral to flow theory (Esteban-Millat et al., 2014; Moneta and Csikszentmihályi, 1996).

I invite a student (Josh) to go to the front and lead the explanation to the other students (see transcript). I stand at the back of the class. Students are engaged and in dialogue with Josh.

In the flow self-assessment questionnaire, both Caleb and Kyle scored ‘all the time’ on feeling the right amount of challenge (Josh did not respond). This would support the flow markers that they were in group interactive flow (right challenge being a characteristic of flow theory).
Memo 7.22

Although not silent teaching, I am deliberately silent at this point, enabling students to peer teach and encouraging independent learning.

Transcript (image 10/9 1.3)

(Josh) here we have a number, this number is provided here, these three numbers. Yeah. So, this divided by one I mean, one divided by that doesn't work.

(Kyle): Its 17 add zero

(Josh): Doesn't work okay so we use 7 to make 17 how many go into 17

(Kyle) 2 remainder 5

(Josh) and we will use that here on the eight. And now, how many goes into ...

(Caleb) 9 remainder 0, and then add 4 before 0

(Josh) And because 6 * 9 is 54, the remainder will be 4 and I will divide that into a decimal. And so, you put a zero here, and use that 4 and the decimal point here.

(Caleb) the decimal point behind 8

(Josh) How many 6's go into 40.

(Unknown) 5 oh no 6

(Josh) And then we have 4 left over, and then basically it will be... it will be that

(unknown) 6 recurring

(Josh) 6 recurring 7 okay.

Two students are having a break (Paige and Lili) and not participating. The field journal entry below has more detail.

Memo 7.23

Looking at the video (rather than the images) the two students (Lili and Paige) although they break for a few minutes, then go back into flow. Reasons for this may be that flow is not unlimited quantity; a notion supported by flow theory. The journal entry below substantiates this.

Field journal entry 10th September 2015

I noticed in the lesson today that when students had a break they then got back into flow. I wonder if breaks are needed. Flow isn't infinite ...
Flow markers include: (solitary) focused on the task, writing, head close to the task (Nicky); (interactive), discussing the work involved with energised focus (Kyle).

Josh is at the front of the class at the whiteboard. Fae, Caleb, Kyle and Josh are having a conversation about remainders and where to put them. I am standing at the back of the class, not contributing to discussion. The field journal below illustrates this further.

**Memo 7.24**

Although a silent teaching approach and enabling students to learn more independently appears to engender flow, a drawback to the approach is the amount of planning that it takes. Nonetheless after a while and some practice this type of teaching, becomes more intuitive. This is substantiated below in the journal entry.

*Field journal 10th September 2015*

Power of peer teaching, empowering students creates conditions for a flow state. Needs to be a certain amount of organic planning, this can’t all go in a lesson plan as it very much depends on the moment.
Josh gets to the end of his explanation of the question. Lili and Paige are now concentrating and involved in the lesson (after taking the break, see Memo 7.24). Lili cheers when Josh completes.

Memo 7.25

Lili cheering when Josh completes is possibly a sign of enjoyment, an intrinsic benefit to flow.

There seems to be solitary flow and group interactive flow being demonstrated (see also Walker, 2010). They are being shown by different members of the class and also by the same members but at different times.

I recognise and acknowledge a limitation of ‘flow markers’ in that they are not always definite and are can be open to misinterpretation. For example, Nicky could be in solitary flow previously and now in co-active, or he could have been taking a break. (head close to the work or head on the desk).

Josh steps away from the board and ‘gives the pen’ to Kyle, who now goes to the board and answers another question.

\[
44 \div 7 =
\]

He uses the bus stop method (see transcript below).
Findings (2) – series 2 and 3

Figure 7.25 image Lesson 3 at (6:27)
Flow markers include (co-active) looking at another but not obviously talking or engaging, resting head on hands, writing (e.g., Amber, Benji, Reuben).

Transcript (image 10/9 1.6)

(Kyle): Can’t do seven into 4 so you put the 4 there. Seven into 44 is 6, the remainder, we put it in front of zero, remainder two and 7 into 20 is 2 so and then ... yay.

Kyle seems to talk more to the class, giving his explanation, rather than involving the class.

Memo 7.26

Interactive flow occurs when all are involved in the conversation about the mathematics. If a student doesn’t involve others then it may become co-active flow at best, and possibly flow does not occur at all for the other students.

Figure 7.26 image Lesson 3 at (8:33)
I intervene and try to move the task on. I go to the board and explain the method that Kyle has used. (See transcript below).

*Transcript (image 10/9 1.7)*

(ME) I think there were a few who didn't get it... let's recap what you have done here.

You have said 7 into 4 goes zero, and 7 into 44 goes six. Now we can all get up to there. I've come around I've had a look at some of your work. We can all do the ones without the remainders. The clue here or the trick here is to put the zero here because 44 is the same as 44.0 and that's what Josh assumed everybody knew. And Kyle similarly assumed everybody knew. Okay, we must not assume that people will know this. So not everybody knew that if you didn't know that...

(ME) Yes, the zeros go off. So, I take that remainder of two.

[I ask them to write down in book].

Although some students are working (e.g., Benji, Kit) some appear disengaged (e.g., Fae). The flow marker of Teo could be that he was in solitary flow, however he wrote in the self-report; ‘I was not in flow today’, and so it could be that he was off-task, resting his head on the desk at this point.

**Memo 7.27**

Flow theory suggests that in order for flow to occur the challenge of the activity should match the perceived skill of the students; the challenge of the task itself was maybe too high for all of the students (dividing where the dividend was greater than the divisor and leaving the answer as a terminating decimal).

The disinterest could also have been the teaching approach; possibly too passive; talking at students, effecting the challenge.

The self-assessment questionnaire responses showed that 56% felt the right amount of challenge. 35% felt their thoughts ran fluidly and smoothly in this lesson, which was a lower score than in other lessons (typically 60%+). This could be the difficulty of the concept, equally how the concept was introduced. Paige responded in her questionnaire ‘I got out of flow because I started to not understand the remainder to decimal part which got me angry’ (see Figure 7.27 below).
Fae wrote a comment; ‘When I’m in flow and the teacher talks it distracts me’. Paige also added ‘I enjoy the lesson when there [is] silence’. Teo wrote that he was not in flow in this lesson but did not elaborate as to why. The journal entry below illustrates further.

**Memo 7.28**

This (distractions caused by me) had been brought to my attention previously. When I bring the class together, (usually with best intentions – to discuss a common question or to move the class on to another task), some students would break out of a flow state. Complete concentration / concentration on the task is a characteristic of flow theory.

By focusing on flow and how it manifests in student I improved my teaching; the journal entries enabled a reflectiveness on how I introduced and taught certain topics. The journal entry below supports this.

**Journal entry 10th Sept 2015**

*Before I changed tasks today, there were quite a few students who didn’t seem to be getting it. They were off task and I guess I did the right thing and moved them on. It was interesting because I really thought that division that moves into decimals would have been easier for them – is it a topic that is quite algorithmic? How can I teach it differently?*
Lesson 4, Lesson 5, Lesson 6

These three lessons did not have video due to technical problems, nevertheless I have included a brief description to show the story. The student self-assessment dataset was collected (results are discussed in the following chapter). All three lessons continued with the binary concept introduced previously.

Lesson 4 Setting the scene and task description

Students were required to create counting numbers from 1-31 using the method in Figure 7.28 below. Learners worked in groups in sizes two to four.

<table>
<thead>
<tr>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>In teams of 6 allocate numbers 16, 8, 4, 2, 1 on big pieces of card to 5 members of the team, sitting on chairs in a slight arc. The sixth member to be the conductor. Ask those who make a total of 19 to stand up. Ask students to form counting numbers 1 to 31 using the method above. Choose a team to present to the group. Show them that this can be presented in a table like this. The process is converting binary to decimal numbers.</td>
</tr>
</tbody>
</table>

Towards the end of the lesson students started a problem-solving task called ‘binary monsters’ (Figure 7.29 below).

binary monsters
Set up a grid with binary column headings and create monsters like this:

Notice how the code requires translation to base 10.

New swap codes with other students and recreate monsters.

What happens if we add one to all the code numbers? Or double them?

Figure 7.29 ‘binary monsters’ task
Examining the self-assessment responses from this lesson, 67% (n=21) marked ‘very much’ on the question centered around absorption, and 74% (n=21) answered similarly to the question on concentration. The group work may have affected this high, along with the task design which was investigative and required discussion and focus.

14% (n=21) said they had the correct challenge none of the time, not matching their perceived skill; Fae responded she ‘wasn’t in flow at all since it wasn’t very challenging or exciting’. Could this be the groups they were in? Groupwork and its effect on flow is an area outside of the remit of this study and for possible future research.

Lesson 5 Setting the scene and task description

A continuation from the previous lesson of a problem-solving task, called binary monsters (Figure 7.29 above).

Self-assessment results from this lesson indicated 84% (n=20) answered ‘very much’ to the question centered around time distortion. 47% (n=20) answered ‘very much’ to the question centered around challenge. Possibly time was more of an obvious flow characteristic to identify.

Lesson 6 Setting the scene and task description

At the start of this lesson, I used a silent teaching approach, writing questions on the dry-wipe board and using the questions in Figure 7.30 below as prompts to assist me.
Findings (2) – series 2 and 3

Figure 7.30 a series of questions that I used in a silent approach

Students also were asked to complete the task below in Figure 7.31 towards the end of the lesson.

Figure 7.31 binary conversion task

Memo 7.31

69% of students (n=20) in this lesson responded that they felt just the right amount of challenge ‘very much’ (1). (in the two previous lessons it had been 39% and 47% respectively) This could have been due to the silent teaching approach, the main difference in teaching approach in this lesson.
Lesson 7 (1)

There are two excerpts presented from this double lesson, and self-assessment responses were collected along with video.

Setting the scene, task description and planimetric

This short extract is from the start of the lesson. Students are watching a video. They also have a sheet of question with tasks that can be answered by watching the video. This is being used as an ‘entrance activity’; to calm students and to engage them quickly into the lesson.

I am circulating the classroom for most of the extract. The seating plan is shown in Figure 7.32, and is similar to the previous lesson.

The topic is BIDMAS. The question sheet is shown in Figure 7.33. There is an element of ‘met before’ which may have an implication on the challenge of the task for students. The phrase ‘met-before’ is describing a previously established schema relating to a concept (McGowen and Tall, 2010; Tall, 2004). I am attempting for learners to use the acronym as a memory aid not as a ‘rule’ as a way of increasing the curiosity and challenge of the topic.
Lesson narrative - excerpt Lesson 7 (1)

Andrew and I are in dialogue, he has asked me to check his understanding. It is not possible to transcribe what I said as the audio was inaudible and there was not a relevant field journal entry.

**Memo 7.32**

Hassan’s response in the self-assessment to the question concerning an autotelic experience was ‘very much’. This is congruous to the flow marker; in several images he has his head in hands, seemingly absorbed.

56% of the class overall answered ‘very much’ to the same question, and 100% answered ‘very much’ or ‘partly’ in this lesson.

I am talking with Paige, discussing her homework. Andrew and Brandon are discussing the work. It is not possible to know what they are talking about due to the audio not being of good enough quality, nevertheless both are looking at Brandon’s work, an implication they are probably talking
about mathematics rather than off task. Josh is trying to attract Jack’s attention, who is focused on the task and video.

**Memo 7.33**

Although many of the mathematical conversations were not possible to be transcribed because the audio was of poor quality, it could be seen when students were talking about work, they could be indicating at it in some way, sometimes with a pen or their hands (an identified flow marker).

**Figure 7.36 image Lesson 7 (1) at (3:47)**

*Flow markers: (interactive), talking about the work/topic/task absorbed in that conversation (Josh, Omar, Caleb and Amber); (solitary), concentrating on the work, engaged energy, keenly writing down onto the paper (Liam, Layla, Paige, Brandon).*

Josh is in conversation with Omar. I am now conversing with Hassan, checking on his homework. Liam appears to be concentrating on the work. In the self-assessment Liam responded that felt the challenge was correct ‘all of the time’.

**Memo 7.34**

Despite flow markers indicating she seemingly was involved in the work in much of this extract Layla’s response to the question around an autotelic experience was ‘partly’. This could be because the questionnaires were given at the end of the lesson, and she concentrated her self-assessment remarks later on, when she may not have been as absorbed. This was a known limitation of issuing the questionnaire only once in the lesson.
Lesson 7 (2)

Setting the scene and task description

Next, an extract from the same lesson, the second part of a double lesson. It starts from 73:00. Students were given the opportunity to practice their number skills utilising BIDMAS and are given a series of tasks, the first is shown in Figure 7.37 below. Each of the questions are timed to take two minutes. I asked the students to work in pairs and talking about the mathematics was encouraged. The seating is the same as previous (Figure 7.32).

Question 1

Use the numbers 100, 4, 6, 7, 8 and 8 to make 601

Figure 7.37 number skills practice task (UKMT, 2014)

Lesson narrative - excerpt Lesson 7 (2)

The first task is displayed on the board and is the central perspective (Figure 7.37, above). Some students start already (e.g., Jack, Paige, Hassan). I am talking to the group from the front of the class and give them instructions about the task. I do not give any more mathematics ‘content’, rather letting the students know about the timings of each task.
Findings (2) – series 2 and 3

Memo 7.35

In a sense I did give the mathematics content to the students, although solely through a written medium (the task explicitly had written ‘use the numbers … to make ….’). The ‘telling them (orally)’ was absent and this had the effect of engendering social flow as it suggested that students should communicate mathematically to one another orally.

Memo 7.36

According to the self-assessment responses, for a majority in the lesson overall, the tasks seemed to be at the correct challenge vs perceived skill in this lesson. 81% of students scored ‘partly’ or ‘very much’ to the question centred around challenge. The questionnaire responses from this lesson (n=16) also showed that 75% of those that answered scored ‘very much’ when experiencing temporal distortion. Similarly, 75% scored highly for a merging of action-awareness and clear goals. 74% scored ‘very much’ for complete concentration. 13% in the self-assessment questionnaire (n=16) wrote they had control ‘not at all’ (a 3 on the Likert scale). 50% had control ‘very much’.

Andrew was the only student who responded with a ‘not at all’, when describing the challenge, but scored a ‘1’ (‘very much’) in the questions around ‘clear goals’; ‘loss of self-consciousness’ and ‘merging of action-awareness’ (the other he responded ‘partly’). Possibly the questions and the task were not challenging enough for him, yet he was in flow as he participated actively in the lesson, and his perceived challenge was gained from teaching others and actively participating? He also recorded; ‘it was interesting’.

Figure 7.39 image Lesson 7 (2) at (0:21)
Flow markers include: co-active looking at the board (Layla); solitary, writing with focused energy (Brandon).
Some students have finished the question and are taking a ‘break’; For example, Liam is taking his jacket off, Lili is looking in her jacket at something. Josh has also completed the task. (Josh responded in the self-assessment questionnaire that the challenge was correct for him ‘some of the time’). The field journal entry below elucidates further.

**Memo 7.37**

In a situation where some students complete the work before others, I often give another question to students who finished. I didn’t on this occasion; I was not prepared. However, the incident gave more thought to the notion that breaks are important for flow and laid the foundations for further enquiry. The journal entry below confirms this.

*Field journal entry 22nd September 2015*

In today’s lesson I let students talk on purpose, that is talking about other things. I didn’t interfere with this, and it was only for a few mins. It seemed to be helpful as nearly all then entered another period of flow.

---

I bring the class together and ask students to stop working on the task. I only give ‘process’ authority at this point, moving to the side and indicating for students to be quiet (see Oyler, 1996b). I ask Kyle to go to the front of the class and present his solution, and he starts writing his answer to the task on the whiteboard.

**Memo 7.38**

At this part of the class, I am assisting the interactive flow that is appearing. I am enabling others and consciously and constantly ‘manipulating’ the situation to allow students to experience social flow.
Findings (2) – series 2 and 3

Flow markers include: (interactive), talking about the work with focused energy, (e.g., Paige and Saffron, Kaitlin & Ava); (co-active), looking at the board part way through conversation (Andrew, Brandon); solitary flow (Katya, Benji) focused engagement.

Students are working with each other and are using large (A3) whiteboards.

Memo 7.39

The self-assessment scores from Benji confirm the flow markers. Benji had high scores in the self-assessment for the flow characteristics; he responded with ‘very much’ 8 out of the 9 characteristics, apart from the question centered around having everything under control which he marked ‘not at all’. It would have been good to have talked with him about this, to unpack these apparent inconsistencies; the main role being his teacher rather than researcher I did not get a chance to do this.

Benji also wrote a comment that he had short bursts of flow 30 seconds max.

Memo 7.40

Fae wrote ‘I was in flow because I really wanted to be first, so I tried really hard and concentrated’. This highlights the idea of being able to ‘turn flow on’ by ‘trying really hard and concentrating’ (or at least intend/want to). It also emphasised an (extrinsic) motivator of competition.
Lesson 8

Setting the scene, planimetric and task description

This lesson had a smaller group of students (n=11). Many were absent due to a school trip. The seating changed in this class because of the smaller size and is shown in Figure 7.42. Students were sat next to others whom they didn’t normally sit next to, and all of the students seemed happy to sit next to one another and to discuss mathematics. A student led class discussion took place. There are two camera views, and the excerpt is taken from towards the end of the lesson.

The task focused on the conversion from decimal to binary and vice versa. A question prompt: ‘what is the same and what is different about binary and decimals’, is the central perspective and is on the board.

Memo 7.41

An advantage of silent teaching and encouraging peers to have mathematical conversation was the camaraderie and cooperation that students gained with one another. Students who typically did not want to sit next to one another or would bring in issues from outside the classroom would work mathematically together.
Lesson narrative - excerpt Lesson 8

Kyle is standing at the front of the class writing on the dry wipe whiteboard. He is talking with the class and describing converting a number into binary. The discussion is shown in the transcript.

*Transcript (image 24/9 0:33)*

(Kyle) It’s still 16, 8, 4, 2, 1 so to make 5 I go maybe two there because 0 0 0 2 1

(Josh) You have used 2

(Kyle) okay my bad I need that okay, ok if you do 4,0,0 ... nine. I don’t know.

*Memo 7.42*

The response from Josh ‘you have used a 2’ was not picked up on by me at the time (or any of the students). The comment shows he has an understanding of the task, and (taking the images as a whole sequence) his expression/facial features/body language in the next three images may indicate the challenge was not at the right level to match his skill. The self-assessment was not completed by him for this lesson.

*Memo 7.43*

A recognised limitation of the self-assessment was that it assessed the entire lesson. In this particular case the self-assessment response from Benji corresponds with the markers that he displays.

Benji’s responses in the self-assessment were that when in flow he experienced all nine characteristics ‘all of the time’.
Guiding the dialogue so students have control of the mathematics, yet not taking over the lesson and stifling students' creativity and flow experience is a skill can be seen as something of an art form and not a science, and something that I did not always get right.

Kyle is talking and I am standing at the back but involved in the discussion. He is wanting to answer another question, that is off topic and I intervene to keep him ‘on topic’. (see the transcript).

Transcript (image 24/9 1.2):

(ME) different questions are coming up. Let's Stop. What's the difference between base 10 and base 2 or binary- which is what the original question was? Yeah. Okay. That's what we try to answer. So, there's not trying to answer other questions. We will get to the other questions in a bit. So, let's look at what is base 2 and what is base ten.

(Kyle) okay. So, if you wanted to make five with base 3.

(ME) No, base 2. we are not interested in base 3. What is base 2 on what is base 10?

(Kyle) Base 2 is 1's and 0's.

The journal entry below elaborates on the conversation.
Findings (2) – series 2 and 3

Memo 7.45

It is difficult to know without further questioning whether the challenge for Kyle was too high or too low. It may be that is was too high, as it appears, he deflects and does not answer the question on binary. His response in the self-assessment to the question ‘I felt just the right amount of challenge’ was a ‘2’ (some of the time).

Field journal entry 29th September 2015

Kyle was wanting to look at other bases not base 2. Is this because for him the challenge is too much to convert to binary, and he is deflecting and wanting to go to his ‘comfort zone. It did feel like there was an interactive flow occurring nevertheless – Josh and Kyle appeared quite excited.

Kyle has gone back to his seat. I am at the board writing down students’ contributions and interjecting where I see necessary (see transcript below).

Transcript (Lesson 8 2:55):

(ME): I think this is what you (pointing to Nicky) were talking about here when you were saying it’s the column. If I said to you, can we then use these numbers in here? Is that what you were suggesting?
Liam’s responses to the characteristics section of the self-assessment questionnaire were all high apart from loss of self-consciousness (‘I was completely lost in thought’); possibly he was in social flow and felt that he could not be lost in thought whilst conversing with others. Although the class and I had a ‘shared agreement’ on solitary flow characteristics, and the self-assessment questions, this discussion had taken place several months previously during series 1. The class and I had also yet to discuss social flow, and the idea that it may be possible to be ‘in the zone’ with others.

Kyle is answering a question about base 10 (see transcript). Some of the class are working on their own on the task, some are discussing amongst themselves.

Transcript (image Lesson 8 3:36)

(Kyle) base 10 is one zero two wait 012345678 and 9.
(Several students) Yeah.
(Kyle) To make 13 could you use 9?
(Me) (no answer, I wait)
(Kyle) I remember. I actually remember
Memo 7.47

Kyles proclamation ‘I remember. I actually remember…’ points towards the interactive flow marker of ‘talking animatedly and excitedly about the maths work’. It also points towards the positive benefits of flow, happiness and joyfulness.

The proclamation comes after I give Kyle time to think, wait time, allowing him time to process the information. I deliberately do this, counting silently in my head.

Memo 7.48

The different ‘modes’ of learning and teaching in the classroom, e.g., dialogue using the dry wipe whiteboard, working in small groups or pairs, working on own was possible mainly because of the small group size.

In the self-assessment to the question ‘I had everything under control’, 80% of students responded, ‘very much’ and 20% responded ‘partly’. There were no students who responded with ‘not at all’. The group dynamic due to the small size may have had an effect on this (n=10), could students have had more control of their learning with a smaller group size? Perhaps the small group size meant I was a much more dominant figure and so the students had less control?

Students were often leading the discussion and I was attempting to give guidelines only, using my process authority and holding back on content authority. (Oyler, 1996a).

70% scored ‘very much’ for a merging of action-awareness, implying flow states during the lesson. Liam commented ‘When I am in flow, I get the right thoughts at the right time

Memo 7.49

Ostensibly, all three categories of flow occurred at this point; interactive, co-active and solitary (see also Walker, 2010).
Lesson 9

Setting the scene, planimetric and task description

This lesson continues the topic of binary. Students start with a short task, multiplying in binary and then there is silent teaching for about five minutes. I ended my silence when students had demonstrated they could solve the examples. The seating is shown in Figure 7.47.

Students are multiplying binary numbers, answering the questions in Figure 7.48.

Calculate the binary numbers:
(a) $1011 \times 100$  (b) $110110 \times 1000$  (c) $11011 \times 10000$

Lesson narrative - excerpt Lesson 9

Students are working on the ‘starting’ task multiplying binary numbers (Figure 7.48.). I have talked very little about how to do the task, students are using their previous knowledge of binary. Most are converting to counting numbers before multiplying.
Findings (2) – series 2 and 3

Memo 7.50

Students are adapting the challenge to their own perceived skill level – some are converting to decimal before multiplying, an easier skill. Others are not converting first. They are guiding themselves, what is it that is causing some to take an easier path and some to take a harder one.

Figure 7.50 image Lesson 9 at (0:55)
Flow markers include: (interactive social flow) talking about the work with focused energy, (Caleb, Josh, Amber, Fae); (solitary flow) head bent over work writing (Benji, Nicky).

I travel around the room checking in individually with students, providing feedback.

Figure 7.51 image Lesson 9 at (1:38)
Flow markers include: (interactive social flow) talking animatedly and excitedly about the maths work (Josh, Omar, Liam, Hassan); (solitary flow) looking at the work on their own (Jack and Katya).

I am talking with Caleb about the task. There is also a discussion between three students (Josh and Omar are joined by Hassan), although the detail of the discussion is not known as audio transcription was not possible and there were no specific journal entries.
I have not set up any expectations for talking, some students are talking with each other, some are in solitary flow. In both of these images solitary flow and social flow seem to both be occurring within the classroom at the same time (see also Walker, 2010).

I bring class together, and a ‘silent section’ to the lesson begins, by writing a question (in binary) on the board.

\[ 11 \times 10 = \]

I give the pen to Liam. He answers correctly (1100) and asks me for feedback (in the image). I do not say anything and indicate to the class to say if it is the correct solution (which they do) and give him back the pen.

I deliberately do not say anything to Liam when he asks for feedback. It was clear that he was seeking validation of his answer rather than having a problem with the question. By not answering him and asking the class to validate his answer instead I use my role as an ‘expert’ to enable students’ understanding and independence of learning.
Liam is talking to the class while he is explaining how he did the binary multiplication. He then writes his explanation and the next question for the students which he makes up (11*11). The other students applaud. Liam now ‘hands the pen’ to Caleb.

Caleb is now at the board. He first converts both of the previous questions to counting decimal numbers before writing an answer. The students are now becoming self-sufficient in their learning. I have not contributed anything in terms of content or process.
Findings (2) – series 2 and 3

Caleb has put another question on the board, shown in Figure 7.55. His hand is up and is deciding who gets the pen next.

Caleb gives the pen to Maruf and then he sits down. Maruf takes over and starts to solve the new question. Initially I sit at the back of the class. Maruf makes a mistake, I intervene by indicating it.
is a mistake on the board. I do not talk and offer no assistance or feedback as to why it is wrong or the correct solution. There is a dialogue that starts again in the class, around the question. Maruf perseveres and writes the solution correctly, by converting to decimals first.

Memo 7.55

Not always intervening when students make mistakes is a way of empowering students independent learning. Sometimes it is necessary to provide feedback; in this case it was not.

I share some content knowledge. I am showing students that they can solve the problem without changing to base 10 first, using the long multiplication format that was started by Caleb. I do not complete the question, leaving some out. I am not talking.

Flow markers include (interactive social flow) talking about the work with focused energy (Andrew and Maruf, Liam, Katya, and Sadie).
Katya and Nicky now have hands up wanting to volunteer, and I hand the pen to Nicky. Flow markers include (co-active) hands up wanting to answer a question (e.g., Katya, Nicky). The silent part to the lesson ends. Books are handed out. There is short break. There are some students who are still engaged in the binary multiplication task talking about the previous questions.

Memo 7.56

94% of students who completed the self-assessment (n=18) marked ‘partly’ or ‘very much’ for the question centred around challenge (‘I felt just the right amount of challenge’). Nevertheless, Fae wrote ‘I wasn’t in flow when multiplying binary because I didn’t know the methods well (long multiplication)’. Lili also commented on the challenge ‘When the questions were hard it made it harder to concentrate and I couldn’t keep a rhythm’, and ‘I was in flow in the easy parts of the questions’. She scored a 2 in the question around the characteristic of challenge. Liam wrote, ‘I was in flow when I have a task I know’.

Memo 7.57

I am increasing the challenge appropriate to the skill of the class, moving the class on, acting as the lead, ‘stepping up to the plate’. Nevertheless, I am not telling students how to solve the mathematical task.

Memo 7.58

An intentional break, nonetheless, I allow some to continue with the work. Difficult to know when to end the section of the lesson. Different students have different interests in the work, most have finished the task on the board, but not the increased challenge of resisting the change to decimals and base 10 first.
Lesson 10

Setting the scene and task description

Students were required to complete an investigative task (Figure 7.60 above). They did this in groups of three to four. It was an assessment lesson, although students were working in groups. This was a new approach to assessment both for students and me, that we were trialling across the mathematics department. The self-assessment garnered responses from 14 students.

Memo 7.59

91% said that the activity helped them get into flow. 67% responded ‘very much’ to the right challenge’. Particular to this lesson, more students said that the teacher helped them get into flow than their peers (64% vs 55% overall figure). Perhaps not surprising as this was an assessment lesson, and also a new type of assessment (being investigative) and so my input was valued (e.g., students wanted help because of the value placed on formal assessments).

Andrew commented ‘I liked it’.

Another suggestion as to the higher response to teacher assisting with flow could be because there was a lot of detail put into the lesson planning. For example, I had planned out the questions/possible responses.
Lesson 11

Setting the scene, planimetric and task description

This lesson continued with the concept of dividing binary numbers. Seating is similar to previous lessons (shown in Figure 7.61). There are two cameras in this lesson. The excerpt takes place between the start and the middle of the lesson; it is after students have entered and have done a ‘warm up’ activity which was a series of arithmetic questions using both counting numbers and binary numbers. I have written an example on the board as the initial task: 10100 ÷ 10

The assessment feedback from the previous lesson indicated that several in the class are finding the concepts of binary operations difficult to understand, particularly without converting into counting numbers (decimal notation) first. Nevertheless, there may be some who can move straight into operations on binary numbers without conversion. To encourage students to keep the numbers as binary notation I have copied a ‘crib’ sheet of binary number bonds (Graham, 2010) that I share with students if appropriate (Figure 7.62). This was an effort to ensure the challenge matched the perceived skill of the individual student. Students are also asked to complete a two more binary division problems, that I have printed onto a separate sheet.

1100 ÷ 100
1100000 ÷ 110

It is possible to add and subtract binary numbers in a similar way to base 10 numbers. For example, $1 + 1 + 1 = 3$ in base 10 becomes $1 + 1 + 1 = 11$ in binary. In the same way, $3 - 1 = 2$ in base 10 becomes $11 - 1 = 10$ in binary. When you add and subtract binary numbers you will need to be careful when ‘carrying’ or ‘borrowing’ as these will take place more often.

**Key Addition Results for Binary Numbers**
- $1 + 0 = 1$
- $1 + 1 = 10$
- $1 + 1 + 1 = 11$

**Key Subtraction Results for Binary Numbers**
- $1 - 0 = 1$
- $10 - 1 = 1$
- $11 - 1 = 10$

Figure 7.62 adding and subtracting binary (Graham, 2010)
Memo 7.60

Minimising distraction seems to be encouraging flow states (see journal entry below), and in an effort towards this the two extra problems are on my desk for students to take themselves when they are ready. This also gives students an opportunity to have ownership and control over their learning, another flow characteristic.

Field journal 28th September 2015

When Mr Murphy (a senior teacher) entered the class today it was only for a minute or less but it seemed to take ages for students to get back into the lesson, also when class were distracted by the bee in the last lesson it seemed to take ages again for students to get back to learning. Thinking of what Paige said about me being a distraction I’m thinking that if I can minimise these distractions then flow can continue more freely.

Lesson narrative - excerpt Lesson 11

![Image](imageless11.png)

Flow markers include: (solitary) hand on chin denoting thinking (e.g., Hassan); (co-active) looking at the task (e.g., Paige, Benji, Ollie, Caleb); (interactive) talking about the task animatedly (Kyle).

Kyle is at the front of the class, using the dry wipe whiteboard and explaining the first question \((10100 \div 10)\). There is some discussion within the class around the question while Kyle is explaining (Amber and Fae).

There are two students at the front Maruf and Dru whose facial expressions suggest they may be unengaged.
Although throughout the study I concentrated on the occasions when flow was demonstrated, and used theoretical sampling to achieve this, there were times when learners were clearly not in a state of flow (Maruf and Dru). Possible reasons to do with the challenge not an appropriate match for perceived skill (a fundamental element of flow).

Journal entry 1st October 2015
Not such a great lesson today ... Could I have involved Dru? Possibly and maybe, I should have ...was the challenge appropriate - probably not. I just didn’t notice him.

Nonetheless, 64% of students (n=11) scored 'very much' to the question centred around challenge, and there were no students who marked 'not at all'. 73% scored 'very much' to the question about the activity being autotelic and Hassan wrote, 'the class was noisy, but I coped' and 'I enjoyed the activity'.

Involving students in the lesson may engender interactive social flow. A possible reason why I did not involve Paige and Dru in the explanation and asked questions to other students may be to do with them outside my peripheral vision. When addressing a group, the speaker (at the front of the audience/apex of the triangle in Figure 7.64) can concentrate their attention on those inside the triangle and leave out those who are on the outer sides of the group (Jaffe, 2007).
Findings (2) – series 2 and 3

I asked the class for suggestions of where to go next in the problem. Josh suggests using long division as a method rather than short. I ask Josh to come to the board to explain.

**Memo 7.63**

Allowing students to teach each other and I ‘step aside’ seems allow the blending of egos, a prerequisite of social flow. It also encourages independent learning.

Flow markers include: (solitary), head resting on hands (Hassan); (co-active) watching the task/speaker/board (Maruf, Paige, Benji, Jack).

I stand at the rear of the class. Josh explains up until 8.17 with no further input from me. Maruf is beginning to be involved (taking the images as a series, he was not involved in the previous image). Ollie is working on the problem by himself.

**Memo 7.64**

Maruf becoming involved in the lesson may indicate the challenge of the task is now appropriate to his perceived skill. The task was also being explained by his peer.
Lesson 12

Setting the scene, planimetric and task description

This excerpt is from the start of the lesson. I have given students a worksheet as they entered the class based on arithmetic questions. There is one camera, and it is now in a high position, looking over the class from the front. The seating plan is shown in Figure 7.67.

![Seating Plan](image)

*Figure 7.67 seating plan Lesson 12*

The task is shown in Figure 7.68. All of the questions are on division. Students have to complete the boxes using the horizontal divided by the vertical numbers. The questions are of varying complexity, for example there are many with remainders, and students are encouraged to find the answers as decimals.

![Division Task](image)

*Figure 7.68 division task*
Previous to this, students had their morning break and students can often demonstrate inappropriate behaviour because of this and take several minutes to become involved in the lesson. One of the purposes of this activity was to settle students down. For this reason, I have designed the activity to be deliberately repetitive and start with questions I am aware are accessible to all. It also has deliberate variation that enables learners to concentrate on the common structure between the questions (see also Mason and Johnston-Wilder, 2004a). Calm starts to lessons, which, this activity promotes, along with my teaching approach, appear to engender flow.

Lesson narrative - excerpt Lesson 12

Students working in pairs, on the ‘entrance activity’ (Figure 7.68). The ‘noise’ level is low, students are talking quietly in pairs.

**Memo 7.65**

I have encouraged learners to talk with their peers about the mathematics, particularly if they are finding difficulty with the task. Talking and interaction contributes to the emergence of social flow.
Findings (2) – series 2 and 3

A majority of students are engaged in the lesson. There is a small amount of talking in the lesson at this moment, which is about the mathematics.

Memo 7.66

In this lesson, there was only a small amount of returns of the self-assessment (n=10), and the comments may be more reliable than any percentages. Nevertheless 80% of students (8/10) marked a ‘very much’ for the ‘temporal distortion’ question and in the ‘space to record the moments I am in flow’ Teo wrote ‘I was not noticing time pass’.

The time distortion characteristic of flow was an easier characteristic to share with the class, most could associate with a time when doing a mathematics task and the time had gone by much quicker than expected. This could be seen as a reliable indicator that flow occurred.
Flow markers include: (solitary), writing with apparent concentration (Caleb); (interactive), discussing the work involved with energised focus (Omar and Teo), writing/pointing to the exercise book, looking at the writing (Reuben, Shurik, Kit, Layla).

Two students have both finished the task (Kyle and Benji). Reuben and Shurik and also Kit and Layla are in conversation, it appears centred around the task.

Several students appeared to enjoy the lesson, although there was not a specific reason given. Hassan wrote, ‘today I had fun and I enjoyed today’s lesson’. Andrew wrote ‘I liked today’s lesson’.

An improvement on further research would be to analyse the questionnaire comments closer to the time of the lesson, and appropriate follow up questions could be asked.
I ask Josh to complete an example that I have put on the board. Students are contributing to the discussion; I am silent and not involved.
Kyle takes a turn and does another example at the board, talking with students. I did not direct him to go up to the board, students are in effect teaching each other and my role has diminished. Some students are working out the task on their own and not involved in the conversation from the front. There are some learners working on their own, or with one or two others talking quietly. Caleb is using a calculator to check the answers (providing a feedback loop).

I have stood back from the group interaction and students are teaching one another. It appears that social flow is occurring. A student teaching their peers may be more likely to involve students than a teacher and this could be a possible reason for the prevalence of social flow when student/student dialogue.
Section 2: Series 3

There had been a considerable time, 9 months, between series 2 and this series. This was due partly to pressures of teaching and also the writing up of series 1 and 2. In the interim 9 months there were 2 stimulated recall lessons, and students had been informally writing flow journals in their mathematics books. The results from these are enmeshed in the discussion chapter that follows this chapter and also in Appendix A. Subsequent to this lesson there was a follow up stimulated recall interview with 2 students (Josh and Fae).

In line with previous sections, the memos are a way of differentiating between the description of what happened and the discussion of why what happened. It is an attempt at an understanding through pictures rather than just about pictures. The red/orange boxes were written at or around the time of the lesson, and the blue boxes written after.

Lesson 13

Setting the scene

Series 3 showcases a single lesson 50 minutes in length (Series 3) that took place in Summer 2 half term of 2015. During this series, I worked collaboratively with two mathematics teachers, Ms Taylor and Mr Bruce, in the planning of the lesson and the planning of the worksheets. The additional opinions and discussion garnered around the research questions introduced and stimulated new ideas and notions, with the hope of adding further value, transparency and trustworthiness to the research. Collaboration is in line with the methodological approach of teacher research, resonating with Stenhouse (1975, p. 155) who fluently writes; ‘A teacher who wishes to take a research and development stance to his own teaching may profit at certain stages in the development of his research by the presence of an observer in his classroom’.

A focus of this series was the connection between breaks and flow and the impact on students. Within a classroom situation there are inevitable (and normal) social interactions. In the previous series I had become aware that breaks seemed to be having a positive impact on students subsequent and immediate flow experience. The journal below has more detail:
The intention was to observe how students took breaks and identify connections to the research questions.

**Task description**

The topic adding and subtracting fractions was chosen with an attending to understanding and process. My previous knowledge of the students’ mathematical understanding was that most could specialise adding and subtracting fractions and I hoped that most students would be able to generalise adding and subtracting fractions after completing the worksheets.

My colleagues and I collaboratively planned the worksheets (Figure 7.74 above), and considered possible misconceptions connected with fractions that I had observed in the group previously (such as ignoring the denominator when adding e.g., $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$).

The direction in which we intended the lesson to go (from specialising to generalising) was also thought about. Students would start with consolidating their knowledge of numerical fractions, and then compare these to fractions with variables.

The main focus was on the similarities (and differences) that fractions held, which emphasised an understanding of why it was necessary to change to a common denominator.
The questions requiring a text response on the right of each worksheet were planned to highlight the differences and similarities between numerical and subsequent algebraic fractions and enable learners to become aware of those similarities and differences. I hoped that learners would be able to solve questions and tasks involving algebraic fractions with minimal teacher input. It would allow students to stress their understanding, which would be in contrast with learning an algorithm like cross multiplying.

Series 1 and 2 also indicated that with (unnecessary) teacher input at a minimum flow may be more likely to occur (e.g., by utilising a silent teaching approach). My two colleagues and I agreed, that during this lesson, we would take a less active role in directing the learning process, and the worksheet was to be the main content authority as opposed to the teacher. For example, there would be no formal introduction to the topic despite students not having any previous exposure to algebraic fractions.

We made allowances for individual help in the lesson, although I very much let the students answer questions. For example, a student would ask, ‘how do I do …(this)’? to which the response might be something similar to, ‘I don’t know, how do you do … (this)?’. This was in response to series 1 and 2 findings that allowing students to answer their own questions, encouraged more mathematical talking, encouraging and engendering interactive social flow.

The grammar (how the images, text and memos connect)

There is a nuance to the grammar of this imagining in comparison to previous sections. The term grammar is used here to communicate how the evidence connects, and is a notion from multi-slice imagining, discussed earlier in chapter 3. I wanted to see how flow occurred across an entire lesson in response to the focus on breaks; and I have taken a snapshot of the entire lesson, concentrating in main on the first 15 minutes. A timeline of the lesson is shown in Figure 7.75 below.
Visual content and style

In this section, I have placed more emphasis on the text. The images are used more to illustrate a connected commentary text, placing emphasis and attending to the moving aspect of the images (video). As far as I can tell from the existing literature, Multi-slice imagining has not
been utilised with video previously, and this slightly different way of analysing was a response to this, providing another way of adding rigour to the existing and emerging thoughts and ideas and the study as a whole. It was also in line with the iterative nature of teacher research.

Planimetric

The seating plan is shown in Figure 7.76. The changes of seating of students from series 2 were in response mainly to behaviour considerations.
Lesson narrative - Lesson 13

The whiteboards (central perspective) are both blank. Students each have a worksheet. The (sole) instruction given was to complete the worksheet and to talk about the mathematics. There was no other content knowledge initially communicated to the group as a whole, other than the written worksheet.

Memo 7.89

This was a new topic; previously I would always introduce a new topic. Instead, the worksheet was used as a sole primer. The move away from an introduction by me, was a risk, and some students were uncomfortable with it, citing that the challenge was too high (or too low). For example, Josh in the post lesson interview commented ‘I wasn’t in flow during the first fifteen minutes.’

Perhaps interestingly in the self-assessment however, Josh scored ‘very much’ for the challenge characteristic.

Brandon commented ‘I wouldn’t get into flow because the work on the first worksheet was easy’.

Nonetheless the challenge was appropriate for some, and for this question, 33% scored ‘very much’ and 67% scored ‘partly’ (n=15). Nobody scored ‘none at all’.

Notwithstanding Brandon’s comment above, the ‘uncomfortableness’ around challenge could be to do with the move away from familiarity, rather than an issue with mathematical understanding. 63% said their mathematical skills helped them get into flow.

(0:05) Liam is engaging with the mathematics. He is briefly talking with his neighbour, Omar, both looking at the worksheet, and then writes in his mathematics exercise book, hunching over the book. Omar looks around and does not engage for another couple of minutes. Josh is also writing on the worksheet, engaging with the mathematics. He continues until 5:20.
Solitary flow markers include oblivious to noise and distraction, working very quietly not talking of anything else, writing with apparent concentration (e.g., Josh). Interactive (group) flow markers include talking with one another about the mathematics with energised focus (e.g., Fae and Katya, Teo and Ms. Taylor), pointing at the work whilst talking (e.g., Liam), engaging in a conversation to the exclusion of all else (e.g., Denzel, Benji Omar).

Liam appears to start on the task (see memo below) and turns to Reuben who looking at the first worksheet. They discuss ‘something’ to do with the worksheet. Next to Reuben is Josh who is still engaged, bent over the desk and writing furiously.

Memo 7.70

The self-assessment results for Liam indicated that he was in flow for some of the lesson; he scored ‘partly’ for all flow characteristics. He also indicated that talking with friends started the flow experience.

Unfortunately, the audio was not possible to understand what was being said by Rueben and Liam. The description of the interaction is my perception of what can be seen on the video. This is a known limitation of this analysis and dataset and a reason why other datasets were used alongside the video.
Solitary flow markers include head bent down, moving closer to the work (e.g., Omar and Reuben), looking at the work on their own, concentrating on the work (e.g., Liam).

(1:58) Liam turns back around to his desk and continues to note something on sheet. Benji and Denzel are not yet engaged, not working on a mathematical task and talking about something else. Omar has got involved with the work now - he is talking with Liam briefly.

Memo 7.71

An example of limitations of flow markers being a sole dataset. If the image was taken on its own, then possibly Denzel and Benji are discussing the mathematics. However, watching the video, it is possible to see that they are not talking about the mathematics (at this point) and are engaged in a social interaction.

(2:18) Caleb’s attempts to ask for help but is ignored (deliberately) by Mr Bruce and myself initially.

(2:25) Caleb talks with Teo who is sitting adjacent to him, and the problem seems to be overcome, as they are now working again.

(2:52) Fae’s attempts to ask for help but is ignored by Mr Bruce and myself at least initially. Reuben is also stuck.
(3:23) Fae appears to be perplexed and has now temporarily ‘given up’. Reuben is talking with Liam, and then Omar and they appear to be overcoming the challenge and talking about the worksheet. The discussion lasts until 3:32 and then they turn around and start working on their own again. A group naturally starts to form here, of these three students, Reuben, Liam and Omar; Josh also is involved sometimes.

**Memo 7.72**

By ignoring calls for help initially (although this wasn’t done always done with forethought), as teachers we were enabling students to take back their mathematical rights and allow their natural powers and mathematical instincts to happen. The purposeful waiting by my colleagues and I assisted with students independent learning and the manifestation of flow.

The subsequent field journal entry shines additional light on this.

**Field journal entry 1st July 2016**

*Does tacitly ignoring and not helping assist in fostering an independence, better learning and ultimately a flow state? Certainly, did in the case of Liam and Reuben. Perhaps not with Fae. There are different ways students become ‘unstuck’. Flow (through the implied creativity) can help becoming unstuck, and students communicating the ‘stuckness’ to each other can contribute to being in interactive flow. Talking to a teacher may not always assist in manifesting flow as although it may be of help with answering a problem it can also take away control, ownership and independence from learners.*

**Memo 7.73**

Overall, 38% reported talking with their teacher helped them get into flow, whereas 94% reported that talking with their friends got them into flow.
Findings (2) – series 2 and 3

(3:28) Benji and Denzel now both engaged in the mathematics. Denzel sat back and appeared to do nothing outwardly, then suddenly started writing on the sheet. Josh was furiously concentrating writing from close to the start of the lesson straight through until 4:16 when he put up hand, question not answered, hand put down and he went back into it. He converses briefly with Reuben (4:37 - 4:59) and then continues writing. (4:56) Denzel is off task; another student asks to borrow a pen which he throws across room to them. Benji is fiddling with pen (sitting next to Denzel).
(5:24) Josh has his hand up, I wait for 40 seconds before discussing with him. (5:33) Benji starts to get engaged with work. (5:45) Denzel begins to be engaged. 

Memo 7.74

Denzel, Benji and Josh are entering flow, and then exiting it again. It appears that there is a period where these 3 learners stop their mathematics, take a break and go in and out of flow. Is this behaviour that teachers should address? Something that could be for further research; if talking is to be encouraged in the mathematics classroom because of the benefits that it brings, should breaks be also encouraged? What should these breaks look like? The journal entry below elucidates further;

Field journal 1st July 2015

Allowing them to have this freedom, to take 5/6 mins to get into work can enable flow to occur more readily, and halts the anxiety and attention from expert teacher? Of course, this should be taken in context of the individual, sometimes students will not settle in without encouragement

Memo 7.75

Denzel appears to stay in flow from 2:03, until a break between 4:56-5:45. In the self-assessment Denzel commented ‘I was in flow when I started doing the worksheet’.

Benji, who was beside him was not in optimal state for same amount of time, or so it appeared, taking short breaks in between his flow experiences.

Field Journal 3rd July 2015

If a student encountered flow and made progress irrespective of whether they have had a break period is this to be deemed as learning? Are breaks at inevitable part of flow?
Findings (2) – series 2 and 3

Solitary flow markers include in ‘thought’ focused, looking at the work on their own (e.g., Benji, Denzel). Interactive flow markers include pointing at the work or indicating at it in some way, talking about the work (e.g., Liam, Omar, Josh, Me)

(6:08) I answer Josh’s question, until 6:51. He wanted to know if the work he had completed was correct, wanting reassurance and validation. I asked him if he could check his answers himself - he was very keen on me checking them for him. I asked/prompted if he knew/ had any ideas on how he could check the work. He suggested subtraction (inverse).

Memo 7.76
Josh was not necessarily asking for help because he was stuck on the question. Instead, he was asking for a validation of his answers that he already knew were correct.

(6:37) Fae has her hand up again, having spent the last three minutes working out the problem with Katya.

(6:52) I answer a question from Fae, validating her answer.
Findings (2) – series 2 and 3

(7:47) The second worksheet given to Josh. He has decided to check his answers himself to the first worksheet, using an inverse method.

(8:14) Caleb asks me a question around converting the fractions to decimals to assist in the arithmetic. (Figure 7.81 above shows an example of Caleb’s work). Josh is now working with Reuben, until 8:31.
Solitary flow markers include head bent down, moving closer to the work (e.g., Denzel and Benji). Interactive flow markers include engaging in a conversation to the exclusion of all else, talking with one another about the maths with energised focus (e.g., Reuben and Josh, Omar and Liam).

(8:51) Liam, Omar, Josh and Reuben are talking quietly and intensely and to each other about the mathematics. Benji and Denzel are working intently on their own.

Solitary flow markers include concentrating on the work, writing (e.g., Josh, Maruf, Teo).

(9:40) Liam, Omar, Josh, Reuben are talking and off task taking a ‘break’. Josh indicates to the work and seems to want to go back to it, ignoring the other students after about ten seconds.

(9:02 -10:15) I finish answering a question from a student and walk by Liam & Omar who are now looking around, not engaged obviously in the work. I do not interfere, and neither ask for help.
Memo 7.77

It may be possible to include markers ‘not in flow’. For example, smiling and laughing (Liam and Omar).

Memo 7.78

The off task ‘ness’ was not necessarily something that detracted from the lesson, all four students had been concentrating intently for nearly 10 minutes, and after the ‘break’ the group begin immediately to discuss the mathematics again.

Figure 7.85 image Lesson 13 (10:25)
Solitary flow markers include head bent down, moving closer to the work (e.g., Benji and Denzel). Interactive flow markers include discussing the work to the exclusion of all else (e.g., Josh, Omar, Reuben Liam, Ms. Williams also Katya, Fae and Mr. Bruce). Co-active flow markers include wanting to ask a question about the work, hand up (e.g., Lili).

(10:15) – Mr Bruce gives Katya a second, more challenging worksheet. Katya explains what she has done so far, seeking validation like Josh earlier with myself. There is a period of intenseness and they are in a highly engaged conversation (interactive social flow marker). The circles denote those students who are engaged together in interactive flow.

(10:23 - 11:42) Mr Bruce is speaking individually with Katya and Fae. Fae is asking a question about the work. Lili has her hand up, wanting to ask a question.
Findings (2) – series 2 and 3

Interactive flow markers include pointing at the work or indicating at it in some way, talking about the work (e.g., Benji and Denzel), discussing the work with energised focus (e.g., Ms. Williams, Josh, Reuben, Omar), engaging in a conversation to the exclusion of all else (e.g., Mr. Bruce and Fae).

(10:50) Benji and Denzel start a conversation that lasts until 11:11 - both students seem to be bent over the work, a flow marker.

(11:05) Katya now joins in conversation, the three appear to be in a flow experience at this point.

Solitary flow markers include bent over work, looking at the task, writing, (e.g., Benji, Denzel).

(11:21) - Maruf is off task. He is calling out to Caleb. After a few seconds of talking to Maruf, Caleb turns back and engages with the mathematics again.
Findings (2) – series 2 and 3

Memo 7.79
Caleb is distracted only momentarily and then turns back to the work. Doing mathematics and the associated flow experience is having a stronger influence than talking with a friend.

Memo 7.80
It would be possible to mistake Caleb and Maruf as showing interactive flow markers, but Maruf was distracting Caleb; this was identified through my recollection from the lesson and field journal.

Memo 7.81
From 13:55 onwards, for the group that included Liam, Omar, Reuben and Josh there was very little or no teacher input. Ms. Williams left the class at this point. The drive to work, was self-generated and probably intrinsic. There was no ‘must’ for them to work – Mr. Bruce, Ms. Taylor and myself left them alone for a majority of the rest of the lesson. They had breaks in their work, however the getting back into flow was driven by the mathematical activity (and vice versa) and not by the teacher. None of these students asked for help from a teacher after 13 minutes.

(12:01) Caleb now gets back to working on the fractions task. Benji and Denzel are absorbed (individually) about the work.

(12:54) Benji and Denzel are out of flow state and off task, taking a break. The maximum period of flow at this time is approximately four to six minutes at a time (more detail in the later section in this chapter on flow and breaks). Students then seem to break, albeit briefly.

Memo 7.82
Benji and Denzel are also talking to one another from time to time and this seems to be aiding their flow state. It may be that there is a combination or a synergy of social and individual flow, one may be helping the other. Could the two states could be merging? A possible topic for future research.

(13:04) Mr Bruce gives Benji and Denzel the next worksheet and the three of them have a discussion.
Findings (2) – series 2 and 3

Interactive flow markers include engaging in a conversation to the exclusion of all else (e.g., Mr. Bruce, Benji, Denzel also Omar, Reuben, Josh and Liam).

(13:14) Reuben, Liam and Omar are working as a group and Josh is working on his own, contributing to the discussion from time to time. All four are engaged with work, which is holding their interest. Liam is focusing on the worksheet. Benji and Denzel are talking with Mr Bruce about the new worksheet he has given them.

(14:27) Omar, Liam, Josh and Reuben are out of flow briefly, and then it appears the focus on the mathematics and the worksheet holds their attention.

Memo 7.83

For Katya this (taking breaks) seems to work and assist her flow state. In the self-assessment Katya responded; [I’m in flow when I am] ‘doing a specific amount of questions then having a small break before I do another set of questions.’

(15:45) Many members of the class are going in and out of flow, for example Katya and Fae in particular are out of flow and taking a break.
(23:30) Mr Bruce looks over Katya and Fae’s work, over their shoulder, and does not interrupt. He stays for a brief time, approximately 10 seconds. The circles denote the grouping of students who are showing flow markers.

Memo 7.84

By suspending judgement, and not commenting Mr. Bruce allows Katya and Fae independence and control (flow characteristics). The 'non-interference' could be seen by some students as a form of validation, i.e., that teachers only interrupt when the answer is wrong.

(23:53) - I interrupt and ask those talking to be quiet. This did not appear to interfere with the flow states of groups (Benji and Denzel) and (Katya and Fae – now back in flow).
Solitary flow markers include oblivious to noise and distraction, working very quietly not talking of anything else (e.g., Benji and Denzel, Omar, Liam).

(24:11) – The circles show the groupings of flow. I am talking to the group as a whole from the back of the class. I am reminding them to ask for the next worksheet, if they are finding the one they are working on easy.

**Memo 7.85**

On this occasion, the interruption from myself appeared to have less of a distracting effect. For example, Omar and Liam, Benji and Denzel continued working on the mathematics, apparently in flow. What I did differently was to let students carry on working if they wanted to, while I was talking as long as they
Findings (2) – series 2 and 3

Solitary flow markers include head bent over work writing (e.g., Denzel) looking the work, in 'thought' focused (Benji). Interactive flow markers include discussing the work involved with energised focus (e.g., Katya, Fae and Reuben).

(28:59) – Katya and Fae and Reuben, appear to be discussing work.

(15:17 - 43:20) Denzel and Benji are experiencing flow, working on the task the entire time, apart from break between 26:03 and 31:09. Part of the time is 'individual flow' and part social flow. Denzel had help/reassurance on three occasions during the lesson from two different teachers. His response, in the self-assessment to the question ‘To get into flow I will:’ was, ‘concentrate on the work’. This he did, and it seemed to work for him.

(41:23) Josh is interacting with Ollie, excited look on face from Josh who gets out of seat to confirm something with Ollie.

(41:26) Denzel is interacting with Reuben, also now in social interactive flow.

Memo 7.86

During the lesson in general some students were in ‘individual flow’ and working on the sheet on their own, (e.g., Josh and Ollie). They had some conversation, but their progress did not seem to depend on the conversation. Ollie commented in the self-assessment; ‘I was in flow for the whole lesson until 11:55. I went back into flow at 12:00 and now it is 12:13’.

Whereas there were others, Benji, Denzel, Reuben, Omar, Liam, Katya and Fae with whom mathematics talk and any social flow that occurs as a result seems to be integral to their work in this lesson.
Interactive flow markers include talking animatedly and excitedly about the maths work (e.g., Josh & Ollie), writing/pointing to the exercise book, looking at the writing (e.g., Teo and Caleb).

(42:05) - I interrupt, asking students to write on a post it ‘what they had learned/picked up that was new’. I also asked them to write an example that they could/should make up themselves. Denzel and Josh seem to continue in their flow state until end, according to flow markers. Katya, Fae, Benji, Caleb and Liam all have their energies focusing on me.

**Summary of chapter 7**

In this chapter I have presented and made an initial analysis of the evidence from series 2 and 3, chronologically using images from the videos. The included memos, consistent with situational analysis, suggest possible conjectures with reference to the demonstration of flow in the classroom; including flow markers (also dealt with in chapter 6); social flow; and breaks from flow. In addition, within the memos I begin to analyse and examine the role I can play as their teacher in engendering flow, including methods and teaching approaches and through task and activities. Additionally, this chapter has suggested some of the limitations of the study and areas for additional research. The following chapter 8 discusses the entirety of the findings, specifically examining the results of the self-assessment, looking at the structure of flow.
CHAPTER 8: The structure of the flow experience – a discussion

In this chapter I develop the conjecture started in chapter 6, that students manifested and demonstrated flow over the study. I look at the extremes of flow in lessons and why they may have happened. This is in response to RQ1. RQ2 is discussed in chapter 9. I analyse the results from the questionnaire looking at the predictive and descriptive framework (conditions and characteristics) suggested by flow theory. I examine the student responses from the self-assessment indicating flow being demonstrated and utilise an indicative measure of flow, the ‘flow score’. I look at the study overall, the class, and for individuals. I utilise the written comments from the self-assessment, the stimulated recall lesson and the student flow journals. The full results from the self-assessment questionnaire are contained in Appendix D.

Manifesting and demonstrating flow

Conjecture: over the entirety of the 18-month study many students experienced flow in their mathematics lessons.

Learners were invited to make written comments throughout the study, and these are analysed and synthesised in this chapter. They were asked to complete question stems (see Appendix A) and also encouraged to freely write their ideas around flow and mathematics in student flow journals (Coles and Banfield, 2012; Moon, 2006). Excerpts from the transcript of the stimulated recall interview are also used as evidence. The interview took place after series 1 and 2, with eight of the students (see Appendix A).

Students were asked nine questions concerning the characteristics underlying the processes of a student flow experience in Part A of the self-assessment. These were based on the theoretical model of flow following an adaptation of the flow state scale FSS (Jackson and Marsh, 1996).

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4 Previously, in chapter 6 I conjectured that during a single lesson taking place in May 2015 (series 1), many students were in a flow state. I now develop this conjecture.
Students answered Part A using a Likert scale. Part A was also used to calculate a flow score; an indicative measure of how many students experienced flow.

In Part B of the self-assessment students were asked to respond ‘yes’ or ‘no’ to 10 questions examining the emergence and protraction of the flow experience.

*Placing a number on flow - a caveat*

The complexity of the flow phenomenon should be born in mind during the breakdown and analysis of the self-assessment numerical responses. To place a number on flow is indicative and although it may indicate when flow took place and should not be taken as an accurate breakdown of how flow occurred. Flow states are a complex phenomenon and reducing it to a number may subtract from its meaning (Jackson and Eklund, 2002; Csikszentmihalyi, 1992). Written comments may be more useful than numerical scores (Cohen et al., 2013) and as stated earlier, additional evidence from other datasets often were utilised in conjunction with numerical analysis where possible.

In the coming paragraphs, I examine pertinent elements of flow that were demonstrated indicating how the manifestation of flow occurred for learners.

**Happiness**

Happiness was demonstrated by students in lessons whilst doing mathematics. For example: in lesson 8 Kyle shouted a cry when he remembered how to convert from decimal to binary (Memo 7.47); during lesson 3 Lili cheered when Josh, in front of the whole class, solved the division question (Memo 7.25); in lesson 1 a student cries out ‘oooh’ when suddenly there was an understanding of the algebraic generalisation during a class discussion (Memo 5.9). Although not a direct characteristic or condition of the theoretical framework of flow, happiness is a known outcome of the flow experience and a suggestive indicator (e.g., Nakamura and Csikszentmihályi, 2002; Csikszentmihalyi, 1997; Goleman, 1996; Seligman, 1995).

Comments in the self-assessment included; ‘the activity was joyful’ [Ollie 9th May 2015]; ‘today I had fun’ [Hassan Lesson 12]; ‘I enjoyed the activity’ [Hassan Lesson 11]; ‘I liked it’ [Paige 61]; ‘It [the lesson] was very interesting’ [Benji 1/7/15], and; ‘I am having a good time’ [Liam Lesson 5]. Student journal comments included; ‘I enjoyed the task’ [Lili (15/9/16)], and; ‘I had fun with it’ [Teo 10/16]. 60% of students suggested their positive mood assisted in their flow manifestation (part B of the self-assessment).
The structure of the flow experience – a discussion

Also, 67% of students said ‘very much’ to the question centred on an autotelic experience, (part A of the self-assessment). An autotelic experience is when the (mathematical) activity is enjoyed for itself, the outcomes were not the aim. The moments of being excited, curious and interested while discussing mathematics I suggest were a part of happiness and joyfulness. This may be because the shared experience of social flow provided a ‘joyful’ element to the mathematics classroom (Walker, 2010).

Challenge – perceived skill balance

Challenge – perceived skill balance, is an individual flow condition, corresponding to the social flow elements of familiarity, equal participation and moving it forward. Over the entirety of the study, 51% of students responded they felt the right amount of challenge ‘very much’, 39% answered ‘partly’ and 10% answered ‘not at all’ (see Appendix D). By combining very much or partly, 90% of students responded they felt the right challenge conceivably suggesting that the challenge need to only be partly met (or fully) for flow to manifest.

Most of the lessons were around 50% or higher for ‘very much’ responses to the question set around challenge-perceived skill. Lesson 9 appeared lower (22%) compared to the others. Could a reason for this be the task set was inappropriate for the majority of students, either too high or too low challenge vs perceived skill, thus effecting the flow states of students? Certainly, in this particular lesson I pitched the challenge of the task relatively high, hoping they would be able to multiply in binary without changing to decimal notation, and I left students without a solution, however, this was on purpose, as a teaching point to be brought up at the following lesson.

Students may have found the lesson difficult, and have got stuck at times, nonetheless still demonstrated flow and persevered with the task. Although in the self-report 22% said they felt challenge ‘very much’ in the lesson 94% of students felt the challenge was just right ‘very much’ or ‘partly’, and similar to the overall combined result discussed above flow manifested when the challenge was only partly met. Conceivably learners changed the problem to suit their perceived skill and adjusted the challenge for themselves (see Mason et al., 2010). For example, in lesson 9, students are working on a binary to decimal conversion task, and many adapt the challenge to their perceived skill, by instead of keeping the numbers as binary while multiplying, they change them to normal counting numbers first, even though this was not what I asked them to do in the task (Memo 7.50). Caleb, when teaching the class, adapts the challenge to make it more difficult; he
varies the number of digits in the question, changing it to a 3-figure by 2-figure, thus increasing the arithmetical complexity of the question (see Memo 7.54).

*Holistic*

The phenomenological nature of flow infers the entirety of the elements of flow do not need to all be present for flow to manifest and should be taken holistically (Csíkszentmihályi, 1990a; Csíkszentmihályi, 1975). Although the challenge-skill perception was scored lower at 22%, 92% of the students who completed the self-reports in lesson 9 said they felt ‘very much’ that they were doing the activity for an intrinsic purpose (an autotelic experience). The other scores in the self-assessment for that lesson were in line with other lessons across the research. Furthermore, flow markers were observed in lesson 9, indicating both individual and social flow appeared to occur. Markers such as *talking about the work with focused energy* (Maruf and I); *looking at the speaker with concentration and focus* (Liam, Katya); *talking animatedly and excitedly about the maths work* (Josh, Omar, Liam, Hassan); (solitary flow) *looking at the work on their own* (Jack and Katya).

*Smaller group effect*

Lesson 8 had an exceptionally high score for students experiencing the correct challenge ‘very much’ (80%), and this could be attributed to the dynamics of a smaller group size. This lesson had a smaller group size than the rest of the observed lessons (12 students were present out of a full class of 32). Conceivably, the higher challenge-perceived skill score may be because in a small group, students are more likely to communicate, describe their mathematics and share their understandings. For example, there was easier access to the dry wipe whiteboard. If the challenge of the task was not matched to their skill, then they/I could have addressed it more easily in a small group. As well as having an effect on challenge-skill perception, the smaller group seemed to also effect the response to the flow elements of being in control, and transparent feedback.
Being in control – making mistakes

Similarly, in lesson 8 there was a comparatively low score for students responding ‘very much’ to a control over their learning (20% compared to 54% of students overall). Control over learning can be grouped with the social flow element of ‘potential for failure’ as when there is control there is also room to make mistakes and learn from them; a ‘potential for failure’. The flexibility that control over learning precipitates means mistake making becomes a positive experience. Could the small group imply mistakes were highlighted more than in a larger group? There is scope for future research into group sizes and the connection between control over learning/ potential for failure, and the value of errors for learning, creating an ethos of ‘positive mistake making’.

Transparent feedback

Students answering ‘very much’ to the question centred on transparent feedback was lower in this lesson (30% in lesson 8 compared to 56% overall figure). Transparent feedback is an individual flow element that corresponds to the social flow element of communication. To probe further I would want to question students, for example if feedback was an aspect of communication students felt disadvantaged in, and what type of feedback from me that was required. Communication is also discussed later in the next chapter under social flow.

Teacher vs researcher

During analysis, there were some occasions, like the example above concerning feedback, when querying apparent incongruences may have led to further discoveries. Clarifications would need to be raised close to the time for accuracy of answer, due to the nature of phenomenon and retrospective bias (Hektner et al., 2007). However, I was not able to because of the constant pressures of everyday teaching. I was their teacher first and then the researcher and so timely follow up was not always possible. An instance of this occurred in lesson 7 with an apparent inconsistency with Benji’s responses to the self-report; he had responded ‘very much’ to eight of the nine characteristics, except for characteristic ‘having everything under control’ in which he responded, ‘not at all’. I could not unpack this with him due to time constraints (see Memo 7.39).
The structure of the flow experience – a discussion

The corollary of this was that many of the discoveries would not have been made if I were not their teacher, for example the rapport and relationships that added reliability and honesty to self-assessment responses, the organic twists and turns the research took, and the multi-slice imagining analysis was given another dimension through my inside knowledge of individuals within the class.

Standards of comparison of the student’s understanding of flow

Ensuring the students and I had a common understanding of the meaning of flow and its theoretical model was important because differing standards of comparison may affect the transparency and trustworthiness of written comments and the numerical analysis. On occasion students had disparate views as to the definition of flow. For example, when discussing the ‘doing’ element of flow students confused sleep with flow; Caleb wrote in his flow journal ‘I was in flow when I was sleeping, and I woke up and had an answer’; Liam wrote ‘Is sleep considered as flow?’ [student flow journal Lesson 4]. As a response the shared agreement that students and I had as to the definition of flow, was revisited and reinforced.

Nevertheless, the condition challenge-skill perception appeared to have a clear common meaning. Written evidence from students suggests there was an awareness of the significance of challenge according to the theoretical flow model. The self-assessment comments referred to challenge appropriately; ‘The questions seem easy (kind of) when I am in flow’ [Josh, Lesson 9]; ‘I am in flow when I have a medium levelled task, not easy but not hard’ [Liam Lesson 4]; and ‘I wouldn't get into flow because the work was easy’ (Brandon Lesson 13).

In the stimulated recall interview (Feb 2016) Jack and Ollie described the effect of the challenge of the task on their flow experience, again in line with flow theory;

**Jack:** I feel it’s easier to get everyone along when you like really know the subject like if you really know it well and you can like look at a bunch of questions and you like I can do this but you still have to get into flow. So then depending the questions can’t be too easy they got to be just right as well

**Ollie:** When you understand something you get more into flow like what Jack said but when you don’t understand something like if that something that you don’t understand like you curious about and you excited like you want to learn like you would get in to flow but like if something that you don’t know and you like whatever I don’t really care about this like you won’t get into flow.
This awareness of the definition may have increased across the study, certainly there were many more comments about challenge late on in the research. Many of the comments in the student journals (written in the last two months of the study) referred (accurately) to challenge and perceived skill, for instance; [I’m in flow when] ‘it was challenging’ [Vinton Sept 2016]; [I’m in flow when] ‘challenging questions that push me beyond my limit’ [Omar Sept 2016]; [I’m in flow when there is] ‘just the right amount of challenge as I understood it, but I had to think about it for a while’ [Lili Sept 2016]; [I’m in flow when there is] ‘Right level of skill’ [Sophie Sept 2016]; [I’m in flow when there is] ‘challenge, just above my skill set’ [Tahir Sept 2016]; and [I’m in flow when I] know exactly what I have to do to find the answer but getting there is challenging for my brain [Fae Sept 2016].

The understanding and focus on challenge-perceived skill condition from flow theory inferred students became more aware of the challenge of the mathematics, and so more aware of the mathematics itself. Some suggest that an awareness of the mathematics and mathematical processes is a central way that students learn (e.g., Young and Messum, 2011; Gattegno, 1971).

Clear goals

Lesson 7 had a higher score of students answering ‘very much’ than other lessons for the question centred on goals (75% vs 52% overall). This may be because I set clear external goals for students in this lesson; there was a worksheet with clear questions students had to go from, and also a clear series of number skills practice tasks. Lesson 10 had a score of 67% and also had very clear external goals set by me. Many of the other lessons that were scored less were less prescriptive and learners were not given ‘objectives’ or ‘success criteria’.

Nevertheless, external goals set by me, although assisting with the objectives of the lesson and fostering a longer-term interest in the mathematics are not necessarily the emergent specific challenges that arrive from doing the activity and proximal goals implied by flow theory (Jeanne, 2009).
The structure of the flow experience – a discussion

Loss of self-consciousness

Generally, over the entire study a smaller number of students answered, ‘very much’ to the question centered on loss of self-consciousness (44%); and a much higher number (24%) answered ‘not at all’. Why would students suggest this, yet answer much higher to time distortion, a characteristic which would imply some loss of self-consciousness? Conceivably some of the students and I may not have had a common understanding of non-duality and a loss of self-consciousness.

A loss of self-consciousness was possible during social flow (Sawyer, 2007); however, students may not have realised it is possible to lose awareness when in social situation. Liam wrote; ‘You don't realise you are in it [flow] until you get out of it unless you are working with someone else’ [student flow journal 14/9/16, my italics].

Complete concentration on the task at hand

This was the confirmation question. Limitations and advantages of confirmation bias questions are detailed in chapter 3, section 2. Concentration was one of the higher percentages that students answered ‘very much’ to (66% overall). Concentrating also appears to have an influence on a learner’s experience of flow emergence; in section B of the self-assessment, 79% reported that concentrating assisted in the emergence of flow (illustrated in Table 8.1 below).

<table>
<thead>
<tr>
<th>How does flow start?</th>
<th>‘yes’ response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrating</td>
<td>79%</td>
</tr>
<tr>
<td>Ignoring distractions</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 8.1 emergence of flow (concentration characteristic)

Additional evidence to the self-assessment data, were comments made in the stimulated recall interview; Layla and Jack reported that if I talked to the class when they were in flow, the distraction from their concentration would cause them to break out of it.

**Layla:** Like let’s say you give us a task and if you see people focusing like for example let’s say if you say we got like five minutes for this task while people are focusing like maybe you can make it longer we will be in flow and concentrate for longer.
The structure of the flow experience – a discussion

**Jack:** This is something that you can’t really stop it takes everyone out flow like when our head teacher maybe comes into a room (over talking) that does stop us, and it’s something you can’t stop.

Concentration may also be a component student have less difficulty identifying and thus a characteristic with clear shared meaning. Concentrating in flow theory has the same definition as it is used in everyday language and is a common word (at least more common than, for instance ‘consciousness’). The word concentrating was used extensively in the comments that students wrote in the self-assessment and was a foundation for flow markers (see Table 6.3).

**Temporal distortion**

Time passing quickly (similar to concentration) was also a verifiable measure of flow for students. During the background lesson to orient students to the definition of flow, several could associate with a time when doing a mathematics task and the time had gone by much faster than expected. The question centered on time distortion had a high number of students who responded, ‘very much’ (71%). Written comments from the self-assessment included; ‘When I’m in flow I feel different and I don’t notice time and anything else’ [Paige Lesson 5]’ and; ‘I go out of flow when I notice time’ [Liam Lesson 4]. In the stimulated recall, Fae reported that knowing the time assisted her to go into flow state, although now there was then a pressure to complete the work. The clock became a part of the feedback, a different flow condition.

**Fae:** I love the clock. Because I like I don’t know like I use to know what the exact time is and how much I’ve spent on like each thing like I don’t know why, I find it like I don’t know the time left see how much you have left and I think ok I have to get this done still have enough time and I like focus on it more to get it done

**Fae:** I feel like to be in flow I need like motivation I need like a set time that I have to finish

Other comments in the student flow journal included: ‘Flow kills time’ [Caleb 13/9/16] ‘I lost track of time and when I realised I forgot where I was’ [Denzel 20/9/16] ‘The 1 hour of maths passed by in a minute’ [Benji 20/9/16] ‘I did not realise I was in flow until I had finished the exercise’ [Fae 23/9/16].
Did students achieve flow more easily after 18 months of repeated exposure to it?

Examining the flow score (Appendix D), it would appear that the first lesson had more flow than the end lesson (1.33 vs 1.44\textsuperscript{5}). Such a comparison, however, does not hold merit as the lessons were all different, both in terms of content and students and therefore, their flow experiences would have been different. Different branches of mathematics were looked at throughout the study and there were different topics that were looked at within for example number and algebra. Some students had a higher perceived skill than others for specific topics. Some students preferred different teaching methods at differing times.

Additionally, students’ knowledge of what constitutes flow would have changed and learners may have had higher demands on what it feels like to be in flow as the study progressed. Being in flow may have become the norm and only exceptional flow experiences would be enough to be noticed. Students also arrived in the class with their own emotional ‘baggage’, and it would be different every lesson, even change during the lesson. The mood of the student would affect not only their flow experience but the experiences of those around them. An interpretivist approach would suggest that students made their own interpretation of the social life world (Crotty, 1998), and thus my analysis would be concerned with the individual case.

Thus, it may be more useful to examine individual lessons, for example looking at extreme values of flow score or using the score to pinpoint if and when flow occurred, as a comparison would have less transparency for the reasons outlined above.

All flow scores were below 1.5, substantiating the other datasets, confirming that flow manifested during the study.

*Extremes of ‘flow score’*

Lesson 10 had a lower flow score, (1.24), which would imply a manifestation of flow; students responded ‘very much’ for more of the characteristics. Although there were less students that responded to the self-assessment in this lesson, (53\%) which may have had an effect on results, in this lesson there was an investigative assessment, different from other lessons.

\textsuperscript{5} The flow score was an indicative score that summarised the different characteristics of flow (from part A of the self-assessment). It should be read inversely, the lower the bar the higher the flow score because if a student scored 1 it meant ‘very much’ (flow characteristic) and 3 meant ‘not at all’.
The structure of the flow experience – a discussion

Investigative tasks encouraged talking and communication about mathematics (and social flow), and possibly more students participated fully in the lesson because the lesson was also an assessment, meaning more communication and more interactive social flow.

Lesson 3 had a higher ‘flow score’ (1.47) implying that flow occurred (it was less than half the maximum score of 3) but less students responded ‘very much’ to less of the nine characteristics. There were no obvious differences to the teaching in this lesson. Students were teaching one another for some of the time and flow markers were observed.

Nevertheless, at one point I intervened and talked ‘at’ students trying to impart a division method. The task was not investigative. There was also a point in the lesson when some appeared disengaged, and this appeared to be because the challenge may have been too high for these students. This may have contributed to the lower score.

What can I say about the variability between individuals?

In addition to looking at a flow states being demonstrated in different classes; I used the flow score as a measurement to examine how flow manifests in individuals. The flow score is a self-assessed indicative measurement of the characteristics and conditions of flow. (see Appendix D for a further detail of these scores and the flow score for individuals across the lessons.)

The scores were not utilised as a measurement of comparison. As discussed previously, lessons were not identical, with different and unknown variables, such as content, environment and context. The ephemeral nature of flow also makes comparisons difficult as experiences are interpreted differently, by the observer (me) and the observed (students). The sample is incomplete, due to absences and non-returns which would also affect a comparison of individuals.

Nonetheless, with the caveat outlined above and in the previous section, the individuals flow scores are useful to look at individuals. Fae, Ike, Teo, Denzel and Benji (and possibly Paige) in particular appeared to have low overall flow scores, suggesting that in these instances flow states may occur more readily and immediately for them.

Additionally, it appeared as the study progressed and as students entered flow with greater frequency, they seemed to become more prepared and willing to enter flow. The intrinsic motivation experienced by students in the research lessons may have nurtured and influenced learners into flow experiences and developed autotelic personalities (Nakamura, 1988).
An autotelic personality is not something that is innate within a learner, rather a learned disposition (e.g., Kotler, 2014; Jeanne, 2009). The feelings of joyfulness that were intrinsically created by learners and the flow experienced during lessons was a reciprocal cycle that led a student to develop an autotelic personality and enter flow states more readily. I recognise that more information is needed to identify students who may have an autotelic personality and the remit of the study did not allow for a development of this finding, for example there may be ethical reasons in attempting to single out specific students who were or were not showing a propensity towards flow.

To conclude this chapter, flow manifested in the mathematics classroom and the characteristics suggested in the theoretical flow model were demonstrated on multiple occasions by multiple students in the lessons observed. In this chapter I have presented and discussed the results from the self-assessment to substantiate this. In the following chapter, I further discuss RQ1, how is the notion of flow attested, manifested and demonstrated by mathematical learners, examining flow markers, social flow, the impact of breaks on lessons. I also discuss RQ2, what pedagogy elicits flow, exploring teaching approaches and task design.
CHAPTER 9: Further Discussion, Interpretation and Reflection

This chapter continues from the previous chapter answering the research questions using my analysis and synthesis of the data.

Section 1 primarily answers RQ1; How is the notion of flow attested, manifested and demonstrated by mathematical learners? Conjectures discussed in section 1 include; attesting flow with markers; students demonstrating flow; the manifestation of social flow; and breaks having an impact on students demonstrating flow.

Section 2 of this chapter contains a short discussion of what I, as teacher, was doing when flow occurred. It incorporates the mathematical tasks set and what pedagogical routines were utilised. Section 2 is predominantly a response to RQ2; What pedagogy (including task design, and questioning techniques), elicits flow?

Section 1: Aspects of flow – research question 1

Attesting flow

Conjecture: The manifestation of the flow experience in the classroom was demonstrated by learners displaying distinctive physiological behavioural features, which I have termed ‘flow markers’.

Flow markers are empirical indicators of the manifestation of the flow experience. Initially presented in chapter 6 this notion is established and substantiated throughout the data with the analysis leading to further development of individual flow markers and inclusion of the classifications interactive and co-active social flow markers.

See chapter 3 and chapter 6 for more detail on flow markers.
Interactive flow emerges when learners are interacting and in a dialogue. Conversation is essential for interactive flow to occur. It is not simply individual flow in a social situation, as individuals behave differently in groups than in isolation (Lee, 2006; Pimm, 1989). Interactive social flow has a theme of involvement, when students are actively involved with another’s thought processes, contributing and discussing.

Co-active flow occurs when students’ individual activities emerge within a group. Although the phenomenon may demonstrate as individual flow, other individuals are present and may impact the experience (Walker, 2010). Co-active social flow has a theme of involvement but not contributing and has some limitations in its identification (which will be discussed later in the chapter). Using a sporting analogy, golf would be (in the main) a co-active social flow activity and football would be (in the main) an interactive social flow activity.

Individual flow is an experience which is had alone, although others may be present, they do not impact the experience as in co-active flow. Individual flow has the theme of being alone in the mathematics, working out solutions and answers individually.

List of updated markers

Table 9.1 below details a final list of updated markers, organised into social (group) flow (interactive and co-active) and individual (individual) flow. Any overlap to the descriptions is purposeful as my attempt at attesting a student’s experience was not to reify flow, rather to celebrate the complexity of the phenomenon (Csikszentmihalyi, 1992). Thus, the markers in Table 9.1 below are inherently descriptive to minimise any loss of detail in the individual experiences. Moreover, the nuanced descriptors are in line with multi-slice imagining methodology; attempting to look ‘through the pictures rather than at the pictures’ (e.g., Konecki, 2011; Bohnsack, 2008) and utilising the characteristics and conditions resonating with the theoretical models of individual and social flow proposed by Csikszentmihályi (1975) and Sawyer (2007).
Further Discussion, Interpretation and Reflection

<table>
<thead>
<tr>
<th>Interactive Social flow</th>
<th>Co-active Social flow</th>
<th>Individual Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discussing the work to the exclusion of all else.</td>
<td>Looking at the speaker, listening and not contributing, with concentration and focus.</td>
<td>Head bent over work writing; head close to the task.</td>
</tr>
<tr>
<td>Talking animatedly and excitedly about the maths with focused energy. Watching the board and contributing.</td>
<td>Looking at the task, and not contributing actively. Hands up wanting to answer a question.</td>
<td>Looking at the work on their own, in ‘thought’, focused. Head resting on hands, in ‘thought’ focused, or hand on chin denoting thinking. Oblivious to noise and distraction, working very quietly not talking of anything else.</td>
</tr>
<tr>
<td>Excitement is evident, a heightened mood.</td>
<td>Hand on chin, listening but not participating.</td>
<td></td>
</tr>
<tr>
<td>Focused on the speaker but also engagement and participating in the conversation. A focused attention on the discussion with talk contribution from 2 or more.</td>
<td>One person talking, another or others passively listening.</td>
<td>Interaction is not occurring and there are others present who are interacting.</td>
</tr>
</tbody>
</table>

Table 9.1 final update of flow markers

The theoretical model of flow originally proposed by Csíkszentmihályi (1975) does not categorise the elements into conditions and characteristics. This categorisation was made later (Kotler, 2014; Jeanne, 2009) and is made on the basis that some elements (conditions) are causes and predictors of flow, (challenge vs perceived skill, clear goals and transparent feedback), and some elements (characteristics) are displayed and observed during flow (merging of action and awareness, a loss of self-consciousness; concentration on the task; being in control; temporal distortion; and autotelism). Jeanne (2009, p. 10) considers these dimensions can be; ‘organised in several models of phenomenology of flow that start with antecedent conditions and end in outcomes.’ Sawyer (2007) suggests that there are 10 slightly different yet analogous and corresponding components of social flow. Using this, social flow can be incorporated into the flow model of predicative and descriptive elements. This model assisted in the development of the flow markers; the characteristics (outputs) were used initially and in main as guidance to the ‘markers’ of flow. Figure 9.1 below illustrates this.
Further Discussion, Interpretation and Reflection

<table>
<thead>
<tr>
<th>Conditions (predictive ‘inputs’)</th>
<th>Solitary flow elements</th>
<th>Social flow elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenge vs Skill</td>
<td>Familiarity / Equal Participation</td>
<td></td>
</tr>
<tr>
<td>Goal Setting</td>
<td>Moving it forward (probably closest to challenge vs skill)</td>
<td></td>
</tr>
<tr>
<td>Transparent Feedback</td>
<td>The group goal</td>
<td></td>
</tr>
<tr>
<td>Merging of action and awareness</td>
<td>Communication</td>
<td></td>
</tr>
<tr>
<td>Non-duality and a loss of self-consciousness</td>
<td>Blending of egos</td>
<td></td>
</tr>
<tr>
<td>Concentration on the task</td>
<td>Close listening</td>
<td></td>
</tr>
<tr>
<td>Being in control</td>
<td>Complete concentration</td>
<td></td>
</tr>
<tr>
<td>Temporal distortion</td>
<td>Being in control / Potential for failure</td>
<td></td>
</tr>
<tr>
<td>Autotelism</td>
<td>Both of these are elements of social flow but are not named as such by (Sawyer, 2007).</td>
<td></td>
</tr>
</tbody>
</table>

Characteristics (descriptive ‘outputs’)

<p>| | |</p>
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<tr>
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<tbody>
<tr>
<td>Merging of action and awareness</td>
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<td>Temporal distortion</td>
<td></td>
</tr>
<tr>
<td>Autotelism</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9.1 flow conditions and characteristics; a restructure to include group flow.
This diagram illustrates the descriptive and predictive framework of the theoretical model of flow. It is a copy of Table 2.2 in chapter 2. Inputs are those elements that can predict flow and influence its manifestation. Outputs are those elements which are easier to observe. Social flow conditions can also be categorised similarly, specifically due to the analogous replication in Sawyers model of social flow components. It appears from my observations that elements are fluid and can overlap.

Interchangeability to the characteristics and conditions of flow

There appeared to be some scope for the interchangeability and overlapping of conditions and characteristics (see Kotler, 2014), in particular concentration and time distortion. For example, Katya reported that she would ‘force herself’ to concentrate in order to get into flow, ‘I told myself -okay there is 15 minutes left, let’s do some hard-core maths’ (Error! Reference source not found.); ‘I was in flow because I really wanted to be first, so I tried really hard and concentrated’ [Fae, self-report Lesson 7]. Taken over the entire study, 79% suggested concentration was a factor in starting flow (in part B of the self-assessment).

Time distortion is another example of a characteristic that may be manipulated purposely in order to engender flow. In the stimulated recall interview, Reuben and Ollie commented on time, and how to manipulate it.

Reuben: don’t tell us how long we have left

Ollie: I got an idea like you should like cover the clock, so we don’t look at the time, so we don’t know like how much we have left
Manipulating and influencing concentration and time distortion meant they became a condition of flow (alongside an observable characteristic). There is room for future research on how this could be done.

**Limitations of markers**

The flow markers are a perception by an external observer of an individual’s complex psychological state using an image. In other words, an interpretation of the state of mind of an individual in an image by someone other than the individual in the image. This may present issues as the interpretations may differ. To mitigate this (bracket), wherever possible they were not taken as the sole indicator. The analysis of the combined datasets forms the evidence which contributes to any conclusions. The self-assessment was often utilised as additional information, alongside conversations recorded in the field journal. For instance, in lesson 3 (Memo 7.21) Kyle and Caleb were *focused on the speaker but also were engaged and participating in the conversation*, an interactive flow marker. Both also responded ‘very much’ to the question on feeling the right amount of challenge in the self-reports. Examining the congruency of flow markers is an example of bracketing methodology; clarifying an understanding and interpretation of the phenomenon (Drew, 2004; Laverty, 2003).

Use of the markers was advantageous, for example, when audio was not able to be transcribed, they were a useful dataset to identify possible instances of flow in lessons. To provide an illustration; in lesson 7, there is poor audio quality, but it appears that Andrew and Brandon are *discussing the work to the exclusion of all*; an interactive flow marker. It is this marker that identifies flow occurring despite the lack of audio. See Memo 7.33.

Co-active markers were harder to determine when used on their own as a dataset. An awareness of the vulnerability of this category in particular I found apposite. The vulnerability stems from the definition of co-active flow; others are impacting the experience when a student is in co-active flow (see also Walker, 2010). For instance, in lesson 3, after Josh had explained a particular solution to the class, and Paige was applauding, utilising only markers, Nicky could have been in individual flow while Josh was explaining, and then in co-active flow, or he may not have been in flow at all (his head was close to the work). This is shown in Memo 7.25.
Also, if co-active markers were mistaken for classroom interactions not concerned with the activity, then flow may not be occurring such as a student not engaged with the activity nonetheless looking at the teacher. An example of this is demonstrated in Lesson 1; Caleb is at the board explaining and Caitlin is looking at him; it is not clear if the marker is accurate; looking at the speaker, listening and not contributing, with concentration and focus (see Memo 5.18).

When a student appears to be working on the mathematics alone it may be problematic to reliably attest if the student is being impacted by another and thus distinguish between co-active and individual flow. In Lesson 3, for instance, I was engaging the class in a conversation about whether it was possible to concatenate as a solution, and Ollie was either looking at the work or at me (Memo 7.15). In these circumstances any doubt lies not in whether flow states were either seen by me or reported by students, rather in if the flow was categorised co-active or individual.

In summary, flow markers are a set of qualitative descriptors which provide the means to identify flow states demonstrated by learners in my mathematics classroom.

**Social (group) flow**

Conjecture: social flow characteristics and conditions manifested within the classroom; in particular interactive social flow was demonstrated during the study.

Social flow characteristics are familiarity; equal participation; moving it forward; the group’s goal; communication; blending egos; close listening; complete concentration; being in control; and the potential for failure (Sawyer, 2007). Series 1 lessons demonstrated several incidents that supported the manifestation of social flow (e.g., Memos 5.4; 5.5; 5.7; 5.16) and series 2 and 3 lessons demonstrated further that social flow manifested in the mathematics classroom in the study (e.g. Memos 7.19;7.28;7.36;7.38;7.40;7.60;7.63). Characteristics are detailed next. Being in control, close listening, blending of egos, complete concentration and moving it forward are also discussed in chapter 6 as outcomes of a silent teaching approach.
Equal participation

When I observed this element, there was enough familiarity with the mathematical concepts that the class were able to participate in a group mathematical discussion. One example of this was shown in lesson 1; Ike is given a board pen with no further input from me. He writes on the board and there is enough familiarity with the mathematics to engage in a mathematical conversation with Paige, Kyle and Caleb (see Memo 5.6). Moreover, often a negotiation as to the meaning of the activity occurred whilst students talked about issues relating to the mathematics, which would sometimes be in feedback of a corrective nature. As an instance, in lesson 2, students and I engaged in a conversation about money to illustrate division. (see Memo 7.4). Negotiation of meaning is a part of ‘equal participation’ (Armstrong, 2008).

Moving it forward

In order to foster social flow, the theoretical social flow model suggests the leadership of the group needs to be fluid in order to ‘move it forward’ (Kotler and Wheal, 2017). Where possible, I deliberately attempted to embrace this concept when I led the class. I attempted to provide openings for collaborative and dialogic learning (as opposed to transmission and didactical) in order to foster fluidity in the leadership of the mathematics. The silent teaching approach would often lend itself to this. During lesson 1 for example, Ike got stuck on a question, and the class began to flounder and the interactive flow that was occurring was lost for a while. Immediately on seeing this I transferred the pen to Caleb, as I knew he had some preconceived knowledge and could continue the mathematics (see Memo 5.14).

Separating aspects of leadership into mathematical leadership and an organisational leadership was useful as students could more easily lead mathematically. Often these facets were intertwined. Oyler (1996a, p. 6) calls these dimensions ‘content authority’ and ‘process authority’. She powerfully states; ‘when I speak of process, I mean the teacher is in authority for classroom procedures: who gets to do what, when, where and how. In contrast the content dimension of authority pertains to the teacher as an authority on classroom knowledge: what counts as knowledge and who is validated as a knower.’
Process authority assisted with the focus on studying, and an equanimity amongst students. An instance of this is demonstrated in lesson 7 (Memo 7.35); I gave the timings of the number skills ‘countdown’ task (process) but no mathematical oral instructions (content).

There were examples of students providing content authority, for instance in lesson 11 when Josh is using long division as a method of dividing binary numbers and he explains to the class. (Memo 7.63). There were also occasions when learners were able to self-govern in both these dimensions, for example in lesson 9, Liam writes his correct solution and explanation of multiplying binary on the dry wipe board, writes another question on the board (11 *11) and then gives the pen to Caleb to solve it (see Memo 7.53).

**Group goal**

During the research lessons I would often ask students to collaboratively solve a mathematical activity. An instance of this is shown during lesson 1; I provide a slide that has multiple solutions to an algebraic problem and rather than modelling an answer, instead invite a student to provide their solution to the class (see Memo 5.6). Collaborative activities provide a group goal, another suggested element of social flow (Sawyer, 2007). Generally, I worked on establishing a classroom atmosphere of co-operation, where collective learning could take place.

The journal entry below further sheds-light;

*Journal entry October 19th, 2016*

*Mrs Mieville is a senior teacher and Head of Year.*

Mrs Mieville had a conversation with me today. I was not in lesson yesterday and her words about the lesson were both encouraging and interesting: ‘You have a bunch of geniuses. They were all helping each other. Although there was noise, I thought it was justified, they were all working hard, and really interested’; ‘They were all keen and participating, working with each other’.

The feeling I got was that my work with them, encouraging them to question, to talk about the mathematics (with most of them it had been nearly 2 1/2 years of encouraging them to talk), has had real benefits. Students are questioning processes, and methods. Students are talking about generalising, about specialising, and other mathematical vocabulary. Students are experiencing flow and associating this with mathematics.
Blending egos

When I noticed a blending of egos (characteristic of social flow) in an observed lesson, a single individual was not dominating, instead all participants were contributing to the situation and moment in time. For example, in lesson 7 Caleb was at the dry-wipe board explaining how to multiply two binary numbers by converting to counting decimal numbers first. Liam, Kyle, Josh, Katya, Layla and others are listening and contributing (Memo 7.53). In order to encourage collaboration and the blending of egos I actively stopped and discouraged put downs, rebuffs, insults and disparaging remarks.

Communication

Conceivably when interactive social flow was demonstrated by students it was because of the communication through social interaction that occurred and was encouraged. For instance, in lesson 2 I encourage Reuben, Shurik and Hassan to verbalise their difficulties with the mathematics, and I encouraged this by developing their criticality through questions (Memo 7.17). Social interaction is important if social flow is to manifest (see also Walker, 2010; Sawyer, 2007). The conversation adds to the task, something that the task alone cannot provide. Supportive evidence of this notion additional to the flow markers includes the many written comments about communicating and social interaction as a means of getting into flow in the self-assessment. Written responses to the question, ‘To get into flow I will…’ included; ‘talk about the maths’ (Hassan), Work with people around me (Ava), Focusing and talking about the activity (Teo), Talk about the work (Omar). (Lesson 13). The question stem ‘[I’m going to get into flow by…] was responded ‘talking about the work’ in by different students throughout the study [e.g., Fae 14/6/16; Benji 03/03/16; Josh 10/3/16]. Examples are shown below in Figure 9.3.
Other comments made about communication were written in the student flow journals: ‘Stay in flow by talking to those who stay on topic and know what they’re doing’ [Paige 03/03/16]; ‘I was in flow for over 45mins and I think it was because I was talking with Teo’ [Hassan, 10/03/16].

In the stimulated recall interview, Ollie reported a description of his experience of interactive social flow which consisted of talking, discussion and communication.

*Me:* Tell us about a time you were in flow?

*Ollie:* like when we having like a normal discussion not like too loud but we just talking about the math and something that like excites us and like everyone wants to like put their hand up and speak that puts us in flow and then when you talking about something and then like you curious about it we like want to talk like to our partners next to each other like we talk about like it and add more like information on it

*Noisy classrooms*

Social interaction can lead to noisy classrooms, and there were occasions because of the talking when there was opportunity for students to behave in such a way that their learning was compromised. As an example, in lesson 13 there is a lot of mathematical conversation being encouraged amongst students, and for a while Benji and Denzel are off task; Denzel throws a pen across the room to another student (see Memo 7.74).
Rather than trying to stop this interaction and conversation, an alternative I explored was to welcome and embrace conversation as inevitable and expected, channelling the conversation towards mathematical activity and social interactive flow. Through the emphasis on conversation and interaction students learnt how to work with one another in the classroom, fostering an atmosphere of collaboration and co-operation. In lesson 12 for instance, students work on a short numerical task as they enter the classroom, and although the purpose is to settle learners as they arrive from break, I encourage them to talk about the mathematics, attempting to foster mathematical co-operation (Memo 7.76).

**Feedback**

The content of the conversation itself will have an effect on the flow occurring, and what others are inputting is important. When involved in the conversation, I would need to match the skills of the student(s) and ensure the feedback is accessible. For example, in lesson 9, whilst at the board teaching the class, Maruf makes an error in his solution when converting to decimals from binary. My feedback at this point is to indicate it is a mistake, but not what the mistake is. Maruf perseveres and solves the problem correctly, but without an explanation; I then share with the class some of the conceptual ideas, as it appeared not all of the class understood at that moment in time. (Memo 7.55). Another instance was in lesson 2, when I engaged in a dialogue with Liam and Jack and used money as a way of illustrating division of the number 1; making my feedback accessible and bespoke to their skill needs (Memo 7.4).

Transparent feedback is a central condition to flow (e.g., Csikszentmihalyi, 2014). It could be provided internally; a student would correct the mistake or move forward themselves without external input (see Figure 9.4 below). The feedback could also be provided externally, by another student or myself. It could also be provided by a mechanical means, for example a worksheet, a textbook, or another media source such as a video or internet page. In lesson 11, for example I provide a crib sheet for students to use as feedback, in lesson 7 a video is used to provide feedback.
Figure 9.4 a student flow journal entry written by Caleb. He used internal feedback (his previous experience with his brother) to assist his mathematical activity.

Sometimes the feedback students required was regarding the corroboration and approval of their working. This would sometimes be necessary, for example when checking a solution is correct but at other times would be a validation that is not the feedback suggested by flow theory. Students would seek an authorisation for questions that they quite rightly knew the answer to. As much as possible, I would deliberately ignore such approaches; for example, students didn’t ask me if the chair was really black in colour, or the table had legs, but would ask me if they had the answer right to a specific mathematics question. An instance in lesson 13 illustrates this; Josh asked me if the work he had completed was correct, which I felt he knew was right already and instead of providing an immediate textbook answer, I asked him how he could check his work himself, he suggested using an inverse method and went ahead, without any further input from me (see Memo 7.76). This resonates with Gattegno (1971, p. 59) who eloquently states; ‘clearly, if we know how to remain in contact with our own sense of truth during our meetings with students, they will understand what we are offering them and take responsibly for the integration of this knowledge in their self…’
A student may also find it necessary to talk the question through externally, as in the previous vignette and suddenly the answer arrives for them; it is the act of talking out loud the question that suffices. In lesson 2, Layla asks me a specific question about the activity; ‘could she use the same number more than twice’. I ask her to justify why she is asking (instead of providing an immediate answer) and Layla correctly answers her own question. She knew the answer already. (see Memo 7.13). This is substantiated by Mason et al. (2010, p. 98) who write, ‘I have often found that the act of trying to express my problem to someone else produces sudden insight into what is blocking me, without my listener having to utter a single word!’.

It would not only be me, as teacher that would be providing external feedback. Peers would offer feedback in their dialogue, as in lesson 2 Ollie and Reuben are quietly talking and comparing answers (see Memo 7.2). Is there a differing effect on the flow that is manifested depending on who provides the feedback? I discuss this aspect next.

Talking with teacher and talking with peers

62% of students reported talking with their peers engenders the start of flow. Less than half of this number (29%) suggested that talking with me (as their teacher) started a flow state. (Table 9.2 below).

<table>
<thead>
<tr>
<th>How does flow start?</th>
<th>‘yes’ response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talking with my friends</td>
<td>62%</td>
</tr>
<tr>
<td>Talking with my teacher</td>
<td>29%</td>
</tr>
</tbody>
</table>

Table 9.2 the emergence of flow (talking with friends and teacher). 12 mathematics lessons (n=154-178)

Comments in the self-assessment supported this; ‘It was fun working on the whiteboard and the maths made it easy. When I'm in flow which I experienced the right thoughts just came and I didn't know I was doing it. And because I got to do it with a best friend it made it more enjoyable’ [Paige (Lesson 1)]; ‘Because I got to work with my best friend it made it more enjoyable and comfortable and we managed to do the task extremely quickly’ [Lili (Lesson 1)]; To get into flow I will; ‘Communicate with my peers (Amber 5/2/16).

Peer teaching was reported as engendering flow during the stimulated recall interview;
Further Discussion, Interpretation and Reflection

**Ollie:** In that one, on that one when I was talking to him he was confused and I was telling him how to like do an order when order was not in the question and because we talking and he still can’t understand it so I had to like talk more and like get into more detail until he understood it after that I think they was the fuel that got me into flow.

Like talking about the work and am explaining to someone that got me into flow.

**Reuben:** Talking to Fae cause yes, it’s easier to talk ... it’s easy for me to get in flow then.

**Layla:** I think the best place for me to concentrate is like to sit to a person that you know ... I can ask her anything and it gets easier

Why would many students report that flow was likely when they talked with their friends? Conceivably because of challenge-perceived skill condition (Morrison, 2018). Students that are communicating mathematically with peers can often share their skillsets and also produce more challenging tasks (e.g., Lee, 2006).

![Figure 9.5 a student flow journal entry written by Katya. She attaches an importance to her friend as a way of obtaining feedback and engendering a flow experience.](image)

Comments in the stimulated interview from Paige, Fae and Layla, substantiated the notion, that me talking would sometimes cause a distraction rather than an assistance to flow states;

**Paige:** When you talk a lot, it sorts of confuse me

**Fae:** When you are like explaining I don’t really get in the flow then because it’s not really, we not really doing anything we just listening but when you actually like doing something like it’s easy to get in to flow if you like fix on what you do if you doing something you probably go into flow I think.

**Layla:** When you start talking sometimes it might make a bit more noise to like to distract you
Why would ‘talking with my teacher’ appear to engender flow far less than ‘talking with my friends’ (62% vs 29%)?

As their teacher, often I was the dominant player in the classroom. I may have presented a barrier of authority; the source of the very knowledge that the student was seeking was a barrier to that knowledge. However, students talking with their friends was not a panacea, and there were instances in which talking with friends did not always cause or protract flow states. In lesson 13 for example, Josh and Ollie seemingly did not place great emphasis on talking with friends, moving between individual and social flow; their progress did not seem to result necessarily from interactions with others and the associated social flow (Memo 7.86). Another instance was in lesson 1; Caitlin scored low in the self-report to the question centered around talking about the mathematics as a cause of flow (see Memo 5.7).

In addition, my role as teacher did not exclude me from experiencing social flow, and I would also at times be a part of the social flow. There is some evidence to suggest that secondary and primary teachers experience flow in high frequency, (Jeanne, 2009), and the journal entry below illustrates some of my earlier thoughts on this.

Journal entry 27th May 2015

When in a class, and I am interacting and talking with a group of students, and I am experiencing a flow experience, are the students too in that experience? Are all of the students in that experience? Is it necessary for flow to be experienced by everyone in the group for social flow to happen?

Co-existence

Social flow is more than a series of individuals manifesting individual flow, the interaction and dialogue adding a distinct dimension (Kotler, 2014). As discussed previously, there were occasions during lessons when individual flow and social (and co-active) interactive flow co-existed in the classroom (e.g., memo 5.18; 7.3; 7.6; 7.26; 7.49; 7.86). In addition, a student seemed to be able to move between the categories, for example starting in individual flow, then in social flow and then back to individual. For instance, in lesson 13, Benji and Denzel would talk about the worksheet with one another, and be in social flow, then work individually on the same worksheet; in solitary flow, and then compare answers, in social flow (see Memo 7.82).

The utility of categorising individual and social flow was to be able to draw distinctions but not necessarily to compare. Comparison could a topic for further research and there is also more scope and opportunity for further investigation generally around co-active flow.
Further Discussion, Interpretation and Reflection

The diagram below (Figure 9.6) summarises the apparent effect of students and teachers talking about mathematics.

**Talking and Flow**

- When there is a teacher talking and a student talking about mathematics (top right-hand quadrant), often this will consist of feedback as the focus of attention in the conversation. Flow is likely to occur.
- When there is a teacher not talking and a student talking about mathematics (top left-hand quadrant), this could be when a silent teaching approach would occur. It could also be a group conversation about the mathematics, without a teacher involved. Flow is likely.
- When there is a teacher talking and a student not talking about mathematics (lower right-hand quadrant), this could passive learning; a teacher talking at a class rather than with a class. Flow is less likely. Nevertheless, with carefully thought-out active tasks and planned questioning, co-active flow is possible.

*Figure 9.6 illustrating how talking may affect flow*
• When there is a teacher not talking and a student not talking about mathematics (lower left-right hand quadrant), this could be when a task has been set for students to complete silently. If the task is designed carefully, then flow is possible.

To summarise at this juncture, encouraging talking about mathematics engenders social flow; in particular, talking with peers seems to engender social flow. Group and individual flow are not mutually exclusive in the mathematics classroom, that although different, both can co-exist. Individual flow and social flow occur side by side in many of the lessons examined.

The breaks

Conjecture: The breaks that occur between flow states are beneficial for flow states to manifest.

What is a break?

I am defining a break as an opposite of flow. The mathematical activity is not providing challenge stimulus and flow is not present; in other words, the interrelationship with challenge and perceived skill is absent. A break could be a social situation, for example, talking with a peer, laughing and joking. In lesson 13 for instance, Omar and Liam appear to be taking a break, both are smiling, talking with one another and it appears they are not working on a mathematical task (Memo 7.77).

There is some literature on the subject of breaks and the flow experience (although not specifically relating secondary mathematics students to flow and breaks). For example, Benson and Proctor (2003) posits a four-stage cycle consisting of preparing (struggle), entering (release), being in (the zone), and exiting (recovery). Flow is analogous to the ‘being in the zone’ stage and the classroom breaks in the mathematical activity I observed would be analogous to the ‘entering’ phase.

Exemplification

A representation of how breaks are demonstrated by a pair of students during series 3 is shown below in Figure 9.7.
Figure 9.7 timeline showing breaks and flow periods in series 3 for Benji and Denzel
This diagram illustrates how students can move in and out of flow. I used flow markers to determine when the students were in flow.

Figure 9.7 above is a timeline of two students, Benji and Denzel moving in and out of flow. It illustrates a pair of students and the times they were in flow and the times they were out of flow. I
have included this timeline as further evidence that breaks and flow co-existed within my classroom. The timeline illustrates these two students are not in flow for the entire lesson, rather they have 5 breaks interspersed with periods of individual and interactive flow. The data was taken from flow markers and memos in series 3. I chose these students to show this conjecture, in part because they had more obvious markers to show both in and out of flow and in part the locality of the camera to their seating position in the classroom meant there was a clear view as to what they were doing. A break which takes a student out of flow was seemingly an important part of the experience and assisted flow manifesting with consistency and regularity. For example, written comments from students endorsing breaks as a beneficial feature included [I’m in flow when I am] ‘doing a specific amount of questions then having a small break before I do another set of questions.’ [Self-assessment, Katya (Lesson 13)]; [I am in flow] ‘when I do something entertaining/fun or stare at a dot and try to clear my mind’ [student flow journal, Josh (14/9/16)].

There was also evidence that students would return to flow after breaks. For example, near the start of lesson 13, Denzil, Benji and Josh take a break, and then enter flow again (see Memo 7.74). A student flow journal entry by Fae illustrates this notion; ‘…it helps when I have a short break when Paige gave me [sic] a city to find it took my mind off the work for a minute and then I got back to the work and it was easier to get in flow.’ (Figure 9.8 below.)

Figure 9.8 flow journal showing the value of breaks
Transition

From watching the video footage of the series, it seemed that some students take longer than others to get engaged with the mathematics and they appear to possibly need a ‘warming up period’. Some students get straight into the work. Before a student goes into flow, or emerges into a break, there often occurs an interval of transition. Transition looks like somewhere in-between the two states. This would be for future investigation.

Structure

On some occasions I provided a structure for students to have breaks, for example in lesson 2, I enabled a break by asking several students to hand out books (Memo 7.7). This assisted in the defining of the boundaries between mathematics and non-mathematics. Sometimes there was not a structure; in lesson 2 students were distracted by the camera (Memo 7.5), or in a different lesson, Mr Murphy (another teacher) entered the room distracting students and creating a break (Memo 7.60). It was also difficult to co-ordinate timing of breaks due to differing needs of students. In lesson 9 I called the segment of the lesson to a close, and some were part-way through the activities and wanted to continue working, either on the initial task set or on an increased challenge multiplying binary numbers without changing to decimal counting numbers first. There were others who had finished the initial task and may have already started a break or wanted to start a break (see Memo 7.58).

Providing purposeful and structured breaks can provide opportunity for the ‘entering’ state by ‘interrupting preceding mental patterns’; prompting an emotional state similar in description to flow (Benson and Proctor, 2003, pp. 40-43). Kotler (2014, p. 120) describes the break in between flow states as ‘to take your mind off the problem’. Mulling, a notion proposed by Mason et al. (2010) is additionally similar to the notion of breaks; requiring a learner to distance themselves from the problem through mathematical reframing.

Breaks have an effect on the flow experience, and there is scope for future investigation. For example, what would be the optimum structure for a break in a lesson; could adding ‘non-flow’ markers assist in identifying breaks; what could a break look like; could the length of time a student is in flow be increased; could there be ‘learning’ that could lengthen the duration; is the amount of times that a student enters flow during a lesson related to the length of time and so on.
It may be pertinent to examine the length of time of a break, the effect on the frequency of flow through breaks. Is there an optimum number of breaks that exists for the maximum period of flow to occur? Is it context dependent? E.g., how does the complexity of the task relate to the length and frequency of breaks?

In summary, empirical evidence from the observed lessons, including an indicative timeline of two students’ experiences during series 3, flow markers and comments from self-assessments, suggest students ‘go’ in and out of flow, and flow experiences were often juxtaposed with break periods (e.g., memos 7.5;7.23;7.36;7.75;7.78;7.83). I suggest flow states cannot be sustained indefinitely, and breaks are necessary for it to occur without restraint.

Section 2: What was happening in the classroom when flow manifested?

Although the research concentrated in main on the occurrence of flow and its manifestation in students (RQ1), it also seemed apposite to examine the mathematics and pedagogy that was happening at the times flow was observed in order to reflect on RQ2; what pedagogy (including task design, and questioning techniques), elicit flow? In this section I look at various teaching approaches utilised to promote independent learning and the type of activity when flow was demonstrated.

Teaching approaches

Conjecture: characteristics of flow were engendered by encouraging student independence and ownership of learning.

Independent learning

The term independent learning is used here in line with learner autonomy; described by Holec (1981, p. 3) as ‘the ability to take charge of one’s own learning’. Nevertheless, there are distinct interpretations of the term (Ivanovska, 2015). In my experience coaching both beginning and more experienced teachers, often students that are encouraged to independently work on a task are directed to work on the task as a solo venture.
The definition I prefer to use here distinguishes from individualistic connotations, emphasising the value of interaction and interdependence and the shifting of mathematics into the learner’s world. A student who is engaged in independent learning is active in their learning, possibly working with other peers and adults but seeking initiative and solutions for themselves. When mathematics is active then effective learning takes place. There may be input from a teacher, although often it would be to encourage initiative.

This definition aligns closely with subordination of teaching to learning; a concept based on the premise that learning is a transformation from teacher to learner and the most effective learning occurs when a student transposes a teacher’s ideas into their own (Gattegno, 1971) Knowledge is not preconceived, is not something that is passed on by a teacher to a learner, and memory is not a channel for instruction. It opposes the idea that knowledge is a gift, to be ‘given’ by a teacher rather there is space for natural mathematical powers to flourish (Gattegno, 1971; Mason and Johnston-Wilder, 2004a).

Conceivably independent learning engenders flow states because it gives students control over their learning, a characteristic of flow. Autonomous students may also have more control over challenge complexity and be able to adjust the challenge to match perceived skill. Independent learning implies an intrinsic motivation and thus an autotelic experience (another flow characteristic) would be accented.

Many of the routines that I used in my lessons encouraged independent learning, and subsequent flow states were observed in students. This resonates with Walker (2010) who suggests rituals to establish flow are a characteristic of social flow. I detail and describe these routines below.

**Classroom routines to establish flow**

**Silent teaching approach**

During series 2 and 3 I continued using a silent teaching approach and flow characteristics were observed. For example, during lesson 9 there is a silent section of the lesson and social flow characteristics such as ego blending, concentration, no fear of failure can be observed. Liam is at the board explaining, he finishes his explanation and without any prompt from me he gives the pen to Caleb who then continues with the explanation, both of them involving the rest of the class with the mathematics; the class becoming self-sufficient (see Memo 7.53).
Comments from the stimulated recall interview also confirmed the effectiveness of a silent teaching approach in enabling flow characteristics.

**Paige:** You know when you go up to a white board and you don’t say anything, and you just write down I concentrate more.

**Layla:** You know about the white board thing that’s really useful as well cause like for example if am doing my work and your talking I have to stop my work so like I know what you saying whereas if you put on the white board I can carry on with my work and then it’s still going to be there and I can look at it later.

I suggest that a major significance of the silent teaching approach\(^7\) was to encourage independence in students leading to an emergence of flow states. For example, the ‘noise vacuum’ that I created by the removal of my oral communication is filled by learners, which allows an independence of learning and a ‘flourishing’ of natural powers. This is shown in lesson 3, in which I do not talk or contribute orally, and I invite a student (Josh) to go to the front and lead the explanation, around division. He engages with members of the class e.g., Kyle, Caleb, Fae, Layla and completes a solution and explanation, modelling to the class. The input from me is displaying the initial activity the board, the students provide the rest (Memo 7.25). A part of encouraging independence and autonomy and engendering flow is involving learners in the mathematics and in the decision making. A silent approach contributes; and in focusing on student-led learning, peer conversation and dialogue is encouraged; engendering interactive social flow.

Although involving students in the lesson may stimulate interactive flow, if a student does not involve others, there is no collaboration and the interactive flow is less likely to happen; there isn’t the blending of egos suggested by social flow theory (Sawyer, 2007). Continuing the vignette above, in lesson 3 after Josh had completed his explanation, he passed the pen to Kyle, who tried to explain \(44 \div 7 = \); however, Kyle did not involve other members of the class in his teaching, and the lesson began to stagnate. I intervened to attempt to keep the flow states going (Memo 7.26).

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\(^7\) Note: there is a more discussion and reflection on the silent approach in chapter 6.
Further Discussion, Interpretation and Reflection

**Holding back**

I nonetheless endeavoured to hold back responses to students, trusting students to discover solutions on their own; resonating with the framework ‘who is active’ (Mason and Johnston-Wilder, 2004a). By doing this I attempted to suspend judgment on learners, encouraging a deeper acknowledgement and awareness of the necessity and applicability for rules and conventions in mathematics. An instance of this occurred during lesson 2, when Shurik asked if he should repeat the numbers, and rather than telling him he could not, I asked Shurik how did he know that he could not repeat the numbers (Memo 7.17).

I would often acknowledge ‘wait time’ by not answering (ignoring) or refusing to answer a student’s question, which could sanction students the time and space to furnish a correct solution and provide independence to students and subsequent influence on flow states. For example, during lesson 8 Kyle asks me a question, about how to change from decimal counting numbers to binary numbers. I do not reply, waiting for a few seconds instead, and Kyle arrives with the answer by himself (Memo 7.46).

This routine wasaccented when I used a silent teaching approach, as the non-oral response necessitated more thought, both from me because I was not speaking and the student’s initial question because I would not be replying.

**Calm periods of the lesson**

I sometimes attempted deliberately to foster a period of time in which students were quiet, focusing on the mathematics and working without communicating orally with others. This was in order to foster individual flow. An example of this was during lesson 2 when I had set up an activity for students to complete upon entering the class; they had just arrived from break and as well as encouraging individual flow I wanted to calm the students. Kyle, Caleb, Teo, Layla, Kit, Ava were all showing individual flow markers at this point of the lesson (Memo 7.1). The task design was important during these quiet periods; it would need to be accessible to as many as possible without any teaching points as flow would be more likely to occur if both students and I were not talking.

A flow journal entry from Fae, in October 2016 shows that she appreciated the quiet times; ‘...I find it easier to concentrate when I am in silence …’ [Fae 4/10/16]; (see Figure 9.9).

Paige reported in the stimulated recall interview that working silently assisted her flow state;
Further Discussion, Interpretation and Reflection

Paige: Well sort of you know we have those one-minute silence for people who die and all you
know soon as we start talking a lot, I feel like you stop us and give a one-minute silence it sort
of calms me down and when I get back to work am in flow more.

Figure 9.9 an example of a flow journal, written by Fae.
She seems to like silence as a way of ‘getting into flow’.

These routines were born of experience and risk; sometimes there could be occasions where
progress would be less than what was expected. For example, in September 2015 Paige made a
comment to me about how my talking too much took her out of the flow state, and I also reflected
a few days later in my field journal that I felt by talking to the class (opposed to with the class) I
was sometimes preventing flow (Memo 7.16). Guiding the dialogue so students have control of
the mathematics, yet not taking over the lesson and stifling students’ creativity and their flow
experience is a skill can be seen as an art form and not a science. Adjusting challenge, to match
perceived skill was an ongoing undertaking, often involving extra planning. During lesson 3 for
example, I had seemingly set a challenge that was too high for the perceived skill of the students,
dividing and leaving the answer as a terminating decimal. As a response I adapted and changed
the activity after I noticed several learners i.e., Teo and Fae were disengaged (see Memo 7.27).
To summarise at this juncture, different teaching approaches influence the flow states in the lesson
and shape students’ knowledge of mathematics. Teaching approaches include my decision to allow
a student led lesson or part of a lesson. Moreover, it is not only student-led vs teacher-led; the
issues also encompass student agency, self-direction, and self-regulation (see Boaler, 2002).
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Task design

Conjecture: The type of mathematical task contributed towards the flow that occurred within the lesson, in particular investigative tasks. Certain tasks were more likely to engender flow, particularly interactive social flow. This would agree with flow theory, which states it is the activity that causes flow through its associated challenge (Jeanne, 2009). Walker (2010, p. 9) writes ‘tasks that provide “interdependence, co-ordination and cooperation” can promote social flow’. The stimulus of the activity is what allows flow to manifest, flow has a doing element to it, provided by the activity itself.

Evidence to support this includes the flow markers in the observed lessons (e.g., memo 5.16; 5.19; 7.60; 7.75), and self-assessment responses. 87% of students responded that the activity had a positive influence on the flow states experienced (Table 9.3 below). Other written comments included: ‘My flow starts depending on my different tasks’ [Liam, self-assessment Lesson 8].

<table>
<thead>
<tr>
<th>How does flow start?</th>
<th>‘yes’ response</th>
</tr>
</thead>
<tbody>
<tr>
<td>The activity itself</td>
<td>87%</td>
</tr>
<tr>
<td>The mathematical challenges</td>
<td>58%</td>
</tr>
<tr>
<td>My mathematical skills</td>
<td>50%</td>
</tr>
</tbody>
</table>

*Table 9.3 the emergence of flow (mathematical activity). 12 mathematics lessons (n=154-178)*

Investigative tasks

Open ended and investigative tasks that allowed for discovery necessitated collaboration, communication and mathematical conversations, and facilitated conditions for social flow. For example, in lesson 2 I asked students to complete an activity that was open ended with minimal instruction. The class started working in groups and flow markers such as *discussing the work, looking at the task while talking, animated, pointing at the work or indicating at it* were observed (see Memo 7.11). In lesson 10 I asked students to complete an investigation as a part of an assessment, in small groups. 91% of students who answered the self-report (n=14) said that the activity helped them get into the flow state (Memo 7.59).
Further evidence to support investigative and open-ended tasks engendering flow states amongst students includes the stimulated recall interview. Caleb talks about investigative tasks and their assistance in triggering flow states, describing flow characteristics in his answers;

*Caleb:* What was I going to say? oh! The investigation we did like the last lesson that really help because like the note is saying finding pattern and it’s like it’s really helpful to get you in a good state of mind working say you get another topic it’s you know what to do, you know what to experiment with, you know what to try out and not try out.

*Caleb:* The most common thing that gets me in to flow is like one you brought on a topic it’s just like an interest or ambition I get am like oh! What’s this and then like say if there’s like a question on the board the scenario am trying to give an example like say investigations just like when you introduced them, when you introduced something that gets me in to flow because it’s gets me curious like what it means and it gets me like I want to experiment I want to try, I want to try a lot of stuff, I want to try match my skills and challenge to do that I like have to find something that drives me like in this case like the ambition or the interest in the class.

Investigative tasks garnered comments in the self-report such as ‘I liked this lesson because it got my mind working’ (Kyle, 14/915).

**Summary**

Table 9.4 below contains a summary of the conjectures discussed in this chapter, along with references to the relevant memos in chapters 5 and 7.

<table>
<thead>
<tr>
<th>Conjecture (findings)</th>
<th>Relevant Memo (Series 1, 5.x; series 2, 7.1-7.68; Series Three 7.69-7.85)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The manifestation of the flow experience in the classroom was demonstrated by learners displaying distinctive physiological behavioural features, which I have termed ‘flow markers’ Over the entirety of the 18-month study many students experienced flow in their mathematics lessons.</td>
<td>Taking into account other forms of evidence; (5.4;7.16;7.25;7.32;7.39;7.71;7.80) Markers are useful when audio could not be transcribed (7.33;7.70) Interactive social flow (5.16) Joyfulness (5.9;7.25;7.47;7.67) Intrinsic motivator (7.81)</td>
</tr>
<tr>
<td>Flow manifested in the mathematics classroom and the characteristics suggested in the theoretical flow model were demonstrated on multiple occasions by multiple students in the lessons observed.</td>
<td>Group size effects the flow experiences (7.48) Students in flow in with a small group size (7.48) Common understanding changed/ mitigation of common meaning (5.7 7.14;7.46) Possible confusion between categories (7.6; 7.9;7.25) ESM adaptation (self-assessment issued only once) (7.34;7.43) I was their teacher first, then researcher (7.39;7.67)</td>
</tr>
<tr>
<td>Conjecture (findings)</td>
<td>Relevant Memo (Series 1, 5.x; series 2, 7.1-7.68; Series Three 7.69-7.85)</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Social (group) flow characteristics and conditions manifested within the classroom;</td>
<td>Combinations of flow – co-active/interactive (5.18; 7.3; 7.6;7.25; 7.49; 7.51; 7.82; 7.86)</td>
</tr>
<tr>
<td>in particular interactive social flow was demonstrated during the study.</td>
<td></td>
</tr>
<tr>
<td>Encouraging talking about mathematics</td>
<td>Familiarity and Equal participation</td>
</tr>
<tr>
<td>engenders social flow; in particular,</td>
<td>Moving it forward/moving it on (5.13;7.19)</td>
</tr>
<tr>
<td>talking with peers seems to engender</td>
<td>Blending of egos (5.6;5.12;5.16;7.38;7.53;7.63)</td>
</tr>
<tr>
<td>Social and individual flow are not mutually exclusive in the mathematics</td>
<td>Concentration (5.4; 5.5;5.16; 7.36 (in a group situation)</td>
</tr>
<tr>
<td>classroom, that although different, both can co-exist.</td>
<td>Distractions (7.28; 7.60)</td>
</tr>
<tr>
<td></td>
<td>Concentration becomes a condition rather than a characteristic (7.40)</td>
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<td></td>
<td>Control over learning (5.6; 7.36)</td>
</tr>
<tr>
<td></td>
<td>No worry of failure (5.16;5.17)</td>
</tr>
<tr>
<td></td>
<td>Feedback (5.7; 7.2;7.10)</td>
</tr>
<tr>
<td></td>
<td>Validation/checking of answers (7.13;7.52;7.76)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Conjecture (findings)</td>
<td>Encouraging talking (5.3;7.65)</td>
</tr>
<tr>
<td></td>
<td>Talking doesn’t work for all, some prefer not to talk, (5.8)</td>
</tr>
<tr>
<td></td>
<td>although different times different contexts (7.3)</td>
</tr>
<tr>
<td></td>
<td>Collaboration (5.10;7.64)</td>
</tr>
<tr>
<td></td>
<td>Noisy class (7.61)</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>What is communicated (7.4)</td>
</tr>
<tr>
<td></td>
<td>Talking and communication about the activity encourages social flow (5.3: 7.12)</td>
</tr>
<tr>
<td></td>
<td>Motive was flow but communication has other added social cooperation benefits (7.41)</td>
</tr>
<tr>
<td></td>
<td>Talking at rather than with (7.27)</td>
</tr>
<tr>
<td></td>
<td>Who communicates – peers vs me (teacher role) Me not assisting the flow experience through talking (7.15;7.27;7.72), Solution to me being a distraction (7.85)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Talking with friends starts the experience (7.70;7.73)</td>
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<tr>
<td></td>
<td>Planning carefully what was said has a positive effect on teacher assisting with flow (7.59)</td>
</tr>
<tr>
<td></td>
<td>Student talking too much (7.26)</td>
</tr>
<tr>
<td></td>
<td>Students go back into flow after a break</td>
</tr>
<tr>
<td></td>
<td>(7.5;7.23;7.36;7.75;7.78;7.83)</td>
</tr>
<tr>
<td></td>
<td>Structure for breaks necessary, providing intentional breaks (7.5;7.58; 7.74)</td>
</tr>
<tr>
<td></td>
<td>Complications of breaks, students with different needs and wants realities of the inner-city classroom (7.6;7.19;7.36)</td>
</tr>
<tr>
<td></td>
<td>Benefits of breaks, what do they look like; should they be encouraged (7.74)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Characteristics of flow were engendered through encouraging student independence and ownership of learning.</td>
</tr>
<tr>
<td></td>
<td>Difficulties in silent way – equanimity of all playing an active role, class size consideration (5.1; 5.18)</td>
</tr>
<tr>
<td></td>
<td>Teacher skills of moving it on (5.14; 7.19)</td>
</tr>
<tr>
<td></td>
<td>Ensuring students have a control over their mathematics balanced with behaviour issues (7.44)</td>
</tr>
<tr>
<td></td>
<td>5.15 (Teacher skills of organic planning)</td>
</tr>
<tr>
<td></td>
<td>5.15 (consideration of the task)</td>
</tr>
<tr>
<td></td>
<td>5.17 (no worry of failure – absence of voice contributes to less judgement)</td>
</tr>
</tbody>
</table>
Further Discussion, Interpretation and Reflection

<table>
<thead>
<tr>
<th>Conjecture (findings)</th>
<th>Relevant Memo (Series 1, 5.x; series 2, 7.1-7.68; Series Three 7.69-7.85)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenge and its importance (7.30;7.56;7.64;7.69)</td>
<td>Challenge not only from the task – also from engaging with others in the lesson (7.36).</td>
</tr>
<tr>
<td>Lack of challenge can be to do with the ‘move from familiarity rather than the mathematics itself. (7.69)</td>
<td>Student start to provide their own challenge, peer teaching (7.50;7.54;7.64;7.65;7.68) Temporal distortion (7.30;7.367.66)</td>
</tr>
<tr>
<td>Silent approach (5.2; 7.1; 7.22)</td>
<td>Ignoring and; suspending judgement (5.11;7.17;7.18; 7.2; 7.31; 7.72;7.84)</td>
</tr>
<tr>
<td>Taking a less dominant role leads to social flow amongst students (7.53)</td>
<td>Involving students - ‘attention’ triangle (7.62) Not intervening when student making mistakes and not ‘telling answers’ allowing learners to flourish and correct themselves to keep challenge ((7.55;7.57)</td>
</tr>
<tr>
<td>Written medium of content authority (7.35)</td>
<td>Adjusting challenge of the task (7.20); too high or too low (7.27; 7.42;7.45) Consideration of body language (5.15; 5.2) Lessons did not always have positive benefit for all (7.61)</td>
</tr>
<tr>
<td>Using process authority to establish quiet to engender flow (7.1)</td>
<td>Process authority different in a smaller group size (7.48) Peer assessment influences conversation which influences interactive social flow (7.8)</td>
</tr>
<tr>
<td>Complication of extra planning (7.24;7.37)</td>
<td>Benefits of extra planning (7.59) Open ended task creating complexity (5.12; 7.11;7.60) encouraging ‘worker bee’ (7.29)</td>
</tr>
<tr>
<td>Mathematics provides the activity for flow to occur (5.16;7.60)</td>
<td>The activity causes the experience (5.19;7.60;7.75) Activity stronger influence than off-task behaviour (7.79) Groupwork (7.29)</td>
</tr>
</tbody>
</table>

The type of mathematical task contributed towards the flow that occurred within the lesson.

Table 9.4 a summary of the findings with references to the relevant memos

To summarise this chapter, I have answered how the flow phenomenon is attested, manifested and demonstrated by mathematical learners (RQ1). Additionally, I have begun to look at what was happening in the classroom when flow occurred which begins to answer RQ2; what pedagogy (including task design, and questioning techniques), elicit flow. The limitations to research question 2 are addressed in the final chapter that follows.
CHAPTER 10: Conclusion

In this final chapter I summarise my contribution, consider limitations of the study and discuss the possibilities of future research.

Key points

1. My research has developed a list of flow markers, grounded in empirical evidence, which are a detailed development of the existing theoretical work. (e.g., Williams, 2002; Custodero, 1998). Flow markers have enabled me to identify and promote flow amongst the learners in my classroom. They are a means of allowing flow to become more visible and they increased in their complexity as the study progressed.

2. Additionally, in order to empirically grasp the situation, I have extended the existing academic research involving the method of multi-slice imagining (Konecki, 2011), moving it into the realm of video.

3. I have also made a contribution to the existing theoretical work concerning the predictive and descriptive model of antecedent conditions and outcomes (e.g., Quinn, 2005) to encompass group flow and have identified evidence that inputs and outputs are fluid and can overlap (see Kotler, 2014), in particular concentration and temporal distortion. I have begun to achieve insight into that which is within my control to influence flow and additionally further insight into that which impacts my teaching practice but nonetheless indicates flow is occurring (Morrison, 2018).

4. I have shown the characteristics and conditions suggested by the theoretical flow model (Csíkszentmihályi, 1975), were demonstrated by students in the lessons, in particular challenge versus perceived skill, clear goals, transparent feedback, loss of self-consciousness, complete concentration and temporal distortion.
I utilised numerical analysis from the results of the self-reports made by learners to corroborate this assertion. Additional evidence supporting how students manifest flow included the flow markers established from multi-slice imagining, written comments from the self-assessment, excerpts from the stimulated recall interview, lesson transcripts, filed journal entries, and memos from the previous chapters.

5. Social flow (Sawyer, 2007) was a key part of the flow demonstrated in my mathematics classroom. Through promoting communication and talking amongst students, group flow manifested with positive results. Intrinsic enjoyment became a feature of my classroom. An ethos of collaboration and co-operation amongst students was engendered by the emphasis on interaction and mathematical dialogue. Current general research on group flow (e.g., Kotler, 2014; Salanova et al., 2014; Armstrong, 2008; Sawyer, 2007; Sawyer, 2003) is limited (Nakamura and Csíkszentmihályi, 2002); research on mathematics classrooms and group flow even more so (Armstrong, 2008). Thus, the work carried out is a contribution to existing theoretical work.

6. I started to look at breaks in lessons and the impact that they make. It appears, that these may be a positive feature and necessary for flow to take place and if so, could shine new light on how learning could be organised in the classroom. This conjecture was limited in its conclusion by the time available and an understanding that the depth of other areas of the study would have been affected by concentrating on this hypothesis. It is a notion that may well be an area for further investigation.

7. Characteristics of flow were engendered through encouraging student independence and ownership of learning. In particular, a silent teaching approach facilitated flow states amongst learners.
8. The type of mathematical activity contributed towards the optimal experience that occurred within the lesson, having a positive effect on flow amongst students, specifically constructs such as investigative and open-ended tasks appeared to engender the phenomenon. When students were given activities that were suggestive of action despite risk, i.e., active learning, flow was observed. The frameworks that I used, e.g., specialising and generalising, aspects of variation theory (Mason and Johnston-Wilder, 2004) would seem to agree with the theoretical model of flow, in as much as challenge complexity was provided that took a learner past a frustrated can’t do it response and encouraged flow experiences.

In summary so far, I have answered the research questions: RQ1) How is the notion of flow attested, manifested and demonstrated by mathematical learners; RQ2) What pedagogy (including task design, and questioning techniques), elicits flow.

Limitations

It could be argued that RQ1 was answered in greater detail than RQ2, nonetheless, from the start this was always a possibility (described in the introduction); the depth of the study was felt to be of greater importance than the breadth, and a focus on RQ1 was always intended to take a priority.

In order to answer RQ2 in greater detail, a tighter focus on observing patterns in certain activities where flow is present would be required. From this, the question arises whether it is possible to ‘hack flow’ (Kotler, 2014, pp. 100-103); i.e., bring about the optimal state of flow deliberately and intentionally. Some researchers have proposed that starting flow deliberately is not possible, (Csikszentmihályi, 1990; Jackson and Marsh, 1996)), nevertheless improving the pre-conditions for flow can increase its likelihood (Wrigley and Emmerson, 2011). There may also be a danger that by isolating particular mathematical pedagogies, we become reductionist (Foster, 2013), in the sense of teaching mathematics is a holistic venture, and flow theory exists not solely because of the way the task was designed, or the pedagogic approach.

Whilst bearing the above in mind, a sole focus on RQ2 is a possible future research opportunity. A further research question could be around the plasticity of flow, i.e., the extent to which flow can be enhanced/achieved better, quicker and with more effect.
Investigation into the notion of autotelic personality, that certain individuals engender flow more easily than others (Csíkszentmihályi, 1990), was disregarded part way through the study. I felt that the effort necessary to investigate autotelic personalities would detract from the main research questions, changing and broadening the direction. The methods may also have had to have changed towards specific students and their flow relationships. Additionally, by pursuing this avenue there could have been ethical issues (e.g., would students have been treated differently to others if they were or were not showing a greater propensity to flow than others). With careful consideration this could be an area for further opportunity however in the remit of this study it was inappropriate.

There were limitations brought about by the time constraints of teaching. Although my role as the class teacher had many advantages, indeed the whole nature of the research would have changed were I not the class teacher, it also limited the time that I could spend on following up queries and questions arising from the analysis. The measurement of the flow phenomenon is time-sensitive due, in main, to recency bias (Csikszentmihalyi and Larson, 1987), and it was often not possible, because of the pressures of teaching, to analyse the data until considerable time had elapsed after the lesson observations took place.

The sample is not necessarily representative of the population at large, and conclusions cannot be necessarily generalised. Despite some aspects of the research not necessarily being generalised, this does not take away from the transparency of the results (e.g., Clarke et al., 2018) and moreover, the research is potentially transferable to the classroom practice of other mathematics teachers. Supporting this claim is: first, that my analytic method offers rich contextual detail that permits other practitioners an understanding of the pedagogical and institutional circumstances of the classroom setting, to make judgements around whether and how to incorporate my findings. Second, that the teaching approaches and pedagogies I utilised to engender and foster flow experiences in the classroom amongst learners were not extremely dissimilar from my own past practice and other teachers practice and accordingly could be applied in a range of settings.

An argument could be made that the characteristics of flow theory occur anyway during mathematics lessons, and this research is merely pointing to the obvious, possibly attempting to explain something that does not need an explanation. I would argue in order for something to become obvious it needs to be articulated (DeGeest, 2006); until the obvious is pointed out, it will remain implied.
Moreover, it may be beneficial to look at the research teleologically; in terms of the purpose that the research serves rather than the cause by which it arises. For example, the ways of measuring the flow phenomenon had a positive effect on students in the classroom. Students journaled about their mathematics in ways that they had not done previously, the self-assessment questionnaires and the flow journal entries made by the students enabled a reflectiveness amongst learners that may not have been seen otherwise. Students became very aware of the focus on challenge (vs perceived skill) of the mathematical activities because of the flow focus. Learners talked extensively about the mathematics. In carrying out the research, my teaching improved. The prior planning of lessons had to be more detailed, in particular the focus on the silent teaching approach as a means of engendering flow meant I had to think differently and there was an improvement in my organic planning. I planned many activities that involved students talking about mathematics, which encouraged social flow amongst learners.

As can be seen, the focus on flow generated a positive effect on student learning and the quality of my teaching. By making flow states real in the mathematics classroom, mathematics learning provided enjoyment and happiness thorough engagement and intrinsic motivation.

**Future opportunities**

A further opportunity could be concentrating on students who appeared to demonstrate flow, and a deeper exploration of the notion of an autotelic personality. Also, socio-cultural demographics could be examined, as in this research they were used as descriptive notions rather than ideas for investigation. Assumptions were made in analysis that the differences in flow scores were due to variances in activities and delivery rather than cross-cultural or gender differences. For example, what effect does race, gender or socio-economic status have on flow? Do, for example, black boys experience less flow than their non-black counterparts? I started to examine co-active flow and recognise that this could be an area that could be researched in greater depth, in particular the differences and similarities with other categories of interactive social flow and individual flow.

In addition, the relationship of social flow to the size of student groups, where students were seated, and with whom they were seated has scope for future research and the highlight on discussion and collaborative working groupings of learners and its influence on flow states is an area that could be expanded in any future work.
Conclusion

Personal consequences

I have grown personally through carrying out this study. Embarking on this thesis reinvigorated my enthusiasm for classroom practice and has assisted in a career change; from an advanced skills teacher in 2012 to a teacher educator, joining University College London, in 2017. In 2016 I completed the classroom research, and I would often take theoretical knowledge into practice, something I did not previously do. My performance management observations in 2012 at the start of this journey were graded satisfactory/good and towards the end were consistently rated as outstanding for the previous two years. By undertaking this doctoral study, there has been an improvement in my knowledge of pedagogy, alongside an improvement in my writing ability and reading of complex academic texts.

Final conclusion

Ultimately, by starting to define the quality of experience within the mathematics classroom I have engaged in the conversation about improving teacher education and thus pupil learning. Students studying mathematics in my classes, exhibited the intrinsic motivation, the happiness, the resilience and creativity suggested by flow theory. There were demonstrable improvements in my teaching and equally in student learning. The study was never intended to be a comparison, considering it was relatively small-scale, however it was possible to gain an insight into the experiences of learners, and demonstrate practical methods designed to make mathematics enjoyable in the classroom. Without doubt, and nonetheless, the research, knowledge and the experiences gained, will be useful to myself on a smaller micro scale in my own present and future roles. The results of this study may be of interest to other mathematics teachers and the commonalities between the secondary subjects may enable the findings to be applied more generally.
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References


References


References


References


Udana I: 10


References


Appendices
Appendix A. Student interview and Journal dataset

In this appendix is a description of the stimulated recall interview providing a context of the interview itself. I have included comments from the transcript within the discussion. Also is a description of the journal dataset, a sequence of lessons in which only student flow journals were used to record the learner flow experiences. Excerpts from these are also contained in the main text.

Stimulated recall interview

The stimulated recall interview took place in November 2015, after series 1 and 2, and followed a mathematics lesson. It was 50-minutes in length. Students were shown clips of previous lessons which focused on the class in flow. All of the clips were from series two and described in section two of this chapter.

I had a prepared series of indicative prompt questions with the intention of focusing on the quality of the experience (shown below). I also tried to incorporate ideas that had surfaced so far, for example it seemed that students demonstrated flow when they were peer teaching and/or working and talking about mathematics in groups.

A part of the discussion centred around how they were experiencing flow in lessons, and what could be done more of and less.
Questions prompts for stimulated recall

- What does flow mean to you?
- How do you get in/out of flow?
- Does quiet help?
- Does noise help?
- Restricting/enlarging distractions
- Do mini whiteboards help?
- What else can help?
- How does a teacher (me) break a flow state?
- How does a teacher (me) help in a flow state?
- How does feedback from teacher help flow state?
- How does my input increase/decrease flow experience?
- How does the challenge of the task help?
- What puts you off?
- How does it help flow state to listen to others explain mathematics?
- How does a video help?
- What about where you are sitting in the class?

Students were given their completed questionnaire response forms, from the lesson clips that they watched, as another aide memoire.

The journal dataset

This series took place from September through to October, the Autumn 1 term of 2016. The data collection method concentrated solely on journals, in order to gather qualitative comments from students. This was in part due to class population changes (around a third of the group were changed) and also it was felt that enough had been gleaned from questionnaire self-assessment and video. In addition, the study was drawing to a close.

The lessons were planned to encourage interactive social flow and focused on a sequence of lessons, all on algebra. They were all modified from a proprietary scheme of work (see Butterworth et al., 2014). Group work was encouraged in all of the lessons.
The student journals had an element of free writing, nevertheless I gave students some key question stems. They included:

- I am going to get into flow by …
- I was in flow today because …
- I was not in flow today because …

Figure A.2 student flow journal entry by Kyle.
Describing what flow meant to him, his ‘being in the zone’.

Figure A.3 student flow journal entry by Katya.
She found the challenge too much and also her state of tiredness had an effect.
Appendix B. Consent form, information sheet and form

<table>
<thead>
<tr>
<th>INFORMATION SHEET FOR PARTICIPANTS (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open University Ethics Committee Ref: HREC/2015/1782/Morrison/1</td>
</tr>
</tbody>
</table>

**Flow and the relationship with the mathematics classroom**

We would like to invite you to participate in this postgraduate doctoral research project. You should only participate if you are willing to do so. You will not be disadvantaged if you decide not to take part, it is important for you to decide whether you want to take part. The purpose of the research is to investigate mathematical teaching as part of a PhD in Mathematics Education.

The investigation is focused on my professional practice and the notion of Flow and its relationship with mathematical learning. I am interested in students working in ways that are more likely to engender flow practices.

I am involved in action research on my teaching and learning practices continue to improve. I can also try out new projects to provide all students involved with sound teaching practices.

The possible benefits of the dissertation will be to raise awareness of successful mathematics teaching in practice, and to improve your child's attention.

If you agree to take part then up to fifteen of your child's mathematical lessons will be video recorded for analysis. The entire study will take place during normal maths lessons.

**What is Flow?**

Flow is defined as a psychological state in which the person feels simultaneously cognitively efficient, motivated, and happy.

Comes easy

Get into it

I'm on a roll

I can't stop

I feel like a machine

**Please complete this form after you have read the Information Sheet and/or listened to an explanation about the research.**

**Title of Study:** Flow and its relationship with the mathematics classroom

Thank you for considering taking part in this research. The person organizing the research will explain the project to you before you agree to take part. If you have any questions arising from the information sheet or explanation given to you, please ask the researcher before you decide whether to join in. You will be given a copy of this Consent Form to keep and refer to at any time.

- I understand that if I decide at any time during the research that I no longer wish to participate, I can withdraw from it at any time.
- I consent to the processing of my personal information for the purposes explained to me in the Information Sheet and on the Form.
- The investigation is to be observed in a series of lessons, without prejudice.
- This is a free study and all data will be treated anonymously.
- The study is focused on my professional practice and on the notion of Flow and its relationship with mathematics teaching. I am interested in students working in ways that are more likely to engender flow practices.
- Three of the main parts are; an element of challenge, definite goals and a learner 'connects' with the tasks.

**STUDENT NAME:**

Parent/Guardian’s Statement (please select as appropriate):

[ ] I agree that the research project I have shown above has been explained to me by my satisfaction and agrees to my child to take part in the study. I have read both the notes written below and the Information Sheet about the project, and understand what the study actually involves.

Signed ____________________________ Date __________

**Please tick all boxes only if you wish to take part.**

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**Flow in the classroom**

The flow experience has been described as being in the zone. Flow refers to the state that occurs when the learner is totally absorbed in a task to the exclusion of all else with a complete concentration and successful outcome.

A group of students who are interested in the mathematics classroom described flow as:

- Interesting
- Fun
- Challenging
- Exciting
- Can't stop
- In the zone
- Flow is what I'm doing now
- Get into it
- I feel like a machine
- Feel virtually unbeatable

**Please complete this form after you have read the Information Sheet and/or listened to an explanation about the research.**

**Title of Study:** Flow in the classroom

Thank you for considering taking part in this research. The person organizing the research will explain the project to you before you agree to take part. If you have any questions arising from the information sheet or explanation given to you, please ask the researcher before you decide whether to join in. You will be given a copy of this Consent Form to keep and refer to at any time.

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Signed ____________________________ Date __________

**Please tick all boxes only if you wish to take part.**

**Figure B.4 example of consent form and information sheet**
Memorandum

From: Dr Duncan Banks
Chair, The Open University Human Research Ethics Committee
Email: duncan.banks@open.ac.uk
Tel: 01234

Subject: The Open University Human Research Ethics Committee (HREC) approval

Date: 30 January 2015

This memorandum is to confirm that the research protocol for the above-named research project, as submitted for ethics review, has been given a favourable opinion by the Open University Human Research Ethics Committee. Please note that the OU research ethics review procedures are fully compliant with the majority of grant awarding bodies and their Frameworks for Research Ethics. Please make sure that any question(s) relating to your application and approval are sent to ResearchREC@open.ac.uk quoting the HREC reference number above. We will endeavour to respond as quickly as possible so that your research is not delayed in any way.

At the conclusion of your project, by the date that you stated in your application, the Committee would like to receive a summary report on the progress of this project, any ethical issues that have arisen and how they have been dealt with.

Regards,

Dr Duncan Banks
Chair OU HREC

The Open University is incorporated by Royal Charter (number RC 000391), an exempt charity in England & Wales and a charity registered in Scotland (number SC 038302).

Figure B.5 ethics form approval (HREC)

October 2014

To whom it may concern

Open University, Milton Keynes

Dear Sirs,

Ref: Sipho Morrison, PhD Mathematics Education

I hereby give permission for Sipho Morrison to undertake research at Hampstead School westbere road, London N11 as part of his Doctoral degree.

I understand that this research may include lesson observations, interviews and discussions with staff and students. I am aware these may be recorded, using audio and video equipment for the purpose of analysis.

Yours faithfully,

Head

Head: Jacques Szemelikovskis
Hampstead School, Westbere Road, London N11
T: 020 7794 8133
F: 020 7435 8243
E: enquiries@hampsteadschool.org.uk
W: www.hampsteadschool.org.uk

Figure B.6 school approval
Appendix C. Self-report questionnaire

Several adjustments were made to the question over the first five months of the research and each version of the questionnaire built on the previous one (consistent with teacher research). For example, remarks in the field journal suggested students wanted a space to freely record comments and the first questionnaire (A1) was adjusted so that formative comments could be recorded by students in the next questionnaire (A2). Table C.1 summarises the various changes to the questionnaire.
Appendix D. Self-report results

Bar charts

Figure D.8 and Figure D.9 below are bar charts illustrating the number of students responding ‘very much’ for each of the flow characteristics (expressed as a percentage of the total number of students that answered the question).

Figure D.8 bar chart showing variance across lessons
Highlighting the possible disparities across the study. Referred to in chapter 8. Note, lesson 2 did not have a self-assessment.

Figure D.9 bar chart showing the variance in flow characteristics
Grouping the characteristics across the observed lessons and highlighting the possible disparities in flow elements.
This chart (Figure D.10 above) is a measure of flow that manifested in the lessons. The flow score was an indicative score that summarised the different characteristics of flow (from part A of the self-assessment), using a geometric mean to accent the differences. It should be read inversely, the lower the bar the higher the flow score because if a student scored 1 it meant ‘very much’ (flow characteristic) and 3 meant ‘not at all’. The closer the flow score number got to 1 the more students scored ‘very much’ for each of the characteristics; the highest score could be 3; if students marked ‘never’ for experiencing all of the characteristics of flow.

**Individual flow scores**

Table D.2 below shows the flow scores across the study for individuals. The flow score is a self-assessed indicative measurement of the characteristics and conditions of flow. Blank entries refer to students who did not fill out a self-assessment, because of absence or a non-response. The scores are colour coded from green to red, green being the best score, and red being the worst. A 1.00 would indicate a student marked ‘very much’ (1) for all nine of the elements, and a score closer to 3 would imply a student marked ‘not at all’ (3) for all of the nine elements.

The scores were not utilised as a measurement of comparison. As discussed in the main text, lessons were not identical, with different and unknown variables, such as content, environment and context. The ephemeral nature of flow also makes comparisons difficult as experiences are interpreted differently, by the observer (me) and the observed (students). The sample is incomplete, due to absences and non-returns which would also affect a comparison of individuals. Also, there are 12 lessons shown, as the second lesson did not have a self-report.
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Table D.2 flow score of individuals