The role of nonlinear dynamics in interactions with digital and acoustic musical instruments

The role of nonlinear dynamics in musicians' interactions with digital and acoustic musical instruments

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Abstract

Nonlinear dynamical processes are fundamental to the behaviour of acoustic musical instruments, as is well explored in the case of sound production. However, such processes may have profound and under-explored implications for how musicians interact with instruments. While nonlinear dynamical processes are ubiquitous in acoustic instruments, they are present in digital musical tools only if explicitly implemented. Thus, an important resource with potentially major effects on how musicians interact with acoustic instruments is typically absent in the way musicians interact with digital instruments. 24 interviews with free improvising musicians were conducted to explore the role that nonlinear dynamics play in the participants' musical practices, and to understand how such processes can afford distinctive methods of creative exploration. Thematic analysis of the interview data is used to demonstrate the potential for nonlinear dynamical processes to provide repeatable, learnable, controllable and explorable interactions, and to establish a vocabulary for exploring nonlinear dynamical interactions. Two related approaches to engaging with nonlinear dynamical behaviours are elaborated: edge-like interaction which involves the creative use of critical thresholds; and deep exploration which involves exploring the virtually unlimited subtleties of a very small control region. The elaboration of these approaches provides an important bridge that connects the concrete descriptions of interaction in musical practices on the one hand, to the more abstract mathematical formulation of nonlinear dynamical systems on the other.

Introduction

Although they can be simple in construction, acoustic instruments are often incredibly complex in their operation. Acoustics research continually unveils additional layers of complexity, nonlinearity, sensitivity and nuance in the processes governing the behaviour of reeds (Almeida et al. 2010), the vibration of strings (Desvages et al. 2016), the interactions of strings with fingers, fingerboards and bows (Ducceschi et al. 2016), the behaviours of two dimensional membranes (Torin and Bilbao 2013), and so on. This research is gradually highlighting what many musicians will already know: interactions with acoustic instruments can be complex, difficult and unpredictable, but simultaneously rich and subtle. A fundamental component of acoustic instruments is their nonlinear dynamical behaviour (Fletcher 1999). Fletcher (1999) distinguishes between musical instruments that are "essentially nonlinear" and instruments that are "incidentally nonlinear". The former category implies that the nonlinear nature of interaction with the instrument is a fundamental aspect of playing that instrument, and is associated particularly with blown and bowed instruments. In the latter category, the interaction may include a range of nonlinearities, but these are not as prominent and linear approximations can still be effective. This category typically includes struck and plucked instruments.

Digital music tools generally do not exhibit nonlinear dynamical behaviours unless they are explicitly implemented. Such implementations can be found in areas of music influenced by cybernetics, such as ecosystemic composition (Anderson 2005), or in chaotic sound synthesis processes (Slater 1998; Stefanakis et al. 2015). While nonlinear dynamical processes are the rule in acoustic musical instruments, they can be considered the exception to the rule in digital musical tools and instruments. It seems important then to consider the roles that nonlinear dynamical processes play in musical interactions, how these processes are engaged with in musical practice, what kinds of interaction they afford, and therefore, how digital interactions that don't include such processes might be engaged with in different ways.

These questions are approached in this article through the presentation of a set of 24 interviews conducted by the first author into how free improvising musicians engage with their musical tools (whether acoustic or digital). Definitions of free improvisation can vary, but for the purposes of this article the salient features are: the general willingness to engage with the intricate detail of instrumental interactions, the focus on exploring the broadest possibilities of how instruments can produce and control sounds, and the tendency to pay close attention to the timbral nuance of sounds produced. See Lewis (2002), or more recently Clark (2012) for accounts of London-based improvisation particularly relevant to many of the interview subjects in this article. These interviews build on an earlier lab-based study that explored the use of digital musical tools with and without nonlinear dynamical processes (Mudd et al. 2015). This earlier study provided evidence that the use of nonlinear dynamical processes in musical interfaces can lead to unpredictable interactions, but that the interfaces tended nevertheless to remain controllable and explorable. The interviews expand on these connections between unpredictability and control in relation to different characterisations of musical exploration in free improvisation. Accounts are given of particular kinds of engagement between the musician and their instrument that nonlinear dynamical processes appear to facilitate. Firstly, *deep exploration*, where the musician is able to investigate a very small control region of their instrument in great detail. Secondly *edge-like interactions*, where the musician can explore the complexity of behaviours found close to an abrupt transition in the instrument's behaviour.

Nonlinear dynamics are viewed here as an important consideration in relation to musical gestures, particularly when considering differences between acoustic and digital musical instruments. An important consideration is the *timing* of these gestures: small differences in the timing of inputs have the potential to lead to very different outcomes. For this reason, the precise timing of musical gestures can afford subtle and expressive control over essentially nonlinear instruments, in a way not possible with many digital instruments. This aspect of gestural interaction is one that risks being overlooked in research that focuses solely on the physicality or embodiment of gestures .

In what follows, instrumental interactions are viewed through the lens of nonlinear dynamical systems. Real-time interactions with such systems are considered, and examples are presented of the complexity of behaviours found at and close to critical thresholds. We link these perspectives with contemporary musical practices, as well as with relevant research on acoustics, interaction, and parameter mapping, and we examine them in the light of the participant interviews.

Interactions with nonlinear dynamical processes

Making sounds with an acoustic musical instrument is considered here as an interaction with a nonlinear dynamical system. Playing an instrument is therefore the navigation of the phase space of this system, the governance of the forces affecting the behaviour of a specific trajectory through the possible states of the instrument. This comparison is not presented merely as an analogy: many current digital models of musical instruments are based on nonlinear differential equations that very much describe systems of this kind Bilbao et al. (2018). The elaboration of the complexity of these systems is presented here as a way in to discussions on the inherent complexity and nuance often found in interactions with acoustic musical instruments.

Mathematical perspective

For our purposes, we will consider nonlinear dynamical systems to be represented by nonlinear differential equations (or difference equations for discrete implementations) of the kind shown below in equation 1, that describe changes to particular variables in relation to changes over time. The equations can be thought of as mapping out the behaviours of particular trajectories in a multidimensional phase space.

$$\frac{dx}{dt} = F(x, u) \tag{1}$$

The change in the system over time is a nonlinear function, F, of both the current state of the system, x, and an input vector, u, describing the current state of the various input controls. In these examples, and in this paper, specific trajectories through the phase space described by nonlinear dynamical systems are considered as literal renderings of waveforms that can be emitted as sound. While this move from pure mathematics to real life interactions with acoustic instruments may appear to some to be quite a jump, this is a routine aspect of current acoustics research.

Nonlinear dynamical systems in this form can exhibit a range of interesting behaviours that do not generally occur in linear systems. They can be chaotic, that is, very small adjustments to initial conditions (or input parameters) can lead to highly divergent outputs (Wiggins 1990). Perhaps more significantly from an interaction perspective, they exhibit hysteresis meaning that the behaviour of the system is dependent not only on the current input, but also on the current state of the system (Lakshmanan and Rajasekar 2003, p. 23), and therefore prior inputs to the system can also be highly significant in determining the output.

Real time interactions and timing

Acoustic musical instruments can be described as nonlinear dynamical systems that are, of course, controlled in real time. Viewing instrumental control as the real time management of a nonlinear dynamical system involves a subtle but important distinction: the musician is not in direct control of the instrument's output, but only manages the settings of the system that generates this output. Control is in a sense less direct: the player manages a system, and that system produces sound. Thompson and Stewart (2002) provide a detailed description of real-time interactions with a particular nonlinear dynamical system—a damped, forced Duffing oscillator, based on the research done by Ueda (1980)—which helps to paint a picture of instrumental control as the management of a nonlinear dynamical system.

> The regions in parameter space are delimited by various arcs. To interpret the meaning of these arcs, it is helpful to think of the parameters as *controls*, like a throttle or rheostat used to adjust the operating regime of a real dynamical system such as an airplane, a motor, or a simulation device. We may then imagine this dynamical system running at high speed while the controls are slowly adjusted; we gradually change the controls, and let the system settle to final behaviour in each new regime. As the control settings cross one of the arcs [...], we observe the system settling to a qualitatively different behaviour: the motion may change from periodic to chaotic, or the previously stable motion may become unstable, in which case the system settles to a different attractor; or the change may be more subtle, as when the subharmonic number of a stable periodic motion changes. In any case, there has been a *qualitative* change in the long-term behaviour, associated with a change in (or disappearance of) an attractor. (Thompson and Stewart 2002, p 12, italics retained from original).

Although the idea of leaving an instrument to settle into different behaviours may sound strange, the time-dependent behaviour of musical instruments has been demonstrated (Almeida et al. 2010). That is, even with unchanging input, an instrument's behaviour can change over time, sometimes dramatically —, and acoustic instruments are capable of locking into different states (Menzies 2002). This time dependence and the possibility of different system states may be harder to perceive in musical instruments than in the given examples of aeroplanes or motors, but this is perhaps due to the much more shorter timescales that interactions with musical instruments require.

The relevance of time dependence and state locking behaviour in instrumental interactions are discussed further later in the article, in the light of the interviews with improvisers.

Interaction close to critical thresholds

An important aspect of interaction with nonlinear dynamical processes for the research presented in this article is their behaviour close to critical thresholds or bifurcation points. These are precisely defined points at which the system undergoes a discontinuous change: attractors can appear, disappear, or change their behaviour, influencing trajectories through the phase space in very different ways (Lakshmanan and Rajasekar 2003, p 75). In real time, these points can be in some sense explorable, that is, a user can spend time discovering a range of interesting behaviours for particular trajectories. A simple example can be seen with the Lorenz attractor, a well-studied nonlinear dynamical system (Sparrow 1982; Wiggins 1990). The famous butterfly-like pattern created by the movement of trajectories through the three dimensional phase space occurs as a particular threshold of the system parameters is passed. The trajectories trace a constantly changing path around one wing of the butterfly before hopping to the other wing and tracing a path there. As the system parameters are reduced below the critical threshold, the trajectories end up stuck in one or the other wing of the butterfly shape. If the system is being run at a very high rate of iteration, then it can be very difficult to predict which wing the trajectory will end up in. This opens up a particular approach to experimenting with the system: the user can push the system into the orbit-hopping regime, then pull back from the threshold as an attempt to hop the system from one wing to another. This is a very simple example of an interaction affordance close to a critical threshold, demonstrating state locking behaviours. The complexity and variety of behaviours found around these thresholds opens up many other possibilities for complex interactions, exploiting instabilities (Pomeau and Manneville 1980, p 130), period-doubling behaviours (Lakshmanan and Rajasekar 2003, p 97), and other complexities that emerge from the sudden appearance, disappearance or change of attractors (Grebogi et al. 1987, p 5366).

Musical instruments and nonlinear dynamical processes

Nonlinear dynamics have been explored in musical contexts in a variety of forms. They have sometimes been explicitly identified and leveraged by artists and researchers, as with ecosystemic kinds of composition, physical modelling, chaotic synthesis, and dynamical parameter mapping. They are also used less explicitly, as with the use of loudspeaker and guitar feedback, and with the specific nonlinear dynamical aspects of acoustic musical instruments where there may be less awareness that these kinds of processes and interactions are present. This section examines these different uses of nonlinear dynamical processes in musical contexts with a focus on how the processes influence the nature of musical *exploration*.

Explicit uses of nonlinear dynamical processes in musical practices

The explicit uses of nonlinear dynamical processes are often tied to cybernetic-like approaches: artists creating nonlinear feedback networks either with circuitry (Kuivilla 2004; Nakamura 2000; Mudd et al. 2014), with microphone and speaker feedback (Davies 2002), with digital processes both for note-based composition (Pressing 1988; Spasov 2015) or for synthesis (Choi 1994; Stefanakis et al. 2015), or with combinations of the above (Meric and Solomos 2009; Sanfilippo and Valle 2013; Pirrò 2017). The term *ecosystemic*—associated with composers such as Agostino Di Scipio and Simon Waters—links in closely with the emergent nature of nonlinear dynamical systems (Anderson 2005; Waters 2007), allowing for the chaotic and time-dependent properties to play a significant role in the artists' approaches to developing and structuring musical outputs. In these situations, the system is often deliberately "unknowable" in some sense (Haworth 2014). Those interacting with the systems can exercise control, and can attempt to push the systems in different directions, but the results of their actions are not always predictable, even in strictly deterministic systems.

A more commonly encountered version of this kind of process can be found with microphone-loudspeaker feedback, or (essentially the same process) guitar feedback. These kinds of feedback have been relatively common in pop and rock music, particularly in areas with heavily distorted elements, such as metal and grunge (distortion is always a nonlinear processes). Minimal and postminimal music from the 1960s onwards has frequently engaged with feedback explorations as part of the compositional process (Oliveros 2003; Glover 2013). The feedback is sometimes tamed and used in very focused ways (e.g. Brian Eno's *Discrete Music*), but is often a deliberately chaotic element that will vary from performance to performance, from gig to gig, and can be explored (or not) by the performer in the moment yielding unpredictable outcomes (e.g. Lou Reed's exploration of feedback across his career, particularly on the *Metal Machine Music* album (Petrusich 2007)).

Nonlinear dynamical processes and parameter mapping in digital musical instruments

Considering musical interactions in terms of nonlinear dynamics provides a useful perspective on mapping research for digital musical instruments. The potential benefits of complicated mappings rather than straightforward one-to-one connections between digital controls and sound parameters were investigated by several authors (Rovan et al. 1997; Hunt and Kirk 2000; Wanderley and Orio 2002). Cross-mappings, where individual inputs can control multiple sound parameters and individual sound parameters can be affected by multiple inputs, were found by Hunt and Kirk (2000) to be associated with the potential for exploration and a sense of fun. Hunt and Kirk point to the incomprehensibility of the complex mappings as a factor in them necessarily being intuitively explored rather than being engaged with in an analytical fashion. Menzies (2002) extended this work into linear dynamical processes, showing how the deterministic but complex nature of dynamical processes can provide a “rich field for experimentation”.

Nonlinear dynamical processes in musical instruments complicate the mapping between input and output in a more involved manner than either the linear systems or the cross-mappings discussed above. As with the cross-mappings, the intricate detailed and nuanced relationship between action and reaction may be something that supports explorative rather than analytical engagements (discussed below in relation to an earlier study by the authors). The temporal aspect may be particularly significant in this vein — nonlinear dynamical processes make it possible for interactions that can lead to very different outcomes just by varying the speed and the dynamics of a particular input gesture.

There are existing examples of musicians and interaction designers explicitly leveraging nonlinear dynamics for mapping processes. Bowers and Hellström (2000) describe two of their own musical systems explicitly in terms of both nonlinear and dynamical elements, citing the desire for supporting “usability at the edge of control” as the motivating factor. They also express a strong interest in exploration: “we intend an interface which not merely supports exploring a soundscape but incites it,” (ibid).

Kiefer (2014) describes a method for deploying dynamical control mappings through the use of echo state networks. These networks are a specific type of recurrent neural network consisting of a set of input units, a set of output units, and a set of interconnected internal units. Each connection has an associated weighting coefficient, as with other neural networks, but echo state networks are unique in that only the output weightings are trained, while the other weightings are randomised. The dynamical aspects of the reservoir are therefore exploited through the calibration of the output weights and the system can make use of a gradually fading memory of its input. The degree to which the output is determined by the history of the inputs can be adjusted through scaling the weightings of the internal elements. Kiefer demonstrates a range of approaches to eliciting nonlinear behaviour from these systems, and shows how they may be applied to music, again citing unpredictability as a central motivation: "compelling, unpredictable and strangely lifelike behaviours for music and interaction" (ibid. p. 297).

Nonlinear dynamical interactions with acoustic musical instruments

Nonlinearities are usually fundamental in the behaviour of instrument excitation mechanisms: e.g. plucking, blowing, striking (McIntyre et al. 1983). From an interaction perspective, nonlinearities become even more significant when the excitation is not percussive but rather sustained, and the mechanism is coupled with other parts of the instrument, e.g. the reed with the bore in wind reed instruments, the bow with the strings in bowed instruments. As noted earlier, Fletcher (1999) terms these instruments "essentially nonlinear", distinguishing them from instruments where the nonlinearity is present in less central ways. Even in what are sometimes considered the linear elements of musical instruments, nonlinearities are increasingly being shown to play an important role. Bilbao (2014) shows how the nonlinearities of vibrating strings, membranes, and tubes, and the nonlinear aspects of collision interactions between strings, frets, fingers, and fingerboards are important components of understanding the behaviour of musical instruments, particularly in relation to timbral aspects. The importance of these nonlinear dynamical aspects is well understood in the domain of musical acoustics and physical modelling (McIntyre et al. 1983; Smith 2010; Bilbao 2014). They are rarely considered in relation to musical interaction or musical practice however, beyond some of the specific musical areas described above.

Links to surprise and exploration

A recent study by the present authors explicitly investigated the use of nonlinear dynamical processes in digital musical instruments (Mudd et al. 2015). The study focused on how including these processes in digital instruments changed the way that musicians engaged with these instruments, and how controllable, unpredictable and explorable they found the instruments. The study demonstrated that the inclusion of nonlinear dynamical processes did lead to significant increases in the sense of unpredictability and in the scope for exploration and discovery (at least for the specific implementations used in the study).

A second notable aspect of the study was that participants did not feel that there was a corresponding lack of control with these nonlinear dynamical instruments, despite their potential for unpredictability. This result can be connected to the experience of playing acoustic instruments: the instruments can often be initially unpredictable, but they can also be tamed and controlled with immense nuance and precision.

A limitation of the study was that both exploration and unpredictability are multifaceted, and can mean very different things to different musicians in different contexts. In short post-study interviews, one of the participants eloquently described two perspectives on surprise, with a clear preference for one over the other:

> "What I want is a surprise that leads somewhere, rather than a surprise that's a dead end."

Whilst there is likely a subjective aspect to whether a surprise can lead somewhere or whether it can't, the more detailed interviews presented in this article show how nonlinear dynamical processes can help to leverage surprises that do lead somewhere, that open up new territory for exploration rather than being unhelpful dead ends.

Study: Interviews with improvisers

To investigate how nonlinear dynamical interactions are engaged with by musicians in practice, 24 interviews were conducted with musicians engaged to some extent in free improvisation. The goal was to untangle the relationships between attitudes to surprise, exploration and control in instrumental interactions, and to look closely at how the nonlinear dynamical nature of the instruments influenced these attitudes. This study was conducted as part of the first author's doctoral thesis (Mudd 2017). The focus here is on the ramifications for computer musical practices and the relationships between musical practices and nonlinear dynamical processes. The interviews were restricted to participants with some level of experience of free improvisation, as it can be viewed as an area where musicians are particularly attentive to the specific behaviours of their instruments, especially behaviours that are largely unexplored in more conventional performance techniques (Keep 2009; Prévost 2008; Bailey 1992; Krekels 2019). In this area of practice, there often appears to be a movement towards the use of nonlinear aspects of the instrument, even where the instruments may be more firmly in Fletcher's "incidentally nonlinear" category. For example, pianists interacting directly with the strings inside the piano, using bow hairs to bow the strings, bouncing objects on the strings, placing objects across multiple strings, and so on. They are often more open to the specific timbral aspects of their playing, and less likely to view the results in terms of discrete note events. These timbral specifics are often where the subtleties of the nonlinear dynamical aspects are most prominent.

Methodology

The approach taken in this study to collecting and analysing data draws on ethnographic research methods and aspects of grounded theory as described initially by Glaser and Strauss (1967) and refined by Strauss and Corbin (1998). The relatively exploratory nature of the study and the lack of any definitive initial hypothesis is well suited to the open nature of grounded theory (Muller and Kogan 2010).

The participant interviews were semi-structured: the conversations were generally kept on the topic of improvisation, exploration, surprise, and control in relation to their specific instruments, but space was provided for participants to deviate to allow for unforeseen topics to emerge. Thematic analysis, following the specific approach described by (Braun and Clarke 2006, p 87), was used to examine the interview data. The initial data coding was made with a view to addressing relationships between the aforementioned topics of nonlinear dynamics in musical tools, surprise, exploration, control and improvisation, but as with the interviews, was still kept relatively open (in the sense described by Strauss and Corbin 1998). Codes and themes that might initially seem to be connected to these central questions could therefore be considered more thoroughly before being either put to one side, or incorporated in the study.

Participants

24 participants with different instrumental practices were interviewed across a two month period in 2016. The vast majority of these were London-based musicians (22 out of 24). This was in part a practical consideration due to the location of the researcher, but London is a valuable location to explore a broad range of improvised musical practices, as there is a diverse range of players and attitudes across the city. The community is far from homogenous in the approaches taken towards instrumental interactions, and interacting with other improvisers. Participants were recruited individually with a view

Participant	Electronic	Participant	Acoustic	Participant	Mixed
A	Laptop/samples	H	Objects	Q	Flute/electronics
B	Electronics/theremin/radios	I	Piano/objects	R	Objects/voice/effect pedals
C	Laptop/radios/samples	J	Cello	S	Saxophone/objects/electronics
D	Modular synthesiser	K	Trombone	T	Piano/samples/objects/effect
E	Modular synthesiser/laptop	L	Saxophone	U	Wine glasses/objects/effect pe
F	Sine tones (laptop)	M	Violin	V	Objects/laptop sampling
G	Digital feedback networks	N	Double bass	W	Violin/drums/laptop samplin
		O	Double bass	X	Electric guitar/feedback/objec
		P	Viola		

Table 1. List of tools and instruments used by the study participants, categorised as acoustic, electronic or mixed for each participant.

to including players of a wide range of different tools and instruments. Table 1 shows a breakdown of the different instruments played by the participants in three categories: participants using primarily electronic instruments, participants using primarily acoustic instruments, and participants regularly using a mixture of the two.

Participant Interviews

Although musicians may have an intuitive understanding of the behaviour of their instrument, most are likely unaware of the nonlinear dynamical processes involved. This makes it difficult to ask the study participants directly about interaction in these terms. Their engagement with nonlinear dynamical processes is therefore approached through wider questions about unpredictable aspects of working with instruments, and situations in which there appears to be the complex kinds of instrumental interactions outlined in the introduction. An initial categorisation of surprise was proposed—based on the discussions of practice that emerged from interviews with participants involved in the earlier lab-based study discussed above— in order to begin to address these issues. These categories were presented to the participants to see whether they were reflected in their own practices. The initial categories were as follows. Firstly, genuinely or effectively random aspects. This could include randomised functions on hardware or software devices, but also interactions that were effectively random such as radios, autonomous motorised movements (e,g. vibrators moving around by themselves), dipping into recorded media at unknown points (as described by Wessel and Wright 2002) or other chance-based methods. Secondly, situations that are deterministic but impossible to control accurately. This might include situations where tiny, almost imperceptible movements lead to varying output, or where musicians are pushing against their physical limitations of strength, endurance and accuracy. Thirdly, unstable interactions that may change abruptly at unknown thresholds. Feedback provides an example: a performer may slowly increase the gain of an amplifier knowing that at some threshold it may abruptly feedback, but not knowing at exactly what point. Fourthly, changing situations that result in surprises, such as playing in a different acoustic space or using new tools and/or new combinations of tools. These categories are not mutually exclusive, and some examples may fit multiple categories. For example, dipping arbitrarily into recorded media (e.g. a record or an audio file) at various points is technically deterministic, but is limited by accuracy and memory of what is where in the recording. The categories nevertheless provide a useful starting point for discussing the nature of surprises with the participants.

The participant interviews were semi-structured, involving a pre-designed question list (below) but leaving room for deviation and development as necessary. The interviews were conducted individually with each participant, focusing on their particular performance practices. The structured questions attempt to draw out attitudes to surprise, exploration and control, and to encourage the participant to consider these elements in relation to their specific musical tools and instruments. Interviews were conducted in person, or via remote video connection. Audio from the sessions was recorded and transcribed for subsequent analysis. The semi-structured questions were as follows: (1) What tools and instruments do you use in your practice? (2) Could you describe the role of exploration in your performance? (3) Are you often surprised by your instrument/tools (as appropriate)? (4) Do you actively search for unpredictable elements, and if so (or if not) what motivates this? (5) Is there anything that has been in your mind during this interview that has not been said, or anything that you wish to add?

Where possible, a video or audio recording of the participant performing, rehearsing or recording was found in advance that could be played to the participant, with certain parts of the recording being identified that appeared to show the participant engaging with their instrument/tool in a way that was somewhat surprising, or seemed to show the participant exploring unknown territory of some kind. Solo recordings were preferred where they were available. These recordings helped to provide concrete situations for the participants to talk about, and even if the musical sections selected proved not to be surprising situations, they could still serve as jumping off points to investigate the participant's thought process in relation to the instrument while playing.

Key Findings

Certain key aspects of the study are explored here. An exhaustive breakdown of the thematic analysis, showing the themes and codes that emerged from the participant interviews, is given in the first author's thesis (Mudd 2017). The focus in this article is on the results relevant to musical exploration with nonlinear dynamics, and how these results pertain to the nature of electronic and digital musical tools.

Exploration in relation to instruments

A first perspective is provided by looking at the participant replies to the question "what role does exploration play in your practice?". The replies were sorted into four categories: (1) those in which the participants felt that exploration was a central element in what they do, whether practising or performing (15 out of the 24 participants), (2) those in which they felt that exploration was important but had some caveats (4 participants), (3) those in which they felt that the question didn't fit or in which their answers didn't provide a relevant perspective (2 participants), and (4) those in which they explicitly distanced themselves and their practice from exploration (3 participants). All three participants in the fourth category described taking a more compositional approach to their performance and improvisatory practice.

It is interesting to look at how players of different instruments are distributed across these categories. Firstly, all players of blown instruments (4) were in the first category (two saxophonists, a flautist and a trombonist). Five out of six bowed instrument players were in the first category (violin, viola, cello, double bass). Almost all participants who were not in the first category used electronics in some form (8 out of 9). Finally, just over a third of those in the first category did use electronics (6 out of 15). This data is not sufficient to draw specific conclusions about these links between choice of instrument and attitude to exploration. While it might be the case that the instrument suggests particular approaches to exploration to the musician, the results could equally indicate the converse, that the choice of instrument is itself determined by the attitude to exploration.

Deep exploration

As suggested above, the interviews demonstrated a range of attitudes to exploration. Different participants often seemed to have slightly different definitions of what exactly exploration might mean in a performance or rehearsal situation. A significant distinction was between an idea of exploration as covering quite a wide terrain very quickly, finding extremities and boundaries, and the idea of exploration as mining a very narrow region very thoroughly. Although this distinction was not brought up by the interviewer, several participants brought up the subject and were keen to point to the deep model of exploration as something particularly relevant to their practices:

"it's more rewarding because there's more depth to focusing on one set of

> material and really going deeper and deeper into it." (participant N, a double bass player)

> "I like the idea of the solo of just going right into, very deep, narrow and deep, and just holding on to something and staying with it" (participant K, a trombone player)

> "there's a depth of information that comes out of every corner of that instrument" (participant W discussing the violin).

> "For me at this point, it's much more rewarding to concentrate on one thing, and to deliberately not move, to go deep." (participant B, who typically plays various electronic devices, often incorporating feedback processes).

An example of this kind of exploration also emerged during the lab based study discussed above, where one of the participants felt very much as though they were exploring, despite using only a very tiny region of the available parameter space (Mudd 2017, p 149). Participant B discussed a sense of depth in the context of their use of nonlinear feedback processes, created by feeding an analog mixer back on itself. They explicitly contrasted deep exploration with a broader notion of exploration, describing a book written by Marco Polo (likely *The Travels of Marco Polo*) that describes relentlessly each place he visited and what he saw. Although the participant described the book as interesting and beautifully written, he found that the endless variety became dull: new territory is constantly uncovered, but when the new territory is just one step on a rapid overview of a great many new territories, it can feel less satisfying than the more subtle exploration of a single territory. Participant N discusses how the double bass feels like a resource that can be endlessly mined:

> "You actually find yourself looking for the surprise, for the minute detail [...] that detail goes on forever you can keep digging it seems. I've not got to the bottom anyway."

The participant describes a narrowing of focus where there is a lot of potential for surprise and discovery when initially encountering an instrument, but over time "the rate of surprise diminishes - it slows and becomes more complicated", and the surprises and the areas for exploration become more subtle and more focussed on "minute detail".

Participants J and X also note the room for endless exploration in their existing instrumental set-ups - cello and guitar/feedback/preparations respectively. While it is not clear whether this corresponds to exploration in a broad or deep sense, the fact that this depth is obtained without needing to make changes to their instrumental set-ups suggests the potential to explore in a deep rather than a broad sense. For example, Participant J states that they:

> "rarely have used preparations because [...] there's so much to discover that I don't feel like I need some other tools to create new layers of possibilities of sound."

Similarly for participant X, although they use a variety of objects and preparations that augment and interfere with the guitar, these objects are rarely changed, and the guitar itself provides an endless resource:

> "In my mind [the guitar is] not completely mapped out, not a completely mapped out field, and it never will be."

Participants B, K, J, N and W all use instruments that are in the "essentially nonlinear" category outlined above (and arguably participant X, once the regular involvement of guitar string preparation and amp feedback is taken into account). The properties of nonlinear dynamical processes do appear to afford deep exploration of this kind. In particular, the range of different behaviours found close to critical thresholds, the chaotic sensitivity to very minor adjustments, and perhaps most significantly, the fact that the *timing* of different input adjustments can lead to very different results, all seem to suggest that there can be a whole world of subtlety and variation to discover in what might appear to be very limited interaction spaces with the instrument. By contrast, if one thinks of interfaces that are either linear or "incidentally nonlinear" (e.g. a piano keyboard, or a digital sampler), it is harder to picture exactly how this approach to deep exploration might be fostered (although musicians such as Chris Abrahams and Charlemagne Palestine have both demonstrated ways of accessing and exploring the nonlinear properties of struck strings via fast, repeated strikes on a single key). The relation between nonlinear dynamics and deep exploration traced in this section suggests that nonlinear dynamical processes may provide a useful method for fostering deep, exploratory engagements with digital tools.

Edge-like interactions

In participant B's description of deep exploration, they provide a useful overview of a particular approach to interacting with their mixer feedback apparatus: "I would go to this border of feedback [...] so you put everything on the edge, and this is where things start to happen, and this is where pleasant surprises start to happen." The "edge" in this sense seems to be partly metaphorical and partly literal. It is metaphorical in that the edge is a zone of tension, unpredictability, instability; things are on edge. The mathematical descriptions of interactions with nonlinear dynamical processes given in the "Interaction close to critical thresholds" section of this article show how there is also a literal manifestation of these edges, in the abstract mathematical systems, in nonlinear acoustic musical instruments, and in the kinds of feedback systems used by the participant. This literal aspect is important as it shows how the specific material properties of the musical device afford this approach to musical exploration. The participant is clear that “the edge is not a goal, it’s a method,” but nevertheless highlights the importance of this method in their musical practice:

> “Yes, after all it all comes to why I’m doing this and why I’m doing art. [...] Ok, so I want to create this artefact, this something, this is an interaction with something, this is about understanding something, this is about creating something new or remembering something, or I don’t know what is it, but this is something, and I find the best way to do it for myself is to find this threshold.”

Edges—or “thresholds” in the above quote—are described as resources, as regions that can be explored and that can suggest avenues for development and provide inspiration to the musician. Edge-like interactions were discussed explicitly by participant B, but they also fit descriptions given by many of the other participants (C, D, E, G, J, L, M, N, Q and T), often approached through a variety physical metaphors. Participant M describes aspects of their approach in terms of stretching a rubber band: “it’s like how far can you pull this invisible rubber band before it snaps. And you can almost feel the tension.” Several participants characterised situations in terms of balance and stability in unstable regions: “when something starts to develop then you might follow it for a while or keep it stable for a bit, and [...] it’s got an element of just balance about it, and there’s always a question of “what if I do this, what if I do that?” So that exploratory component drives the next thing that you do.” (participant Q), “it’s like with [a] current of water: I kind of like controlling that current and it can spill out sometimes [...] I somehow feel more comfortable, or I feel better in that territory.” (participant D), “it’s more like a feedback thing where you’re surfing with it and you’re playing with the edges, and it might fall out underneath you but then you can get out there [...] your goal is to sort of try to keep it, so you're surfing, but it's shifting, you're like surfing something that's going down rapids. So it's moving, but you have to stay afloat" (participant C).

In describing their practices with their instruments, the participants touched on a range of specific interactions that illustrate their more figurative comments. Participants J and N described bowing very close to the bridge as a way of exploring very high, unstable harmonics, sometimes across multiple strings. Participant M describes affixing a paperclip to a violin string and bowing the paperclip itself. Participants B and Q both describe the deliberate use of feedback to create unstable situations, the former with mixer feedback and the latter by placing a small microphone inside a bass flute. Participant L gives a rich account of exploring multiphonics on the saxophone when asked to give an example of an unpredictable situation with the instrument:

> "[...] there are certain sounds now that I can more or less access when I want to, and they're increasingly stable which means you can add a new layer of instability to them. So say there are all these harmonics that you can do with the left hand — it's hard to talk about — they're quite harsh blocks that bleat, they're not those gentle kind of juicy multiphonics, they're really quite harsh ones where there are a lot of pitches that are quite close together and they fizz about quite a lot. There's maybe about four or five of them where I now know more or less exactly where they are and I'm trying to figure out how to add a layer of stuff on in different ways, and I can do that now, but before I couldn't do that at all, and that just comes from playing, pushing, not necessarily practicing those, just playing."

This description gives a useful insight into how unstable instrumental behaviours can be found and developed—even on a familiar instrument—providing scope for deep exploration. The comment on the difficulty of finding language to discuss such processes is notable: even for an experienced player used to talking about their practice, the specific behaviours and control relationships can be hard to articulate.

While not all of these situations describe the exploration of critical thresholds as clearly as the example from participant B, they demonstrate a similar mixture of agency, where the tool is driven to a state where it is liable to do its own thing, fight back against the musician, and the musical outcome is a negotiation of the affordances of these somewhat unpredictable behaviours.

Discussion

This section examines the interview data in a broader context, considering surprise, exploration, control and nonlinear dynamics in computer music interactions and broader musical contexts. Four specific discussions are taken up below: the significance of the default absence of nonlinear dynamical processes in digital musical interfaces, nonlinear dynamics supporting deep exploration with digital musical interfaces, edge-like interactions and questions of agency, and the influence of nonlinear dynamics in musical practices beyond free improvisation.

Nonlinear dynamics and digital interfaces

Digital tools for musicians are often distinguished from acoustic musical instruments by the lack of physical interface, and by the lack of an inherent mapping between physical inputs and sonic outcomes (Winkler 1995; Wanderley and Malloch 2014). We would suggest—following both the interviews detailed in this article, and the prior work discussed above—that the disappearance of nonlinear dynamical processes is an important part of this distinction, and that digital interfaces, by default, will tend to preclude the possibility of the kinds of exploratory edge-based interaction described by participant B and others. This is, of course, not to say that digital instruments cannot be explored in lots of other meaningful ways, some of which are perhaps made possible precisely by the *absence* of nonlinear dynamical processes (e.g. digital instruments can be accurate and precise, in a one-parameter-at-a-time fashion, without anything unstable or unexpected occurring). Certain musical practices explicitly reintroduce these processes into digital situations, as discussed above in relation to ecosystemic musical practices and chaotic synthesis processes.

The richness of physical gestures with acoustic instruments appears to be closely connected to the rich nonlinear dynamical nature of the instruments. With a nonlinear dynamical process, the precise articulation of an input gesture can have significant consequences. Timing becomes critical: the same gesture done at different rates, or with different rates of change, can lead to completely different results. The potential for very divergent results goes beyond the basic mapping of rates of change of input parameters discussed by, for example, Hunt and Wanderley (2002). As an example, consider the input gesture as moving a microphone in front of a speaker, generating feedback. If the microphone lingers for too long in a particular region, the specific frequency associated with that particular distance from the speaker may become strong enough to become the dominant frequency in the sound produced by the speaker. If that region is passed more quickly however, that particular frequency may not come to prominence. Even with a single, low-resolution input to a digital nonlinear dynamical system, there may be endless possibilities for exploring variations in the timing of input gestures to drive the system into novel states. Moreover, to continue the microphone-speaker example, if the new frequency does come become the predominant frequency, the whole system is now in a different state, and may have an entirely new set of affordances. The potential for hysteresis in nonlinear dynamical processes opens the door to the possibility of the system being driven into different states. At different points in time, the input from the user may be the same, but the output may be very different depending on the state of the system.

Deep exploration, edge-like interactions and nonlinear dynamics

The time-based variations noted above may be a vital part of both deep exploration and the kinds of edge-like interactions outlined in this article. The potential for accessing different system states with very subtle parameter adjustments and—as shown above—subtle variations to the dynamics of parameter adjustments, shows how nonlinear dynamical processes can facilitate the kind of deep explorations discussed by the study participants. This mode of exploration appears to be particularly prevalent in free improvisation. The existence of different states in musical interactions is easier to think through in the microphone-speaker interaction discussed above, but it can be shown to exist in purely acoustic too. Overblowing with wind instruments provides one such example: breath pressure is increased to the point where the system jumps to a higher-frequency regime. Once the system is in this new state, the breath pressure can be reduced without the system immediately reverting back to the lower frequency. This new state—of being in the higher harmonic—comes with a different set of affordances. The same inputs to the instrument will yield different outcomes compared to when the instrument was producing the lower harmonic.

The percussionist Eddie Prévost's use of the bow on the tam-tam (a very large gong) provides another example, that is slightly easier to perceive. See for example 9'40 of the documentary *Eddie Prévost's Blood* (Hajdukiewicz 2013). Prévost uses the bow to elicit a surprisingly varied set of harmonics and timbres. This setup is useful for highlighting the temporal aspect of the interaction. Cymbals are highly nonlinear with thousands of interacting modes contributing to the final inharmonic result (Ducceschi and Touzé 2015). The bow acts as a navigational tool, constraining the instrument to resonating in sympathy with certain modes, in a similar manner to the reciprocal relationship between the resonances of a bowed string, and the stick-slip motion of the bow (Fletcher 1999). The complexity of interaction between the bow behaviour and the nonlinear resonances of the tam tam provide a rich landscape of affordances for sonic exploration.

This idea of deep exploration provides a contrast with the view of exploration put forward by Tubb and Dixon (2014), where exploration is supported through fast access to high dimensional parameter spaces. In Tubb and Dixon's model, the user can very quickly navigate through the many different regions of this high dimensional space via a simple two dimensional input. The significant difference in the use of the term *exploration* may highlight differences in engagement between free improvising musicians, on the one hand, and the kinds of electronic musicians being considered by Tubb and Dixon, on the other: the former placing a greater focus on finding interesting behaviours that relate to the interaction itself—often putting exploration at the centre of their practice—and the latter perhaps being more results-oriented. This is more in line with the description of electronic music composition provided by Gelineck and Serafin (2009), who identify three phases: exploration, editing, and a final pragmatic phase where exploration is not important and unpredictable results are not welcome.

Nonlinear dynamics in other musical domains

Free improvisation was examined in the study presented above, as the engagements with nonlinear dynamical processes appear to be closer to the surface, more direct, and more tangible than in many other domains of music. Areas of music that explicitly engage with feedback processes present another example where nonlinear dynamical processes play a central role, such as minimal and postminimal music, ecosystemic composition, and algorithmic music that utilises iterated nonlinear functions. We suggest that although the influence may be harder to trace, the nonlinear dynamical nature of musical instruments plays an important role in other musical domains. The almost magical role ascribed to tone in jazz brass and wind playing (Hasbrook 2005; Kleinhammer 1963; Campos 2005) presents another example. Tone is discussed as a very personal dimension, and each player is often encouraged both to study the tones of others and to find their own tone, potentially over a lifetime of playing (Kleinhammer 1963, p 36). The descriptions of tone present it as a site for almost limitless refinement and exploration. As with the discussion of deep exploration outlined above, this endless refinement seems likely to be afforded, at least in part, by the nonlinear dynamical aspects of the interaction — the different possible ways of navigating nonlinear phase space are potentially infinite, even within finite regions of parameter inputs.

Conclusion

This article has examined how nonlinear dynamical aspects of musical tools and instruments are leveraged in free improvisation, showing how they can be drawn on as resources for creative exploration. Participant interviews help in particular to show how the complex behaviours found close to critical thresholds in nonlinear dynamical processes may be key aspects of how instruments are played and engaged with, and the role that these processes may therefore play in informing musical practices. These edge-like explorations of critical thresholds, together with the notion of deep exploration, help to link musical practice with current models of how acoustic instruments work. Although the study is limited to the domain of free improvisation, it is suggested that musical interactions in other domains may also draw on the nonlinear dynamical nature of instrumental interaction for the subtleties of expression and creative exploration. These findings are put forward as of particular relevance to the fields of computer music, digital lutherie, and HCI more generally, as the default absence of nonlinear dynamical processes in digital interfaces marks a significant difference between digital and real-world interactions. This is not to paint a technologically deterministic picture of the role of the physical artefact in musical practice, but to draw attention to the back-and-forth relationships between musical practices and their material underpinnings.

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