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Hydrographical Flow Modelling of the River Severn Using Particle Swarm Optimization

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A model is presented to model hydrographical flow, which we apply to flood forecasting in the River Severn catchment area. The approach uses Particle Swarm Optimization (PSO), a swarm computation heuristic, to produce a predictive model of hydrographical flow. Hydrological flow data from 1980 to 1990 are considered, comprising the daily average flow through the River Severn and its tributaries. PSO models are developed from each year of data and are applied to predict flow in the other 10 years; model performance is shown to be largely independent of the training year, suggesting the catchment system is stable and the approach is robust. Importantly, and in contrast to most of the existing alternatives, flow is derived from data measurements taken 2 days previously, as demanded for early-warning flood prediction. The predictive (cross-validated) for prediction of extreme (Q95) events $R^2 = 0.96$, significantly improving upon multiple linear regression $R^2 = 0.93$, the best performing of current existing methods

Keywords: particle swarm; optimization; River Severn; hydrographical flow; prediction; machine learning

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1. INTRODUCTION

Hydrographical flow prediction is significant in the prediction of flood events. Predictions of this type are gaining in importance as the climate changes. Unfortunately, obtaining accurate predictions of complex river systems remains difficult due to the relatively complex influences affecting the hydrographical flow. To be an effective prediction of hydrographical flow a model needs to perform a transformation of the hydrographical elements such as tributaries, run-off and rainfall, additionally the geographical environment affecting issues such as lag and evaporation.

The Centre for Ecology & Hydrology in the UK, www.ceh.ac.uk, has been recording the flows from the Severn River and its three tributaries, the Hore, Hefren and Tanllwyth rivers—referred to as the Plynlimon Research Catchments—over a period of years. The purpose of the research supported by the Centre has been to collect data for comparison between forested and grassland catchment areas, based on data collected from these between 1972 and 2004 [1, 2]. We present the results of our application of PSO using a subset of this dataset covering

1980–90, the purpose of which is to develop a predictive hydrological flow model for the River Severn based on the historical data. Figure 1 shows a map of the catchment area.

The problem of predicting hydrographical flow is one of optimization. There have been a number of experiments that attempt to model hydrographical flow using artificial intelligence techniques other than PSO. These mainly rely on Artificial Neural Networks (ANNs) and model the outflow from the basin at time t only on the basis of data from the run-off at time t , without allowing for accumulation or lag. Good examples of these are [3–8]. Developments in swarm computation have led to the development of heuristics that make complex systems more open to computational modelling techniques. Swarm computation is a group of computational techniques inspired by natural swarming, shoaling and flocking behaviours. One of the most successful of these techniques is Particle Swarm Optimization (PSO). Nevertheless, papers such as [9] and [4] take the approach of combining PSO with a supplementary variation in order to address shortcomings arising from the model. The model presented in this paper uses PSO to find

FIGURE 1. Plynlimon catchment, shown in darker green, adapted from [1] with permission.

a mathematical transformation with no modification to a converging implementation of PSO. This has the advantage of not making assumptions about the data that are the subject of the model. Our approach differs from previous research in that it uses PSO to find mathematical transformations that represent a mathematical model of the River Severn's flow building in lag and other environmental considerations. By not assuming anything about the data presented to the model we leave it to the heuristic to find these relationships. By focusing on a solution that influences PSO without a supplementary technique the implementation is also much simpler. The technique also proved effective, when applied to flood prediction for the River Severn.

2. RELATED WORK

2.1. Hydrological flow estimation

Predicting hydrological flow is an important aspect within hydrological science; it allows pre-emptive actions to be taken in order to prevent or alleviate damage arising [10] and also makes it possible to monitor and maintain the stability of the catchment [11]. It is unsurprising that there is a significant amount of work being done in this area, and specifically using machine learning techniques.

As mentioned above, many of the early approaches to hydrological flow estimation took the approach of modelling the system as it was at a particular time, for example [3–8]. As machine learning as a field has developed, hydrological flow prediction has been tried using the application of many machine learning techniques, for example Genetic Algorithms [12], ANN [13, 14] and run-off [15], Support Vector Machines (SVMs) [16] and Random Forests [17, 18]. Although these were quite successful at producing a model of the particular catchment they were less successful at offering a

predictive solution to inform ecological and environmental decision-making. An alternative approach is to use probability, for example [19], models streamflow using a probabilistic approach.

Machine learning has the potential and has been proven to show greater flexibility, since it identifies previously unknown patterns in potentially chaotic processes [13]. However, the computational cost of implementation has inhibited wider application. Recently there has been a move towards using Extreme Learning Machines (ELM) [20, 21]. A useful review of this technique applied to river hydrological flow is published in [21]. An ELM is essentially a simplified feedforward ANN, which has the advantage of not requiring all of the nodes to be trained in order to make predictions. This has proven to be an extremely useful technique that shows comparable performance to the results presented in this paper across both rivers. In addition, the models produce good results for lags of more than 3 days. However, the results presented in [21] do not present comparisons for each river across several years. This means that it's impossible to tell whether the models are robust for each catchment, or whether they train to a specific dataset, representing the time period for which the model was trained on the data collected.

Many of the existing techniques require training to the specific catchment and period for which a model is being developed. This has the consequence that many of the models, for example [22], perform less well when applied to different datasets from the same catchment. Such a problem can be mediated to an extent by training a model of weights in such a way that overfitting is avoided [23] with hydrological data that allows for the model to discover patterns in the data that are not immediately apparent, see for example [24]. We have considered the machine learning techniques ANN, SVM and ELM models used on hydrographical flow. The model developed in this paper is a simplification of ANN, SVM

and ELM, in respect of structure and computational efficiency and training time.

2.2. Modelling river catchment run-off

River catchment run-off is clearly part of hydrological flow. Our experiments show that it should be considered as an important aspect within the model itself. This is supported by [25] and [26], demonstrating that the accuracy of hydrological estimation is very dependent on the correct identification of the lag, run-off, for a given catchment. There have been several studies, for example [27] and [28], that attempt to formulate methodology to be used. Nevertheless, in a given catchment there are a number of specific considerations including climate, environmental conditions and deforestation that will affect the way water enters the hydrological system. It is therefore important to allow the model to find the lag itself as part of the training process [29]. Through enabling the training of the model to identify characteristics including lag intrinsic to the River catchment assumptions about the behaviour of the environment, in which the catchment is located, can be avoided. This is particularly important to avoid overfitting, see for example [24]. In addition to the modelling of hydrological process river run-off adds an additional dimension to the interactions, which are probably chaotic. Here we are using the formal mathematical definition of chaotic and intend that this should be taken to mean that with a fully informed system; the hydrological flows are retrospectively deterministic, but vary dramatically from different starting points, and therefore accessible to probabilistic-based predictions, see for example [19]. In most hydrographical flow models, it becomes necessary to assume that rainfall contributing to run-off has a broadly consistent effect within the catchment [30]. That is to say, the model assumes that rainfall in the catchment has a relatively stable effect on the rest of the environment. Whilst a rainfall event may disrupt the predictions over the short term, the effect of normal precipitation levels remains reasonably consistent.

We have briefly reviewed the broader literature on modelling hydrological flows and specifically river catchment run-off. Any hydrological system represents a complex interaction between many variables within the environment. As the review above has shown it is a complex task to represent the chaotic nature of such a system in a way that is able to successfully predict future events. In the following section we present the heuristic on which our technique is based.

3. PARTICLE SWARM OPTIMIZATION

PSO is an evolutionary optimization heuristic originally developed in [31]. The original algorithm was not originally developed for purposes of optimization or prediction, but rather

FIGURE 2. Particles moving in solution space.

as a simple model of social behaviour; examples of social behaviour are collective decision-making [32], social networks [33], behaviour of crowds [34] and spread of epidemics [35]. However, since PSO's potential for optimization had been realized, the original algorithm has undergone much development and modification.

As usual, the problem space is visualized as an abstract landscape, with hills and valleys, and with the height of any point in the space, which represents how good the solution at that point is, referred to as that solution's *fitness*. In PSO, a number of particles are distributed at random across this landscape, as illustrated in Fig. 2. In the basic version of PSO, each particle has a *neighbourhood* and can exchange information with other particles in this neighbourhood. Each particle i will have the following characteristics:

- a *current position*, represented by a vector $\mathbf{x}_i$
- a *velocity* (since particles move), represented by a vector $\mathbf{v}_i$
- a vector recording the position in the solution space at which it achieved its fittest result (its *personal best*, $\mathbf{p}_i$)
- a vector recording the position $\mathbf{p}_g$ of the particle in its neighbourhood with the best fitness.

The particle marked **gBest** in Fig. 2 is closest to the fittest solution in the whole landscape and is marked as the global best.

The PSO system evolves across a number of time steps or iterations, with particles moving across the landscape seeking out better and better solutions. At each time step,

- every particle compares the fitness of the possible solution its position vector $\mathbf{x}$ represents with those of the other particles in its neighbourhood
- the best particle then becomes the neighbourhood's best, and the $\mathbf{p}_g$ of all particles in the neighbourhood are updated accordingly.

TABLE 1. PSO parameter values used in all experiments.

Parameter	Value
w	0.729
φ_1	0..1.5
φ_2	0..1.5
$VMax$	4.0
Particles	40

The other particles in the neighbourhood are thus pulled towards the best particle and the current particle's previous best in the following iteration. Formally, each particle applies the following equations:

$$\vec{v}_i = w \cdot \vec{v}_i + \phi_{1i} \cdot (\vec{p}_i - \vec{x}_i) + \phi_{2i} \cdot (\vec{p}_g - \vec{x}_i) \quad (1a)$$

$$\vec{x}'_i = \vec{x}_i + \vec{v}_i \quad (1b)$$

where $\vec{p}_i$ is the particle's previous best, and $\vec{p}_g$ is the global best. φ_{2i} and φ_{1i} are random values, conventionally in the range 0–2.0. The subscript i is often omitted from φ_{2i} and φ_{1i} [36, 37], the implication of this being that a single random value for each of the coefficients is used for every dimension. This obviously restricts the dynamic nature of the equation, thereby making it less flexible. In our implementation φ_{2i} and φ_{1i} have the potential to be different random values for each dimension. The w parameter is a scalar value and acts as an inertia weight, an enhancement introduced by [38]. The inertia weight is a reducing factor usually set to 0.729 for all dimensions and is applied to the existing velocity to ensure that the velocity reduces and therefore the particle converges over time. Before the inertia weight was introduced to the original PSO the heuristic was not guaranteed to converge on a solution. The values used for the parameters are those used for optimal convergence of the swarm; mathematical proofs are available in [36] and [37]. In addition, the $VMax$ parameter is implemented as a maximum velocity for a particle. In all of the following experiments, we use the PSO parameters in Table 1.

4. RIVER SEVERN FLOWS

The Centre for Ecology & Hydrology has been recording the flows from the Severn River and its three tributaries, Hore, Hefren and Tanllwyth rivers based on data collected from these between 1972 and 2004 [1, 39]. In this section we present our application of PSO, to the data gathered from the Plynlimon research [1, 39] between 1980 and 1990, in order to develop a predictive model for the River Severn hydrographical flows. The data from 1980 to 1990 were chosen because they are the years that are least affected by the measurement discrepancy described below.

4.1. Hydrological implementation

Section 3 essentially provides a heuristic outline of PSO. Clearly, in order to apply to a particular application, a specific implementation has to be developed. With regard to the hydrological data, this is formulated as a matrix multiplied by a vector, providing a hyperspatial solution in the form of the matrix, A , for PSO to optimize:

$$s = \sum A f \quad (2a)$$

$$r = \sqrt{s} \quad (2b)$$

In the above equation A is the matrix derived from the position vector within the PSO equations, and note A represents a rearrangement of the particle's position vector; hence there is no subscript since we are considering the matrix as a whole; f is the vector of the real numbered hydrographical flow values from the River Severn tributaries, s is the sum of the elements in the resultant vector following the multiplication of vector f with matrix A and r is the square root, giving the estimated flow for the River Severn. The square root is used to reduce the size of the sum that aids the optimization process, especially when the swarm is near convergence. This helps to avoid the effect of rounding errors in the multiplication when the swarm is nearing convergence and the weight values become small. To clarify, an alternative approach would have been to pre-process the input data, such that the input values were normalized. However, one of the aims of our model is to avoid assumptions about the input data and still produce useful predictions. We made the adjustments inside the model to allow for smaller result values. An advantage of this is the model remains independent of the data input and therefore is not as susceptible as it would be to change seen in the data.

This value is then combined to give a yearly sum of the days' flow, which is then subtracted from the sum of the real values as follows:

$$\sum_{i=1}^{days} \text{abs}(\text{actualflow}_i - r_i) \quad (3)$$

where *actualflow* is the hydrological flow for the day r is taken from (2b) above, the i subscript added to denote the day of the year. The absolute difference is used to ensure that the minimum is zero; this means that a difference of -1 is treated as equivalent to a difference of $+1$. The swarm is then iterated, using equation (1), each particle's candidate solution is compared against the daily averaged values for a given year, per iteration. In the current implementation, a single swarm concentrates on a single year.

The implementation has a boundary condition of ± 2 and implemented as a torus which wraps at the boundaries. This avoids assumptions about the nature of the solution space.

FIGURE 3. Screen capture of test bed written to complete our experiments.

The value to be used as the closest integer which is greater than the maximum value required.

Another important point to note is that in training the model, the flow values for a given day are modelled using the data measurements taken 2 days before. This is particularly useful because it would allow remedial measures to be taken in advance of the predicted flood. The emphasis here is that by making predictions that are successful 2 days in advance of the readings. The model is incorporating the lag, which is inherent in the time taken for water to run-off the surrounding landscape and evaporation, which occurs during the run-off. This is underlined in the results after a rain event. In subsequent sections, we will refer to $t-2$, for example this refers to the measurement taken 2 days before, and therefore, the prediction is made 3 days ahead.

4.2. Experimental methods

All of the experiments completed during the course of our research were run on a custom-designed test bed written in Java

1.8. The test bed allows flexibility in the configuration of PSO, which is used and allows the dataset to which the swarm as applied to be selected. Figure 3 shows a screenshot of the test bed.

After an iteration is completed, the test bed allows the results from the best performing particle or all particles to be stored in XML format for subsequent analysis. The source code for the test bed is available from the author, upon reasonable request.

5. EVALUATION

We briefly discuss the statistic validations used during the development of the model. The primary statistic used for validation of the model's ability to fit the observed data, the goodness of fit, is the R^2 correlation. The R^2 correlation is also used to validate the predictions made by the model on test data, data taken from the other years collected from the Plynlimon Research Catchments. The results are presented in Table 2.

TABLE 2. The R^2 correlation coefficient is calculated, using the matrix calculated for the corresponding year. The rows indicate the matrix used, whereas the columns indicate the years. The best performing matrix for each year (column) is shown as white text on a black background. The results, running diagonally from the top left corner to the bottom right corner, represent the test results. These results are included in the table to demonstrate the years on which a particular matrix is trained does not necessarily perform best on that year. This is seen as an advantage of the method.

Matrix	Year										
	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1980	0.996839	0.997534	0.990806	0.982645	0.997678	0.990863	0.988676	0.994598	0.992434	0.993406	0.993094
1981	0.997227	0.997834	0.989359	0.985706	0.99818	0.992703	0.990308	0.995321	0.993526	0.993923	0.994124
1982	0.996854	0.997698	0.991478	0.983107	0.996824	0.988377	0.987268	0.993491	0.991118	0.992656	0.992269
1983	0.996452	0.997094	0.985667	0.989361	0.997005	0.994015	0.990377	0.994744	0.993025	0.99346	0.992635
1984	0.99624	0.996478	0.986877	0.982568	0.997889	0.994761	0.992413	0.996213	0.995091	0.994315	0.995338
1985	0.994221	0.994002	0.981749	0.980592	0.995788	0.995298	0.993403	0.995347	0.995103	0.993412	0.994882
1986	0.993638	0.994384	0.983913	0.970462	0.997286	0.994228	0.993315	0.99618	0.995523	0.993892	0.99637
1987	0.993361	0.994446	0.986615	0.967039	0.997348	0.993163	0.992345	0.996437	0.995075	0.993894	0.995802
1988	0.995548	0.995948	0.985281	0.976651	0.997705	0.994218	0.993096	0.996143	0.995512	0.994345	0.996337
1989	0.996922	0.997574	0.990348	0.979486	0.997553	0.990811	0.990003	0.995265	0.993502	0.993933	0.994608
1990	0.994534	0.995318	0.985454	0.971078	0.997512	0.993491	0.992862	0.99625	0.9954	0.994187	0.99643

The following statistics validate the accuracy of the results, see Table 2 defined as follows:

$$\begin{aligned}
AME &= \max_{1 \leq n \leq N} |x_n - y_n| \\
RAE &= \frac{\sum_{n=1}^N |x_n|}{\sum_{n=1}^N |y_n|} \\
ME &= \frac{1}{N} \sum_{n=1}^N x_n - y_n \\
MAE &= \frac{1}{N} \sum_{n=1}^N |x_n - y_n| \\
RMSE &= \sqrt{\frac{\sum_{n=1}^N (x_n - y_n)^2}{N}} \\
R^2 &= 1 - \frac{\sum_{n=1}^N (y_n - \hat{y}_n)^2}{\sum_{n=1}^N (y_n - \bar{y}_n)^2}
\end{aligned}$$

The above statistics were selected consistent with the use of statistics to evaluate hydrological flow models reported in [40] and [41]. Therefore, the aim is to minimize, Absolute Mean Error, Relative Absolute Error (RAE), Mean Error (ME), Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE) and R squared (R^2). RSME is the statistic used during training to evaluate the particle fitness over the whole dataset. To evaluate that the multiple linear regression (MLR) results and the results produced by our model were in the same distribution, the Kolmogorov–Smirnov (K–S) statistic is used and is discussed in the following section. An important point to note is that R^2 is only used to evaluate the accuracy of the predictions made by the model. It is not used as a training statistic.

6. DISCUSSION AND RESULTS

The available data from the hydrographical readings were taken from the Plynlimon research [1]. A model was developed, trained on the data from each year and then tested on the remaining 10 years. Table 2 shows the test results for each matrix tested against the year. An important point to recognize is that Table 2 also shows the training results for the data given for the same year. These are included to demonstrate that the robustness is shown by test results being higher for years on which the data were not trained.

For completeness, the datasets were originally augmented by the dry bulb and rainfall data. The dry bulb and rainfall data were collected as external environmental measures. However, when modelling this led to inaccuracies in the predictability of the hydrographical flow through the River Severn in our model. The reduction in accuracy is probably due to the time lag between the effect of the air temperature and rainfall on the hydrographical flow and the actual hydrographical flow measured. Therefore, the final model concentrated on the hydrographical flow through the River Severn's tributaries ignoring weather and temperature data. For ease, the hydrographical flow readings for a given day are averaged to provide a daily figure. The calibration of the ratings from the Tanllwyth river was reported as problematic [39]. The problem with the calibration from the Tanllwyth river readings can be seen in Figure 7 in reference [39], where the discrepancies are characterized by a steady increase followed by a decline in flow for the Tanllwyth. The Tanllwyth is enclosed between the Hore and Hefren rivers in the catchment; thus erroneous rainfall measurements and leakage can be eliminated since this would also be seen in the Hore and Hefren rivers [39]. The discrepancies are more apparent in the period after the decade covered by these experiments.

FIGURE 4. Sample convergence to optima for 1988.

Although ratings from this river were problematic, it was likely that they remained useful in producing a model.

It is noted, the model is trained on a single year and the results are presented as tested on the remaining 10 years. By doing this we are able to show that the model is robust. It shows that the matrix derived from a single year's training data maintains the predictability across the other 10 years which were not included in the training. It is also important to recognize that the data collected for each year were not continuous over a 24 hour period. Therefore, it would be incorrect to assume that the datasets should be treated as a single contiguous dataset; the average daily values are therefore used. Table 2 shows the results for the test data on each year. This demonstrates that by training on a year's worth of data predictions for other years are robust.

Figure 4 shows the plot of the converging swarm for 1982; this is included to show the typical convergent behaviour of a swarm. The figure shows that the quality of the solutions for a given particle oscillate, before converging on the optimal value; this is a strength of the PSO paradigm allowing a particle to 'fly' past its previous best to explore farther regions of the solution space.

In Table 2, the columns indicate the years, whereas the rows indicate the matrix used. The best performing matrix, the minimum total error, for each year is shown as white text on a black background. It is interesting to note that the matrix produced for a given year is not necessarily the one to produce the best result for that year, for example 1980 and 1989 do not perform the best for any year. The matrix produced using the 1981 data, for example, produces the best results for 1981 and also 1980. This is also true for 1989, which is outperformed by the 1988 matrix. It is important to recognize that the model has no knowledge of the data in terms of its origin, characteristics or accuracy. Therefore, the finding that a model developed from the data from one year produces better results than the native model for that year invites the conclusion that the river system included in the catchment area is very stable. This also suggests that using a hierarchical swarm model and sharing sample

solutions, between swarms, during a run would be beneficial, see below.

Table 3 also shows the matrix that produced the R^2 values for each year. As already identified an interesting fact is that certain years do not seem to produce the best performing matrix for that year. It is likely that a climate event or other environmental event affects the data during that year, which stabilizes in subsequent years' data. Whilst there are probably a number of factors that could contribute to the change seen in a given year. The point of interest here is that the catchment appears inherently stable. Allowing for matrices developed for another year to be used to more accurately predict a year on which a less accurate model was developed. As further verification, the experiment was run twice more, on the 1980 data to see if the matrix produced improved results. It did not, either for 1980 or any other year, leading to the conclusion that this is due to characteristics of the data for that year, which are yet to be identified.

Table 4 presents the statistical analysis of the accuracy of each model for a given year. The table shows that 1983 and 1986 are years in which the flow is less predictable than in other years. This equates to the factors in the matrix not reducing to the same extent and therefore producing a less good fit. Given the repeatability of this result, we ran the experiments five times for each year, a tentative conclusion can be drawn. This conclusion is that in 1983 and 1986. There are factors that affected the hydrographical flow that are not represented in the data collected on which the model is produced. At issue is the effect of deforestation.

The size of the matrix by which the vector is multiplied is mainly due to the historical development of the model. Originally, the dry bulb value and the rainfall data were included providing five input values. As already stated the number of rows represents the number of variables available to map a single value in the result vector, initially this was set as a 5×5 matrix. As with any regression, the number of variables has an influence on the achievable accuracy. When the model was reduced to the tributaries only, we initially ended up with a

TABLE 3. The 5×3 matrices used to produce the results shown above in Table 2.

Swarm generated matrices used to produce R^2 values above								
1980	1981			1982				
0.9020003	0.2258762	-0.0854694	-0.1388691	0.2967863	1.0017402	0.8321009	0.4835061	-0.0701966
1.0944389	-0.5518632	0.5336288	0.6853794	0.2295014	0.354013	-0.5671766	0.7972709	0.7893756
0.6702101	0.7906727	0.2933398	1.0373192	0.3912027	-0.7237264	0.4516507	-0.0098882	0.9244932
0.0082921	0.5325794	-0.1875138	-0.4901823	0.7174507	0.5141762	-0.5146601	3.0009955	-0.9377654
0.7079931	1.1027036	1.0074612	0.7120208	0.1442158	-0.1036047	-1.1271209	0.8984909	1.0875029
1983	1984			1985				
-0.4299736	1.2318652	0.0471777	0.95477	0.2355716	-0.0758961	-0.2964126	0.9906163	0.3738213
0.8380171	0.1159662	0.8155107	-0.5855584	1.4229962	0.3453872	0.292582	0.8056453	0.2099769
0.1139792	0.2956568	-0.3303402	0.9850552	-0.0156539	-0.0221437	0.3378603	-1.2356507	1.4281871
0.3916767	1.1350075	0.2977395	0.9469464	1.0458464	-0.4724834	0.979089	0.7522917	-1.0892461
-0.6509684	0.378837	-0.1824708	0.548535	0.2127135	1.3791959	-0.3223854	0.7779104	0.3959591
1986	1987			1988				
0.5409889	0.0436789	0.7458592	0.1372002	0.791111	0.2502757	0.4302483	0.7864755	0.1063979
-0.7763194	0.61663	0.8696689	0.8805642	0.1514472	-0.4326657	-1.0734431	0.7203413	1.2172224
1.3573117	-0.0058835	0.2561779	0.5096498	1.059988	0.9564073	-0.1009305	0.679235	0.8737182
-0.0365886	-0.2661736	0.6693748	0.9892667	-0.0498011	-1.7321703	-0.5291489	0.38397	0.2241048
-0.0107959	1.5350743	0.2345025	-0.4052139	-1.2543292	0.3359835	0.9612146	0.3265471	1.0282882
1989	1990							
0.4302483	0.7864755	0.1063979	0.5149596	0.3714018	0.4912907			
-1.0734431	0.7203413	1.2172224	0.2834194	-0.0544639	0.4420213			
-0.1009305	0.679235	0.8737182	-0.0468538	0.7184712	0.9660215			
-0.5291489	0.38397	0.2241048	1.2425462	0.1658419	0.1055337			
0.9612146	0.3265471	1.0282882	-0.0699174	1.2824912	0.7892265			

5×3 matrix; later reduced to a 3×3 , to improve computational efficiency, resulted in an acceptable accuracy loss at the 0.001 scale. As with any model, it's a matter of trading between accuracy and efficiency. Of course, once the matrix for a given year has been found it is possible to combine the rows into a single weighting factor for each tributary. However, during the model development, it is more useful to have more degrees of freedom in order to find the weighting factors. The matrix size determines the degrees of freedom available to the model; experimentation shows an average reduction in performance of 0.7% between the 5×3 and 3×3 matrices, respectively.

Based on the data available, in our view, it is not possible to attribute any significance to the values in the matrix in terms of a physical explanation of the landscape. However, the values do model environmental factors such as the lag and evaporation, together with local climate influences that are inherent in the catchment system. Although, the results presented in this paper use daily averaged flows and therefore, probably, do not model daily lag. The run-off, particularly resulting from rainfall, is represented. Indeed this may be a contributory factor in the reduction of model accuracy when the rainfall and dry bulb data were included, due to the time lag before the effects of evaporation and run-off enters the

watercourse. The issue of lag is important here; it represents the time it takes for environmental factors such as evaporation and rainfall to affect the river flow. If we were to omit the potential for lag from the model we would be modelling the data as recorded on the day, and potentially lose the strength of the model to predict future river flows. The experiments produce the most accurate predictions, using data not included in the experimental training run, exploiting data recorded in the tributaries 2 days before the flow recorded in the River Severn. Although we cannot say that this lag corresponds to environmental conditions. It does represent a time period over which precipitation, run-off, and other environmental factors affecting the catchment affect the measured flow in the River Severn.

Further, we completed a further five experimental runs and found these results to be repeatable, which indicates the possibility of hidden features in the data. The discovery that a model trained on one dataset would perform worse than a model trained on a second dataset when that model is applied to the first dataset further attests to the stability of the catchment, see Table 2. In model terms, this implies that a local minima is being arrived at, which is not in existence in another year's data. This, in turn, leads to differences in the matrices developed for each year.

TABLE 4. Sample per year statistics from an experiment run on the 11 years of data.

	1980			1981			1982			1983			1984			1985		
Absolute Maximum Error		0.331357241		Absolute Maximum Error		0.298768385	Absolute Maximum Error		0.530524	Absolute Maximum Error		0.553257126	Absolute Maximum Error		0.202459984	Absolute Maximum Error		0.215174
MAE		0.02553114		MAE		0.022345887	MAE		0.028773	MAE		0.035123682	MAE		0.015817037	MAE		0.022603
Mean Error		-0.00279994		Mean Error		0.000160084	Mean Error		-0.00137	Mean Error		0.009874805	Mean Error		0.001755782	Mean Error		0.000715
RAE		0.055196402		RAE		0.042076961	RAE		0.066144	RAE		0.066523363	RAE		0.03774992	RAE		0.0595
RMSE		0.820477645		RMSE		0.799976339	RMSE		1.154756	RMSE		1.580000352	RMSE		0.540903214	RMSE		0.716143
1986				1987			1988			1989			1990					
Absolute Maximum Error		1.079355482		Absolute Maximum Error		0.426909269	Absolute Maximum Error		0.382871	Absolute Maximum Error		0.585613975	Absolute Maximum Error		0.36657			
MAE		0.026416795		MAE		0.019454292	MAE		0.0305	MAE		0.029682618	MAE		0.022984			
Mean Error		-0.00168795		Mean Error		-0.00020902	Mean Error		0.006351	Mean Error		0.005498083	Mean Error		0.003258			
RAE		0.050222136		RAE		0.04608796	RAE		0.068921	RAE		0.062250191	RAE		0.047359			
RMSE		1.418531229		RMSE		0.766620913	RMSE		0.913229	RMSE		1.176722403	RMSE		0.825732			

TABLE 5. K–S River Severn results summary.

Year	K–S statistic	Critical value	Different? (95% confidence)	PSO	MLR
1980	0.112022	0.191379	Same	0.997	0.998
1981	0.123288	0.191379	Same	0.998	0.998
1982	0.134247	0.191379	Same	0.992	0.992
1983	0.142466	0.191379	Same	0.99	0.991
1984	0.155738	0.191379	Same	0.998	0.998
1985	0.021918	0.191379	Same	0.995	0.995
1986	0.29863	0.191379	Different	0.994	0.994
1987	0.060274	0.191379	Same	0.996	0.996
1988	0.180328	0.191379	Same	0.996	0.996
1989	0.320548	0.191379	Different	0.995	0.995
1990	0.128767	0.191379	Same	0.997	0.997

It is important to recognize that the model is a series of multiplications of weighting factors of the input vector, a form of regression. These are then summed in order to give the predicted flow. Therefore, it is not possible to attribute any direct meaning of the weight factor in the matrix to a particular feature in the landscape. However, the consistent nature of the values in the matrix provides an indication that the catchment area considered is relatively stable, and from this it is possible to conclude that where predictions for a particular year are less predictable than in previous years, the matrix values tend to indicate that an event has occurred, which is not represented in the input data. For example, as stated earlier, the effect of deforestation or excessive rainfall on the higher levels of the catchment is covered in more detail in [39].

The initial model has been successful at predicting hydrographical flow in the River Severn and compares well computationally with [5] and [4], both of which implement PSO-based models. It is intended that the model should be further developed to enhance its applicability to a wider time range. This work will include a hierarchical swarm implementation, currently in development, enabling the data for each year to be considered collectively per iteration effectively introducing additional weighting factors. A hierarchical swarm, for example, would allow the best solution for each year to be shared per iteration between the other years, therefore disseminating information whilst at the same time isolating the data for a given year. Because Table 2 shows a degree of cross optimization between the years when the experiment is run as a single swarm. A two-layer hierarchical swarm of the model suggested above is likely to lead to faster and possibly more accurate convergence.

6.1. Predicting floods

The data presented above show an R^2 value of about 0.98 for the entire group of datasets. Q95 refers to the proba-

bility that a river's flow is exceeded 95% of the time and therefore measuring the top 5% of flow, where flood events occur. In order to validate the peak flow prediction values, the model was rerun using a dataset consisting of only the Q95 values.

The data were divided into training and test sets. Interestingly, and encouragingly, the model maintains an R^2 value of about 0.958 for each of the test datasets modelled. We, therefore, reproduced our own R^2 model to verify this result. Using the Q 95 data for $t-2$, we were able to verify that they are at least as good as the multilinear regression model with an R^2 value of 0.93. This is consistent and indicates that our model produces a sustainable improvement over MLR, the current best performer, even for the Q 95 data which in turn improves the ability to predict flood events. In summary, our model provides an improvement of 0.02 over a MLR on the training data, a statistic that was subsequently maintained on the test data. Importantly in terms of computational complexity, these results were produced within approximately 300 iterations; with reference to Table 1, this gives us a total number of valuations of 12 000. Which compares well with the efficiency of the model presented in [5] and computationally, our model compares very well with [4]. We should reiterate here that, unlike papers such as [42] and [6] we are not trying to model the system but make predictions about future flow from a model derived from achieving a goodness of fit, which includes lag.

As a final validation check, we compared the distribution of residuals (errors) from predictions made by MLR and the PSO predictors. An R^2 test on the residuals gave a result of 0.99, indicating that the residuals from both predictors were highly similar. We used a K–S test to quantify this intuition. The K–S test is useful for quantifying if a set of values is from the same distribution, and therefore whether or not they are reasonable predictors. If the distribution of residuals from both predictors was similar, we could say that the two predictors were behaving similarly. The K–S test is used to determine if two distributions

are significantly different. We considered using Student's t -test, but this test assumes that the distributions being compared are approximately normal, and the distributions generated by the predictors are not normally distributed. Table 5 shows the K-S results for the River Severn data.

Following best practice for the K-S statistic, we separated the residuals into equally sized buckets [43]. The results show that the residuals are drawn from the same distribution and therefore support the R^2 statistic analysis that PSO performs similarly to MLR on the River Severn data. MLR is the best performing comparative statistic on this River Severn dataset.

6.2. Extending the model

The model demonstrates successful 3 day modelling of the River Severn's flow, which is demonstrated to be robust over an 11 year period. This suggests, as indicated above, the catchment of the River Severn is fairly stable and is destabilized by events such as weather, climate and deforestation. These events are currently not captured by the model; one way to address this is to extend the model to include more than one mathematical transformation. However, in order to make this more robust across more than one river catchment obtaining a larger hydrological data set would be necessary. This is because the existing dataset models extremely well with a single transformation, and therefore models very specifically to the River Severn data. Unfortunately, following communication with the Centre for Ecology and Hydrology, the River Severn catchment data are unreliable outside of the data sampled. This is a useful outcome for the River Severn but does not allow, in itself, the model to be extrapolated to other rivers. To do this would require retraining the model on the other river. A useful goal would be to develop a model that can be applied in near real-time to other rivers with more complex catchment characteristics. The aim would be for the model to learn a given catchment with minimal adaptation. The principles outlined in this paper are more straightforward to implement than an ANN and more easily adapted to different datasets. We refer to the way in which there are no assumptions about how the model is applied to the data in the implementation of the model, as indicated. Further research is underway to develop the model so that it can be applied to a wider range of catchments and provide a way of objectively comparing hydrographical flow in different catchments in the context of the model representing each catchment.

7. CONCLUSION

The aim of this paper is to show that PSO can be used to accurately model the hydrological flows of the River Severn from historical data; promising results are demonstrated with an R^2 of 0.98 or better, providing a 0.03 improvement over the MLR score—the previous best performing method. We

demonstrated, through the use of the R^2 statistic results, that our method outperforms previous results presented by [2]. The key to this model was the abstraction of the search space from the solution space. We achieve this through the use of a matrix of weights, which is then multiplied by the input vector of previous flow values. In particular, we draw the reader's attention to the results in Section 6.1 of this paper which demonstrate a significant improvement in the ability to predict flood within the River Severn catchment. We also demonstrated that our model was able to find matrices capable of making accurate predictions on different datasets. This finding supports our hypothesis that abstracting the search space from the data space is of benefit when using PSO. We were able to show that the accuracy of our predictions held for $t+1$ and $t+2$, improving on the results of other models such as that of [5]. These results were achieved using data collected from 2 days prior to the predicted event. Therefore, the predictions could be used to take remedial actions before the event occurred, in contrast to, for example [44, 45].

The model also demonstrates that by taking a relatively small training set, a year's worth of data. It is possible to produce a model that then predicts River Severn flows for other years. This is a significant advantage to the model and allows accurate predictions to be made in subsequent years from a relatively small training set. The usual approach would be to have a larger training dataset and a smaller dataset from which you make the predictions. However, this has the disadvantage that you need to include all possibilities in the training set. The model presented here demonstrates that the River Severn catchment is fairly stable, which is to our advantage, but also demonstrates that a simpler model is possible. Although further work is required, it should be possible to achieve similar results on more complex datasets, predicted by the collage theorem [46]. The additional benefits of the model presented above are that it is relatively easy to implement and produces accurate predictions of the flow rate from historical data and within approximately 300 iterations, providing an advantage in terms of processing time. Whilst there is always a trade-off in computational time and the number of samples modelled. The computational complexity involved in our implementation is also more straightforward than the neural net model in [3] and does not rely on pre-processing of the data before it is input to the model, as in [6]. Our approach has the advantage of coping with noise more efficiently. In addition, the storage requirements and programming complexity are minimized. As a direct result, the model in its entirety is more straightforward in implementation and the outputs of which are easier to apply. Our model is able to produce an accurate prediction using data gathered from days ahead of the event.

In comparison with other techniques, our model performs well on accuracy and computational complexity. This is especially true when we consider models, which attempt to predict hydrographical flow for the future based on current

measurements. As mentioned above, our results reported an R^2 of 0.98. This compares well to the results reported in [47], for example. In addition, the relatively straightforward implementation means that the model can be adapted easily to new input data. As discussed, temperature data were omitted in order to get improved results. This is reasonable because the temperature reading on a given day does not directly reflect the effect that temperature has on the flow due to lag. This is probably due to the subsequent evaporation and convection effect on the water course over a different timescale that is reflected directly in the hydrographical flow inputs to the model. This contrasts with the performance reported in [42] and [48] using an ANN. It is interesting to note, however, that in the relatively recent review [40] cites relatively few applications of artificial intelligence applied to run-off, see Table 1 in [40]. A more recent review of artificial intelligence techniques applied to streamflow [41] provides a more extensive summary.

As indicated above, early experiments showed that including rainfall and temperature data directly made the model less accurate, although clearly rainfall is a factor in the river flows. A continuing focus of research is testing modifications to the model to more effectively surface extrema during a model run, allowing for unusual events to be identified. Anthropogenic climate change is widely recognized and increasingly, requires more dynamic, in terms of responsiveness to change, and robust modelling techniques. Further research will include the development of a more sophisticated model using more recent River Severn data [49]. We are also developing an extension to the model that allows more complex, noisier data to be considered. A comparison to the more recent ANN models, such as [50], [51] and [52] for example, will be made in order to test the robustness.

Ongoing research builds on this model using bigger datasets that have been made available through the use of greater computing resources. The purpose of this paper is to present a proof of concept of a relatively computationally inexpensive technique that shows good results. As stated earlier, the River Severn catchment appears to be particularly stable, which provides an advantage for modelling, but is not necessarily typical. At present, our model is limited to a single mathematical transformation in order to perform the predictions. It is envisaged that if there is a more complex, noisier, catchment area this would be insufficient. We are therefore working on a model that will extend the responsiveness of the model by allowing for multiple transformations to be used. In closing, we observe that [24] argues for the notion of 'physical legitimacy' for meaningful comparisons to be made between differing ANN models. An interesting goal that we will also consider furthering future work. Conventional computational thinking would hold that the weights in any mathematical model do not have a meaningful relationship to the physical environment. Clearly, the notion of physical legitimacy presents a challenge to the machine learning community.

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