“Knowledge gained by experience”: Olaus Henrici—engineer, geometer and maker of mathematical models

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Abstract
The German mathematician Olaus Henrici,\(^1\) who was born in Denmark in 1840, studied engineering and mathematics in Germany before making his career in London. Initially, and for only a short time, he worked in an engineering business. He subsequently took on academic positions, first at University College London and then, from 1884, at the newly formed Central Institution (later Central Technical College) where he established a Laboratory of Mechanics. While at University College he became an active promoter of pure geometry and a producer of models of surfaces. In this paper I explore the geometrical side of Henrici’s work, setting it into the context of his career and arguing that his interdisciplinary background was a key factor in his success as a creator of models.

Keywords: Olaus Henrici; models of surfaces; geometry in 19\(^{th}\) century Britain

MSC: 01A55; 51-03; 97-03

Zusammenfassung:
Der deutsche Mathematiker Olaus Henrici,\(^2\) der 1840 in Dänemark geboren wurde, studierte Technikwissenschaften und Mathematik bevor er in London Karriere machte. Anfangs arbeitete er kurze Zeit als Ingenieur. Dann fand er akademische Stellungen, zuerst am University College London und dann, aber 1884, an der gerade entstandenen Central Institution (später Central Technical College) wo er ein Laboratorium für Mechanik gründete. Am University College wurde er ein aktiver Forscher in reiner Geometrie und Entwickler von Modellen mathematischer Flächen. In der vorliegenden Arbeit untersuche ich die geometrische Seite von Henricis Wirken, ordne es in den Kontext seiner Karriere ein und argumentiere, dass sein multidisziplinärer Hintergrund ein entscheidender Faktor für seinen Erfolg als Schöpfer mathematischer Modelle war.

\section{Introduction}

At a meeting of the London Mathematical Society (LMS) on 13 May 1869, the President, Arthur Cayley, “drew attention to a model, constructed by Dr. Henrici, of the cubic surface

\(^1\) To avoid any confusion, Olaus Henrici was not related to Julius Henrici (1841–1910), the Heidelberg high schoolteacher and co-author of a geometry textbook with Peter Treutlein (1845–1912), or to the Swiss numerical analyst Peter Henrici (1923–1987).

xyz = 1, pointing out the lines of curvature and other singularities.”

It was the first of several models of surfaces that Henrici exhibited in front of the LMS, a number of which the Society subsequently had copied for their own collection. Other members of the LMS exhibited geometric models in front of the Society but nobody else exhibited such a range of models or did so as often as Henrici. While geometric model-making was very popular in Germany during the latter third of the 19th century, no-one anywhere else in Britain was producing models like Henrici. This raises several questions. What prompted Henrici’s interest in this type of model? To what extent was he involved in the physical process of production? How did he use them? How were the models received? To answer these questions, I shall look at Henrici’s life and the different phases of his career, including his education, his time as an engineer, and his time as an academic mathematician. Next, I shall look at the production of models of surfaces in Britain and the environments in which these models were created, used and displayed, before finally looking at the individual models created by Henrici.

2. Overview of Henrici’s life

Olaus Henrici was born in 1840 in Meldorf, a small town on the west coast of Holstein, where his father, who had studied science and engineering in Berlin, worked on the development of canals. As a schoolboy Henrici showed little promise in classics with the result that his father did not consider him suitable for a university education and it was decided that he should become an engineer. At the age of 16 he was apprenticed to a company which made agricultural equipment and small steam engines in Flensborg, the second largest port in Denmark (after Copenhagen) and about 100km north of Meldorf (Henrici, 1911, 70). But the work ran dry and Henrici who, as he put it himself, was “not particularly efficient as a working mechanic” (p.71), had nothing to do.

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4 Of the models commissioned or donated to the Society, only 14 boxwood models of quartic surfaces given by Julius Plücker to Thomas Archer Hirst in 1867 still exist (see Section 5 of this paper). In 1894 the Society extended its Memorandum and Articles of Association to allow for the purchase of models and instruments and the forming of “a library and museum for the use of the Members” (Proceedings of the London Mathematical Society 26 (1894), p.x) indicating they were keen to build up a collection. No such museum ever materialised, presumably because the Society did not have a permanent home until 1988, which also explains the loss/dispersal of its other models.

5 For example, on 10 November 1870, Cayley exhibited a model of Steiner’s surface (Cayley, 1870, 190), and on 14 November 1872, William Kingdon Clifford exhibited several models of polyhedra (Proceedings of the London Mathematical Society 4, p.147).

6 Meldorf was at that time part of Denmark which is why Henrici is occasionally referred to as being Danish.

7 Most of the information about Henrici’s career prior to joining UCL is obtained from an autobiographical note he wrote on his retirement (Henrici, 1911). All future references to this publication in this section will be by page number only.

8 As Henrici described, the system was such that nobody was ever dismissed which meant that the manager had a difficult time trying to keep the men employed (p. 70).
Eventually, in 1859, Henrici persuaded his father to send him to the Polytechnische Schule (later Technische Hochschule) in Karlsruhe, one of the leading schools for engineering in Germany. It was an auspicious choice. Alfred Clebsch was professor of mathematics and Christian Wiener was professor of descriptive geometry. The head of mechanical engineering was the mathematically inclined Ferdinand Redtenbacher who had initiated Clebsch’s appointment in 1856, and openly championed the use of mathematics in engineering, as he made clear in a lecture in 1859:

I still hope to show people the proof that mathematics is no luxury and that by applying it to mechanical engineering, progress will be achieved provided that one understands the practice and exactly knows what is necessary for its use in one’s daily life.9

Furthermore, Redtenbacher placed great store on the use of models in teaching, as his former student Karl Keller observed:

In his lectures he did not underestimate the value of the presentation of examples in the form of models and engineering drawings.10

Redtenbacher’s model collection, which was one of the most extensive in its day, contained about 100 different models, and these would have been available to Henrici. Redtenbacher not only constructed his teaching models he also documented their construction so that they could be copied and used elsewhere, and copies were purchased by several museums, including the Science Museum in London (Wauer et al, 2009, 1619).11 Although Redtenbacher’s models were quite different to those produced by Henrici, the use of models in teaching was something Henrici would later advocate.

When Henrici arrived in Karlsruhe in 1859, he was initially not allowed to enter Redtenbacher’s engineering course because he “was ignorant of the calculus” (p.71) so he joined the second year’s Mathematical Course which included lectures on “descriptive geometry” by Wiener and “applied mathematics” by Clebsch. At the time Clebsch was interested in problems of mathematical physics, especially hydrodynamics and elasticity theory—his monograph on the latter appeared in 1862—so it is likely he lectured to Henrici on these subjects. This idea is supported by the fact that a few years later Henrici published a book that drew significantly on elasticity theory (Henrici, 1866a).

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9 Quoted in (Wauer et al, 2009, 1611).
10 Quoted in (Wauer et al, 2009, 1614). Keller later became professor of agricultural engineering in Heidelberg. Felix Klein singled out Redtenbacher’s lectures on the theory of machines as especially influential in the rise to prominence in Germany of the combination of mathematical and technical education (Klein, 1893, 107-108). Herbert Mehrtens, when considering the use of geometric models within a 19th century educational context, observed that interest in these models coincided with the rise of technical education to an academic level, citing the Karlsruhe and Zurich polytechnics, together with the École Polytechnique in Paris, as “the ‘model’ institutions” in this regard (Mehrtens, 2004, 297).
11 The entire collection is available to view at http://wayback.archive-it.org/2566/20180418122228/http://kmoddil.library.cornell.edu/redtenbacher.php
As Henrici recalled, he got on well with both Wiener and Clebsch, but it was Clebsch—his elder by only seven years—who proved to be the decisive influence for his future career:

I remember how in the Drawing Class room I liked to invent my own constructions in connection with Descriptive Geometry, and how I had in consequence violent discussions with the assistant, who was of the opinion that any line drawn different from what the Professor had done on the blackboard must be wrong. The Professor [Wiener] thought differently. Of greater importance to me was the fact that Clebsch took notice of me. He induced me to devote myself exclusively to Mathematics. During the three month summer vacation in 1860 I remained at Karlsruhe earning a little money by private teaching. I was honoured by seeing much of Clebsch. Practically every morning I called for him at 10 o’clock for a long walk during which much Mathematics was talked. It was only much later that I realised how very much I had learned during these lessons without paper or blackboard.

On the advice of Clebsch I joined the third year’s Mathematical Course, and also attended Redtenbacher’s [engineering] lectures (p.71).

But although it was Clebsch who nurtured Henrici’s interest in mathematics, it was Wiener, albeit encouraged by Clebsch, who would become a pioneer in the making of geometrical models, and in this particular respect it may well have been Wiener who was the more formative influence.

By 1862 Henrici’s conversion to mathematics seemed complete. That year he went to Heidelberg to study for a PhD in algebraic geometry with Clebsch’s former teacher, Otto Hesse; he also studied mathematical physics with the famous Gustav Kirchhoff. The move was a success and Henrici’s dissertation (Henrici, 1865a), which extended work of Hesse’s was considered to be of such high scientific value that the University recommended the government of Baden to recognise its importance by conferring a special public distinction on Henrici. Henrici then spent a few months in Berlin attending lectures by Kronecker and Weierstrass, before setting out for Kiel as a privatdozent in the hope of earning a living by tutoring at the University.

Kiel was then somewhat of a backwater with only one chair in mathematics, which makes it a rather surprising choice for an up-and-coming young mathematician. But the fact that Kiel was close to his birthplace may have played a role. In 1865, unable to get enough students

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12 The same year, 1862, Kirchhoff was awarded the Rumford Medal of the Royal Society of London for his research in spectroscopy.
13 *Nature* 2, 30 June 1870, p.168. In his autobiographical piece, Henrici did not mention the “public distinction”. The thesis was published in *Crelle’s Journal*, having appeared the previous year as a stand-alone publication. It was Henrici’s second paper to appear in *Crelle’s Journal*. His first paper in the journal, which overturned a conjecture of Hesse’s concerning the discriminant of a conic given as the intersection of a plane and a quadric surface, appeared in the previous volume (Henrici, 1865b).
14 In the winter of 1864/5 Weierstrass lectured on the theory of elliptic functions (Weierstrass, 1903, 356). It is not known what Kronecker lectured on—at that time he was lecturing in his capacity as a member of the Berlin Academy (he was not then a professor of the university)—but it is likely to have been an aspect of number theory.
to make life in Kiel financially viable, he accepted an offer from a fellow engineering student from Karlsruhe to join him working as an engineer in London (see Section 3). However, the work proved profitless and, rather than return penniless to Germany, he stayed in London tutoring mathematics to schoolboys, his spare time spent in the British Library studying mathematics.

In 1868, with a mathematical paper ready for publication and needing advice on where to publish,¹⁵ Henrici obtained from Hesse an introduction to James Joseph Sylvester, then professor of mathematics at the Royal Military Academy at Woolwich. Through Sylvester he met Thomas Archer Hirst, William Kingdon Clifford and Cayley. Hirst was professor of pure mathematics at University College London (UCL), while Clifford and Cayley were then both in Cambridge. Thus within a relatively short time of arriving in London, Henrici had made the acquaintance of the leading pure mathematicians in England. His meeting with Hirst was especially propitious. For Hirst, unusually for a British mathematician at that time, was German educated, having been awarded his PhD in Marburg in 1852 and later studying geometry with Jakob Steiner in Berlin. Like Henrici, he had originally begun his working life as an engineer, so the two had much in common.

Significant for Henrici’s relationship with Hirst was the fact that Hirst was seriously interested in and published on the new line geometry of the German mathematician and physicist Julius Plücker.¹⁶ In the 1860s, after a 20-year break, Plücker had returned to the study of geometry “encouraged by the friendly interest expressed by English geometricians [presumably Cayley, Hirst and Sylvester]” (Plücker, 1866, 204). He had responded to their interest with an article in the Philosophical Transactions of the Royal Society (Plücker, 1865), and by a talk at the 1866 meeting of the British Association for the Advancement of Science (BAAS) (Plücker, 1867).¹⁷

In 1869 Hirst appointed Henrici as his assistant at UCL and helped him to get a position at Bedford College, a higher education college for women.¹⁸ The next year, while Hirst was ill, Henrici took over Hirst’s classes. With three lecture courses at UCL, one lecture course at Bedford College, an evening class at Siemens Brothers in Woolwich,¹⁹ as well as private

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¹⁵ In his autobiographical note (p.72), Henrici mentions one paper without naming it, but he is presumably referring to (Henrici, 1869a) and possibly the connected paper (Henrici, 1869b).
¹⁶ In Plücker’s line geometry, the line (not the point) is fundamental, and the four-parameter family of all lines in space the basic elements, see (Rowe, 1994). For an example of Hirst’s work in this area, see (Hirst, 1879).
¹⁷ Plücker’s talk will be discussed further in Section 5 of this paper.
¹⁸ In his diary for 6 October 1869, Hirst described Henrici as “a very accomplished mathematician”. (Hirst, 1980). Bedford College, established in 1849, was the first higher education college for women in Britain.
¹⁹ Siemens Brothers was a large electrical engineering, design and manufacturing business which included a large cable factory. The company was involved in the Indo-European telegraph line, laying the submarine cable through the Black Sea, which was completed in 1870, and it manufactured the Transatlantic cable of 1874. It began its life as a branch of the Berlin-based company Siemens and Halske. It is likely that Henrici found employment there through German connections.
pupils, Henrici was now fully occupied with teaching. At the end of the summer term he
was appointed to succeed Hirst as Professor of Pure Mathematics. During his tenure of
the professorship, Henrici also busied himself with matters of administration, at one point
taking on the role of Dean of Arts and Laws (the faculty to which mathematics belonged).

Two years into his position at UCL, Henrici was joined by Clifford who had been appointed as
Professor of Applied Mathematics and Mechanics, and the two became close colleagues,
both earning reputations as excellent teachers as well as sharing a deep interest in
geometry. For Henrici the arrival of Clifford, the most innovative British geometer of his
generation, was a stroke of good fortune. While Henrici might not have been as radical as
Clifford in his mathematical ideas, he was not shy of originality, and a productive and
harmonious relationship developed between the two.

In 1880, after the untimely death of Clifford, Henrici transferred to the chair of Applied
Mathematics and Mechanics. While in this position he introduced graphical statics to the
training of engineers (Hill, 1918, xlii), a subject he had earlier promoted at a meeting of the
LMS (Henrici, 1871) and one which harked back to his own early work as an engineer. After
four years, he left UCL to become the founding Professor of Mechanics and Mathematics at
the Central Institution in South Kensington where, unlike UCL, all his students were fledgling
engineers, and where he remained until his retirement in 1911. Precisely why Henrici
made the move from UCL is not known but the evidence suggests he was attracted by the
guaranteed salary of £1,000 p.a., with the potential for more depending on the number of
students. At UCL his income, which was calculated from student fees and therefore

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20 Hirst had resigned from the chair, citing the heavy teaching load, and had taken an administrative post in the
College in order to have more time for research.

21 In an incomplete paper of Clifford’s on “The Classification of Loci”, written shortly before Clifford died,
Henrici, who had refereed the paper, added explanatory notes which were graciously acknowledged by
Clifford (1878, 664–665). Clifford had also proposed to Henrici that together they should write “a series of
books on Mathematics, beginning at the very commencement and carrying the subject in each case rapidly to
the most advanced stages” (Tucker, 1882, xxi). Although they often discussed the project, it never came to
fruition partly due Henrici being busy with examining duties and partly due to Clifford’s poor health.

22 Clifford had earlier given a course on “Synthetic Geometry and Graphical Statics”, the notes for which were
identified by Henrici and included in (Clifford, 1882, 637–639).

23 The Central Institution was established to provide training for technical teachers, mechanical civil and
electrical engineers, architects, builders and decorative artists, principals, superintendents, and managers of
chemical and other manufacturing works. It was renamed the Central Technical Institute in 1893 and in 1907
it became one of the constituent colleges of the new Imperial College of Science and Technology. It was one of
two new technical colleges founded in London in the 1880s, the other being Finsbury Technical College which
opened in 1883. Henrici had a sympathetic counterpart at Finsbury in John Perry, the Professor of Mechanics
and Mathematics there. Perry, an engineer by training, was ardently anti-Euclid and in his teaching
emphasised the practical use of geometry. A leading reformer of mathematics education in Britain, he was the
originator of the eponymous ‘Perry Movement’, see (Mock 1963).

24 In 1885 Karl Pearson referred to the fact that UCL did not have “the power of the purse” to retain “first-class
men” (Pearson, 1885, 358). Although not mentioning him by name, he was presumably referring to Henrici
(among others).
variable, is unlikely to have reached more than £500 p.a.\textsuperscript{25} Another likely source of attraction was the opportunity to set up and teach in a new specifically designed laboratory of mechanics. As well as providing a practical environment for teaching, the laboratory also provided Henrici with a space for producing instruments himself. Not long after he moved to the Central, he invented a new form of harmonic analyser, considered by his obituarist, Micaiah Hill, to be his most original piece of work (Hill, 1918, xlvi).

After meeting Sylvester and Hirst, Henrici quickly became integrated into the British mathematical community, becoming active in several spheres. In 1868 he joined the recently founded London Mathematical Society and was a regular contributor to the Society’s meetings, being elected President (1882–1884).\textsuperscript{26} He was elected a Fellow of the College of Preceptors in 1871, and of the Royal Society (RS) in 1874, the latter on the strength of his research in algebraic geometry and construction of models,\textsuperscript{27} and he was a member of the RS Council (1882–1883). He was also a founder member of the Physical Society.\textsuperscript{28} He regularly attended the annual meetings of the BAAS, including their meetings in Montreal (1884) and Toronto (1897), using the occasions to visit California in 1884 and to attend a meeting of the American Mathematical Society in 1897. In 1883 he was President of Section A (Mathematics and Physics) when the BAAS met in Southport, his presidential address on the teaching of geometry in Britain attracting a large audience (Henrici, 1884).\textsuperscript{29}

In 1875 he joined the BAAS committee on the teaching of elementary geometry,\textsuperscript{30} as well as the committee on mathematical notation and printing; and in 1894, the year in which he was a Vice-President, he produced a \textit{Report on Planimeters} (Henrici, 1895).\textsuperscript{31} In addition to these activities, he was also involved in the 1876 \textit{South Kensington Museum Special Loan Exhibition of Scientific Apparatus} and in the 1893 \textit{Mathematical Models, Apparatus and Instruments Exhibition} in Munich (organised by the Deutsche Mathematiker-Vereinigung), both exhibitions playing a role in the promotion of his work (see Section 5).

\textsuperscript{25} In 1870 when the chair of applied mathematics became vacant—the professor, T.B. Moore, having resigned due to the falling number of students—the lack of candidates had resulted in the income for the chair being augmented by a guarantee of £200 p.a., after which Clifford was appointed (Hale Bellot, 1929, 323).

\textsuperscript{26} Henrici had also been nominated for the Presidency of the LMS in 1872. Then the other candidates were Cayley and Hirst, and although Henrici obtained the least number of votes of the three, the fact that he was nominated as early as 1872 shows the speed of his assimilation (Hirst 1980, 10 October 1872). (Hirst was elected but Cayley had already served as President (1868–1870) so the outcome is not surprising.)

\textsuperscript{27} Henrici’s Election Certificate cited his publications in \textit{Crelle’s Journal} and the \textit{Proceedings of the London Mathematical Society}, and it described him as a “Constructor of Models of Surfaces of the second, third, sixth and ninth order”. Among those who endorsed his candidature were Henry Smith, Sylvester and Hirst.

\textsuperscript{28} The Physical Society merged with the Institute of Physics in 1960, the merged society taking on the latter’s name.

\textsuperscript{29} For a brief description of this address and its place in the context of British geometry at the time, see (Barrow-Green & Gray, 2006, p.319). The latter incorrectly states that Henrici received a PhD from Berlin.

\textsuperscript{30} For more information about this Committee, see Section 4.1 of this paper.

The mathematical education of women was another arena in which Henrici was active. As well as teaching at Bedford College, he gave courses of lectures for the Ladies Educational Association of London, he signed the 1880 memorial to the Vice-Chancellor of the University of Cambridge “praying that the Tripos examinations may be opened and degrees granted to properly qualified women” (Aldis, 1880), and he was one of the nine signatories to Hertha Ayrton’s candidacy for Fellowship of the Royal Society in 1902. On a personal level, he acted as an intermediary on behalf of Felix Klein for Grace Chisholm, helping her with introductions to mathematicians in London when she returned to England from

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33 The “memorial” was produced in the wake of Charlotte Scott’s success (equal to the 8th wrangler) in the Mathematical Tripos earlier in the year. It achieved partial success: women were granted the right to sit the Tripos examinations but were not allowed to be awarded degrees. That had to wait until 1947.
34 For a list of Ayrton’s supporters, see ‘Certificate of a Candidate for Election for Fellowship of the Royal Society, Hertha Ayrton’, Royal Society Archives, GB11, CD/312. Hertha Ayrton’s candidacy was refused. The Royal Society elected its first female Fellows in 1945.
Germany in 1895, writing to Karl Pearson that she deserved “all encouragement and especially for her love of geometry” which, he noted, was “only too rare in England.”

Although it was as a teacher that Henrici’s reputation was principally made, he produced several original works. In addition to papers on algebraic geometry, geometric models and harmonic analysers, he wrote several reviews for *Nature* and other periodicals, reports for the BAAS, and articles for the *Encyclopaedia Britannica* with his article on geometry being particularly successful (Anon., 1880, 119).

In 1877 Henrici married Helen Stodart Kennedy, the sister of Alexander Kennedy, Professor of Engineering at UCL. They had one son, Ernst Olaf, who went into the Royal Engineers, having been the top student at the Royal Military Academy in Woolwich, and with whom Henrici collaborated on a paper on the theory of measurement using metal tape (Henrici & Henrici, 1912). Henrici retired to Hampshire and died there in 1918.

### 3. Engineering and instrument making

When Henrici arrived in London in 1865 it was to work with a fellow student from Karlsruhe, Theodor Reuter, who had been there since 1862 employed as an engineer. It seems likely that Reuter was hoping to set up his own civil engineering business since in 1866 a patent for an invention which “consists in building bridges, arches and roofs as skeleton structures” was granted to Reuter, Henrici and Edward Kochs, with Reuter as the lead applicant. As well as a description of the invention, the specification includes five pages of diagrams showing structures of different types of bridge as well as arches and roofs, and details of links and brackets (see Figures 2 and 3).

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35 Henrici to Karl Pearson, 26 November 1895. Henrici also reported to Pearson that Klein considered Chisholm to have “true scientific insight in mathematics” and to possess “great geometrical gifts”. Karl Pearson Papers, General Correspondence 11/1/8/49. UCL Special Collections, London.

36 For a list of Henrici’s main publications, see (Hill, 1918; Lindemann, 1927).

37 Kennedy was Professor of Engineering at UCL from 1874 to 1889. Later he became an internationally renowned electrical engineer.

38 The paper, which was the first in a new series of Professional Papers of the Ordnance Survey (described as marking a new epoch in the history of the Survey), concerns the first use in Britain of invar wire for geodetic work (A.R.H., 1915, 460–461). See also (Hill, 1918, xlii).

39 Information about Reuter’s early career in Britain is scanty but it is probable that when Henrici reached London, Reuter was working for Alexander Chaplin & Co., a Glasgow engineering firm with administrative offices in London.

40 The patent was granted for “Improvements in the Construction of Bridges, Arches, and Roofs”, Patent Number 913, 29 March 1866. Nothing further is so far known about Kochs except that he was a civil engineer and presumably a good friend of Reuter’s since on the patent application they are listed as living at the same address.
Two points of novelty were claimed for the invention:

Firstly, skeleton structures are composed of single members, bars or rods of suitable section and suitable material, which only meet at their extremities, where they are connected by a single round bolt; continuous beams are therefore never used in skeleton structures, because these, ..., are all composed of independent members only, ... [S]econdly the single members are so
arranged that the whole structure is one rigid body, in which the position of each connecting point of the members amongst the others is unchangeably fixed. Thus a triangle composed of three single bars connected at their extremities by three round bolts is completely rigid, while a quadrangle likewise composed of bars is not rigid without introducing one diagonal.\textsuperscript{41}

By dispensing with superfluous bars, i.e. having no continuous beams, the material throughout the structure could be arranged optimally to bear the maximum strain with least weight of material, and the various strains could be distributed the most uniform fashion. The advantages of the invention were listed as:

- most favourable use of material leading to economy and lightness, and hence capability of erecting structures on a larger scale;
- labour efficient in making, packing, transport and erection of structures;
- better adapted than other systems for the use of steel;\textsuperscript{42}
- the order of the connecting points offers greater facility for security and strength to such points that in other systems are generally the weakest parts.

The patent does not reveal how much each of the three applicants contributed to the specification, but given its points of similarity with Henrici’s book, \textit{Skeleton Structures, especially in their Application to the Building of Steel and Iron Bridges}, published the same year (Henrici, 1866a), it seems likely that Henrici was largely responsible for it. In the preface to the book, Henrici mentions that he had earlier been asked by a friend, presumably Reuter, for his opinion on the theoretical element of a patent of Kochs’ for “Improvements in the Construction of Beams or Supports applicable to the building of Bridges, Viaducts, Roofs, Arches”. He goes on to say that he did not see much of significance in Kochs’ patent except for the part concerned with the construction of bridges which contained a “decidedly novel thought even although Mr. Kochs had not fully recognised its whole importance” (Henrici, 1866a, 3), and that it was this “thought” which had led him to his new principle of construction which he called ‘skeleton structures’. Since no trace has been found of Kochs’ patent, it is probable that it was either never submitted or not granted, and that it was subsumed by the patent granted to Reuter, Kochs and Henrici. The principle of skeleton structures—that the structures are composed of elementary bars connected at their extremities by round bolts, as shown in Figure 4 (Henrici, 1866a, 5)—is essentially the same as the first point of novelty in the patent noted above. Both the patent and the book are dated March 1866, and the first 11 figures in the patent are common to the first 11 figures in the fold-out plates at the back of the book

\textsuperscript{41} Patent Number 913, 29 March 1866, p.2.

\textsuperscript{42} The Bessemer process for making steel was patented in 1856, and the open-hearth process was first used for making steel in 1865. The time was therefore ripe for inventing construction methods involving the use of steel which due to its elastic properties was particularly suitable for building bridges,
(which Henrici claimed as his own (Henrici, 1866a, 60)), although their ordering is slightly different.43

![Diagram](image)

**Figure 4.** Each line from one connection-point to the next represents an independent bar and at each end there is a bolt.

A brief discussion of the analytical details of Henrici’s book can be found in a short article of 1989 by the civil engineer and historian, T.M. Charlton, who points out that Henrici’s method for analysing what today are known as ‘statically indeterminate bar-frameworks’ (i.e. bar-frameworks which lead to more unknowns than equations) is essentially the same, although somewhat less elegant, as that of Claude-Louis Navier of 1819, whose work was unknown in Britain at the time. In his discussion of Henrici’s work, Charlton also refers to a well-known paper by James Clerk Maxwell of 1864 which treats the same topic but in a different way (Charlton, 1989, 199).

Henrici’s book was reviewed in *The Engineer*, a technical periodical for engineers, where it was criticised for being too theoretical for the practical engineer, with doubts being raised about the applicability of Henrici’s structures for the construction of railway bridges, although the overall judgement was favourable (Anon., 1866). Shortly afterwards Henrici provided a polite but firm rebuttal to the finer points of the critique (Henrici, 1866b). The review seems to have had little negative effect as the book was republished in New York in the following year, where it appears to have been a success. According to architectural historians Sarah Landau and Carl Conduit who considered it a “seminal” work:

> Henrici’s treatise was useful to all builders [of New York skyscrapers] because it provided accurate descriptions of truss action, of the geometric basis of rigidity of the triangulated truss, and of the distribution of stresses in the deflected simple beam. ... the concept of truss rigidity that it presented was later useful in understanding the need for and the behavior of wind bracing. (Landau & Conduit, 1996, 22).

43 Although the figures in each publication are drawn by different hands, they are identical in every feature. As an aside, it is notable that Henrici in his book showed a remarkable mastery of the English language for someone who had been in England only a year.
Unfortunately, efforts to uncover the evidence which led to this rather remarkable claim have so far proved unsuccessful. But since Landau and Conduit were not engineers, it is possible that they were rather extravagant in their assessment.

After the collapse of the engineering business, Henrici continued to maintain an interest in linked structures, albeit of a more modest type. In 1873, he provided models of Peaucellier’s linkage (or cell)—the first planar linkage capable of transforming circular motion into perfect straight-line motion discovered by Charles-Nicolas Peaucellier in 1864—to support Sylvester’s “warmly applauded” address on the subject to the LMS (Morley, 1919, 370). When Sylvester spoke on the topic at the Royal Institution, Henrici was again on hand to provide the necessary model (Sylvester, 1873–75, 190). In 1894 at the BAAS meeting in Oxford, Henrici exhibited some linkage models at an evening soirée in the University Museum, presumably to tie in with his Report on Planimeters which he delivered at the meeting and in which he discussed what he called linkage integrators using a Peaucellier linkage as a simple example (Henrici, 1894).

In 1889, Henrici produced his first harmonic analyser, a machine for mechanically calculating a given number of coefficients of a Fourier series for a given curve. A few years earlier William Thomson (later Lord Kelvin) had produced the first such device in order to analyse tidal curves but his machine was large and difficult to manoeuvre. With the growing number of applications of harmonic analysis in engineering and mathematics, there was a need for a much smaller and transportable machine and Henrici had worked to fill the gap. Three years later, in conjunction with Archibald Sharp, his assistant at the Central, he

**Figure 5.** Peaucellier exact straight-line linkage (Kempe, 1877, 12)

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44 I am grateful to Bill Addis, historian of engineering, who investigated this question for me but could find no supporting evidence for the quote.
45 In his paper, Sylvester also noted other occasions in which Henrici had been involved in the construction of related models, remarking on Henrici’s “skill” in design (Sylvester, 1873–75, 188). Linkages were an object of intense study in Britain during the 1870s, not least due to Sylvester, see (Morley, 1919).
47 At the meeting, Henrici presented his report to both Section A (Mathematics and Physics) and Section G (Engineering), the latter presided over by his brother-in-law Alexander Kennedy. It was very well received and ordered to be printed *in extenso*.
designed a more advanced machine which calculated more coefficients, and which was manufactured by Coradi in Zurich. Although well received, it was rather expensive and not as many were sold as Coradi had hoped. Purchasers included universities in Europe, Russia and the United States, as well as the Tokyo Earthquake Investigation Committee. It was generally recognised as being a leader in its field. Vannevar Bush, the noted inventor of the Differential Analyser (Bush, 1931), considered it to be the “most convenient and precise” of all such analysers (Bush, 1936, 659)—and it remained in use up until the Second World War, mainly in the field of acoustics.

4. Henrici as a teacher

4.1 Geometry

In 1869, when Henrici took over from Hirst at UCL, the teaching of geometry in Britain was the subject of much debate. The argument was not about whether, or indeed which, geometry should be taught—all were agreed that the study of geometry improved facility to reason—but about how it should be taught, and in particular the suitability of Euclid’s Elements as a textbook. On one side were the traditionalists, the adherents to Euclid, such as Augustus De Morgan and Cayley, while on the other were the reformers, such as Sylvester and Hirst who, to quote from Sylvester’s 1869 address to the BAAS in Exeter, would “rejoice to see … Euclid honourably shelved or buried ‘deeper than did ever plummet sound’ out of the schoolboys’ reach” (Sylvester, 1870, 6). But if Euclid was to be rejected then an alternative standard, together with examinations, had to be developed. For this purpose, in 1871, the Association for the Improvement of Geometrical Teaching (AIGT) was formed. It is no surprise that Henrici, having been educated in Germany, was in the reformers’ camp. He was an early member of the AIGT; he joined in 1872 and was elected an Honorary Member in 1885. His “fondness for modern geometrical methods” was well

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48 Gottlieb Coradi of Zurich, founded in 1880, was a leading manufacturer of high precision mathematical instruments.
49 The cost was £60 in 1911, roughly equivalent to £7,000 in 2020. See https://www.in2013dollars.com/uk/inflation/1911.
50 For further information on Henrici’s harmonic analyser, see (Barrow-Green, 2017).
51 The 1868 report of the Endowed Schools (Taunton) Commission had drawn attention to the unsuitability of Euclid for schools and in response James Wilson, mathematics master at Rugby School, had published a textbook which covered the contents of the first two books of Euclid in a new way (Wilson, 1868).
52 For details of the debate, see (Howson, 1982, Chapter 7; Richards, 1988, Chapter 4; Price, 1994, Chapter 2).
53 The BAAS responded by establishing a 12-strong committee to investigate improving the teaching of elementary geometry, members of which included Sylvester, Cayley, Clifford, Hirst, Smith and George Salmon. (This was the committee which Henrici joined in 1875.) Report of the Thirty-ninth Meeting of the British Association for the Advancement of Science Held at Exeter in August 1869, London: John Murray (1870), p.lxxvii.
54 The AIGT was the forerunner of today’s Mathematical Association, the UK subject association for mathematics teachers.
55 Prof. Henrici on Mathematical Teaching, The Dublin Review 11 (January 1884), p.202. The term “modern geometrical methods”, here refers to pure or elementary projective geometry, i.e. not ‘modern’ in the sense of Pasch or Hilbert.
known, and his 1883 BAAS address (Henrici, 1884), in which he tackled head on the problem of using Euclid as a textbook, even to the extent of being critical of the AIGT whom he gently castigated for not being bold enough to embrace modern methods, was widely reported. The 1883 BAAS meeting was a well-judged occasion for Henrici to air his views since Cayley was the President of the BAAS that year and an unusually large number of mathematicians were present.

The description of Henrici’s geometry course at UCL shows the strong emphasis he placed on geometrical drawing and on the construction of geometrical models:

In this Course Geometry will be treated by modern as distinguished from Euclidean methods ... [the lectures] will supplement those on Coordinate Geometry; and all students who desire to become acquainted with the Theory of Conics and Quadric Surfaces and with the higher branches of Geometry, are strongly recommended to attend them. One of the great advantages of the purely geometrical methods is that all operations are performed by constructions, mostly in three dimensions. Thus the student learns to realize figures in space, whilst in Coordinate Geometry the geometrical meaning of the algebraical operations is too easily lost sight of.

The full benefit of this, however, can be obtained only by applying practically on the drawing-board the theorems and methods given in the lectures to the solution of geometrical problems. Students attending this class are therefore strongly recommended to join the Class of Geometrical Drawing which will be conducted throughout in connexion with it.

In order still to promote this object facilities are provided in the Work-room for the construction of Geometrical models.

The ‘Work-room’ where students constructed geometrical models was sufficiently novel to warrant two reports in Nature. In the first report, written during 1876 while the Special Loan Collection of Scientific Apparatus was on display in South Kensington, the anonymous author describes the rationale for the room’s existence:

Various models to illustrate the theorems of modern and higher geometry, of kinematics and mechanics, so difficult to understand theoretically, — such models are so largely represented in the South Kensington Exhibition collection—will be made in a simple manner by the students themselves, side by side with theoretical study. The best models, and such as require more time and accuracy for their construction, will be preserved in a small educational collection. It would hardly be possible to insist too strongly on the usefulness, or rather on the absolute necessity of such work for the successful study of science. It is only when the student has not only seen and handled various practical illustrations and applications of the theorems of geometry and mechanics he is studying, but when he has himself constructed them—however roughly

56 Here again Henrici meant the methods of pure or elementary projective geometry. The President of the AIGT, Robert Hayward (1829–1903), provided a well-considered response to Henrici’s address at the AIGT annual meeting the following year (Hayward, 1884, 23–29).
57 University College Calendar, 1876–77, p.31. Quoted in (Richards, 1988, pp.140–141).
58 The South Kensington exhibition is discussed in Section 5 of this paper.
approximate they may be—that the mathematical truths will be permanently impressed on his mind.\textsuperscript{59}

Some two years later, the same author paid a second visit to the Work-room and reported in detail on what he saw:

Following the order indicated in the syllabus, we first examined the models in the mathematical section. Here we were specially interested in the models illustrative of most of the propositions of modern geometry;\textsuperscript{60} pencils of planes and of lines (to show the simple contrivances employed, we may say these models were made of knitting-needles with small spherical ends of sealing-wax of different colours, thus enabling the student to see their different directions; in other cases joins were indicated by ties of different coloured wool, thus showing that the corresponding points of two perspective triangles meet in a line). Projective rows of points made of pricked wood, the corresponding points joined by India-rubber threads; models exhibiting the generation of ruled surfaces of the second order, movable models made of silk threads stretched by weights, parallel pencils of lines making the paraboloid. The generation of curves by the intersection of pencils of lines; this was shown by two flat pencils of lines made of coloured silk in mahogany frames, one of which moved upon the other; at the intersection of certain pairs of threads were placed small indices which clearly showed to the eye various forms of ellipses and hyperbolas. This model we remember attracted considerable attention at the conversazione in June last, while Prof. Henrici was manipulating it so as to give the curves named.\textsuperscript{61}

Henrici’s idea of encouraging undergraduates to learn geometry by working with models appears to have been unique in Britain at the time. No-one else stressed the empirical study of objects in space the way Henrici did. But Henrici’s background, embracing practical engineering and mathematics, as well his experience of Redtenbacher’s teaching with models, was different to that of other teachers of geometry. It gave him an eye for making geometry intelligible as well as for demonstrating its usefulness. A similar stress on the practical is evident in his writings. In his elementary geometry textbook, \textit{Congruent Figures} (1879), in which geometrical drawing was systematically combined with the teaching of geometry, the emphasis was on “knowledge gained by experience”.\textsuperscript{62} It was deliberately designed so students could:

\begin{quote}
... realise the geometrical contents of the propositions as properties of space through actually seeing their truth by the mental or physical inspection of figures, instead of being convinced of their truth by a long process of logical reasoning. (Henrici, 1879, x)
\end{quote}

\textsuperscript{59} \textit{Nature} 15, 16 November 1876, p.69. The Work-room was not solely for the use of geometry students but was also open to physics and engineering students who were encouraged to construct models to help with the study of kinematics and engineering.

\textsuperscript{60} The term ‘modern geometry’ here and elsewhere means geometry from a projective standpoint rather than from a Euclidean one. Not all geometers of the period liked the term, e.g., Luigi Cremona in the preface to his textbook on projective geometry, objected to it on the grounds that it expresses a relative idea: the methods could be regarded as modern while the matter is largely old (Cremona, 1873, vi).

\textsuperscript{61} \textit{Nature} 18, 23 May 1878, pp.95–96.

\textsuperscript{62} The book was part of The London Science Class-Books series edited by George Carey Foster, professor of physics at UCL, and Philip Magnus, the educational reformer. Another aspect of the book, the use of folding as a method of logical reasoning, is discussed in (Friedman, 2018, 109-110). The book was reprinted in 1888 and 1891, and it was published in Japanese in 1892.
Students would thus be prepared “from the very first for those modern methods of which the method of projection and the principle of duality are the most fundamental” (Henrici, 1879, xv).

Although the book ostensibly covered most of the first four books of Euclid, it was a far cry from a standard Euclidean presentation. As one reviewer observed:

> What we have here is not an amended or modified Euclid; not even a freer handling of Euclid’s matter on a similar general plan; but a wholly fresh exposition from the point of view suggested by modern developments of the science.63

Although, as the reviewer acknowledged, students were likely to find learning geometry by using Henrici’s book harder than by using Euclid, not least because rote learning was impossible. Nevertheless, the rewards were deemed to be greater: the student who mastered the difficulties of Henrici was “on the high road to modern geometry” while the student who mastered the difficulties of Euclid “as commonly taught” was not. The same reviewer also noted that had Henrici’s book appeared a few months earlier it would assuredly have been a target for Charles Dodgson, whose humorous defence of Euclid, *Euclid and his Modern Rivals*, had been published shortly before.64 Whether encouraged or not by this review, Dodgson added *Congruent Figures* to his group of *Rivals* in the second edition (Dodgson, 1885, 71–96).

In the preface of *Congruent Figures*, Henrici said that he intended to write a second volume on similar figures, but it never appeared. Judging from his address to the British Association it seems he was unable to find a satisfactory way of completing the project (Henrici, 1884, 400). There he drew attention to what he described as Clifford’s suggestion—although the idea goes back much earlier than Clifford—that Euclid’s parallel postulate be replaced by the postulate of similar figures, i.e. that in a plane similar figures exist, and expressed the desirability of working out a syllabus of plane geometry based on such a postulate. By embracing the advantages of the projective standpoint while at the same time considering ideas of congruence and similarity, concepts from metric geometry, he was adopting a fresh, mixed pragmatic, approach to the teaching of geometry as opposed to a traditional, systematic, one. This approach would certainly have been supported by Clifford and it seems probable that Clifford’s death in 1879, the year *Congruent Figures* was published, was a factor in Henrici’s inability to produce the second volume.

For students in technical education, Henrici was ardent in his belief that using Euclid to teach geometry was inappropriate, and that it was the teaching of pure (or elementary projective) geometry that should be prioritised:

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64 Charles Dodgson, mathematical lecturer at Oxford, is better known as Lewis Carroll, author of the *Alice* books.
It is by the neglect of pure geometry and its applications to geometrical drawing that Cambridge has lost, or rather never has had, contact with the practical needs of the nation. All the marvels of modern engineering have sprung into existence without its help. The great engineers have had to depend to a degree, now unheard of, upon costly experiments, until they themselves gradually discovered mathematical methods adapted to their purpose.\(^{65}\) (Henrici, 1884, 397)

While it might not seem so surprising that Cambridge was guilty of such neglect, it is perhaps more surprising to learn that Oxford was not. Henry Smith, the Savilian Professor of Geometry, had been teaching pure geometry in Oxford since the 1860s, as his student John Russell attested.\(^{66}\) In 1884, the year after Smith’s death, Charles Leudesdorf began to lecture in Oxford on projective geometry, taking the subject to a more advanced level, while at the same time working on a translation of Cremona’s *Elementi di Geometria Proiettiva*. The latter was a project initiated by Sylvester, who had succeeded Smith to the Savilian chair and had also begun to lecture on projective geometry. In Cambridge, lectures on projective geometry began only in 1888. They were given by Richard Pendlebury who, apart from being senior wrangler in 1870, appears to have had an undistinguished career.

At the Central Institution, where Henrici did have “contact with the practical needs of the nation”, he put his ideas into practice. His methods might not have been appreciated by those of a more conservative or less practical persuasion such as Dodgson, but they found favour elsewhere, as, for example, among architects, as Lawrence Harvey observed:\(^{67}\)

> Owing to [the] common-sense way of treating the definitions, and also the consideration of the whole extent of lines and planes from infinity to infinity, instead of small parts, modern geometry as taught by Professor Henrici at the City and Guilds’ Central Institute, was to Euclid what the sewing-machine was to the needle. ...

> After following Professor Henrici’s lectures, the author, instead of advocating geometry as an introduction to masonry, which he had previously thought advisable, recommended lessons on masonry as the best preparation for entering on the study of geometry. (Harvey, 1888)

In an age of increasing industrialisation, Harvey’s sewing machine analogy made his view of Euclid abundantly clear.

### 4.2 Vector Analysis

Another area in which Henrici made a mark as a teacher and showed himself to be in the vanguard in Britain was in the promotion of vectors. As he wrote to Pearson in 1891:

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\(^{65}\) As a counterexample, Henrici did note that Maxwell had a great appreciation for modern geometry (Henrici, 1884, 397).

\(^{66}\) See the preface to Russell’s textbook on pure geometry (Russell, 1893, vi). Smith’s “captivating” lectures on geometry created a taste for the subject among Oxford students (Elliott, 1925, lxviii). See also the preface to John Hatton’s textbook on projective geometry in which the author acknowledges the use of Smith’s lecture notes (Hatton, 1913, vi). For information on Smith’s lectures, see (Hannabuss, 2013, 250–251).

\(^{67}\) Lawrence Harvey was an architect and a teacher, although mostly distinguished as a journalist. Educated in Zurich and in France, he was a promoter of descriptive geometry in England and taught stereotomy to masonry students at the Central Institution (Thibault, 2019).
What is original in my course is the consistent use of vectors whenever space-relations are in question. ... Altogether I find the use of vectors makes students attend more to the concrete things to which they apply and makes more rapid progress possible.

I believe if you used vectors in the same manner you would like them and come to the same conclusion about their use. The great drawback is that one cuts oneself somewhat adrift from the ordinary textbooks etc. and that there is no textbook in vector analysis. I therefore am engaged in writing one.68

But Henrici’s textbook on vector analysis did not fully materialise. The first part of it, which was an elementary text based on his “charming lectures” to first-year students, appeared as Vectors and Rotors compiled by his assistant (Henrici & Turner, 1903).69 The major part was never completed. Nevertheless, his courses on vector calculus were very successful (Turner, 1911, 76). In 1903, the same year Vectors and Rotors was published, Henrici gave An Address on the use of Vectorial Methods in Physics to both Section A and Section G (Engineering) at the British Association meeting in Southport (Henrici, 1904). This address, which was deemed to be one of the most important on physics at the meeting, generated a lively discussion on the lack of a standardised notation. As a result it was agreed that a committee should be appointed to consider the issue, although nothing seems to have come of it, and in 1906 Henrici again spoke at the BAAS on the use and notation of vectors.70

4.3 Laboratory of Mechanics
A novel element of Henrici’s teaching at the Central, was his laboratory of mechanics. Inspiration for the laboratory may have come from the engineering laboratory at UCL set up in 1878 by his brother-in-law, Alexander Kennedy71 although Henrici may well have been buoyed by his experience with the geometry workroom at UCL. A notable feature of the laboratory were the “ingenious devices” which Henrici had arranged to be fitted to pieces of apparatus in order to make extreme accuracy of measurement possible.

On 12 June 1896, the Central threw open its doors to the public so they could see for themselves the laboratory and the equipment:

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68 Henrici to Karl Pearson, 3 June 1891. Karl Pearson Papers, General Correspondence 11/1/8/49. UCL Special Collections, London.

69 See the review in The Mathematical Gazette (Anon., 1904). The book that did become the standard text is (Wilson, 1901) which was based on lectures delivered by Josiah Willard Gibbs at Yale. It ran into seven editions up to 1943.

70 Physics at the British Association, Nature 68, 22 October 1903, p.609. The standardisation of vector notation was an issue that exercised the minds of many mathematicians during this period. In 1903 the German Mathematical Society set up a commission to tackle the question but with little success, and in 1908 the International Congress of Mathematicians in Rome resolved to set up a similar commission under the leadership of Jacques Hadamard. An exchange of views took place, but it proved impossible to arrive at any conclusions (Hobson & Love, 1913, 40). For a standard history of vector analysis see (Crowe, 1994), although some of Crowe’s conclusions have since been challenged (Caparrini, 2003).

71 Kennedy’s laboratory was the first in Britain to provide facilities for experimental work on an engineering scale (Gibb, 1938, 217).
The object of the conversazione at the Central Technical College was to enable the scientific public to witness the ordinary working life of the students there. Consequently, while in deference to the lady visitors, the staircases and corridors were rendered attractive with arches of palms, arm chairs, arc lights, and Hungarian airs, neither the blossoms, the lamps nor the band, were allowed to intrude on the apparatus and machinery, which were left in every-day use.

Laboratory sinks remained sinks, and were not converted into make-believe flower boxes; while no green baize covers hid the traces of oil and tools on the workshop benches, nor made them into extemporised billiard tables.

An examination of the laboratories and workshops by the visitors at the conversazione, made it clear that the aims initially laid down for the working of the Central Technical College have been kept in view. The department of Mechanics and Mathematics showed the apparatus which had been developed for familiarising students with the laws of motion and force. ... Apparatus for measuring centrifugal forces, studying impact, finding moments of inertia, timing the vibrations of pendulums ...

In the same department were seen calculating machines, and quadric surfaces, planimeters and plaster of Paris models, integraphs for solving differential equations, and integrators for evaluating area; while the smooth working of the latest form of Prof. Henrici’s harmonic analyser, led the engineer to speculate on the time when all calculations, however complex, would be done by turning a handle, and when the brain would be left quite free to think and originate.72

From this rather vivid report, it is evident that the laboratory did not only contain equipment for experiments in mechanics but that Henrici’s models of surfaces and the harmonic analyser had their places in it too. It is evident too that the reporter was very impressed by what he saw, and indeed he went on to rue the fact that such a laboratory had not been available when he was a student some thirty or forty years before.

Henrici’s views about the importance of the use of experiment in teaching mathematics also extended to school-teaching. In 1894 he weighed into a debate in Nature about science teaching in schools, arguing that if mathematics were “taught through experimental science” pupils could “grasp in the concrete form the meaning of a formula or an equation which in the abstract form of pure mathematics remained a mystery to them.” (Henrici, 1894).

4.4 Reputation as a teacher

Henrici was recognised as an excellent teacher and several of his former pupils paid tribute to him in this respect.73 At UCL, the physicist Oliver Lodge who studied Riemann surfaces and other parts of higher mathematics with Henrici, found his lectures “admirable”, and considered him “a great teacher of mathematics, of the German school” (Lodge, 1931, 85). For the mathematician George Ballard Mathews it was Henrici’s lectures on projective geometry which stimulated his interest in the subject and made him “realise that mathematics is an inductive science, and not a set of rules and formula” (Mathews, 1914, vii), while the English scholar, Walter Raleigh remarked on the “masterful ease and

73 See (Hill, 1918, pp.xlvii–xlviv).
freedom” with which Henrici overturned the conventional teaching of Euclid (Raleigh, 1917, 4). In the case of Edgar Walford Marchant, the first holder of the chair of electric power engineering at the University Liverpool, it was Henrici’s teaching of Maxwell’s theory and especially the use of vector notation that was “extremely useful” (Turner, 1911, 75).

Under Henrici’s guidance, the teaching of mathematics to engineers at the Central acquired a high reputation, as the engineering correspondent of The Times attested:

At the present time the outstanding feature of the Central College is the department of mathematics. There may be better facilities for the training of mathematicians elsewhere but there is surely no place where engineering students are taught mathematics in such an interesting and suitable fashion. For the sake of the coming generation of engineers it would be well if every mathematical lecturer who has to do with engineering students would visit the laboratories in which the elementary principles of mechanics and mathematics are taught at this college (Anon., 1910).

When Henrici retired from the Central an extended account of his lecturing style was published in the college journal where he was described as “a man of the most persuasive eloquence” and “one of the most popular—if not the most popular—lecturers at the Central”. The author, Henrici’s colleague George Turner, concluded:

Though Professor Henrici is a Mathematician of European reputation, his chief title to fame rests on his teaching powers. He had the supreme gift of making his subject interesting to all, of rousing the sluggish imagination, and of making the intellectually purblind see clearly. ... With utmost confidence it may be stated that all old students have nothing but pleasant memories of their work—sometimes hard, but always fruitful and suggestive—under Professor Henrici. (Turner, 1911, 79-80).

Although the account was written as a tribute to Henrici and should be read in that light, it nonetheless conveys the warmth of a man who was both admired and held in affection by his colleagues and students alike.

5. Models of surfaces in Britain

The first sustained interest in the building of models of surfaces arose in Germany in the early 1860s with the work of Ernst Kummer who in 1862 began constructing models of the remarkable quartic surfaces he had discovered. He presented these ‘Kummer surfaces’, as they are now known, at meetings of the Royal Prussian Academy of Sciences in Berlin in 1863, 1866 and 1872, where they were much discussed (Fischer, 2017, 149). Meanwhile mathematicians in Britain had begun to discuss models of surfaces.

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74 Germany would rapidly become the main centre for models both for research and for commercial production, the latter led principally by the firm founded in Munich in 1877 by the mathematician Alexander Brill and his brother Ludwig. In 1875 Alexander Brill, who had been a student of Clebsch in Karlsruhe at the same time as Henrici, had joined Klein at the Munich Technische Hochschule, the latter becoming the first and most important place for model building in Germany and remaining so until Brill left for Tübingen in 1884. For examples of models developed by Clebsch, Klein and others, see the Göttingen collection of models, http://modellsammlung.uni-goettingen.de/index.php?lang=en&s=1, and (Fischer, 2017). For a discussion of Klein’s research relating to models, see (Rowe, 2013a).
In 1861, Sylvester proposed the idea of constructing a model in wire of the 27 lines of a cubic surface, although he never carried it through (Sylvester, 1861). Four years later, in March 1865, he was in communication with Cayley and Hirst about the possibility of building a model of his “amphigenous surface”, the ninth order surface which had emerged from his proof of Newton’s Rule for the discovery of imaginary roots of equations of the fifth degree (Sylvester, 1864). Sylvester was hoping for a grant from the Royal Society to pay for the model’s construction even though he himself was unable to imagine what it would look like. Cayley, who encouraged him to apply for funds, was keen for the model to be made and voiced the hope that “a few more algebraical surfaces could be modelled”, noting that he was aware of only two: Fresnel’s wave surface and Steiner’s Roman surface (Crilly, 2006, 293). (As it turned out, a model of the amphigenous surface was not made until 1870 and then it was Henrici who produced it, as will be described below.) Three years later Cayley’s hope of more algebraic surfaces being modelled had still not been realised, and he, together with Sylvester and Maxwell, made an approach to the Royal Society for a grant for the construction of such models, although it is not known whether they obtained it.

One of the first to make a public presentation of models of algebraic surfaces in Britain was Plücker who in his talk at the 1866 BAAS meeting in Nottingham used a series of boxwood models of quartic surfaces to illustrate his theory of quadratic line complexes (Plücker, 1867). Hirst, having seen the models at the meeting, asked Plücker if he could have a selection of them for the Royal Society. In the event, Plücker had a new set of fourteen

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75 In 1849 Cayley and George Salmon showed that every smooth cubic surface contains exactly 27 straight lines: Cayley discovered that there are lines and Salmon counted them (Cayley, 1849). Cayley had difficulty in envisaging what a model of such a surface would look like although he did attempt to draw some of the lines, (Cayley, 1849, 129–132), see (Crilly, 2006, 148).

76 For a discussion of Sylvester’s lengthy and important but disorganized paper, see (Parshall, 2006, 180–184).

77 Fresnel’s wave surface, which describes the propagation of light in an optically biaxial crystal, is a particular case of Kummer’s quartic surface. Cayley wrote about it in (1846). Steiner’s Roman surface, a one-sided quartic surface, was later studied by Cayley (1873a). For a discussion of the latter’s history, see (Rowe, 2013b, 53–55). For 19th century examples of both these surfaces, see the Oxford Mathematical Institute collection, https://www.maths.ox.ac.uk/about-us/history/models-geometric-surfaces.

78 Unfortunately, the Royal Society records do not document whether the grant was awarded but given the status of the applicants it would seem likely it was unless the application was withdrawn.

79 Plücker called these quartic surfaces ‘complex surfaces’ because they are enveloped by lines in a quadratic line complex. He described linear and quadratic complexes in detail in his Neue Geometrie der Raumes (1868–1869), the second volume of which was completed after his death by Klein who was then Plücker’s assistant. According to Klein, Plücker was given the impetus to make models by Michael Faraday who had used models to help with his study of mathematics (Rowe, 2013a). For an account of Plücker’s line complexes written for an English audience, see (Jessop, 1903). For a modern account, see (Rowe, 2018b) or (Rowe, 2019, 14–19).

80 The early history of these models is documented in the Plücker-Hirst correspondence, September 1866–October 1867 held in the London Mathematical Society archives. For a transcription and translation of the letters, see https://www.lms.ac.uk/sites/lms.ac.uk/files/library/Plucker%20letters%20to%20Hirst.pdf. For a detailed mathematical description of them, see (Cayley, 1869). The models were exhibited at the South Kensington Exhibition in 1876 and, although owned by the LMS, they remained on the South Kensington site and were displayed in the London Science Museum until 2015. They are now on permanent display in De Morgan House, London, the home of the LMS. See also (Rowe, 2013a, 6–7).
models made in Germany (by Epkens of Bonn) for Hirst who subsequently donated them to the LMS (of which he was then a Vice-President) where they are now on display (Figure 6).

Figure 6. Plücker quartic surfaces. Courtesy of the London Mathematical Society

In 1873 there was a flurry of activity in Britain concerning models of surfaces, catalysed by the BAAS annual meeting held in Bradford that year. The meeting had brought together several mathematicians with an interest in geometry, including Cayley, Clifford, Henrici, Maxwell, Smith, and Sylvester. Significantly Klein was there too since it was only the previous year, on his appointment to a professorship at Erlangen, that he had established the first collection of models in a German mathematical seminar (Mehrtens, 2004, 298).

Also present was the Irish mathematician and astronomer Robert Stawell Ball, professor of applied mathematics and mechanics at the Royal College of Science, Dublin. Ball, whose interest in geometry was related to his work on the theory of screws, reported that “Klein had told them about Plücker’s linear complex, and certain recent developments in the fascinating field of geometrical relations which it involved” and described how “he and Clifford captured Klein after the meeting, and sat up half the night exchanging ideas” (Ball &

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81 Smith was President of the Mathematics and Physics section, and the meeting was notable for the amount of mathematics discussed. So much so that Oliver Lodge was moved to describe it as “a mathematical orgy” (Lodge, 1931, 135).
A few weeks later, doubtless inspired by Klein, plans for what would become the Cambridge Modelling Club were afoot. The Club, which had its first meeting in February the following year, was founded “to promote the making of models, machines and drawings, illustrative of geometry”, and its members included Cayley (president) and Maxwell (custodian of the models) as well as staff of the Cavendish laboratory and some undergraduates.

To be accepted as a member of the Modelling Club, the applicant had to “have made or assisted in making a model, machine or drawing approved by the officers”. Models made by members included cubic scrolls, ellipsoids and ray surfaces. Among the models made by Cayley for the Club were “a cubic cone with a nodal line, a model of Charles-Nicolas Peaucellier’s parallel motion, linkwork made from zinc, models of two of Riemann’s triply connected surfaces, and to illustrate how models could be made of paper, a skew surface.” (Crilly, 2006, 332). Other models made by Cayley and shown elsewhere include a rough model of Steiner’s Roman surface which he showed at the LMS and which was later displayed at the 1876 exhibition (Cayley, 1870; 1873a), and a string model on a cardboard frame showing all 27 lines of Christian Wiener’s celebrated model of a cubic surface which he showed at the Cambridge Philosophical Society (Cayley, 1873b).

Some years later Cayley’s paper on the latter caught the eye of Henrici’s former student, Mathews. Cayley had approached the construction in a pragmatic fashion, using measurements and approximating equations, rather than tackling it projectively (which would have involved more work). This seemingly perverse approach, prompted Mathews to describe the paper as being “so quaint in its topsy-turvydom as almost to suggest Mr. W.S. Gilbert as joint author” (Mathews, 1898, 217).

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82 1873 was the year in which Clifford first published on his theory of biquaternions which has close connections to Ball’s theory of screws.

83 University of Cambridge Reporter 55, 7 October 1873, p.19. It is conceivable that the Club benefitted from the Royal Society grant mentioned above but there is no record of it. No official paperwork of the Club appears to exist, but the Club’s activities can be traced through announcements in the University of Cambridge Reporter.

84 University of Cambridge Reporter 55, 17 February 1874, p.236. For a list of active members see (Crilly, 2006, 538).

85 University of Cambridge Reporter 55, 17 February 1874, p.236.

86 In 1868/69 Wiener produced a gypsum model of a non-symmetric cubic surface displaying all 27 lines; it was the first model on which each of the 27 lines could be seen (see footnote 75). Such was Wiener’s achievement that Sylvester was led to remark (in rather typical florid style) that the realisation of this model was one of the scientific events which “must forever make 1869 stand out in the Fasti of Science.” (Sylvester, 1869, 155). This was followed in 1871 by Clebsch’s discovery of the beautiful highly symmetric surface, now known as the ‘Clebsch diagonal surface’, which displays all 27 lines and makes the structure of the lines clear (Labs, 2017). An example of the model today referred to as ‘Cayley’s ruled surface’ (given by the equation $3z – 3xy + x^3 = 0$) is in the collection at the Oxford Mathematical Institute, see https://www.maths.ox.ac.uk/about-us/history/models-geometric-surfaces. In the 1860s Cayley wrote extensively about such surfaces in general but to date no evidence has emerged to show that he ever made any. I am grateful to Tony Crilly for help with trying to resolve this issue.

87 The allusion is to the work of the librettist William Gilbert, best known for his collaboration with the composer Arthur Sullivan. As Crilly describes, this paper illustrates Cayley’s hands-on approach to the teaching
Given Maxwell’s interest in geometry, especially its applications to physical problems, an interest which he held throughout his career,⁸⁸ it was fitting that he should be the Modelling Club’s ‘custodian of the models’. He made several models for the Club, including a surface of constant negative curvature and a model of Gibbs’ thermodynamic surface of water. As his lifelong friend Peter Guthrie Tait later wrote, Maxwell “preferred always to have before him a geometrical or physical representation of the problem in which he was engaged; and to take all his steps with the aid of this: afterwards, when necessary, translating them into symbols.”⁹⁹

On 12 May 1876, Queen Victoria opened the Special Loan Collection of Scientific Apparatus at the South Kensington Museum. The Exhibition, which was open for six months and attracted over a quarter of a million visitors, had some 20,000 scientific artefacts on display, among which mathematical models “were abundantly represented” (Lindemann, 1971, 61).⁹⁰ Many of these models came from Germany and France, but Britain exhibited several, with the Cambridge Modelling Club playing its part.⁹¹ Cayley’s Steiner surface was on display, as were stereograms of the lines of curvature of surfaces drawn by Maxwell together with his real image stereoscope for showing them. Another exhibit was Ball’s cylindroid, a surface of importance for kinematics and a subject of discussion in Ball’s recently published Theory of Screws (1876). Henrici’s amphigenous surface was there too, as well as one of his cubic surfaces (described below). The Plücker models of quartic surfaces were also on show, having been restored for the occasion by Henrici. Among the German exhibits was a series of cardboard models of quadric surfaces made by Alexander Brill of Munich which derived from a model Henrici had earlier sent to Klein for presentation at a mathematical meeting in Göttingen in 1873 (Brill, 1892, 298; Rowe, 2013a, 89).⁹² When Henrici saw Brill’s models on display, he regretted he had none of his own available to

and learning of geometry, an approach further exemplified by the several instruments he designed and made to help with the drawing of curves (Crilly, 2006, 332).

⁸⁸ To give but two examples: Maxwell’s article on the cyclide (Maxwell, 1867) and the chapter on confocal quadrics in his Treatise on Electricity and Magnetism (Maxwell, 1873, 181–190). For a discussion on the status of geometry in Maxwell’s mathematical physics, see (Harman, 1985).

⁹⁹ Quoted in (McCartney, 2014, 218). Maxwell’s models are held in the Cavendish Laboratory in Cambridge, see https://cudl.lib.cam.ac.uk/collections/cavendish/1.

⁹⁰ I am grateful to David Rowe for drawing my attention to the memoirs of Ferdinand von Lindemann in which Lindemann describes his visit to the Exhibition. The memoirs, which also describe Lindemann’s friendship with Henrici, were edited by Lindemann’s granddaughter (Lindemann, 1971). For a description of all the models on display, see the Catalogue of the Special Loan Collection of Scientific Apparatus at the South Kensington Museum (3rd edition), 1877. An introduction to the models is given by Smith in the exhibition Handbook (Smith, 1876, 45-53).

⁹¹ The Modelling Club had established a relationship with the Museum two years before the Exhibition, having visited it to view the Fabre de Lagrange models of ruled surfaces and to meet with the Director (University of Cambridge Reporter 55, 24 March 1874, p.236).

⁹² Lindemann, in his obituary of Henrici, claimed that the model that inspired Brill had come to the latter through Clebsch’s Nachlass although no other evidence has been found which supports this (Lindemann, 1927, 159).
exhibit (Lindemann, 1927, 159). Since Smith mentions these models in his introduction to the models in the Exhibition (Smith, 1876, 46–47), Henrici had obviously intended to exhibit them. Smith also mentions other models by Henrici which are not in the catalogue: a model showing the distribution in space of the 27 lines on a cubic surface but unaccompanied by the surface itself (Smith, 1876, 47–48), and models of constant positive and constant negative curvature (Smith, 1876, 52–53). No mention of the former model appears in Henrici’s writings so perhaps it was never made,93 but with regard to the latter Lindemann reports that when he met Henrici in April 1876 Henrici was still working on them in his laboratory at UCL (Lindemann, 1927, 158–9). Presumably they were never finished. Another Henrici lacuna at the Exhibition was his planned talk on models—Plücker’s and his own—which for reasons unknown did not take place.94

British geometric models were also exhibited outside Britain. In 1893 the Deutsche Mathematiker-Vereinigung (DMV), as part of their annual meeting which was held in Munich that year, put on an exhibition, *Mathematische und mathematisch-physikalische Modelle, Apparate und Instrumente*, which contained several British models of surfaces.95 Apart from Germany, Britain sent more models to the exhibition than any other country. That this was so was due largely to Henrici who was the mainspring of a committee charged with organising the British contribution, the other members being George Greenhill, Professor of Mathematics at the Royal Military Academy in Woolwich, and Lord Kelvin.96 Two of the British geometric models exhibited in South Kensington made a reappearance in Munich: Henrici’s amphigenous surface, and Ball’s cylindroid. And on this occasion Henrici’s role in the development of cardboard models of quadric surfaces was explicitly acknowledged by Brill (Dyck, 1892, 258). Henrici also exhibited the sixth-order surface he had first shown in front of the LMS in 1871 and his movable hyperboloids of one sheet (both described below).97

The British models of surfaces on display in South Kensington were produced by mathematicians to augment their research and as such were atypical of most of the models of surfaces in use in Britain in the period. Like many of her European neighbours, Britain imported models from Germany for teaching or demonstration purposes. Oxford provides a

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93 In 1882 the Cambridge geometer Percival Frost described how to make a thread model of the 27 lines but the method was tedious and, by his own admission, imperfect since several of the lines were too far away to appear. See (Frost, 1882) and (Henderson, 1915, 59).
95 The Exhibition had been planned for the DMV’s 1892 annual meeting which was supposed to be held in Nuremberg but which was cancelled due to an outbreak of cholera.
96 William Thomson had been elevated to the peerage in 1892, choosing for his title Baron Kelvin of Largs.
97 As well as geometric models, Henrici had two other exhibits in the Exhibition: a model for explaining the mechanism of Edmondson’s circular calculating machine, and his first harmonic analyser. Although Henrici’s second and more sophisticated harmonic analyser had been developed by the time the exhibition opened, it had not been completed in 1892 when the exhibits were sent out from England for the (cancelled) Nuremberg exhibition. As well as providing exhibits, and encouraging others to do likewise, Henrici wrote a substantial article on harmonic analysers for the catalogue (Henrici, 1892a).
typical example. In 1886 Sylvester, who had returned to England from the United States in 1883 to take up the Savilian chair, announced an undergraduate lecture course on “Surfaces, illustrated by plaster, string and card-board models”, with models being supplied by Brill.\textsuperscript{98} The course had to be postponed because the university grant he had applied for in order to purchase the models had not been sanctioned in time. He put on the course the following year, and in later years, renaming it “Surfaces of second order, illustrated by models”.\textsuperscript{99} To Sylvester’s disappointment students did not attend, probably because the subject was non-examinable (Parshall, 2006, 302).

In Edinburgh, the professor of mathematics, George Chrystal, gave an address to the Edinburgh Mathematical Society in 1884 on surfaces of second order in which he advocated their study, lavishly illustrating his address with “a large number of beautiful models in wood, plaster, cardboard and thread”.\textsuperscript{100} Thirty years later, an extensive collection of models was displayed at the Napier Tercentenary Exhibition in Edinburgh.\textsuperscript{101} Many of models of surfaces were listed as coming from the Edinburgh Mathematical Laboratory established by Edmund Taylor Whittaker, Chrystal’s successor at the university.\textsuperscript{102} No maker’s name is given in the catalogue but it is likely that they were Brill or Schilling models. The exhibition included some of Chrystal’s models of surfaces together with others from J.E.A. Steggall of University College Dundee, as well as a model made by Maxwell of Gibbs’ thermodynamic surface.

The Cambridge Modelling Club appears to have fizzled out in the year following the South Kensington Exhibition.\textsuperscript{103} It is not clear why this happened, but a contributory factor may have been Maxwell’s poor state of health. But even without the Club, an interest in models was maintained in the university. In 1907, when the Chancellor, the 8th Duke of Devonshire, made a public appeal for more funds for the University, he identified: “Two lecture-rooms with an adjacent library and a museum of mathematical models” as one of the most pressing needs of the mathematics department,\textsuperscript{104} although the call fell on deaf ears.

On the construction of models of surfaces in Britain, little textual evidence exists. An exception is the short book by William Henry Blythe which sets out to “give an outline of analytical and geometrical methods that are used in treating of cubic surfaces, not taking

\textsuperscript{98} Oxford has a beautiful collection of 19\textsuperscript{th} century plaster models which are available to view on their website: https://www.maths.ox.ac.uk/about-us/history/models-geometric-surfaces. It seems likely that these models are the ones ordered by Sylvester.

\textsuperscript{99} Oxford University Gazette, 15 October 1886.

\textsuperscript{100} Nature 29, 17 January 1884, p.278.

\textsuperscript{101} See (Crum Brown, 1915).

\textsuperscript{102} The Edinburgh Mathematical Laboratory was founded for the purpose of teaching aspects of numerical analysis, see (Maidment, 2020).

\textsuperscript{103} The last publicised meeting of the Club took place on 5 March 1877. See University of Cambridge Reporter 58, 13 March 1877, p.303.

\textsuperscript{104} The Times, 16 February 1907, p.8.
the more advanced part of the subject, but considering mainly anything that may help to the construction of models.” (Blythe, 1905, v). Blythe, who had a rather unremarkable career—he had come 7th in the second class of the Cambridge Mathematical Tripos of 1878 and was a teacher in London and Milford Haven for a short while before returning to live in Cambridge—had been interested in cubic surfaces for some time, publishing several articles on the topic. But the book was not a great success. It essentially fell between two stools. The theoretical part was considered “an unsatisfying mix of rudiments and quotations and references to difficult theorems”. It was based on the work of Cayley, Ludwig Schlafli and Theodore Reye, but, as the German reviewer, Stanislaus Jolles, pointed out, it ignored the fundamental works of other important contributors to the subject, such as Clebsch, Cremona and Steiner. The practical part on constructing models received a better press, although a fuller account with more diagrams would have fared better. Meanwhile in Cambridge, Henry Martyn Taylor, made an extensive study of the 27 lines on a cubic surface, work which culminated in a paper on successfully constructing the lines and models, both of which he had himself produced after he had gone almost blind (Taylor, 1900).

On the theoretical front, and forming rather a stark contrast to Blythe’s book, was the (now) classic text on Kummer’s quartic surface by the young mathematician Ronald Hudson, published posthumously after Hudson’s death in a mountaineering accident (Hudson, 1905). Hudson, whose father was one of the original members of the Cambridge Modelling Club and whose two sisters were mathematicians, was considered one of the most gifted British geometers of his generation. Although the subject of Hudson’s book is beyond the scope of this article, it is relevant to note that Mathews, in his review of the book, singled out the photograph of the plaster model of the general surface, which formed the frontispiece to the book, as being especially helpful to the reader (Mathews, 1905, 229). Hudson himself had an interest in model-making, using coiled steel wire to make models of different forms of Kummer’s surface (Hudson, 1905, 22). It was on his initiative that there was a display of geometric models at the 1904 BAAS meeting held in Cambridge.

As is evident from the above, there was considerable interest in models of surfaces in Britain in the half-century between 1860 and 1910. But equally evident is the fact that only

105 Nature 74, 28 June 1906, p.197.
107 Nature 74, 28 June 1906, p.197.
108 Taylor, who was 3rd wrangler in 1865 and spent most of his life in Cambridge, was the author of several geometrical papers. He became blind by 1897, and subsequently developed Braille so that it could be used in mathematics textbooks. He was elected a Fellow of the Royal Society in 1898.
109 Hudson, who was senior wrangler in 1898, was a Lecturer in Mathematics at University College, Liverpool at the time of his death in 1904.
110 The correspondent from Nature was impressed by the exhibit and singled out for special mention a model of the 27 lines on a cubic surface and Ball’s cylindroid. See Nature 70, 18 August 1904, p.368.
a small number of mathematicians were involved in making such models, and those that did so, did so only occasionally. Nowhere in Britain was there a place like Göttingen with an academic programme of constructing such models. Nor was there a specialist manufacturer of models of surfaces. Businesses like that of G. Cussons & Company of Manchester, which opened in 1876 as manufacturers of educational and scientific apparatus, produced geometric models but these were of an elementary nature. However, the lack of a British Brill is not surprising given the expertise required to set up such a business and the limited size of the market, locally and internationally. There was not enough room for more than one commercial maker of specialist models.

Against this backdrop, Henrici stands out as exceptional in his production and use of models. He not only made some of his own models with his own hands, he also made models a prominent area of his research—witness the fact they appear as part of his Royal Society citation—and he, more than anyone in Britain, encouraged their use in teaching. It is to Henrici’s models we now turn.

6. Henrici’s geometric models

Although only one of Henrici’s original models survives today, it is possible to reconstruct others using 3D printing. This process has the advantage, due to the strength of the material (nylon plastic), that the models do not need a solid base like 19th century models and so both sides of the surface can be seen.

Henrici provided little information about the construction of his models. Rather tantalisingly, Lindemann, in his obituary of Henrici, lists among Henrici’s writings “On the Construction of Cardboard Models of Surfaces of the second Order. Professorial Dissertation for 1871–1872” but this “Dissertation” is not mentioned elsewhere and nothing even vaguely matching its title has come to light. Given that Henrici took over the chair of mathematics at UCL in 1870, that no other examples of UCL professorial dissertations have been found, and that Lindemann’s obituary was published in 1927, nine years after Henrici’s death, it is possible that Lindemann was confused about the status of the document.

All the models described below were exhibited by Henrici at the LMS unless stated otherwise.

111 The University of St Andrews owns two ellipsoids made by Cussons which are thought to date from around the turn of the 20th Century. In 1912 Cussons produced a catalogue of ‘Apparatus For Practical Plane and Solid Geometry’. For more information about the company, see (Wetton, 1997).

112 Lindemann referred to the dissertation by its English title (Lindemann, 1927, 161). There is no trace of a manuscript or publication in the UCL archives that fits the description.

113 Henrici did give an ‘Introductory Lecture’ at the start of the 1873 session at UCL when he was Dean of the Faculty. But the contents of this lecture relate to the distinction between German and English mathematical education. See Nature 8, 9 October 1873, p.492.
6.1 Quadric surfaces
Judging from the date of the lost professorial dissertation, Henrici was making cardboard models of quadric surfaces from at least 1871 and probably earlier. The one he sent to Klein in 1873 was of an elliptic paraboloid which he had constructed using a series of half-circles spliced together, and presumably he used the same method to make his models of other quadric surfaces. Shortly after seeing Henrici’s model in Göttingen, Brill adapted Henrici’s method and used it to make a series of cardboard models of all the quadric surfaces. A great advantage of these models was that they were inexpensive to produce and, since they folded flat, they were easy to transport. Little wonder that they were the first series of models to be sold by Brill once he had set up in business with his brother (Rowe, 2013a, 89). Figure 7 shows some of the Brill models on display in the Science Museum in London with, in the centre foreground, one folded flat.

![Figure 7. Brill models on display at the Science Museum, 2015](image)

6.2 Cubic Surfaces
As mentioned in the introduction, the first of Henrici’s models to be shown at the LMS was the cubic surface:

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114 See footnote 67.
115 The photograph was taken in 2015 before the remodelling of the Mathematics Gallery at the Science Museum and the models are no longer on display.
\[ xyz = 1, \]

which was shown at a meeting on 13 May 1869. Although Henrici was present and had earlier delivered a paper (Henrici, 1869b), it was Cayley, then President of the Society, who drew attention to the model, pointing out the lines of curvature and singularities although neither he nor Henrici gave an indication of how, or of what material, the model was made or indeed of its size.\(^{116}\) No 19\(^{th}\) century examples of the surface have survived, but Figure 8 and Figure 9 each show a 3D printed model of it.\(^{117}\) Figure 8, which is possibly more like Henrici’s model, shows the four pieces of the surface connected together as a solid object, whereas Figure 9 shows the four separated pieces or ‘shells’, of the surface held together by a small additional support structure in the centre which holds the four pieces together.

![Figure 8. The surface \(xyz = 1\) as a solid model](image1)

![Figure 9. The surface \(xyz = 1\) as four separate pieces](image2)

Later the same year, Henrici displayed another cubic surface:

\[ xyz - \left(\frac{3}{7}\right)^3 (x + y + z - 1)^3 = 0, \]

and, unusually, provided a description of how it was constructed:\(^{118}\)

\[ [\text{It}] \text{ has three biplanar nodes (to a scale of } 2\frac{1}{2} \text{ inches as unit). A sufficient number (eleven) of sections, } x + y + z - 1 = \text{ a constant, cut out in cardboard, are connected in a horizontal position, and kept at their proper distance by three vertical sections } y = z, z = x, x = y, \text{ with regard to which the surface is symmetrical. The model contains the central part of the surface with the three nodes, and is bounded by a sphere of eight inches' radius, with its centre at the origin, large enough to show the position of the three straight lines in the surface (each counting for nine), and to give an idea how the surface extends to infinity. The interstices between the cardboard are intended to be filled up with plaster of Paris [gypsum plaster], so as to form a solid model.}^{119} \]

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\(^{117}\) I am very grateful to Oliver Labs for producing all the 3D printed models discussed in this paper. For other examples of Labs’ 3D printed surfaces, see [http://oliverlabs.net/math-objects/](http://oliverlabs.net/math-objects/).

\(^{118}\) It is rare to find a description of a particular model’s construction. Another known example is that of Christian Wiener’s cubic surface (see Footnote 86).

With a bounding sphere of 8 inch radius, this is a relatively large model but, as Henrici had explained, he had made it this size in order to make it easy to see the position of the three straight lines, each of which is counted nine times, in the surface. Six months later, Henrici produced a plaster model of the same surface, a copy of which was later ordered by the Society.\textsuperscript{120}

![Figure 10.](image1.png) ![Figure 11.](image2.png) ![Figure 12.](image3.png)

Figure 10 shows the solid model of the cubic surface \( xyz - \left( \frac{3}{7} \right)^3 (x + y + z - 1)^3 = 0 \), on display at the Science Museum in 2015. Figures 11 and 12 show both sides of a 3D printed model of the same surface. The three straight lines are not marked on the 3D model but visible in this model, and not visible in the solid model, is the triangle-shaped singular region.

As mentioned earlier, one of Henrici’s plaster models of this surface was exhibited at the 1876 South Kensington Exhibition. After the Exhibition, the model remained on display at the South Kensington Museum, part of which later became the London Science Museum. In 1936 the model was seen by the Vorticist artist Edward Wadsworth who used it as the inspiration for the central design of a London Underground poster advertising the South Kensington museums (Figure 13).\textsuperscript{121} The model remained at the Science Museum until 2015 when it was returned to the mathematics department at UCL where it is now on display together with a copy of the poster.\textsuperscript{122}


\textsuperscript{121} As one might expect, Wadsworth’s design is not devoid of artistic licence. The ‘three straight lines’ are incorrectly placed, and the caption incorrectly reads “MODEL OF A CUBIC SURFACE XYZ=K3(X+Y+Z-1)^3”. Nevertheless, there cannot be many other transport posters that include an expression for a cubic! The poster can also be seen at: https://collection.scinemuseumgroup.org.uk/objects/co8357321/south-kensington-london-transport-poster-poster.

\textsuperscript{122} The model presumably became the property of UCL because Henrici was employed there at the time of the Exhibition.
Henrici also produced a series of models of cubic surfaces but almost nothing is known about them. He showed them at a meeting of the LMS in 1872 where he pointed out their singularities and explained how they were made but unfortunately no details of his presentation were published.\textsuperscript{123}

It is not known what prompted Henrici to start constructing cubic surfaces in 1869 but he could well have been stimulated by the surface constructed a few months earlier by his former teacher, Wiener. A further stimulus could have been Cayley’s memoir which extended results obtained by Schläfli on the classification of cubic surfaces which was published that year (Cayley, 1869).

6.3 Sixth Order Surface
In February 1871 Henrici exhibited a gypsum model of a tubular surface of sixth order, a copy of which was ordered for the Society’s collection.\textsuperscript{124} As Henrici described, the surface can be generated in two ways: “Either a sphere of constant radius moves with its centre on a parabola, or it rolls along the same parabola, always touching both branches. The two envelopes thus produced differ in position only.” The surface has a double curve, a parabola congruent to the leading parabola but running in a plane perpendicular to it,


Emerging from the surface at two points A and becoming an isolated curve, and a cuspidal curve consisting of two finite branches which the double curves passes through and which unite in the points A forming peaks.

The equation of the surface is:

\[(27py^2 + 9zK - x^3)^2 = (x^2 + 3K)^3\]

where

\[K = (x + 2p)^2 + y^2 + z^2 - r^2\]

and r is the radius of the sphere and 4p is the parameter of the parabola. Henrici also gave the equations to the parabola on which the centre of the sphere moves:

\[y^2 = 4p(x + 2p), \quad z = 0;\]

those to the double curve:

\[y = 0, \quad z^2 = -4pz + r^2 - 4p^2;\]

and those to the cuspidal curve:

\[27py^2 - 4x^3 = 0, \quad x^2 + 3K = 0.\]

He constructed the model to the scale of \(p = 1/25\) inch, \(r = 2\) inches, choosing the parameters \(r\) and \(p\) to ensure the singularities are fully evident. If \(r\) is too small with respect to \(p\), the singularities effectively disappear. Figure 14 shows a 3D model of the surface, and Figure 15 shows the envelope for the two-dimensional variant of the surface in which the singularities can be more easily seen.

![Figure 14. Henrici’s sixth order surface.](image)

![Figure 15. Envelope for the two-dimensional variant of Henrici’s sixth order surface.](image)

The year after Henrici showed the surface, Samuel Roberts referred to it in a paper classifying properties of the parallel surfaces of quadrics and conics. Of the many surfaces described by Roberts, it was the only one for which he stated that a model had been constructed (Roberts, 1872, 77). A London resident and LMS stalwart, Roberts would have
known Henrici and seen several of his models. Although the surface is a natural one to construct due to the fact that the two-dimensional variant of the surface is one of the simplest examples with respect to computing an envelope, Henrici himself gave no motivation for constructing it, so discussions with Roberts were very possibly the prompt.

6.4 Sylvester’s Ninth Order “Amphigenous Surface”

On 8 December 1870, Henrici “exhibited a large model of Dr. Sylvester’s amphigenous surface” at a meeting of the LMS. Rather curiously, the report of this event, which included mathematical details of the surface, appeared in *Nature* but not in the Society’s *Proceedings*.\(^{125}\)

In March 1865 Sylvester had met with Hirst and with a mechanician, P. Perigall, on at least two occasions at the Royal Society to discuss the construction of a model of this surface (Hirst 1980, 16–18 March 1865), writing to Hirst shortly afterwards:

> I have thought a good bit upon this wonderful surface since last seeing you. I now think it can be constructed – but I wish first to gain a general idea of its nature. This I propose to do by cutting the different parallel plane sections out of squares of pasteboard and filling up these like the slides in a show. ... ... [Its] form ... seems to be gradually growing up in my mind but it requires a prodigious effort beyond my present powers of conception to realize it in its totality; one glance at the real form projected into actual outward space, and the idea will be gained for all time to come.\(^{126}\)

These discussions appear to have come to nothing, and it was only with the arrival of Henrici a few years later that Sylvester was able to see his surface materialise. There is no record of how the collaboration came about but it is probable that Sylvester having seen Henrici display his models at the LMS put to him the idea of constructing the model.

When exhibited in South Kensington in 1876 the model attracted attention, being mentioned in newspaper reviews of the exhibition\(^{127}\) and singled out by Smith in the Exhibition Handbook as of “great importance” (Smith, 1876, 52). Lindemann considered it particularly instructive and was disappointed that it was not available to purchase, although Henrici did give him a stereoscopic image of it (Lindemann, 1971, 61). When the surface was exhibited in Munich in 1893, the detailed description provided by Henrici in which he included the scale (Henrici 1892b), shows that it was not the same model as the one he exhibited at the LMS. Since it is known from a remark in a letter by Sylvester that a plaster model of the surface had been made by the Royal Society, it is possible that this was the one shown in Munich.\(^{128}\) The catalogue contained an essay by Klein on the use of geometry for

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\(^{125}\) *Nature* 3, 27 December 1870, p.178.

\(^{126}\) Sylvester to Hirst, 28 March 1865. LMS Archives, Hirst Correspondence.

\(^{127}\) See, e.g., *The Times*, 16 May 1876, p.4.

\(^{128}\) Sylvester mentions the plaster model of the surface in a letter to Simon Newcomb of 14 January 1878 (Archibald, 1936, 134). At the time of writing Sylvester was at Johns Hopkins University in Baltimore.
counting the real roots of algebraic equations and although the model itself was not mentioned, Klein made reference to Sylvester’s work on Newton’s rule (Klein, 1892, 9).

The amphigenous surface has for its equation:

\[ JK^4 + BLK^3 - 2J^2LK^2 - 72JL^2K - 432L^3 + J^3L^2 = 0 \]

where

\[ K = \frac{J^2 - D}{128}. \]

The equation is obtained by substituting \( x = 1024L, y = 2D/3, z = 6J \), with \( x, y, z \) as rectangular coordinates. \( J, D, \) and \( L \) are invariants, and in particular \( D \) is the discriminant.

There are two cuspidal curves on the surface, a parabola and an ordinary cusp, which touch each other at the origin, and it is these two curves that give the surface its character. The surface divides space into two congruent parts—this can be seen by turning it through an angle of \( 180^\circ \) about the axis of \( D \), which is wholly on the surface. The plane \( D = 0 \) touches the surface along a curve \( 2048L = J^3 \), and divides each half of the space separated by the surface into three distinct parts \( A, B \) and \( C \) which are well defined. It is this property which connects the surface with the theory of binary quintics, and which Sylvester used to classify the roots of a quintic equation as real or imaginary.\(^\text{130}\) If the point

\(^\text{129}\) The equation for the surface appears in (Sylvester, 1864, 622 & 630). In the Nature report, the equation for \( K \) had an incorrect sign which was corrected in the Munich catalogue.

\(^\text{130}\) It appears that it was only in the 19th century that mathematicians parameterized the coefficients of algebraic functions and then studied the curve or surface associated with the discriminant in order to discuss the number of real roots.
corresponding to a quintic equation with real coefficients lies in space $A$, then all roots are real, in space $B$ two roots are imaginary, and in space $C$, four roots are imaginary.

No original models of the surface or stereoscopic images of the model have come to light but a 3D printed model can be seen in Figure 16. At least two different models were made. In Henrici’s original model the unit was taken as $\frac{1}{6}$ of an inch whereas in the model exhibited in Munich the unit was $\frac{1}{5}$ of an inch. The latter, as well as being larger than the former, had in addition a loosely attached piece of cardboard which represented the discriminant plane $D = 0$ on which various named curves were marked.\textsuperscript{131}

The amphigenous surface is the most complex of Henrici’s models of surfaces and the only one prompted by the work of someone else. That no 19\textsuperscript{th} century example of it has been found suggests that only a very few were made, and since the surface was only ever mentioned in connection with the models made by Henrici, apart from references by Sylvester (who did not make a model), it seems likely that Henrici was involved in the construction of all of them.

\textbf{6.5 Movable Hyperboloids of One Sheet}

Of all Henrici’s models, this is the one that relates most closely to his early work as an engineer working with statical structures. Henrici exhibited two varieties of it in front of the LMS, the first in 1875 and the second in 1892 prior to its going to Germany for the DMV exhibition.\textsuperscript{132}

The model originated in 1873 when Henrici gave a student the problem of constructing a model of a hyperboloid of one sheet by “fixing three sticks anyhow, placing others so as to cut these, and tying them together wherever they meet”.\textsuperscript{133} He had expected the system to become rigid but to his surprise found not only that it was movable but that the different positions of the model formed a system of confocal hyperboloids.

The properties of the model became better known when, in 1878, George Greenhill set the following question in the Cambridge Mathematical Tripos examination:

\begin{quote}
Prove that, if a model of a hyperboloid of one sheet be constructed of rods representing the generating lines, jointed at the points of crossing; then if the model be deformed it will assume the form of a confocal hyperboloid, and prove that the trajectory of a point on the model will be orthogonal to the system of confocal hyperboloids.
\end{quote}

and the following year Cayley published the solution in the \textit{Messenger of Mathematics}, but with no mention of Henrici (Cayley, 1879). Since Greenhill was a member of the LMS and based in London, he would almost certainly have learnt about the model from Henrici. It is

\textsuperscript{131} For further details, see (Henrici, 1892b).
likely that Cayley too would have been aware of Henrici’s model although he doesn’t mention it. Cayley’s paper was noticed by the French mathematicians Amédée Mannheim and Gaston Darboux, each of whom applied the properties of the deformable hyperboloid to the theory of rotating bodies but without knowing of Henrici’s contribution (Darboux, 1884, 389-390; Mannheim, 1894, 189-200).

![Figure 17. Movable hyperboloid of one sheet (Wiener & Treutlein, 1905, 13)](image)

In Germany, Henrici’s model was known to Hermann Wiener (Christian’s son) who, in 1905, together with Peter Treutlein included it in a catalogue of models (Wiener & Treutlein, 1905, 13–14), where, over a century later, it was seen by Hellmuth Stachel who used it as an illustrative example in an article on overconstrained frameworks (Stachel, 2014).134

The second version of Henrici’s model consisted of two copies of the original model fastened together in such a way that both could move together and remain confocal. Details of its construction, and theory behind it, are given in the DMV catalogue (Henrici, 1892c).

7. Conclusion

Among academic mathematicians working in Britain in the second half of the 19th century, Henrici, due to his German education and his early work experience, was certainly unusual.

134 An overconstrained framework has more degrees of freedom (or flexibility) than is predicted by the so-called mobility formula.
His apprenticeship at an engineering works in Flensburg, combined with his engineering and mathematics study at Karlsruhe under the tutelage of Redtenbacher, Clebsch and Wiener, meant that when he came to London he was well equipped to tackle a job in engineering design, even though with a PhD in algebraic geometry from Heidelberg he had originally hoped to make a career as a mathematician in Germany. That he had the opportunity to return to mathematics so soon after arriving as a putative engineer in London was due to a providential confluence of events: the failure of his friend’s engineering enterprise and an introduction to Hirst who was on the look-out for someone to assist him with his teaching at UCL.

Once settled at UCL, Henrici was able to pursue his interest in geometry. With Hirst and Clifford as close colleagues, and with Sylvester living in London, UCL was the ideal place for him; and with his mathematical background, his German connections, and his practical experience, he was ideally equipped to embark on the construction of models and to demonstrate his geometrical dexterity. This he did very successfully both with the models he showed in front of the LMS and with his construction of linkages, as well as in his teaching where he encouraged students to make geometric models of their own. An early advocate in Britain for the teaching of elementary projective geometry and promoter of the subject’s practical applications, he won plaudits from his students. Mathematically versatile, Henrici used his versatility to good effect as the variety in his research and the creativity in his teaching show. Utilising his heritage, he acted as an intermediary between British and German mathematicians and emerged unique in Britain as a creator of surface models. Furthermore, it is evident that Henrici himself was involved in the actual making of models, as witnessed by his friend Lindemann when he visited Henrici in London in 1876. As Lindemann said later, “[Henrici’s] practical mind made him ideally suited for the production of surface models as they were used in Germany by Plücker, Kummer and Klein for research and teaching.”

Although none of Henrici’s models acquired by the LMS have survived, the fact that the Society amended their Articles of Association in 1894 to allow for the possibility of purchasing models and for the formation of a museum indicates that they intended to preserve and display the models they owned. The idea of providing a space for display would have been a natural one for the LMS at the time, coming as it did in the wake of the Munich exhibition and during a period when university departments were actively purchasing collections of models. Indeed, such was the enthusiasm for models in the 1890s that the American Arnold Emch went so far as to (wrongly) predict that collections of mathematical models would become for university mathematics departments what museums of natural history had already become for biology departments (Emch, 1893, 93).

135 Sein praktischer Sinn machte ihn vorzüglich zur Herstellung von Flächenmodellen geeignet, wie sie in Deutschland durch Plücker, Kummer und Klein für Forschung und Unterricht verwertet wurden (Lindemann, 1927, 158).
Emch had been impressed by the emphasis on models in Klein’s Evanston Lectures in which Klein had stressed the value of models for developing geometric intuition (Emch, 1893, 91; Klein, 1894, 41–42). Meanwhile, Klein himself, a vigorous and long-time promoter of models (Rowe, 2013a, 89), was also of the opinion that such models were “calculated to disarm, in part at least, the hostility directed against the excessive abstractness of the university instruction” (Klein, 1893, 109). A sentiment with which Henrici would surely have agreed.

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136 The Evanston Lectures were given immediately after the Congress of Mathematics held at the Chicago World’s Fair in August 1893. The German Universities Exhibit at the World’s Fair included a display of plaster and string models prepared by Dyck and mostly manufactured by Brill (Parshall & Rowe, 1994, 307–309). Emch is not listed as attending the lectures.


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