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A supersymmetric $U_q[osp(2|2)]$-extended Hubbard model with boundary fields

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Abstract

A strongly correlated electron system associated with the quantum superalgebra $U_q[osp(2|2)]$ is studied in the framework of the quantum inverse scattering method. By solving the graded reflection equation, two classes of boundary-reflection K-matrices leading to four kinds of possible boundary interaction terms are found. Performing the algebraic Bethe ansatz, we diagonalize the two-level transfer matrices which characterize the charge and the spin degrees of freedom, respectively. The Bethe-ansatz equations, the eigenvalues of the transfer matrices and the energy spectrum are presented explicitly. We also construct two impurities coupled to the boundaries. In the thermodynamic limit, the ground state properties and impurity effects are discussed.

Key words: Hubbard model; Yang-Baxter equation; Reflection equations; Algebraic Bethe ansatz
PACS: 71.10.-w; 71.10.Fd; 75.10.Jm

1 Introduction

In recent decades, there has been considerable interest in the study of strongly correlated electron systems in reduced dimensions exhibiting non-Fermi-liquid behaviour [1–3]. This has been motivated by the surprising properties of high-T_c superconductors [4], heavy-fermion alloys and compounds [5], and the concept of Luttinger liquids [6–8]. Similar non-standard behaviour that lies outside the realm of Fermi liquid theory was also observed in the magnetic properties of systems displaying the Kondo effect [9]. The Kondo problem was solved exactly by means of the Bethe ansatz and the quantum inverse scattering method (QISM) [10]. Following this approach, many one-dimensional integrable quantum systems with impurities [11] have been constructed as inhomogeneous solutions of the Yang-Baxter equations (YBE) [12].

A particularly intriguing situation corresponds to perfect backscattering impurities which can be realized conveniently by integrable open boundary conditions (BC). A systematic approach to handle quantum systems with backscattering boundaries is provided by Sklyanin's work on the reflection equations (RE) [13]. In analogy with the YBE in the bulk, the RE guarantee the factorization of the scattering matrices at the boundaries. Thus, starting from a solution of the YBE, which yields a solvable model with periodic BC, one constructs suitable integrable boundary conditions such that the RE are fulfilled [14–17]. Due to integrability, the bulk impurities obtained by inhomogeneous solutions of the YBE are pure forward scatterers. Thus their combination with the backscattering boundaries may be expected to model physically relevant impurity systems. In the context of boundary integrable quantum field theories [18], a model with impurities can be mapped onto a model with certain boundary conditions. Because impurity effects play a decisive role for the transport in quantum wires, general boundary conditions for strongly correlated electron systems open many opportunities to investigate the transport properties in Luttinger liquids or quantum wires. A proper boundary field may have a feasible realization by applying boundary external voltages and external magnetic fields in experiments on quantum wires [19]. Moreover, we may expect that the local state induced by the boundary fields inherits signatures of the bulk Luttinger liquid [6–8]. The physical quantities such as magnetization, the compressibility, susceptibility, and specific heat, etc., may be manipulated by the Bethe-ansatz equations. For this reason, integrable models that combine bulk and boundary impurities have recently attracted much attention [20–23].

Of particular interest are strongly correlated electron systems associated with *supersymmetric* solutions of the YBE. Models corresponding to non-exceptional Lie superalgebras as, for instance, gl(2|1) and osp(2|2) [24–26], have provided interesting non-perturbative information [27,28] for generalizations of well-known models such as, e.g., the Hubbard model [29]. A further generalization was achieved by considering the solution of the YBE related to the quantum superalgebra $U_q[osp(2|2)]$ [25]. This model has two fermionic and two bosonic degrees of freedom. Furthermore, the Hamiltonian [26] corresponds to a lattice regularization of the integrable double sine-Gordon model [30]. The continuum version of this model with boundary fields is known to describe tunneling effects in quantum wires [31]. These wires are believed to represent a realization of Luttinger liquids. We remark that the coordinate Bethe ansatz for an open $U_q[osp(2|2)]$ chain has recently been studied [32].

In the present work, we perform the algebraic Bethe ansatz for a supersymmetric open chain associated with the quantum superalgebra $U_q[osp(2|2)]$. Due to the supersymmetric structure of the model, we use the graded version of the QISM [28]. Starting from the $U_q[osp(2|2)]$ solution of the graded YBE, we construct a supersymmetric correlated electron system with boundary fields by solving the graded RE. We find that the model contains a hidden anisotropic XXZ open chain characterizing the spin degrees of freedom [1]. This plays a crucial role in our solution, which proceeds in two steps, treating charge and spin degrees of freedom separately by a nested graded Bethe ansatz. This structure also suggests a natural way to incorporate impurities coupling to charge and spin degrees of freedom while preserving the integrability of the model. From the Bethe-ansatz equations, we obtain the ground state energy at half-filling in the thermodynamic limit. We also discuss integrable impurities coupled to the boundaries.

The paper is organized as follows. In section 2, we present the $U_q[osp(2|2)]$ solution of the YBE and solve the corresponding RE. Furthermore, we give an explicit expression of the Hamiltonian in terms of fermionic operators. Section 3 is devoted to the derivation and the solution of the Bethe-ansatz equations by means of the graded QISM. In section 4, the ground state energy in the thermodynamic limit is obtained for the model with boundary fields and impurities. We conclude in section 5.

2 The solutions of the graded RE

We begin by considering a two dimensional classical lattice model, where, to each bond of the lattice, two bosonic and two fermionic degrees of freedom are associated. It is a vertex model, hence interaction takes place at each vertex, and the energies of the various local configurations determine the statistical weight of the configuration. The corresponding Boltzmann weights form a $2^4 \times 2^4$ quantum R-matrix with grading 'bffb', where 'b' stands for *bosonic* and ‘f’ for *fermionic*. It has the form

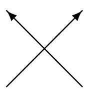
$$
R_{12}(\lambda) = \begin{pmatrix}
w_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & w_3 & 0 & 0 & w_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & w_3 & 0 & 0 & 0 & 0 & 0 & w_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & w_7 & 0 & 0 & w_{10} & 0 & 0 & w_{10} & 0 & 0 & w_{11} & 0 & 0 & 0 \\
0 & w_2 & 0 & 0 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & w_6 & 0 & 0 & w_9 & 0 & 0 & w_8 & 0 & 0 & w_{10} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & w_3 & 0 & 0 & 0 & 0 & 0 & w_4 & 0 & 0 \\
0 & 0 & w_2 & 0 & 0 & 0 & 0 & 0 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & w_6 & 0 & 0 & w_8 & 0 & 0 & w_9 & 0 & 0 & w_{10} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_3 & 0 & 0 & w_4 & 0 \\
0 & 0 & 0 & w_5 & 0 & 0 & w_6 & 0 & 0 & w_6 & 0 & 0 & w_7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & w_2 & 0 & 0 & 0 & 0 & 0 & w_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_2 & 0 & 0 & w_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_1
\end{pmatrix} \tag{1}
$$

where

$$
\begin{aligned}
&w_1 = \frac{x-q^2}{1-x\,q^2}, && w_2 = \frac{1-q^2}{1-x\,q^2}, && w_3 = \frac{q(x-1)}{1-x\,q^2}, \\
&w_4 = x\,w_2, && w_5 = \frac{2\,w_2}{1+x}, && w_6 = \frac{1-x}{1+x}\,w_2, \\
&w_7 = -\frac{1+q^2\,x}{1-q^2}\,w_6, && w_8 = -\frac{2x\,w_2}{1+x}, && w_9 = -\frac{x+q^2}{1-q^2}\,w_6, \\
&w_{10} = x\,w_6, && w_{11} = \frac{2x^2\,w_2}{1+x},
\end{aligned} \tag{2}
$$

with $x = \mathrm{e}^{\lambda}$ characterizing the difference λ of the pseudo-momenta of the particles whose two-body scattering is described by the quantum R-matrix (1); q is the deformation parameter.

In what follows, we are going to use also the standard diagrammatic representation [10], where the R-matrix corresponds to the two-particle scattering picture

$R_{12}(\lambda) =$  . (3)

The quantum R-matrix (1) satisfies the graded YBE

$$
R_{12}(\lambda-\nu)R_{13}(\lambda)R_{23}(\nu) = R_{23}(\nu)R_{13}(\lambda)R_{12}(\lambda-\nu) \tag{4}
$$

guaranteeing the integrability of the model with periodic BC, i.e., the factorization of the scattering matrices into two-body scattering matrices [12]. Above, R_{ij} with $i = 1, 2$ and $j = 2, 3$, denotes on which of the ith and jth spaces of $V_1 \otimes_s V_2 \otimes_s V_3$ the R-matrix acts. In the remaining space, R_{ij} acts an identity. Here $\otimes_S$ denotes the graded tensor product

$$[A \otimes_S B]_{\alpha\beta,\gamma\delta} = (-1)^{[P(\alpha)+P(\gamma)]P(\beta)} A_{\alpha\gamma} B_{\beta\delta}, \tag{5}$$

with the Grassmann parities obey $P(1) = P(4) = 0$ and $P(2) = P(3) = 1$ with respect to the grading 'bffb'.

For other BC such as twisted and open BC, the graded YBE will still account for the bulk part of the model, but the boundary terms have to be chosen appropriately in order to preserve the integrability of the model. In particular, solutions of the RE yield integrable vertex models with open (reflecting) boundaries or the equivalent integrable quantum spin chains with boundary fields. In the context of two-body scattering, the RE characterize the consistency conditions for the factorizations of the two-body boundary scattering matrices at boundaries. Taking into account the grading 'bffb', the left and right reflection matrices, K_- and K_+, are required to satisfy the following RE

$$\begin{aligned} & R_{12}(\lambda - \nu) \overset{1}{K}_-(\lambda) R_{21}(\lambda + \nu) \overset{2}{K}_-(\nu) \\ & = \overset{2}{K}_-(\nu) R_{12}(\lambda + \nu) \overset{1}{K}_-(\lambda) R_{21}(\lambda - \nu), \end{aligned} \tag{6}$$

$$\begin{aligned} & R_{21}^{\mathrm{St}_1\overline{\mathrm{St}}_2}(\nu - \lambda) \overset{1}{K}{}_+^{\mathrm{St}_1}(\lambda) R_{12}^{\mathrm{St}_1\overline{\mathrm{St}}_2}(-\lambda - \nu) \overset{2}{K}{}_+^{\overline{\mathrm{St}}_2}(\nu) \\ & = \overset{2}{K}{}_+^{\overline{\mathrm{St}}_2}(\nu) R_{21}^{\mathrm{St}_1\overline{\mathrm{St}}_2}(-\lambda - \nu) \overset{1}{K}{}_+^{\mathrm{St}_1}(\lambda) R_{12}^{\mathrm{St}_1\overline{\mathrm{St}}_2}(\nu - \lambda), \end{aligned} \tag{7}$$

respectively. Here, we used the conventional notation

$$\overset{1}{X} \equiv X \otimes_S \mathbf{I}_{V_2}, \qquad \overset{2}{X} \equiv \mathbf{I}_{V_1} \otimes_S X, \tag{8}$$

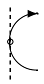
where $\mathbf{I}_V$ denotes the identity operator on V, and, as usual, $R_{21} = \mathbf{P} \cdot R_{12} \cdot \mathbf{P}$. Here $\mathbf{P}$ is the graded permutation operator which can be represented by a $2^4 \times 2^4$ matrix, i.e.,

$$\mathbf{P}_{\alpha\beta,\gamma\delta} = (-1)^{P(\alpha)P(\beta)} \delta_{\alpha\delta} \delta_{\beta\gamma}. \tag{9}$$

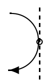
Furthermore, superscripts St_a and $\overline{\mathrm{St}}_a$ denote the supertransposition in the space with index a and its inverse, respectively,

$$(A_{ij})^{\mathrm{St}} = (-1)^{[P(i)+P(j)]P(i)} A_{ji}, \qquad (A_{ij})^{\overline{\mathrm{St}}} = (-1)^{[P(i)+P(j)]P(j)} A_{ji}. \tag{10}$$

We found two solutions of the RE (6) and (7) for diagonal boundary $K_{\pm}$-matrices



$$K_-(\lambda) = \quad = \begin{pmatrix} K_-^{(1)}(\lambda) & 0 & 0 & 0 \\ 0 & K_-^{(2)}(\lambda) & 0 & 0 \\ 0 & 0 & K_-^{(3)}(\lambda) & 0 \\ 0 & 0 & 0 & K_-^{(4)}(\lambda) \end{pmatrix}, \tag{11}$$



$$K_+(\lambda) = \quad = K_-^{\mathrm{St}}(\lambda \to -\lambda - \mathrm{i}\pi,\, \xi_- \to -\frac{1}{\xi_+}). \tag{12}$$

In the first solution, the entries are given as

$$\begin{aligned} K_-^{(1)}(\lambda) &= (x + \xi_- q)(x - \xi_- q^{-1}), \\ K_-^{(2)}(\lambda) &= K_-^{(3)}(\lambda) = (x^{-1} + \xi_- q)(x - \xi_- q^{-1}), \\ K_-^{(4)}(\lambda) &= (x^{-1} + \xi_- q)(x^{-1} - \xi_- q^{-1}), \end{aligned} \tag{13}$$

whereas the second solutions corresponds to

$$\begin{aligned} K_-^{(1)}(\lambda) &= (x + \xi_- q)(x - \xi_-^{-1} q)(x - \xi_- q^{-1}), \\ K_-^{(2)}(\lambda) &= (x^{-1} + \xi_- q)(x - \xi_-^{-1} q)(x - \xi_- q^{-1}), \\ K_-^{(3)}(\lambda) &= (x + \xi_- q)(x^{-1} - \xi_-^{-1} q)(x - \xi_- q^{-1}), \\ K_-^{(4)}(\lambda) &= (x + \xi_- q)(x^{-1} - \xi_-^{-1} q)(x^{-1} - \xi_- q^{-1}). \end{aligned} \tag{14}$$

Here $x = \mathrm{e}^{\lambda}$ as defined in (2) and $\xi_{\pm}$ are free parameters characterizing the boundary fields and the boundary interactions. We emphasize that the permutation-transposition symmetry

$$R_{12}^{\mathrm{St}_1 \overline{\mathrm{St}}_2}(\lambda) = \mathbf{P} R_{12}(\lambda) \mathbf{P}, \tag{15}$$

the unitarity

$$R_{12}(\lambda) R_{21}(-\lambda) = \mathbf{I}, \tag{16}$$

and the graded crossing symmetry

$$R_{12}(\lambda) = \overset{1}{V} R_{12}^{\mathrm{St}_2}(-\lambda - \mathrm{i}\pi)\, \overset{1}{V}{}^{-1} \frac{\zeta(-\lambda - \mathrm{i}\pi)}{\zeta(\lambda)}, \tag{17}$$

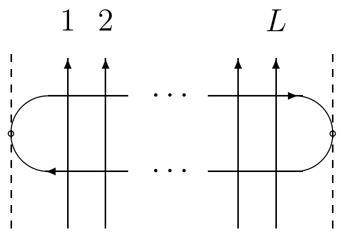
with

$$V = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \qquad \zeta(\lambda) = \frac{(1 - xq^2)}{(1 - x)}, \tag{18}$$

not only result in the isomorphism (12) between $K_+(\lambda)$ and $K_-(\lambda)$, but also constitute the necessary ingredients for the integrability of the model with boundaries. We also remark that the grading does not play a role for the diagonal $K_-(\lambda)$-matrix (13), which coincides with the one for the non-graded R-matrix given in Ref. [32]. However, its companion $K_+(\lambda)$ given by (12) is different from that in the non-graded case because it has to obey the graded crossing symmetry (17). It is worth emphasizing that the boundary parameters ξ_- and ξ_+ should inherit the same crossing property as that imposed on the pseudo-momenta λ such that the boundary terms of the corresponding Hamiltonian are completely symmetric. In Appendix A we present an ansatz to work out the solutions (13) as well as (14).

A more important object in the context of the QISM is the transfer matrix of an integrable system, which can be considered as a generating function of the infinite integrals of motion due to its commutativity for different values of the spectral parameter. Actually, the RE (6) and (7) together with the YBE (4) and the symmetries of the quantum R-matrix ensure the commutativity of the double-row transfer matrix

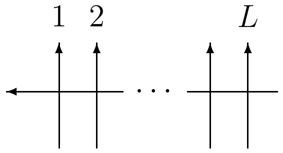
$\tau(\lambda) =$



$$= \mathrm{Str}_0 \left[K_+(\lambda) T(\lambda) K_-(\lambda) T^{-1}(-\lambda) \right]. \tag{19}$$

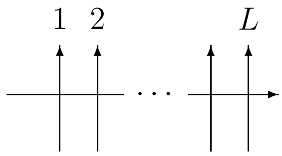
Here Str_0 denotes the supertrace carried out in auxiliary space V_0. The monodromy matrices $T(\lambda)$ and $T^{-1}(-\lambda)$ are defined by

$T(\lambda) =$



$$= R_{L,0}(\lambda) R_{L-1,0}(\lambda) \cdots R_{2,0}(\lambda) R_{1,0}(\lambda), \tag{20}$$

$$T^{-1}(-\lambda) =$$



$$= R_{0,1}(\lambda)R_{0,2}(\lambda)\cdots R_{0,L-1}(\lambda)R_{0,L}(\lambda), \tag{21}$$

respectively. The Hamiltonian associated with the quantum R-matrix (1) is related to the double-row transfer matrix (19) as

$$\tau(\lambda) = c_1\lambda + c_2(H + \text{const.})\lambda^2 + \ldots, \tag{22}$$

where c_i, $i = 1, 2$, are scalar functions of the boundary parameters $\xi_\pm$. Taking into account the Grassmann parity of the host vertices, after some lengthy algebra, one can present the Hamiltonian explicitly in terms of the fermionic creation and annihilation operators $c^\dagger_{j,\sigma}$ and $c_{j,\sigma}$ acting on the site j and carrying the spin index $\sigma = \pm$. It is given by

$$\begin{aligned} H = {} & \sum_{j=1}^{L-1} H_{j,j+1} + U(\xi_-)n_{1,+}n_{1,-} + U(\xi_+)n_{L,+}n_{L,-} \\ & + \sum_{\sigma=\pm} (V_{1,\sigma}\, n_{1,\sigma} + V_{L,\sigma}\, n_{L,\sigma}), \end{aligned} \tag{23}$$

where $n_{j,\sigma} = c^\dagger_{j,\sigma}c_{j,\sigma}$ is the fermion number operator, and the bulk Hamiltonian is chosen as

$$\begin{aligned} H_{j,j+1} = {} & 2\sum_{\sigma=\pm} \left[c^\dagger_{j,\sigma}c_{j+1,\sigma} + \text{h.c.}\right] [1 - n_{j,-\sigma} - n_{j+1,-\sigma} + n_{j,-\sigma}n_{j+1,-\sigma}] \\ & -(q - q^{-1}) \sum_{\sigma=\pm} \left[c^\dagger_{j,\sigma}c_{j+1,\sigma} - \text{h.c.}\right] [n_{j+1,-\sigma} - n_{j,-\sigma}] \\ & +(q + q^{-1}) \left[c^\dagger_{j,+}c^\dagger_{j,-}c_{j+1,-}c_{j+1,+} - c^\dagger_{j,+}c^\dagger_{j+1,-}c_{j+1,+}c_{j,-} + \text{h.c.}\right] \\ & +(q + q^{-1}) [n_{j,+}n_{j,-} + n_{j+1,+}n_{j+1,-} + n_{j,+}n_{j+1,-} \\ & \qquad +n_{j,-}n_{j+1,+} - n_j - n_{j+1}] \\ & +2(q + q^{-1})\mathbf{I}, \end{aligned} \tag{24}$$

with $n_j = n_{j,+} + n_{j,-}$ and $\mathbf{I}$ the identity operator. The remaining terms in (23) are the on-site Coulomb coupling $U(\xi_\pm)$ and the chemical potentials $V_{1,\sigma}$ and $V_{L,\sigma}$ at the ends of the chain. For the first solution (13), we find

$$U(\xi_\pm) = \frac{2(q^2 - q^{-2})q\xi_\pm}{(q - \xi_\pm)(1 + q\xi_\pm)}, \tag{25}$$

$$V_{1,\sigma} = -\frac{(q - q^{-1})(q + \xi_-)}{(q - \xi_-)}, \quad V_{L,\sigma} = -\frac{(q - q^{-1})(q + \xi_+)}{(q - \xi_+)}; \tag{26}$$

whereas the second solution (14) yields the same expression (25) for $U(\xi_\pm)$, $V_{1,-}$ and $V_{L,-}$ but

$$V_{1,+} = -\frac{(q-q^{-1})(q\xi_- - 1)}{(1+q\xi_-)}, \quad V_{L,+} = -\frac{(q-q^{-1})(q\xi_+ - 1)}{(1+q\xi_+)}, \tag{27}$$

as presented in Appendices A and B. The Hamiltonian (23) contains hopping terms with occupation numbers, double hopping terms, on- and off-site Coulomb interaction in the bulk, as well as on-site Coulomb interaction and chemical potentials at the boundaries. We notice that the boundary terms corresponding to the first solution (13) act as boundary chemical potentials, whereas in the second case (14) they act as boundary magnetic fields. The two cases could provide four possible classes of boundary conditions, which lead to different boundary shift factors in the Bethe-ansatz equations of the model. In order to keep the Hamiltonian hermitian, we restrict ourselves to $q = \mathrm{e}^{\mathrm{i}\gamma}$ and boundary parameters $\xi_\pm = \mathrm{e}^{\mathrm{i}\xi^\pm}$ with real γ and $\xi^\pm$. We note that the Hamiltonian in Ref. [26] differs from the bulk part (24), but that they are related to each other by a canonical transformation. Nevertheless, it will be shown that this transformation does not change the Bethe-ansatz equations in the bulk.

So far, we finished the first step towards the algebraic Bethe-ansatz solution for the model associated with the quantum R-matrix (1). Next we shall proceed with the factorization of the transfer matrix (19). In this paper, we restrict ourselves to the first solution (13) of the RE, which leads to a perfect factorization of the transfer matrix (19) acting on the pseudo-vacuum state. The second class of solutions of the RE, given in (14), leads to a very complicated factorization of the transfer matrix (19) acting on the pseudo-vacuum state and the multi-particle states. However the ansatz formulated in the following section works in a similar way for the model with other boundary terms.

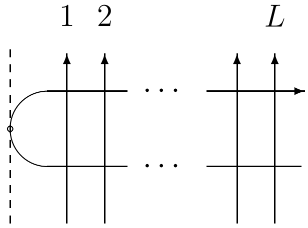
3 The algebraic Bethe-ansatz approach

In order to accomplish the algebraic Bethe ansatz for an integrable system with boundaries, we first need to diagonalize the transfer matrix of the model acting on the pseudo-vacuum state. As usual, we rewrite the transfer matrix (19) in the following form

$$\tau(\lambda) = \mathrm{Str}_0 \left[K_+(\lambda) U_-(\lambda) \right], \tag{28}$$

where $U_-(\lambda)$ is defined by

$U_-(\lambda) =$



$$= T(\lambda)K_-(\lambda)T^{-1}(-\lambda). \tag{29}$$

One can verify that $U_-(\lambda)$ also satisfies the graded RE (6). With regard to the structure of the R-matrix (1), we choose the standard ferromagnetic pseudo-vacuum state [34]

$$|0\rangle = |0\rangle_L \otimes \cdots \otimes |0\rangle_i \otimes \cdots \otimes |0\rangle_1, \tag{30}$$

where $|0\rangle_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_i \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_i$ acts as a highest-weight vector. Following Refs. [34,35], we label the elements of the monodromy matrix $T(\lambda)$ by

$$T(\lambda) = \begin{pmatrix} B(\lambda) & B_1(\lambda) & B_2(\lambda) & F(\lambda) \\ C_1(\lambda) & A_{11}(\lambda) & A_{12}(\lambda) & E_1(\lambda) \\ C_2(\lambda) & A_{21}(\lambda) & A_{22}(\lambda) & E_2(\lambda) \\ C_3(\lambda) & C_4(\lambda) & C_5(\lambda) & D(\lambda) \end{pmatrix}, \tag{31}$$

and further

$$T^{-1}(-\lambda) = \begin{pmatrix} \bar{B}(\lambda) & \bar{B}_1(\lambda) & \bar{B}_2(\lambda) & \bar{F}(\lambda) \\ \bar{C}_1(\lambda) & \bar{A}_{11}(\lambda) & \bar{A}_{12}(\lambda) & \bar{E}_1(\lambda) \\ \bar{C}_2(\lambda) & \bar{A}_{21}(\lambda) & \bar{A}_{22}(\lambda) & \bar{E}_2(\lambda) \\ \bar{C}_3(\lambda) & \bar{C}_4(\lambda) & \bar{C}_5(\lambda) & \bar{D}(\lambda) \end{pmatrix}, \tag{32}$$

$$U_-(\lambda) = \begin{pmatrix} \tilde{B}(\lambda) & \tilde{B}_1(\lambda) & \tilde{B}_2(\lambda) & \tilde{F}(\lambda) \\ \tilde{C}_1(\lambda) & \tilde{A}_{11}(\lambda) & \tilde{A}_{12}(\lambda) & \tilde{E}_1(\lambda) \\ \tilde{C}_2(\lambda) & \tilde{A}_{21}(\lambda) & \tilde{A}_{22}(\lambda) & \tilde{E}_2(\lambda) \\ \tilde{C}_3(\lambda) & \tilde{C}_4(\lambda) & \tilde{C}_5(\lambda) & \tilde{D}(\lambda) \end{pmatrix}. \tag{33}$$

From the structure of the R-matrix (1), the relation (16) — which one uses to construct the inverse R-matrix — and the definitions (20) and (21), one can deduce that the operators $B_a(\lambda)$ and $\bar{B}_a(\lambda)$ $(a = 1, 2)$ act as creation fields acting on the reference state, creating particles with pseudo-momenta λ and $-\lambda$, respectively. While $E_a(\lambda)$ and $\bar{E}_a(\lambda)$ are the 'dual' creation fields to $B_a(\lambda)$ and $\bar{B}_a(\lambda)$, the operators $C_i(\lambda)$ and $\bar{C}_i(\lambda)$ $(i = 1, \cdots, 5)$ behave as annihilation fields. Furthermore, using an invariant of the Yang-Baxter algebra

$$\overset{2}{T}{}^{-1}(-\lambda)R_{12}(2\lambda)\,\overset{1}{T}(\lambda) = \overset{1}{T}(\lambda)R_{12}(2\lambda)\,\overset{2}{T}{}^{-1}(-\lambda), \tag{34}$$

we obtain, apart from an overall factor $Q(\lambda) = K_-^{(1)}(\lambda)K_+^{(1)}(\lambda)$, the eigenvalue of the transfer matrix as

$$\tau(\lambda)|0\rangle = \left\{ W_1^+(\lambda)\tilde{B}(\lambda) + \sum_{a=1}^{2} W_{a+1}^+(\lambda)\hat{A}_{aa}(\lambda) + W_4^+(\lambda)\hat{D}(\lambda) \right\} |0\rangle, \quad (35)$$

where we introduced the transformations

$$\hat{A}_{aa}(\lambda) = \tilde{A}_{aa}(\lambda) - \frac{q^2-1}{q^2-x^2}\tilde{B}(\lambda) = W_{a+1}^-(\lambda)\bar{A}_{aa}(\lambda)A_{aa}(\lambda), \quad (36)$$

$$\begin{aligned}\hat{D}(\lambda) &= \tilde{D}(\lambda) - \frac{q^2-1}{x^2q^2-1}\sum_{a=1}^{2}\hat{A}_{aa}(\lambda) - \frac{2(q^2-1)}{(q^2-x^2)(x^2+1)}\tilde{B}(\lambda) \\ &= W_4^-(\lambda)\bar{D}(\lambda)D(\lambda). \quad (37)\end{aligned}$$

Here,

$$W_1^-(\lambda) = 1, \quad (38)$$

$$W_1^+(\lambda) = \frac{(x^2+q^2)(x^2-1)(x+q\xi_+)(xq-\xi_+)}{(x^2-q^2)(x^2+1)(x-q\xi_+)(xq+\xi_+)}, \quad (39)$$

$$W_2^+(\lambda) = W_3^+(\lambda) = -\frac{xq(x^2-1)(x\xi_+ + q)(xq-\xi_+)}{(x^2q^2-1)(x-q\xi_+)(xq+\xi_+)}, \quad (40)$$

$$W_2^-(\lambda) = W_3^-(\lambda) = \frac{q(x^2-1)(x\xi_- + q)}{x(x^2-q^2)(x+q\xi_-)}, \quad (41)$$

$$W_4^+(\lambda) = -\frac{x^2(xq\xi_+ - 1)(x\xi_+ + q)}{(x-q\xi_+)(xq+\xi_+)}, \quad (42)$$

$$W_4^-(\lambda) = -\frac{(x^2-1)(x^2q^2+1)(xq\xi_- - 1)(x\xi_- + q)}{x^2(x^2+1)(x^2q^2-1)(xq-\xi_-)(x+q\xi_-)}, \quad (43)$$

$$\tilde{B}(\lambda)|0\rangle = \varepsilon_1(\lambda)|0\rangle = W_1^-(\lambda)w_1^{2L}(\lambda)|0\rangle, \quad (44)$$

$$\hat{A}_{aa}(\lambda)|0\rangle = \varepsilon_{a+1}(\lambda)|0\rangle = W_{a+1}^-(\lambda)w_3^{2L}(\lambda)|0\rangle, \quad (45)$$

$$\hat{D}(\lambda)|0\rangle = \varepsilon_4(\lambda)|0\rangle = w_4^-(\lambda)w_7^{2L}(\lambda)|0\rangle. \quad (46)$$

In Appendix C, we give some useful relations for the eigenvalue problem (35), the factorization of the transfer matrix (28) acting on the pseudo-vacuum state (30). We note that the operators $\tilde{B}_a(\lambda)$, $a = 1, 2$, constitute a two-component vector with both positive and negative pseudo-momenta still playing the role of the creation fields acting on the pseudo-vacuum state. In Eq. (33), $\tilde{E}_a(\lambda)$ are the components of the dual creation fields, whereas one can show that the operators $\tilde{C}_i(\lambda)$ are still the annihilation fields acting on the pseudo-vacuum state. The integrability of the model leads to a perfect factorization of the transfer matrix acting not only on the pseudo-vacuum state but also on the multi-particle states. Conversely, the factorization of the transfer matrix on the pseudo-vacuum state reveals the consistency between the boundary reflection $K_{\pm}$-matrices and the integrability of the model.

In order to make further progress we return to the graded RE (6) and derive commutation relations between the diagonal fields and the creation fields. In general, the algebraic Bethe-ansatz solution for an integrable model with open BC is more complicated than that in the case of periodic BC due to the appearance of positive and negative rapidities. As in the case of the open Hubbard and Bariev chains [35], we find — substituting (33) into (6) — that the eigenvectors of the transfer matrices are generated only by two classes of creation fields. The first class consists of the non-commuting vectors $\tilde{B}_a(\lambda)$ satisfying the commutation relations

$$\begin{aligned}
&\tilde{B}_a(\lambda) \otimes \tilde{B}_b(\nu) \\
&= \frac{1}{w_1(\lambda-\nu)} \left[\tilde{B}_c(\nu) \otimes \tilde{B}_d(\lambda) + \frac{w_{10}(\lambda+\nu)}{w_3(\lambda+\nu)} \vec{\eta} \tilde{F}(\nu) (\mathbf{I} \otimes \tilde{A}(\nu)) \right] r(\lambda-\nu) \\
&\quad - \frac{w_{10}(\lambda+\nu)}{w_3(\lambda+\nu)} \vec{\eta} \tilde{F}(\lambda) (\mathbf{I} \otimes \tilde{A}(\nu)) + \frac{w_7(\lambda+\nu)}{w_3(\lambda+\nu)} \left[\frac{w_{10}(\lambda-\nu)}{w_7(\lambda-\nu)} \tilde{F}(\lambda) \tilde{B}(\nu) \right. \\
&\quad \left. - \frac{w_6(\lambda-\nu) w_7(\lambda-\nu) + w_{10}(\lambda-\nu) w_5(\lambda-\nu)}{w_1(\lambda-\nu) w_7(\lambda-\nu)} \tilde{F}(\nu) \tilde{B}(\lambda) \right] \vec{\eta}, \qquad (47)
\end{aligned}$$

where the functions $w_i(\mu)$ are those defined in (2) with $x = \mathrm{e}^{\lambda}$. The second class of creation fields contains $\tilde{F}(\lambda)$, of (32), which commute among themselves, i.e., $\left[\tilde{F}(\lambda), \tilde{F}(\nu)\right] = 0$. Here, $\vec{\eta}$ is a vector defined by $\vec{\eta} = (0,1,1,0)$, $\mathbf{I}$ is a 2×2 identity matrix, and $\tilde{A}$ denotes a submatrix given as

$$\tilde{A}(\nu) = \begin{pmatrix} \tilde{A}_{11}(\nu) & \tilde{A}_{12}(\nu) \\ \tilde{A}_{21}(\nu) & \tilde{A}_{22}(\nu) \end{pmatrix}. \qquad (48)$$

Performing the standard procedure, which is to keep the diagonal fields always on the right-hand sides in the commutation relations, and after several steps of substitutions, we arrive at the following commutation relations

$$\tilde{B}(\lambda)\tilde{B}_a(\nu) = \frac{w_1(\nu-\lambda)w_3(\lambda+\nu)}{w_3(\nu-\lambda)w_1(\lambda+\nu)} \tilde{B}_a(\nu)\tilde{B}(\lambda) + \text{u.t.}, \qquad (49)$$

$$\begin{aligned}
\hat{D}(\lambda)\tilde{B}_a(\nu) = &\frac{w_3(\lambda-\nu)\left[w_3^2(\lambda+\nu) - w_6(\lambda+\nu)w_{10}(\lambda+\nu)\right]}{w_7(\lambda-\nu)w_7(\lambda+\nu)w_3(\lambda+\nu)} \tilde{B}_a(\nu)\hat{D}(\lambda) \\
&+\text{u.t.}, \qquad (50)
\end{aligned}$$

$$\hat{A}_{ab}(\lambda)\tilde{B}_c(\nu) = \frac{w_1(\nu+\lambda) - w_2(\lambda+\nu)w_4(\lambda+\nu)}{w_3(\nu-\lambda)w_3(\nu+\lambda)w_1(\lambda+\nu)} \sum_{d,e,f,g=1}^{2} \left\{ r(\lambda+\nu-2\mathrm{i}\gamma)_{gf}^{ea} \right.$$

$$r(\lambda-\nu)_{cb}^{df}\tilde{B}_e(\nu)\hat{A}_{gd}(\lambda)\Big\} + \text{u.t.} \quad . \tag{51}$$

Here $a, b, c = 1, 2$. In the commutation relations (49)–(51), we omit all unwanted terms (u.t.) because they consist of a complex mixture of creation and annihilation fields and need a lot of space to display. The complexity of the unwanted terms plus the appearance of negative pseudo-momenta makes it very hard to perform the algebraic Bethe ansatz in a systematic way, in contrast to the case of the 1D Hubbard model with periodic BC [34]. However, we notice that the first term in each of the commutation relations (49)–(51) contribute to the eigenvalues of the transfer matrix which should be analytic functions of the spectral parameter λ. Consequently, the residues at singular points must vanish. This yields the Bethe-ansatz equations which in turn assure the cancellation of the unwanted terms in the eigenvalues of the transfer matrix. Hence we prefer to use the analytical properties rather than an analysis of the unwanted terms to derive the Bethe-ansatz equations. Fortunately, Eq. (51) reveals a hidden SU(1)-symmetry structure of the nesting transfer matrix, which is realized by the auxiliary r-matrix given by

$$r(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \varrho_1(\lambda) & \varrho_2(\lambda) & 0 \\ 0 & \varrho_2(\lambda) & \varrho_1(\lambda) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{52}$$

with

$$\varrho_1(\lambda) = w_8(\lambda) - \frac{w_6(\lambda)w_{10}(\lambda)}{w_7(\lambda)}, \quad \varrho_2(\lambda) = w_9(\lambda) - \frac{w_6(\lambda)w_{10}(\lambda)}{w_7(\lambda)}. \tag{53}$$

This hidden symmetry, leading to a factorization of the spin sector, plays a crucial role in the exact solution of the model (23). For $q = \mathrm{e}^{\mathrm{i}\gamma}$, $0 \leq \gamma \leq \pi$, the matrix (52) is nothing but the scattering matrix of the anisotropic Heisenberg XXZ model with Boltzmann weights

$$\varrho_1(\lambda) = \frac{\sinh 2\mathrm{i}\gamma}{\sinh(\lambda + 2\mathrm{i}\gamma)}, \quad \varrho_2(\lambda) = \frac{\sinh\lambda}{\sinh(\lambda+2\mathrm{i}\gamma)}. \tag{54}$$

Thus, the deformation parameter γ also plays the role of an anisotropy parameter in the hidden XXZ open chain. We also note that the commutation relation (47) exhibits an important symmetry, i.e.,

$$\vec{\eta}\, r(\lambda) = \frac{[w_6(\lambda)w_7(\lambda) + w_{10}(\lambda)w_5(\lambda)]\, w_7(-\lambda)}{w_7(\lambda)w_{10}(-\lambda)}\vec{\eta}, \tag{55}$$

leading to a symmetrization of the multi-particle states.

Following the argument of Refs. [34,35], we phenomenologically construct the n-particle state,

$$|\Phi_n(\nu_1,\ldots,\nu_n)\rangle = \Phi_n(\nu_1,\ldots,\nu_n)F^{a_1,\ldots,a_n}|0\rangle. \tag{56}$$

Here $F^{a_1,\cdots,a_n}$ are the coefficients of arbitrary linear combinations of the vectors reflecting the 'spin' degrees of freedom with $a_i = 1, 2$ $(i = 1,\ldots,n)$. The n-particle vector Φ_n satisfies the recursion

$$\begin{aligned}\Phi_n(\nu_1,\ldots,\nu_n) = {} & \tilde{B}_{e_1}(\nu_1)\otimes\Phi_{n-1}(\nu_2,\ldots,\nu_n) \\ & -\sum_{j=2}^{n}\Big\{\Big[\vec{\eta}\otimes\tilde{F}(\nu_1)\Big]\,\Phi_{n-2}(\nu_2,\ldots,\nu_{j-1},\nu_{j+1},\ldots,\nu_n) \\ & \Big[\tilde{B}(\nu_j)G^{(n)}_{j-1}(\nu_1,\ldots,\nu_n) - \Big(\mathbf{I}\otimes\tilde{A}(v_j)\Big)\,H^{(n)}_{j-1}(\nu_1,\ldots,\nu_n)\Big]\Big\},\end{aligned} \tag{57}$$

where the indices $e_i = 1, 2$ $(i = 1,\ldots,n)$ have been suppressed on the left-hand side for brevity. We remark that $\vec{\eta}$ excludes the possibility of two up- or two down-spin particles residing at the same site. $\tilde{F}$ creates a local particle pair with opposite spins. The coefficients $G^{(n)}_{j-1}$ and $H^{(n)}_{j-1}$ in turn can be determined from the symmetry of the wave functions

$$\begin{aligned}&\Phi_n(\nu_1,\ldots,\nu_j,\nu_{j+1},\ldots,\nu_n) \\ &= \frac{1}{w_1(\nu_j-\nu_{j+1})}\Phi_n(\nu_1,\ldots,\nu_{j+1},\nu_j,\ldots,\nu_n)\cdot r(\nu_j-\nu_{j+1})\end{aligned} \tag{58}$$

and the constraint arising from the cancellations of the unwanted terms in the corresponding eigenvalues of the transfer matrix. Explicitly, the one- and two-particle vectors read

$$\Phi_1(\nu_1) = \tilde{B}_{e_1}(\nu_1), \tag{59}$$

$$\begin{aligned}\Phi_2(\nu_1,\nu_2) = {} & \tilde{B}_{e_1}(\nu_1)\otimes\tilde{B}_{e_2}(\nu_2) + \frac{w_{10}(\nu_1+\nu_2)}{w_3(\nu_1+\nu_2)}\vec{\eta}\tilde{F}(\nu_1)\left[\mathbf{I}\otimes\tilde{A}(\nu_2)\right] \\ & -\frac{w_7(\nu_1+\nu_2)w_{10}(\nu_1-\nu_2)}{w_3(\nu_1+\nu_2)w_7(\nu_1-\nu_2)}\vec{\eta}\tilde{F}(\nu_1)\tilde{B}(\nu_2),\end{aligned} \tag{60}$$

respectively. Again, spin indices e_1 and e_2 are assumed implicitly on the left-hand side such that (56) is fulfilled.

According to the algebraic Bethe ansatz, the requirement that the unwanted terms in the eigenvalues of the transfer matrix cancel exactly yields the so-called Bethe-ansatz equations, which are quantization conditions for the rapidities. The property that the eigenvalues of the transfer matrix should have no poles suggests an alternative way to derive the constraints on the rapidities. It turns out that the Bethe-ansatz equations obtained by the latter way indeed imply the integrability of the model as can be seen by checking the consistency of the two-level transfer matrices. Letting the diagonal fields act on the state (56) and using the commutation relations (49)–(51), we get

$$\begin{aligned}
&\tilde{B}(\lambda)|\Phi_n(\nu_1,\ldots,\nu_n)\rangle \\
&= \varepsilon_1(\lambda)\prod_{j=1}^{n}\frac{\sinh\frac{1}{2}(\lambda-\nu_j+2\mathrm{i}\gamma)\sinh\frac{1}{2}(\lambda+\nu_j)}{\sinh\frac{1}{2}(\lambda-\nu_j)\sinh\frac{1}{2}(\lambda+\nu_j-2\mathrm{i}\gamma)}|\Phi_n(\nu_1,\ldots,\nu_n)\rangle + \text{u.t.}, \quad (61)
\end{aligned}$$

$$\begin{aligned}
&\hat{D}(\lambda)|\Phi_n(v_1,\ldots,v_n)\rangle \\
&= \varepsilon_4(\lambda)\prod_{j=1}^{n}\frac{\cosh\frac{1}{2}(\lambda-\nu_j)\cosh\frac{1}{2}(\lambda+\nu_j-2\mathrm{i}\gamma)}{\cosh\frac{1}{2}(\lambda-\nu_j+2\mathrm{i}\gamma)\cosh\frac{1}{2}(\lambda+\nu_j)}|\Phi_n(\nu_1,\ldots,\nu_n)\rangle + \text{u.t.}, \quad (62)
\end{aligned}$$

$$\begin{aligned}
&\hat{A}_{aa}(\lambda)|\Phi_n(\nu_1,\ldots,\nu_n)\rangle \\
&= \varepsilon_{a+1}(\lambda)\prod_{j=1}^{n}\frac{\sinh\frac{1}{2}(\lambda-\nu_j+2\mathrm{i}\gamma)\sinh\frac{1}{2}(\lambda+\nu_j)}{\sinh\frac{1}{2}(\lambda-\nu_j)\sinh\frac{1}{2}(\lambda+\nu_j-2\mathrm{i}\gamma)} \\
&\quad\times r(\lambda+\nu_1-2\mathrm{i}\gamma)_{g_1 f_1}^{e_1 a} r(\lambda-\nu_1)_{a_1 a}^{d_1 f_1} r(\lambda+\nu_2-2\mathrm{i}\gamma)_{g_2 f_2}^{e_2 g_1} r(\lambda-\nu_2)_{a_2 d_1}^{d_2 f_2} \\
&\quad\times \ldots r(\lambda+\nu_n-2\mathrm{i}\gamma)_{g_n f_n}^{e_n g_{n-1}} r(\lambda-\nu_n)_{a_n d_{n-1}}^{d_n f_n}|\Phi_n(\nu_1,\ldots,\nu_n)\rangle + \text{u.t.} \quad (63)
\end{aligned}$$

In the last equation above, the summation convention is implied for the repeated indices d_j, f_j and g_j except for the indices a and e_j. The n-particle state with indices e_j on the right-hand side, which is as defined in (56), should be equivalent to the one with indices a_j on the left-hand side. It is easily found that (63), the eigenvalue of the submatrix $\hat{A}_{aa}(\lambda)$, involves a nesting double-row transfer matrix consisting of the inhomogeneous Lax operators $r(\lambda+\nu_n-2\mathrm{i}\gamma)$ and $r(\lambda-\nu_n)$. For convenience, we shift the rapidities, $\lambda = u+\mathrm{i}\gamma$ and $\nu_j = v_j + \mathrm{i}\gamma$, and obtain

$$\begin{aligned}
&\tau(u)|\Phi_n(v_1,\ldots,v_n)\rangle = \Lambda(u,\{v_j\})|\Phi_n(v_1,\ldots,v_n)\rangle \\
&= \Bigg\{ W_1^+(u+\mathrm{i}\gamma)\varepsilon_1(u+\mathrm{i}\gamma)\prod_{j=1}^{n}\frac{\sinh\frac{1}{2}(u-v_j+2\mathrm{i}\gamma)\sinh\frac{1}{2}(u+v_j+2\mathrm{i}\gamma)}{\sinh\frac{1}{2}(u-v_j)\sinh\frac{1}{2}(u+v_j)} \\
&\quad + W_2^+(u+\mathrm{i}\gamma)\varepsilon_{a+1}(u+\mathrm{i}\gamma) \\
&\quad \times \prod_{j=1}^{n}\frac{\sinh\frac{1}{2}(u-v_j+2\mathrm{i}\gamma)\sinh\frac{1}{2}(u+v_j+2\mathrm{i}\gamma)}{\sinh\frac{1}{2}(u-v_j)\sinh\frac{1}{2}(u+v_j)}\Lambda^{(1)}(u,\{v_j\}) \\
&\quad + W_4^+(u+\mathrm{i}\gamma)\varepsilon_4(u+\mathrm{i}\gamma)\prod_{j=1}^{n}\frac{\cosh\frac{1}{2}(u-v_j)\cosh\frac{1}{2}(u+v_j)}{\cosh\frac{1}{2}(u-v_j+2\mathrm{i}\gamma)\cosh\frac{1}{2}(u+v_j+2\mathrm{i}\gamma)}\Bigg\} \\
&\quad \times|\Phi_n(v_1,\ldots,v_n)\rangle \quad (64)
\end{aligned}$$

provided that

$$-\left.\frac{W_1^+(u+\mathrm{i}\gamma)W_1^-(u+\mathrm{i}\gamma)w_1^{2L}(u+\mathrm{i}\gamma)}{W_2^+(u+\mathrm{i}\gamma)W_2^-(u+\mathrm{i}\gamma)w_3^{2L}(u+\mathrm{i}\gamma)}\right|_{u=v_j} = \Lambda^{(1)}(u=v_j,\{v_j\}), \tag{65}$$

for $j=1,\ldots,n$. Here $\Lambda^{(1)}(u,\{v_j\})$ are the eigenvalues of the nesting transfer matrix (67):

$$\tau^{(1)}(u,\{v_j\})F^{e_1,\ldots,e_n} = \Lambda^{(1)}(u,\{v_j\})F^{e_1,\ldots,e_n}. \tag{66}$$

The nesting transfer matrix reads

$$\tau^{(1)}(u,\{v_j\}) = \mathrm{Tr}_0\left[T^{(1)}(u)\bar{T}^{(1)}(u)\right], \tag{67}$$

where

$$T^{(1)}(u) = r_{12}(u+v_1)^{e_1 a}_{f_1 g_1}\ldots r_{12}(u+v_n)^{e_n g_{n-1}}_{f_n g_n}, \tag{68}$$

$$\bar{T}^{(1)}(u) = {T^{(1)}}^{-1}(-u) = r_{21}(u-v_n)^{d_n f_n}_{d_{n-1} a_n}\ldots r_{21}(u-v_1)^{d_1 f_1}_{a a_1}. \tag{69}$$

In the previous two expressions, we used the standard notation $r_{12}(u) = p{\cdot}r(u)$. Here p is a standard permutation operator, which can be represented by a $2^2\times 2^2$ matrix, i.e., $p_{\alpha\beta,\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma}$. We also note that $r_{12} = r_{21}$ for the r-matrix (52) and the trace operation in (67) leads to the identification $g_n = d_n = a$. It can be seen that the coefficients $F^{e_1,\ldots,e_n}$ act as the multi-particle vectors for the inhomogeneous transfer matrix (67), which characterizes the spin sector of the model.

So far, we managed to solve the charge degrees of freedom. The next task, the diagonalization of the anisotropic Heisenberg XXZ model with open boundaries, was done previously [13]. Thus we immediately obtain the eigenvalue of the nested transfer matrix (67) as

$$\begin{aligned}
&\Lambda^{(1)}(u,\{u_1,\ldots,u_M\},\{v_1,\ldots,v_n\}) \\
&= \frac{2\sinh(u+2\mathrm{i}\gamma)\cosh u}{\sinh 2(u+\mathrm{i}\gamma)}\prod_{l=1}^{M}\frac{\sinh(u-u_l-2\mathrm{i}\gamma)\sinh(u+u_l)}{\sinh(u-u_l)\sinh(u+u_l+2\mathrm{i}\gamma)} \\
&\quad+\frac{2\cosh(u+2\mathrm{i}\gamma)\sinh u}{\sinh 2(u+\mathrm{i}\gamma)}\prod_{j=1}^{n}\frac{\sinh(u-v_j)\sinh(u+v_j)}{\sinh(u-v_j+2\mathrm{i}\gamma)\sinh(u+v_j+2\mathrm{i}\gamma)} \\
&\quad\times\prod_{l=1}^{M}\frac{\sinh(u-u_l+2\mathrm{i}\gamma)\sinh(u+u_l+4\mathrm{i}\gamma)}{\sinh(u-u_l)\sinh(u+u_l+2\mathrm{i}\gamma)}
\end{aligned} \tag{70}$$

provided that

$$
\frac{\cosh^2(u_k+2\mathrm{i}\gamma)}{\cosh^2 u_k}\prod_{j=1}^{n}\frac{\sinh(u_k-v_j)\sinh(u_k+v_j)}{\sinh(u_k-v_j+2\mathrm{i}\gamma)\sinh(u_k+v_j+2\mathrm{i}\gamma)}
$$

$$
=\prod_{\substack{l=1\\ l\neq k}}^{M}\frac{\sinh(u_k-u_l-2\mathrm{i}\gamma)\sinh(u_k+u_l)}{\sinh(u_k-u_l+2\mathrm{i}\gamma)\sinh(u_k+u_l+4\mathrm{i}\gamma)},\quad k=1,\ldots,M, \tag{71}
$$

where M is the number of itinerant electrons with spin down, and n is the total number of itinerant electrons. The eigenvalue (70) and the constraint (71) on the spin rapidities u_k and u_l have paved the way to diagonalize the transfer matrix (28) completely. Making a further shift of the spin rapidities, i.e., $u_k=\mu_k-\mathrm{i}\gamma$ and $u_l=\mu_l-\mathrm{i}\gamma$, the eigenvalues of the transfer matrix (64)

$$
\tau(u)|\Phi_n(v_1,\ldots,v_n)\rangle=\Lambda(u,\{\mu_l\},\{v_j\})|\Phi_n(v_1,\ldots,v_n)\rangle \tag{72}
$$

are given by

$$
\begin{aligned}
\Lambda(u,\{\mu_l\},\{v_j\})=&\prod_{j=1}^{n}\frac{\sinh\frac{1}{2}(u-v_j+2\mathrm{i}\gamma)\sinh\frac{1}{2}(u+v_j+2\mathrm{i}\gamma)}{\sinh\frac{1}{2}(u-v_j)\sinh\frac{1}{2}(u+v_j)}\\
&\times\Bigg\{W_1^+(u+\mathrm{i}\gamma)W_1^-(u+\mathrm{i}\gamma)w_1^{2L}(u+\mathrm{i}\gamma)\\
&+W_2^+(u+\mathrm{i}\gamma)W_2^-(u+\mathrm{i}\gamma)w_3^{2L}(u+\mathrm{i}\gamma)\\
&\times\prod_{l=1}^{M}\frac{\sinh(u-\mu_l-\mathrm{i}\gamma)\sinh(u+\mu_l-\mathrm{i}\gamma)}{\sinh(u-\mu_l+\mathrm{i}\gamma)\sinh(u+\mu_l+\mathrm{i}\gamma)}\Bigg\}\\
&+\prod_{j=1}^{n}\frac{\cosh\frac{1}{2}(u-v_j)\cosh\frac{1}{2}(u+v_j)}{\cosh\frac{1}{2}(u-v_j+2\mathrm{i}\gamma)\cosh\frac{1}{2}(u+v_j+2\mathrm{i}\gamma)}\\
&\times\Bigg\{W_4^+(u+\mathrm{i}\gamma)W_4^-(u+\mathrm{i}\gamma)w_7^{2L}(u+\mathrm{i}\gamma)\\
&+W_2^+(u+\mathrm{i}\gamma)W_2^-(u+\mathrm{i}\gamma)w_3^{2L}(u+\mathrm{i}\gamma)\\
&\times\prod_{l=1}^{M}\frac{\sinh(u-\mu_l+3\mathrm{i}\gamma)\sinh(u+\mu_l+3\mathrm{i}\gamma)}{\sinh(u-\mu_l+\mathrm{i}\gamma)\sinh(u+\mu_l+\mathrm{i}\gamma)}\Bigg\}.
\end{aligned} \tag{73}
$$

The rapidities of the charge and spin degrees of freedom satisfy the following Bethe-ansatz equations

$$
\zeta(v_j,\xi^+)\zeta(v_j,\xi^-)\frac{\sinh^{2L}\frac{1}{2}(v_j-\mathrm{i}\gamma)}{\sinh^{2L}\frac{1}{2}(v_j+\mathrm{i}\gamma)}
=\prod_{l=1}^{M}\frac{\sinh(v_j-\mu_l-\mathrm{i}\gamma)\sinh(v_j+\mu_l-\mathrm{i}\gamma)}{\sinh(v_j-\mu_l+\mathrm{i}\gamma)\sinh(v_j+\mu_l+\mathrm{i}\gamma)}, \tag{74}
$$

$$\frac{\cosh^2(\mu_k+\mathrm{i}\gamma)}{\cosh^2(\mu_k-\mathrm{i}\gamma)}\prod_{j=1}^{n}\frac{\sinh(\mu_k-v_j-\mathrm{i}\gamma)\sinh(\mu_k+v_j-\mathrm{i}\gamma)}{\sinh(\mu_k-v_j+\mathrm{i}\gamma)\sinh(\mu_k+v_j+\mathrm{i}\gamma)}$$
$$=\prod_{\substack{l=1\\l\neq k}}^{M}\frac{\sinh(\mu_k-\mu_l-2\mathrm{i}\gamma)\sinh(\mu_k+\mu_l-2\mathrm{i}\gamma)}{\sinh(\mu_k-\mu_l+2\mathrm{i}\gamma)\sinh(\mu_k+\mu_l+2\mathrm{i}\gamma)}, \tag{75}$$

for $j = 1,\ldots,n$ and $k = 1,\ldots,M$, respectively. Here, we introduced the notation

$$\zeta(v_j,\xi^{\pm})=\frac{\cosh\frac{1}{2}(v_j-\xi^{\pm})}{\cosh\frac{1}{2}(v_j+\xi^{\pm})}. \tag{76}$$

From (22) and (73), we obtain the eigenvalues of the Hamiltonian (23) as

$$E=\sum_{j=1}^{n}\frac{2\sin^2\gamma}{\cos\gamma-\cosh v_j}+2L\cos\gamma-\frac{2}{\cos\gamma}-\sin\gamma\left[\cot\frac{1}{2}(\gamma-\xi^+)\right.$$
$$\left.+\tan\frac{1}{2}(\gamma+\xi^+)+\cot\frac{1}{2}(\gamma-\xi^-)+\tan\frac{1}{2}(\gamma+\xi^-)\right]. \tag{77}$$

Apart from a shift of the boundary parameters $\xi^{\pm}$, the Bethe-ansatz equations (74) and (75) coincide with those obtained by the coordinate Bethe ansatz in Ref. [32]. Notice that, besides the obtained Bethe-ansatz equations, we in addition presented a systematic way to formulate the algebraic Bethe ansatz for Hubbard-like models with open boundary fields and obtained the eigenvalue of the transfer matrix (73), which is essential for the investigation of finite-temperature properties of the model [33]. Furthermore, the two-level transfer matrices, characterizing the charge and spin sectors separately, allow us to embed different impurities into the system. From the Bethe solutions (74), it is found that the boundary fields characterized by $\zeta(v_j,\xi^{\pm})$ act indeed nontrivially on the densities of roots for spin rapidity μ_l and charge rapidity v_j, and thus change the ground state properties as well as the low-lying energy spectrum. The function $\zeta(v_j,\xi^{\pm})$ contributes a phase shift to the density of roots of the rapidities. Though the first factor on the left-hand side of the Bethe-ansatz equation (75) originates from the pure boundary effect of the spin sector, it contributes to both charge and spin rapidities in a nontrivial way. Of course, we may treat other boundary conditions for the model in a similar way. In the following section, we are going to discuss the boundary and impurity effects on the ground-state properties of the model at half filling.

4 The ground-state properties

Due to the boundary effects, the Bethe-ansatz equations (74) and (75) are not merely a doubling of the Bethe equations for the periodic $\mathrm{U}_q[\mathrm{osp}(2|2)]$ chain [26]. Thus the boundary fields contribute nontrivially to the ground-state properties of the model. If we consider the ground state at half filling, corresponding to the case that the v_j are real, while the spin variables form strings of type $\mu + \mathrm{i}\frac{\pi}{2}$, the discrete Bethe-ansatz equations (74) and (75) may be written as

$$\begin{aligned} &2L\theta_1(\tfrac{1}{2}v_j, \tfrac{1}{2}\gamma) - \theta_2(\tfrac{1}{2}v_j, \tfrac{1}{2}\xi^+) - \theta_2(\tfrac{1}{2}v_j, \tfrac{1}{2}\xi^-) \\ &= 2\pi I_j - \sum_{l=1}^{M} \left[\theta_2(v_j - \mu_l, \gamma) + \theta_2(v_j + \mu_l, \gamma)\right], \end{aligned} \tag{78}$$

$$\begin{aligned} &2\theta_1(\mu_k, \gamma) \\ &= 2\pi J_k - \sum_{\substack{l=1 \\ l \neq k}}^{M} \left[\theta_1(\mu_k - \mu_l, 2\gamma) + \theta_1(\mu_k + \mu_l, 2\gamma)\right] \\ &\quad - \sum_{j=1}^{n} \left[\theta_2(\mu_k - v_j, \gamma) + \theta_2(\mu_k + v_j, \gamma)\right], \end{aligned} \tag{79}$$

for $j = 1, \ldots, n$, and $k = 1, \ldots, M$, respectively. Here we introduced the shift functions

$$\theta_1(v, \gamma) = -\mathrm{i} \ln \frac{\sinh(\mathrm{i}\gamma - v)}{\sinh(\mathrm{i}\gamma + v)} \equiv 2 \arctan\left(\tanh v \cot \gamma\right), \tag{80}$$

$$\theta_2(v, \gamma) = -\mathrm{i} \ln \frac{\cosh(\mathrm{i}\gamma + v)}{\cosh(\mathrm{i}\gamma - v)} \equiv 2 \arctan\left(\tanh v \tan \gamma\right). \tag{81}$$

The integers I_j and J_k may be regarded as quantum numbers associated to the Bethe-ansatz equations. If we define $I_{-j} = -I_j$, $J_{-k} = -J_k$, $v_{-j} = -v_j$, and $\mu_{-l} = -\mu_l$, and pass to the thermodynamic limit $L \to \infty$, $n \to \infty$, and $M \to \infty$, with n/L and M/L kept finite, the thermodynamic Bethe-ansatz equations read

$$\begin{aligned} \rho^{\mathrm{c}}_\infty(v) = \frac{1}{\pi} \Bigg\{ &\frac{\mathrm{d}}{\mathrm{d}v}\theta_1(\tfrac{1}{2}v, \tfrac{1}{2}\gamma) + \frac{1}{2L}\frac{\mathrm{d}}{\mathrm{d}v}\theta_{\mathrm{cb}}(v, \xi^\pm) \\ &+ \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}\mu \frac{\mathrm{d}}{\mathrm{d}v}\theta_2(v - \mu, \gamma)\rho^{\mathrm{s}}_\infty(\mu) \Bigg\}, \end{aligned} \tag{82}$$

$$\rho^{\mathrm{s}}_\infty(\mu) = \frac{1}{2\pi} \Bigg\{ \frac{1}{L}\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_{\mathrm{sb}}(\mu, \gamma) + \int_{-\infty}^{\infty} \mathrm{d}v \frac{\mathrm{d}}{\mathrm{d}\mu}\theta_2(\mu - v, \gamma)\rho^{\mathrm{c}}_\infty(v)$$

$$+\int_{-\infty}^{\infty} \mathrm{d}v \frac{\mathrm{d}}{\mathrm{d}\mu}\theta_1(\mu - v, 2\gamma)\rho^{\mathrm{s}}_{\infty}(v)\Bigg\} . \tag{83}$$

Here $\rho^{\mathrm{c}}_{\infty}(v)$ and $\rho^{\mathrm{s}}_{\infty}(\mu)$ denote the densities of roots of the charge and spin rapidities at half filling, respectively. The term

$$\theta_{\mathrm{cb}}(v, \xi^{\pm}) = -\theta_2(v, \gamma) - \theta_2(\tfrac{1}{2}v, \tfrac{1}{2}\xi^{+}) - \theta_2(\tfrac{1}{2}v, \tfrac{1}{2}\xi^{-}) \tag{84}$$

characterizes the charge contributions to the densities of roots from the boundary potentials, whereas

$$\theta_{\mathrm{sb}}(\mu, \gamma) = 2\theta_1(\mu, \gamma) - \theta_1(\mu, 2\gamma) - \theta_1(2\mu, 2\gamma) - \theta_2(\mu, \gamma) \tag{85}$$

denotes the boundary contributions in the spin sector. We would like to stress that, although $\theta_{\mathrm{cb}}(v, \xi^{\pm})$ arises completely from the charge degrees of freedom, and $\theta_{\mathrm{sb}}(\mu, \gamma)$ only from the spin degrees of freedom, both terms contribute nontrivially to the densities of roots $\rho^{\mathrm{c}}_{\infty}(v)$ and $\rho^{\mathrm{s}}_{\infty}(\mu)$. We also find that the ground state of the system is a singlet, which means that the variables μ and v occupy the entire interval from $-\infty$ to ∞. By Fourier transformation, the solutions of (82) and (83) have the form

$$\rho^{\mathrm{c}}_{\infty}(v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\rho}^{\mathrm{c}}_{\infty}(\omega)\, \mathrm{e}^{-\mathrm{i}\omega v}\, \mathrm{d}\omega, \quad \hat{\rho}^{\mathrm{c}}_{\infty}(\omega) = \hat{\rho}^{\mathrm{c}}_{0}(\omega) + \frac{1}{L}\hat{\rho}^{\mathrm{c}}_{\mathrm{b}}(\omega), \tag{86}$$

$$\rho^{\mathrm{s}}_{\infty}(\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\rho}^{\mathrm{s}}_{\infty}(\omega)\, \mathrm{e}^{-\mathrm{i}\omega \mu}\, \mathrm{d}\omega, \quad \hat{\rho}^{\mathrm{s}}_{\infty}(\omega) = \hat{\rho}^{\mathrm{s}}_{0}(\omega) + \frac{1}{L}\hat{\rho}^{\mathrm{s}}_{\mathrm{b}}(\omega). \tag{87}$$

Here, $\hat{\rho}^{\mathrm{c}}_{0}(\omega)$ and $\hat{\rho}^{\mathrm{s}}_{0}(\omega)$ denote the bulk densities of roots for the charge and the spin rapidities, which are given by

$$\hat{\rho}^{\mathrm{c}}_{0}(\omega) = \frac{2\sinh(\frac{\pi}{2} - \gamma)\omega}{\cosh\frac{\pi}{2}\omega}, \quad \hat{\rho}^{\mathrm{s}}_{0}(\omega) = \frac{1}{\cosh\frac{\pi}{2}\omega}, \tag{88}$$

respectively. The remaining parts $\hat{\rho}^{\mathrm{c}}_{\mathrm{b}}(\omega)$ and $\hat{\rho}^{\mathrm{s}}_{\mathrm{b}}(\omega)$ contain the contributions caused by the boundary terms, i.e.,

$$\hat{\rho}^{\mathrm{c}}_{\mathrm{b}}(\omega) = -\frac{[\sinh\xi^{+}\omega + \sinh\xi^{-}\omega]\cosh(\frac{\pi}{2} - \gamma)\omega}{\sinh(\pi - \gamma)\omega\, \cosh\frac{\pi}{2}\omega} - \frac{\sinh\frac{\gamma}{2}\omega}{\sinh(\frac{\pi}{2} - \frac{\gamma}{2})\omega}, \tag{89}$$

$$\hat{\rho}^{\mathrm{s}}_{\mathrm{b}}(\omega) = -\frac{[\sinh\xi^{+}\omega + \sinh\xi^{-}\omega]}{2\sinh(\pi - \gamma)\omega\, \cosh\frac{\pi}{2}\omega} + \frac{\sinh(\frac{\gamma}{2} - \gamma)\omega}{2\sinh(\frac{\pi}{2} - \frac{\gamma}{2})\omega\, \cosh\frac{\pi}{2}\omega}, \tag{90}$$

respectively. In the expressions (89) and (90), we separated the effects of the boundary potentials and the pure boundary effects, i.e., corresponding to the first and the second terms on the right-hand sides of (89) and (90), respectively. It is straightforward to recover the result for free BC for the model (23) by switching off the boundary potentials via $\xi^{\pm} \to 0$. Because the open BC do not spoil the symmetries $\hat{\rho}^{\rm c}_{\infty}(\omega) = \hat{\rho}^{\rm c}_{\infty}(-\omega)$ and $\hat{\rho}^{\rm s}_{\infty}(\omega) = \hat{\rho}^{\rm s}_{\infty}(-\omega)$, the energy per site of the singlet ground state ($n = \frac{L}{2}$), calculated from (77), reduces to the form

$$
\begin{aligned}
E = & -4\sin\gamma \int_0^\infty \frac{\cosh(\frac{\pi}{2}-\gamma)\omega\,\sinh(\pi-\gamma)\omega}{\cosh\frac{\pi}{2}\omega\,\sinh\pi\omega}\,{\rm d}\omega + 2\cos\gamma \\
& -\frac{1}{L}\Bigg\{4\sin\gamma\int_0^\infty \frac{\hat{\rho}^{\rm c}_{\rm b}(\omega)\sinh(\pi-\gamma)\omega}{\sinh\pi\omega}\,{\rm d}\omega + \frac{2}{\cos\gamma} + \sin\gamma\Big[\cot\frac{1}{2}(\gamma-\xi^+) \\
& + \tan\frac{1}{2}(\gamma+\xi^+) + \cot\frac{1}{2}(\gamma-\xi^-) + \tan\frac{1}{2}(\gamma+\xi^-)\Big]\Bigg\}. \qquad (91)
\end{aligned}
$$

The first term in the ground-state energy (91) is the bulk ground-state energy which coincides with the result of the periodic chain [26]. We emphasize that the boundary potentials do not only enter in the expression for the ground state energy explicitly as $\cot\frac{1}{2}(\gamma-\xi^{\pm}) + \tan\frac{1}{2}(\gamma+\xi^{\pm})$, but also implicitly via $\rho^{\rm c}_{\infty}(v)$ and $\rho^{\rm s}_{\infty}(v)$ of (86) and (87).

Before closing this section, we discuss the problem of embedding integrable impurities into the open chain [20–22]. The algebraic Bethe ansatz provides us with a natural way to incorporate different kinds of impurities. If we embed two impurity vertices at the boundaries, namely, extend (20) to

$$
T(\lambda) = R_{{\rm r},0}(\lambda+p_{\rm r})R_{L,0}(\lambda)R_{L-1,0}(\lambda)\ldots R_{2,0}(\lambda)R_{1,0}(\lambda)R_{\ell,0}(\lambda+p_\ell), \quad (92)
$$

the impurity Hamiltonian with the impurity-host charge interactions and exchange coupling between the impurities and boundaries can be determined by

$$
\begin{aligned}
H_{\rm bi} = & \frac{1}{{\rm Str}_0 K_+(0)}\Big\{{\rm Str}_0[K_+(0)R'_{{\rm r},0}(p_{\rm r})R^{-1}_{{\rm r},0}(p_{\rm r})] \\
& + {\rm Str}_0[K_+(0)R_{{\rm r},0}(p_{\rm r})R'_{L,0}(0)R^{-1}_{L,0}(0)R^{-1}_{{\rm r},0}(p_{\rm r})]\Big\} \\
& + \frac{1}{2}R_{1,0}(0)R_{\ell,0}(p_\ell)K'_-(0)R^{-1}_{\ell,0}(p_\ell)R^{-1}_{1,0}(0) \\
& + R_{10}(0)R'_{\ell,0}(p_\ell)R^{-1}_{\ell,0}(p_\ell)R^{-1}_{1,0}(0). \qquad (93)
\end{aligned}
$$

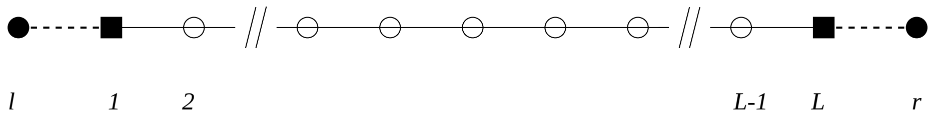


Fig. 1. Impurities coupled to each of the boundaries.

Due to rather lengthy algebra, we schematically present the interactions between boundaries and impurities by Figure 1 instead of presenting $H_{\rm bi}$ explicitly in terms of fermionic operators. The quantities $p_{\rm r}$ and p_ℓ shifting the pseudo-momenta characterize the impurity rapidities or impurity strength. As discussed previously [11,21], this shift preserves the integrability of the model and allows one to continuously vary the strength of the impurity coupling to the boundaries revealing different impurity effects. This embedding of the impurities, carrying both charge and spin degrees of freedom, leads to an additional factor

$$\prod_{m=\ell,{\rm r}} \frac{\sinh\frac{1}{2}(v_j+p_m-{\rm i}\gamma)\sinh\frac{1}{2}(v_j-p_m-{\rm i}\gamma)}{\sinh\frac{1}{2}(v_j+p_m+{\rm i}\gamma)\sinh\frac{1}{2}(v_j-p_m+{\rm i}\gamma)}, \tag{94}$$

on the left-hand side of the Bethe equation (74), thus contributing to the densities of roots for the charge and the spin rapidities in (86) and (87) at order $1/L$, namely,

$$\hat{\rho}^{\rm c}_\infty(\omega) = \hat{\rho}^{\rm c}_0(\omega) + \frac{1}{L}\left[\hat{\rho}^{\rm c}_{\rm b}(\omega) + \hat{\rho}^{\rm c}_{\rm i}(\omega)\right], \tag{95}$$

$$\hat{\rho}^{\rm s}_\infty(\omega) = \hat{\rho}^{\rm s}_0(\omega) + \frac{1}{L}\left[\hat{\rho}^{\rm s}_{\rm b}(\omega) + \hat{\rho}^{\rm s}_{\rm i}(\omega)\right]. \tag{96}$$

The contributions to the densities of roots from the impurity terms are

$$\hat{\rho}^{\rm c}_{\rm i}(\omega) = (\cos p_{\rm r}\omega + \cos p_\ell\omega)\,\hat{\rho}^{\rm c}_0(\omega), \tag{97}$$

$$\hat{\rho}^{\rm s}_{\rm i}(\omega) = (\cos p_{\rm r}\omega + \cos p_\ell\omega)\,\hat{\rho}^{\rm s}_0(\omega). \tag{98}$$

In this case, the ground state energy (91) has to be changed slightly by $[\hat{\rho}^{\rm c}_{\rm b}(\omega) + \hat{\rho}^{\rm c}_{\rm i}(\omega)]$ replacing $\hat{\rho}^{\rm c}_{\rm b}(\omega)$, which affects the surface energy and finite-size corrections.

From the above discussion, one can easily distinguish the effects of the impurities, boundary potentials, and the free edges of the system, i.e., the effects of dynamic magnetic impurities, the external scalar boundary fields, and open boundary conditions. In addition, the integrability of the open chain also allows us to embed a forward scattering impurity (without any reflection scattering amplitude) into the bulk part. Although the resulting impurity Hamiltonian is different from the boundary Hamiltonian, the Bethe ansatz shows that the impurity effects do not depend on the position of the impurity due to the pure forward scattering that is required by integrability [11]. Another interesting embedding of the impurity would be given by operator-valued boundary $K_{\pm}$-matrices [22] which lead to Kondo-impurity-like terms in the Hamiltonian. These different embeddings of the impurity would provide results essential to the study of the thermodynamic properties such as low-lying excitations, finite-size corrections, magnetization, etc. The other string solutions to the Bethe equations (74) and (75), which form the charge and spin bound states, give an independent approach to the investigation of the low-lying excitations for the model. We intend to consider this situation in the future.

5 Conclusion and discussion

In summary, we have discussed the algebraic Bethe-ansatz solution for the extended Hubbard model with boundary fields associated with the quantum superalgebra $U_q[osp(2|2)]$ in terms of the graded QISM. Two classes of solutions of the graded RE leading to four kinds of possible boundary terms in the Hamiltonian were obtained. The Bethe-ansatz equations, the eigenvalue of the transfer matrix and the energy spectrum were given explicitly. The ground-state properties in the thermodynamic limit were also studied. We found that the model exhibits an anisotropic Heisenberg XXZ open chain as its nesting transfer matrix characterizing the spin sector. This nesting structure seems to be different from that of other extended Hubbard models [22]. The Bethe-ansatz results allow us to embed impurities at the boundaries of the model. Our results provide a useful starting point for studying the thermodynamic properties and correlation functions for the model. The boundary potentials, pure boundary effects and impurity effects contribute nontrivially and separately to both the density of roots of the charge rapidity and the spin rapidity at order $1/L$. The impurity strengths p_ℓ and p_r, the boundary parameters $\xi^{\pm}$ and the q-deformation parameter γ change the asymptotic behavior of the thermodynamic Bethe-ansatz equations, i.e., the band filling, magnetization, susceptibility, compressibility, finite-size corrections, etc. Our computations could open an alternative way to study the thermodynamic properties for the integrable double sine-Gordon model [30] as well as to investigate the tunneling effects in quantum wires [31].

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A The boundary $K_{\pm}$-matrices

Let us consider an algebraic ansatz which fixes the boundary $K_{\pm}$-matrices. Due to the isomorphism between K_+- and K_--matrices, we need to solve the RE (6) to determine them. Let us first fix the left boundary K_--matrix (11). By substituting it into the RE (6), one may find that the RE (6) involves two variables λ and ν which make the functional equations involving $K_-^{(i)}(\lambda)$, $i = 1, \ldots, 4$ much more complicated. Nevertheless, taking into account the structure of the R-matrix (1) and the RE (6), we find that the functional equations of the RE arising from the positions corresponding to the permutation operator $\mathbf{P}_{\alpha\beta,\gamma\delta} = (-1)^{P(\alpha)P(\beta)}\delta_{\alpha\delta}\delta_{\beta\gamma}$ provide us with simpler equations than the ones arising from other positions in the RE. These simpler equations allow us to separate $K_-^{(i)}(\lambda), K_-^{(i)}(\nu)$ into factorized forms as follows

$$\frac{K_-^{(1)}(\nu)}{K_-^{(2)}(\nu)} = \frac{w_3(\lambda+\nu)w_2(\lambda-\nu)K_-^{(1)}(\lambda) + w_3(\lambda-\nu)w_4(\lambda+\nu)K_-^{(2)}(\lambda)}{w_2(\lambda+\nu)w_3(\lambda-\nu)K_-^{(1)}(\lambda) + w_4(\lambda-\nu)w_3(\lambda+\nu)K_-^{(2)}(\lambda)}, \tag{A.1}$$

$$\frac{K_-^{(1)}(\nu)}{K_-^{(3)}(\nu)} = \frac{w_3(\lambda+\nu)w_2(\lambda-\nu)K_-^{(1)}(\lambda) + w_3(\lambda-\nu)w_4(\lambda+\nu)K_-^{(3)}(\lambda)}{w_2(\lambda+\nu)w_3(\lambda-\nu)K_-^{(1)}(\lambda) + w_4(\lambda-\nu)w_3(\lambda+\nu)K_-^{(3)}(\lambda)}, \tag{A.2}$$

$$\frac{K_-^{(3)}(\nu)}{K_-^{(4)}(\nu)} = \frac{w_3(\lambda+\nu)w_2(\lambda-\nu)K_-^{(3)}(\lambda) + w_3(\lambda-\nu)w_4(\lambda+\nu)K_-^{(4)}(\lambda)}{w_2(\lambda+\nu)w_3(\lambda-\nu)K_-^{(3)}(\lambda) + w_4(\lambda-\nu)w_3(\lambda+\nu)K_-^{(4)}(\lambda)}. \tag{A.3}$$

Substituting the Boltzmann weights of the R_{12}-matrix (1) into the equations above, and analyzing the structure of these equations, we can infer that the functions on the right-hand side of each of the equations (A.1)–(A.3) involving the variable λ should cancel because the left-hand sides of (A.1)–(A.3) are functions of the variable ν only. This cancellation leaves us with the following expressions for $K_-^{(i)}(\lambda)$

$$K_-^{(1)}(\lambda) = \left(x + C^{(1)}\right)\left(x + C^{(2)}\right)\left(x + C^{(3)}\right), \tag{A.4}$$

$$K_-^{(2)}(\lambda) = \left(x^{-1} + C^{(1)}\right)\left(x + C^{(2)}\right)\left(x + C^{(3)}\right), \tag{A.5}$$

$$K_-^{(3)}(\lambda) = \left(x + C^{(1)}\right)\left(x^{-1} + C^{(2)}\right)\left(x + C^{(3)}\right), \tag{A.6}$$

$$K_-^{(4)}(\lambda) = \left(x + C^{(1)}\right)\left(x^{-1} + C^{(2)}\right)\left(x^{-1} + C^{(3)}\right), \tag{A.7}$$

with a minimal number of coefficients that are to be determined. We regard Eq. (A.4)-(A.7) as the main structure of the K-matrix. Here $x = \mathrm{e}^\lambda$ as in section 2. Employing the RE again with (A.4)–(A.7) it is easily found that only one coefficient is free, and we have the relations $C^{(1)} = C^{(2)} = -C^{(3)}q^2$ or $C^{(1)} = -C^{(3)}q^2 = -q^2/C^{(2)}$. The first case yields the solution (13) while the second case corresponds to (14). The corresponding boundary terms have been given by (25)–(27), respectively. Due to the fact that the left and right boundaries are independent of each other, the two cases of boundary terms allow four possible combinations. In particular, we find a spin-degenerate situation with chemical potentials at the boundaries (i) identical for spin-up and spin-down electrons, of (26), or (ii) different yielding (27). Or, the spins are distinguished either (iii) at the right boundary, i.e., $V_{L,\pm}$ as in (27), or (iv) at the left boundary, i.e., $V_{1,\pm}$ as in (27), and other chemical potentials given by (26). In all cases, $U(\xi_\pm)$ is given by (25). These potentials constitute the general integrable boundary terms corresponding to the diagonal boundary K-matrices. In this paper, we restrict our discussion to the physically most realizable, symmetric case given by (13).

B The Hamiltonian

Because of $\mathrm{Str}_0[K_+(0)] = 0$, we have to consider the second-order expansion of the transfer matrix (19) with respect to the spectral parameter λ in order to construct the Hamiltonian with boundary fields. Following [15], we derive

$$\begin{aligned} H = &\sum_{j=1}^{L-1} H_{j,j+1} + \frac{1}{2}K_-'(0) + \Big\{\mathrm{Str}_0\left[K_+'(0)R_{L0}'(0)P_{L0}\right] \\ &+\frac{1}{2}\mathrm{Str}_0\left[K_+(0)R_{L0}''(0)P_{L0}\right] + \frac{1}{2}\mathrm{Str}_0\left[K_+(0)(R_{L0}'(0)P_{L0})^2\right]\Big\} / \Big\{ \\ &\mathrm{Str}_0\left[K_+'(0)\right] + 2\mathrm{Str}_0\left[K_+(0)R_{L0}'(0)P_{L0}\right]\Big\}, \end{aligned} \tag{B.1}$$

where

$$H_{j,j+1} = P_{j,j+1}\frac{\mathrm{d}\,R_{j,j+1}(\lambda)}{\mathrm{d}\lambda}\bigg|_{\lambda=0}. \tag{B.2}$$

The prime denotes the derivative with respect to the spectral parameter λ. Using (B.2), we can write the bulk Hamiltonian in terms of two commuting species of Pauli matrices σ and τ, i.e.,

$$
\begin{aligned}
H_{j,j+1} = &\left(\sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+\right)\left(\frac{\sigma_j^z\tau_j^z + \sigma_{j+1}^z\tau_{j+1}^z}{4} - \frac{1+\tau_j^z\tau_{j+1}^z}{2(q-q^{-1})}\right) \\
&+ \left(\tau_j^+\tau_{j+1}^- + \tau_j^-\tau_{j+1}^+\right)\left(\frac{\sigma_j^z\tau_j^z + \sigma_{j+1}^z\tau_{j+1}^z}{4} - \frac{1+\sigma_j^z\sigma_{j+1}^z}{2(q-q^{-1})}\right) \\
&+ \frac{q+q^{-1}}{2(q-q^{-1})}\left[\sigma_j^+\sigma_{j+1}^-\tau_j^-\tau_{j+1}^+ - \sigma_j^+\sigma_{j+1}^-\tau_j^+\tau_{j+1}^- + \text{h.c.}\right] \\
&- \frac{q+q^{-1}}{8(q-q^{-1})}\left[\sigma_j^z\tau_{j+1}^z + \sigma_{j+1}^z\tau_j^z + \sigma_j^z\tau_j^z + \sigma_{j+1}^z\tau_{j+1}^z + 4\,\mathbf{I}\right] \\
&+ \frac{1}{4}\left(\sigma_j^z - \sigma_{j+1}^z + \tau_j^z - \tau_{j+1}^z\right). \qquad \text{(B.3)}
\end{aligned}
$$

For the solution (13) of the RE, the second term in (B.1) gives the boundary terms at site $j=1$

$$
H_1 = -\frac{(1+q^2)\xi_-\sigma_1^z\tau_1^z - (2q-\xi_- + q^2\xi_-)(\sigma_1^z+\tau_1^z)}{4(q-\xi_-)(1+q\xi_-)}. \qquad \text{(B.4)}
$$

The boundary terms at site $j=L$ are then given by the remaining terms of (B.1) as

$$
H_L = -\frac{(1+q^2)\xi_+\sigma_L^z\tau_L^z - (2q\xi_+ + 1 - q^2)\xi_+(\sigma_L^z+\tau_L^z)}{4(q-\xi_+)(1+q\xi_+)}. \qquad \text{(B.5)}
$$

Using the Jordan-Wigner transformation [36], apart from a factor $-1/(q-q^{-1})$ — absorbed in the constant c_2 in (22) — the bulk Hamiltonian (B.3) together with the boundary terms (B.4) and (B.5) has the form presented in Eqs. (23)–(26). The last term in (B.3) should be taken into account in the boundary terms.

C Useful commutation relations

For the factorization of the transfer matrix (19) acting on the pseudo-vacuum state, we need the following commutation relations

$$
\begin{aligned}
w_1(2\lambda)C_1(\lambda)\bar{B}_1(\lambda) &= w_2(2\lambda)\left[\bar{B}(\lambda)B(\lambda) - A_{11}(\lambda)\bar{A}_{11}(\lambda)\right], \qquad \text{(C.1)}\\
w_1(2\lambda)\bar{C}_1(\lambda)B_1(\lambda) &= w_4(2\lambda)\left[B(\lambda)\bar{B}(\lambda) - \bar{A}_{11}(\lambda)A_{11}(\lambda)\right], \qquad \text{(C.2)}\\
w_1(2\lambda)C_2(\lambda)\bar{B}_2(\lambda) &= w_2(2\lambda)\left[\bar{B}(\lambda)B(\lambda) - A_{22}(\lambda)\bar{A}_{22}(\lambda)\right], \qquad \text{(C.3)}\\
w_1(2\lambda)\bar{C}_2(\lambda)B_2(\lambda) &= w_4(2\lambda)\left[B(\lambda)\bar{B}(\lambda) - \bar{A}_{22}(\lambda)A_{22}(\lambda)\right], \qquad \text{(C.4)}
\end{aligned}
$$

$$w_1(2\lambda)C_3(\lambda)\bar{F}(\lambda) = w_5(2\lambda)\left[\bar{B}(\lambda)B(\lambda) - D(\lambda)\bar{D}(\lambda)\right]$$
$$-w_2(2\lambda)\left[C_4(\lambda)\bar{E}_1(\lambda) + C_5(\lambda)\bar{E}_2(\lambda)\right], \quad (C.5)$$
$$w_4(2\lambda)C_3(\lambda)\bar{F}(\lambda) = w_2(2\lambda)\left[\bar{A}_{11}(\lambda)A_{11}(\lambda) - D(\lambda)\bar{D}(\lambda)\right] - C_4(\lambda)\bar{E}_1(\lambda)$$
$$+w_5(2\lambda)\bar{C}_1(\lambda)B_1(\lambda) - w_8(2\lambda)C_5(\lambda)\bar{E}_2(\lambda), \quad (C.6)$$
$$w_4(2\lambda)C_3(\lambda)\bar{F}(\lambda) = w_2(2\lambda)\left[\bar{A}_{22}(\lambda)A_{22}(\lambda) - D(\lambda)\bar{D}(\lambda)\right] - C_5(\lambda)\bar{E}_2(\lambda)$$
$$+w_5(2\lambda)\bar{C}_2(\lambda)B_2(\lambda) - w_8(2\lambda)C_4(\lambda)\bar{E}_1(\lambda), \quad (C.7)$$

which can be derived directly from (34). From these relations, we obtain

$$C_3(\lambda)\bar{F}(\lambda) = \frac{(q^2-1)^2}{(x^2q^2-1)(x^2-q)}\left[\bar{A}_{11}(\lambda)A_{11}(\lambda) + \bar{A}_{22}(\lambda)A_{22}(\lambda)\right]$$
$$-\frac{2(q^2-1)}{(x^2+1)(x^2-q^2)}B(\lambda)\bar{B}(\lambda) + \frac{2(q^2-1)}{(x^2q^2-1)(x^2+1)}D(\lambda)\bar{D}(\lambda), \quad (C.8)$$

$$C_4(\lambda)\bar{E}_1(\lambda) = \frac{q^2(q^2-1)(x^4-1)}{(x^4q^4-1)(x^2-q^2)}\bar{A}_{11}(\lambda)A_{11}(\lambda)$$
$$-\frac{x^2(q^2-1)^2(q^2+1)}{(x^4q^4-1)(x^2-q^2)}\bar{A}_{22}(\lambda)A_{22}(\lambda) - \frac{q^2-1}{x^2q^2-1}D(\lambda)\bar{D}(\lambda), \quad (C.9)$$

$$C_5(\lambda)\bar{E}_2(\lambda) = -\frac{x^2(q^2-1)^2(q^2+1)}{(x^4q^4-1)(x^2-q^2)}\bar{A}_{11}(\lambda)A_{11}(\lambda)$$
$$+\frac{q^2(q^2-1)(x^4-1)}{(x^4q^4-1)(x^2-q^2)}\bar{A}_{22}(\lambda)A_{22}(\lambda) - \frac{q^2-1}{x^2q^2-1}D(\lambda)\bar{D}(\lambda). \quad (C.10)$$

In addition, one also can show that $C_i(\lambda)\bar{B}_j(\lambda) = 0$ for $i \neq j$, $i = 1, 2, 3$, and $j = 1, 2$. With the help of (C.8)–(C.10), it is not difficult to derive (35).

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