Can changes in my practice have a positive impact on how my pupils solve problems in mathematics?

Thesis

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Can changes in my practice have a positive impact on how my pupils solve problems in mathematics?

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Abstract

This Action Research came about as a result of my concerns with the difficulties my young pupils experienced when applying their mathematical skills in other contexts. Initially my research questions focused on pupils learning and asked if problem based learning could improve thinking skills in mathematics, if collaborative learning was a more effective approach than working independently and what strategies were used by pupils when solving problems.

My focus changed from examining my pupils’ learning to examining my practice and as a result, four action-reflection cycles were carried out in order to bring about changes in my practice. Further key questions focused on the extent to which I used higher order questions to extend pupils’ learning in mathematics and asked if increasing the number of higher order questions had an impact on their learning. I also examined the effect changing my practice had on the strategies pupils used to solve problems and the extent to which it enabled them to engage in discussions while doing so. My final research question focused in particular on generalising and the opportunities I provided for my pupils to specialise and generalise in mathematics lessons.

My research found that when my very young pupils were given an appropriately challenging problem, they could engage in collaborative dialogue with the purpose of solving that problem. Evidence from this research also demonstrates how some very young pupils have the ability to make general statements about mathematics when given a rich mathematics task and challenging questions. However, there is a gap in the literature relating to how very young pupils generalise and specialise in mathematics and my research makes a contribution towards filling this gap as it describes how I enabled my pupils to get to this stage. It also contributes to the body of knowledge available on
action research as it describes how my research journey travels alongside my mathematical journey. Although it may not be generalizable in the wider context it shows what can be achieved with action research.
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Structure of the Thesis

This thesis is an Action Research report on how I reflected on, and changed, my practice in an effort to improve my pupils' learning in mathematics. It is written over eight chapters and consists of four cycles of action and reflection carried out over a three year period.

The first chapter describes the context and focus of the research which took place in a two-form entry infant school in an outer London Borough. At the time of the research my pupils were experiencing difficulty with problem-solving in mathematics which is why I became interested in researching this area. Chapter one also looks at problem-solving in the National Numeracy Strategy (DfEE, 1999) and what the Leverhulme Numeracy Research Programme 1997-2003 (Brown, Askew, Millett, and Rhodes, 2003) had to say about it.

In chapter two the literature in three important areas is reviewed. The first of these areas is problem-solving in mathematics and some of the works of Boaler (1998, 2009), Nunes and Bryant (1998) and Skemp (1989) are used to establish the importance of problem-solving in developing pupils' mathematical ability. The second area to be reviewed is that of collaborative versus independent learning. The literature in this field strongly supports collaborative learning as a more effective way of learning and includes work by researchers such as Bennett and Cass (1989) and Vygotsky (1978) to name but a few. The third area explores the research literature on the strategies pupils use to solve problems and includes work by experts such as Burton, Mason and Polya.

In chapter three, I describe the methods I used to collect and analyse the data and it includes an explanation of why it changed from Practitioner Research to Action Research. The literature review includes work by noted action researchers such as McMahon (1999)
and McNiff and Whitehead (2006) and stresses the importance of strategic action to bring about changes to one’s practice.

In chapter four I discuss the first action research cycle involving reflection on my practice. The literature review focuses in particular on Mason’s (2002b) *Researching Your Own Practice: The discipline of noticing* and Schön’s (1995) *The Reflective Practitioner* and how they influenced me and led to my becoming a more reflective practitioner.

In chapter five I describe the second action reflection cycle where I investigated the questions I used in my practice and the extent to which I used higher order questions. Research by Boaler and Brodie (2004), Gall (1970), Khisty and Chval (2002) and Mason (2002a) for example, highlights the importance of asking appropriate and challenging questions in order to develop pupils’ thinking skills and also of allowing them enough time to respond.

A third cycle of action and reflection is described in chapter six and focuses on the strategies pupils use when solving mathematics tasks. The literature describes a range of strategies from simple counting to more advanced methods of specialising and generalising and includes work by researchers such as Anghileri (1989), Carpenter and Moser (1984) and Houssart and Evens (2005).

Chapter seven describes in more detail the strategies of specialising and generalising and looks at the extent to which my pupils engage in these strategies. The literature reviewed in this area includes researchers such as Kaput and Blanton (1999), Mason, Drury and Bills (2007) and Warren (2004) and stresses the importance of providing pupils with appropriate tasks which will enable them to make generalisations. It also highlights the
link between generalising and algebra and the need for algebra to be introduced in the early years.

The final chapter summarises the main points made in the thesis and how my practice changed and highlights the need for more research into the learning of younger pupils in mathematics.
Chapter 1 - Introduction

Context of the Research

The research took place in a two-form entry Infant school with a Nursery attached. The school is located in an outer London borough in an area considered to have high social deprivation. The main research data were collected from a Year 2 class of approximately 27 pupils during the academic year 2009-2010, while data for the Initial Study came from the Year 2 class of the academic year 2007-2008. Two further lessons were recorded at the beginning of academic year 2011-2012. The data were collected in the classroom for which I am the teacher.

This research arose out of my interest in problem-solving which I wished to investigate further. I was keen to consider any new and innovative strategies that would maximize the effectiveness of my teaching and further enhance my pupils' learning. I therefore embarked on a doctorate in education as a means of developing my professional practice and broadening my knowledge and understanding of this aspect of mathematics education.

Focus of the Research

The National Numeracy Strategy (DfEE, 1999), which came into being in 1999, stressed the need for more pace in mathematics lessons. This resulted in mathematics topics being visited for short periods of time when pupils were taught the relevant skills for that particular topic. The rate of change of topic and the brisk pace meant that pupils had a greater workload to cover in each lesson. This, in my experience, was leading to shallow, passive learning where our pupils were acquiring knowledge on the surface but not
retaining it for any length of time. They were not being given the time to assimilate this knowledge on a deeper level and consequently they found it difficult to use and apply their skills in problem solving situations. Evidence from the Leverhulme Numeracy Research Programme 1997-2003 (Brown, Askew, Millett, and Rhodes, 2003) claimed that although there had been an improvement in pupils’ mental calculation strategies following the implementation of the National Numeracy Strategy (NNS), there had been a decline in their ability to use these strategies to solve problems in context. In my experience, the focus of mathematics teaching, as a result of the NNS, was more on the skills to be learned rather than how to apply them. Our pupils were finding it increasingly difficult to solve simple problems like these.

Find pairs of numbers with a difference of 3.

How many numbers can you make using the digits 1, 3 and 4?

In 2005, OfSTED (OfSTED, 2005) published its evaluation of the impact of the National Strategies on the primary curriculum and found the using and applying strand of mathematics to be the weakest. This resulted in the new Primary National Strategy: Primary Framework for literacy and mathematics which was introduced in 2006. Greater emphasis was put on using and applying and objectives were made ‘more explicit’ (DfES, 2006, p67) in an effort to improve pupils’ use of their skills. Problem solving was no longer to be regarded as an activity to be practised at the end of a topic but more as a means of extending learning. In my school, these changes in emphasis had no impact on the teaching of problem-solving. Although I can only speak in terms of my school, I would be very surprised if we were alone in our experience.
On a personal level I was more concerned about the learning taking place in my classroom. Other areas of the curriculum did not give me such cause for concern as introductions to lessons, for example History, Science or Design & Technology, always elicited exclamations such as 'yes!' or 'yippee!' from the class. My pupils could not wait to get involved in whatever activity or task was being discussed, and wished to share their ideas with their partners. This did not often happen in mathematics lessons and although when asked, my pupils would say that they liked maths, there was not the same spark of engagement that was so evident in other lessons.

I wanted my pupils to get the same enjoyment and learning experience from mathematics as they did from other subjects and I believed that this would be possible if a more problem based approach was adopted. I tried this idea in one of my lessons with my Year 2 class early in the academic year of 2007-2008, where I introduced money by asking the pupils to work in pairs to solve a particular problem. They were given small white-boards on which to record their working out so they did not have to worry about making mistakes in their books. Their attitude was considerably different. They were busy chatting and drawing different ways of finding the answer which, for many, were inappropriate and unhelpful methods but they were engaged and making attempts. My reflections on this lesson included the note, 'This is how a maths lessons should be'. I wanted to achieve this level of interaction in all of my mathematics lessons and I believed that a problem based approach could provide this.

I was keen to investigate this further and my research focused on the following questions.

1. What strategies do young pupils use when solving problems?

2. Can problem based learning improve thinking skills in mathematics?
3. Is collaborative learning a more effective approach to problem solving than working independently?

In order to be able to address the research questions I had to find out what was being said about these specific areas so I undertook a review of the relevant literature and the research that had been carried out on these topics, which is discussed in the following chapter. Further research questions arose during the research process and they, along with further literature reviews, will be discussed in subsequent chapters.

I carried out an Initial Study (Appendix 1), the focus of which was to test methods of data collection, to trial some problem solving tasks with my pupils and to see how they worked with other pupils. Although there was evidence that some pupils were able to work collaboratively when given appropriate tasks, it was also clear that working co-operatively did not come naturally to them. I wanted to change the learning environment of the classroom to one of collaboration, where pupils would be encouraged to discuss and share their ideas and be able to reason with and explain their strategies to others. I am aware that distinctions can be made between co-operative learning as being more teacher directed and collaborative learning as being more pupil-driven, as outlined by Panitz (1997). In this research, I use the terms collaboration and co-operation interchangeably with the knowledge that although collaborative learning is the ultimate goal, young pupils need to learn to co-operate with each other before being able to collaborate in the sense described by Panitz (1997).

I had not expected that the initial study would cause me to reflect on my practice and to 'notice', as Mason (2002b) described, aspects of it that needed to change before I could expect improvements to occur in my pupils' learning. As a result of this initial reflection, I realised that Action Research would enable me to engage in a focused cycle of reflection
and action. This is explained in detail in Chapter 3 on Methodology. Figure 1.1 below summarises the four cycles of my research, with the fourth one focusing solely on generalising. The literature review that follows in Chapter 2 does not include research on changes to practice. This is addressed in the literature reviews that accompany each action research cycle.
Figure 1.1: The Four Cycles of Action Research
Chapter 2 - Literature Review

Introduction

I carried out this literature review in order to establish a theoretical basis for my research, explore the findings of previous research in the field and to identify the gap that my research might fill. The literature review included here examines research that is significant in:

- problem solving in mathematics;
- collaborative learning versus working individually; and
- strategies used by children to solve problems.

Problem Solving in Mathematics

According to Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Werne (1999), the debate between acquiring knowledge and applying it is one that has been in existence since the early 20th century and has not yet been resolved. The National Curriculum, in its present format (1999-2013), considers the skill of problem solving to be an integral part of learning and describes mathematics as a ‘creative discipline’ stimulating ‘moments of pleasure and wonder when a pupil solves a problem for the first time, discovers a more elegant solution to that problem, or suddenly sees hidden connections’ (DfEE/QCA, 1999, p60). Included in the document are quotes from several mathematicians supporting this view and describing the enjoyment they experience from such ‘intellectual challenges’ (ibid, p61).
The National Curriculum document described the inception of the NNS as being ‘put in place to raise standards in all primary schools in England’ and claimed that if schools followed the NNS completely they would ‘fulfil their statutory duty in relation to the National Curriculum subject of mathematics for Key Stages 1 and 2’ (DfEE/QCA, 1999, p23). Although it was not mandatory for schools to implement the NNS, most did so (Askew, Millett, Brown, Rhodes and Bibby, 2001). The teachers in the school in which I work implemented the NNS and found it an advantage to have the learning objectives given for each half term as well as examples of what pupils were expected to do. However we found that the pace at which we were expected to cover these objectives was a definite disadvantage. This was particularly true with regard to the lower attaining pupils who were not given time to grasp one idea before being moved on to something else. We were not alone in our experience, as the results of the Leverhulme Project demonstrated (Brown, Askew, Millett and Rhodes, 2003). Brown et al (2003) reported how schools in their research also experienced a decline in the progress of their low attainers and had difficulties with the pace at which they had to work.

Another disadvantage of the NNS was that problem solving appeared to be tagged on at the end of a mathematics topic rather than being central to it. The Primary Framework (2006) on the other hand believed that solving problems lay ‘at the heart of mathematics’ and ‘should be integrated into mathematics teaching and learning’ (DfES [online], 2006, p9). It was more in favour of ‘embedding’ problem solving into regular lessons than having it as a ‘Friday-only’ activity’ (DfES [online], 2006, p9). This crucial point, however, had not been highlighted by subject leaders in our Local Authority or passed on to our staff following curriculum meetings. In my opinion there appeared to be a contradiction in what the National Curriculum and the Primary Framework promoted in theory and what was actually happening in the classroom. The then Secretary of State, Ed Balls,
called for a review of the primary mathematics curriculum, resulting in the Williams Review (2008), which emphasised using and applying, problem solving through dialogue and strong CPD as being important for learning in mathematics. As a result of a change of government in 2010 a further review of education was undertaken, the results of which are due to be introduced in schools in 2014 as a new curriculum.

The Primary Framework reflected the views of many educationalists including Boud and Feletti (1997) who stated that problem-based learning was not simply:

...the addition of problem-solving activities to otherwise discipline-centred curricula, but a way of conceiving of the curriculum which is centred upon key problems... (Boud & Feletti, 1997 p2).

This is no easy task. To adopt a problem based approach to any area of the curriculum necessitates a complete change in focus for those involved in its instruction especially teachers. Williams (1999) referred to research from the QUASAR project (Williams and Baxter, 1996), in which a problem based approach was used in the teaching of mathematics at middle school level in the USA. He discussed the positive and negative aspects which were synonymous with this approach to learning. On the positive side, there was an increase in both the quantity and quality of pupil talk as pupils worked on problems and shared their ideas in what was a 'discourse-oriented' style of teaching (Williams, 1999 p212). They discussed their solutions and questioned each other in an effort to construct mathematical learning, with the teacher becoming less involved in their activities. One of the negative aspects of this type of teaching was that pupils came to know what was expected by the teacher in discussions and group work and performed these activities because it was expected behaviour rather than because of the benefits to their learning.
As a result of personal experience and some initial reading I decided to explore the possibilities of changing my approach to teaching mathematics from being skills based to focusing more on problem solving. A skills based approach emphasises the teaching of procedural skills and pupils are then taught how to apply these procedures in various situations. A problem based approach begins with a given problem and looks at ways of solving it. It is through the process of solving these everyday problems that the mathematical skills are taught. These skills are therefore taught in context and pupils, in my opinion, might gain a greater understanding of how they are used. I believed that a problem based approach might give my pupils the opportunities to think and to work things out for themselves before being introduced to formal methods. This could then enable them to deal with questions which are presented in a different manner and thus lead to more ‘intelligent learning’ in mathematics, as advocated by Skemp (1989).

Skemp believed that there were two forms of mathematical understanding — instrumental understanding and relational understanding. Instrumental understanding is not really understanding in the truest sense of the word but more ‘doing’ by rules and procedures. Relational understanding, on the other hand, involves not just knowing how to do something but more importantly why it is being done. It is concerned with relationships between ideas and concepts and building on these concepts to create more abstract knowledge structures or ‘schemas’ (Skemp 1989). This is what Skemp referred to as ‘intelligent learning’ and a person who learns in this way is dealing with their knowledge on a much deeper level. The idea of this deeper more intelligent learning was what interested me and it was the path along which I wished to take my young learners because I believed they would develop skills that would enable them to deal with mathematics problems in a more flexible way.
The type of problem solving approach as described above was reported by Boaler (1998) in her research in secondary schools on 'alternative approaches to teaching, learning and assessing mathematics'. In this research she described two contrasting models for teaching mathematics employed by two secondary schools with similar demographic catchments, Amber Hill and Phoenix Park. At Amber Hill a more traditional method of teaching was employed, where students were taught procedures then practised them, working diligently through their textbooks. Phoenix Park School adopted a different approach where students were given a problem to work on for a few weeks and when new procedures were needed they were taught in context. The more problem based approach of the second school seemed to result in more effective learning and greater subject retention by students. The fact that procedures were taught in context gave them greater meaning for the students.

Forman and Ansell (2001) described a similar approach to the teaching of mathematics in a third grade classroom of pupils aged 8-9 years, where the teacher did not believe in teaching procedures as methods for problems but preferred pupils to work them out using their own strategies. She felt that this problem based approach was a more positive way of teaching mathematics and that pupils should not be taught procedures until they understood how to use them. Although I agree with the teacher’s view that pupils, particularly very young learners should be encouraged to use problem solving strategies which they find most effective I do not believe that one should avoid the teaching of more efficient strategies. I believe that the teaching of certain procedures is necessary and introducing them through problem-solving, in a way that is meaningful for the learner, is a valid approach.
Although the students involved in both articles are older than the pupils with whom I work I feel this research can have relevance for my situation and as yet I have not found a great deal of literature that deals with mathematics problem solving in early years settings. There appears to be a gap in the literature when it comes to research on very young pupils’ learning in mathematics and I hope that my research will begin to address this gap. Nunes and Bryant (1998) carried out research into the teaching and learning of mathematics among younger learners. They debated the issues that affect young pupils’ understanding of mathematics and the reasons why they seem to be more successful when solving problems outside the classroom than they do within the school setting. This is due in part, they believe, to how pupils are taught, in the classroom, the strategies which they are expected to use. They stressed the importance of involving young pupils in problem-solving and the fact that they might need ‘visual representations’ in order to do this should not be a deterrent. Nunes and Bryant (1998) also stressed the importance of teachers recognising that pupils come with a certain amount of prior mathematical knowledge when being introduced to new topics. They believed that this knowledge, although incomplete, should be valued and acknowledged as legitimate mathematical learning. If this prior knowledge is ignored in favour of knowledge acquired or constructed within the classroom then it sends a message to pupils that classroom mathematics is more ‘worthy’ than what they have learned ‘outside’. According to Nunes and Bryant,

The classroom environment pushes them towards a definition of mathematics where the ways in which solutions are obtained takes precedence over their understanding (Nunes and Bryant, 1998 p247).
Some pupils in Nunes and Bryant's (1998) research set aside their mathematical reasoning in favour of 'school-taught algorithms', resulting in their being incorrectly labelled as failing mathematically. This is something that can be prevented Nunes and Bryant (1998, p248) believe, if pupils are allowed to 'bring their understanding from everyday life into the classroom'. I agree with this idea and believe it is echoed in the research by Forman and Ansell (2001) above, in which the teacher preferred pupils to work problems out using their own strategies, rather than focusing on set procedures.

Up until this point I had always believed that pupils should work independently when engaging in a mathematical task because it was the way I was taught mathematics both at school and while studying for a mathematics degree. However, since researching a more problem solving approach, this view changed and I started to consider the ways that my pupils might work together on mathematics. This collaborative approach to learning is explained in the next section.

Collaborative learning versus working individually

Most of the research available appears to promote collaborative learning as a more effective way of learning than individual. Williams (1999, p212) found that a more 'discourse-oriented' style of teaching was indicative of a constructivist approach to learning in mathematics where pupils were more engaged in their tasks. Research by Wegerif, Littleton, Dawes, Mercer and Rowe (2004) from their 'Thinking Together' project, demonstrated how using 'Exploratory Talk' not only had a positive effect on reasoning and interaction but also improved behaviour and the performance of pupils, particularly those labelled as low attaining. The findings of research by Rojas-Drummond and Mercer (2003) and Mercer and Sams (2006) supported this evidence and endorsed...
Vygotsky's (1978) view that when pupils work on a problem socially as a group their ability to think for themselves also develops and improves.

However, enabling pupils to work collaboratively may not be as easy as it might seem. According to Bennett and Cass (1989) the research available about group work in British schools showed that although pupils worked in group situations they did not actually work collaboratively. From personal experience, I would agree with this to some extent especially when talking about younger pupils. Just because six pupils sit around a table sharing the same resources and working on the same task does not necessarily mean that they are working together in a collaborative sense. I have seen 6-7 year olds supposedly working with a partner performing the same actions and writing the same answers without speaking to each other except for the odd comment such as, 'I'm on number 4, which one are you on?’. I came to believe that just getting pupils to talk is not enough. It is the quality and purpose of the talk which is important.

A significant amount of research in the field (Bennett and Cass, 1989; Dawes and Sams, 2004; Dekker, Elshout-Mohr and Wood, 2006; Goos, Galbraith and Renshaw, 2002; Pratt, 2006; Rojas-Drummond and Mercer, 2003 and Williamson, 2006) deals with junior or secondary school pupils and there is very little information available which examines collaboration among younger learners. This is not really surprising since collaborative learning is something which does not come naturally and needs to be actively encouraged. Since young pupils are still learning and developing many of their social skills the task of encouraging them to work constructively with a partner is even more challenging. Although it is true that they can work together and collaborate when engaging in playground activities many seem to experience difficulty in applying the same attitudes to their learning. Pupils need to be encouraged to share ideas, discuss strategies
and question and support each other throughout mathematical tasks. According to Mercer (1995, quoted in Rojas-Drummond and Mercer 2003):

...children are not commonly taught about ways of talking effectively together, or helped to develop specific dialogue strategies for thinking collaboratively (Rojas-Drummond and Mercer 2003, p102).

If pupils are not taught these dialogue strategies or ways of talking then much of their talk will not be of benefit and will instead be what is known as off-task talk (Bennett and Cass, 1989). Pupils consolidate their learning and improve their attainment when they discuss their work and explain the strategies used (Pijls, Dekker and Van Hout-Wolters, 2007). It shows that they have a good understanding of the concepts involved if they can explain their methods and reasoning to their partner (Webb 1991). This is not something they could do if they were working independently.

When assessing the performance of pupils working collaboratively, their achievement seemed to be dependent on the group in which they worked. Research by Bennett and Cass (1989), Wiegel (1998), Webb (1991) and Fawcett and Garton (2005) demonstrated that pupils who worked in mixed attainment groups with a more ‘expert’ partner showed significantly greater achievement as a result of working collaboratively than pupils who worked independently or with a partner of similar attainment. The reason for this is believed to be the level and frequency of the dialogue in which they were engaged. Lower attaining pupils who worked in these groups appeared to have made the greatest improvement, benefiting from the ‘expert’ knowledge, while high attainers did not appear to have put in as much effort as their peers.
These findings had implications for my research and how I organised my groups. At that time, pupils worked in groups of similar attainment, as assessed by me. According to the research, in order for them to achieve higher levels of attainment I would need to change these groups. However, I did not want the progress of the higher attainers to be adversely affected in the process. Contrary to this, the studies carried out by Bennett and Cass (1989) found that groups consisting solely of high attainers performed far better than all the other groups. With this in mind I reorganised my groups so that I now have two groups of higher attainers and three mixed attainment groups and pupils are regularly moved within and between these groups so that they have the opportunity to work with different partners.

Having considered the above research on collaborative learning it is not surprising that Qin, Johnson and Johnson (1995), when comparing co-operative and competitive efforts at problem solving, found the former to be the more effective way of learning. They stated that:

...the past research has found that cooperative efforts produce higher quality problem solving than do competitive efforts on a wide variety of problems that require different cognitive processes to solve (Qin et al, 1995, p139).

The reasons they gave for co-operative efforts being more effective were primarily because ideas are shared within the group and learners work together to develop a range of strategies which can be used to solve the problem. Competitive efforts on the other hand are more 'ego oriented' (Nicholls, Cobb, Wood Yackel and Patashnick 1990) where the individual is more concerned with competing against others and proving superiority over their peers. Pupils who learn collaboratively are more 'task oriented' (ibid.) and
motivated by a desire to complete the task and develop an understanding of the process involved.

Pupils need to be taught how to collaborate, according to Dawes and Sams (2004), and not just grouped together in the expectation that they will know how to work cooperatively. This is true in my opinion, and is what I experienced when I attempted to have my pupils collaborate on a task. Many did not seem to know what to do or where to begin when asked to share their ideas. It was clear to me that they would need some guidance if they were to be expected to work in this way. Chamot, Dale, O'Malley, and Spanos, (1992, p24) in their study of ESL pupils concluded that it was important to teach 'students how to become strategic in their approach to problem solving'. I believe that if pupils are to 'become strategic', as Chamot et al (1992) suggested, they should have a range of strategies to hand which they can use in order to begin solving a problem. My experience of teaching very young pupils has shown that this is not always the case and quite often they have difficulty getting started. In my opinion, it is important for pupils to be taught a range of strategies any one of which they can then use to solve a particular problem.

Polya (1957) stressed the importance of teachers guiding pupils in their development of problem solving skills by demonstrating these skills and providing pupils with opportunities to practice them. He described four phases which help to develop these problem solving skills. The following section outlines Polya's (1957) phases for solving problems and discusses some of the strategies used by young pupils to find solutions to problems.
Strategies pupils use to solve problems

Polya, in his book *How to solve it* (1957), explained the four phases which he believed were integral to effective problem solving.

1. Understanding the problem

2. Devising a plan

3. Carrying out the plan

4. Looking back

*Understanding the problem* involves reading the problem and understanding what is being asked. It involves knowing what information is relevant to the solution and any key words which will assist in reaching that solution.

*Devising a plan* involves searching for methods or heuristics that will help solve the problem. Using analogous problems may be helpful or using the same problem with smaller numbers may lead to a solution. Representing the problem visually with drawings or apparatus can also be an effective strategy. Fan and Zhu (2007) also included guessing and looking for patterns among the list of heuristics which can be used as ways of solving the problem.

The third phase concerns *carrying out the plan* and is when the methods devised in the plan are put into practice in order to find the solution. This phase also involves checking each step of the method and checking for errors. If a successful solution has not been reached then it may necessitate going back to the planning stage and starting again.
Looking back is the final phase of the problem solving process where calculations are checked and solutions are reviewed against the original problem.

I believe it can be difficult for very young pupils to follow these steps and some of my pupils have struggled to get past the first phase of understanding the problem. Many more omit the last phase of looking back and fail to recognise that their solution is incorrect because they have not reviewed it or checked that it works within the original problem. I agree with Polya’s (1957) point that teachers should demonstrate these phases and allow pupils to practise them so that they can be internalised and used independent of the teacher. Polya stressed the importance of teachers’ questioning as a means of scaffolding pupils learning until they are in a position to ask these questions of and by themselves.

Boaler (2009) described how 7th and 8th grade pupils, aged 12-13, who attended a summer school in America in an effort to improve their learning in mathematics, went about solving problems. Those pupils considered to be low-attaining were less likely to use Polya’s strategies than those considered to be high-attaining. They also worked in a less organised and systematic way and failed to check if their final solutions were correct. Boaler believed that it was worthwhile to teach pupils strategies ‘directly’ and also that the very best way to teach children helpful strategies is to provide interesting settings, problems and puzzles that require the strategies and then to share and discuss successful methods and strategies (Boaler, 2009, p186).

Research by Lee (1982) also found that some 4th grade pupils were more able to apply Polya’s phases after they had received intense instruction on how to do so. However teaching strategies alone is not sufficient to developing effective problem-solving. Boaler
(2009) described the negative effect a teacher had on their pupils' learning in mathematics when they expected them to work in total silence during lessons. The pupils in Boaler's (2009) research showed significant improvement in their problem-solving skills during the summer school not only because they were taught how to use Polya's phases but also because they were allowed to discuss and share their ideas. Unfortunately for most, this situation was reversed when they went back to their classrooms and to their teachers' ineffective teaching methods.

Polya's work on problem solving still has relevance for how we teach mathematics today and is quoted by many researchers including Chamot et al (1992), Fan and Zhu (2007), Gick (1986), Mason and Johnston-Wilder (2004) and Thom and Pirie (2002). Burton's (1984) phases of entry-attack-review-extension are along similar lines to those of Polya (1957). The entry phase is where pupils try to gain an understanding of the problem, what it is asking and the key information being given. The attack phase, like Polya's devising a plan, involves looking for patterns, attempting a simpler version of the problem or drawing visual representations in order to reach a solution. Burton's (1984) review phase is where the solution is discussed and checked and if incorrect then a return to the entry and attack phases is required until a solution is reached. It is linked with the final extension phase which explores the solution in greater detail and can generate other problems, resulting in new phases of entry, attack, review and extension. These phases are what Burton (1984, p41) described as 'affective responses' to the 'cognitive processes' central to mathematical thinking. In other words, pupils who engage in mathematical thinking are not only using their powers of reasoning but also their emotions when solving problems. The cognitive processes Burton was referring to are the processes of specialising, conjecturing, generalising and convincing.
Specialising is the process of testing specific examples, which leads to a conjecture being made when enough examples have been tried that some relationship or pattern is spotted. On recognising this pattern or relationship, the learner explores it further and makes a general statement for all cases, which then needs further testing so that they and others can be convinced of its authenticity. Mathematical thinking, according to Burton (1984, p36), is distinct from thinking about mathematics because ‘the operations on which it relies are mathematical operations’. It is, in Mason, Burton and Stacey’s (2010, p144) words, a dynamic process in which the learner expands their understanding because it allows them to increase the complexity of the ideas with which they are able to cope.

Figure 2.1 below shows Mason et al’s (2010, p144) representation of aspects of the dynamics of mathematical thinking, known as manipulating – getting a sense of – articulating, as a helix in which the ‘processes and the emotional states are linked together dynamically’ (Mason et al, 2010, p142).
When a learner starts working on a problem during the *entry* phase their curiosity is aroused and they *manipulate* ideas or objects in order to gain an understanding of what is involved. They may also try *specialising* with examples as a way of *attacking* the problem in order to *get a sense of* what the problem is about. Motivated to continue, they may engage in further phases of *attack* through *conjecturing* and *convincing* as they work towards a solution. This allows them to make a *generalisation* which can be *articulated* in any suitable way and *reviewed* against the original problem. The looped shape of the diagram allows for movement forwards and back through the stages as articulations, which can happen at any time, are reviewed and manipulated further until the final solution is reached. The emotional states run throughout the process as the learner is first *curious* during the entry phase then *motivated to continue* during the attack and review phases. Sometimes learners become 'STUCK', as Mason *et al* (2010) described, which can lead to frustration and giving up. The importance of being stuck, according to Mason *et al* (2010), is to recognise and acknowledge it as an unavoidable part of problem-solving and use it as a learning experience in the future.

However if pupils are not introduced to these problem-solving processes then their attempts to find solutions are more likely to be unsuccessful. De Corte and Verschaffel (1985) carried out research into the strategies used by elementary school children when solving numerical addition and subtraction problems and arithmetic word problems. They found that young pupils had little difficulty with the numerical problems. They did however have greater difficulty with problems such as:

\[ ? = 4 + 8 \text{ and } 5 = ? - 7. \]

It appears, this was due to a misunderstanding of the meaning of '='. I had similar experiences with my pupils when working on problems such as \[ 6 + 4 = ? + 2. \] A common
error was to write 10 as the unknown number indicating that they had added 6 and 4 but
had not engaged in Polya's (1957) looking back phase. Neither, it would seem, had they
engaged completely with the first phase of understanding the problem as they appeared
not to have fully understood what was being asked of them.

When solving the arithmetical word problems the pupils in De Corte and Verschaffel's
(1985) research appeared to have difficulties understanding the problems due to their
interpretations of the language used. The strategies used by pupils to solve these word
problems were their own informal methods of counting rather than formal addition and
subtraction and most of the errors made were due to using an incorrect strategy to find a
solution. The pupils in De Corte and Verschaffel's (1985) study did not appear to apply
any 'verification actions' to check their answers which, according to the researchers,
would have greatly reduced the number of incorrect answers. As well as not checking
their solutions, as Polya (1957) recommended, Boaler (2009, p180) found that a common
error among pupils is that they 'rush in and do something with the numbers without
really considering what is being asked of them'. This reinforces the point made that pupils
do not always understand what they are being asked to do when solving problems.

This highlights the importance of actually teaching pupils how to engage in the process of
solving problems whether they are word problems, logic problems or puzzles. Burton
(1995) claimed that

Problem solving cannot be taught. It happens in an environment where skills
which have already been acquired are exercised (Burton, 1995, p10).

It may be true that problem solving cannot be taught but I think it is important that pupils
are instructed in applying phases such as Polya's (1957) so that they can engage in
mathematical thinking in order to solve problems efficiently and effectively. Mason et al (2010, p145) made an important point when they stated that

Mathematical thinking is supported by an atmosphere of questioning, challenging and reflecting (Mason et al, 2010, p145).

Boaler (2009) has earlier made the same point about the importance of such an environment in promoting learning in mathematics but also stressed the fact that pupils were becoming less questioning as they progressed through school. Her research showed that older pupils were becoming disillusioned with mathematics as they did not feel that their ideas were valued. Boaler found that

When students are asked to give their ideas on mathematical problems they feel that they are using their intellect and that they have responsibility for the direction of their work (Boaler, 2009 p44).

This supports the belief by Burton (1995, p1) that when pupils take responsibility for their own mathematical thinking then problem-solving becomes real for them. It is up to us as teachers to provide opportunities that not only will allow, but also will challenge pupils to take this responsibility and thus become 'intelligent learners'.

**Reflections on the Literature Review**

Reviewing some of the literature relevant to my area of research has helped me to locate my research in relation to that literature and to gain a better knowledge and understanding of the topics I was investigating. The literature on problem-solving overwhelmingly supports it as a more effective strategy for improving mathematical thinking and learning. The National Curriculum considers it to be integral to learning in mathematics although there is a conflict, I believe, in what it promotes in theory and
what happens in reality in the classroom. Skemp (1989) distinguished between instrumental and relational understanding with the latter being the more important because it results in the learner not only knowing how to do something but also the reasons why. It leads to more ‘intelligent learning’ and a person who learns in this way is dealing with their knowledge on a much deeper level.

Having reviewed some of the literature on problem solving and collaboration I realised that for effective problem solving to occur it must take place in a collaborative environment. I also realised that collaboration and problem solving do not just happen by arranging pupils in a group with a problem and letting them get on with it. Pupils must be encouraged to interact with a partner by taking turns, reasoning and by sharing and discussing ideas. Research (Bennett and Cass, 1989; Dawes and Sams, 2004) has shown that this is not something that happens easily. Very young pupils find it difficult to take turns and to reach a mutual agreement when they have differing ideas and they need to learn how to overcome this in order to work more effectively. Using different strategies to solve a problem can also lead to disagreement among pupils as to which strategy to use.

Boaler (1998, 2009) and Forman and Ansell (2001) described how teachers who used a problem-based approach to teaching mathematics had a more positive impact on their pupils’ learning. They allowed pupils to use their own strategies to solve the problems and introduced new procedures in context, making them more relevant for pupils. Polya’s (1957) work on problem-solving still has relevance for how we teach mathematics today. He described four phases of understanding the problem, devising a plan, carrying out the plan and looking back, which he believed were integral to effective problem solving. Burton’s (1984) entry, attack, review and extension phases are similar and are an integral
part of the ‘dynamics of mathematics’, as described by Mason et al, (2010). If pupils become accustomed to using these phases, then they engage in what Burton (1984) referred to as mathematical thinking, resulting in more effective problem-solving and intelligent learning.

Having reviewed the literature on problem solving and how pupils work collaboratively, discussing their strategies, I was keen to examine my practice and look at ways of developing a problem-solving collaborative culture in my classroom. The following chapter describes the research methodology used in an attempt to achieve this aim.
Chapter 3 - Methodology

Introduction

My reason for carrying out this research was to explore the impact on my pupils' learning of using a problem based approach to teaching mathematics. In this chapter I consider how I could carry out the research in order to explore the following research questions.

1. Can problem based learning improve thinking skills in mathematics?

2. Is collaborative learning a more effective approach to problem solving than working independently?

3. What strategies do young pupils use when solving problems?

In this chapter, I set out the methodology by which I planned to address these areas and my reasons for choosing this approach. My key consideration was whether to employ a purely quantitative or qualitative methodology as a means of carrying out this research.

There is much debate (Bryman, 1988; Hammersley, 1992) concerning the qualitative/quantitative divide and whether one type of research is more worthy of being classed as research over the other. Both Bryman (1988) and Hammersley (1992) discussed the differences between both traditions and claimed these differences are not as clear-cut as one might think. Hammersley explored what is distinctive about qualitative and quantitative research and stated that elements of one can be found to a certain extent within the other. It is not unusual in educational research for researchers to adopt a mixed methodology approach to research where one method of data collection is used to ‘facilitate’, ‘complement’ or to ‘verify’ the findings of another (Burgess, 1994; Bryman and Burgess, 1999). According to Bryman (1988), Denscombe (2005) and Hammersley (1992)
qualitative research is concerned more with the interpretation of events and with understanding how and why people think and act as they do. It is also concerned with natural settings where the researcher has little or no effect on the normal day to day situations being researched. Hammersley however, does not agree with this distinction between natural and artificial settings but claims it is more a case of ‘the degree to which the researcher shapes the data’ (Hammersley, 1992, p74). Qualitative research emphasises the generation of theory from data as opposed to testing theories. It is important that the data are collected in the field so that the theory is ‘grounded’. As theories emerge during ongoing analysis, more data are collected which then help to further develop the original theories (Corbin & Strauss, 2008; Denscombe, 2005).

I chose to do a qualitative study because I believed this methodology best fitted the type of research I intended to do. I was concerned with my young pupils’ interpretations of problems in mathematics, how they began to solve them and the strategies they used. The naturalness of the setting was maintained as much as possible during the data collection, bearing in mind the views of Hammersley (1992). Some of the collected data were analysed regularly in order to generate themes which dictated the next set of data to be collected. The following section deals with the research strategy and the methods of data collection that were used.

A desire to improve the learning of mathematics in my classroom was what originally led me to begin what started as practitioner-research. McNiff and Whitehead (2006, p51) believed that ‘as a practitioner-researcher, your real work is to improve learning, both your own and others’, in order to improve practice’ and improving my practice has always been of significant importance to me. However when I initially began this research project my sole concern was with improving my pupils learning and I did not consider there to be
an issue with my practice. I was convinced that practitioner-research was an appropriate research strategy for my purposes. Consequently, I did not consider the merits of any other research methodology at this point.

**Research Strategy**

Some researchers, such as Groundwater-Smith and Mockler (2005), use the terms practitioner-research and action research interchangeably as referring to the same research strategy. Jacobson (1998), when defining the qualities of practitioner-research was also describing what are considered to be the characteristics of action research. McNiff (1995, p6) claimed that 'because action research is done by you, the practitioner, it is often referred to as practitioner based research' but McNiff and Whitehead (2006, p259) made the point that although action research 'must be done by practitioner-researchers, not all practitioner-researchers do action research'. I agree with researchers, such as Burton and Bartlett (2005), Dadds (1998), McWilliam (2004) and Noffke (1999), who described practitioner-research as having many forms, one of which is action research. I also see action research as being just one of the many forms of practitioner-research, but a much more focused and rigorous form with a strong emphasis on reflection both in and on action (Schön, 1995) in order to bring about changes in one's practice.

Since action research is concerned with improving practice, as discussed in the works of Denscombe (2005), Ebbutt (1985), Haggarty and Postlethwaite (2003), Kelly (1985), Kemmis (1988) and McNiff & Whitehead (2006), and specifically the researcher's own practice, I considered it to be a more appropriate strategy for this research than practitioner-research. Action research however involves more than just a concern with improving practice. It is 'practitioner-driven' according to Denscombe (2005, p77) and...
involves the researcher reflecting on and changing their practice as a continuous and cyclical process. It is centred on self-reflection, according to McNiff (1995, p6) and is an 'enquiry conducted by the self into the self' but Somekh (1995) warns of the danger of focusing too much on 'self-exploration and personal growth' rather than the real purpose of the research. The real purpose of my research was to improve my pupils' learning in mathematics and the way for me to achieve this was to look at how I could improve my practice. I believed action research to be the best strategy to help bring about these changes. The following section gives a brief overview of action research and how it emerged.

**Action Research**

Kurt Lewin (1946) is considered by many researchers (Hammersley, 2004; Kemmis, 1988; McNiff & Whitehead, 2006) to be one of the early instigators of action research. Its significance as a research strategy has grown in recent years, especially in the field of education. Lewin's research in the 1940s was concerned with inter-group relations within communities in America. He believed that social research should 'proceed in a spiral of steps each of which is composed of a circle of planning, action and fact-finding about the result of the action' (Lewin, 1997). This process has developed into what is known today as 'action research'.

The development of action research in Britain was largely influenced by Lawrence Stenhouse in the 1970s and his idea of the Teacher-Researcher (Stenhouse 1981). He made an important distinction between research in and research on education. By research in education he meant research carried out in the field which promotes educational development. He contrasted this with research on education which can be conducted by any discipline other than education without having any impact on
educational development. Stenhouse (1981, p113) stressed the importance of having teachers ‘intimately involved in the research process’ and of researchers ‘justifying themselves to practitioners, not practitioners to researchers’. I believe that these last two points reinforce this distinction that needs to exist between research in and research on education. As the practitioner, involved in all aspects of their daily practice, the teacher is in the best position to carry out research into their practice, knowing what is most likely to work and seeing the impact of changes first hand. This ‘bottom-up’ approach is more likely, in my opinion, to be beneficial to those involved because it is ‘grounded’ (Corbin and Strauss, 2008) in real practice and is true research in education. The ‘top-down’ approach of research on education, in my opinion is less effective because teachers feel it is being imposed on them from researchers who are ‘outsiders’ (Smetherham, 1978) to their practice. Research conducted to improve or develop education should involve research in and research on education. This, I believe, is what Stenhouse (1981, p113) meant by ‘researchers must justify themselves to practitioners’. Since teacher-researchers are conducting research into their field of expertise, i.e. their practice, their research should be considered to be valid and worthy of recognition by the wider educational research community.

Kemmis (1988) further developed the idea of action research and its use particularly in the field of education. He interpreted Lewin’s cycle of planning, action and fact-finding as a ‘spiral of cycles of planning, acting, observing and reflecting’ (Kemmis, 1988, p184) and stressed the importance of this ‘spiral of self-reflection’ in bringing about changes in practice. Kemmis (1988) discussed three approaches to action research which he referred to as technical, practical and emancipatory action research some of which involve outsiders acting as ‘facilitators’ in the research.
Technical Action Research uses techniques and methods influenced by outsider-researchers who have co-opted the practitioner into investigating or testing an aspect of practice. With this approach opportunities for practitioners to be reflective may not exist and there is the danger that the practitioner may be in some way subordinate to the external researcher.

Practical Action Research involves the practitioner reflecting more on how they can improve their practice but this time with the external researcher having more of a consulting and supportive role. With this approach changes to practice are more likely to occur but there is no implication for the practitioner to take responsibility for these changes.

Emancipatory Action Research, by definition, frees the practitioner to be responsible for researching their own practice and for the changes that result. The practitioner is the researcher at the heart of the research who reflects on and challenges their existing practice in order to improve it. There is no need for ‘outsiders’ but practitioners may work in collaboration where each has equal standing and a democratic voice in the research process.

According to Kemmis (1988) emphasis has shifted over the years to a more emancipatory approach to action research where the practitioner initiates and is responsible for the research process and if outsiders are involved the relationship between them is one of collaboration. I consider the research strategy I am using to be emancipatory action research because I have initiated the research as a means of reflecting on and improving my practice and I am responsible for the changes that result.
Action Research has been discussed and written about by numerous researchers, (Denscombe, 2005; Ebbutt, 1985; Hopkins, 2008; Kemmis, 1988; McNiff and Whitehead, 2006) all of whom agree on it having the following main characteristics.

**Identifying a problem**

Action research begins with the identification of a problem or issue within a practice that is causing concern and needs addressing.

**Improving practice**

It is concerned with bringing about change or improvement to practice as part of the research process rather than at the end of it. This is what is referred to as ‘strategic action’ by McMahon (1999) where the participant actively brings about changes to their practice.

**Involves participants**

Practitioners play a central role as participants in the research and as such, are directly involved in all aspects of the research process.

**Practical**

Action research by definition is practical. It involves every-day, real-life problems that occur in the practitioner’s setting and which s/he is concerned with solving.

**Reflection**

The practitioner-researcher reflects on what has emerged from the first stage of the data collection and uses this to plan the next stage of the research. An important element of
action research is this reflection on practice which is part of its distinctive cyclical or ‘spiral’ format.

These main characteristics are what make action research the most appropriate research strategy for my purposes. As well as attempting to improve my practice I was also directly responsible for all aspects of the research from data collection and analysis to writing the report and disseminating the findings. The research took place in the field during the course of normal teaching practice and alongside my role of researcher was that of practitioner, responsible also for teaching the lessons.

Outlined below in Figure 3.1 is my interpretation of what a cycle of action research involves and the process I was planning to go through in an effort to change my practice. I developed this diagram based on what I considered to be a cycle of action and reflection from my reading of the work of Denscombe (2005), Macintyre (2000), Kemmis (1988) and McNiff and Whitehead (2006).

![Figure 3.1: My interpretation of an Action Research Cycle](image-url)

Figure 3.1: My interpretation of an Action Research Cycle

- **Plan the action**
- **Act:** implement changes
- **Reflect:** analyse data evaluate changes
- **Identify the problem**
- **Modify the plan in light of reflections**

Modify the plan in light of reflections

Plan the next action

Identify the problem

Act: implement changes

Reflect: analyse data evaluate changes

Modify the plan in light of reflections

Plan the next action

Identify the problem

Act: implement changes

Reflect: analyse data evaluate changes

Modify the plan in light of reflections
The first problem I identified in my practice was the need to reflect on what I as a practitioner was doing in lessons to develop my pupils' mathematical thinking. The action plan involved reading literature, changing aspects of my practice based on what I had learnt from the literature and producing data which I could analyse. The reflective aspect of the cycle involved data interpretation and analysis, reviewing literature based on the findings of this analysis, leading to modification of the plan and a new research question to continue the cycle.

Methods of Data Collection

Kemmis (1988) made the point that action research is not defined by any particular 'technique' for collecting or analysing data. Denscombe also stated that action research:

is clearly a strategy for social research rather than a specific method. It is concerned with the aims of research and the design of the research, but does not specify any constraints when it comes to the means for data collection that might be adopted by the action researcher (Denscombe 2005, p74).

The most important aspect of action research is the ongoing process of reflection and action which leads to changes in practice. The methods of data collection used during this process depend on what is most appropriate in the research situation and on the personal preferences of the researcher.

Outlined below are the main methods of data collection which I planned to use in order to investigate each research question and my reasons for selecting them.
Lesson Observations

The majority of the data was to be collected by means of lesson observations. In the course of carrying out what Hammersley (1993) called ‘teacher research’ I planned to adopt the role of participant-as-observer as described by Gold (1958) and Burgess (1980). The general consensus among many researchers including Burgess (1980) and Kelly (1985) is that this type of research, described by Hellawell (2006) and Smetherham (1978) as ‘insider research’, is not without its problems. It is difficult to adopt the role of observer while one is engaged in the practice of teaching. Burgess explained that ‘the role adopted would be that of teacher-as-researcher whereby research activities would be subordinate to teaching duties’ (Burgess 1980, p166). Although this might ease my dilemma to some extent it would not solve the practical problem, discussed by Burgess (1980) and Kelly (1985), of trying to teach a lesson and observe it at the same time. One solution to this issue would be the use of a tape-recorder with which to record aspects of the lesson relevant to the research. Burgess (1985, p185) recalled how ‘...the use of a tape-recorder proved invaluable for keeping a full and accurate record...’ of the interviews carried out as part of her research. The advantages of audio-recorded data are that they depict events accurately without any interpretation or bias, as explained by Cohen, Manion and Morrison (2005), they can show events that might have gone unnoticed by the teacher during the lesson, as noted by Richards (1995), and they can be revisited time and again to allow for in-depth analysis. For these reasons I decided to use tape recordings to provide data.

Since qualitative research is concerned with ‘process’ as discussed by Gubrium and Holstein (1997), the observation of pupils’ activities would be of vital importance. These activities were to be recorded with small groups of six pupils using audio equipment in
order to achieve the most accurate representation of events possible in this situation. The transcripts which would result from these recordings were to be used in the analysis stage of the research which will be discussed later.

**Field Notes and Research Diaries**

Alongside the recordings of the lessons I planned to keep a research diary and to record field notes which I would make as soon as possible after the lessons. I planned to write some brief notes as I observed pupils working but as researchers such as Burgess (1980) and Kelly (1985) stated, this is difficult for a teacher-researcher to do whilst teaching. There can be disadvantages to writing up observations after the lessons. It can be difficult to find time immediately after teaching a lesson to record what has been observed. The problem with leaving it until later is that much of the important detail is likely to be forgotten. Burgess (1985) experienced similar problems in her research when she decided against writing notes during the lessons she was observing and wrote them afterwards instead. She however, made the point that her ability to recall detail improved as her research progressed.

Burgess (1981) discussed the importance of keeping a research diary which, he suggested, should contain the following.

**Substantive account**

The substantive account should include detailed information of what has occurred, where it has occurred and who has been involved in the event.
Methodological account

The methodological account should take note of the methods used to collect the data and the role played by the researcher. It should also explain the criteria used to select the informants.

Analytic account

The analytic account may contain questions which the researcher had in mind at the start of the research and notes on how these questions may have changed over time. It may also contain ideas or initial analyses made by the researcher at various stages in the research process.

In keeping a research diary I hoped that it would contain more of the analytic account where I question why certain events may have occurred in my practice. However I understand that keeping a research diary is a habit that I as a first time researcher need to develop so that sufficient and worthwhile data are collected. The research diary can then be used as another source of data to feed into the analysis stage of the research (Appendix 2). The methods I have described so far are those involved in qualitative research and following below is a brief outline of the more quantitative methods which I planned to use to complement the qualitative data.

Quantitative Data Collection

Scrutiny of work

Evidence from the pupils’ written work during the recorded lessons would provide information on the strategies they used (Appendix 3). This would help ascertain if there were any changes in pupils’ approaches to problem solving. Time was also to be spent
scrutinising work produced during the year in order to analyse the strategies used. A comparison would also be made between their attainment at the end of KS1 and that of the previous year’s class in order to measure any differences in achievement. The results of these quantitative methods of data collection will be discussed in the analysis sections of later chapters. Table 3.1 below gives a brief overview of how the data would be collected with a view to answering the research questions.

**Table 3.1 Data collection and the research questions**

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Lesson Observation</th>
<th>Discussions with the children</th>
<th>Scrutiny of children’s work</th>
<th>Research Diary/field notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- What strategies do pupils use when solving problems?</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2- Can problem based learning improve thinking skills in mathematics?</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3- Is collaborative learning a more efficient approach to problem solving than working independently?</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Triangulation**

A means of testing the validity of the research is through supporting it with more than one method of data collection or with ‘perspectives drawn from other research methods’ as described by Altrichter, Feldman, Posch and Somekh (2008). This is known as **triangulation**. It is defined by Cohen *et al* (2005, p112) as ‘the use of two or more methods of data collection in the study of some aspect of human behaviour’ and is considered to allow for more in-depth insights into what is being studied. Burgess (1985) described how combining methods of data collection helped him to acquire a more
accurate picture of what was happening in his research situation. The use of multiple methods would also be of assistance to me in validating my research. I planned to triangulate my data using lesson observations, taped discussions, field notes and my research diary and examples of pupils’ work.

Lesson observations, using audio equipment, would provide information pertinent to what was happening in the lesson and would provide evidence that would address all of my research questions as outlined in Table 3.1 above.

The data collected through lesson observations were to be corroborated by means of discussions with some of the pupils. The discussions would add extra depth to what had been observed and interpreted in the lessons and would help validate the data. Evidence from discussions with pupils would also help to address some of my research questions in Table 3.1.

The use of evidence collected by means of field notes, research diaries and a scrutiny of the pupils’ written work would add an extra dimension to the data collected by observations and discussions. Samples of pupils’ work during the lessons would be particularly useful in addressing the research questions in Table 3.1 relating to the strategies pupils used and the opportunities allowing them to generalise and specialise.

Anticipated Problems

Prior to starting data collection I considered the possible problems that could occur.

Access

Being an ‘insider’ in the school, employed to teach, meant that the question of access to the setting was not an issue as is the case for many practitioner-researchers e.g.
Cummings (1985) and Griffiths (1985). However I still had to obtain permission to carry out the research within the school because, as Griffiths (1985, p201) discovered, 'automatic presence' did not necessarily 'signify freedom to research'. This permission had been obtained in writing from both the Head Teacher and the Board of Governors at the start of the research, and was valid for the duration of the research project.

Ethics

In order to ensure the ethical status of the research, permission was sought from parents allowing their children to participate in the research. Not to have done so would have resulted in 'covert' or 'semi-covert' research, as described by Burgess (1980), which could have lead to further problems at the writing up stage. The request for permission included a guarantee that all recordings and data collected would be solely for research purposes, would not be shown to anyone else and would be destroyed following completion of the EdD. The anonymity of the school and those participating in the research was guaranteed. The children were also given an explanation of the process and the reasons behind it and were given the opportunity to choose their own pseudonyms, an idea I took from Allen (2009). This allowed them to feel that they were part of the research rather than just having it 'done' to them. Mauthner (1997, p17) argued that children should be seen as 'subjects rather than objects of research' and gave examples of the ways in which they could participate in the research process.

The question of whether to have a control group was dismissed for ethical reasons since I believe all pupils are entitled to the best possible education. There was the possibility of comparing the study group to the parallel class but this would have involved issues of comparing teachers as well as teaching styles and was beyond the scope of this study.
Bias

The question of objectivity was stressed by both Eisner (1992, p50) and Phillips (1989, p69). It is important that research is without bias and follows a clear framework for all observations if it is to have credibility. As the researcher observes and records the activities they have to interpret what is happening. They have to make judgements based on their perceptions. They are constructing the data based on their experiences of the situation. This I believe is where the researcher’s ‘self’ comes in. How they interpret and analyse their findings depends on the type of person they are. How the researcher deals with their values and beliefs when constructing the research is what is important not the fact that they bring these beliefs and values to the research situation. Action Research involves studying the ‘self’ and one’s practice which, according to Dadds (1993), can be a highly emotive experience leading to episodes of negativity and self-doubt.

Although I agree with Smetherham’s (1978, p99) point that ‘every participant can be a stranger’ within their setting, I do not believe it is always possible to do so. In my roles as Senior Teacher and Student Mentor I regularly observe lessons of colleagues and student teachers in order to assess performance. It is not difficult in these instances to be an observer and a ‘stranger’ in their classroom. It is however, more difficult to apply the same strangeness when observing my own pupils performing a task. My natural reaction as their teacher is to intervene when I see them having difficulties with their work. Burgess (1980, p169) described this teacher-researcher problem as one of ‘detachment’. Cummings (1985, p220) described it as one of mentally ‘stepping outside the familiar setting’. Gans (1968, p42) believed that the researcher should not be ‘emotionally involved’ but rather should ‘surrender any personal involvement’ they ‘might have in the situation in order to be free to observe it, and the people who are creating it’.
Reason and Bradbury (2001, p8) claimed that in action research ‘objective knowledge is impossible, since the researcher is always a part of the world they study’. I agree that it may be impossible to achieve complete objectivity but I believe that there are degrees of objectivity that can be reached, which according to Dadds (1993), may come with experience. Dadds made a more practical point that although it may not be possible to remove ‘self’ from ‘judgement to allow judgement-free analysis to prevail’ (Dadds, 1993, p298) if we understood how to separate ‘negative judgements’ from the ‘analytical process’ then it may be possible to ‘minimise emotional disturbance’ and allow for ‘new questions and insights’ (Dadds, 1993, p299).

Recorded Data

Access to audio recording equipment would not be a problem and having tested the equipment I was confident of achieving recordings of a suitable quality. There are obvious disadvantages with audio-recordings, one being the fact that it could be difficult to record the whole class as they worked in small groups. Another disadvantage as described by Altrichter et al (2008) is the fact that non-verbal behaviour is not recorded so can be missed if not picked up by direct observation. Another more serious problem, as experienced by Smith and Strahan (2004) while transcribing the data, was an excess of background noise and an inability to distinguish individual voices on more than one recording. This can be a serious issue for a researcher especially if the recordings account for a major part of the data as they would in my case.

Reactivity

There could be an issue with reactivity when using recording equipment, in that pupils could behave differently than they would under normal circumstances. There is the
likelihood that some might 'play up' or be non-responsive when they see the recorder. Although this might not have any effect on the 'validity of the findings' (The Open University, 2006, p48) it could disrupt the lesson or cause a distraction for those participating in the research. In order to minimize the effects of reactivity during my research, mathematics lessons would be recorded regularly so that pupils were more relaxed and at ease when being recorded. Richards (1995, p63) claimed that 'once the initial novelty wears off' recording a lesson becomes an accepted part of the classroom routine. Cummings (1985), on the other hand, found that the recording equipment continually caused a reaction although it is possible that this reaction lessened the more the equipment was used. I decided to observe and record one group at a time due to limited amounts of recording equipment which meant that more lesson observations would be necessary in order to ensure that sufficient data were collected and that no critical incidents were overlooked.

The potential problems just discussed are concerned with access and data collection. Of major concern also was the process of data analysis which is not without its problems, one of which would be the amount of time needed to complete the process. It is recommended by many researchers especially the grounded theorists Corbin and Strauss (2008) that the analysis of data occurs throughout the research process rather than being left to the end. This idea of simultaneous and continuous analysis is also a central aspect of action research as discussed by Altrichter et al (2008) and Somekh (1995). The next section discusses the methods of data analysis which I planned to use in my research.

Data Analysis

Analysis of data is the process by which data are examined and explored for meaning and interpretation. There are many ways of analysing qualitative data and, according to
Patton (1990), whichever one is chosen by the researcher is very much dependent on the research questions and the purpose of the research. For the purposes of this research I planned to use theme analysis when analysing the lesson transcripts, the field notes and research diary and samples of pupils’ work. I also planned to use a comparative analysis to compare incidences that were common across the various types of data collected.

Theme Analysis

This type of analysis involves sifting through the data in search of themes in relation to the research questions and coding them accordingly. Subsequent coding of the data results in the emergence of more developed themes or categories which are then scrutinised for any relevant relationships which connect the themes. The process of theme analysis is as follows.

The first stage of data analysis is the organization of the data in such a way that it can be worked on, for instance interviews and audio recordings must be transcribed carefully. Patton (1990) described the problems facing one student when she failed to ensure that her data were transcribed ‘verbatim’. She had to repeat the transcription process which had initially taken weeks to complete. It is important to check the transcripts against the original data to ensure they are as accurate as possible before beginning any analysis.

Coding the Data

After transcribing and organizing the data it is important to read and become familiar with the transcripts. Once familiar with the data I planned to read them again and to begin ‘open coding’ them (see Appendix 4).

Open coding is the process referred to by Corbin and Strauss (2008) and Denscombe (2005) whereby the researcher initially goes through the raw data looking for themes and
codes them accordingly. Following this initial coding the data would be revisited and further coding, known as 'axial coding', would be carried out.

**Axial coding**, according to Corbin and Strauss (2008) and Denscombe (2005), is the process of looking for relationships between the initial themes and combining them under a wider category or highlighting a category that appears to be more significant than the others. During the coding process of the data I planned to write memos to record the emerging themes and my initial interpretations (see Appendix 5).

**Writing Memos** or theoretical notes acts as an aide memoire during the analytic process and helps the researcher record their interpretations of the data and their reasons for coding them in a particular way. Many researchers emphasised the importance of writing memos (Delamont 1992, Denscombe 2005) or theoretical notes (Altrichter et al, 2008) during the coding process and Corbin & Strauss (2008, p140) believed that memos ‘stimulate and document the analytic thought process and provide direction for theoretical sampling’. After having coded and re-coded the data I planned to compare the codes looking for similarities and differences.

**Comparative Analysis**

Comparative analysis is an important aspect of grounded theory analysis. Corbin & Strauss (2008, p195) refer to it as ‘comparing incident against incident for similarities and differences’. When incidents are found that are similar in concept to those which have already been categorized they become part of that category. This helps to deepen and expand the existing categories and also assists the researcher in locating from where the next data should be collected. It is important that comparisons are made between different data collected as well as within data, as a means of triangulation as described by...
Delamont (1992). Comparative analysis would help me to become more deeply immersed in the data by looking for the negative case ‘that does not fit the pattern’ (Corbin & Strauss, 2008, p84). I would have to think about data in a more abstract way in order to develop theory and searching for the negative case, according to The Open University (2006, p112), can act as ‘a rigorous test of the developing theory’. As I analysed and compared the data I would become aware of other supportive data that may need to be collected.

The idea that analysis ‘takes place at the same time and in interaction with data collection’ (The Open University, 2006, p102) is in order to avoid collecting too much data and to make sure that whatever data is collected is relevant and worthwhile. Nias stressed the benefits of revisiting the data and looking for different themes.

A...benefit of returning again to the data over such a long period and in pursuit of different themes is that [...] one rakes it into such a fine tilth that ideas germinate easily (Nias 2003, p144).

This echoes the views of Corbin and Strauss (2008) that analysis of data should occur simultaneously with its collection in the field. The reason for this is so that theories can be developed from the data which in turn lead to the collection of more data until no new theories can be generated. This is known as theoretical saturation and is ‘the point in the analysis when all categories are well developed in terms of properties, dimensions and variations’ (Corbin & Strauss 2008, p263). I therefore planned to regularly analyse some of the data collected which would generate themes and categories relating to the research questions and subsequently lead to further research questions being asked. In Action Research it is important that the research is constantly moving forward through
stages of data collection-analysis-reflection-changes in practice-more data collection until no more data are needed and the desired changes in practice have taken place.

**Questioning**

Throughout the analytic process it is important to interrogate the data and ask questions such as *why/why not, how, when, which*. Answering these questions leads away from description and more into interpretation of the data. In order to analyse the data in more depth a greater understanding of why or how particular incidents occurred is important for the researcher. The types of questions which I planned to ask of my data were as follows.

How did s/he arrive at this particular solution?

Why did s/he choose this strategy?

What does it tell me?

How can I develop this further?

Questions like these would be asked of all of the data and would help me to delve more deeply into them in order to look for answers and to reflect on the concepts that emerged.

**Research Diary and Field Notes**

Having analysed the lesson transcripts and noted the themes which emerge I then planned to read through the field notes and entries in my research diary relating to the lesson and look for corresponding themes. I planned to code these themes as
recommended by Corbin and Strauss (2008) and Denscombe (2005), comparing them to what emerged from the transcripts and use them to triangulate the data.

**Pupils’ Work**

Samples of pupils’ work would be analysed with respect to the same themes which emerged from the lesson transcripts. They would be used to provide evidence to support, or refute, what emerges from the lesson observations and contribute significantly in triangulating the data, helping to provide a more accurate picture of what was happening in the research situation as it did for Burgess (1985) when he combined different methods of data collection.

**Summary**

Qualitative methods would be more appropriate for my research because of their concern with meaning and the interpretation of events. Practitioner-research was the original strategy I had planned to use for this research but carrying out an initial study made me realise that it was not rigorous enough for my needs. Action research may be a form of practitioner-research but it involves more than the practitioner wanting to investigate and change their practice. The most appropriate research strategy for me would be action research because of its emphasis on change through action and the importance of the participant as researcher. As well as advocating that the participant should reflect on their practice, action research emphasises the need for ‘strategic action’ as described by McMahon (1999), whereby the practitioner actively plans and implements steps to bring about changes in their practice. It is this ‘spiral of cycles of planning, acting, observing and reflecting’ (Kemmis, 1988, p184) that makes action research so distinctive and rigorous.
Data would be collected using both qualitative and quantitative methods namely observations, field notes, research diary and pupils' work. Collecting data through different methods would help to triangulate the data in the later stages of the analytic process and validate the research. Anticipated problems were explored with regard to access and ensuring the research was carried out ethically. It is important that research is without bias and follows a clear framework for all observations if it is to have credibility.

In action research, because the researcher plays a central role in researching their practice, complete objectivity may be an impossible achievement. I agree that it may be impossible to achieve complete objectivity but it is important that the researcher does everything in their power to ensure the research is without bias as far as is possible.

Once the data were collected the analysis would begin and would be carried out by means of a grounded theory approach. Open and axial coding would be used to locate themes within the data and relate themes and categories to each other through theme analysis. Comparisons were to be made between categories and incidents that occurred in all of the data and questions asked in order to develop a greater understanding of the developing concepts. A full and detailed analysis of the data will be provided in later chapters at the end of each action-reflection cycle along with a report on the findings of the research which will be included in the final chapter.

The next chapter describes the first cycle of action research in which I focus on reflecting on my practice. It describes how and why I amended my original plan and the changes that occurred as a result.
Chapter 4 Cycle 1 - Reflective Practice

Introduction

My initial plan at this stage was to examine the strategies used by my pupils when solving problems, to review the relevant literature and to look at how I could change my practice so as to further develop these strategies. I decided instead to focus on how I reflected on my practice which I had not planned to do until later. This was an area of higher priority which needed considerable development if I was to carry out rigorous action research and improve my practice.

The first part of this chapter looks at reflective practice and some of the literature which supports it, while in the second part I concentrate on how I reflected on my practice in order to change and improve it in line with action research (McNiff and Whitehead, 2006). I recorded and transcribed some lessons at the beginning of the following academic year (2009-2010) and looked at themes that were common across all of them. I have examined these themes and reflected on their significance both to my practice and the pupils' learning and considered what action I could take as a result.

Reflective Practice

The idea of teachers reflecting on their practice gained strength in the 1980s as a result of Schön's (1995), work entitled The Reflective Practitioner, which was first published in 1983. He distinguished between 'reflection-in-action' and 'reflection-on-action'. By 'reflection-in-action' he meant reflecting in the moment so that any changes to the action can have immediate effect. 'Reflection-on-action' on the other hand is reflection on the action that has taken place but with less immediacy. This latter form of reflection can occur at a much later date well after the action has occurred. Convery (1998) suggested...
that teachers interpret ‘reflection-in-action’ as occurring during their classroom practice. I agree with this and have tried to reflect on or to ‘notice’, my actions as I teach, as Mason (2002b) advocated, with varying degrees of success. There have been times when I have reflected during an action or a statement I was making and successfully altered the next action. However there were far more times when I had planned to be reflective but had become so involved in the task that I had forgotten to focus on the particular action on which I wanted to reflect. I found that it is more difficult than it seems to reflect-in-action and Mason (2002b) suggests ‘marking and recording’ the moment so that it can be reflected on in more detail later.

Convery (1998) and Dadds (1993) support the idea that if reflective practice is to be positive and supportive then it should be a collaborative process. I agree that this should be the case if one is ‘reflecting-on’ one’s practice as it is important to have a critical friend or colleague who can be more objective and positive than one’s self. However I do not see how it is possible to ‘reflect-in-action’ collaboratively with anyone else unless they are in the ‘action-present’ (Schön 1995, p62) with me, which would not be possible if it occurred in one of my lessons as Convery (1998) suggests. For most of my lessons, I am the only adult in the classroom. It would however be possible to collaboratively reflect on what I had ‘reflected-in-action’ but it would have to take place after the action and would not therefore influence the action in process.

Sometimes when I have reflected on my practice and asked myself why I acted in a particular way I have found my actions difficult to explain. Is this because I have become ‘over-learned’, as described by Schön (1995)? Am I becoming habitual and routine? Reading through the lesson transcripts made me realise that there were other elements in my practice that needed addressing before I could explore the learning in mathematics.
What follows is an initial analysis of what I noticed occurring in my classroom. The data used in this analysis came from lessons recorded in the autumn term 2009, between the end of September and the end of November. I wanted to identify features of my practice that might have become habitual or routine. This involved me in looking at the transcripts through different eyes and not allowing myself to be distracted by what the pupils were saying.

**Data Analysis**

I read through the lesson transcripts and carried out an initial coding of the texts as Corbin and Strauss (2008) suggested. I then compared the transcripts and looked for common themes in my practice. The most obvious common themes were as follows.

**Behaviour Management**

**Questioning**

**Instructions**

**Mathematics tasks**

**Behaviour Management**

As a teacher I believe I have good classroom management skills and I try to use positive approaches when dealing with issues of behaviour. Strategies such as; praising positive behaviour and ignoring inappropriate behaviour, reminding the child of what they should be doing, giving choices, giving warnings of consequences if inappropriate behaviour continues, all form part of my approach to managing the behaviour in my classroom and have proven effective in most instances. Consequently I had not considered the need to examine behaviour management within my classroom or the fact that it could be
interfering with the flow of learning until I looked closely at some of the lesson transcripts. I can be heard using class rules as a means of reminding the children of appropriate behaviour.

Teacher: Okay, hands up, now when people are speaking, what do the rest of us need to do?

Children: Listen

Teacher: Listen. That includes sitting down and sitting quietly. Okay, Daniel...

This is a positive strategy that has proven successful for me especially, according to Evertson, Emmer, Sanford and Clements (1983), if the children are aware of the rules and if they are used consistently. However not all behavioural comments I made were as positive as I had originally believed. There were recorded instances where I reprimanded pupils for not being on task which in some cases were then echoed by other pupils.

Teacher: Children, it's too noisy!

Teacher: You two boys turn around and work quietly.

Teacher: Adam, don't touch theirs. (counters)

Angela + Charlie: Stop it Adam!

Adam: I can touch mine?

Teacher: Adam, don't play with them.

These negative comments were less effective in achieving the desired result and emphasised the inappropriate behaviour not only to the pupils involved but to the others.
who were listening. How could I have dealt with the situation more effectively so as to focus these pupils back on the mathematics task? Perhaps reminding them that they should work quietly so that others can think better might have been a more positive way of reducing the noise level. The two boys who had been talking to the pupils behind them might have refocused more readily if asked to explain their progress so far, as might Adam if I had asked him to show me how he had sorted his counters rather than telling him not to touch the other pupils’. My reason for wanting to reduce the noise level was not because I wanted pupils to work silently. This class was a particularly loud and vociferous group and in order for me to be able to interact with groups or individuals and for the lesson recordings to be audible I needed to reduce the noise to a manageable level. It seems in some instances that my concern was focused more on dealing with the behaviour itself than on getting pupils back on task. I appeared to be concentrating more on classroom management than on the mathematics task. A clear example of this was an exchange I had with Adam during the Sorting Counters lesson (Appendix 6).

Teacher: What are you doing now? How have you sorted them now Adam?

Adam: Brrrh! ...Minding my own business.

Teacher: Pardon?

Adam: Minding my own business.

Teacher: Pardon?

Some children: He said, “Minding my own business”.

Teacher: Adam!
Adam: Sorting with shapes, and I'm...

Teacher: What did you say, when I asked you, Adam?

Michelle: He said he was minding his own business.

Teacher: What did you say, when I asked you?

Adam: I'm minding my own business.

Teacher: I asked you what you were doing. Are you being rude to me?

Adam: I'm sorting

Teacher: Then that's all you had to say. How are you sorting them?

Adam: In the colours.

Teacher: Thank you, that's all you had to say to me.

During the lesson I had not heard Adam's response to my question so I had wanted him to repeat what he had said. The others at the table seemed more than willing to repeat his answer despite the fact that I repeatedly addressed the question to Adam. I had been somewhat surprised by the other pupils' replies and wanted to hear for myself what he had said, not believing that he would be rude to me. As I engaged in this conversation part of me wanted to let it go as he had answered my initial question by saying he was sorting shapes. However another part of me wanted to know what he had really said and if the other children had heard him correctly, so I pressed on until he answered my question. Why had I acted in this way? My knowledge of Adam as a very intelligent boy, with English as a second language and an interest in words and phrases, should have told me that there was more to the situation than first appeared. On transcribing the lesson I
discovered what had instigated Adam’s response to my question. An interaction between other pupils at the table had resulted in the following comment from Michelle.

Michelle: I told Charlie how to spell something and he said I was to mind my own business.

It would appear that Adam had overheard this comment and found the phrase interesting. I unwittingly gave him his first opportunity to use this phrase by asking him what he was doing, to which he replied “minding my own business”.

If I am focusing on the mathematics task rather than the behaviour, the comments I use should reflect this and should then focus pupils away from the behaviour and back onto the task. Asking questions such as ‘Show me what you have done so far?’ or ‘How are you sorting the objects?’ might have avoided this confrontation and kept the focus on the learning instead of the behaviour. This aspect of my practice has changed considerably as a result of these reflections. My focus is now more on the mathematics tasks and on how my pupils are engaging with them rather than their behaviour which makes for a more positive learning environment with fewer instances of inappropriate behaviour. Minor issues are dealt with more effectively with comments such as, Explain what you have done so far, which maintains the focus on the task while still managing the classroom.

Questioning

Many of my lessons began with a review of previous work especially if it was part of a series of lessons on a particular topic and usually involved a short questions and answers session before introducing the task for that day. At various stages in the lesson I asked pupils to review their progress by explaining what they had done so far and in the plenary we reviewed the task as a class. As a result a significant amount of teacher talk in my
lessons consisted of questioning. Consequently I wished to explore in more detail the
types of questions I asked and whether or not they challenged pupils' thinking. As well as
questioning them about their work, I noticed that in the course of the lessons I used
questioning to manage behaviour, for example,

*Are you listening?*

*Who is ready?*

*Can I just ask people to sit still please?*

I had used this approach for some time, almost 'routine(ly)' Schön (1995) might say, but I
was aware that this was what I did and I believed it to be a more positive approach than
telling the children to *listen* or to *sit still* which, to me, sounded more reprimanding. I also
used closed questions, such as,

*How many horses?*

*How did you sort them?*

*How many centimetres in a metre?*

With these questions pupils had to give particular answers. I felt that this type of
questioning was useful at the beginning of a lesson as a means of reviewing previous
work or finding out what pupils knew about a new topic. It also gave an idea of how they
were getting on with a task, but questions like these could not be considered to challenge
pupils' thinking and extend their learning. In order for this to happen, higher order
questions needed to be asked which encourage pupils to reason and to think beyond the
basics. There was evidence of such questions in my lesson transcripts for example,
Can you think of another way to sort it?

When you sorted the animals which had the most?

How did you work it out?

Why did you do that?

Did anyone do it a different way?

By asking these questions I challenged my pupils’ thinking and encouraged them to focus more on how and why they used certain strategies to work out a problem rather than focusing solely on what the answer might be. Because I employed this type of questioning more readily in my practice, pupils appeared to be developing more confidence in explaining their reasoning and many could explain some of the strategies used by others, as can be seen in the example from Measuring Strings (Appendix 7) below.

Teacher: So if this piece of string is 20 cm, how long will half of this piece of string be?

Jasmine: 10 cm

Teacher: Why do you think it is 10 cm?

Jasmine: Because I halved the string and got 10 cm.

Teacher: So you did that yes? (Folding string in half), halved the string, and what did you say?

Jasmine: I got 10 cm

Teacher: But how do you know that's 10?
Jasmine: Because...(pause)

Teacher: Can anybody help Jasmine explain? Lexa

Lexa: Because half of 20 is 10.

(Appendix 7: Measuring Strings)

It was important for me to develop further use of 'why' and 'how' questions so that pupils come to expect that a reason or explanation is necessary when they give an answer. This should then help them to develop their questioning to a point where they expect their partners to justify their responses. This did not appear to happen at the time and only in very few instances had I heard some pupils question the answers of others. The example below, where Jack questioned Joseph’s answer, showed that Jack knew what the correct answer should be but Joseph did not justify his answer or explain why he had written 6.

Jack: Why are you going... why are you doing 3 and 4?

Joseph: To score 6

Jack: Because that will make 7.

This apparent absence of questioning on my pupils’ part was a concern for me and one which needed addressing. How could I develop a culture of questioning within my classroom? If I challenged their thinking enough would it in turn lead to their challenging their partners’ answers and consequently their thinking? This was something which needed further investigation. I wanted to hear pupils engaging in more ‘exploratory talk’, as described by Wegerif, Littleton, Dawes, Mercer and Rowe (2004), which is self-initiated and not solely in response to my questioning, which seemed to be the situation. Much of
the whole class talk seemed to be dominated by me and as well as questioning I appeared
to spend a lot of time instructing pupils, which is the focus of the next section.

Instructions

Generally I instructed the children on routine matters such as reminding them to write
the date and the learning objective, to work with their partner or the use of certain
resources. I also instructed them on the tasks they were expected to do in the lesson. I
examined the statements I used in some of my lessons while giving instructions and was
surprised by the complexity of some of them and also by the fact that I appeared to give
more than one at a time.

Now children you all have your objects that you have to sort. Once you've
written... sshh... once you've written the date and the learning objectives
you've got to discuss with your partner how you're going to sort them. Once
they are sorted, how are you going to record the information? Okay?

The above example begins with an instruction pertaining to the mathematics task of
sorting (Appendix 6), followed by classroom management (sshh). It is then followed by an
instruction relating to writing the date and learning objective combined with another
instruction asking them to discuss with their partner and explain what they were doing.
This is then followed by a further instruction relating to the mathematics task and
recording their work. Given individually these instructions would be manageable but
given together in quick succession makes them complex and difficult for young children to
follow, particularly according to Robinson and Whittaker (1985) and Speer (1984), if they
are ambiguous. I did not realise at the time that I had given so many different instructions
at the same time. If this was common practice in my lessons then it was hardly surprising
that I have to repeat myself in order for pupils to follow what I have said. I noticed that at times when I gave an instruction or asked a question I immediately repeated it. Was this because subconsciously I was aware that it may be too complex and by repeating it I was attempting to simplify what I had said so that pupils would have a clearer understanding of what they had to do? In order to avoid this tendency for complexity I needed to ‘reflect-in-action’, as Schön (1995) advocated and to ‘notice’, as recommended by Mason (2002b), the types of statements I was using so that my ‘talk’ would be pitched at the children’s level of understanding. I needed to make my instructions clear and avoid overloading them with too many at a time. If pupils did not fully understand the instructions then their inability to follow them was likely to cause a misunderstanding of the mathematics task which could hinder its successful completion. The following section examines the mathematics tasks given to the pupils and their level of engagement. I was interested to know if the tasks were appropriate for what I was expecting my pupils to do. The said tasks were called Four-pin Bowling (Appendix 8), Sorting Counters (Appendix 6) and Measuring Strings (Appendix 7) and were all recorded in Autumn 2009.

Mathematics Tasks

As I coded the transcripts I noticed that considerably more of the pupils’ discussions involved talking about the mathematics tasks than I had originally expected and this seemed to increase the more they engaged with different tasks. The type of talk varied from simply reading the question together, to counting objects, to describing the strategies they used to find the answers. The latter type of talk, which seemed to me to have features of mathematical thinking, as described by Burton (1984), was for me the most important and therefore the one I wanted to hear occurring most often in my
lessons. Unfortunately at the time it was the type with the fewest occurrences so I needed to think about how I was going to address this issue.

Some examples of the type of talk pupils engaged in while working on the tasks are as follows.

Jack: Oh, we just put them together.

Patrick: We can do a line, do a big line.

Jack: A thick line

Patrick: Which pins must Joshua knock down to score exactly 5 ... 2 different ways?

(Appendix 8: Four-pin Bowling)

Lexa + Aisling: We can do 1, 2, 3, 4, 5, pictures, so, 1, 2, 3, 4, 5, 6

Aisling: Okay, what we'll do is we'll do little circles for it.

Lexa: Yeah, because these are the things that we have sorted.

Aisling: 1, 2, 3, 4 Red

Aisling + Lexa: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Aisling: One ... look, Lexa, we'll do it that way. So that's 6, yeah?

(Appendix 6: Sorting Counters)
The examples above show how pupils were on task, discussing various aspects of what they were doing, with many working collaboratively. There were some instances where pupils engaged in distracting talk or social comments, but these episodes did not last for long and in many cases they got back on task by themselves or were prompted by their partners.

The instances of talk which contained features of mathematical thinking relates to the times when pupils were explaining the strategies they used to solve the problems or where they were reasoning as they worked out the answers. The following examples show how some pupils could explain the strategies they used to solve parts of a problem on measuring (see Appendix 7: Measuring Strings).

Teacher: If the string is 40 and you double it. You get...

Julia: 80

Teacher: 80, good girl. How did you do that?

Julia: Because 4 + 4 equal 8 so 40 + 40 equals 80.

Children: How much longer would I need to...

Teacher: ... make the string to make it 25 cm long?

Angela: 5 cm

Teacher: Why do you say, 5 cm?

Angela: Because if you have 20...because if you have 20 and you add 5 more you get 25.
It is clear from the above examples that these pupils had a good understanding of what the problem was asking and were able to explain their reasoning. One pupil, Julia, demonstrated evidence of generalising as part of her strategy for solving the problem as follows.

Julia: Because 4 + 4 equal 8 so 40 + 40 equals 80.

These explanations, however, were in response to my questions and there were many more examples of this type of teacher-pupil exchange. I examined the transcripts more closely looking for instances where pupils justified their strategies to each other and unfortunately did not find very many.

Camille: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, no...

Niamh: It's wrong.

Camille: 30...

Niamh: No look, 21, 22, 23, 24, no start again. 21, 22, 23, 24, 25, 26, 27, 28, 29, 30... Look 30! So 20+10 equals 30. Twenty plus ten equals...

(Appendix 7: Measuring strings)

Angela: We need to make... 4 + 1 makes 5

Lexa: Anyway you have to do some adding. Do you understand?

You have to do some adding how to make 5.

Angela: Wait! Let me just do the 7... 2, 3, 4, 5, that makes 5.

Emily: 4 + 3 equal 7, it is. Count 4 in your head, 5, 6, 7.

(Appendix 6: Sorting Counters)
The above examples show how pupils could explain their reasoning and how, at times, they were willing to challenge their partner's ideas. There was some evidence of this in all lessons but it was less evident where pupils found the task more challenging such as the measuring strings activity (Appendix 7). Why did some pupils go quiet when they were having difficulties? There are a number of possible reasons for this, one of them being insecurity and another being competition. Both of these aspects can cause what is referred to as *maths anxiety* by Tobias and Weissbrod (1980). By this they meant the stress and panic felt by some learners when faced with a mathematics problem which can lead to their avoidance of the mathematics in order to avoid this stress. Newstead (1998), Quilter and Harper (1988) and Tobias and Weissbrod (1980) suggested that because anxiety in mathematics affected how pupils participated in mathematics it also had an effect on their performance. Some pupils in my classroom were afraid of getting the answer wrong and would rather remain quiet than risk the possibility of others laughing at their efforts. One particular boy, who called himself Patrick, was very competitive and found it difficult to deal with activities that challenged him. At the beginning of the year he would not attempt a task with a partner if that task was demanding and he would not voluntarily ask for help. He appeared to lack confidence in his ability to successfully complete these tasks and was very concerned with his standing among his peers. His immediate response in situations like this was “I'm no good at this” or “Does it matter if we can't do it”. Nicholls, Cobb, Wood, Yackel and Patashnick (1990) distinguished between *task orientation* which involves learning for its own sake and the challenge it provides, and *ego orientation* which involves learning and engaging with a task because of the wish to be competitive and demonstrate superior ability. Patrick appeared to be more ego oriented and displayed some of the symptoms of anxiety about mathematics which only surfaced when faced with more challenging tasks. According to Dweck (1986),
A strong orientation toward this goal can thus create a tendency to avoid challenge, to withdraw from challenge, or to show impaired performance in the face of challenge. Ironically, then, an overconcern with ability may lead children to shun the very tasks that foster its growth (Dweck 1986, p1043).

As a high achieving pupil, Patrick seemed unwilling to ‘risk failure’ (Quilter and Harper, 1988) and therefore avoided engaging in challenging tasks rather than appear unsuccessful in front of others. Newstead (1998), Quilter and Harper (1988) and Tobias and Weissbrod (1980) suggested that this anxiety about mathematics could possibly have been caused by past classroom experiences or inadequate teaching where the emphasis is on practising skills and low level tasks. A possible solution to mathematics anxiety put forward by Newstead (1998) is,

an approach which includes a more personal and process-oriented teaching method emphasising understanding rather than drill and practice (Newstead 1998, p55).

Changing the ethos of my mathematics classroom, during the course of the year, to one where the emphasis was more on the thinking and reasoning than on the answer to the question, gradually began to have an effect and led to a change in Patrick’s behaviour as well as that of the other pupils. He became more willing to attempt challenging tasks and to discuss possible strategies with his partner and although he displayed instances of anxiety they were less frequent and intense.

A possible reason for some pupils’ apparent reluctance to interact with others when faced with a difficult task may have been the fact that they did not yet possess the skills necessary for engaging in this type of interaction. I recall when Camille told James the
answer to a question and urged him to write it down; he sat looking at his work in apparent doubt as to what to do. I suggested that if he was not sure of Camille’s answer he should ask her to explain how she got it, which he did. They repeated the activity together and came up with a different answer with which they both seemed satisfied. In order to develop the skills of interactive discussion with their partners I encouraged pupils always to give a reason for their response or to justify their strategy. This was an idea I picked up having attended some professional development courses on mathematical thinking and problem-solving provided by BEAM (Be a Mathematician) in 2007. I used questions or phrases such as,

*How do you know?*

*Prove it*

*Does anyone agree/disagree?*

I also told them to ask their partner to explain how they got their answer rather than just accepting what they had been told. Pupils became more used to these comments and some became more confident at giving their explanations, while one or two gave reasons unprompted. There were, however, some who needed to further develop this ability to explain their reasoning and justify their answers so that they could more confidently collaborate with others on problem-solving tasks. Having explored pupils’ responses when explaining their strategies I decided to look more closely at my questioning to see how effective it was at developing pupils’ mathematical thinking. Outlined below are some of the conclusions I have drawn and the changes I needed to make to my practice in order to address these issues.
Conclusion

The conclusions I drew from my initial analysis of data were as follows.

- I sometimes focused more on the distracting behaviour that occurred in a lesson than on the mathematical task.
- Pupils talked about the task but not enough about their mathematical thinking.
- I needed to challenge pupils thinking by asking a greater number of higher order questions.
- My instructions to pupils were often complex and too many at once.

I realised from reading the lesson transcripts that I focused more on the pupils' behaviour than on their engagement with the task. In order to rectify this I needed to draw their attention back to the task by focusing on what they were doing with the mathematics rather than on highlighting the inappropriate behaviour. Engaging pupils more with an appropriately challenging task addressed the issue of behaviour and led to fewer distractions.

Although pupils spent a considerable amount of their talk time discussing the task much of it was managerial or instructional in nature and not enough time was spent actually discussing the strategies to be used. This could have been because so much of it was going on in their heads rather than not happening at all but they were not verbalising it. It was up to me to encourage them to verbalise their thoughts and to get their mathematical thinking out in the open so that they could construct their knowledge in collaboration with others as Vygotsky (1978) believed. I could encourage this by thinking about the type of questioning I used. My questions should challenge pupils' thinking,
make them justify their answers and find alternative ways of finding solutions. This should lead to a more questioning environment where pupils can develop the confidence and the skills to challenge the thinking of others.

Asking challenging questions does not mean that they should be asked in a challenging way. Pupils must be able to understand what is being asked. My tendency to overcomplicate questions and instructions would not help my pupils to develop as thinkers if they had to first work out what I was saying. My questions and instructions needed to be straightforward and uncomplicated if pupils were to focus on what was being asked rather than on their interpretation.

**Changes to my Plan**

I had initially planned to focus on the strategies used by pupils when solving problems but developing skills of reflection and reflecting on my practice needed more immediate addressing, so I decided to focus instead on this aspect first. As a result I have read literature on reflective practice which I had not planned to read until later on.

My initial plan had been to regularly collect and analyse the data and read the relevant literature as I went along. I decided instead to collect data for one month at a time and then spend the following month analysing the data, examining changes to my practice and reading the relevant literature. I felt this was a more realistic approach since trying to continuously collect and analyse data, as well as research and read literature, was becoming unmanageable in the time available to me. By alternating the recording and data analysis each month I could fit more time in for reading the literature which I had previously found difficult to manage.
The plan was then to examine how the issues which arose in the first cycle had been addressed, to look at any changes to my practice and how these related to the literature. I also intended to focus again on the strategies children used when problem-solving and review the relevant literature. I decided instead to leave pupil strategies until later and to focus in more detail on my classroom as a questioning environment as I was concerned with the lack of questioning by my pupils. I wanted to find out if the type of questions I asked had any impact on my pupils' learning and their use of questions so I asked the following research questions.

4. To what extent do I use higher order questions to extend pupils' learning in mathematics?

5. Does increasing the number of higher order questions I ask impact on my pupils' learning in mathematics?

The next chapter explores the literature on questioning and its importance in developing mathematical thinking. I examine in detail the types of questions I used in my mathematics lessons and the effect that changes in my practice had on the learning that took place as a result.
Chapter 5 Cycle 2 - Questioning

Introduction

In this chapter I consider the extent to which my classroom was a questioning environment. Reflecting on my practice was an important turning point for me at this stage and helped me to see the need for change in this aspect of my teaching.

The importance of reflecting on one's practice was described by Schön (1995), and supported by Mason (2002b) who went further to suggest 'marking and recording' the action to allow for more detailed reflection later. Having begun to reflect on my practice, as described in Chapter 4, I was concerned by the fact that my pupils did not ask questions during mathematics tasks and I wondered if my questions were challenging their thinking. In an effort to improve the quality of the questions I asked, I spent the next cycle of action research looking at the types of questions I asked my pupils in lessons and how I used them to develop pupils' thinking in mathematics. The research questions I used to help me focus on this aspect of my teaching were as follows.

4. To what extent do I use higher order questions to extend pupils' learning in mathematics?

5. Does increasing the number of higher order questions I ask impact on my pupils' learning in mathematics?

Pratt (2006) made the point that the NNS (DfEE, 1999) made little impact on the interactive discourse that occurred in the mathematics classroom. He claimed that the 'massive investment in in-service training' (Pratt, 2006, p223) had little effect on how teachers interact in class. There may have been large investments in teacher training but
how this training was disseminated may, in my opinion, have been partly responsible for this lack of change. My experience of the training in the lead up to the launch of the NNS was not very positive. All primary schools in the Borough attended a training session with the Local Authority to discuss how to create our own short and medium term plans and how to implement the three-part lesson. Mathematics co-ordinators attended extra training sessions which they were then required to deliver to their own staff in schools. This was done with the aid of video clips and group discussions but very little if anything was mentioned about the importance of questioning and developing more discursive, collaborative practice. The emphasis, in my experience, appeared to be more on mental skills and the pace of the lesson rather than on how to challenge learners’ thinking through effective questioning. According to Tanner, Jones, Kennewell and Beauchamp (2005) the NNS videos featured questioning that demanded instantaneous or very rapid responses from pupils recalling prescribed number facts, rather than high quality dialogue, discussion and strategic thinking (Tanner et al, 2005 p722).

This was an aspect of my practice I wished to change as my use of questions did not appear to be effective at encouraging my pupils to ask their own questions. I first looked at my use of questions in the lessons I had already recorded in early 2010. Following this I planned to review the literature relating to the types of questions I used and follow this up with more lesson recordings which would show any changes implemented in my practice. The next section examines the importance of questioning in mathematics and the questions I used when engaging with my pupils during lessons.
The importance of questions in the mathematics classroom

Khisty and Chval (2002) discussed the importance of the role played by teacher's talk in pupils' learning and particularly stressed the significance of questioning in promoting and developing children's mathematical understanding. They provided evidence gathered from one particular teacher's (Ms Martinez) use of language in her fifth grade classroom in support of this. Ms Martinez introduced her students to the appropriate mathematical language from the beginning and modelled its correct use by repetition and by correcting the students' omissions in a positive and supportive manner. The research showed that by the end of the year her pupils could confidently discuss their strategies, appropriately using the correct mathematical language. The benefits of this approach were that her pupils became more confident mathematicians with a secure knowledge and a firm base for further learning.

I regularly used this strategy of repeating what my pupils said, both as a means of clarifying their responses for others and like Ms Martinez, as a way of demonstrating correct use of mathematical language. The example below, from a lesson on reading a scale recorded in May 2010, shows some evidence of this in my classroom.

Teacher: Now, how much would it hold if the liquid went all the way up to there? (Pointing to a mark on a jug) Charlie

Charlie: A thousand metres?

Children: Huh?... No...

Teacher: Sssh, let Charlie have another go. Think about what Mala said we measure with. We don't measure with metres that's for
length and height. If we are measuring how much liquid is in
the container, what do we use?

Charlie: Litres

Teacher: Litres. So how much is in that?

Charlie: 1

Teacher: 1 Litre, yes. It went all the way up to 1, which, yes, is a
thousand millilitres.

In the above example I clarified Charlie’s use of mathematical language with regard to
measures and demonstrated what should be used instead and why. However sometimes
when I thought I was repeating what pupils said I was actually repeating what I wanted
them to have said, as the following example shows.

Rose: I got 10 and I...

Teacher: Okay.

Rose: and I counted 6 more on and I got 16.

Teacher: Right, so Rose got 10 in her head and counted on 6 more and
she landed on 16.

In hindsight, had there been any need for me to elaborate on what Rose had said? Her
explanation had been clear enough not to warrant any teacher clarification and by
changing her words had I implied that what she said was insufficient? Consequently
repeating what pupils say can have its disadvantages. Gall (1970, p717) referred to this as
a ‘bothersome teaching habit’ which she classed as poor questioning along with

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'repetition of one's questions' and 'answering one's own questions'. It could be seen as a means of controlling the discussion that takes place if pupils perceive that their responses have to be repeated or clarified by the teacher. This, in my experience can lead to non-participation in lessons by some pupils as they know their teacher will repeat the important information. Khisty and Chval made the point that 'the strategy of repeating causes students to listen less to their peers since they know that the teacher will say it again' (Khisty and Chval, 2002, p162). What could I do to change this attitude within my classroom? Instead of repeating Rose's response in the above example I could instead have asked another pupil to explain her method as a means of checking their understanding. This approach, I believe, of using other pupils to clarify or repeat a response would then send out the message that I considered what other pupils say as being important. That is not to say that I did not value their answers but that may have been the impression I gave. If my repeating what pupils said was to have the positive effect that it had with Ms Martinez's class then I needed to consider what elements of their responses needed to be repeated and why. If it was for the purpose of correcting errors in language or mathematical understanding then I believe teacher repetition was necessary. If it was to check other pupils' understanding of a response then simply asking another pupil to repeat what they thought was said could be more useful than just managing behaviour.

My use of Questions

Collecting and coding the data

I reflected on my reasons for asking questions of my pupils and listed what I thought to be the main ones.
1. To extend pupils' learning in mathematics.

2. To find out what they knew.

3. To find out something from them that I did not already know.

4. To draw their attention to something new that I had noticed - I asked a series of focusing questions until they give me the answer I was looking for.

5. To make sure they were listening and following the lesson.

6. As a means of managing behaviour and to bring them back on task if they appeared to be distracted or were distracting others.

The first four reasons are to do with pupils' understanding in mathematics. The last two relate to classroom management. I then looked at how I used questions in my mathematics lessons to see why I asked these questions. I examined the transcript of a lesson (Appendix 9: The Slipper Problem) which had been recorded in January 2010 and which I had used previously when reflecting on my practice. I highlighted every question that I had asked and coded it according to the type of question I believed it to be. I then grouped similar questions together under the following headings:

1. Open questions

2. Closed questions

3. Behaviour managing questions

4. Clarifying questions

Open questions are designed to extend pupils' thinking, to encourage them to give reasons for their replies and to engage them in discussion about their work. They are also
referred to as higher cognitive (Cotton, 2001; Gall 1970) or higher order questions (Kilpatrick, 2003; Martinello, 1998) and are used to develop learners' reasoning and critical thinking skills. They include questions such as;

Why did you do that?

What do you think?

How did you work it out?

Closed questions are questions which are limited to a single answer or a yes/no response. Also referred to as lower cognitive (Cotton, 2001; Gall 1970) or lower order questions (Kilpatrick, 2003; Martinello, 1998) they focus on pupils' recall of facts or information. Although these questions did not extend pupils' thinking beyond a basic answer I considered them to be appropriate for the responses that were being sought. They included the following types of questions;

What is the answer?

How many pairs are there in 10?

Would you agree with that?

Behaviour managing questions contained both open and closed questions but the reason I separated them from the others and put them into a category of their own was because of my intentions when asking the questions. I used these questions to refocus pupils back on the task they were doing when it appeared to me they were going off task or becoming excessively distracted. They were questions related to the task they were doing and seemed to have the desired effect in most cases as pupils continued with their work after responding to me. They included questions of the type;
How are you getting on with your work?

What have you done so far?

What will you do next?

The last category contained what I call clarifying questions. These were questions I used to clarify a pupil's response either for my own benefit or because I did not think it was clear enough for the other pupils. Some examples of clarifying questions are as follows.

Are you sure?

You did another 2 didn't you?

So do you mean, 8 + 8?

In the transcript from the Slipper Problem (Appendix 9) lesson I counted 112 teacher questions asked. Table 5.1 below shows the number and percentage of occurrences for each question.

Table 5.1: The Slipper Problem (Appendix 9)

<table>
<thead>
<tr>
<th>Type of question</th>
<th>Number of occurrences</th>
<th>% of occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open questions</td>
<td>43</td>
<td>38</td>
</tr>
<tr>
<td>Closed questions</td>
<td>42</td>
<td>37.5</td>
</tr>
<tr>
<td>Behaviour managing questions</td>
<td>14</td>
<td>12.5</td>
</tr>
<tr>
<td>Clarifying questions</td>
<td>13</td>
<td>12</td>
</tr>
</tbody>
</table>

It appears from the table that I had asked similar numbers of open and closed questions. This surprised me as I believed at the time that I had asked far more open ended
questions. I believe that some closed questions are useful and appropriate but was it necessary for me to have asked quite so many? I reviewed some of the literature that was written about the use of questioning in order to find out what the research said and in an effort to better understand how to use questioning more effectively in my mathematics classroom. According to Gall (1970)

about 60% of teachers’ questions require students to recall facts; about 20% require students to think; and the remaining 20% are procedural (Gall 1970, p713).

More recently, research by Boaler and Brodie (2004) reported incidents where more than 95% of questions asked by teachers using what they termed ‘traditional’ methods, were fact finding questions, whereas research by Smith, Hardman, Wall and Mroz (2004) found that only 10% of questions asked by Key Stage 1 teachers were open-ended. It would seem that the emphasis was still on teachers asking lower order questions and I appeared to be no different. In comparison to what the literature said, 38% of the questions I asked in the Slipper Problem lesson were open-ended questions but I did not consider this to be sufficient if I wanted to create a questioning environment in my classroom.

I agree with Mason’s view that ‘not all utterances with a question mark are questions and some assertions are actually questions’ (Mason, 2002a, p248) which was a point also made by Boaler and Brodie (2004). Sometimes I made a statement which in actual fact was a question because I was seeking a response, for example:

Teacher: So you knew that 16 is made up of ...10 + 6 makes 16

Angela: (nods)
In this instance Angela appeared to know that a response was expected because she nodded in agreement but there are times when children do not respond and a follow up question is needed in order to elicit an answer. So in cases like this perhaps I needed to make it clearer to pupils that I required an answer by asking an appropriate question.

When I asked a question was I always looking for a response? There were instances in some of my lessons where I asked a question but did not expect a response because I knew the answer already, for example;

Teacher: Who is making all that noise, Charlie?

I knew when I asked this question that it was Charlie who was making excessive noise but I used the question as a way of getting his attention and controlling his behaviour. This is the type of controlling question, referred to by Mason (2002a), which teachers use to impose their authority in class. My reason for using a question to control behaviour instead of a statement such as Charlie, stop making that noise, was that, to my mind, it seemed a more positive approach. Whether it was any more effective than using the direct reprimand is debatable but at the time I believed it was. Mason (2002a) described the different types of questions as follows.

Controlling questions are used to control pupils' behaviour or learning or to impose the teacher's authority. Ruzlan (2007) used the term closed-routine to categorise questions that were asked in order to manage the classroom. I tended to use controlling questions when I wanted my pupils to refocus on a task. Why did I use this type of questioning as a way of maintaining or controlling behaviour? Why did I not use assertions or statements instead or just give them a direct instruction as Mason (2002a, p252) recommended. Using questions to maintain control can confuse pupils and according to Mason (2002a)
they quickly become aware that they are being controlled by these questions. He believed this hinders the 'creation of a questioning, conjecturing atmosphere in the classroom, one which supports rather than obstructs mathematical thinking' (Mason, 2002a, p252). An example of this in my practice was when I asked Joseph how he was getting on with his work, knowing from his behaviour that he wasn't actually getting on with his work.

Teacher: Joseph how are you getting on? Children, remember you need to show me underneath the problem what you did to work it out.

I did not press Joseph for a verbal response and he appeared to know one was not expected since he refocused on his task. Joseph seemed to know that my question was a way of controlling his behaviour and that I was not really asking about his work. In hindsight it would have been more effective if I had followed through with my initial question and actually found out how Joseph was getting on with his work. This then would have led to a discussion about mathematical thinking and perhaps created the questioning atmosphere referred to by Mason (2002a).

Funnelling questions are described by Mason (2002a, p253) as a series of questions which become more 'precise and detailed' as the teacher tries to simplify the problem in order to elicit some response from the learner. The questions become increasingly focused with the aim of directing the learner to a particular answer. Smith et al (2004) and Tanner et al (2005) described funnelling questions as low-level questions where the teacher selects the thinking strategies and controls the decision making process to lead the discourse to a predetermined solution (Tanner et al, 2005 p723).
I recently noticed myself engaging in this type of questioning during a lesson in Religious Education. I reflected in and on action as Mason (2002b) advised and changed what I had planned to say, to instead giving them the answer I had been looking for. It had been quite a surprise to find myself asking funnelling questions and I wondered if I did much of this in mathematics lessons. In order to determine if this was the case I reviewed two transcripts and found no evidence of funnelling in either one, which did not necessarily mean that I never used this type of questioning, just that it was not present in those lessons. The fact that I was now aware of funnelling meant that I could notice when I used it in future and amend my practice accordingly.

Mason (2000) and Tanner et al (2005) distinguish between low-level funnelling questions and more skilled focusing questions. The latter type of questioning draws pupils' attention to some important part of a response or problem which has emerged and merits discussion. Focusing questions are effective if pupils are made aware of this shift in focus and the reason behind it. Facilitating the discussions that follow such teacher questioning demands significant skill from the teacher, according to Tanner et al (2005) and if used effectively can enable pupils to engage in problem-solving (Boaler and Brodie, 2004).

Genuine enquiry questions are used when teachers genuinely do not know the answer and are seeking information for example, 'How did you work it out?' or 'Why did you do it that way?'. Mason (2002a) argued that this type of questioning could lead to a defensive stance unless the culture of the classroom encouraged this type of questioning and justification of results. My pupils were becoming more used to the fact that they had to justify their answers in mathematics lessons. Some were at the point where they would immediately explain their reasoning without any prompting while others, as Mason
(2002a) observed, became defensive and reluctant to say anything further. I believed that continuing to use this type of questioning and encouraging pupils to justify their responses would help the more reluctant learners to become more confident about explaining their reasoning.

**Meta questions** are questions about ‘process’, for example, ‘What led you to choose this approach?’ These types of questions encourage pupils to think about and explain how they reached a particular solution, placing the emphasis on the strategies used rather than the answer itself. It had taken most of the academic year for my pupils to accept the fact that how they arrived at an answer was more important for me than the answer itself. The example below shows how I encouraged pupils to explain why they chose a particular strategy thus further extending their thinking.

Teacher: Where did the 8 come from? What gave you the idea that it was 8 you had to take away Chloe?

Boaler and Brodie (2004) and Tanner et al (2005) referred to this type of questioning as probing, where a teacher questions a pupil further in order to gain a more detailed or clear response or explanation. Moyer and Milewicz (2002) used the term *competent questioning* to describe the way pre-service teachers in their research asked probing questions to gain a better insight into pupils’ responses. These questions could be considered to be genuine enquiry questions as the teacher would not already know the answer and would be interested in what the pupil had to say.

Table 5.2 below gives a summary of the how the types of questions I asked and my reasons for asking them relate to what the literature says about questioning. It is worth noting that numbers 3 and 4 in the table may also contain some closed questions. For
example, *did you know the answer was 8* may be a genuine enquiry question but it is closed in so far as it only allows for a yes/no answer. The questions referred to in number 5 in the table could become closed questions if they changed from focusing pupils' attention on a new concept to funnelling them towards a required answer.

Table 5.2: Relationship between my questions and those found in the literature

<table>
<thead>
<tr>
<th>Types of questions I asked and why I asked them</th>
<th>Questions discussed in the literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 To extend pupils' learning in mathematics – open questions</td>
<td>Open questions (Cotton, 2001; Gall 1970; Kilpatrick, 2003; Martinello, 1998)</td>
</tr>
<tr>
<td>2 To find out what pupils knew – closed questions</td>
<td>Closed questions (Cotton, 2001; Gall 1970; Kilpatrick, 2003; Martinello, 1998)</td>
</tr>
<tr>
<td>3 To find out something from pupils that I did not already know – open questions</td>
<td>Genuine enquiry questions (Mason, 2002a) Meta questions (Boaler and Brodie, 2004; Moyer and Milewicz, 2002; Tanner et al, 2005)</td>
</tr>
<tr>
<td>4 To draw pupils' attention to something new that I had noticed - I asked a series of focusing questions until they gave me the answer I was looking for – open questions</td>
<td>Focusing questions (Mason, 2000; Tanner et al, 2005) Could lead to funnelling questions (Mason, 2002a; Smith et al 2004; Tanner et al, 2005)</td>
</tr>
<tr>
<td>5 To make sure pupils were listening and following the lesson – clarifying questions</td>
<td>Could be regarded as controlling questions (Mason, 2002a; Ruzlan, 2007)</td>
</tr>
<tr>
<td>6 As a means of managing behaviour and to bring pupils back on task if they appeared to be distracted or were distracting others – behaviour managing questions</td>
<td>Controlling questions (Mason, 2002a; Ruzlan, 2007)</td>
</tr>
</tbody>
</table>

I was keen to develop my use of open questions as I believed this type of questioning would develop my pupils' thinking skills. Therefore I planned to record some more lessons in the second half of the spring and the first half of the summer terms 2010, with a view to changing my questioning. During these lessons I focused on developing my use of open questions and on cutting down on those I used to manage behaviour and to clarify what the pupils said. I examined the transcripts of two lessons (Appendix 10 and 11) that I had recorded, in February and at the beginning of May 2010 respectively. Table 5.3 below
shows the type of questions asked in each lesson and the number of occurrences of each type. I compared the questions across all three lessons to see if there were any significant changes in the number of occurrences of each type. The percentage of occurrences is included for ease of comparison.

Table 5.3: Type of Questions Asked in Lessons

<table>
<thead>
<tr>
<th>Type of Question</th>
<th>Slipper Problem</th>
<th>What is the Question?</th>
<th>Word Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n = 112</td>
<td>%</td>
<td>n = 82</td>
</tr>
<tr>
<td>Open questions</td>
<td>43</td>
<td>38%</td>
<td>26</td>
</tr>
<tr>
<td>Closed questions</td>
<td>42</td>
<td>37.5%</td>
<td>35</td>
</tr>
<tr>
<td>Behaviour managing questions</td>
<td>14</td>
<td>12.5%</td>
<td>10</td>
</tr>
<tr>
<td>Clarifying questions</td>
<td>13</td>
<td>12%</td>
<td>11</td>
</tr>
</tbody>
</table>

In the Word Problems lesson (Appendix 11) I had asked considerably more questions than the previous two lessons and there appeared to be a slight decrease in the percentage of behaviour managing and clarifying questions asked. However the number of open questions decreased by 11 percentage points while the number of closed questions saw a noticeable increase of 18 percentage points. I was very surprised by these findings and found them difficult to explain. I had expected a small increase in the number of open questions used or at the very least no noticeable change but this drop in the number of open ended questions puzzled me. It appeared that I was asking different questions to those I thought I was asking. A study by Nicol (1999), on how prospective teachers in Canada learned to teach mathematics, noted how the questions they asked of their students in reality differed to the beliefs they held about the type of questions they should ask. Mason's (2000) suggestion, to 'notice' and to 'attend' to what we as teachers are asking, could help to overcome this issue and ensure more effective questioning strategies on my part.
I noticed when reading these transcripts that there were episodes during the lessons when my voice was not picked up by the microphone which was suspended above one of the tables during all of the recorded lessons. As a result when I moved away from this area my comments were inaudible on the recording. I was curious to know if this made any difference so I planned to record another lesson which would focus more on what I was saying. I addressed the issue of the microphone by attaching it to me and recording myself separately during the *Mother Goose* lesson (Appendix 12). I transcribed the recording and coded the questions as per the previous transcripts, setting out the number of occurrences of each type in Table 5.4 below.

<table>
<thead>
<tr>
<th>Type of Question</th>
<th>Mother Goose n = 258</th>
<th>Mother Goose %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open questions</td>
<td>93</td>
<td>36</td>
</tr>
<tr>
<td>Closed questions</td>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>Behaviour managing questions</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>Clarifying questions</td>
<td>85</td>
<td>33</td>
</tr>
</tbody>
</table>

The number of questions picked up by the recording during the *Mother Goose* lesson (Appendix 12) was considerably higher than that recorded on the previous lessons and the percentage for the open questions was more in line with those of the first two lessons. The percentage of 'clarifying' questions showed a noticeable increase compared to the previous lessons and the percentage of questions I used to manage behaviour had decreased to 6%. I was interested in the fact that my efforts at developing my use of challenging questions did not appear to have had an impact on the types of questions I asked. Although I was disappointed by this discovery, I noticed from the literature that the percentage of open questions I used was well above those used by other teachers.
Boaler and Brodie, 2004; Cotton, 2001; Gall, 1970; Smith et al, 2004). Cotton (2001) advocated asking predominantly lower order questions of young learners and ensuring they were questions which they could answer correctly. Gall (1984) had earlier drawn the conclusion that fact questions might be more effective for younger learners but to also 'include some higher cognitive questions to stimulate development of their thinking skills' (Gall, 1984, p41). Although I can understand that the reasoning behind Gall’s (1984) and Cotton’s (2001) views may be related to the fact that younger pupils may still be learning basic skills, I do not accept that they could not also cope with higher level thinking. Research by Hamm and Perry (2002) provided evidence that first-grade pupils could 'be engaged successfully in coming up with creative solutions and in presenting and defending their ideas to others’ (Hamm and Perry, 2002, p130). I agree with Hamm and Perry’s (2002) point and also the belief expressed by Jeffcoat, Jones, Mansergh, Mason, Sewell and Watson (2004), that appropriate questioning strategies which promote mathematical thinking should be open to all pupils. With this in mind I wanted to explore in more detail what effect asking higher order questions would have on my pupils.

Does increasing the number of higher order questions I ask impact on my pupils' learning in mathematics?

I looked again at the transcripts of the lessons in Appendix 9-13 so that I could examine how my pupils' responded to any open-ended questions I asked. In the following example, pupils had to make up their own problems based on the Slipper Problem (Appendix 9). In response to my question, Lexa was able to explain how she had worked with her partner using a 100 square to work out the answer. Her strategy was confidently and clearly explained and received agreement from Emily.

Teacher: Emily, read out one of your problems please.
Emily: Pat had 44, he...

Teacher: 44, what? ... 44 apples, bananas, shoes, buttons? What?

Emily: Seeds

Teacher: Seeds, that's fine, that will do.

Emily: Pat had 44 seeds he got 17 more. How many does Pat have?

Teacher: Okay, how are you going to work out that answer?

Lexa: It could be 61.

Teacher: Why? ... Was that a guess or did you work it out?

Lexa: Worked it out

Teacher: I want to know what you do to work it out.

Lexa: Me and Aisling went on 44, then we went down 10 on 54 and then we added on 7 and led us to 61.

Teacher: Okay, is that the right answer?

Emily: Yes.

My asking pupils how they were going to work out the answer was a genuine enquiry question (Mason, 2000, 2002a) and the fact that it was followed up with I want to know what you do to work it out stressed the importance of the process rather than the solution. It encouraged Lexa to explain her reasoning and the process in which she engaged in order to reach her solution. In the example below, from Class Teams.
(Appendix 13), when asked what information was important in the problem, Rose responded as follows.

Rose: She sorts them into 4 groups

Teacher: She sorts them into teams of 4. Now that's different, you said she sorts them into 4 groups. Is that the same thing as sorting them into teams of 4?

Children: No.

Teacher: What's the difference between teams of 4 and 4 teams?

Patrick: That groups... that groups have got 1, 2... they have got how many numbers in each.

Teacher: How many numbers in each group?

Patrick: 4

Her reply was different to the actual problem which stated that the teacher sorted the pupils into teams of 4. My question, which asked if this was the same as sorting them into 4 teams, was appropriate and designed to challenge pupils' thinking. However I had unwittingly given them the response by saying 'now that's different' before asking the question. It would have been more appropriate if I had just asked the question without commenting on how Rose's response had differed from the original statement. Having given pupils time to work on the problem, I asked for someone to explain their strategy and Aisling volunteered with an appropriate method and explanation.
Teacher: Okay, who would like to explain what they did?

Aisling: First we drew 20 circles and then we pretended they were children and we put circles to know that there were 4 in the groups and the answer was 5.

Teacher: So you drew your 20 and you put 4 in each group?

Aisling: Yes.

Teacher: and then you put a ring around them so that that was the group.

Aisling: Yes.

Teacher: Okay, so that is what it looks like ... (drawing on the board) ...the question is how many teams will there be? So, what was the answer?

Children: 5

I asked for other ways of working out the answer and Jack replied with the following strategy.

Jack: 2 x 4 equals 8.... 8 times 2 equal 16

Teacher: Where did you get 8 and 2 from? What has that got to do with the question?

Jack's reply was not picked up on the recording but it appears from the dialogue that followed that he had worked out how many in two teams first then multiplied it by 2 to get the value for 4 teams.
Teacher: So you got 2 teams at a time. So you got 2 teams first...

Jack: And then I got 4 teams.

Teacher: So, how many teams is that altogether?

I used probing questions here, as recommended by Boaler and Brodie (2004) and Tanner et al (2005), in an attempt to gain a deeper insight into the thinking behind Jack's strategy. He appeared to have difficulty verbalising his thinking so I asked other pupils to put forward their ideas as to what Jack did.

Teacher: Let's look at it altogether. If I put it on the board can anybody else explain where Jack got his answer from? So, the first thing you had was...

Jack: 2 times 4

Teacher: 2 x 4... so where did he get 2 x 4 from? Jessica.

Jessica: He took away the 0 from the 20.

Teacher: Did you?

Jack: No.

Teacher: What are the important numbers we're looking at?

Children: 4

Teacher: There are going to be how many in each team?

Children: 4
Teacher: 4... so there is 4, and he has got 2 of them, so he has got two teams of 4. So, that is 2 times 4, which is...

Children: 8.

Teacher: 8. And underneath he has got 8 times 2. Where did he get that from? Angela

Angela: He got that from the 2 x 4. He got the 8 from the 2 x 4. It's an extra 2.

Teacher: Well, no, it is not an extra 2. It is this again isn't it? Because he got 2 groups of 4 here and that makes 8, and then he got another 2 groups of 4 which makes another 8. So altogether, he has got 2 8s, can you see that? And what is 2 8s altogether?

Children: 16

Teacher: 16. Then, what did he do after that?

Jack: Added another 4.

Teacher: Added another 4, which is another... team. How many is that altogether?

Patrick: 20

Teacher: 20 ... So he had 2 teams here... 4 + 4 ... then he had 2 more teams here. So he had 4 teams altogether plus he had enough for another team. So, how many teams? Count.
Children: 1, 2, 3, 4, 5

Teacher: 5 teams.

The example of dialogue above began with my attempts at engaging other pupils in the discussion of Jack's strategy but I then appear to have fallen into the trap of what Gall (1970, p717) referred to as a 'bothersome teaching habit' of interrupting pupils and answering the question myself. Although my questioning was open to begin with as I focused on aspects of Jack's solution, by putting forward my explanations I was not allowing pupils to engage in an open discussion. This was a recurring theme in my lessons. Asking pupils to explain their strategies was having a positive effect and they were doing so more readily but I was still the one asking all the questions and they were responding but not asking questions themselves. This may have been because the questions I was asking were not always as open as I had wanted them to be. I wanted to change this so that lessons were more interactive, with my pupils feeling confident enough to share and discuss their strategies and go beyond just responding to my questions.

Mason (2002a, p252) claimed that creating a 'questioning, conjecturing atmosphere in the classroom' 'supports mathematical thinking' and this is done by encouraging pupils to put forward their conjectures or ideas which can then be modified if necessary in the light of other conjectures. In a conjecturing classroom learners are taught to be supportive of each other and to seek 'modifications to conjectures' rather than just state that an answer is 'wrong'. I agree with Mason's (2002a, p257) view that it is necessary to 'establish a practice where learners are willing to struggle out loud because they know that others will help them (not mock them)' but this in my experience is easier said than done. The class I had then contained a small group of pupils, namely, Joseph, Patrick, Charlie, John and Ben, who were not very supportive of others and seemed to derive
pleasure from what they perceived to be others’ mistakes. Where did this attitude come from and what was I doing in order to change it? My initial response had been to reprimand these pupils but when this had little effect I tried ignoring them and focusing on the child who was speaking. A more effective strategy I discovered was to direct the question to them and seek their ideas which then took the focus away from the behaviour and placed it on the task under discussion. Another effective strategy, according to Allen (2010), was to challenge and confront these pupils, questioning them about the reasons for their negative attitude to others, otherwise their negativity could continue. Mason’s idea of getting pupils, who disagree with the ideas of others’, to say something like ‘I disagree with your conjecture’ or ‘I invite you to modify your conjecture’ would, I believe, lead to a more supportive learning environment.

Although the language used in these statements is more suitable for older pupils I believed the ideas could be incorporated into my classroom practice. I adopted this approach to some degree by using statements such as ‘prove it’ and ‘have another think about it’ when children have given responses which may not have been ‘accurate’. My aim however was for my pupils to use statements like these with each other rather than them being initiated by me. There was evidence of this happening on a small scale in one particular lesson when two girls were working together to make up problems of their own based on one we had done as a class. The following extract from the lesson transcript of the Slipper Problem (Appendix 9) shows how the girls shared their strategies and ensured that both played an active role in finding the solution.

Aisling: 16 and we have 25... Where is 25? 25, go down, and add 6.

Lexa: I know...
How many slippers does Lexa have now?

Okay, let's get it so we have... 25

No, but how do we work out...?

No, but I know what we do. We go 10 down and then 1, 2, 3, 4, 5, 6.

41 is the answer.

Prove it

You do it

Okay, 61+...

61? 16 not 61

25 + 10 and 6... 1, 2, 3, 4, 5, 6... 41.

Proved.

So we do...

25 + 10 + 6 =

Lexa’s request that Aisling ‘prove’ her answer was more in terms of convincing her than a proof in any mathematical sense and may have seemed unnecessary since Aisling had already explained her strategy. Aisling’s request that Lexa do it instead was appropriate in this instance because it meant that both girls shared their strategies and they were satisfied with the result.
Interactions like those described above are what contribute to the 'conjecturing classroom' that Mason (2002a) is referring to and are the types of interactions I wished to hear more often among the learners in my mathematics lessons. Although the two girls did not engage in serious questioning, they were clearly supportive of each other and a statement like 'prove it' is a simple way for young learners to ask 'How did you work it out?' or 'What strategy did you use to find the answer?'. Research by Martino and Maher (1999) discussed the importance of teachers asking more open-ended questions and how effective use of these can build a 'classroom community that invites active participation, confidence and further learning' (Martino and Maher, 1999 p75). They described how pupils were encouraged to share their ideas with their peers particularly when they disagreed over strategies or their solutions differed. It was the teacher’s strategic intervention and appropriate questioning that helped to develop this community of learning. It is my belief that once young pupils begin to develop a questioning frame of mind they can gradually be introduced to appropriate strategic questions and taught how to use them.

One of the issues which arose for me, as a result of reading through my transcripts, was how to encourage my pupils to engage in the use of questions especially higher order questions. Mason (2002a, p248) also raised this issue when he asked the question 'How do we stimulate learners to ask their own questions?'. He suggested asking pupils to say what question they thought the teacher would ask. The purpose in doing this, he suggested was

...to signal a metacognitive shift from current content to the sorts of questions I habitually ask, in order to encourage the students to ask themselves those questions (Mason 2000 p14).
As part of a mini-cycle of action research I tried this out with my pupils at the beginning of a lesson when I asked what question they thought I was going to ask. Five pupils put their hands up immediately with a response while the others looked and said nothing. The interaction that resulted went as follows.

Teacher: Okay, what question am I going to ask you about it?

Joseph: How... how are you going to work it out?

Teacher: Interesting question. Does anybody think I'm going to ask a different question?... Mala

Mala: What did you do to get the answer?

Teacher: Another interesting question. Tell me Joseph, why did you think I would ask that question?

Joseph: Because that's what we're going to work out.

Teacher: Okay. Why did you ask that question Mala?

Mala: Because when we did partitioning questions you asked us, how did we work it out?

When pupils were discussing their work at the beginning of a lesson the first question I asked them was either, how will/did you get the answer? or how will/did you work it out? Both Joseph and Mala appeared to have internalised this question and were now able to ask it for themselves. I asked the question, what question am I going to ask you about it? again a few days later and some more pupils were able to tell me what they thought I would ask. When they worked in pairs I reminded them to ask each other a question and
most could be heard asking their partner *how will you work it out?*. I wanted to get pupils to a stage where they could ask this question themselves without prompting by me. Mason (2000) suggested that when pupils are familiar with one question to then introduce a new question and work on it in the same way. By doing this, learners build up a repertoire of known questions which they can ask of themselves, thus developing their questioning skills. Evidence from research by Martinello (1998) and Boaler and Brodie (2004) demonstrated how pupils who were regularly exposed to higher-order questioning by their teachers began to ask these types of questions when working with their peers.

Martinello (1998) stressed the importance of teachers being good models of questioning for their students and being trained in these skills. However this idea of training teachers in skills of questioning is not new. In 1970, Gall also stressed 'the need for effective teacher training programs to implement desired questioning strategies in the classroom' (Gall, 1970 p718). Cotton (2001) claimed that despite the significant role played by questioning in teaching and learning, trainee teachers in the United States received little or no training in developing questioning skills. Research by Smith *et al* (2004) demonstrated how insufficient in-service teacher training, when implementing the NNS, failed to have any significant effect on 'encouraging and extending pupil contributions to promote higher levels of interaction and cognitive engagement' (Smith *et al*, 2004, p408).

It would seem that this is an ongoing issue which is likely to continue for the foreseeable future as my own experience, when observing lessons as Induction Tutor for student and newly qualified teachers, leads me to believe. Student teachers who complete their final teaching practice in our school focus predominantly on closed, fact-finding questions and need a lot of practice to move on to probing questions and questions which extend pupils' thinking.
Although it is widely acknowledged that asking higher-order questions can lead to the development of critical thinking skills (Kilpatrick, 2003; Martino and Maher, 1999; Mason, 2002a; Wimer, Ridenour, Thomas and Place, 2001), asking higher-order questions does not always result in higher-order responses, according to Gall (1984). Issues such as the clarity of the question asked, pupil’s attention to or understanding of the question asked and the amount of time allowed for a response can all affect the level at which the pupil responds to the question. The importance of wait-time is highlighted in research as playing a crucial role in how pupils respond to questions (Cotton, 2001; Ewing, 2005; Gall, 1984). The more difficult the question the more time the teacher should wait for the pupil’s response. Three seconds was considered sufficient for low level questions whereas pupils performed better on higher level questions when there was no time limit (Cotton 2001). Tobin (1986) found that increasing the wait-time after a question led to teachers asking more ‘appropriate’ questions. Gall (1984) and Wimer et al (2001) claimed that both higher and lower order questions have value and are necessary and the role of the teacher is to know when and how to use both effectively. Boaler and Brodie (2004) highlighted the importance of using a range of different questions as a more effective strategy and Sorto, McCabe, Warshauer and Warshauer (2009) suggested we look at how we use questions. They claimed that by focusing on the big idea of the lesson, the context of the question, and how the question fits into the overall flow of the lesson teachers will be able to understand how to use questions to enhance student understanding and build better lessons (Sorto et al, 2009, p50).
The link between questions and tasks

Alongside questioning as a central element of a critical thinking learning environment is the mathematics task on which pupils work. Kilpatrick (2003), Martino and Maher (1999) and Sullivan, Warren and White (2000) suggested tasks should be open-ended to allow for extended pupil thinking and teacher questioning. This is an important point. If the mathematics task is closed then it would be more difficult for a teacher or pupil to ask open questions and engage with it at a higher cognitive level.

When I began looking at my use of questioning my main concern was about asking more open ended questions but I now realise there is a lot more involved. The issue is not just about asking more open ended questions but about asking better questions in general. Research by Tobin (1987) has shown that asking more higher cognitive level questions coupled with extended wait time can often lead to fewer questions in total being asked as more time is allowed for cognitive thinking. However if the task involved is at a low cognitive level then it makes it difficult for teachers to engage in higher order questioning which in turn reduces pupils' higher cognitive thinking. This was a point I needed to consider in relation to my own practice. I was aware from previous reflection that I did not always allow sufficient time before expecting a pupil response as discussed by Cotton (2001), Duell (1994), Ewing (2005) and Tobin (1987), which may have affected how my pupils responded as well as the type of questions I asked. I had already begun to change the level of task on which pupils worked and I was now aware that I needed to extend my wait time, so in order for me to develop more pupil interaction I must also further develop my use of higher cognitive questions. I felt that my use of open questions was not having the effect I desired, which was to create a conjecturing classroom, as described by Mason (2002a), where my pupils could freely engage in mathematical
thinking. I realised that this was going to be more difficult than I first anticipated and would need to continue through the next cycle of action and reflection.

**Summary**

In this chapter I focused on my use of questioning in my mathematics lessons. The reason for this was due to my concern over the lack of mathematical questions asked by my pupils of each other. I focused on the following research questions and attempted to answer them.

4. To what extent do I use higher order questions to extend pupils’ learning in mathematics?

5. Does increasing the number of higher order questions I ask impact on my pupils learning in mathematics?

I looked at the types of questions I asked over a number of lessons and categorised them into four groups namely; open, closed, clarifying and behaviour managing questions. Analysing the data showed that I asked more open questions than was common for many teachers, especially of younger pupils (Boaler and Brodie, 2004; Cotton, 2001; Gall, 1970; Smith et al 2004). However I was not using them consistently or as effectively as I wished.

Reading some of the literature on questioning helped highlight for me some of the aspects I needed to change with regard to how I used challenging questions in my own practice and how I could in turn develop my pupils’ use of questioning. I believe I was able to answer the first of these questions regarding the extent to which I used higher order questions but the second question was more difficult to answer as measuring impact over such a short period was not easy.
Gall (1984) and Cotton (2001) and Hamm and Perry (2002) appeared to have conflicting views as to whether younger pupils could cope with higher order questions. My research supports that of Hamm and Perry (2002) and Jeffcoat et al (2004) and provides evidence showing how some of my learners, although very young, were able to respond to questions that were more challenging. It contributes to research on the subject of questioning because it demonstrates how it is possible for very young pupils to be challenged through appropriate questioning. I discovered that it was not the quantity of higher order questions that was important but their quality. Of equal importance was the quality of the tasks on which pupils were expected to work and the impact of these would require further research.

The subject of teacher questions was an important one which needed addressing as a result of the findings of my last research cycle. My initial plan had been to record progress made with regard to the strategies used by my pupils when solving problems but because of the need to focus on my use of questioning I had to defer this to a later stage. My plan for the next action research cycle was to examine how changes to my practice through developing a more questioning environment would impact on my pupils' learning in mathematics. I also planned to examine the development of learners' strategies for solving problems resulting from the changes to my practice and the conclusions I could draw from the research to date.
Chapter 6 Cycle 3 - Strategies

Introduction

The purpose of this chapter is to examine the strategies my pupils used when solving mathematical problems. The previous action-reflection cycle focused on my use of questioning as a means of challenging my pupils' thinking and developing their use of questioning in class. This next cycle of action-reflection focuses on the impact, if any, that changes to the use of questioning or tasks in my practice had on the strategies the pupils used when solving problems. I was also interested to find out if they engaged in more questioning or discussion of their strategies when working with a partner than they had done previously. I used the following research questions as a focus for this action-reflection cycle.

6. What impact does changing my practice have on the strategies pupils use when solving problems?

7. To what extent does changing my practice enable pupils to engage in discussions when solving problems?

Before researching the literature on strategies I first wanted to find out what strategies my pupils used when solving problems so I carried out an initial analysis as described in the following section.

Data Collection

I recorded four more mathematics lessons towards the end of the school year in July 2010, in order to find out if there were any changes in the way my pupils worked as a result of the changes in my practice. This was perhaps not the best time of the year to
record as pupils were restless and excited about the holidays and everything was winding down in preparation for the summer closure. The last recorded lesson took place on the second-last day of term and continued over two sessions in order to allow pupils time to complete the task.

I transcribed these lessons and carried out an initial analysis in order to see what themes would emerge. I focused in particular on the last two recorded lessons because of the way in which pupils engaged with these tasks. I have used these to form the basis of this chapter in order to make comparisons between how pupils worked at the beginning and at the end of the school year.

Initial Analysis

My initial analysis of the transcripts looked at pupils' talk and the extent to which they focused on the mathematics. I was interested to find out if the changes to my practice had impacted on the way pupils worked in class. Much of their discourse at the beginning of the year had consisted of chit-chat, bickering and comparing how far ahead on the task they were of others in their group. This reflected the findings of Bennett and Cass (1989) that pupils in British schools were often being unproductive, uncooperative and frequently coming off task when they worked together. There had been very few instances picked out in those earlier lessons that could be identified as genuine mathematical talk where they discussed their strategies or reasoning. The lessons recorded at the end of the year were evidence of the way in which pupils' behaviour during mathematics lessons had changed over the year.

The initial analysis showed that pupils remained on task for longer periods than they had done earlier in the year and there were also fewer instances of 'bickering' and 'off-task
talk'. Most of pupils' discussions focused on the activity in which they were engaged, unlike lessons earlier in the year where some found it difficult to interact with others as the following example shows.

Emily: I was before you, you just snatched

Joseph: Oh shush! You never even listen to anyone by the way

Jack: How do we write anything we use to help you?

Joseph: I don't know. I'm not you.

(Appendix 14: Time Problems)

This class was not very cohesive as a group and there were a number of assertive characters, one being Joseph (above), who constantly vied for dominance. Towards the end of the year these pupils had developed a more positive attitude towards each other and were more open to assisting others in the group as can be seen in the following example.

Joseph: Oh yes, we have extras

Mala: We have extras as well.

Jack: How many extras?

Mala: We have 7.

Joseph: We have 3. We have these... these...

Chloe: I have one extra
There were also fewer instances of teacher talk. In earlier mathematics lessons a significant amount of teacher talk was used to manage behaviour and it focused on the behaviour in question rather than on the mathematics task. The following extract from a lesson recorded at the beginning of the academic year highlights the way attention was focused away from the task to deal with a minor behavioural issue.

Teacher: Mala, are you making that noise?

A child: No it's Rubin

Teacher: No it wasn't Rubin. It was coming from your table.

Sophie: Adam

Teacher: It wasn't Adam.

One of the changes I have made in my practice is to focus pupils' attention back on the mathematics task as a means of managing any minor behavioural issues. Instead of reprimanding them for their inappropriate behaviour, I ask them questions such as:

_How are you getting on with your work?_ 

_Can you explain what you have done so far?_ 

_What are you going to do next?_
I was interested in the apparent increase in mathematical talk and had categorized it as strategies in my initial analysis. I carried out further analysis and subdivided this category into separate identifiable strategies as follows.

Counting/counting on

Counting in steps or skip counting

Adding/Using known number facts

Train spotting

Generalising and Specialising

I researched some of the literature available on these strategies and the following section gives a brief outline of what the research had to say about the strategies pupils use when solving problems.

**Literature Review**

**Counting Strategies**

According to Arnett (1905, p327) counting is considered to be 'the most fundamental of all the arithmetical processes both mathematically and psychologically' as pupils need, not only to know the number names, but also that each number stands for a quantity of objects, before they can progress to manipulating these numbers. There are different levels of counting strategies which progressively become more complex and lead to more efficient solution strategies. They are; direct modelling, counting on or back, rhythmic counting and skip counting or use of number patterns.
Direct Modelling Strategy/Counting-all

Carpenter, Ansell, Franke, Fennema and Weisbeck (1993) discuss the importance of modelling as a process by which young children can solve problems. Carpenter et al (1993) and Carpenter and Moser (1984) described this as a *direct modelling strategy* which involves pupils using counters, objects or pictures as representations of the problem situation. This strategy is widely considered by researchers such as Carpenter et al (1993), Carpenter and Moser (1984), Mulligan and Watson (1998), Secada, Fuson and Hall (1983) and Steinberg (1985) to be the most basic of problem-solving strategies and involves pupils counting all of the counters, objects etc. in a representation in order to find the answer.

Counting-on/back

Counting on or back from a given number is considered by many, including Nunes and Bryant (1998), to be a progression from counting-all. By using this strategy, pupils demonstrate that they understand the cardinality of the first number and need only count on or back from this number to reach the answer. According to research by Carpenter and Moser (1984), Mulligan and Watson (1998) and Secada et al (1983) counting on/back strategies need not involve direct counting of objects but may ‘involve more abstract methods’ (Secada et al, 1983, p48) such as, counting hops on a number line, or keeping mental count while saying the number names in sequence. A progression within counting-on/back is considered to be counting from the larger number and Carpenter and Moser (1984) found that pupils who were able to use this strategy did not always choose to do so. In fact their research showed that pupils, instead of adopting the most efficient strategy to solve a problem, will often use a range of strategies ‘interchangeably’.
Rhythmic Counting

Rhythmic counting, as described by Anghileri (1989) and Mulligan and Watson (1998), is particularly used by pupils when they are attempting to solve problems involving multiplication or division. It is considered to be a progression from unitary counting (Anghileri 1989) in multiplication as counting-on is considered to be a progression from counting-all in addition. When pupils engage in rhythmic counting they count each object or number by either stressing certain numbers in a pattern or pausing after a group has been counted (e.g. 1, 2, 3, 4, 5 ... 6, 7, 8, 9, 10 ... or 1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

Skip or Step Counting

A further progression from rhythmic counting is skip counting where pupils count in steps or multiples following a sequence of numbers (e.g. 5, 10, 15...) as described by Mulligan and Watson (1998). Anghileri (1989) describes this strategy as one of using a number pattern where pupils are applying their knowledge of number facts. This strategy is considered to be more advanced than rhythmic counting as pupils do not need to count every number and demonstrate an understanding of the fact that each number in the pattern represents the total for each group being counted.

Using Known Number Facts

Using known number facts is considered by many (Anghileri, 1989; Carpenter and Moser, 1984; Houlihan and Ginsburg, 1981; Mulligan and Watson, 1998; Steinberg, 1985) as being a further development in skills from counting and according to Anghileri (1989) it follows on from using number patterns or skip counting when solving multiplication problems. Pupils progress from counting in steps of 5 for example, as in 5, 10, 15... to knowing that $5 + 5 = 10$ and $10 + 5 = 15$ and ultimately to the fact that $5 \times 3 = 15$. 
Knowledge of known number facts can lead pupils to derive facts for other numbers with which they are less familiar. Carpenter and Moser (1984) refer to derived facts strategies (DFSs), Thornton (1978) describes thinking strategies and Houlihan and Ginsburg (1981) use the term invented strategies when explaining the process of using a simpler known fact to derive a more difficult or unknown fact. They discuss the importance of instructing pupils in the use of these strategies as a means of improving their learning of number facts and how they use them in problem solving situations.

Although deriving number facts from known facts is considered to demonstrate a greater understanding of number concepts, research by Carpenter and Moser (1984) and Steinberg (1985) showed that the use of these strategies was not confined to higher attaining pupils but helped many pupils to learn number facts which they did not already know. Thornton (1978) however, makes the point that although many pupils could learn and use them successfully lower attaining pupils chose not to when given the choice. This point is further elaborated on by Houlihan and Ginsburg (1981) when they suggested that children do not always use the most efficient strategies when solving a problem but instead 'choose their strategies according to the size of the problem's addends' (Houlihan and Ginsburg, 1981, p104).

**Train Spotting**

Train spotting is the term used by Hewitt (1994) to describe how pupils become more focused on a pattern when solving a problem than on the problem itself. They become sidetracked and see the pattern as being more important, consequently they lose track of what they were meant to be investigating. According to Hewitt,
spotting patterns in the numbers becomes an activity in its own right and not a means through which insights are gained into the original mathematical situation (Hewitt 1994, p49).

In Hewitt’s (1994) research the problem lay with the way the pupils were being asked to record their work. More emphasis, he believed, was placed on recording tables of results and the patterns that emerged than on the mathematics involved in the problem situation. Train spotting is discussed in more detail in Chapter 7.

Specialising and Generalising

When pupils are engaged in legitimate pattern identification, as opposed to ‘train spotting’, it can lead to generalisation of mathematical concepts. Much has been written about the importance of generalising in algebra by researchers such as; Carpenter and Levi (2000), Houssart and Evens (2005), Kaput and Blanton(1999), Mason, Drury and Bills (2007) and Warren (2004) to name but a few. Mason has ‘long promoted the conjecture that expressing generality lies at the heart of school algebra’ (Mason et al, 2007, p1) as have Schliemann, Carraher, Brizuela, Earnest, Goodrow, Lara-Roth and Peled (2003) who are of the opinion that ‘generalising is at the heart of algebraic reasoning’ (Schliemann et al, 2003 p2). Mason et al (2007) also acknowledge the importance of generalising as a higher order skill that shows a deeper understanding of mathematics but it should not be solely confined to number patterns. Mitchelmore (2002) described three ways, outlined below, in which generalisation could be taught.

The Abstract Before Concrete or ABC method requires that pupils first learn the generalisation in its abstract form before it can be applied in a concrete situation. The
disadvantage with this method is that it results in ‘abstract-apart knowledge’ (Mitchelmore, 2002, p5) which can be quickly forgotten by learners.

The *Exploratory method*, on the other hand, focuses on a series of concrete examples and leads the learner to form a possible generalisation based on these examples. The danger with this approach, according to Mitchelmore (2002), is that pupils may make inaccurate generalisations.

The *Problem-solving method* is perceived by Mitchelmore (2002) as allowing pupils to acquire a ‘deeper understanding’ of mathematical concepts and any generalisations they make will be theoretical, which according to Davidov (1990, cited in Mitchelmore 2002 p6), provides a superior method of teaching mathematics’.

I believe that Mitchelmore’s (2002) problem-solving method to be the more effective and it is what I am aiming for in my practice. It allows learners to work in contexts which are relevant and concrete, helping them to develop deeper and more secure understanding of the generalisations that emerge. Evidence from the *Underwater Treasures* lesson (Appendix 15) demonstrated to me how my pupils could engage with a problem involving the abstract concept of multiples by using concrete representations to help them.

Generalising and specialising, according to Mason (1999), are opposite sides of the same coin. Whereas generalising deals with the wider context and encompasses all possibilities, specialising deals with a specific case or particular situation which may be an example of a generalisation. Mason (1999, p6) stated that,

the purpose of specializing is to gain clarity as to the meaning of a question or statement, and then to provide examples which have some general properties
in common, so that you can begin to see and appreciate those common properties – the process of generalizing.

Although Zazkis, Liljedahl and Chernoff (2008) agree with Mason’s view, they urge caution and make the point that ‘not every set of examples will lead to a successful generalisation’ (Zazkis et al, 2008, p140). This echoes the point made by Mitchelmore (2002) in his explanation of the exploratory method of teaching generalisation.

Research shows that young pupils are capable of making generalisations (Carpenter and Levi, 2000; Mitchelmore, 2002; Schliemann et al, 2003; Warren, 2004,) from patterns observed while engaging in a mathematics activity. However, it is also common for young learners to give particular examples when asked to explain the concept in general terms (Carpenter and Levi, 2000; Mason et al, 2007). This does not necessarily mean that they cannot generalise but may be due to their inability to express the generalisation in the correct mathematical terms. Carpenter and Levi (2000) in their research found that as well as being able to generalise a concept young learners were able to exemplify the generalisations of others. This is not always the case, as Houssart and Evens (2005) reported in their research on early algebraic thinking. Although some of the pupils in their report were able to explain and justify a general statement many more than was expected were unable to give examples. This was surprising, since giving examples or specialising is considered by many (Carpenter and Levi, 2000; Houssart and Evens 2005) to be the less challenging task.

**An analysis of the strategies used by my pupils when solving problems**

For this analysis I used the last two lessons which were recorded at the end of the school year and will be referred to as Patterns (Appendix 16) and Underwater Treasures
Having categorised the strategies the pupils used in those lesson and read some of the relevant literature, I looked at the strategies which they had used at the beginning of the year and compared them to those used at the end of the year in order to see if there were any changes. The lessons used from the beginning of the year will be referred to as *Four-pin-Bowling* and *Time Problems* (Appendices 8 and 14).

**Counting Strategies**

I focused initially on the counting strategies which the pupils used in each lesson and have subdivided them as follows.

**Direct Modelling Strategy/Counting-all**

There was evidence of the use of a direct modelling strategy in two out of the four lessons. In *Time Problems* some pupils used card clocks with moveable hands to help them work out the answers to the problems and moved the minute hand from one number to the next as an aid to counting the minutes and hours. In *Underwater Treasures* the pupils were given 30 cut out pictures of shells (Appendix 15) to share between two to assist them in working out the problem. All of the pupils used the pictures as a direct modelling strategy described by Carpenter et al (1993), organising them in different ways but only some used counting-all as a means of finding totals. The following example, taken from *Underwater Treasures*, exemplifies Carpenter and Moser’s (1984) view and demonstrates how all pupils will use counting-all as a strategy regardless of their level of attainment and their ability to count at a higher level.

Jack (Higher attaining):  
So, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Mala (Average attaining):  
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23...
The fact that pupils were working with numbers below 10 in *Four-pin-Bowling* was a possible reason why they had not needed to use this strategy in the first lesson as most of them had worked confidently with numbers up to 10 in Year 1.

**Counting-on/back**

There was evidence of this strategy being used in all four lessons with the greatest number of occurrences to be found in the earlier lessons. In the excerpt below from *Four-pin-Bowling*, Emily appeared to be using the count-on-from-first strategy but her question, *What did I say? 3 plus 4*, suggests that she may have had some understanding of the commutative properties of addition and if so why did she not begin with the larger number as suggested by Carpenter and Moser (1984)?

Emily: 4 plus 3 equal 7, it is. Count 4 in your head, 5, 6, 7.

Emily: What did I say? 3 plus 4

*Patterns* appeared to have the fewest instances of counting-on and the pupils appeared to be counting in steps as the following example demonstrates.

Jack: I would say 8, 14, 20

It seemed that Jack had spotted the pattern but this was not clear from the transcript as counting-on, according to Secada *et al* (1983) may ‘involve more abstract methods’ (Secada *et al* 1983 p48) and Jack could have been counting on mentally from each number but only saying the pattern numbers.
Rhythmic Counting

The use of rhythmic counting as a strategy occurred most often in Underwater Treasures. This could be due to the fact that the activity in this lesson was more conducive to this type of counting than those carried out in the other three lessons. Because pupils had to group objects in multiples of given numbers many used skip counting as a more advanced strategy than counting-all to help them reach the answer. The example below demonstrates how Mala grouped the shells in threes and used rhythmic counting to find out the total.

Mala: 1, 2, 3... 4, 5, 6... 7, 8, 9... 10, 11, 12... 13, 14, 15... 16, 17, 18...
      19, 20, 21... 22, 23, 24...

The following extract demonstrates Chloe and Mala’s progression to using repeated addition as a further step from rhythmic counting, as described by Anghileri (1989), and shows their use of a second count to highlight the number of groups they had counted.

Chloe: 4 plus 4 plus 4 plus 4...
Mala: 4 plus 4 plus 4 plus 4 plus 4 plus 4 plus 4 takeaway 1
Chloe: Yes that's right, 1, 2, 3, 4, 5...
Mala: 6

Skip or Step Counting

Counting in steps or skip counting was confined to the latter two lessons out of the four and demonstrates pupils’ awareness of repeated addition. Some progressed further and
were using the vocabulary of multiplication as the following example shows, although it is not clear from this example if Mala was aware of this.

Mala: You have to do eight 4s?... It is eight 4s, do eight 4s

This second example from Mala showed that she had a good understanding of the concept of multiplication and could recall some multiplication facts.

Mala: Just write 5 times 5 because 5 times 5 is 25

Charlie, also, had a clear understanding and when asked was able to give the answer using mental recall of number facts.

Charlie: I put 5... I had... I put 5 in each group and then what I did is, first I put 5 in each group.

Teacher: How many groups of 5 did you have?

Charlie: 5 groups of 5

Teacher: What do 5 groups of 5 make altogether?

Jack: 25

These examples support Anghileri's (1989) view that knowledge of number patterns can lead to the development of known number facts.

Using Known Number Facts

There was evidence of this strategy being used in all four lessons but with significantly greater use evident in *Four-pin-Bowling* than in *Time Problems*, where most pupils demonstrated a secure knowledge of number facts to 10 which they had learnt in Year 1.
In *Underwater Treasures* the distinction between counting in steps and using known number facts was blurred and some pupils were found to use both, which again confirms Anghileri’s (1989) belief that children progress from knowing number patterns to knowing number facts.

Joseph: ...2, 3, plus 3

Jack: 6

Mala: 3 plus 3

Joseph: plus 3.

Jack: You have to write 2

Joseph: That’s 2

Mala: 3 plus. How many threes are done?

Chloe: 3 plus 3 is 12... no is 6

Jack: 3... 3... 3

Mala: plus another 3 equals 6

**Train Spotting**

There is a danger that if pupils become too focused on finding number patterns they will lose the focus of the problem which is what happened in three out of my four lessons. In *Four-pin-Bowling* pupils became so concerned with finding ways to make 5, 6 and 7 that they overlooked the fact that they could only use the numbers 1 to 4 and that each number could be used only once. This meant that answers such as 3 + 3 = 6 and 0 + 7 = 7,
although factually correct, could not be accepted. The following excerpt shows how Daniel, in *Time Problems* (Appendix 14), lost sight of the problem when asked to exemplify the problem he had been working on.

Daniel: 53 clocks... no, pictures, hanging on a wall. 25 fell off.

Emily: Fell off... 25 fell off.

There were more instances of this train spotting behaviour (Hewitt, 1994) in *Underwater Treasures* when the pupils became so caught up with grouping in multiples that they neglected the clues which had been given to help them as the following exchange between Chloe and Mala demonstrates.

Chloe: That's why you should do it in like twos because if you do it in fours it might not work...2

Mala: Where's the 2?

Chloe: 2

Mala: That's 4

Chloe: You are meant to be helping me. You do half. Do some at the top but don't go past this line

Mala: It looks like they are in stairs.

Chloe: Just put it down there

Mala: 2, 4, 6.

Chloe: See, it works with 2s
Mala: 2, 4, 6.

Chloe: No, you start from, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

Chloe: How many twos? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12... 12 twos

The two girls had found the answer but because they were so focused on the pattern they had not realised this and went on to spend more time grouping in other multiples before finally reaching a solution.

**Specialising and Generalising**

Spotting a pattern can lead to a generalisation being made and there were instances of specialising and generalising in both of the later lessons. Specialising occurred where pupils selected a specific number to test it as a multiple of 3, 4 or 5 as can be seen below.

Mala: We'll test the 19 first.

Aisling: Let's find 24... (?)

Joseph was the first pupil to express a generalisation as follows.

Joseph: Because if we did it in fours last we would have to have an even number and 29 is an odd number

Later when asked to generalise about the answer he made the following statement.

Joseph: It is in the 3 times tables, and the 4 times tables and it is not in the 5 times tables.

When asked to use more mathematical language his reply was to say, **24 is a multiple of 3 and a multiple of 4, but it is not a multiple of 5.** He had made an incorrect generalisation
earlier when he thought the answer was a multiple of 5. This can happen, according to Mitchelmore (2002) and Zazkis et al (2008) and can be as a result of pattern spotting. Joseph demonstrated an ability to specialise and to move from the particular to the general. Charlie also demonstrated an ability to generalise when he stated earlier in the lesson that he thought the answer was a multiple of 3. However when asked to show this with examples he was unable to do so despite having a clue that told him to group in 3s. This demonstrates the point made by Houssart and Evens (2005) that not all pupils find exemplifying easy.

The fact that the Patterns lesson focused on finding number patterns made it more conducive to generalising and more pupils were able to make generalisations based on the patterns they had made. Jack particularly demonstrated an ability to make generalisations from patterns on the 1-25 square (Appendix 16). He observed that all numbers going to the right had a difference of 6. When asked to explain his statement Jack gave some examples as a justification for his generalisation. Joseph expanded on this with the observation that all patterns going diagonally to the left had a difference of 4 and made a further generalisation as follows.

Joseph: If the pattern starts on one, they are all odd numbers. If it starts on 2 they are all even numbers.

The fact that there were more instances of generalising in the last two lessons may have been due to the fact that both the activities and the teacher questioning encouraged pupils to engage in such behaviour. This supports the views of Carpenter and Levi (2000), Kaput and Blanton (1999), Mitchelmore (2002) and Zazkis et al (2008) that pupils must be given opportunities and the right activities to enable them to make their own generalisations which will have greater meaning for them.
**Summary and Conclusions**

The focus of this cycle of action research was to explore the strategies used by pupils at the end of the year in order to see if there were any significant developments as a result of the changes in my practice. I was also interested to find out if there had been any increase in the amount of mathematical talk as they engaged in an activity. The research questions I used were as follows.

6. What impact does changing my practice have on the strategies pupils use when solving problems?

7. To what extent does changing my practice enable pupils to engage in discussions when solving problems?

The second of these questions proved easier to answer than the first. I focused on lessons recorded at the beginning and at the end of the year and noticed a decrease in the amount of teacher talk and a considerable increase in the amount of pupil talk focusing on the task. This, I believe, was partly due to a change in the tasks the pupils were being asked to do and a change in the types of questions I was asking.

In response to the first of these research questions, in my opinion the nature of the activities given to pupils to work on had, in some way, influenced the strategies which they used to solve the problems. Although I appreciate that it is difficult to measure impact on pupils particularly over such a short period of time there were recognisable changes in their behaviour which I believe were due to the changes in my practice. The activity in *Four-pin-Bowling* (Appendix 8) required pupils to work with numbers below 7 so it was not surprising that they should be confident using known number facts as most were familiar with them from the previous year. The activity was not conducive to
generalising and so did not afford pupils the opportunity to demonstrate their ability to engage in this strategy. The activities in the final two lessons on the other hand were more conducive to generalising and challenged pupils in a way that previous activities had not done. My research shows what young learners are capable of achieving when they are given mathematically challenging tasks and questioned appropriately.

Reading the literature also made me aware of a greater need to encourage a culture of generalising and conjecturing, as described by Mason (1999), which I had not fully appreciated before. In the next chapter I explore the literature on specialising and generalising in more detail and the extent to which pupils engage in such problem solving strategies during my mathematics lessons.
Chapter 7 Cycle 4 - Generalising in Mathematics

**Introduction**

In Chapter 6 I briefly discussed the importance of generalising in mathematics as part of a discussion on strategies. The purpose of this chapter on generalising is to elaborate on this aspect of mathematical thinking and examine its significance in terms of its existence in my practice and the extent to which my pupils engaged in it. In my last cycle of action research I focused on the questions I used in class and how changing the types of questions I asked might influence my pupils' mathematical thinking. During my initial analysis of the strategies my pupils used to solve problems I noticed instances when some pupils made general statements about, or specialised with numbers in the course of solving a problem. Subsequent analysis of my questioning to see how changes to it affected pupils' thinking in mathematics also highlighted for me a greater use of specialising and generalising by some pupils.

I wanted to investigate this idea of generality and how the literature related to my findings. This chapter provides a more detailed discussion on generalising in mathematics based on the research available and the extent to which my pupils generalised when solving problems. In this chapter I attempt to address the following research question.

8. To what extent do I provide opportunities for my pupils to specialise and generalise in mathematics lessons?

Evidence supporting this research question is provided from data collected from lesson transcripts, samples of pupils' work and my research diary. Evidence from my research demonstrates how young pupils in early years' education can specialise and generalise and fills a gap which appears to exist in the literature available on this topic.
The Literature Review

The following literature review examines some of the research available on the subject of generalising in mathematics and the important role it plays in the development of mathematical thinking. Different types of generalising are discussed and suggestions are given as to how pupils might be taught to generalise.

Generalising in Mathematics

Generalising in mathematics is considered by researchers such as Burton (1984), Mason (1999, 2007) and Mason, Drury and Bills (2007) to be a crucial element in mathematical thinking. Burton (1984, p35) described this mathematical thinking as a 'process' or a 'means through which mathematics is derived', rather than just thinking about mathematics. She highlighted the four processes of specialising, conjecturing, generalising and convincing as being intrinsically part of mathematics. Mason (1999, 2007) and Mason, Drury and Bills (2007) believed that these processes were so much a part of mathematical thinking that

when teachers and texts do the specialising and generalising, the conjecturing and even the convincing for students, they enculturate students into parking their own powers at the classroom door as 'not wanted here' (Mason 2007, p 916).

Specialising

Specialising is when specific examples or cases are examined in relation to the problem being solved. These examples are more 'concrete and confidence-inspiring' for the learner (Mason, 1999, p1), and allow them to simplify the problem or to 'see' it in their own terms. Specialising can also be used to exemplify a generalisation which is
considered to be the opposite process of specialising and it involves anything from numbers to diagrams or apparatus. It must be noted, however, that what works for one learner may not necessarily work for another so it is important that the examples used have relevance for the individual. Mason (1999, p5) highlighted the importance of specialising \textit{appropriately}, by which he meant, to specialise ‘drastically enough so that what you obtain makes use of confidence-inspiring, manipulable objects’. Zazkis, Liljedahl and Chernoff (2008) discussed the importance of the types of examples used when specialising and claimed that the ‘choice of example is crucial in creating experiences that enable students to generalize’ (Zazkis \textit{et al.}, 2008, p132). As well as supporting a generalisation, examples can also, according to Zazkis \textit{et al.} (2008), be used to disprove a false generalisation. They referred to these examples as \textit{pivotal examples} as they bring about a change of viewpoint in the learner’s thinking, which I think is necessary if an appropriate generalisation is to be made.

\textbf{Conjecturing}

When the learner explores sufficient examples, a pattern or relationship may emerge which enables a theory or hypothesis to be made regarding the relationship. This hypothesis may be tentative or provisional and may need amendments but of greater importance is the act of putting forward conjectures because as Mason (1999, p8) stated ‘mathematical thinking is best supported by adopting a conjecturing attitude’. However, in order for learners to develop this conjecturing attitude, they must be allowed to express their thoughts and ideas in a supportive and questioning environment without fear of ridicule or rejection. This may not always be the case in a classroom situation as pupils may not be confident enough to put forward a conjecture for fear of being wrong. An environment that supports conjectures fosters a conjecturing attitude among learners,
helping them learn from their mistakes rather than being afraid of making them (Mason, 1999).

Carpenter and Levi (2000), in their research with first- and second-grade pupils aged 6-8 in the United States, discussed how one particular teacher developed her pupils' ability to conjecture by using open number sentences as a focus for discussion. The pupils were encouraged to develop 'rules' relating to these number sentences which then became conjectures. Carpenter and Levi's (2000) study found that although some pupils were able to make general statements when explaining, others were not always able to justify their conjectures, often relying on using examples as a means of explaining or simply restating the conjecture. I have noticed this to be the case with some of my pupils and in one particular instance, during a lesson on multiples, Molly made the general statement that multiples of ten end in zero.

**Generalising**

A generalisation is made when a particular pattern or relationship between elements is recognised or identified. The learner then applies this pattern or relationship in a wider context. Mason (1999, p18) described generalising as 'the twin process of specializing' and claimed that because it stems from an attempt to work in a more general context it is 'intimately connected with understanding'. His view was that

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generalizing is about making connections, and capturing them in a succinct statement from which particular instances can be retrieved by specializing
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(Mason, 1999, p19).
Mason (1999 p19) however warned against being so enamoured with generalising that one neglects the process of specialising since he believed that familiarity with ‘specific examples’ leads to ‘powerful generalizations’ (ibid).

### Convincing

When a generalisation is made, the learner must first prove its validity to themselves and then to the wider community. They must convince themselves and others that it stands up to scrutiny, going beyond simple explanation to more robust justifications. This assists the learner in recognising misconceptions or flaws in their theories and ultimately in their thinking. Evidence from Blanton and Kaput’s (2000) research demonstrated how one pupil’s engagement with another through ‘argumentation and justification’ helped correct misconceptions when constructing general statements about odd and even numbers.

Burton (1984, p38) outlined two approaches in which these four processes of specialising, conjecturing, generalising and convincing can be used as follows.

**Inductive learning:** SPECIALIZING → CONJECTURING → GENERALISING

**Deductive learning:** GENERALIZING → CONJECTURING → SPECIALIZING

Inductive learning involves approaching the problem by testing specific examples first then making a conjecture and building from there to a general statement. An example of inductive learning would be if pupils were asked to add two odd numbers together and say what they noticed each time. They could test their conjectures and come up with a general statement for all pairs of odd numbers. Deductive learning approaches the problem from the general statement and breaks it down by testing specific examples. An example of deductive learning would be if pupils were given a general statement to
explore such as; adding two odd numbers will always give an even answer. They would then make conjectures as to whether it was true or false and investigate their conjectures by testing specific examples. Mason's (2002c) view is that both approaches are necessary for mathematical thinking and if teachers promote one approach over the other they are in danger of leading pupils to believe that one is superior to the other. Amit and Neria (2008) provided evidence from their research with 11-13 year-old pre-algebra students of inductive/deductive learning in action. The students involved worked on linear and non-linear pattern problems and demonstrated

a process of induction to find the general case, followed by deduction to find specific cases, pointing to a strong intuitive understanding of the importance and power of generalisation (Amit and Neria 2008, p127).

There is clear evidence from the literature which highlights just how important generalising is to mathematical thinking. Burton made the point that

offering overt opportunities for specializing, generalizing, conjecturing, and convincing enables the thinker to encounter aspects of his or her own thinking more deeply (Burton, 1984 p45).

I agree with Burton as I believe that in order for learners to reach the level of making general mathematical statements, they need to have a greater understanding of the mathematics involved. Mathematical understanding, however, exists on different levels, as does the ability to generalise.

The following section outlines some of the various types of generalising used by learners when solving problems involving generality.
Types of Generalising

Different types of generalisations are used by pupils at different stages in the problem-solving process. The terms near (Stacey, 1989), local (Amit and Neria, 2008) and empirical (Bills and Rowland, 1999; Dörfler, 2008) generalisations are used to describe the initial stages of generalising, whereas terms such as far (Stacey, 1989), global (Amit and Neria 2008) and theoretical (Dörfler, 2008) generalising describe more advanced levels of mathematical thinking.

Near and far generalising

Stacey (1989) in her research with 9-13 year-old students used the terms near and far generalisations when describing how they solved generalising problems. By near generalisation she meant finding an element in a pattern by ‘step-by-step drawing or counting’ (Stacey, 1989 p150), for example, finding the number of eyes for 10 dogs (Blanton and Kaput, 2004). There is a danger at this stage that pupils may become engaged in what Hewitt (1994) called train spotting, which will be discussed in more detail later on, and which means that they become more focused on numbers and pattern-finding than the solution they are seeking. A far generalisation, according to Stacey (1989), refers to a problem that is difficult to solve by such practical means and involves a greater understanding of general rules and relationships, for example finding the number of eyes for 100 dogs (Blanton and Kaput, 2004). Pupils, who engage in far generalising in this instance, demonstrate an awareness of the relationship between the number of dogs and the total number of eyes and can find the larger values without having to follow the vertical pattern of counting in twos. Research by Barbosa, Vale and Palhares (2009 p12) with 11-12 year old students found that using ‘visual strategies’, where students could ‘see the structure of the pattern’, led to greater success in solving
far generalisations. Mason (2007) made the point that when pupils are moved on too quickly to symbolic representations they fail to gain a full understanding of the purposes of algebra and how it can be used to solve problems. From my experiences with my young pupils, I have to agree with Mason’s point and believe it is important therefore that pupils are allowed to explore visual and physical representations of the problem in order to retain a view of the problem as a whole.

Recursive-local and functional-global generalising

The term recursive-local generalisation was used by Amit and Neria (2008) in their research with ‘talented pre-algebra students’ aged 11-13 to describe how repeating patterns were used in number sequences to find a near or local generalisation. However this approach did not help them to find what Amit and Neria (2008) called a global generalisation. The students who visualised the problem and used diagrams to gain an understanding of the structure of the pattern were more successful at finding the global generalisations because of their ability to express the relationship between the elements in the pattern as a function. Amit and Neria (2008, p126) believe that it is this ‘functional connection that is at the heart of generalization’ and students who engage in it are expressing functional-global generalisations. I found it interesting that these students had no previous experience of algebra and yet were engaging in algebraic thinking as a result of the tasks they were given. I believe that if pupils were given the appropriate tasks and opportunities to generalise at an earlier age then their ability to express functional-global generalisations would be more advanced by the time they reached the same age as the students in Amit and Neria’s (2008) research.
Empirical and theoretical generalising

Bills and Rowland (1999) and Dörfler (2008) used the term *empirical generalisation* to describe a generalisation that is based on recognising patterns and is dependent on a 'number of cases'. It can also lead to *train spotting*, as described by Hewitt (1994), if the focus is allowed to remain on the pattern rather than on the relationship between the elements within the pattern. If the learner focuses instead on the structure of the pattern as Mason *et al.* (2007) advised then it can lead to a higher level of generality. This is what Dörfler (2008 p147) called *theoretical generalisation* which deals with the 'generic case' and because it is a result of 'actively interacting with or transforming the situation' it is relational in character. Bills and Rowland (1999, p3) called it *structural generalisation* and it results from 'looking at the underlying meanings, structures or procedures' rather than just the initial patterns as is the case with an *empirical generalisation*. A pupil who explored the sequence 3, 6, 9... for example, and worked out the subsequent values by adding three to the previous values could be said to have made an empirical generalisation, whereas a pupil who recognised the pattern as increasing in multiples of three and recorded it as \( f(n) = 3n \) could be regarded as having made a structural or theoretical generalisation. I agree with Dörfler's (2008) view that pupils can and should be taught how to make theoretical generalisations and that pattern tasks have an important part to play as they enable pupils to recognise relationships and begin to generalise.

Train Spotting

Looking for patterns is important but focusing on pattern-spotting rather than the *relation* between the elements in the pattern can lead to what Hewitt (1994) termed 'train spotting'. He described how helping students to focus on 'how they counted, rather than the results of counting' (Hewitt 1998, p20) enabled them to arrive at a general
statement more efficiently. In Hewitt's (1994) research the problem lay with the way the pupils were being asked to record their work. More emphasis, he believed, was placed on recording tables of results and the patterns that emerged than on how the patterns were achieved. As a result of pattern-spotting, pupils can become sidetracked and see the pattern as being more important, consequently they can lose sight of the generality they are seeking. According to Hewitt,

> Spotting patterns in the numbers becomes an activity in its own right and not a means through which insights are gained into the original mathematical situation (Hewitt 1994, p49).

Warren (2005) and Warren and Cooper (2005) noted from their research that when young learners record patterns on a table they have a tendency to look down the table for the next value in the sequence rather than finding the pattern or relationship across the table. I do not necessarily see this as a problem but more as developmental stage that very young pupils need to go through when they are first learning to generalise. The problem comes, I believe, if this pattern-spotting is just seen as an end in itself rather than as a means to an end, that is, that of making a general statement. It is up to the teacher to make pupils aware of the wider picture and that recognising a vertical pattern is just one aspect of that wider picture.

It is clear from the literature that when pupils engage in near, local or empirical generalisations they are using the pattern they have spotted to find the next elements or values. This in my opinion is perfectly acceptable as a starting point and a way into the problem but if pupils are allowed to focus solely on this then there is a danger of becoming sidetracked and losing sight of the task in question. Pupils who focus instead on the structure of the pattern and the connection between its values are more likely to
reach a far, global or theoretical generalisation. Amit and Neria (2008) and Barbosa et al (2009) stressed the importance of allowing pupils to use visual strategies when generalising and demonstrated in their research how pupils who engaged in such visualisations were more likely to be successful when making generalisations than those who focused on recursive patterns. When I work on a task I find it helpful to manipulate objects or to draw a representation that will enable me to 'see' the problem more clearly. As a result I encourage my pupils to do the same and provide them with visual representations whenever possible.

Warren and Cooper (2005, p160) suggested using 'change' activities as a way of focusing attention away from vertical pattern. They are random and therefore more important as they 'focus children on generalising between terms (not generalising along a sequence of terms)' thus developing functional thinking. I found this idea interesting and I looked at evidence from my practice of where I had attempted such an activity. One pupil rearranged the terms so they formed a pattern while the other continued making his own pattern from the random arrangement. I agree with Warren and Cooper's (2005) idea of using activities to focus pupils away from vertical patterns but perhaps in this instance the terms were not random enough to allow this to happen.

The Importance of Generalising in Mathematics

One of the questions raised by Mason et al (2007) is why teachers appear to be so resistant to generalising in mathematics lessons. Considering the views of researchers such as Hargreaves, Shorrock-Taylor and Threlfall (1998); Kaput and Blanton (1999); Mason (2002c); Mason et al (2007); that generality is so intrinsic to mathematics education one would expect that it would play a greater part in teaching and learning. I do not agree with Mason et al (2007) that teachers are being deliberately resistant. It is
my belief that many, and I included myself in this category until recently, are unaware of just how important the ability to generalise in mathematics really is for learners. The National Curriculum (DfEE/QCA 1999, p62) programme of study for mathematics at Key Stage 1 states that

Pupils should be taught to:

Understand a general statement and investigate whether particular cases match it.

It is listed as the ninth item under Using and Applying Number and is in the subsection of Reasoning. At Key Stage 2 (DfEE/QCA 1999, p67) it states that

Pupils should be taught to:

Understand and investigate general statements.

This statement is listed as the tenth item again under Using and Applying Number in the subsection of Reasoning. Generalising is mentioned in The National Curriculum attainment targets (DfEE/QCA 1999, p9) in AT 1 at level 3 when it states that

Pupils show that they understand a general statement by finding particular examples that match it.

There is no further mention of generalising as a requirement at subsequent levels in the Primary National Curriculum. If generalising is considered by researchers such as; Hargreaves et al (1998); Kaput and Blanton (1999); Mason (2002c) and Mason et al (2007); to be so central to mathematics one would expect it to have a higher profile in the National Curriculum being delivered in primary schools. It is my view that in its current state, as part of the mathematics’ curriculum, generalising is being overlooked by
teachers as more emphasis is placed on learning procedures and acquiring mathematical
skills. This has certainly been the case in my school and in the twenty years that I have
planned mathematics lessons with colleagues not once has a discussion taken place about
generalising. It had never been a part of my training or discussed by curriculum leaders
during in-service training sessions. Until recently I was unaware of just how important it
was that pupils make and explore general statements and I very much doubt if I was the
only teacher in that position. It was as a result of the work carried out during this action
research project and discussions with my supervisor which highlighted for me the gap in
my pupils learning that I, in my ignorance, was allowing.

The NNS (DfEE 1999 p7) when it was in use, stated in Section 3 that pupils in Year 1
should ‘investigate a general statement about familiar numbers or shapes by finding
test examples that satisfy it’. Pupils in Years 2 and 3 were also expected to ‘generalise and
predict’ whereas pupils in Year 4 were also expected to ‘make’ general statements as well
as investigate them. Years 5 and 6 had the added task of

making general statements about odd or even numbers, including the
outcome of sums and differences (DfEE 1999, p22, p26).

Although generalising was mentioned more in the NNS, when viewed alongside all the
other objectives it did not appear to have any great significance and in my opinion was
still overlooked. The renewed Primary Framework for literacy and mathematics which
was introduced in 2006 moved generalising to Year 4 and stated that most pupils should
‘learn to investigate a statement involving numbers and test it with examples’ (DfES 2006,
p78). Most pupils in Year 5 were ‘expected to propose a general statement involving
numbers or shapes’ and to ‘identify examples for which the statement is true or false’
(DfES 2006, p80). This invites the question, if generalising is so important in mathematics
why was it moved out of the earlier years? Are very young pupils considered too young to make or even understand general statements? There are many researchers who would not agree with this view including Mason et al (2007, p2) who believe that every learner who arrives at school walking and talking has displayed the power to perceive and express generality.

This is true in other areas as well as in mathematics. A recent interaction between our science teacher and one of my six-year old pupils resulted in the following exchange:

Teacher: Why have you coloured the dinosaur purple?
Pupil: Because dinosaurs are purple.
Teacher: No, dinosaurs are a brownish, green colour.
Pupil: No, they are purple. I have a dinosaur and it’s purple.

(Horne, 2011)

The pupils at the time were engaged in a task involving sorting animals from plants and recognising features. The pupil involved had made a general statement about what she perceived to be the true colour of all dinosaurs based on her experience. The fact that it was factually inaccurate is irrelevant in this instance, what is of greater importance is the fact that she was able to make a general statement, doing so in a confident manner and with justification.

Much of the available literature on generalising at primary level, for example Blanton and Kaput, (2000); Cooper and Warren, (2003, 2008); Hargreaves et al, (1998); Houssart and Evens, (2002, 2003, 2005); Kaput and Blanton, (1999); Mitchelmore, (2002); Schliemann
et al, (2003); Warren, (2004, 2005), involves pupils aged between 7 and 12. There is little research which investigates the ability of very young pupils under the age of 7, to generalise. This may be due to the fact that in some countries formal primary schooling does not begin until the age of 7. In Singapore, for example, pupils generally begin primary school at age 7+ (Ng, 2004). It may also be due to the fact that the ability to generalise may be considered by some to be beyond the capabilities of very young pupils.

Carraher, Schliemann, Brizuela and Earnest (2006) during their research into early mathematics education found that historically there was a view that ‘developmental constraints’ prevented pupils of a very young age from learning algebra, of which generalising is a crucial part (Carpenter and Levi, 2000).

Carpenter and Levi’s (2000) study demonstrated how young pupils, aged 6-8 in first and second grade classes in the United States, were able to make general statements about numbers and engage in discussions involving generalisations when encouraged to do so. Warren and Cooper (2005) also presented evidence from their research that showed how young pupils were capable of making generalisations. The research by Carpenter and Levi (2000) and Warren and Cooper (2005) stressed the need to develop algebraic reasoning alongside that of arithmetic rather than at a later stage as is the current situation.

According to Carpenter and Levi (2000)

the artificial separation of arithmetic and algebra deprives students of powerful schemes for thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in the later grades (Carpenter and Levi 2000, p1).

They are of the view that making generalisations is an intrinsic part of algebraic thinking and should be developed at an early age. They believe this would help overcome some of
the difficulties pupils experience with algebra later on in their education. Carraher *et al* (2006, p89) advocated this approach and believed that difficulties experienced by students when they were introduced to algebra later stemmed from ‘missed opportunities and notions’ which occurred early on in their education.

Usiskin (1995, p31) described algebra as the ‘language of generalisation’ and as ‘generalised arithmetic’ (Usiskin 1999, p9) and Molina and Ambrose (2006, p111) regarded an understanding of the equals sign as an integral part of algebra. However, a common misconception displayed by pupils concerns the equals sign and their interpretation of its meaning. Many pupils solely believe that an answer should follow the equals sign resulting from an operation performed on two numbers preceding it (Carpenter and Levi, 2000; Molina and Ambrose, 2006; Schliemann *et al*, 2003; Warren and Cooper, 2005). I have experienced this with some of my pupils at the beginning of this academic year (2011) when I wrote $12 = 7 + 5$ and asked them if it was true. One pupil replied that it was wrong because the equal sign was in the wrong place – it should come after the 5. I find this to be a common occurrence with each new group of pupils and it takes a considerable amount of work to change. Research by Warren (2003, 2006) demonstrated how widespread this narrow understanding of the equals sign is and how difficult it is to put right as learners can carry it with them through many years of school.

How pupils develop these misconceptions is not clear but Carpenter, Franke and Levi (2003) are of the view that over emphasis on one interpretation of the equals sign at elementary level is a contributing factor. I believe it is very important that very young pupils are taught the correct mathematical language and meaning from the outset and any misconceptions should be addressed immediately. According to Carpenter *et al* (2003) misconceptions such as those involving the equals sign not only hinder pupils’ learning of arithmetic but provide greater obstacles later on when algebra is introduced.
They believe that the most effective way of changing pupils’ misconceptions is by enabling them to challenge their own thinking and to justify it to others. I agree with this point and go further to say that teachers also need to challenge their own thinking and be confident about using correct mathematical terms and symbols with very young pupils. In my experience if pupils hear about them often enough in the right contexts then they have a better chance of remembering and using them appropriately.

The role of generalising has long been acknowledged by many including Usiskin (1995), Amit and Neria (2008), Blanton and Kaput (2004) and Hargreaves et al (1998), as not only central to mathematical thinking but more specifically to algebraic thinking. When talking about the importance of algebra Usiskin (1995) described it as ‘the language of generalization’ and ‘the language through which we describe patterns’ (Usiskin 1995, p31). Research by Amit and Neria (2008), Blanton and Kaput (2004), Hargreaves et al (1998), Houssart and Evens (2002), Kaput and Blanton (1999), London McNab (2006), Mason et al (2007), Stacey (1989) and Zazkis and Liljedahl (2002) highlights the importance of recognising patterns as central to generalising in algebra and particularly as a means of introducing algebra in early years (Warren, 2005; Cooper and Warren, 2008). I am not convinced by Mason et al’s (2007 p1) suggestion that ‘a lesson without the opportunity for learners to generalise is not a mathematics lesson’ but I strongly support the view that the ability to generalise is a crucial element of mathematical thinking. The question then arises as to how to develop generalising with young learners and teach them to think algebraically.

**How to teach pupils to generalise**

One of the first and foremost ways of developing generalising with pupils is to give them tasks which afford them the opportunities to make generalisations. Researchers such as
Barbosa et al (2009), Dörfler (2008), Houssart and Evens (2003), London McNab (2006) and Orton (1997) stressed the importance of providing pupils with ‘meaningful’ activities such as pattern tasks which they can visualise and use to see the structure of the pattern from which they can then go on to make general statements about wider cases. Dörfler (2008, p159) saw generalising patterns as ‘a strongly conceptual activity which does not result in a number but in a greater conceptual structure presented with variables’.

In order to avoid pupils becoming sidetracked by the pattern when they find a near generalisation, it is important that their attention is focused on the structure of the pattern and the relationship between the values. Houssart and Evens (2003) advised that teachers should work alongside pupils using visual representations as a means of ensuring structural understanding. They, along with Mason (2007), warned against prematurely introducing symbolic notation as a means of recording generalisations as this can lead to confusion with pupils who have not yet reached that stage of abstraction. Mitchelmore (2002), whose methods of teaching generalisation were discussed in the previous chapter, made a similar point when explaining the ABC method. He believed it to be the least useful of the methods he discussed as knowledge was acquired in an abstract way, out of context and was therefore easily forgotten. The most effective method of teaching generalisation, according to Mitchelmore (2002), is the Problem-solving method because it allows pupils to work in more concrete and relevant contexts thus creating their own more meaningful abstractions. Houssart and Evens (2003) also suggested that pupils be encouraged to share their solution strategies with each other in order to appreciate that there may be more than one way to reach a solution to a problem.
Mason (1999) stressed the importance of creating a *conjecturing atmosphere* in class where learners can share their ideas and conjectures without fear of ridicule by their teachers or peers. He made the point that

everyone should be encouraged to express what they have understood, or what they think might be true, so that others can question, and invite or suggest modifications (Mason 1999, p9).

I agree with Mason (1999) and believe that it is very important for the development of mathematical thinking that pupils should not only feel confident about sharing their conjectures but also know that they can question the conjectures of others without fear of reprisal from or causing offence to their peers. This I believe will only happen when the emphasis is removed from the solution to a problem and focused more on the strategies used to reach that solution. Teachers need to take the lead in creating this conjecturing atmosphere by valuing pupils' responses and helping them to realise that we learn from mistakes and it is not just about getting the correct answer.

Orton (1997) in her study of 9 to 13 year old pupils and their generalising strategies found that there were obstacles which prevented generalising, one of which she called *arithmetic incompetence*. Pupils who were not secure with known number facts or made mistakes with simple calculations found it more difficult to recognise a pattern and hence work out its structure. Another barrier she noted and which was also highlighted by Houssart and Evens (2003) was the fact that pupils did not check their results. This is a very important phase that pupils need to be made aware of and encouraged to do as part of their solution strategy. Another obstacle to generalising which has already been discussed, is that of train spotting (Hewitt 1994), where pupils become focused on the pattern they see and lose track of the wider problem. I believe that if teachers are aware
of these obstacles and work with pupils, even those who are very young, to overcome them, then these pupils can have a better chance of making theoretical generalisations which lead to algebraic thinking.

Most of the mathematics research available on algebra and generalisation involved pupils aged 9 and above. Carraher et al (2006) claimed that little was known about how young pupils are able to make generalisations and called for more research in this crucial area, so that it can be developed further. Their research involved young pupils aged 8-10 years old in an intervention study. Warren and Cooper's (2005) research involved 6-7 year old pupils and that of Blanton and Kaput (2004) involved pupils as young as 3 years of age in long-term intervention studies to develop functional thinking. The research that I have carried out with my young pupils is not an intervention study but an action research project and it will, I believe, begin to address the gap that exists in researching younger pupils' generalising capabilities. I have evidence from my practice involving one teacher and one Year 2 class that 6-7 year old pupils are capable of functional thinking when they are provided with an appropriate task.

The following section explains how changing my practice through Action Research allowed my very young pupils to engage in generalising.

To what extent do I provide opportunities for my pupils to specialise and generalise in mathematics lessons?

Until I read the literature on generalising not only was I unaware of its significance in the development of mathematical thinking but also of how central it was to the development of algebra. I had come through a system of mathematics education which very much emphasised the procedural understanding described by Skemp (1989) and problem
solving had been very much tagged onto the end of topics as a way of testing if we could use and apply what we had learnt. As a result, my teaching style followed the same pattern, which is why I disagree so strongly with Mason et al (2007) and the view that teachers are being resistant to generalising. I think for many it is a lack of awareness of how teaching learners to generalise can lead them into functional thinking. At the beginning of this research journey I would have felt confident in saying that my young pupils did not make general statements; that it was too challenging for their stage of mathematical development. Now it is my belief that generalising may have been occurring in some lessons but I was not hearing it. I was failing to notice, as Mason (2002b) described and was unaware of the levels of mathematical thinking that may have been taking place in my classroom.

This caused me to reflect more on my practice and what was actually happening in my mathematics lessons so I asked myself the question

9. To what extent do I provide opportunities for my pupils to specialise and generalise in mathematics lessons?

**Specialising**

In order to answer this question I looked again at the transcripts from lessons recorded in November 2007, the first year of this research. The very first lesson I recorded involved pupils finding possible ways of making the number 30 (Appendix 17). This task may not be what London McNab (2006) or Orton (1997) would have considered an appropriate activity for developing generalising but it was a much more open task than I would have given them previously. It allowed pupils to specialise as they tried to find numbers that totalled 30, as the following extract shows.
Bonny: Now let's go to 26 and count on. 1 2 3 4 ... 26 + 4 = 30

Shanta: 30 + 0... yeah

Rachel: Yeah... no 0 + 30, yeah

Bonny: 20 + 10

Shanta: I'm good you know. I'm doing these another way. I can do loads like this.

Rachel: I can do 29.

Brigita: I'm going to try 11.

Bonny: Now do you want to do 23?

Brigita: Ok

Shanta: 9... 23 + 9 = 30

However this random selection of numbers did not lead to any generalising for most pupils and resulted in some repetition of answers or in many being omitted as can be seen from Brigita's work below.
Rachel was able to see a pattern that helped her quickly arrive at the end result. However, one mistake in the pattern caused the rest of it to be incorrect as her work demonstrates (see Figure 7.2).

This is an example of Hewitt’s (1994) train spotting which I had not picked up on in my earlier analysis of the transcript as I was unaware of its existence. It appears that Rachel was so focused on the pattern that she lost sight of the problem and what she was meant to be doing. It is also likely that she had not checked her work as recommended by
Houssart and Evens (2003) and Orton (1997). If she had, I believe she would have spotted the discrepancies in the pattern, realised her mistake and corrected it.

The following extract is taken from a lesson recorded a week later with children from the same class and demonstrates another example of specialising, this time with the days of the week. The problem (see Appendix 18) involved pupils finding particular days on which karate lessons took place having been told when the first lesson occurred.

Justin: Anya, look, Anya, look...Monday...Monday, Tuesday, Wednesday, Thursday, Friday.

Anya: If it’s Friday, that’s because you’re not counting Sunday.

Malinka: It’s Wednesday

Diana: No...Sunday, Monday, Tuesday, Wednesday... It’s Wednesday

Anya: It tells you to start from Sunday

Diana: It’s Wednesday because it tells you to start from Sunday...1 2 3 4...it’s Wednesday.

Anya: You can count the week until we get to the 4th day.

These pupils had also lost track of the problem and what they were meant to be doing. Not only had they forgotten that Kevin’s first lesson was on a Monday but also that he had a lesson every 4th day. They appeared to be confusing finding his fourth lesson with counting four days to get to the next lesson. Some pupils were able to work out when the second lesson would be but were unable to explain how they would find the 3rd and 4th lessons. Other pupils did not get so far and although they had spent the time counting
days they had not arrived at any particular solution. These pupils found this task challenging and even specialising a lot of cases as a way of reaching a solution appeared to be beyond them. Because of my lack of awareness at that time of specialising and its significance I did not actively seek to develop this skill with my pupils.

Although these and subsequent tasks given throughout the year were more challenging and focused more on problem solving skills than previous activities, they did not allow pupils the opportunity to develop generalising skills. In response to the question I posed earlier as to whether I provided opportunities for my pupils to engage in specialising and generalising, the answer was clearly a negative one. I did not provide the appropriate tasks and I did not use the sort of questioning that would allow these skills to be developed. Engaging in cycles of action and reflection led to changes in my practice over the next three years with regard to the tasks and questions I used in mathematics lessons. I began to introduce more tasks such as those in Appendix 15, 16 and 19 which allowed my pupils opportunities to engage in specialising and generalising.

**Specialising and generalising**

The following extract taken from a lesson recorded in May 2010 with my Year 2 class is an example of very young pupils specialising with numbers in order to solve a problem about multiples (Appendix 15).

Joseph: We know it works in fours. We know it works in fives, we don't know if it works in threes and it doesn't.

Jack: Remember, we have 8 here plus 3

Joseph: Cross off 20, the answer is not 20
Joseph and Jack's attempt at finding the solution to the problem resulted in their trying out specific numbers using trial and improvement rather than any organised method. They arrived at a solution and Joseph was able to make a general statement about the answer stating that it was a multiple of 3 and a multiple of 4, but was not a multiple of 5.

Other extracts from a lesson on number patterns (Appendix 16), recorded in July of the same year, provide evidence of these pupils making general statements about the patterns they found.

Joseph: If the pattern starts on one, they are all odd numbers. If it starts on 2 they are all even numbers.

Jack: If you go from the 4 to 16 there will be a difference of 6.

Teacher: Is there a difference of 6 from 4 to 16?

Jack: No, if you go from 4 to 8... wait...

Jack: There is only a difference of 6 that way and that way (gesturing diagonally down to the right)
Teacher: Is there a difference of 6 that way? (Gesturing diagonally down to the left)

Jack: No.

Teacher: No, so which direction do you have to go in for there to be a difference of 6? Megan... What number are you starting on?... Right or left?

Megan: Left

Teacher: So if we start on the left and go diagonally to the...

Megan: Right

Teacher: There is a difference of...

Children: 6

Teacher: What happens if you start on the right and go diagonally to the left? Joseph

Joseph: There is a difference of 4.

Although I believed this to be a higher level of thinking than had previously taken place I was not aware at the time that they were specialising and generalising. It was not until later analyses of the transcripts of those lessons that I became aware of specialising and generalising as skills which my pupils were using to further their mathematical thinking.

Having made this discovery I read some of the literature on this topic, as discussed in the literature review above and realised just how significant the development of these skills was for mathematical thinking. I then engaged in a further mini cycle of action research to
see if further changes to my practice would result in my young pupils being able to specialise and generalise as the research suggested.

**Action Research Mini Cycle 3**

**Methodology**

An important point which came through the research literature was how crucial the tasks were which allowed pupils to generalise (Barbosa *et al*, 2009; Dörfler, 2008; Houssart and Evens, 2003; London McNab, 2006 and Orton, 1997). Having read this research I was curious as to how my young pupils would cope with just such a task so I tried the ‘Eyes and Tails’ activity described by Blanton and Kaput (2004), over two lessons (Appendix 20, 21).

**Data Collection**

The data were collected over two lessons with my Year 2 class of 6-7 year olds. The lessons were recorded over a two week period in November 2011 when most of these pupils were still 6 years old. One group of six pupils, considered to be high attaining, was recorded as they worked and transcripts were made of their dialogue. Samples of work from all pupils were taken as were notes made in my research diary after the lessons. The purpose of this was to triangulate the data by looking at it from more than one perspective, as recommended by Cohen *et al* (2005) and Denscombe (2005), in an effort to provide more reliable and valid evidence of my findings.

In the first lesson (Appendix 20) I introduced the first part of the activity where pupils had to work out the number of eyes for the number of dogs. I used the interactive whiteboard to show pictures of dogs so that pupils would have a visual representation of the problem...
as recommended by Amit and Neria (2008) and Barbosa et al (2009). In the second lesson (Appendix 21) they had to work out the total number of eyes and tails for the number of dogs and again pupils were shown a visual representation on the interactive whiteboard. After the initial introduction in the first lesson, pupils worked in pairs and were free to record their working out in pictorial or numerical form.

A further activity of this type involving money was given to pupils in January 2012 where they had to work out possible amounts of money in a purse with only 2p coins (Appendix 22). A visual representation was shown on the interactive whiteboard and pupils also had the use of 2p coins if they wished but just three pairs chose to use them. This lesson continued over two sessions as most pupils, at the end of the first session, had only found the total amount for 10 coins, a near generalisation as described by Stacey (1989). These sessions were not recorded but samples of work were taken from all pupils and some notes were made in my research diary at the end of each session.

The next section describes the analysis of the data and the findings in relation to my pupils specialising and generalising.

**An analysis of specialising and generalising by pupils as a result of changes to my practice**

I examined the transcripts from lessons 26 and 27 (Appendix 20 and 21) with regard to the themes of specialising, conjecturing, generalising, convincing and trainspotting because of the significance placed on them by Burton (1984), Mason (1999, 2007) and Mason et al (2007) for the development of mathematical thinking. I looked for evidence of these themes in the lesson transcripts and in pupils’ work. I now believed that my pupils, although very young, were capable of demonstrating these skills and evidence
from lessons recorded in 2010 had shown this. If some of my pupils were capable of specialising and generalising without my knowledge when given an appropriate task, as recommended by Barbosa et al (2009), Dörfler (2008), Houssart and Evens (2003), London McNab (2006) and Orton (1997), then I wanted to find out what they were capable of doing in lessons where generalising was the objective.

Specialising

In all three lessons (Appendix 20-22) pupils specialised lots of cases (Mason 1999) to find the near generalisation (Stacey 1989) as the following extract taken from the Dogs and Eyes lesson (Appendix 20) demonstrates.

Hannah: I have 10 dogs... so 11 dogs will have... 22... 12 dogs will have 24... 13 dogs will have... 26

Figure 7.3 Hannah’s work from Dogs and Eyes
After finding the near generalisation of 10 dogs, Hannah then continued specialising in an effort to find the far generalisation of 100 dogs. She got as far as 13 dogs and would have continued using the pattern she had found had I not intervened and tried to focus her attention on the relationship between the number of dogs and the total number of eyes.

Teacher: Don't keep going on... I need you to work out a quick way of doing it... so how have you worked out... what did you do to 5 to get 10?

Hannah: I added another 5.

Teacher: Okay, so, what did you do with 6 to get 12?

Hannah: I added another 6

Teacher: So why did you add on the same number?

Hannah: Because it asks you to add the number here on to here...

Teacher: Okay, so, what... if I said to you... what would you do to get 100 dogs? What will you do with the hundred? What will you do to get 100 dogs?

Another example of pupils specialising can be seen in the following extract from Lesson 27 (Appendix 21), where they were trying to work out the total number of eyes and tails.

Isabelle: 2 dogs...

Hannah: 2 dogs will have 6... 3 dogs...

Isabelle: 3 dogs will have 9
Hannah: Will have 9, yes... and then

Daisy: 3 dogs will have 9...

Hannah: ... and then 4 dogs will have...

Isabelle: 13.

Hannah: 13... are you sure?

Isabelle: Yes.

Hannah: Right, let me check.

Hannah appears to have had some doubt as to the accuracy of 13 and decided to check. Later she can be heard to count ‘3, 6, 9, 12, 15’ which suggests they have corrected the error. Evidence from their work (Figure 7.4) shows the amendment they made before proceeding with the pattern to find the near generalisation.

Figure 7.4 Hannah and Isabelle’s work from Eyes and Tails
All pupils were able to specialise to some degree in all three lessons and most were able
to find the near generalisations. However many became focused on the patterns they had
spotted and continued finding subsequent elements in the pattern rather than looking for
relationships between them.

**Trainspotting**

When working on the *Dogs and Eyes* lessons (Appendix 20) some of my pupils were quick
to spot the vertical pattern, as can be seen below. As individual pupils worked out the
number of dogs and the corresponding number of eyes I recorded their answers on the
whiteboard as follows.

\[
\begin{align*}
1 \text{ dog} & \rightarrow 2 \text{ eyes} \\
2 \text{ dogs} & \rightarrow 4 \text{ eyes} \\
3 \text{ dogs} & \rightarrow 6 \text{ eyes}
\end{align*}
\]

Without prompting Jack called out that he had noticed a pattern but was unable to
explain what the pattern was.

Jack: Hey, there's a pattern!

Teacher: Where is the pattern?

Jack: 2, 4, 6, 8.

Teacher: Why did you say 8?

Jack: It's supposed to be 8, because it's a number pattern.

Teacher: It is a number pattern yes, and it is counting in...
Children: Twos/even numbers.

The following week, when looking at the second part of the problem (Appendix 21), Daisy also noticed the vertical pattern and was able to explain how it was made.

Teacher: Can you see a pattern? Daisy

Daisy: 3, 6, 9.

Teacher: What do you notice about that pattern?

Daisy: It is going in the 3 times tables.

All pupils were able to use the vertical pattern to continue the sequence and find a near generalisation (Stacey 1989) for 10 dogs. Some went from knowing 4 dogs to working out 10 dogs without recording any other elements in the pattern. They may have worked out the relationship between the number of dogs and the total number of eyes but at this stage in the lesson they were unable to verbalise it.

Hannah: Yes, that's easy... 3, 6, 9, 12, 15... now we just have to wait until...

Daisy: You are meant to do a pattern going across

Hannah: Oh yes, we have to do the pattern going across.

Isabelle: Across?

Christian: Yes, 1... 3, 2... 6, 3... 9... see I have spotted the pattern going across. It's 1... 3 so odd... 2... 6 even... 3... 9 odd... 4... 12 even... 5... 15 it's odd
I tried using random terms with my pupils in the 2p Coins Lesson (Appendix 22). After discussing the problem together Christian suggested starting with 1 coin and recording it on a t-chart. I then asked how much money there would be if Miss Lynch had 3 coins and then 2 coins, recording pupils’ answers on the whiteboard as they gave them. I then told them to look at the relationship between the number of coins and the total amount of money and use this to work out the value for 10 coins. Stella began by copying the t-chart from the whiteboard as I had written it then re-arranged the number of coins in numerical order (see Figure 7.5).

![Table of 2p Coins](image)

**Figure 7.5 Stella’s work from 2p Coins - a near generalisation**

She then continued the vertical pattern of counting in twos until she reached the near generalisation of 10 coins. She was then able to work out the far generalisation of 100 coins as can be seen from Figure 7.6, and continued the pattern to 600 coins but with an error at 500.
Stella had not been asked to find the terms beyond 100 but chose to continue the pattern to 600. It is quite possible that she was becoming sidetracked, as Hewitt (1994) suggested by the vertical pattern she was generating. When I asked her to write a general statement for any number of coins she claimed not to know how, but when I asked her to tell me what she had to do to the number of coins to give the total amount of money she recorded her response as follows.

Unlike Stella, Jake continued recording the number of coins in what, at first glance, appears random but is in actual fact a pattern in itself as can be seen from Figure 7.8. He recorded the corresponding values correctly until he got to 13 coins by which time he had passed the required 10 coins.
Jake appears to be continuing the pattern by alternating the numbers. It is not clear if this was his intention but it appears to be too organised to be accidental. The fact that the corresponding values are correct may indicate that he recognised the relationship across the table rather than using the vertical pattern. The fact that the last value is 27p may also support this view. If he had been continuing the vertical pattern one might have expected him to write 24 in error rather than 27.

Figure 7.8 Jake’s work on 2p Coins- a near generalisation

I believe, in this instance, that Jake may have been using the relationship between the number of coins and the total amount of money to work out his values, but I cannot be sure. The fact that he was able to record the value for 100 coins (Figure 7.9) when asked, without any difficulty, leads me to believe that this may have been the case. Jake did not get to the point of verbalising this relationship (if known) or of recording a general statement during the course of the lesson.

Figure 7.9: Jake’s work from 2p Coins- a far generalisation

There were other pupils who continued the table past the value for 10 dogs and 10 2p coins using the vertical patterns as a means of finding the values for 100 dogs and coins. It is not surprising that these pupils did not actually reach the far generalisations and many had errors in calculations along the way as Rebecca’s work demonstrates. If she had
known the facts for the 3 times tables it is unlikely she would have written 35 instead of
36 but she could easily have made an error further on if she had continued counting in
threes. This highlights Orton’s point (1997) that errors in calculations can hinder pupils’
generalising. It also supports the point made by Hewitt (1994) that pupils can become
sidetracked by the pattern and likely to lose sight of the generality they are seeking if
their attention is not refocused on the relationship between the values across the table.

![Table Image]

Figure 7.10 Rebecca’s work from *Eyes and Tails*

Some, like Joe below, appeared to use the relationship to work out a near and far
generalisation without any need for a sequential pattern but seemed content to find
more far generalisations in patterns of 200. Joe had become sidetracked by the pattern
rather than the need to express the generalisation. The next step for him is to be able to
express what he is doing as a general statement.
Although I agree with Hewitt’s (1994) point that pupils can become more focused on the patterns they create than on the mathematical situations they are investigating, I believe that recognising patterns is a stage which some pupils need to go through as they develop the ability to make generalisations (Orton 1997). The problem arises if that is all they are doing. When pupils recognise a pattern they must go to the next stage of recognising how that pattern is made and what they did to find each subsequent value in the pattern. Teachers can focus pupils’ attention back on the problem and encourage them to express their ideas as conjectures. Carpenter and Levi (2000), Hewitt (1998) and Mason (1999) stressed the importance of allowing pupils to put forward conjectures as a way of developing mathematical thinking.

**Conjecturing and Convincing**

Evidence from the last three lessons demonstrates how Christian used the pattern he had generated to help him reach a near generalisation of 10 dogs. This led him to recognise some relationship between the number of dogs and the total number of eyes. The following extract shows when Christian realised he had made this connection.
Christian: I know how it works... I know... how... it... works... You have 1, you add 1... you have 2, you add 2... you have 3, you add 3... you have 4, you add 4...

Alice: No, because where is the other 1 coming from then?

Christian: If you have one, you add one. You have 2, you add 2...

Alice: You were saying 1 1, 2 2... where are you getting the other 1 from?

He then went on to make a conjecture, as described by Mason (1999), regarding the relationship between the number of dogs and eyes and is not put off by Alice's questioning. In fact he went on to further explain his conjecture, stressing what he believed to be the correct strategy in an effort to convince Alice.

Christian: You don't add 3... you add the number of dogs!... You add the number of dogs.

Christian: See, you have one dog you add one... you have 2 dogs you add 2. You have 3 dogs, you add 3... you have 4 dogs you add 4... when you have 5 dogs you add 5...

Alice: Oh, I get it now... I think what you're trying to say is that if you have one you add...

Christian: You have to add the number of dogs here. Like if you have one you add one, if you have 2 you add 2... you see?

Alice: Now I get it
The fact that these pupils were able to interact freely in this way is an example of the 'conjecturing environment' described by Mason (1999) which is developing in my classroom, where pupils are encouraged to express what they have understood, or what they think might be true, so that others can question, and invite or suggest modifications (Mason 1999, p9).

Christian was further encouraged to modify his conjecture when I questioned him, as the extract below shows, about how he had found the number of eyes for each number of dogs.

Teacher: What did you do with the 2 to get 4?

Christian: We added 2 on

Teacher: What did you do with 5 to get 10?

Christian: We added 5.

Teacher: What is another way of saying that? So what are you doing to this side to get these?

Christian: Adding each number...?

Teacher: But there is a better way of saying that.

Alice: Doubling

Teacher: You are doubling them, that's right. So, what would you do to get 100 dogs?
Christian: We would...

Alice: Double it

Teacher: And how many dogs... how many eyes would those dogs have?


The reason for this questioning was to focus Christian and Alice’s attention on the relationship between the elements horizontally, as recommended by Warren (2005) and Warren and Cooper (2005) so that they would then be able to make a general statement about that relationship.

Generalising

Christian and Alice were able to recognise the relationship and proceeded to making a generalisation about how the total number of eyes was related to the number of dogs. Evidence of this is demonstrated in Christian's responses below.

Teacher: Who has managed to find out what is happening to the number of dogs to give us the number of eyes? Christian.

Christian: We keep doubling the number of dogs

Teacher: Very good, now did that help you work out how many eyes 100 dogs would have?

Christian: Yes.

Teacher: So, how many eyes would that be?

Christian: 200 eyes
Christian recognised the relationship between the number of dogs and the total number of eyes and was able to use this to work out larger numbers. He had worked with smaller numbers initially in order to find a vertical pattern and used this to help him recognise the relationship between the values in the pattern. Evidence from his work from that lesson demonstrates how he tried to show the relationship between the dogs by writing 2 on the line. He then went on to show the value for the far generalisation of 100 dogs.

Figure 7.12 Christian’s work from Eyes and Tails

Christian also wrote a question mark which he then crossed out. This may have been his attempt to record a general statement for any number of dogs. Evidence from the transcript shows how pupils discussed the practice of using a question mark to stand for an unknown number.

Teacher: I want you to talk to your partner and see if there is an equation you can write down that would explain to us, show us what we are doing... something that will cover any number.
Hannah: Oh, I know... a question mark

Hannah then went on to write $? + ? = ?$ as a general statement for finding the value of eyes for any number of dogs. At the end of the lesson I showed pupils that if I used a question mark and doubled it the equation could be written as $? \times 2 = ?$. I then explained that sometimes in maths a letter can be used to stand for any number and introduced the variable $n$ to stand for any number of dogs so then the solution could be written as $n \times 2$ or $2n = \text{the number of eyes}$.

Evidence from the lesson on eyes and tails (Appendix 21) also shows some pupils able to make a general statement about the relationship between the number of dogs and the total number of eyes and tails.

Teacher: What did you do with the 1 to make 3?

Christian: We added 2 more

Teacher: Mmm, well what did you do with the 2 to make 6? Remember when we were doing the eyes on their own what did we do with the number of dogs to give us the number of eyes?

Christian: We doubled it

Teacher: We doubled it. What do we need to do with the number of dogs to give us the number of eyes and tails?

Christian: Treble it
Teacher: Treble it. Yes, so, what would we need to do with the 10 to give us the number of eyes and tails for 10 dogs?

Christian: We would treble 10

Teacher: So treble 10. What is three 10s?

Christian: 30

Teacher: What do we need to do to the number of dogs to give us the number of eyes and tails? Christian.

Christian: Treble it.

Teacher: Treble it. How could we write that as a general statement? We could write it in words like you said... if we trebled the number of...

Children: dogs.

Teacher: dogs, we get the number of...

Children: eyes...

Teacher: eyes and tails. Okay, now, what did we use last time to stand for any number? We used a symbol, didn't we? Hannah

Hannah: A Letter

Teacher: Very good, a letter. Does anybody remember what that letter was?

Children: n
Teacher: We used the letter $n$. So we have to treble $n$ to give us the number of eyes and tails. How will I write that? Ali

Ali: $2n$...

Teacher: No, not $2$... $2n$ was double last week. We are trebling it. So, what could we write? Alicia

Alicia: $n$ times $3$.

Teacher: $3...n$ times $3$ or $3$ times...

Children: $n$

Teacher: $n...3n$... that tells us we can put any number in for $n$ and when we treble it that would give us the number of...

Children: eyes and tails.

These pupils were assisted in their use of $n$ to record the unknown in the general statement by my questioning. Pupils who use symbols in this way are considered to be working at level 5 in the National Curriculum (DfEE/QCA, 1999).

The 2p Coin lesson (Appendix 22) recorded two months later has evidence not only of pupils making general statements but recording them as written statements as well as equations. In the following example Benjamin used his recognition of the relationship between the elements to find far generalisations for 100, 400 and 800p. He then proceeded to record a general statement, which he abbreviated to g.s, to explain what he had done to the number of coins to find the total amount. When I asked him to record
this as an equation he wrote $800p + 800p = 1600p$ but then wrote $N \times 2 = \? \text{ when I explained that it did not show the value for any number of coins.}$

Figure 7.13 Benjamin’s General Statement

Christian was also able to record the generalisation as $n \times 2 = \text{ but only after several attempts, as can be seen from his work below.}$

Figure 7.14 Christian’s General Statement
Three other pupils recorded the generalisation as a written statement as follows but were unable to write it symbolically.

![Figure 7.15 Isabelle’s General statement](image)

![Figure 7.16 Molly’s General Statement](image)

When I asked Stella to write a general statement for any number of coins she claimed not to know how, but when I asked her to tell me what she had to do to the number of coins to give the total amount of money she recorded her response as follows.

![Figure 7.17: Stella’s work on 2p Coins- a general statement](image)

Although Isabelle was unable to write her generalisation algebraically she was able to write her own example, as the following piece of work shows. The fact that she was able
to find the near and far generalisations shows that she also recognised the relationship between the number of 5p coins and the total amount of money.

![Image](image-url)  
**Figure 7.18 Isabelle's Own Example**

Orton (1997) in her research with 9-13 year old pupils had not expected the younger pupils in the research to express their generalisations using the letter \( n \) as a symbol. The evidence given above from my practice, shows that some younger pupils of 6-7 have the ability not only to recognise that a letter can be used to express an unknown but also to write it as an algebraic equation. The fact that my pupils were engaging in such advanced mathematical thinking is a result of the significant changes which have taken place in my practice.

Orton (1997) was of the opinion that generalising takes place at different levels. Some pupils can make near generalisations only, others can go on to use the relationship between values to make a far generalisation whereas others can go a level further and make a general statement based on this relationship that is true for all numbers. There is evidence of all of these levels in my practice as has been shown above.
Without question generalising is a challenging task and I agree with Orton’s point that there are levels of generalisation but I do not agree that it is solely characteristic of more capable pupils as suggested by Amit and Neria (2008). It is my opinion that although pupils who are considered to be ‘lower attaining’ may not be able to express a generalisation as an algebraic equation or even verbalise the relationship between values, it does not mean that they are not capable of generalising on any level. Evidence from my practice shows some lower attaining pupils considered to have special educational needs recognising a pattern and making a near generalisation. Jack had spotted the pattern at the beginning of the lesson and had used this pattern to find the value for 10 dogs. He may have been at a lower level of generalising than a higher attaining pupil such as Christian, but he was nonetheless able to make some generalisations and with more experience he should continue to develop in this area.

Figure 7.19 Jack’s Near Generalisation
Evidence from my research diary and from his work show Felix was able to tell me what each subsequent element in the pattern would be but was unable to explain how he had worked out the values. Evidence from my practice also suggests that although they sometimes make errors in calculations, pupils who can recall known number facts such as times tables can recognise when errors are made and are able to correct them more readily.

**Summary**

In this chapter I have reviewed some of the literature on generalising in mathematics which highlights its significance in the development of mathematical thinking. The four processes of specialising, generalising, conjecturing and convincing are important stages in developing mathematical thinking according to Burton (1984), Mason (1999, 2007) and Mason *et al* (2007) and should be part of every mathematics lesson so that pupils can practise and develop these skills. An important part of mathematical thinking is the recognition and use of patterns when solving problems which can help pupils progress through different stages of generalising. The terms near (Stacey, 1989), local (Amit and
Neria, 2008) and empirical (Bills and Rowland, 1999; Dörfler, 2008) generalisations describe finding the elements in a sequence by drawing or counting to continue the repeating pattern. The terms far (Stacey, 1989), global (Amit and Neria, 2008) and theoretical (Dörfler, 2008) generalisations involve more advanced thinking because an awareness of the relationship between the elements and the structure of the pattern are necessary in order to work out values that are too large to find by practical means. There is a danger, however, that pupils might become more focused on spotting patterns, as described by Hewitt (1994), that they lose sight of the problem to be solved. Pupils who engage in far, global or theoretical generalisations are considered to be developing functional thinking which is a fundamental part of algebraic thinking according to Warren and Cooper (2005).

Generalising, although an import part of mathematics, particularly algebra according to researchers such as Hargreaves et al (1998), Kaput and Blanton (1999), Mason (2002c) and Mason et al (2007), does not feature significantly in the English National Curriculum (DfEE/QCA, 1999). Some researchers, such as Carpenter and Levi (2000) and Carraher et al (2006), have called for generalising to be introduced to pupils at an earlier age so that they can gain a deeper understanding of algebra and avoid the common misconceptions which can act as obstacles and hinder their learning later on. If young pupils are to engage in generalising then it is important that they are taught how to do so. This can be done primarily by providing them with tasks which allow them the opportunities to generalise and which challenge their thinking as recommended by Barbosa et al (2009), Dörfler (2008), Houssart and Evens (2003), London McNab (2006) and Orton (1997). It is also important, according to Houssart and Evens (2003) that pupils are given opportunities to share their thinking in what Mason (1999) described as a conjecturing atmosphere, where they feel they can put forward their ideas and also question the ideas of others.
I carried out a mini cycle of Action Research into my practice to see if my pupils could engage in generalising and functional thinking such as that described by Blanton and Kaput (2004) and Orton (1997). I used the *Eyes and Tails* problem described by Blanton and Kaput (2004) in their research because I wanted to find out if my pupils could generalise, given a task which focused specifically on that skill. The evidence shows that not only were they able to find the near and far generalisations described by Stacey (1989), some were also able to record the general statement in algebraic form using a letter to represent the unknown element in the equation. Orton (1997) had not expected the younger pupils aged 9 in her research to be able to use a symbol to represent an unknown in an equation and I was even more surprised to find some of my even younger pupils aged 6-7 engaging in this level of thinking. This supports the point made by Warren and Cooper (2005) and Blanton and Kaput (2004), that very young pupils are capable of mathematical thinking at a higher level if they are provided with tasks which allow them to do so.

My research makes an original contribution to knowledge about the strategies younger learners can use when solving problems. It particularly demonstrates the capacity very young pupils have for specialising and generalising when provided with rich, challenging tasks and explains how I, as a teacher, was able to achieve this with my pupils. Although this research is not generalizable in a wider context, as action research I believe it has relevance for all teachers who seek to improve their practice in mathematics and demonstrates quite clearly what can be achieved with pupils as young as 6 and 7 years of age.
Chapter 8 - Summary and Conclusions

This thesis is the result of what began as practitioner research into ways of improving pupils learning in mathematics and developed into action research into how changing my practice could impact on my pupils learning. Through cycles of action and reflection I focused on eight research questions in total and how I could answer them. The way the first three research questions below are worded, shows how my initial concern had been with my pupils' learning and not with changing my practice, since I was unaware at the time that this in fact was what needed changing.

1. Can problem based learning improve thinking skills in mathematics?

2. Is collaborative learning a more effective approach to problem solving than working independently?

3. What strategies do young pupils use when solving problems?

The literature I reviewed in the areas of problem-solving, collaborative learning and solution strategies clearly demonstrates how inter-dependent they are and how important they are for developing mathematical thinking and learning. Although the National Curriculum considers problem-solving to be integral to learning in mathematics, I believe, what it promotes in theory is not translated in reality in the classroom. Boaler (1998, 2009) and Forman and Ansell’s (2001) research highlighted the impact of using a problem-based approach to teaching mathematics on pupils' learning and how teachers allowed pupils to use their own strategies to solve problems and introduced new procedures when they were contextually relevant. Their research was of particular interest to me and caused me to think about my methods of focusing my pupils' attention.
on a particular strategy and my feelings of frustration when they appeared not to understand. Skemp's (1989) work on the distinction between relational and instrumental understanding and the role played by the former in problem-solving also caused me to examine what was happening in my mathematics lessons. I wanted my pupils to become familiar with the different phases of problem-solving described by Polya (1957) and Burton (1984) so that they could engage in what Burton (1984) referred to as mathematical thinking, resulting in more effective problem-solving and intelligent learning.

I discovered, while reviewing the research, that for effective problem-solving to occur it must take place in a collaborative environment which was not an easy situation to achieve, as research by Bennett and Cass (1989) and Dawes and Sams (2004) demonstrated. Very young pupils find it difficult to take turns and to reach a mutual agreement when they have differing ideas and they need to learn how to overcome this in order to work more effectively. Using different strategies to solve a problem can also lead to disagreement among pupils as to which strategy to use, especially if they had not been used to using a range of strategies freely, as was the case with my pupils at the time.

It was at this point that I came to the realisation that it was not simply a matter of improving my pupils' learning in mathematics. There were areas of my practice with which I was not satisfied and I began to feel that perhaps this was, in some way, affecting my pupils' learning. I found Schön's (1995) The Reflective Practitioner and Mason's (2002b) Researching Your Own Practice influential in helping me to reflect on my practice and examine aspects that needed changing. I was also convinced that my research methodology was action research and not practitioner research as I had previously...
believed because, as well as reflecting on my practice, I wanted to actively change and improve it. As a result there was a change in my focus, as can be seen from the wording of my later research questions. Instead of looking at what my pupils did in mathematics lessons I examined my actions and how they impacted on those of my pupils. I had begun introducing more problem-solving tasks because I wanted to provide more opportunities for problem-solving. Engaging pupils more with an appropriately challenging task addressed issues of behaviour and led to fewer distractions as did my focusing their attention on the task rather than highlighting inappropriate behaviour.

I had thought that the introduction of more challenging tasks would lead to pupils becoming more involved in mathematical discussions but much of their talk was managerial or instructional in nature and not enough time was spent actually discussing the strategies to be used. I wanted to get their mathematical thinking out in the open so that they could construct their knowledge in collaboration with others as Vygotsky (1978) advocated. In order to do this I had to think about how I questioned my pupils. My questions should challenge pupils' thinking, make them justify their answers and find alternative ways of finding solutions. This should lead to a more questioning environment where pupils can develop the confidence and the skills to challenge the thinking of others.

I reviewed the literature on questioning (Boaler and Brodie, 2004; Cotton, 2001; Gall, 1970; Smith et al 2004) and examine the type of questioning I asked in my practice using the following two research questions.

4. To what extent do I use higher order questions to extend pupils' learning in mathematics?
5. Does increasing the number of higher order questions I ask impact on my pupils learning in mathematics?

I was concerned over the lack of mathematical questions asked by my pupils of each other so I looked at the types of questions I asked over a number of lessons and categorised them into four groups namely; open, closed, clarifying and behaviour managing questions. In answer to question four, analysis of the data showed that I asked more open questions than was common for many teachers, especially of younger pupils (Boaler and Brodie, 2004; Cotton, 2001; Gall, 1970; Smith et al 2004) but the consistency and effectiveness of their use could be improved. I discovered that it was not the quantity of higher order questions that was important but their quality. Answering question five in terms of measuring the impact on my pupils' learning was more difficult over such a short period but I believe they were becoming more confident about reasoning and discussing their strategies with others.

I examined in more detail how changes to my practice may have impacted on the strategies used by my pupils and on the amount of mathematical talk in which they engaged during a task, by asking the following research questions.

6. What impact does changing my practice have on the strategies pupils use when solving problems?

7. To what extent does changing my practice enable pupils to engage in discussions when solving problems?

A comparison was made between lessons recorded at the beginning and end of the academic year and a decrease in the amount of teacher talk and a considerable increase in the amount of pupil talk focusing on the task was noticed. This, I believe, was partly
due to a change in the tasks the pupils were being asked to do and a change in the types of questions I was asking. In my opinion the nature of the tasks given influenced the strategies which pupils used. Although I appreciate that it is difficult to measure impact on pupils particularly over such a short period of time there were recognisable changes in their behaviour which I believe were due to the changes in my practice. The earlier activities were too simple and did not afford pupils the opportunity to generalise. The activities in the final two lessons on the other hand were more conducive to generalising and challenged their thinking in a way that previous activities had not done. Reviewing the literature helped me to appreciate the need, in my classroom, for a culture of generalising and conjecturing, as described by Mason (1999). I learned of the significance of the four processes of specialising, generalising, conjecturing and convincing in the development of mathematical thinking (Burton, 1984; Mason, 1999, 2007; Mason, Drury and Bills, 2007) and how they should be part of every mathematics lesson.

If I wanted my pupils to engage in generalising then I had to teach them how to do so. I had to provide them with tasks which allowed them the opportunities to generalise and which challenged their thinking, as recommended by Barbosa et al (2009), Dörfler (2008), Houssart and Evens (2003), London McNab (2006) and Orton (1997). Pupils should be given opportunities to share their thinking in what Mason (1999) described as a conjecturing atmosphere, where they feel they can put forward their ideas and also question the ideas of others. In order to see if my pupils could engage in generalising and functional thinking such as that described by Blanton and Kaput (2004) and Orton (1997) I carried out a mini action research cycle using the following research question.

8. To what extent do I provide opportunities for my pupils to specialise and generalise in mathematics lessons?
I used Blanton and Kaput (2004) *Eyes and Tails* problem because it was a task which focused specifically on generalising. The evidence shows that not only were my pupils able to find the near and far generalisations described by Stacey (1989), some were also able to record the general statement in algebraic form using a letter to represent the unknown element in the equation. I was even more surprised than Orton (1997) to find some of my even younger pupils aged 6-7 able to use a symbol to represent an unknown in an equation. This supports the point made by Warren and Cooper (2005) and Blanton and Kaput (2004), that very young pupils are capable of mathematical thinking at a higher level if they are provided with tasks which allow them to do so. It was clear to me, having carried out this research that I had not been giving my pupils opportunities to specialise and generalise because not only were the tasks not suitable but I was unaware that this was what I should be doing.

Changes in my practice have brought about changes in how my pupils now interact and learn in mathematics lesson. Reflecting on my practice and thinking about what I do in a mathematics lesson and why I do it has probably brought about the greatest changes in my classroom. It has caused me to think very seriously about the tasks I give to pupils and the purpose behind them. By giving pupils more challenging tasks I can now provide greater opportunities for pupils to specialise and generalise and this also enables me to ask more challenging questions. I also found that appropriate tasks encouraged pupils to engage in discussions about the task more readily and they remained focused for longer. Instances of inappropriate behaviour were fewer because they were interested in what they were doing and less likely to be bored by the activity. My practice is by no means perfect but it has undergone a significant change for the better as, I believe, has my pupils’ learning in mathematics. When I find myself falling back into previous habits I am reminded by Mason (2000b) to ‘notice’ my actions and to think again about what I should
do. This demonstrates that I still have much to learn about and to change in my practice and this research is just the beginning for me and the pupils in my class.

**Limitations of the Research**

As with any research there are limitations which need to be acknowledged and these were evident in both the methodology and the analysis of this research.

**Methodology**

Action researchers carry out research into their own practice and are active participants driving the research process. I experienced some tensions in the early stages of my role as teacher/researcher and found it difficult to take a more observational position. The fact that I was the sole researcher and practitioner in my classroom could be considered as limiting my research but I believe that my triangulated data are rich enough to preserve both the reliability and the validity of the research and would yield similar findings to those of another researcher.

Although I used audio recording to produce most of my data there were a number of disadvantages involved in its use. The lessons were time consuming to transcribe, which was further exacerbated by the amount of background noise from a range of sources. It was sometimes difficult to distinguish an individual pupils’ discourse in a group conversation and non-verbal interactions were not picked up on tape. There was a degree of procedural bias, as described by Macintyre (2000), caused by the presence of the recording equipment but this was reduced as pupils grew accustomed to it over time.

There were limitations to recording field notes during lessons due to the difficulty I experienced remaining as an observer of a group and being constantly distracted by other pupils and incidents in the classroom. This necessitated writing notes as soon as possible.
after the lesson which according to Burgess (1985) may have resulted in important detail being forgotten or omitted.

**Analysis**

As with methodology, the fact that I was the sole researcher analysing the data could also be considered as a limitation of the research. Therefore, it was important for me to remain as objective as considered possible by Dadds (1993) and Reason and Bradbury (2001). The analysis of my data informed the next stage of action research and I continually built on and drew from the literature. This constant reference to other research by experts helped me to stay focused and kept my research grounded in reality. As a result, I am confident that analysis of my rich, triangulated data by another researcher would yield similar results to mine. Action research is also limited by the fact that 'the findings relate to one instance and should not be generalized beyond this specific ‘case” (Denscombe, 2005). Although I am not claiming my research to be widely generalizable, I strongly believe that it makes a valid contribution to the field of mathematical learning.

**Future research**

I believe there is a place for my research and future research like it in education. There is a gap in the available literature on how very young pupils engage in algebraic thinking. Carraher et al (2006) claimed that little was known about how young pupils are able to make generalisations and called for more research in this crucial area, so that it can be developed further. I have evidence from my practice that pupils as young as 6-7 are not only capable of making general statements but can also express them in algebraic form using symbols as unknowns. I support Carraher et al (2006) in their appeal for more
research into this area of mathematical learning and go even further to suggest that teachers should become involved in researching their own practice. It is my belief that through examining and changing our own practice we can improve our pupils' learning, allowing them to develop as independent mathematical thinkers. If research is carried out by real teachers in real classroom situations, rather than solely through teaching experiments or intervention studies, I believe it will have greater impact in the mathematics classroom. I find it frustrating to think of my pupils' learning being set back if they move on to teachers who do not acknowledge what they are capable of or understand the importance of generalising and problem-solving. This is an issue which should concern all teachers and be part of their professional development. My experience mentoring student teachers highlights for me the need for more emphasis on problem-solving and the use of questioning as part of their training as these are areas with which they have difficulty on their teaching practice.

My research shows how one teacher can carry out research into her own practice and it can be used by others who wish to carry out their own action research project. It clearly explains my starting point, the methodology I used and how changing my practice impacted on my pupils' learning in mathematics. This research makes an original contribution to literature both in terms of methodology and in terms of theory. It describes how my methodological journey travels alongside my mathematical journey and how my theoretical position moved from thinking about what my pupils were doing wrong to what I was doing wrong. The analysis of my data informed each subsequent phase of action research and I was continually building on and drawing from the literature. This constant reference to the literature helped to maintain my focus on reality and keep my research grounded. Another original aspect of my research was how I as a class teacher, through changes in my practice, was able to move my very young pupils'
mathematical learning forward to a point where some of them were beginning to write
general statements in algebraic form at the age of 6-7. Most importantly it demonstrates
what can be achieved by pupils when they are given the opportunities and allowed to
show what they are capable of. Evidence from this research not only has implications for
how we as a school teach mathematics but how mathematics should be taught in all
schools.
References


Allen, B (2010) Private communication


Blanton, M. & Kaput, J. (2000) Generalizing and progressively formalizing in a third grade mathematics classroom: Conversations about even and odd numbers


London: The Falmer Press

Educational Research Journal, 6*(2), 165-173

*Qualitative Methods* (Vol. 2). London: Sage Publications

Press

Research* (Vol. 4). London: Jai Press

Mathematics Education, 15*, 35-49

Nash Pollock Publishing


Cooper, T. & Warren, E. (2008) The effect of different representations on Years 3 to 5 students' ability to generalise. ZDM, 40, 23-37


De Corte, E., & Verschaffel, L. (1985) Arithmetic Problem-solving in Elementary School Children: some findings of a Belgian research project, in A. Bell, B. Low & J. Kilpatrick (Eds.), Theory, Research and Practice in Mathematical Education. Nottingham: Shell Centre for Mathematical Education


Maidenhead, Open University Press


applying_guid_paper.pdf (accessed on 1/10/07)


Mason, J. (2002c) What makes an example exemplary?: pedagogical and research issues in transitions from numbers to number theory, *PME 26*


Nias, J. (2003) ‘Primary teachers talking: a reflexive account of longitudinal research’ in
Open University

theories of success in mathematics: individual and classroom differences’ in *Journal for
Research in Mathematics Education* 21:2 109-122

*Educational Studies in Mathematics*, 37, 45-66.

Noffke, S. E. (1999) What's a nice theory like yours doing in a practice like this? And other
impertinent questions about practitioner research, *Change: Transformations in Education*,
2, 25-35

Oxford

OfSTED (2005) *Primary National Strategy an evaluation of the impact in primary schools
2004/5*, London: DfES

Journal of Primary, Elementary and Early Years Education*, 25, 61-65

concepts which will help us understand the underlying nature of interactive learning.
*Cooperative Learning and College Teaching*, 8, 5-7


Appendices

Appendix 1: The Initial Study

Rationale

In the academic year 2007-2008 I carried out an initial study, the purpose of which was to test my methods of data collection, to get a feel for how my pupils worked on problem solving activities and to see how they worked with partners. The main method of data collection was audio recordings of mathematics lessons with some field observations carried out at the same time.

Methodology

The research strategy I used was that of practitioner-research because I was the practitioner conducting small-scale research into an aspect of my practice with the view to improving or changing it, as described by Dadds (1998). It was more of a practitioner-based inquiry into how my pupils learned mathematics so that I could improve that learning. At this point, although I recognised the central role I would play as the practitioner I saw myself in a similar position to the teachers in Thompson’s (2007) research as just carrying out a small-scale inquiry into my practice in order to change some aspect of it. I did not consider that it would involve such cycles of reflection in and on action as described by Schön (1995) followed by strategic action advocated by McMahon (1999). It is this strategic action which McMahon (1999) believed distinguishes action research from reflective practitioner-research and because of this I considered my initial research strategy to be practitioner-research.
Ethical Issues

I wrote to the Governors asking permission to carry out my research in the school as part of my EdD study. On receipt of their approval I wrote to the parents explaining my intentions and seeking their permission for their children to participate in this research. I also spoke to the children and explained my intention to record the lessons and how it would affect them. I wanted to be open with them about my plans and by eliciting their help I hoped they would be more willing to co-operate. In total 26 out of 27 parents replied and were willing for their children to take part. One family did not respond which meant that I was unable to collect data involving their child although he participated in the lessons as normal. In order to maintain the anonymity of the children all names used are pseudonyms.

Collecting the Data

Audio Recording

Being the class teacher I was responsible for delivering the lessons as well as collecting the data. Consequently the audio recordings were of primary importance with regards to the data. The initial study provided an opportunity for me to practise my observational and note-taking skills in addition to trialling audio recording.

It also allowed me to see how my pupils would react to being recorded. One group appeared to ignore it totally and seemed to get on with their activity without paying any attention to it. It seemed to cause a certain distraction for the other group and they could be heard referring to it on occasion during the lesson. As Cummings (1985) noted, some degree of reactivity can be expected when introducing something different into the classroom setting and it is reasonable to expect pupils to react in different ways.
Field Notes and Research Diary

After setting up the recording and settling the pupils to work on the task I had planned to observe the focus group being recorded and write some field notes. I found this a lot more difficult than I had expected. Because I was the only adult present for Lesson 1 I could not give the focus group my undivided attention and was distracted by activity at other groups. The presence of a classroom assistant in Lesson 2 reduced these distractions somewhat but I still found it difficult to record exactly what was going on. I tried to remain as an observer and not interact with pupils in the group and because of this I felt like Smetherham’s (1978) ‘outsider’ despite being an ‘insider’ in the classroom. This resulted in notes having to be made after the lessons rather than as events occurred during the lessons and may have resulted in some loss of detail similar to that experienced by Burgess (1985).

Working with Problems in Mathematics

Context

As part of routine classroom organisation tables are arranged in five groups which can accommodate six pupils sitting in pairs roughly based on their attainment as assessed by NC levels. Pupils were usually directed to work independently, to work as a group or to work in pairs. This situation was maintained for these two lessons, which were recorded in November 2007 and pupils were told to work with their partner seated next to them. I set the equipment up to record the two highest attaining groups as I felt they might be more vocal and interactive. We did not discuss any strategies or equipment to be used but I told them to have a chat with their partners and decide what they needed to do to work it out. By changing my approach and allowing pupils time to work on tasks by
themselves and engage in dialogue with their partner, I had hoped to encourage and develop their thinking skills (Mercer and Sams, 2006; Rojas-Drummond and Mercer, 2003; Pijls et al, 2007; Wegerif et al, 2004).

The tasks

The first task recorded a group of six pupils in November 2007 and required them to find as many different ways as they could of making the number 30 (Appendix 17). During the course of the mathematics topic pupils had been working on aspects of reasoning about numbers so I felt this was an appropriate task to start with.

The second task (Appendix 18) took place the following day with two groups of six pupils being recorded.

Kevin has just started karate lessons. He has a lesson every 5 days. His first lesson is on a Monday. On which day of the week will his 4th lesson fall?

I chose this task because I believed that it would allow those who were not as confident with number to be more willing to participate. Knowledge of the days of the week was necessary in order to complete this task but they were displayed in class for those who were unsure.

What Happened in the Lessons

Most pupils began the first task immediately, discussing how they could go about solving the problem and seemed on first inspection to be getting to grips with it quite well. I worked with the lowest attaining group, making brief notes as they worked and trying as much as I could to stand back and not get too involved in their discussion. This was more difficult than expected for two reasons. Firstly, being the only adult in the room meant
that I could not give them my undivided attention, and secondly being the person that I am, I could not refrain from asking questions and making encouraging comments to get them started. The term used to describe this type of teacher behaviour is *teacher lust*, according to Mary Boole in Tahta (1972, p12), by which she means that the teacher ‘wants to proselytize, convince, control, to arrest the spontaneous action of other minds’ to the point where they find it difficult to think for themselves. Contrary to Boole’s belief it was not my intention to stifle their ‘investigating faculties’ but instead to help them to think more and to interact with each other. This dilemma of whether to intervene or hold back and just observe was discussed by Mason (2002b) and the choice as to which to do was considered by him to be dependent on the reasons for the observation. This is what I understand is meant by the ‘conflict’ between teacher and researcher and how the researcher’s ‘self’ comes into the research process (Denscombe 2005). I had to learn to do as Mason advocated, which was to ‘sensitize’ myself to ‘notice’ when it was appropriate to intervene with my pupils and when to remain as an observer because ‘the purpose of noticing is to make it possible to choose to act non-habitually, non-automatically’ (Mason, 2003, p8).

The second lesson recorded the following day involved a more challenging task which necessitated pupils talking together more and sharing ideas. In setting this problem my intention was to challenge pupils’ thinking with regard to how to solve a problem rather than their skills with number or calculations. I was confident that most if not all knew the days of the week and those less confident could use the classroom display to help. Having a classroom assistant (CA) for that lesson made it easier for me to spend more time with the group I was observing rather than having to stop regularly to check on the others. The CA then fed back to me at the end of the lesson and explained the notes and
observations she had made on that particular group. She too had found it difficult to observe this group without getting involved in trying to keep them on task. For future observations, relocating the CA to a position within earshot but not at the children's table and, giving her clearer instructions about not intervening, helped improve this situation.

The Analysis of the Data

The data analysed included the two lesson transcripts and the field notes written during and after the lessons and involved a theme analysis. This type of analysis involves sifting through the data in search of themes in relation to the research questions and coding them accordingly.

Theme Analysis

I transcribed the tape recordings and searched for emerging themes or concepts. I allocated codes to these themes and then carried out axial coding, as recommended by Corbin and Strauss (2008). I read through each category and attached memos as advised by Corbin and Strauss (2008), Delamont (1992) and Denscombe (2005). I found writing memos helpful in focusing my attention on interpreting the data and, although they began as brief descriptions of what was happening, as I progressed through the transcripts I began to reflect more on why I thought certain behaviour had occurred. With continued practice, my memos became more analytic in content, interpreting what was happening in the data as well as describing it, thus leading to a more in-depth analysis.

The Categories

The initial themes (Appendix 23) which emerged from all the transcribed data were grouped together under broader categories as follows.
1. Using strategies

2. Working together

3. Working independently

4. Distractions

5. Refocusing

I checked the field notes I had made after the two recorded lessons of 12 pupils and noted similar themes of counting on, guessing, explaining, bickering and working independently. They helped to confirm the themes from the transcripts and became part of the broader categories above.

The first three categories were directly related to the areas I was investigating through the following research questions and helped me begin to answer both of these questions.

1. Is collaborative learning a more effective approach to problem solving than working individually?

2. What strategies do young children use when solving problems?

I explored each category and recorded my interpretations of what had occurred in the lessons.

**Category 1: Using Strategies**

It is difficult to work out which strategy pupils have used for a given problem unless they can tell you or record it in some way. Pupils of this age find it difficult to communicate how they have arrived at a particular solution and I have often been told ‘I used my fingers’ or ‘I thought it in my head’ when I have asked for an explanation of the methods
used. Evens and Houssart (2004) looked at pupils' written answers to problems and found that it was common for children especially those of a young age to experience difficulty in explaining their reasoning. Carrying out the initial study gave me the opportunity to take a closer look at some of the strategies used by two groups during the lessons and evaluate their effectiveness.

Lesson 1: Making 30

The most obvious strategies to emerge from the transcripts of Lesson 1 were counting on, counting back, mental recall of number bonds as discussed by Carpenter et al (1981), Geary et al (1991) and Thornton (1990) and using patterns as outlined by Gray and Tall (1992). The most commonly used strategy appeared to be counting on. The pupils from one group demonstrated that they had a clear idea of what to do. There is evidence that most in the group were randomly suggesting numbers and then counting on from those numbers to reach 30. When they felt they had exhausted the counting on possibilities they began counting back from a chosen number to reach 30. Two pupils immediately suggested getting a hundred square and counting on from the different numbers to make 30.

Bonny: 15+ count on to 30

1,2,3,4,5,6,7,8,9,10,11,12,13,14 15 +15=30

Shanta: Look! 14...14 44 take away 1,2,3,4,5,6,7,8,9,10,11,12,13,14 See!

14

These were appropriate strategies to use in order to solve the problem although some did miscount which resulted in some incorrect answers (see Figure 1.1). However, the
random selection of numbers like those chosen by Bonny and Shanta could result in some
repetition of answers or in many being omitted as can be seen from Brigita’s work below,

![Figure 1.1 Brigita's work from lesson 1](image1)

Two pupils developed their strategy further and were able to see a pattern that would
help them arrive at the end result quicker. If they followed the pattern correctly they
would have no need for the hundred square, however, one mistake in the pattern would
cause the rest of it to be incorrect, which is exactly what happened in Rachel’s recordings
(see Figure 1.2).

![Figure 1.2 Rachel's work from lesson 1](image2)
It is possible that Rachel changed the way she was recording her answers. She started with $30 + 0 = 30$ (working horizontally across the page) and wrote the consecutive numbers to continue the pattern. It is possible that she changed to writing the vertical numbers in first which means that the addition sum was not looked at as an item. The columns would have taken priority. It is difficult to ascertain whether she wrote each column first or completed one row at a time or at what point she may have changed her form of recording the sums. Having successfully completed most of the pairings, it is interesting to note that this did not continue.

Whichever method she chose, I wondered what her reasons might have been for choosing it. Did she spot the pattern and write it in rows or had she written it in columns because she considered it to be a quicker way of working it out? Her error occurred when she wrote $8 + 23 = 30$. The first column continues correctly after 8 as does the middle column after 23 which may suggest she was completing it in columns. If, by this time, she was working vertically then she would not have read $5 + 26 = 30$ in its entirety which I would have expected her to know was incorrect.

However she may also have been completing it in rows and having made the error, continued, using the previous row as a guide, thus perpetuating the error to the end. A discussion with Rachel about her work might have helped to answer some of these questions and without it I cannot be sure what her strategies or her reasons behind them were.

Another strategy that pupils appeared to use in Lesson 1 was mental recall of number bonds, although it is difficult to be certain that this is actually what they were doing. Bonny and Shanta were heard making statements such as:
Bonny: 20 + 10

Shanta: 28 + 2... 28 + 2... yeah.

These statements, which were not followed by counting, could suggest that they knew the answers without having to work them out but they could also have counted on quickly and mentally without the need for verbalisation.

Lesson 2: Karate Lessons

In Lesson 2 there was also evidence of counting strategies being used as well as reasoning and guessing. One group of pupils began by counting the days of the week and trying to decide where to start but they were counting on 4 instead of 5 days. They appeared to be confusing the 4 lessons in the question with 4 days. Anya started counting from Sunday and she had the right idea of counting on 5 days. However she had missed the fact that the first karate lesson was on a Monday and consequently got the incorrect day for the second lesson. Justin knew that starting on Sunday was not the right thing to do and he tried to explain but his efforts were ignored. He had worked out that the second lesson would be on Saturday by counting 5 days from Monday, however, he was not secure enough in this knowledge and became distracted by the others' counting of different days. There was some evidence of reasoning from Malinka, Justin and Anya where they tried to qualify an idea when explaining it to others.

Malinka: Listen...listen, if he's dressed up in his karate suit that means it's Monday

Justin: Anya, look, Anya, look...Monday...Monday, Tuesday, Wednesday, Thursday, Friday.
Anya: If it's Friday, that's because you're not counting Sunday.

These attempts at reasoning did not seem to follow any clear pattern of thought and were mingled with what appeared to be guesses from some individuals not based on any particular strategy.

Some pupils came up with the idea of writing down the days of the week in a list, initially omitting Saturday and Sunday, only adding them when I asked how many days were in a week. They put crosses and ticks beside the days as they counted them thus working out that the second lesson would be on a Saturday (see Figure 1.3). Pupils at another table picked up on this strategy and after much discussion and disagreement arrived at the same conclusion. Justin continued from Saturday and worked out when the third lesson would be and can be heard on the tape to say 'No Thursday's the 3rd lesson'. Unfortunately Justin did not record this on paper apart from writing 'His first lesson was on Saturday' so there is no written evidence to show how he worked out the third lesson.

Figure 1.3 shows how Alex and Shanta organized the days of the week in a list and used this to count on from the first Monday. They both explained that they counted on 5 days and arrived at the Saturday. There is no evidence to suggest that they were able to use this counting strategy to work out the third lesson. In fact neither group seemed able to progress any further and to work out when the fourth lesson would be. Most of them thought that the problem had been solved when the second lesson had been worked out.
Rachel seemed to come to a sudden realisation of how to work out the second lesson when she exclaimed 'I know...I know what the answer is'. Because of this reaction I decided at the end of the lesson to ask her to describe to the others what she had done. She was able to explain clearly her strategy of writing down the days and counting 5 days until she reached Saturday. She was unable however to explain how she might work out days for the other two lessons. It was the same situation with the other groups, although they had spent the time counting days they had not arrived at any particular solution. It appeared that even specialising lots of cases was beyond this group of pupils at this stage.

This task really challenged the children's thinking, especially with working out what the question was asking of them. One group's confusion over the number of days and the number of lessons and the other group's confusion over the number of days in a week got
in the way of their problem solving. One of the disadvantages of 'realistic' problems, according to Cooper and Dunne (2004), is how pupils interpret them and what might appear realistic to one may not be to another. Many pupils go to various lessons and classes after school in the week but they tend to be on the same day every week. The fact that the lessons in the problem were every 5 days may have been 'unrealistic' for some pupils and therefore caused difficulties with interpretation. I decided that I needed to consider possible interpretations when using 'realistic' problems in future questions.

Category 2: Working Together

Themes relating to how pupils interacted together were combined under the broader category of Working Together as described by Bennett and Cass (1989), Webb (1991) and Wegerif et al (2004). These themes included demonstrating, explaining, taking turns, making suggestions and confirming. There is more evidence of pupils working together in Lesson 1 than in Lesson 2. They can be heard suggesting numbers then counting on together to get to 30 and taking turns.

Bonny: Now let's go to 26 and count on.

Bonny and

Brigita: 1,2,3,4

Bonny: 4 26 + 4 = 30

Bonny: Now do you want to do 23?

Brigita: Ok
There were instances during the lesson where they helped others in the group by explaining and demonstrating how they worked out the answer.

Bonny: 10+20 is 30

Bonny: 10 and then 20

Brigita: No

Bonny: It is......Look......I'll show you. 10 and then 1,2,3,4,5,6,7,8,9,10...
11,12,13,14,15,16,17,18,19,20. You see 30!

Brigita: Ooh!

Bonny appeared to be more confident and was able to demonstrate to Brigita how she worked out her answer. She explained her method to her friend to help correct her errors rather than just telling her what the answer is. This was what Webb (1991, p373) referred to as ‘content-related help’ and is considered to be more beneficial in raising the achievement not only of the person being helped but also of the person who is giving the help, as it consolidates their conceptual understanding.

Some pupils seemed to work well together in the first lesson and appeared to be focused on the task for the entire time that they were working which, in this instance, was contrary to the findings of Bennett and Cass (1989) who claimed that there was a lot of off-task talk among young children working in groups. In the second lesson, although many did collaborate and discuss the problem, there were several instances where they were not being quite as co-operative as they had previously been.

The next category relates to how some pupils appear to be working independently in the lesson.
Category 3: Working independently

The themes which are grouped under this category relate to instances where pupils appeared to want to work on their own. Statements of intent such as 'Well I'm not working with a partner' and 'I'm not telling you' show that these pupils were unwilling to share their ideas with others. There were also claims of copying coming from both groups in the second lesson which again showed a reluctance to share ideas and work with others. It also showed that pupils were aware that copying was frowned upon in their previous school years and not considered to be appropriate learning behaviour.

Some pupils in Lesson 2 disagreed over what it meant to work collaboratively with others. In the following extract Justin and Anya seem to have conflicting views of what working with your partner means.

Anya: Justin has...he’s only been thinking of my ideas... he hasn’t gave me any ideas.

Justin: No, it doesn’t matter.

Anya: hey Junior the next idea will be yours

Diana: We’re meant to work as partners.

Anya: I’m not starting to work as partners...if I...

Junior: Fine! We’ll take turns.

Anya: NO! I’ve had all the ideas so you have to do 4.

Justin: WHY?

Anya: That’s why!
Justin: (...) We both do 4.

To these pupils it seemed that working as partners meant taking turns in coming up with ideas rather than discussing and investigating any ideas put forward. They did not have the experience of working collaboratively and were in effect appeared to be working independently but taking turns to do so.

Other evidence where pupils appeared to be working independently came from statements such as:

Anthony: I'm good you know. I'm doing these another way. I can do loads like this.

Rachel: I can do 29.

Anthony: I'm the fastest. Everybody look!

From where did Anthony get the idea that speed is important? Was I subconsciously giving the message that speed is more important than quality of work or was this something that was particular to him? I was aware that I encouraged my pupils to keep on task and get on with their work but I also tell them to take their time and check their work on completion. I quite often say to them 'There are no prizes for finishing first'. I needed to be aware of instances where this message was not getting through to individuals and devise appropriate strategies for dealing with it.

Evidence from my research diary showed that pupils from the lowest attaining group displayed very few instances of talking or working together except for one pair who were taking turns to choose a number from which to start counting. At one point Jason tried to explain to Nonso why \(25 + ? = 30\) had to be 5. Nonso was not keen to write 5 in the space
but after an explanation from Jason, with the help of a 100 square, of how it was correct, he accepted the answer and wrote it in. The other pairs at this table worked individually with the occasional glance at each other's work but with little or no discussion. They were working much more individually and competitively (Nicholls et al 1990 and Qin et al 1995) and appeared unwilling to share resources or ideas, preferring to keep everything to themselves.

The highest number of instances of pupils appearing to work on their own occurred in Lesson 2 and could be related to the fact that the task was more challenging. The fact that pupils had not been taught how to engage in 'exploratory talk', as outlined by Dawes and Sams (2004), could also be a reason why they wanted to work independently of others. They did not possess all the skills necessary for reasoning in a constructive and collaborative way. This could also be the reason why there were more instances of distractions which is discussed in the next category.

Category 4: Distractions

This category consists of instances where pupils were perceived to be distracted from the task by engaging in behaviour that was not related to the work they were doing. Behaviour such as bickering, reacting to the recording equipment and other off-task behaviour, caused pupils to be distracted from their work. There were increased instances of distracting behaviour from all groups in the second lesson. I believe this was caused by the more challenging nature of the task in which they were engaged. Houssart (2002) discussed the effect that task difficulty or presentation can have on how well pupils participate, particularly if they are low attainers. She found that there was a greater tendency for task refusal if the task was too difficult or presented in a certain way. This seemed to be the case with my groups and more so with my low attainers. They
demonstrated very little inclination to work together on this task and the classroom assistant reported very little talk between partners. Pupils who had previously worked very well together appeared not to get along when faced with a different, more difficult task.

Shanta:  You have to work with a partner

Brigita:  Well I'm not working with a partner.

Rachel:  I know! I know what the answer is.

Bonny:  What is it?

Rachel:  I'm not telling you

The general ethos over the years in my school has been for pupils to work independently particularly in mathematics. Collaborative learning for this group was a new concept and presented challenges for those not used to working in this way. There was a need for pupils to be actually taught how to work together which had been part of the focus of my research at the time.

Pupils from another group were heard to disagree and bicker more frequently and for longer periods. They also showed more awareness of the presence of the tape recorder and the fact that it was recording their interactions.

Diana:  Stop it. It's recording... (Gasps) ooh! so Mrs D is going to hear that noise.

According to Bennett and Cass (1989) research showed that when children in British schools worked together they were not actually working collaboratively but were in fact
being unproductive, uncooperative and frequently coming off task. There clearly were instances in my lessons where some pupils from both groups were being uncooperative and engaging in chit-chat, but these instances were by no means extensive and some lasted for mere seconds before those involved resumed the task discussion. They were often helped to refocus by comments from others in the group as the next category describes.

**Category 5: Refocusing**

The themes which are brought together in this category concern the instances where pupils were perceived to be conforming to or following the rules of accepted classroom behaviour. These instances appeared to be used by some as a way of refocusing their peers who had become distracted by someone or something.

Rachel: You guys! You have to do this right.

Shannon: Justin, it's recording! ....Anyway, let's work it out. Kirsty, come on

Justin: Stop playing around.

Categories 4 and 5 sparked my interest because I was curious as to why pupils engaged in such behaviour during a lesson and if it was common in every lesson.

**Pupils' Perspectives**

At the end of each lesson I verbally asked pupils how they felt about working with a partner. The majority claimed to enjoy working together and the general consensus was similar to the views expressed by Laura and Shanta. I focused on Laura and Shanta because they appeared to work particularly well together.
Laura: I think it was more better to work with a partner because working with a partner you can, you can share your ideas and, and the other person can share their ideas as well.

Shanta: And if you get stuck on something you can ask your partner if they know.

Laura: Yeah!

One or two said they preferred to work on their own and their reasons varied. I focused on Alex and Rachel because they appeared to find it difficult to work with their partners.

Alex: Well I think it was a bit tricky.

Teacher: Tricky? What was tricky about it, Alex?

Alex: Well, because Derek is, like, too slow.

Rachel: I thought it was a bit hard working with my partner because we, like, have different ideas and we might... we might... one person might want to write what they want to write and another person might want to write what the other person wants to write.

Rachel appeared to be aware of the difficulties that can arise when working with a partner but had not yet learned the skill of reaching a mutual understanding. When I asked her how she had dealt with this issue her response was 'I wasn't quite sure'.

Quantitative Analysis

At the end of the year I compared the pupils' end of KS1 mathematics results with those of the previous year (Table 1.1). These results were taken from school analysis of KS1 data...
and used with the permission of the Head Teacher and Assessment Co-ordinator. There was a significant increase in the number of pupils who achieved Level 2A and an increase of six percentage points in the number who achieved Level 3. These results do not however indicate why such increases occurred or how they were reflected in the test papers.

Table 1.1: End of KS1 results for 2007 and 2008

<table>
<thead>
<tr>
<th>Total Maths</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>2C</td>
<td>21%</td>
<td>12%</td>
</tr>
<tr>
<td>2B</td>
<td>34%</td>
<td>19%</td>
</tr>
<tr>
<td>2A</td>
<td>17%</td>
<td>33%</td>
</tr>
<tr>
<td>3+</td>
<td>26%</td>
<td>32%</td>
</tr>
</tbody>
</table>

100% 100%

During the year I offered the pupils a number of rich mathematics tasks which was different from my normal skills based mathematics lessons. It was possible that these tasks had enabled my pupils to develop skills in mathematical thinking that resulted in higher scores in KS1 SATs than the previous class I had taught.

Reflections on the initial study

Throughout my education my experience of mathematics had been one of working independently to complete tasks and show understanding. As a result this was how I taught mathematics to my pupils. Through working on this initial study, my view changed and I now saw the importance of allowing pupils the opportunity to learn in collaboration with others. When I began introducing more problem solving activities in class I did not initially appreciate the importance of collaboration. I naively thought I could hand the
pupils a problem and they would just get on and do it. I came to realise that the most
effective problem solving occurs in co-operation with others and working co-operatively
does not come naturally to young children. I experienced many surprises during this initial
study not least of which was the fact that the same pupils can appear to work well
together on one task and strongly conflict over another.

The transcribing process took longer than expected due to an excess of background noise
and several attempts were necessary in order to work out some of the dialogue. There
were however, instances where parts of the discussion were unclear and too difficult to
work out despite my best efforts. Although I understood that transcribing data could take
some time I was not prepared for just how long it actually took to transcribe one hour of
recorded observation. This was something I had to seriously consider when carrying out
the main research.

Making observations and taking notes also proved to be more difficult than I had
anticipated. I had hoped during the lessons to be able to record some detailed
observations of at least one group for the purpose of triangulation (Cohen et al, 2005;
and Denscombe, 2005). However, having to attend to the needs of others meant that I
did not spend enough time with my focus group and could only manage the briefest of
notes.

Transcribing and analysing the data enabled me to hear how pupils were beginning to
work together and the kinds of discourse that occurred when they were working in
partnership with others. It was encouraging to find that there were changes in the way
some pupils worked and there was clear evidence of co-operation between some. The
initial study helped highlight the social interaction skills and problem solving strategies
which needed developing to further progress towards a more collaborative way of
learning in mathematics. It also helped me to focus on my research questions and finding answers to them.

Summary and Conclusions

My reasons for carrying out an initial study were threefold. First of all I wanted to test methods of data collection to see what problems might arise so that I could resolve these before my main data collection began. The second reason was to trial some problem solving activities and see what strategies my pupils engaged in when solving these tasks and the third reason was to see how my pupils worked together on a task when given the opportunity to do so.

The main problem that arose when recording the lessons was an excess of background noise from other classes which made it difficult to hear my pupils' dialogue. This could be lessened to an extent by planning the lessons to take place at a time when one or both of the neighbouring classes were elsewhere. Another problem that arose was the difficulty I had recording field notes during the lessons. This might have improved with practice but I decided it was better to record these notes in my research diary as soon as possible after the lessons, despite Burgess' (1985) concern that some detail might be lost.

Carrying out the initial study helped me to look at my research questions and begin finding answers to them. Category 1 from the data analysis which explored the strategies pupils used helped me to focus on answering the research question,

What strategies do young children use when solving problems?

The strategies pupils used for the task in lesson 1 were mostly counting on, counting back, mental recall of number bonds as discussed by Carpenter et al (1981), Geary et al (1991) and Thornton (1990) and using patterns as outlined by Gray and Tall (1992). In Lesson 2
some pupils used counting on strategies as well as reasoning and guessing but were less
successful in solving the problem than in Lesson 1. Some pupils did not appear to know
what strategy to use and even specialising simple cases seemed to be beyond them. Most
of the strategies pupils used were basic strategies and even though the task in Lesson 2
was more challenging I had expected some of the higher attaining pupils to solve it. It was
clearly not a task they were used to. This led me to believe that the tasks I was giving
pupils were not challenging enough to allow them to use more advanced strategies. I
therefore planned to introduce more rich mathematics tasks which would challenge
pupils’ thinking and develop their use of a wider range of problem-solving strategies.

Categories 2 and 3 from the data analysis helped me to focus on answering the research
question,

Is collaborative learning a more effective approach to problem solving than
working individually?

The fact that fewer pupils appeared to work together in Lesson 2 than in Lesson 1 may
have been due to the challenge of the task in Lesson 2. They were not used to working
collaboratively and when faced with a challenging task many did not seem to know how
to help each other to become ‘unstuck’. Some pupils became protective of what they
considered to be their ideas and accused others of ‘copying’ even though their ideas did
not seem to be getting them very far. They were working much more individually and
competitively as discussed by Nicholls et al (1990) and Qin et al (1995), preferring to keep
everything to themselves. Although there was evidence that some pupils were able to
work collaboratively when given the right tasks, it was also clear that working co­
operatively did not come naturally to them. I wanted to change the learning environment
of the classroom to one of collaboration, where pupils would be encouraged to discuss
and share their ideas and be able to reason with and explain their strategies to others. With this in mind I planned to introduce Thinking Together (Dawes and Sams, 2004) as a programme of regular lessons in 'exploratory talk'. The 'ground rules for talk' would be generated by my pupils and displayed in class to be used regularly.

References


Tahta, D. G. (1972) A Boolean Anthology: Selected Writings of Mary Boole on Mathematical Education: Association of Teachers of Mathematics


Appendix 2: Extract from my research diary

[Handwritten text image]

<table>
<thead>
<tr>
<th>Date</th>
<th>Activity Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/2/2021</td>
<td>Discussed problem</td>
</tr>
<tr>
<td>9/2/2021</td>
<td>Hive problem together</td>
</tr>
</tbody>
</table>

We asked the children to discuss the problem together. Simon and Sam were able to work on it, while the others worked on it. Simon and Sam seemed able to work on it, while the others worked on it. Simon and Sam seemed able to work on it, while the others worked on it.

Joe then quickly said yes, and I suggested to get a mark. I suggested to get a mark. I suggested to get a mark. I suggested to get a mark. I suggested to get a mark. I suggested to get a mark. I suggested to get a mark.
Appendix 3: Sample of work

[Image: Four-pin bowling]

Appendix 4: Example of open coding

Teacher: How did you get 10 cm, what did you do? **Questioning**

Megan: Because I halved the string and got 10 cm. **Maths thinking**

Teacher: So you did that yes? (Folding string in half), halved the string, and what did you say? **Reviewing/clarifying**

Megan: I got 10 cm

Teacher: But how do you know that's 10? **Questioning**

Megan: Because...

Teacher: Can anybody help Megan explain? Lexa **Questioning**
Lexa: Because half of 20 is 10. **Maths thinking**

**Appendix 5: Example of writing memos**

Angela: \( 2 + 3 = \) or \( 4 + 1 = 5 \)

Teacher: How did you get those 2 numbers, what did you do?

Angela: I put them together with a pencil.

Teacher: Why?

Angela: Because...

Teacher: Yes?

Angela: Because it was easier for me

Teacher: But why did you do that?

Silence

Teacher: Did you use the number line, did you use your fingers or did you know the numbers in your head already?

Jack: I knew the numbers already.

Did she understand my question? Her response explains how she recorded her answer rather than how she worked out the solution. I had to prompt her with the question below in order for her to understand that I wanted to know how she worked it out and in the end her partner answered.

**Appendix 6: Sorting Counters**

Children were given an assortment of counters at their tables in the shape of fruit, animals, vehicles etc.

They had to decide how they were going to sort them and then record the results in their maths books.
Appendix 7: Measuring Strings

A piece of string is 20 cm long. How long is half a piece of string?

How long would the string be if I doubled it?

How long could each piece be if I cut it into 4 pieces?

Appendix 8: Four-pin Bowling


Appendix 9: The Slipper Problem

Michelle had 6 slippers and Ronson had 10.

How many slippers did they have altogether?

How many pairs of slippers could they make?
Appendix 10: What is the Question

If the answer is 50 what is the question?

Appendix 11: Word Problems

Rose had 52 oranges. She bought 13 more.

How many oranges did Rose have altogether?

Pupils had to add/subtract 2-digit numbers, making up their own problems similar to the one above.

Appendix 12: Mother Goose

Mother Goose puts her eggs into baskets.

Mother Goose has 12 eggs.

She puts the same number of eggs into each basket.

How many eggs does Mother Goose put into each basket?

Appendix 13: Class Teams

Blue class is getting into teams.

There are 20 children in blue class.

Miss Indigo organises the children into teams of 4.

How many teams will there be?
Appendix 14: Lesson 2 - Time Problems

Year 2 go on the playground at 9 o'clock. They stay out there for 2 hours.

What time is it when they come back into class?

Appendix 15: Lesson 22 - Underwater Treasures

For this activity children were given a copy of the clues and 30 pictures of shells, pre-cut, to share between 2.

Underwater treasures

The mermaid has between 16 and 30 shells.
If she groups the shells in fives there are four left over.
If she groups the shells in threes there are none left over.
If she groups the shells in fours there are none left over.

How many shells does the mermaid have altogether?


Appendix 16: Lesson 21 - Patterns

Kerry has made a diagonal pattern on a 1-25 square.

The first square she coloured was 2.

She colours 4 squares altogether.
What are the other numbers in the pattern?

The children were given squared paper and encouraged to draw anything to help them.

Most of them drew a number square similar to this with numbers 1-25. For the extension activity the children had to use a blank number square.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
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<td>25</td>
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**Appendix 17: Making 30**

Write as many different ways as you can of making the number 30.

(Taken from the National Numeracy Strategy, DfEE 1999)

**Appendix 18: Karate Lessons**

Kevin has just started karate lessons. He has a lesson every five days. His first lesson is on a Monday.

On which day of the week will his fourth lesson fall?

Appendix 19: Lesson 15- Lots of Lollies

Frances and Rishi were given a bag of lollies.

They shared them out evenly and had one left over.

Just as they had finished sharing them their friends Kishan, Hayley and Paul came along. They wanted some lollies too so the children shared them out again between all of them. This time they had two lollies left over.

How many lollies could there have been in the bag?

(http://nrich.maths.org/curriculum)

Appendix 20: Lesson 26- Dogs and Tails

Part 1: Suppose you were at a dog shelter and you wanted to count all the dog eyes you saw. If there was one dog, how many eyes would there be?

What if there were two dogs? Three dogs? 100 dogs?

Do you see a relationship between the number of dogs and the total number of eyes? How would you describe this relationship? How do you know this works?

(Blanton and Kaput, 2004)
**Appendix 21: Lesson 27- Eyes and Tails**

*Part 2: Suppose you wanted to find out how many eyes and tails there were all together.*

*How many eyes and tails are there for one dog? Two dogs? Three dogs? 100 dogs?*

*How would you describe the relationship between the number of dogs and the total number of eyes and tails? How do you know this works?*

*(Blanton and Kaput, 2004)*

**Appendix 22: Lesson 28- 2p Coins**

*Miss Lynch has only 2p coins in her purse.*

*How much money would she have if she had 10 coins? 100 coins?*

*Can you write a statement that would give us the amount for any number of coins?*

**Appendix 23: Initial codes grouped under broader categories**

<table>
<thead>
<tr>
<th>Initial Codes</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning</td>
<td></td>
</tr>
<tr>
<td>Guessing</td>
<td></td>
</tr>
<tr>
<td>Using patterns</td>
<td></td>
</tr>
<tr>
<td>Mental recall of number bonds</td>
<td></td>
</tr>
<tr>
<td>Taking turns</td>
<td></td>
</tr>
<tr>
<td>Making suggestions</td>
<td></td>
</tr>
<tr>
<td>Confirming</td>
<td></td>
</tr>
<tr>
<td>Claims of copying</td>
<td>Working independently (Nicholls et al (1990))</td>
</tr>
<tr>
<td>Affirming ability</td>
<td></td>
</tr>
<tr>
<td>Statements of intent</td>
<td></td>
</tr>
<tr>
<td>Bickering</td>
<td>Distractions (Bennett &amp; Cass (1989), Webb (1991))</td>
</tr>
<tr>
<td>Off-task talk</td>
<td></td>
</tr>
<tr>
<td>Reactivity</td>
<td></td>
</tr>
<tr>
<td>Following rules</td>
<td>Refocusing</td>
</tr>
<tr>
<td>Conforming</td>
<td></td>
</tr>
<tr>
<td>Refocusing</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 24: Analysis of a Lesson Transcript triangulated with Samples of Work and extracts from Research Diary

Lesson- Dogs and eyes

Suppose you were at a dog shelter and you wanted to count all the dog eyes you saw. If there was one dog, how many eyes would there be? What if there were two dogs? Three dogs? 100 dogs? Do you see a relationship between the number of dogs and the total number of eyes? How would you describe this relationship? How do you know this works?

(ELEMENTARY GRADES STUDENTS' CAPACITY FOR FUNCTIONAL THINKING
Maria L. Blanton and James J. Kaput)

The children working on this problem were

Alice and Christian

Hannah and Isabelle

Daisy and Jude

Teacher: Okay, the problem we are going to look at today is... Jack, when you are ready ... Look at the board, what have we got a picture of?

Children: A dog.

Teacher: A dog. Now suppose you were at a dog shelter... suppose you were at a dog shelter and you wanted to count all the dogs' eyes that you saw.

Children: Aaah

Teacher: If there was one dog, how many eyes would there be? Alice

Alice: 2

Teacher: 2... How do you know? What makes you think that? Questioning

Alice: Because animals and people always have 2 eyes. Justifying/ general statement

Teacher: Okay, so if there is one dog it’s always got...

Children: 2 eyes.

Teacher: 2 eyes. What if there were 2 dogs? Wow, all those hands up! Hannah

Hannah: 4
Teacher: Why?

Hannah: Because all animals and humans have 2 eyes and another 2 eyes make 4...2 plus 2 equals 4. Hannah was able to make a general statement to explain her reasoning.

Teacher: Okay, what if there were 3 dogs? Stella

Stella: 6

Teacher: 6... why?

Stella: ?

Teacher: Just a minute, I need to hear Stella. Stella, I need a louder voice from you please.

Stella: If there is one dog, then there is 2 eyes. If there are 2 dogs that's 2 more eyes and 3 dogs, that's 2 more eyes. Strategy/counting 2s

Teacher: Okay, so how could we record that? What could we do to show that on our paper? Molly

Molly: We could draw... we could draw a picture of a dog and we could put... we could put an arrow line with eyes and write 2 eyes. Visual representation

Teacher: Like that? (Drawing → from the picture of the dog to 2).

Children: Yes. See research diary where this is drawn.

Teacher: That's one way. Any other way? Jack

Jack: If we write 3 dogs, they have 6 eyes.

Teacher: Okay, how would we show that?

Jack: You put 3 dogs, and then you put 2 eyes on each dog. Explaining recording

Teacher: So if I get... there is one dog... so, how many dogs have I got now? (Using cloning feature on IWB to get extra pictures of dogs)

Children: 2

Teacher: How many eyes? (writing → and corresponding number) Questioning

Children: 4

Teacher: Then what will I do next? Questioning

257 | Page
Children: Get another dog.

Teacher: I could... then what would I do next.

Children: Write 6

Teacher: I could do that. Is there any other way... is there anything else I could do?

Jack: Hey, there's a pattern! Pattern spotting

Teacher: Where is the pattern? Questioning

Jack: 2, 4, 6, 8.

Teacher: Why did you say 8? Questioning

Jack: It's supposed to be 8, because it's a number pattern. Recognising next element

Teacher: It is a number pattern yes, and it is count in...

Children: Twos/even numbers. Recognising pattern

Teacher: It's counting even numbers... very good, so we know it's even numbers and it's counting in...

Children: Twos

Teacher: It's counting in twos. Now if...Benjamin... if we didn't have the pictures, what other way could we show it? Molly Questioning

Molly: Draw the dogs...

Teacher: Ali please stop making that noise.

Molly: Draw the pictures of the dogs. Way of recording

Teacher: But I said, if we didn't have pictures, what other way could we record it? Hannah

Hannah: We could do 1 plus 1 equals 2... 2 plus 2...

Teacher: What do you mean one plus one... where are you getting the 1 from? Questioning

Hannah: ...
Teacher: Sorry Hannah. Children I need you not to move the chairs because it is going to be very loud on the tape. So just sit still... yes Hannah

Hannah: Em...(Unable to explain)

Teacher: So let's have a look at recording it another way. What could we say? How many dogs do we have? Questioning

Children: 3

Teacher: No, no... when we started off.

Children: 1

Teacher: One dog... how many eyes did it have? Questioning

Children: 2

Teacher: What happened when we had 2 dogs? Questioning

Children: 4 eyes

Teacher: So if there are 2 dogs... then 4 eyes... What about if we had 8 dogs?

Children: Ooh! 8 dogs?

Benjamin: That's easy!

Teacher: I don't want to hear 'that's easy' ... I want to hear you telling me what it is. Put your hands up if you have an answer. Jude

Jude: 16 Mental strategy

Teacher: 16... where did you get 16 from Jude? Questioning

Jude: ...

Teacher: Let Jude answer. Jude worked out that if there were 8 dogs there would be 16 eyes, and he's going to tell us how he got that. Questioning

Jude: ...

Teacher: You worked out that answer straightaway didn't you Jude.

Jude: Yes.

Teacher: You had to do something in your head to work it out. Didn't you?
Jude: Yes.

Teacher: What did you need to do?

Jude: Count

Teacher: How did you count?

Jude: In twos.

Teacher: You count in twos. Tell us exactly what you did.

Jude: ...

Teacher: What did you start on?

Jude: 1

Teacher: I don't think you started on one. Can anyone explain how Jude would have got that answer? Christian

Christian: He could have counted in twos... (demonstrates counting in twos using his fingers) 2... 2... 2... 2... 2... 2... he could have counted in twos 8 times.

Teacher: Okay, what else could he have done? Benjamin

Benjamin: When he got to 4 he could have doubled it and then he could have doubled it again... then you could have doubled it again and got 16.

Teacher: Why would he have doubled it?

Benjamin: Because 2 plus 2 equals 4, and when he doubles it... 8 ... and then he just needs 12 more

Teacher: Why would he have added 8?

Hannah: He could have added 8

Teacher: Okay, yes, he might have done that. Right, what I want you to do now, I want you to work out what you think the pattern is. I want you to decide how you're going to record it and I want you to look at the relationship.
which means how they are connected... how would the number of eyes be connected to the number of dogs. Okay. Now Jack said there is a pattern going down. I want you to look across to see if there is a pattern across. How is one dog connected to 2 eyes? How are they related to each other? See if you can show me how you could record that information if I asked you to work out how many eyes 10 dogs would have.  

**Setting up problem**

**Children:**  
What!... easy, peasy

**Teacher:**  
I want you to talk to your partner and see who can work out... no, I don't want you putting your hands up... listen to me. I want you to talk to your partner. I want you to find out how you work out how many eyes there were if there were 10 dogs, and I want to know, what is the relationship between the dogs and the eyes? Then, I want you to work out... if there were 100 dogs how many eyes would there be?

**Children:**  
Aah! One hundred!

**Teacher:**  
Off you go and talk to your partners.

**Christian:**  
10 dogs...

**Alice:**  
10 dogs...

**Isabelle:**  
So we are counting in twos...

**Hannah:**  
2, 2, 2, 2, 2, 2, 2, ... ah let's count ... 1, 2, 3... wait, 1, 2, 3, 4, 5, 6... 7... 8...

**Isabelle:**  
I should do them

?  
?

**Hannah:**  
I think it is 5

**Isabelle:**  
Why 5?

?  

**Isabelle:**  
it's 20

?  

**Alice:**  
it's 200.

**Isabelle:**  
200 and 20

**Recognising a pattern**

**Counting elements in the pattern**

**Worked out the near generalisation**

**Worked out the far generalisation**

**Near and far generalisation**
Hannah: And how do you get are they connected to the dog? Questioning

Alice: No, it's 100 dogs

Isabelle: How do you connect the dogs' eyes to the dog? Questioning

Daisy: 10 dogs, how many eyes do 10 dogs have? Restating the question

Teacher: Joe, what have you written so far? Questioning

Daisy: 10 dogs, how many eyes do 10 dogs have? Questioning

Jude: We have to work it out and write the answers on the sheet.

Isabelle: We could draw 10 dogs Making suggestions

Hannah: Let's draw 10 dogs Suggesting drawing the dogs as a way of working out the answer - Check work to see what strategies were used.

(Children begin drawing dogs and discussing how they are drawing them as they go along).

Teacher: Okay, stop for a minute, stop for a minute. Right, how are you recording it on your paper?

Isabelle: Because, we draw 10 dogs and then we counted... we draw 2... we draw 2 then 4, then...

Hannah: We are going to draw 10 dogs... Explaining their strategy

Teacher: Just a minute... can you leave the pencils. I don't want to hear pencils being dropped on the tables. Thank you.

Hannah: We are going to draw 10 dogs and then we are going to count how many eyes they all have Explaining strategy

Isabelle: ... many eyes they all have.
Teacher: Okay, how are you going to... is anybody going to do it a different way? They are drawing dogs... is anybody doing it like this? Molly

Molly: Me and Alicia...

Teacher: Sorry... somebody else is talking.

Molly: ... are using the number line and then every time we are counting 2 numbers and Alicia is putting one of her fingers there and we get what number we are on...

Teacher: Okay, now I did show you on the board there were other ways of recording it. Not many people are recording anything. You don't have to do all of this in your head. You have a piece of paper that you can write or draw, anything that will help you. Some people are drawing the dogs and writing it like this (showing pictures and arrows on the white board) one dog has 2 eyes, 2 dogs have 4 eyes. There is another way you can do it that will save you... because drawing dogs is going to take a bit of time. You use a lot of time drawing, when it's not really the point of the lesson so, what we need to do, is to think of a way of recording it. Remember yesterday when we were recording information using a Carroll diagram, well, this is what we call a t-chart...

Children: T-chart?

Teacher: It looks like a t. Okay, on this side we could have the...

Children: Dogs.

Teacher: The dogs. On this side we could have the...

Children: Eyes.

Teacher: The eyes. So, if we have one dog...

Hannah: We have 2 eyes.

Teacher: We have 2 eyes. If we have 2 dogs...

Children: 4 eyes.

Hannah: We have 4 eyes

Teacher: If I have 3 dogs...

Children: 6, 6 eyes.
Teacher: Okay, now, what I want you to do... I want you to work out how many eyes there would be if we have 10 dogs. No, I don't want your hand up to tell me. I want you to work it out. Carry on, and once you have done that... listening ..., I want you to look at these numbers. I don't want you looking down. Jack has already spotted the pattern going down, that's fine, but I want you to look at the pattern that you might see across. How are they related? How will that help you work out 100 dogs? Setting up the problem

Children: Wow...aah!

Teacher: Stop! Don't do that... you don't need to do that. If you were to do it this way (pointing down the pattern) and all the way down to 100 you won't have room on your page and it would be a waste of time. Is there a quick way, using the relationship between these (pointing across the pattern) that would help you to work it out? Don't put your hand up and tell me. Work out first of all 10 dogs then work out 100 dogs. Setting up the problem

Hannah: Okay, so now we have to cross these 2 dogs off.

Isabelle: Like that?

Hannah: Yes, just do that and use a t-chart. Recording


Teacher: Jake, why haven't you started? Lily, why haven't you started?

Lily: I'm being silly.

Teacher: You're being silly, yes. So stop being silly and get on.

Isabelle: One dog has 2 eyes...

Hannah: You have to write dogs and eyes.

Isabelle: I know, look, there's dogs and eyes.

Alice: That's what we're doing.

Hannah: 2 dogs then 4 eyes recording elements in the pattern

Isabelle: 3 dogs have 6 dogs, I mean 6 eyes recording elements in the pattern

Hannah: 4... 8 recording elements in the pattern

Isabelle: Yes, 8.

Hannah: One dog will have how many eyes?... 2... 2 dogs will have 4 eyes. 3 dogs will have 6 eyes.
Isabelle: 4 dogs will have 8 eyes.
Hannah: 5 will have 10 eyes. **Spotting the pattern**
Isabelle: No.
Hannah: Yes, 5 and then 10
Isabelle: 6. is 12 eyes... 7 is... **Continuing the pattern**
Hannah: 14... 8 is 15... 9... 17... 10... 20...
?
Hannah: So that's how much... have you done it? Right 15... I mean 16... then you do 18  **Continuing the pattern/near generalisation**
?
Hannah: Oh yes, I made a mistake I wrote 14, 15... 16
Teacher: So, Daisy, how many eyes would 5 dogs have? **Questioning**
Alice: Look, 2, 4, 6, 8, 10, 12... 14... **Spotting the pattern**
Christian: Oh yeah.
Hannah: Now we have to work out how many eyes 100 dogs have
Isabelle: 10 dogs is going to be 20... see, I'm right. **Near generalisation**
Alice: ... 16, 18... **Continuing the pattern**
Christian: Oh yeah
?
Teacher: Have you noticed... how are these connected? **Questioning**
Hannah: I know... 1 plus 1 equals 2.
Teacher: Where are you getting the other one from? Because this is one dog has got 2 eyes. 2 dogs have got 4 eyes **Questioning**
Christian: (singing) Alleluia, alleluia... alleluia, alleluia)... this is hard
Teacher: Look at it... look... one dog has 2 eyes, 2 dogs have 4 eyes... 3 is 6... 4 is 8.
Isabelle: 10 has 20 eyes. **Near generalisation**
Teacher: What is the relationship? That's what you have to look for because that will help you to work out 100... that's going in the right direction. **Questioning**
Teacher: So 4 dogs have...
Jude: 8.
Teacher: 8... 5 dogs will have...
Jude: 10
Teacher: Why do you say 10?  
Jude: Because 5 plus 5 equals 10  
Teacher: Why do you say 5 plus 5?  
?  
Isabelle: How can you write up to 100?  
?  
Alice: Why don't you draw a dog there and another dog there?  
?  
Teacher: Why don't you explain to Daisy why it should be 5+5... because she doesn't look like she's quite sure. So you explain to Daisy why you think it should be 10.  
?  
Alice: Why don't you... why don't you do a dog then an arrow and write 2?... That's what I'm doing anyway, you can do a different way, but I'm doing that.  
Isabelle:  
Christian: 100 dogs...?  
Isabelle: 120 eyes.
Teacher: Jack, don't do that with the chair... okay, good, carry on.  
?  
Teacher: Well, have you noticed... have a look at... one dog has got...  
Isabelle/Hannah: 2 eyes  
Teacher: 2 dogs have got...
I/H: 4 eyes

Teacher: How are they connected?... 3 dogs have got...

I/H: 6 eyes.

Hannah: You're adding one on

Teacher: Are you? Are you adding one onto 2 to get 4?... Why would you add one when one dog had 2 eyes? Are you saying that you add one eye on each time? Questioning

Hannah: No

Teacher: No.

Hannah: 2 eyes.

Teacher: So if you've got 2 dogs they have...

Hannah: 4 eyes

Teacher: Right, so 3...

Hannah: 6.

Teacher: 4...

Hannah: 8

Teacher: So how are they related? Can you see a relationship between the numbers on this side and the numbers on that side? Extending their thinking

Hannah: If 2 dogs have 4 eyes... we are adding the numbers that are here... Beginning to see the relationship

Teacher: Benjamin, where are you going?

Benjamin: ?

Teacher: Pardon

Benjamin: I need to see the board.

Teacher: Then you need to bring your glasses.

? 

Hannah: We are adding 2

Teacher: What about this one?
Hannah: We are adding 3.
Teacher: What about the next one?
Hannah: We are adding 4.
Teacher: What is actually happening to this number to give you this number?
Hannah: We are adding the same number that is here.
Teacher: Have another look, is there something else?... Speak to your partner and look really carefully. When you find out how they are connected and related that would very quickly help you to work out...
Hannah: 100.
Teacher: 100 dogs... (turning to Christian and Alice). Have you worked out a pattern? Have you worked out a pattern between that and that? Talk to each other about it and try and find a pattern across.
Isabelle: I've got 24
Alice: oh, I know... 1, 2, 3, 4, 5...
Teacher: What is happening?... How many eyes will 9 dogs have?
Isabelle: I know... 1, 2, 3, 4, 5...
Teacher: Okay, stop, put your pencils down. Okay, who worked out, how many eyes 10 dogs would have? Isabelle
Isabelle: 20 eyes.
Teacher: How did you work out 20 eyes?
Isabelle: Because we made a t-chart, and we did one dog has 2 eyes, 2 dogs is 4 eyes and 3 dogs are 6 eyes and 4 dogs are...
Teacher: Ssh, Jack, listen
Isabelle: 8 eyes, 5 dogs are 10 eyes, 6 dogs are 12 eyes, 7 dogs are 14 eyes, and 8 dogs are 16 eyes, 9 dogs are 18 and 10 dogs are 20 eyes.
Teacher: Right, so what you did, was carry on the pattern down. So each time... what did you do each time to the number? Jack you were very good at thinking...
Hannah: 6.
Teacher: 4 dogs.
Hannah: 8.
Teacher: 5 dogs.

Children: 10
Teacher: What you are doing going down? Questioning
Hannah: We are adding the number there Explaining strategy
Teacher: What are you doing here? What is happening each time? Questioning
Hannah: We are adding... we are counting in twos Explaining strategy
Teacher: Yes, you are counting in twos, because each time you are adding another pair of eyes on, aren't you. So that is the pattern going down, and Jack spotted that straightaway at the beginning. He spotted the pattern when we wrote the 3rd one. That's very good. Lots of other people spotted the pattern. Some people said it was even numbers. Why is it even numbers? Questioning

Ali: Because even numbers ends in 0, 2, 4... General statement
Teacher: No, why is... will the number of eyes always be even? Questioning
Children: No/yes.
Teacher: Some children say yes, some people say no.
Benjamin: Yes
Teacher: Benjamin, you say yes, why? Questioning
Benjamin: Because when you plus 2 it's always going to be an even number because 2 is an even number. General statement
Teacher: And why is that? because we're talking about the number of eyes, so every time we add an extra dog on, Molly, we'll always be adding on what?
Molly: We'll always be adding on 2 because every dog has 2 eyes. General statement
Teacher: Excellent. Every dog has 2 eyes. We won't ever have an odd number because there is no dog with one eye, unless it has had an accident but every
dog normally has 2 eyes. So that is the reason why the answers are always even and we are counting even numbers. Benjamin said we were counting in 2s and Molly said they all have 2 eyes so it has to be even. So that’s good, you have spotted the pattern going down. Now in order for you to be able to work this one out quickly without spending all day going down the list all the way down to 100 you need to look at what’s happening to this number to give you this number. What’s happening to this number to give you this number, and what’s happening to this one to give you this one. And all the way across. If I carry on...5... 10... 6 would be...

Max: 12
Teacher: 12, yes, 7 would be? Questioning
Children: 14
Teacher: 8 would be?
Children: 16
Teacher: 9 would be?
Children: 18
Teacher: 10 would be?
Children: 20 continuing the vertical pattern

Teacher: So look, what’s happening across? Now I want you to talk to your partner because there’s not a lot of talking going on. Look at the patterns you have done I can see lots and lots of people have done the t-chart and the pattern going down. Lovely. Now I want you to talk about it and to see what is happening. Now Hannah and Isabelle were starting to do a little bit of that but you need to say okay, what is happening to 2... what are the different things I can do to 2 to give me 4? What are the different things, I can do to 3 so that it will give me 6? You work out. Does that apply to all of them to give you the other number? When you decide and find out what it is, you will, like that, (clicking fingers) be then able to work out 100 dogs. You will know what’s happening to the numbers on this side to give you these without having to count. So talk to your partner now, right now this minute, pencils down and talk to your partner.

Christian: I know how it works...
Alice: You are not allowed to tell other people...
Hannah: I know, I know. I add one and I get 2, and I add 2 and I get 4.
Christian: I know... how... it... works... Beginning to see a relationship

Hannah: See, it's easy.

Christian: You have one, you add one... you have 2, you add 2... you have 3, you add 3... you have 4, you add 4... Beginning to see a relationship

Alice: No, because where is the other one coming from then?

Isabelle: You must add 2 then, because...

Hannah: I said add 2...

Isabelle: No you didn't

Hannah: Yes I did

Christian: Will you two work it out!

Isabelle: No you didn't, you said one add 2 add one.

Christian: I know how we can get it because one plus one equals 2... 2 plus 2 equals 4... 3 plus 3 equals 6 Spotting the relationship

Isabelle: 2, 4, 6 Saying the pattern

Hannah: Maybe we should carry on

Isabelle: 10, 11, 12, 13, 14 Saying the pattern
Alice: 100 dogs... do you know how much that makes?

Christian: ...?... You don't add 3... you add the number of dogs!... You add the number of dogs. Making a connection

Isabelle: Draw 100 dogs... joking!

Hannah: I have 10 dogs... so 11 dogs will have... 22... 12 dogs will have 24... 13 dogs will have... 26 Continuing the vertical pattern

Alice: I have only one square left

Teacher: Don't keep going on... I need you to work out a quick way of doing it... so how have you worked out... what did you do to 5 to get 10? Questioning

Hannah: I added another 5. Beginning to make a connection

Teacher: Okay, so, what did you do with 6 to get 12? Questioning

Hannah: I added another 6

Teacher: So why did you add on the same number? Questioning

Hannah: Because it asks you to add the number here on to here...

Teacher: okay, so, what... if I said to you... what would you do to get 100 dogs? What will you do with the hundred? What will you do to get 100 dogs? You are not talking to each other and trying to work out what the relationship is. Come on, get speaking and share your ideas on how you've got to these numbers

Christian: see, you have one dog you add one... you have 2 dogs you add 2. You have 3 dogs, you add 3... you have 4 dogs you add 4... when you have 5 dogs you add 5... Explaining strategy

Alice: Oh, I get it now... I think what you're trying to say is that if you have one you add... Beginning to see the connection

Christian: You have to add the number of dogs here. Like if you have one you add one, if you have 2 you add 2... you see? Explaining strategy

Alice: Now I get it

Teacher: What did you do with the 2 to get 4? Questioning

Christian: We added 2 on Explaining strategy

It appears as if Fionnuala has made the connection between the number of dogs and number of eyes.
Teacher: What did you do with 5 to get 10?  
Christian: We added 5.  
Teacher: What is another way of saying that? So what are you doing to this side to get these?  
Christian: Adding each number...?  
Teacher: But there is a better way of saying that.  
Isabelle: Adds another hundred.  
Alice: Doubling  
Teacher: You are doubling them, that's right. So, what would you do to get 100 dogs?  
Christian: We would...  
Alice: Double it  
Teacher: And how many dogs... how many eyes would those dogs have?  
Isabelle: We've got 100... that makes 200  
Hannah: We could keep on doubling  
Teacher: ?  
Christian:  
Teacher: What is double 100?  
Christian:  
Teacher: Because you know, double 1 is...  
Christian: 2  
Teacher: 2... so double 100 would be...  
Teacher: 200... so you now know how much 100 dogs have... you can write it on the next page if you like  
Isabelle: Mrs D, can we turn around to get more space?  
Teacher: Yes, of course, you can
Isabelle: Write 100 dogs.

Hannah: 200

Teacher: Why do you think it's 200? Questioning

? Teacher: Okay, stop... stop. Who has managed to find out what is happening to the number of dogs to give us the number of eyes? Christian. Questioning

Christian: We keep doubling the number of dogs. General statement

Nicholas is able to verbalise the relationship between the number of dogs and eyes.

Teacher: Very good, now did that help you work out how many eyes 100 dogs would have?

Christian: Yes.

Teacher: So, how many eyes would that be? Questioning

Christian: 200 eyes

Teacher: 200 eyes. How many people worked out that there would be 200 eyes? So that's about half (counting a show of hands). Okay, very good. Hands down. Now, that's okay for... we've worked it out for small numbers... Joe... We've even worked it out for big numbers without having to count all the way down. So we have made it a bit more general. What we want to do now is think about how we could write something that would tell us how we could work it out for any number of dogs. Because we worked it out for 10 dogs, we knew if we doubled that we got 20. We worked it out for 100 dogs... we knew that would give us... Extending the problem and looking for generalisations

Children: 200.

Teacher: 200 eyes. Now, could we think of a general statement we could say that would cover us for any number of dogs? Hannah Questioning

Hannah: We could do 100 plus 100 equals 200.

Teacher: That's only... excuse me somebody is talking when other people are trying to explain, that's not very nice manners. That is only dealing with 100, we want something that... if we could put any number in it to fit it. Could we have a general statement to cover any number? Molly
Molly: We could have a number then we could have... we could double that and then it would... *general statement verbally*

Teacher: Sorry, just a minute Molly... you're explaining it perfectly but pencil rolling and chair banging is going to cover your voice and I won't hear it. Can you say that again... we could double...

Molly: double any number, and it will always be an even number.

Teacher: Okay, so what we need to do is get the number, double it and it will give us the number of eyes. Yes?

Molly: Yes.

Teacher: okay, very good. Now, what are you doing... that these people were able to tell me at this table... what are you doing to a number if you double it? Joe

Joe: You are making more

Teacher: Yes, you're making more, but what is the correct maths you are doing?

Benjamin: You are multiplying. *General statement*

Teacher: By?

Benjamin: by 2. *General statement continued*

Teacher: You are multiplying it by 2. So, is there an equation we could write that would explain what we are doing so we could put in any number? Hannah

Hannah: We could write a number and then...

Teacher: What could we put to show any number? Molly *Questioning*

Molly: We could write...... we could write... I've forgotten what I was going to say.

Teacher: Well, if you had remembered what you were going to say, what would it be?

Molly: We could make a number...?

Teacher: Well, what I would like you to do... when we do an equation and we don't know the number, what do we put in, in its place? So I want you to talk to your partner and see if there is an equation you can write down that would explain to us, show us what we are doing so that if Miss L came in and said
oh, I've got... in my neighbourhood there are 500 dogs. How many eyes have they? Is there a way we could do it that would work for any number so we don't have to count them all up and work them all out? Is there in equation we can say? Can you write it down some way on your sheet? Talk to your partner, write it down. Is there a general statement we could say that we're doubling the number of eyes for any number of dogs?

**Questioning**

?  
Isabelle: I'm doing 100 dogs. **Far generalisation**

?  
Isabelle: Do 100 dogs

Hannah: Let's just write 200 dogs and write the answer... 200 dogs. **Far generalisation**

Teacher: ... something that will cover any number. So, you can't really have a number there

Hannah: Oh, I know... a question mark. **Early algebra using an unknown**

?  
?  

Alice: We can't tell you what we've done.

Christian: We can double 500... **Far generalisation**

Hannah: if we double 500... **Far generalisation**

Isabelle: 10 hundred...  
Hannah: 10 hundred

Christian: Add 500... we could add 500... weird, weird, weird... so that's how it goes like here. You add... you have one you add one, you have 2 you add 2, you have 3 you add 3, you have 4 you add 4... **Restating the pattern**

Alice: So how are we going to make it?

Christian: and that's how it works. So we have to add 500

Alice: He's just a weirdeo.

Christian: I'm not a weirdeo. I'm not a weirdeo

Teacher: Well?
Isabelle: This is what Hannah put

Teacher: Okay, let's have a look

(Hannah had written \( ? + ? = ? \)) Early algebra-using an unknown in an equation

Hannah appears to recognise using a symbol to represent an unknown in an equation but has not yet worked out how to write 'doubling' as an equation. Check her work for evidence

Teacher: Right, if I do this it could mean that that's going to be the same as that... how is that telling us you are doubling? Remember at that table, they said that doubling was multiplying by... Questioning

Hannah: 2

Teacher: By 2 but if you have different symbols... it's good to use the \( ? \) as a symbol to show that we don't know the number, but if you have the same one I might look at that and think oh its the same number for all of them. You see? And if I put here, 3 dogs, am I adding 3 eyes? You see, I might think that 3+3... that's not telling me that I have to double the number of dogs to give me the number of eyes. So, how could you do it that would show us that? I like the question mark. You can still use that, but how could you use it to show you are doubling to get these? Extending thinking

Teacher: (to Christian and Alice), you are still working with numbers. We need a general statement... something general so that we could put any number in... like Molly said or like Christian said we are doubling the number of dogs to give us the number of eyes. So, we don't know what the number of dogs is because we're going to try it for any number. So, how could we write that?... When I give you an equation on the board and there is a number missing what do we put in its place to show us that a number is missing? Extending thinking

Christian: Sometimes a question mark

Teacher: You can put a question mark. Can you put anything else?

Alice: A box.

Teacher: A box, or a square or a circle or a triangle or whatever. So we can use something in place of the number. So could you write an equation that would show us that we are doubling the number of dogs, but we don't know the number of dogs to give us the number of eyes? Have a chat with each other about it.
Hannah: Okay, let's try again... don't cross that one out.

Teacher: (to Jude and Erika)... +4 that's not telling me you're doubling. Remember, what did we said doubling was? What did Benjamin tell us doubling was the same as? Questioning

Jude: times 2.

Teacher: Times 2... and stop. Okay, was anybody able to write an equation or a general statement that would help us work out the number of eyes for any number of dogs? Luke

Luke: ...

Teacher: Luke, remember I told you before, you have to think first and then put your hand up with something to say. Yes? Molly, a big loud voice now.

Molly: Me and Alicia wrote under any number we wrote 90...Far generalisation

Pupils found it difficult to understand the concept of any number and continued to specialise with numbers as their idea of any number. What did their work show?

Teacher: You see, that's a number. It's not anything. We want it to fit for any number, you see we don't know the number. What could we put instead of the number?

Molly: We could put a line Using an unknown

Teacher: Well, okay, what would it look like if we have got a line? What do you have to do with the line to show us what you are doing?

Molly: Put the line for how many dogs we could have... then we could write... has

Teacher: Remember, the idea is that we are telling anyone who wants to work out any number... like this number 365... we can't do that straightaway doing it this way because that would take us a long time. What we need to do is think about a general equation. Then Molly said, the general statement is that we are doubling the number of dogs and that will give us the number of eyes, but how can we write that like a mathematical equation? There Hannah started like this. I'll show you what Hannah did. She put a ? to show the number of dogs. Now, sometimes we use that, that's fine, but then she added another ? on and then she said it's another ? (? + ? = ?) By doing this, this is not showing me that we are doubling because I look at this and I think oh, 3+3 or 2+2. It is not telling me exactly what I need to do with the question marks is it? If that is the number of dogs, what do I need to do with that to work out the number of eyes? What did you said doubling is...
the same as? What are we doing?

**Extending thinking**

Benjamin: Multiplying.

Teacher: We are multiplying... by what?... Not just multiplying by anything.

Benjamin: We are multiplying by 2. **General statement**

Teacher: Yes, doubling is multiplying by 2. You are getting the same thing twice. Now if... (the recorder stopped a few minutes before the end of the lesson).

I continued on the white board and showed the children that if I used a question mark and multiplied by 2 it would look like this ? x 2 =

I told them that sometimes in maths, we can use a letter to stand for any number and introduced the variable n to stand for any number of dogs so then the equation could be written as n x 2 or 2n.

We will continue in the next recorded lesson to look at the number of eyes and tails and see how the children get on with this.

**End of lesson**

It appeared in this lesson that all pupils recognised the pattern vertically and could continue it. Evidence from their work showed that all were able to find the near generalisation for 10 dogs.

About 37% of pupils were able to find the far generalisation for 100 dogs as evident from their work.

Some pupils went further in the lesson and were able to verbalise what was happening to the number of dogs to give the number of eyes but none of them could record this as an equation at this stage.
Jack’s work shows he recognised the vertical pattern and was able to continue it to find the near generalisation for 10 dogs. His work does not show evidence of finding the far generalisation or recognising the relationship between the number of dogs and the number of eyes.

Alice’s work shows how she began by drawing the number of dogs and eyes then changed to a t-chart to continue the pattern and find the near generalisation for 10 dogs. She appears to have lost sight of the pattern towards the end but then gone on to find the far generalisation for 100 dogs.
Hannah’s work shows she began drawing dogs then changed to a t-chart to find the near generalisations for 10 dogs and just recorded the answer for 100 dogs. It also shows how she initially recorded \( ? + ? = ? \) as noted in the lesson transcript when attempting to write a general statement for any number of dogs.

Benjamin’s work demonstrates how he began by counting in 2s before changing to a t-chart to record the near and far generalisation for 10 and 100 dogs. He started to work out 200 dogs but did not continue. Was this where he was trying to work out ‘any number’? There is no evidence from his work that he was able to make a general statement for any number of dogs.
Christian’s work shows his use of a t-chart to find the near generalisation for 10 dogs. Does the fact that he has drawn a dog with an arrow and 2 on the central line of the t-chart indicate that he is recording the relationship between the number of dogs and number of eyes? Evidence from the lesson transcript indicates that he recognised this relationship as doubling the number of dogs to find the number of eyes. He then records the far generalisation for 100 dogs underneath. The ? at the end may be where he was attempting to record any number of dogs but he has crossed it out and shown no other written evidence of making a general statement.

<table>
<thead>
<tr>
<th>Dogs</th>
<th>Eyes</th>
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<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>18</td>
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<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

The 10 on the right column may be where he was trying to record any number of dogs but he has crossed it out and shown no other written evidence of making a general statement.
Extracts from Research Diary

Molly said to draw a dog on an arrow line and write 2 eyes. I wrote this on the board as children gave answers. I wrote in Table as follows using pictures of dogs:

\[ \begin{array}{c}
\times \times \rightarrow 2 \text{ eyes} \\
\times \times \times \times \rightarrow 4 \text{ eyes} \\
\times \times \times \times \times \times \rightarrow 6 \text{ eyes}
\end{array} \]

Then I wrote this as:

\[ \begin{array}{c}
1 \rightarrow 2 \\
2 \rightarrow 4 \\
3 \rightarrow 6
\end{array} \]

Jack: "I notice a pattern. Multiply by 2 away or say 'Hey, there is a pattern'. I asked where? He said '2, 4, 6, 8. The next one should be 8 because it's a number pattern.

When I asked why the children said it was counting, even number or in 2s. The children were able to use this pattern of counting on 2 to find the number of eyes. The dogs would have only been able to spot the use of the base dictated pattern.

I noticed pupils were recording their work in different ways.
Notes from my research diary describing how I tried to focus pupils to notice the relationship between the elements in the pattern.

Notes where Alice first realised the number of dogs had to be doubled to give the number of eyes.

Christian’s general statement recognising the relationship between the number of dogs and eyes.
In actual fact 37% of pupils had worked out the value for 100 dogs.

About 3 the class claimed to have found the value for 100 dogs - we will have to check this in their work - so I then looked for a general statement that would cover any number of days.

Hannah

Molly

Michael

Hannah

Alice

Benjamin

There was evidence of this in her work.