The inter-relationship of procedural and conceptual knowledge in two- and three-dimensional spatial problem solving of technical drawing students

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William Patrick Bolger

M7137754

The Inter-Relationship of Procedural and Conceptual Knowledge in Two- and Three-Dimensional Spatial Problem Solving of Technical Drawing Students

DOCTOR OF EDUCATION (EdD)

2001
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I wish to thank all the students listed below who took part in the Initial Study and the Main Study.

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Thanks are also due to my colleague T.D. teachers who gave of their time and expertise willingly in the pursuit of my aims.

Thanks to my brother Richard who took the problems of the T.D. classroom into an industrial setting.

Finally, I wish to thank my tutor Professor Robert McCormick, without whose help this work could not have been completed.

Go m'éadaigh Dia bhúr stór.

William P. Bolger
Dedication

This dissertation is dedicated to my long-suffering wife Mary, without whose support and encouragement this work could never have been completed.

It is also dedicated to our children Oisin, Sorcha, Pierce and Ciarán who have had to endure countless hours of paternal neglect over the past number of years.
Abstract

Technical Drawing has been a post-primary subject for almost one hundred years in Ireland. During the first ninety years T.D. was taught as a vocational subject and had strong associations with the other practical (technological) subjects on the curriculum, most notably Woodwork and Metalwork. The emphases in drawing during this period were on the skills associated with draughtsmanship and the interpretation (reading) of drawings. The doing (drawing) was emphasised over the thinking.

In 1990 a new rationale for drawing was introduced as part of the major organisational changes which saw the Day Group and Intermediate Certificate programmes replaced by the Junior Certificate programme, for all post-primary schools. The new syllabus document and teacher guidelines for Technical Graphics (formerly Mechanical Drawing) outlined a rationale for drawing based on the notion of 'Graphicacy'. This perspective of drawing gave prominence to the formation, categorisation and manipulation of mental images formed as the result of perception and effectively relegated the making and interpretation of drawings to a subordinate role to mental activity. Thus, the thinking was given prominence over the drawing which was described in terms of its communicative function.

After many years as a Technical Drawing teacher, operating within the graphicacy system, I began to question the ability of the philosophy to adequately describe and explain some of the inconsistencies I was encountering daily in the classroom. This was especially true of the supposed hierarchical structure of Spatial Ability which was viewed as the chief component of graphicacy.

I wondered why it was necessary to separate drawing from thinking in the way it had been done previously. I began to investigate possible alternative rationales in the literature that could assimilate acting and thinking into one theory of drawing. The study reported in this dissertation is an effort to take this one step further through empirical research in a T.D. classroom.
Chapter 1

Introduction

In Chapter 1 I will outline what my research project is about, why it is of interest to me and why what I have done should be of interest to the general reader in education and my colleagues who teach Technical Drawing (T.D.). I will use the term T.D. to describe both junior and senior cycle drawing (Technical Graphics and Technical Drawing respectively) in Irish post-primary schools.

I begin by giving a brief outline of the history of T.D. from the time of its introduction as a post-primary subject at the turn of the 20th Century to its provision today. I then discuss my perception of some of the difficulties the present rationale poses for the modern teacher before outlining what prompted my research. Finally, I briefly outline the content of the following chapters of the dissertation.

A Brief History of Technical Drawing in Ireland

The passing of the Agricultural and Technical Instruction (Ireland) Act in 1899 heralded the setting up of post-primary Technical Schools - 'Techs' - in many large Irish towns and was an effort both to make post-primary education available to a greater number of children and to introduce technical subjects which would enhance their employment prospects.

The limited access to and concentration on academic education in post-primary schools prior to this development was felt to be neglecting the commercial, industrial and agricultural education (Mulcahy, 1981, p.14) associated with progress in other countries and as such, was hindering Ireland's development as an economic entity.

'Techs' were designed for children over twelve years of age and the programmes delivered were of two years duration. Drawing was introduced as a post-primary subject in Ireland for the first time. The suggested syllabus was to be composed of 'freehand work' and 'mechanical drawing' (Durcan, 1972, p.135), and was designed to
cover a comprehensive range of topics in Plane & Solid Geometry and Applications.

Drawing was provided (usually as a subject for boys) in this form in Technical Schools right up to the Vocational Education Act of 1930, when the first piece of educational legislation was introduced by the new State. Under the 1930 Act, Vocational Schools were set up in practically every large town in the country (often in the buildings that housed the old 'Techs').

These new Vocational Schools (funded by the State) provided a curriculum which was composed of academic and practical subjects (continuation education), similar in many respects to the old Technical School provision. Drawing was part of the curriculum from the outset, but it was not until 1947 that the first Mechanical Drawing (M.D.) examination was scheduled as part of the Day Group Certificate. This examination, taken at the end of the two year programme, was introduced to provide the certification of completion (of a post-primary programme) required by employers. The examination in M.D. reflected its strong connections with Woodwork and Metalwork.

Voluntary Secondary Schools continued to provide the three year Intermediate Certificate Examination programme, leading to university matriculation examinations. Thus, a two strand system developed in Ireland.

This form of provision continued until the introduction of the Vocational Amendment Act of 1962 when a series of sweeping changes were introduced. These included common Day Group, Intermediate and Leaving Certificate Examinations for students in the Voluntary and Vocational sectors, and a two-year senior cycle (after the Intermediate Certificate) in all post-primary schools. Practical subjects and M.D. were introduced into many Voluntary Secondary Schools for the first time and programmes were extended by a year and taken at Intermediate Certificate level by students in both sectors. Many students (still) left post-primary school after the Day Group or Intermediate Certificates and it took some years before the numbers staying on for senior cycle in Vocational Schools increased notably. M.D. now had a senior cycle equivalent in Technical Drawing (T.D.).
Examinations in M.D. and T.D. consisted of a single, common level paper divided to reflect the main emphases of the time, i.e. Plane & Solid Geometry with Engineering or Building Applications.

As alluded to above, M.D. and T.D. of this era were viewed as vocational subjects (taken in conjunction with other practical subjects) and formed the basis for entry into craft apprenticeships. Apart from this vocational emphasis, M.D. and T.D. did not have a guiding theory which described or explained students learning and development while studying. The emphases were on the geometrical content of the syllabus and the skills associated with the preparation and interpretation of drawings.

In 1990 the old junior cycle programmes (the Day Group and Intermediate Certificate) were abolished and a single Junior Certificate examination was introduced. The name M.D. was changed to Technical Graphics (T.G.), the subject was to be offered at ordinary and higher levels, and a new view of learning and development was proposed.

The new rationale for T.G., contained in the syllabus document, associated learning and development in drawing with graphicacy, a notion which emphasised the mental activity associated with the solution of two- and three-dimensional problems. This became the received view of drawing because of its inclusion in the new syllabus and T.G. teacher guidelines. It was reinforced through an intensive in-service programme delivered by the Department of Education to all drawing teachers when the new programme was introduced.

Senior cycle T.D. continued to be provided in its original format until the mid 1980's when the single common level examination was replaced by two three hour examinations at higher and ordinary levels.

Proposals to reform the Junior and Leaving Certificate programmes and examinations are currently being examined.
T.G. and T.D. Today: Structure and Rationale

Today, the T.G. programme is viewed as one which provides a unique range of skills which enable it to act as a foundation course for T. D. at Leaving Certificate level. It is (still) also viewed as a contributing discipline to all technology based subjects because it is regarded as a universal language. The 1990 programme aims to stimulate students' creative imagination by developing their spatial abilities, together with the cognitive and manipulative skills associated with graphicacy. The programme also seeks to develop logical and progressive reasoning and the ability to spatialise and visualise two- and three-dimensional configurations when solving graphical problems. Solutions are drawn at the drawing board with instruments (but there is an aspiration to introduce computer assisted drawing). Finally, the programme seeks to develop an understanding of the importance of communicating information graphically (T.G. Syllabus, 1990, pp.1-5).

At the end of the three year programme students sit a terminal examination. Both higher and ordinary level examinations are in two parts: in Section A short questions must be answered; and in Section B long questions must be addressed. The solutions to short questions are drawn on the examination paper while long question solutions must be prepared on the drawing board using drawing instruments.

A knowledge of junior cycle drawing is assumed for those who study T.D. at Leaving Certificate level where the subject is viewed as providing students with the opportunity to visualise and comprehend information presented verbally or graphically. The aims are to develop the intellectual qualities of comprehension, analysis and problem solving as part of overall design sensitivity - together with the physical skills of manipulation and dexterity. The programme is divided into two parts: Part 1, Plane & Solid Geometry which is purely theoretical; and Part 2, Applications, which emphasises the role of T.D. as the language of technology.

Part 2 of the programme is divided into two distinct sections: Part 2(a) Engineering Applications; and part 2(b) Building Applications. For
the Leaving Certificate examination students are required to study Part 1, and either Part 2(a) or (b). The selection from Part 2 is usually made with reference to the background of the drawing teacher - my association with Building Technology means, for example, that I always teach Part 2(b) of the programme, i.e. Building Applications.

Thus, the structure of the programme with its sections on pure and applied geometry, is seen as providing a broad educational experience for the students where their intellects and creative abilities are developed through topics which are appropriate and meaningful in a technological world (Rialacha & Clár, 2000, p. 252).

Differing Emaphases: Traditional and Modern.

Early courses in Drawing emphasised the acquisition of skills associated with draughtsmanship and interpreting (reading) drawings. These were vocationally directed activities intended to equip students with skills which would improve their employment prospects in Irelands two largest industries, agriculture and building.

The changes in the syllabi for senior and (especially) junior drawing, in the mid 1980's and 1990 respectively, have seen the development of a view of T.D. which apparently seeks to separate the cognition associated with the solution of spatial problems from the draughting skills which were given salience in the earlier programmes. Indeed, while the T.G. syllabus of 1990 has much to say about the development of student cognition, it has very little to say about mechanical drawing in the traditional sense, i.e. the type of cognition that is associated with drawing on the board with instruments. This may be regarded as the separation of acting from thinking or thinking from acting. This is an important distinction since a view of T.D. that implies acting without thinking (i.e. with no cognitive element in the exercise of drawing skill) may follow. In my view (which I will outline in Chapter 2) thought and action are inextricably linked in the production of drawings - one can not be separated from the other.

In the last decade the subject has come to be viewed solely in terms of graphicacy. This is a quite complex notion composed of a number of elements but it emphasises the development and manipulation of
mental images and relegates the process of making drawings to a purely mechanical role. That is, drawings provide a medium through which the geometric or spatial problems which have been solved as mental activities can be communicated to others. Drawing is regarded therefore, as being subordinate to the mental actions associated with the other processes of graphicacy. This may be seen as a reversing of the traditional emphasis, i.e. from acting without (necessarily) thinking to thinking without (necessarily) acting.

Furthermore, the research on which the current graphicacy perspective of T.D. is based was conducted almost exclusively outside the educational settings which it has come to influence so profoundly.

Many of the reasons for the change in emphasis, I believe, are deeply embedded in the historical context of Irish education briefly described earlier. Breen et al (1990, pp. 25-26) for example, describe how the curriculum in vocational schools was explicitly linked to the labour demands of the economy. The curriculum in secondary schools, on the other hand, emphasised the religious, moral and intellectual development of students and was less directly oriented towards the perceived labour demands of the economy. Furthermore, secondary education was available to only a few, led to university education, entry to the professions or the clergy, and was therefore, perceived as a better form of education. The curriculum in such schools was concentrated on academic, non-technical subjects where greater value was attributed to thinking rather than doing. The influence of this historical context on the development of the Irish post-primary education warrants further investigation, but was outside the focus of the study reported here. Changing views of the nature of child development and learning which were prominent at the time the new programmes in drawing were being developed were also influential.

Traditionally, the drawing syllabus has been organised around the knowledge associated with geometric topics. This knowledge, which forms an integral part of both traditional and modern programmes, has received little attention since drawing was first introduced. Indeed, it could be argued that it has been totally ignored because of the emphases on draughting skills or mental actions by traditional and modern syllabi respectively. It appears that T.D. knowledge has come
to be regarded as something which can be transmitted from one
generation of students to the next.

It is worth noting that contemporary views of learning which have
emerged since the current syllabus was introduced have reflected on
the role of knowledge in student learning. Thus, the current view (or
the lack of one) of the role of knowledge in T.D. may be depriving
us of a means of understanding the subject more deeply. In my view,
a rationale for T.D. can be developed that emphasises knowledge by
relating it to the thinking and acting which form an integral part of
drawing solutions to geometric problems.

What Prompted my Research

A number of issues arising from previous and current views of T.D.
have led to the research reported here.

1. The current graphicacy view of T.D. makes assumptions about
the type of cognition involved in the formulation of solutions to
problems. These are based on the evidence of drawings which are
produced. It may not be wise to assume that because a particular
type of drawing is produced that the drawer must be engaged in a
particular kind of thinking. This use of the drawing as evidence of
cognition may be regarded as being similar to the classic issue in
psychology where thinking is inferred from behaviour. My experience
in the T.D. classroom (although not previously the subject of rigorous
investigation) in this regard has been that these assumptions often
appear to be inaccurate. Students (often) produce solutions that appear
to evidence the development of advanced graphicacy while lesser
abilities are not evidenced in the same (or other) solutions. This would
appear to question the (supposed) hierarchical nature of the spatial
abilities which are at the core of the modern graphicacy rationale and
raises questions about whether students' drawn solutions to T.D.
problems can be adequately explained by the received graphicacy view.

2. There appears to be evidence (from recent T.D. Chief
Examiner's Reports for example) of students inability to use pure
geometrical principles in situations where these principles are intended
to be applied. This has serious implications for the received view of T.D. since transfer between pure and applied geometry plays a central role in the design and assessment of the current syllabi.

3. Since students' thinking is assumed to be developing in a particular way their learning is also assumed to have certain characteristics. This raises a further issue since views of learning have changed during the period since the new drawing syllabi were introduced in the mid 1980's to 1990. These new views of learning and thinking can also (arguably) offer satisfactory, if different, explanations for the solutions to T.D. problems students produce. No attempt has been made to assimilate these modern views of learning and development into the received view and they have not previously been explored in the T.D. context.

4. The vast majority of investigations which have taken place into how the thinking associated with graphicacy takes place have been conducted outside the educational settings that they have come to influence. Only Gaughran (1990) has carried out a study in the drawing classroom but his methodology and methods were very similar to those used in earlier studies despite his different focus. Therefore, there was a need to carry out research in the T.D. classroom which would describe and explain the activities which were taking place and which could be used to form a view of the subject based on current practice.

5. In my view, both the earlier skills and the current graphicacy views are flawed for another reason. The skills view of T.D. appears to have implied that it was possible to separate (especially) those activities associated with making drawings, from the knowledge and thought which are associated with the solution of T.D. problems. The doing was emphasised above the thinking. Conversely, the modern graphicacy perspective also separates thinking from acting but with a different emphasis. Here the thinking is given prominence over the action of drawing. I believe that it was necessary to develop a view of T.D. in which thinking and acting are integrated.
6. The presence of a body of drawing knowledge is assumed from both the skills and graphicacy perspectives, but the relationship of this knowledge to either or both has not been explored sufficiently. In my view, the role of knowledge is central to the development of a rationale for drawing which will allow the mental and physical actions to be satisfactorily combined (point five above). Such a rationale would, I believe, have serious implications for T.D. in terms of how the syllabus is designed, delivered and assessed. Before such a rationale could be developed however, a study (or several studies) of the use of knowledge in the T.D. was necessary. The study reported in the following chapters is an attempt to begin developing such a rationale.

7. It is also worth noting that several authors have questioned whether courses in T.D. can influence the development of spatial ability, which is at the core of the modern graphicacy perspective. This will be discussed in greater detail in Chapter 2.

The main inconsistencies between the present rationale and my observations in T.D. classrooms over the last decade have led me to question the descriptive and explanatory validity of the received views. In my view, students' work may not always be adequately explained with reference to the current views of T.D..

The study reported in this dissertation is of interest to me personally for the reasons stated above. I believe that a view of T.D. based on T.D. knowledge will provide a more accurate and comprehensive description and explanation of all aspects of the subject. The study should also be of interest to the general reader in education and my colleague T.D. teachers for the same reasons and also because it is the first study of its kind to be carried out in an ordinary T.D. classroom. I hope that it may lead to other, more focused, studies in T.D.. It may also come to influence future syllabus design and teaching methodologies in T.D..

In Chapter 2, which follows, I describe what others whose research has influenced T.D. have concluded and explain what sense I have made of what has been said, both in terms of how they agree and disagree with one another and how I perceive their strengths and limitations. I then discuss the received view of T.D. in detail. This
discussion outlines how the modern graphicacy perspective developed from the debate about the nature of spatial ability and details how problem solving is viewed in terms of students' ability to work through an hierarchy of spatial ability sub-factors following perception. According to this view, problem solving is carried out as a series of mental actions prior to the drawing of the solution, which is undertaken solely for communicative purposes. I go on to identify difficulties with the received perspective, in terms of how the rationale for the subject was developed, the possible influence of T.D. teachers and programmes on student learning and development, how the development of spatial ability is assessed, and how problem solving is perceived. I then outline a number of possible alternative views which might be adapted to provide a rationale for T.D. before discussing how developing these views led to the research which is reported in ensuing chapters.

In Chapter 3 I discuss how I decided to investigate my research questions together with my justifications for adopting such a research strategy. This chapter includes a discussion of the methodology chosen together with details of the procedures adopted and what influenced my choice. The discussion centres around the contrast between the spatial ability research conducted in a positivist tradition which informs the received rationale and the qualitative research reported here which informs the knowledge perspective I am proposing. This discussion includes details of the differences between how empirical materials were collected and analysed to inform the received graphicacy perspective and my proposed knowledge perspective. Also included in Chapter 3 is a discussion of the ethical issues associated with my role as a researcher while continuing to operate as the class teacher.

In Chapter 4 I discuss my research findings by outlining the sense I made of the data generated during the research process. To do this, I outline how the analytical categories used to interpret the data were formulated and include some raw data and summaries so that the reader may see how this analysis and interpretation were undertaken. Further data are included in appendices to this dissertation. I also discuss how my interpretation can be related to the findings of
previous researchers. Thus, the story of my research is woven back into the plot of the literature review.

In Chapter 5 I reflect on my research and evaluate what I have done. I discuss how different types of T.D. knowledge were defined during the course of the research and how students used these types of knowledge in the preparation of their drawn solutions. I reflect on the relevance of my research in terms of the current and proposed views and discuss how students learn problem solving, and how it may be taught. This discussion outlines how what I have done might effect the practice of others in T.D. education. I go on to identify issues which require further study and discuss the research methods I employed, things I would do differently, together with how the findings and conclusions of my study might be presented to my colleague T.D. teachers (and others) for their consideration.
The review of literature reported hereunder is not meant to be an exhaustive account of all that has been written on the topics of learning, development or problem solving as applied to Technical Drawing (T.D.) or any other subject area.

It is firstly, an account of the received view of T.D.. This view incorporates perspectives of learning, development and problem solving for T.D., which are related to graphicacy and to Piaget's ideas on development. The graphicacy model became very influential in Ireland following the publication of the Technical Graphics (T.G.) Syllabus and Teacher Guidelines in 1990. Much of the content of these documents was based on the work William F. Gaughran carried out for his M.TECH. thesis at the University of Limerick and the work he had done prior to its completion (Gaughran, 1990). During the 1970's and 1980's he had been involved in training T.D. teachers, had acted as a consultant to the Department of Education on matters pertaining to T.D., and was researcher and education officer for the National Council for Curriculum and Assessment (N.C.C.A.). This body was (and still is) charged with developing the curriculum for post-primary schools. He was thus, in the unique position where he could influence the development of national policies on T.D., oversee the training of new teachers on training programmes and influence practising teachers (including myself) through his work on Department of Education & Science T.D. in-service programmes. Gaughran's work during this phase of the development of T.D. as a post-primary subject is to be commended very highly, not just because of its influence on the development of a coherent rationale for T.D. but also because he brought the research process into T.D. classrooms in Ireland for the first time. At the time the new syllabus documentation was being formulated, Piaget's theory of learning and development was still prominent and served to provide a compatible and already well developed view of learning and development to fit Gaughran's views of T.D.. The time was right to bring the views of Gaughran and Piaget together in the Irish context. This was done in the rationale for the T.G. Syllabus which was introduced in 1990. A coherent
philosophy was thus developed which could, for the first time, relate views of learning, development and problem solving in T.D. to those held for other areas of the post-primary curriculum. This marriage of philosophies still guides the subject today.

Secondly, the literature review is a discussion of the difficulties associated with this received rationale for T.D. in the terms described above - i.e. it may be viewed a critique of the received view. Although the received philosophy has been unchallenged for a decade, it is nevertheless, a particular view of learning, development and problem solving. It emphasises the mental actions and abilities associated with (particularly) spatial ability and those on the development of thinking from Piaget's perspective and makes assumptions about learning, development and problem solving based on the ensuing behaviour. As outlined above, at the time of its development this guiding philosophy for T.D. was seen as a good fit between Gaughran's and Piaget's views of development. Presumably, the Syllabus Committee (that included Gaughran), which developed the syllabus, selected Piaget's theory to support Gaughran's views because of the perceived similarities between the two perspectives. Other views of learning, development and problem solving also existed (or were in the process of being formulated) at that time also, but were not used in any official syllabus materials. Gaughran's and Piaget's theories alone were chosen to form the basis of the guiding philosophy. One alternative view, examined philosophically and empirically in other subject areas, could be called the knowledge perspective. This describes procedural, declarative and conceptual knowledge and explains how these types and levels of knowledge influence student performance. To date, a philosophy for T.D. based on this perspective has not been formulated or tested as an explanation of learning, development or problem solving. One of the effects of the concentration on the development of spatial ability as an explanation for learning and problem solving in T.D. has been that the subject of the knowledge used by T.D. students has been somewhat neglected, despite several references to types of knowledge in the T.D. syllabus document itself.

Thirdly, therefore, the literature review is a discussion of how the literature on knowledge, developed through the study of other subject areas, can be used to provide a more satisfactory and complete model for understanding all aspects of T.D.. This knowledge perspective
represents my own analysis of learning, development and problem solving for T.D. based on my reading of studies carried out in other subject areas largely in educational settings, and my experience as a T.D. teacher for the last twenty years and subsequent empirical investigation. Since the research which informs the received view of T.D. is based, almost exclusively, on empirical work which was conducted outside educational settings generally and T.D. classrooms specifically, the literature is an argument for conducting the research which informs views of T.D. in the T.D. classroom. As such it may be seen as an argument in favour of the research described in Chapter 4.

The first part of the literature review below seeks to describe and explain the current views of learning, development and problem solving in T.D. from the received syllabus documentation.

The Received View of Technical Drawing.

These received views of learning, development and problem solving in T.D. are composed of two philosophies which are viewed as complementary. The first of these is Gaughran's graphicacy perspective of learning, development and problem solving in T.D. and the second is Piaget's theory of child development. Each will be explained in turn.

The Graphicacy Perspective

The notion of graphicacy is relatively new, having been introduced to Irish T.D. teachers and their classrooms during the in-service training programme which followed the publication and introduction of the new T.G. Syllabus in 1990. This document was prepared with reference to the work of Gaughran (1990) which concerned the development of spatial abilities through computer assisted learning (C.A.L.). However, many of Gaughran's ideas were used to inform the syllabus material for T.G.. Although his primary focus was on computer assisted learning (C.A.L.) it is his work on the constitution and development of spatial abilities which proved to be the most influential. His M.TECH. thesis may be regarded as a comprehensive review of the research reported on the constitution and development of spatial ability conducted in the previous half century, and in it he also describes his individual interpretation of this earlier work. It is
this interpretation of the earlier spatial ability literature, rather than the earlier work itself, which has proven to be the most influential. Although he later published two post-primary textbooks in 1992 and 1996, his influence on T.D. has resulted from his M.TECH. thesis, his work as a consultant to The Department of Education and Science, and as research and education officer at the N.C.C.A. at a time when the syllabus documentation was being prepared.

Gaughran was responsible for the introduction of the notion of graphicacy into T.D. classrooms in Ireland. It is described as

the ability to encode, spatially perceive and manipulate configurations in two- and three-dimensional space and to communicate these graphically. (T.G. Syllabus, 1990, p.2).

This definition is taken directly from his thesis (1990, p.1.). It is composed of a number of elements involving the perception and manipulation of two- and three-dimensional space, the abilities associated with the encoding of these perceptions, and the communication of these ideas graphically. He argues (1990, pp. 3-83) that there are four main factors which constitute graphicacy as he has defined it - (a) perception, (b) visual perception, (c) spatial ability (S.A.) and (d) communicating graphically. All these elements are essential for overall graphicacy but S.A. is regarded by Gaughran as the 'principal element in graphicacy' (1990, p.39). Below, I have briefly described these four elements from a Gaughranian perspective. This is followed by Figure 2.1. which shows the graphicacy perspective in simple diagrammatic form.

a) He describes perception as the biological processes involved in generating mental images following exposure to an external stimulus. Thus, a two- or three-dimensional stimulus or cue sets in train a biological response which results in the formation of a mental image. He describes how the eyes, as sense organs, change the various environmental energies, which form the stimulus, into nervous impulses that are then sent to the brain. Through the psychological process of perception, the patterns of energies become known as two- or three-dimensional configurations, objects, events, people and other aspects of the world around the perceiver. However, the process of perception in itself, does not reveal images of two- or three-dimensional
configurations. Rather, the eyes and the brain combine to transform the physical energy from environmental stimuli into information about what is being viewed. It should be noted that stimulation of the sense organs alone does not determine the nature of what is perceived. Perception is a dynamic process of working on sensory data to produce perceptual objects and events. The work involved in perception is extremely complex and many physical, physiological and psychological factors contribute to it.

In his discussion of perception he reviews the works of Lebowitz (1965), Gregory (1973), Zusne (1970), Haber (1970), Hochberg (1964), Vernon (1964), Rock (1973), Legge & Campbell (1987), Padgham & Saunders (1975), Beck (1972), all of whom describe how what the eye sees is transformed, in conjunction with the brain, to produce perceptual objects or mental images.

Perception is often characterised, for example in the World Book Encyclopedia (1984, Volume 15; p. 252) as having three levels of complexity: detection, recognition and discrimination. Detection refers to whether individuals can sense that they are being stimulated in some way. Recognition refers to being able to detect and identify particular patterns of stimulation while discrimination refers to being able to perceive patterns of stimulation as different. Gaughran however, has separated these levels of perception into two distinct forms of perception - perception described as in a) above and visual perception described as in b) below.

b) Visual perception, according to Gaughran, allows the mental images formed as a result of the perceptual process to be interpreted and attributed significance. In developing his argument he reviews the work of Vernon (1970), Oatley (1978), and Stillings (1987) in an effort to advance his thesis of how images resulting from perception are categorised and attached significance using 'schemata' (Gaughran, 1990, p. 5). He emphasises this process of interpretation of images rather than their biological construction. He also supports Harris' (1963) thesis that the ability to discriminate and organise images formed during perception develops with age and growth in cognitive experience within a domain. Further, he argues, that the retention of mental images as schemata and interpretation following stimulation requires conscious effort - i.e. it is not automatic - and may be viewed
as a drive for consistency similar to that described by Lunzer (1989, p. 34). Gaughran also argues that the type of work involved here is similar to that involved in perception as described above.

Gaughran's approach to perception may not be unique. However, others might not distinguish so readily between the formation and interpretation of images (see Haber: 1970 for a discussion of contemporary theory in respect of visual perception) during the perceptual process.

c) Following perception and categorisation of images during visual perception, Gaughran describes Spatial Ability (S.A.). This ability is viewed as critically important in overall graphicacy. It is viewed as cognition which involves the manipulation of mental images formed during perception and categorised during visual perception such that these configurations may be imagined in different positions, compared and perhaps mentally dismantled. Thereafter to be mentally re-assembled and perhaps graphically communicated (Gaughran, 1990, p. 37).

This is a view of S.A. which has its roots in a debate that has spanned the last six decades. It originated in early studies of mechanical aptitude, in which test instruments similar in many respects to those employed in current Differential Aptitude Tests were used. For example, the D.A.T.'s published by the Psychological Corporation in New York (1990) contain sections on mechanical reasoning and space relations which are very similar to the tests used in those early studies which prompted the debate on the nature of spatial ability (see McGee, 1979 for a discussion).

As alluded to above, the debate on the nature of spatial ability has been a protracted one to which there have been a great number of contributors. McFarlane (1925), Koosy (1935), Thurstone (1938), Guilford & Lacey (1947), French (1951), Anderson (1964), Smith (1964), Vernon (1964), Piaget and Inhelder (1967 & 1971), DeRenzi (1978), Edwards (1981), Ratcliff (1982), and Lord (1985) have all described its nature, composition, and significance. Potegal (1982) also reviewed many of the psychological and developmental aspects of S.A. and outlined the areas of disagreement on their nature and
number. The studies mentioned above have identified as few as two or as many as ten sub-factors which constitute spatial ability. Although no consensus on their number or nature has emerged in the last 75 years, there does appear to be at least some measure of agreement that S.A. is a composite ability. There is also agreement that the ability to visualise two- and three-dimensional configurations is one such major contributor.

A measure of Gaughran's influence on T.D. in the Irish context, is that his view that five contributing factors to spatial ability 'should suffice' (1990, p.53), has been adopted as the official view published in the T.G. Teacher Guidelines (1990, p. 3). His view, is based on his interpretation of the S.A. literature and his empirical work. In the empirical work he reported in his thesis he used tests with geometric solids mounted on a plane (not dissimilar to Piaget's (1956) mountains). Equally significant, to his interpretation of the debate on S.A. however, is his proposition that (as he describes it) S.A. is the 'principle ability in graphicacy' (1990, p. 39), and that success in the T.D. domain is dependent on its development.

In Gaughran's view, spatial ability develops following progression through an hierarchy of what he calls 'space factors' (S.F.'s) (1990, pp. 53-54). These five S.F.'s are recounted verbatim in the T.G. Teacher Guidelines (1990, p. 2) -

Space Factor 1 (SF1) - image holding and comparing,

Space Factor 2 (SF2) - planar rotation: of two- or three-dimensional objects in a single plane,

Space Factor 3 (SF3) - orientation: the ability to imagine an object in two- or three-dimensions as viewed from various perspectives by a spectator,

Space Factor 4 (SF4) - kinetic imagery: the ability to manipulate or rotate an object in the imagination,
Space Factor 5 (SF5) - dynamic imagery; the ability to manipulate elements within a three-dimensional configuration in the imagination.

From Gaughran's (and The Department of Education and Science) standpoint therefore, paramount importance is attached to those elements of graphicacy which involve the construction, interpretation and manipulation of mental images resulting from external stimuli or emanating from the visual imagination of the individual. Thus, the S.F.'s emphasise the notions of 'internalisation' and 'visualisation' (Gaughran:1990, p. 4) - internalised mental activity involving the manipulation of mental images of geometric configurations.

d) Just as the formation of mental images during perception and visual perception service the mental actions which form spatial ability at the beginning of the process, so also do drawings at the latter stages of the process. The making, reading and use of drawings as a cognitive tool is therefore, viewed as subordinate to spatial ability. Drawings are viewed as the vehicles through which information and ideas of a graphical nature can be communicated. Their skilled production and interpretation is viewed as an important constituent of overall graphicacy because of the need to be able to communicate ideas with others involved in technological pursuits. It is in this context that T.D. has attained its status as the universal 'language of design and technology' (T.G. Syllabus, 1990, p.1), placing it in a role where it 'services' all other technological subjects on the curriculum, and beyond (N.C.C.A., 1987, p.11). From a graphicacy perspective, drawings are sub-servient to the other (mental) activities of which it is composed. They represent the ideas of the one who draws and serve to mediate those ideas (which have been the subject of cognition during the previous stages of graphicacy) in the communicative process. Thus, they allow others access to the ideas of the one who draws. This is sometimes referred to as 'spatial modelling' (T.G. Syllabus, 1990, p.1), i.e. the externalisation of images in the form of drawings, diagrams and constructions.

To ensure the effectiveness of drawings in this role they must be produced to certain agreed standards called conventions. This means that the students must attain a level of drawing skill, called draughtsmanship, which will allow ideas to be represented in a way
that others can access and that these skills must be progressed in parallel with the ability to interpret or encode drawings, otherwise the drawings cannot fulfil their role from the graphicacy perspective. The development of these draughting skills is influenced by students levels of psychomotor skills and dexterity in the manipulation of drawing instruments together with their overall level of development from the graphicacy perspective.
Thus, the Department of Education and Science, within the philosophical framework of the *graphicacy perspective*, promotes T.G. as a subject which develops the 'creative imagination' of its students and enhances their 'natural ability' to perceive and conceptualise space by encouraging them to 'reason in two- and three-dimensions'. It also requires that they apply their 'natural abilities', enhanced during and following study of the subject (quite how they are enhanced is not clear), to the solution of graphical and spatial problems of 'an abstract and practical nature'. The cognitive and practical skills developed as a result of studying T.G. are portrayed as the stimulus which will help students 'to see their environment with a critical and analytical awareness' and which will serve to enhance their 'aesthetic values'. (T.G. Syllabus, 1990, p.1).

Thus, students of T.D. in their Junior Cycle programme, are provided with the opportunity to develop their *graphicacy* - the foundation on which the Senior Cycle programme in T.D. is built. This is achieved as students progress through the hierarchy of space factors which lead to a fully developed spatial ability. Central to this fundamental notion of graphicacy, are 'internalisation' and 'visualisation' - unique modes of thinking graphically viewed as incorporating the intellectual qualities of comprehension, analysis, problem-solving, and design sensitivity, general attributes which can be used in any of the topics covered by the drawing programme. These are further developed at Senior Cycle. Further, Gaughran (1990) and the Department of Education and Science (1990) argue that the development of graphicacy in students allows for the process of generating mental images in the absence of external stimulation - contributing to the development of students 'creative abilities in topics which are appropriate and meaningful in a technological world' (Rialacha & Clár, 1994, p.192).

The official documentation does not recommend a specific pedagogy, which will influence the development of the processes that comprise graphicacy except to say that 'it is important that the students be involved in a variety of activities which are both interesting and relevant' (T.G. Teacher Guidelines, 1990, p.3), and that a variety of teaching aids (building blocks, charts, iso or squared geo-boards, two- and three-dimensional puzzles, colour and computers) should be employed in the classroom. Specific details concerning the development of learning problem solving are however, not referred to,
except to say that strategies developed by the teacher should be staged to address the hierarchy of space factors which lead to a developed spatial ability. Problem solving is therefore, directly linked to the space factors which constitute spatial ability, an internalised form of mental activity.

Piaget's Theory Related to The Graphicacy Perspective

The intended relationship between the graphicacy perspective of learning, development and problem solving in T.D. and Piaget's Theory of development, although referred to in the T.G. Teacher Guidelines (1990, p.3), is not fully explained for the ordinary T.D. classroom teacher in any detail. References are made to the 'majority of students entering second level school at the concrete operational stage of intellectual development' with few having 'any proficiency in formal operational reasoning' (1990, p.3) but the theory of how students progress from this stage of development in the early stages of the T.D. programme to formal operational thinkers in the latter stages is not developed. Brief references are also made to appropriate teaching methodologies for these early stages in the programme (involving the use of geometric models) but appropriate pedagogical strategies to progress student learning as they grow older and become more experienced are not provided in the literature. The following is my interpretation of why Piaget's Theory is used in conjunction with the graphicacy perspective as a rationale for the subject. This is based on the references to Piaget which are made in the Teachers Guidelines mentioned above.

As children develop, according to Piaget's Theory (1952) they become able to acquire knowledge of, and to think about not only objects and events which are close to them in space and time but also about objects and events quite remote from them. This development is continuous, progressing through stages. Within each stage of development patterns of behaviour are seen to occur. These patterns are viewed as having a common structure which explains them and gives each stage its unity. Thus, transition from one stage to another requires that a fundamental mental re-organisation takes place. However, there is no sharp break between the end of one stage and the beginning of the next stage just as there are no completely new beginnings because each stage builds on the previous one. This means
that the earlier construction is necessary for the later one to develop. This may be viewed as a constructivist view of development. The main stages of development are held to be the same for all children. However, the speed at which individuals develop is not. The ages quoted by Piaget for these stages are averages and he recognised that there are wide variations from these averages. The three main stages of development he describes are the sensory-motor period, the concrete operational period and the formal operational period.

This staged, constructivist view of development has some congruence with the views of spatial ability development in the graphicacy model described earlier. Within each S.F. constituting S.A. there is also a recognisable pattern of behaviour and structure which gives it its own unity and character. For example, a student whose level of development is at S.F. 1 (image holding and comparing) would be able to look at a two- or three-dimensional configuration and then identify aspects of the configuration in similar configurations. Having progressed to S.F. 2 (planar rotation) the student would be able to recognise similar two- and three-dimensional configurations when rotated in a single plane, i.e. in a different orientation. At S.F. 3 (orientation) the student would be able to imagine how three-dimensional configurations appear from different viewing positions. A student would not be able to operate at S.F. 3 without first having progressed through S.F.'s 1 and 2. This progression only occurs when students are ready. Thus, a gradual development or re-organisation takes place in the students thinking, when there is a readiness to learn or develop. A distinct end to the development of one space factor or beginning to the next is not necessarily distinguishable. This may explain some of the inconsistencies in findings of studies of S.A. and the debate about how many factors constitute it. From the brief descriptions above the similarity of characteristics and behaviours associated with S.F.'s serve to illustrate the difficulties associated both with progression from one S.F. to the next and from one Piagetian stage of development to the next. These similarities also serve to illustrate the attractiveness of the view of S.F.'s as an hierarchy, with progression from one factor to the next being dependent on the development of the earlier one. It also gives an order for development through which every student must progress to full development, albeit at an individual pace of development.
By the time children enter post-primary school, they have passed through the sensory-motor stage and are coming to the end of the concrete operational stage according to Piaget's age profiles. This view is reflected in instructions given in the T.G. Teachers Guidelines (1990, p. 5). The kind of mental flexibility associated with this stage of development is related to the increase in the ability to centre and the development of operational structures. These operational structures are actions. But they are not physical manipulations - they are carried out in the mind. They do however, have their origins in the physical manipulations associated with the sensory-motor period. These mental acts are also of great generality, serving to order, separate, combine, etc. and they cannot exist in isolation, but only within an organised system of operations. By the end of the sensory-motor period group structures already exist, but only on a practical level, the next step is to internalise them. This means that the work done during the sensori-motor period must be carried out all over again on another plane, through symbols in the mind - acts of thought rather than physical acts. For example, younger children can position objects while older children can think about doing this. However, during the concrete operational stage these symbolic acts are still very closely related to the concrete things on which the original physical acts were performed - the child may still be thinking about doing things to physical objects. Thus, concrete operational thinking is more mobile than sensori-motor thinking. It can deal with transformations between, and relationships between states. Overviews are also possible, where sequential views only were possible previously. Where sensori-motor intelligence is geared toward practical success the operational thinker is more interested in explanation and understanding. This development is related to increased awareness of how goals are achieved. Where sensori-motor intelligence is limited to real actions performed on real objects with a narrow range of space and time the range of acts possible for the operational thinker is unlimited in principle, but in practice, the range continues to be restricted as long as thought is in the concrete period. Thus, in my view, a cognitivist view is advanced by the current T.D. rationale.

In Junior Cycle T.D. these views are reflected in the programme which is recommended for Year I students. The emphasis is on the manipulation of the drawing instruments, the development of basic draughting skills, introductions to drawing systems and work with
simple geometric figures and solids. This reflects the view that these students are not yet ready to progress to formal operations - to adult thinking - and thus, that trying to teach them about topics which require formal operational thinking will be unsuccessful. Recommended teaching methodologies include the use of tactile models to represent plane figures and three-dimensional models to introduce geometric solids. Thus, the fact that the operations involved in students thinking is still closely related to the physical manipulation of concrete objects is recognised and developing operational thinking is catered for. The emphasis on developing drawing skills also reflects the desire to ensure that students attain practical success in the period prior to the development of an awareness of how goals are achieved, a time when explanations can be understood. Once the concrete operational stage has been passed through and the child's thoughts are no longer restricted by ties to physical actions the range of possible mental actions is theoretically unlimited.

The thinking associated with the formal operational period is that of 'the intelligent adult' (Donaldson, 1987, p.139). It is marked by the ability to reason logically, progressing from premises to the conclusions which necessarily follow. It does not necessarily mean that the premises are true - they may be accepted as postulations. Thus, the formal operational thinker can entertain hypotheses, deduce the consequences and put the hypotheses to the test by systematically experimenting with its elements in order to develop general rules based on the conclusions of such actions. The formal operational thinker can thus, manipulate ideas or propositions about two- and three-dimensional configurations where the concrete operational thinker is still thinking about manipulating things, even when this is done in the mind. This, is a very significant development since one of its results is that the formal operational thinker is able to shift between what is real and what is possible.

In T.D. this form of thought leads the student, when faced with a problem, to begin considering possible solutions in a systematic way. The formal operational thinker is one who tries out all possible combinations, not necessarily stopping when a solution which works has been found, but continuing until the whole system of possibilities has been explored. This form of thought also allows for the formulation of previously unexplored solutions to problems being
hypothesised as premises and conclusions. Thus, students are encouraged to be creative in their approach to solving problems and the received view is that spatial abilities 'contribute in a major way to creativity' (T.G. Teacher Guidelines, 1990, p.3). Further, the formulation of hypothetical problems and solutions advances the students thinking - helping her/him to learn and develop further. That is, abstract problem solving is seen as part of the process of learning. Once a T.D. student has reached formal operational thinking and has a fully developed spatial ability she/he is capable of solving problems which are posed in the manner described above and also of developing hypothetical problems and solutions in order to advance thinking or to be creative. Thus, in the first instance, 'action and self-directed problem solving are at the heart of learning and development' (Wood, 1988, p.5). Secondly, the benefits accruing from this form of thinking can be addressed to solving practical problems encountered in everyday life.

The current rationale for T.D. therefore, promotes a view of the subject which is both cognitivist and constructivist. However, it should be noted that active construction and self-regulation are not synonymous with learning from a Piagetian perspective and that the knowledge that an individual constructs must be sourced externally.

Summary

Thus, a cognitive constructivist perspective of learning and development in T.D. is proposed for students in Irish post-primary schools. The official literature associated with the subject advances a view of development and learning which is largely based on Gaughran's (1990) graphicacy work, with its emphasis on the importance of the development of students' spatial ability - a form of internalised activity relating to the formation and manipulation of two- and three-dimensional mental configurations. Spatial ability is developed only through its constituent hierarchical sub-factors, called space factors in the official T.D. literature. Problem solving in T.D. is viewed chiefly in terms of the learning associated with development through space factors. These views were then used in conjunction with those of Piaget (e.g. 1952 and 1957) to define a coherent rationale for T.D.. Piaget's theory was used, in my view, because it offered a staged view of development, one where student behaviour could be

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retrospectively linked to learning which was assumed to have taken place. It is also a cognitivist view of mind and mental images. It is hardly surprising that this view was subsequently used in conjunction with those of the importance of hierarchical development through the space factors (to a developed spatial ability) from Gaughran's graphically model. This match of Gaughran's own private theory, developed following earlier work on spatial ability development, with Piaget's published theory of child development, seems an attractive alliance because of the (supposed) consistencies between the views of development they share (especially in respect of the internalised nature of the problem solving process), the link between problem solving and learning, and the abstract nature of both spatial ability and formal operations. Consequently, the new Syllabus document and Teacher Guidelines for Technical Graphics issued in 1990 linked views of spatial ability development with Piaget's theories of development for the first time in Department of Education and Science documentation. This documentation was then disseminated to teachers countrywide during an intensive in-service programme.

Difficulties Associated with the Received View of Technical Drawing.

Research on Which the T.D. Rationale is Based

One of the main difficulties with the received view of T.D. is that the vast majority of research on which the guiding theory is based has only very indirect connections T.D.. As outlined earlier in the discussion on spatial ability, the earliest research to which Gaughran referred concerned tests which were used to determine mechanical aptitude. This research had only tenuous connections with T.D.. The test instruments used were in diagrammatic form. However, test subjects were not (usualy) required to draw solutions to the problems posed, rather they were required to select a prepared drawing from a number of possible solutions. In the general Aptitude Tests used in Ireland today (e.g. those developed by the Psychological Corporation, 1990) the sections devoted to Abstract Reasoning (pp. 17-22), Mechanical Reasoning (pp. 28-39), and Space Relations (pp. 40-51) have many parallels with the earlier tests used early in the S.A. debate. The format of the tests are still very similar. In the case of Abstract Reasoning for example, students are presented with an example of a
typical problem figure and answer figure. In the test itself, each problem figure is accompanied by a series of answer figures from which the student must select an answer. The correct selection may be made by chance (e.g. if four answer figures are given to a problem figure there is a 25% chance of randomly selecting the correct answer figure) so there is an element of inaccuracy present in assessing the results of such aptitude tests.

The laboratory tasks used by Gaughran to assess the development of S.A. can be directly related to his definition of the constituent S.F.'s. When assessing each S.F. solid and pictorial (abstract) models of the figure or configuration were used. These were regarded as interesting tactile and visual stimuli and therefore, as aids to 'perception' and 'visual perception'. Cognitive images of the configuration are generated and categorised by the student. Each S.F. was tested, with the complexity of the tests increasing for successive S.F.'s, reflecting their hierarchical nature.

The approach adopted in solving such problems is (I would argue) very different from that approach to solving T.D. drawing problems in the classroom or examination context. Therefore, the theory which is being used to guide T.D. today is based on research into spatial abilities which used tests which are markedly different to the types of problems encountered in a modern T.D. programme is, at best, built on shaky foundations. In my view, a guiding theory for T.D. needs to be developed from within the subject itself. Research into the types of problem which must be addressed by T.D. students and the types of solutions they must prepare needed to be carried out in order to develop such a theory. The research reported in Chapters 3 and 4 is an initial effort to investigate these issues in a T.D. classroom.

The Influence of the Teacher in the Classroom?

From the Piagetian perspective currently used to guide T.D., development is viewed as resulting from active construction and self-regulation. This is not regarded as being synonymous with learning, which is viewed as the acquisition of knowledge from an external source. It is recognised that the student must develop her/his own construction of such knowledge. The obvious external source of knowledge in the classroom situation is the T.D. teacher. However,
while recognising that such external sources of knowledge are necessary for learning to occur he stresses that it is the stage of development which significantly affects the learning which takes place. Thus, the stage of development the student has reached at the time of the teacher's intervention is crucial in determining whether the intervention is successful in facilitating learning. Donaldson described this as learning being subordinate to the subject's level of development (Donaldson, 1987, p.45). This view of learning, and by association teaching, makes great demands on teachers. In order to provide each student with the appropriate learning environment they must firstly determine, at the beginning of the programme, each student's stage of development - both in terms of S.A. and Piaget's stages. This process of assessing the stage of development at which the student is at must then continue throughout the T.D. programme. This must be undertaken for, as we have seen earlier, progression to the latter stages of development depends on the development of the earlier ones. In effect, this requires that teachers must act as developmental psychologists in order to fulfil their teaching role. I would argue that teachers are not trained to fulfill such a dual function.

How is Spatial Ability Currently Assessed?

In Gaughran's favour, he has recommended a pedagogy which he designed to address this staged view of the development of thinking in T.D.. However, there is a reliance on assessing ability based on behaviour which can be questioned. He recommends for example, that work with students should comprise 'problems', the solution of which, can be attributed to the abilities associated with the S.F.'s. The teacher should work with solid geometrical models in the first instance before progressing to (abstract) pictorial ones. In this way, for example, S.F. 5 problems would require the student to demonstrate the abilities associated with it by drawing the solutions to problems which require the mental manipulation of elements of two- or three-dimensional configurations. Thus, manipulation of the elements of a mental configuration would have to be carried out as mental actions before any drawing of the solution could be prepared. However, the mental actions which constitute the S.F. are not an observable phenomenon. This requires that an assessment of the students ability must be made on the basis of the drawn solution produced. In many respects this is a throwback to the learning theories developed by
Pavlov (1927) and Skinner (1938), in which assumptions about the nature of learning are made following some observable behaviour, albeit in the absence of a model of mind. Although some would argue that the ability described is adequately represented in the solution drawn, I believe that such an equation of behaviour to the ability to problem solve (at whatever level) should not be made too readily. There may be other factors, from within the received perspective or from alternative perspectives, which can explain the nature of the solution produced.

Can T.D. Programmes Influence the Development of Spatial Ability?

It is also worth noting, given the Department of Education and Science view of problem solving and learning in T.D., that there has been some debate in the S.A. literature (e.g. Mendicino: 1958; Myres: 1958; Blade & Watson: 1955; Brinkman: 1966; Smith & Lipman: 1979; Lord: 1984) questioning the influence on the development of S.A. which courses in T.D. may have. This would appear to question the wisdom of basing a course in T.D. on a perspective which attributes such salience to S.A. development, especially when the protracted debate and disagreement about the nature of S.A. is considered. As described earlier, there is a notable absence of explanations as to how the received view of T.D. actually impacts on the development of students learning and problem solving, beyond references to the stage of development at which students enter second level schools. How current T.D. programmes, with the received rationale, can or do influence student development is not clear.

Piaget's theory, when applied to the classroom, does recognise that the speed at which a student progresses through the stages of development is influenced by the social and cultural environment. It also recognises that the sequence through stages remains constant even though the speed of an individual's development depends on whether her/his constructive efforts allow assimilation of what the environment in the classroom affords. Thus, although the teacher theoretically can influence development, this influence may be viewed as very limited in real terms. Whereas, exchanging ideas is viewed as important in the development of thought, in particular in strengthening the awareness of other points of view, the nature and extent of student development is
restricted by the students constructive processes to which the teacher does not have direct access but can support.

Thus, the research on which the current theory which guides T.D. is based may be viewed as having only tenuous links with the subject as it is currently experienced in post-primary classrooms; the influence which the teacher can exert on student learning is minimal; assessment of the development of spatial abilities is based on observed behaviour because students mental actions cannot be directly accessed; and there is a body of literature which questions the ability of T.D. programmes to influence the development of spatial ability. While recognising the limitations of any theory to describe and explain the nature of its subject, it would appear that there are serious issues raised when the current graphicacy perspective of T.D. is examined closely. In the following paragraphs a number of an alternative theories which might more adequately describe and explain the activities in T.D. classrooms and examinations are outlined.

Possible Alternative Theories for T.D.

Other theories could be used to describe and explain the activities associated with learning, development and problem solving in T.D. One alternative explanation to the current cognitive constructivist view of the solution of T.D. problems for example, would emphasise the critical nature of the social and physical contexts in which the actions that produced the drawn solution were undertaken. This falls within a relatively new theoretical framework which is characterised as 'situated' cognition. The situated view of cognition builds on an earlier perspective called activity theory. I will briefly discuss activity theory before returning to situated cognition.

The cultural-historical theory of activity which was initiated by Vygotsky and his colleagues Luria and Leont'ev in the 1920's and 1930's, known as Activity Theory (Vygotsky, 1978, p.40), may offer an alternative explanatory theory of T.D. to the cognitive constructivist perspective of the received Graphicacy Perspective. Activity theory is an holistic, emergent view of learning based on the idea that human learning is mediated through practical activity; activity is mediated by cultural signs: language, tools, conventions and these technologies themselves are artefacts of practical activity. As the artefacts change,
activity changes along with the consciousness of the participants in a continuous, evolving cycle of learning. It precedes knowledge and there is no understanding apart from it.

Three theoretical generations in the evolution of activity theory may be identified. The first generation centred around Vygotsky (1978, p.40) and created the idea of mediation where the relationship between human agent and objects of environment is mediated by cultural means; tools and signs. The second generation was introduced by Leont'ev (1981, pp. 59-69). He emphasised the division of labour as an historical process underpinning the evolution of mental functions. Mediated by tools, work is performed in conditions of joint collective activity. Cole and his colleagues Engstrom and Vasques (1997) recognised the need for a sensitivity toward cultural diversity (the third generation) where a dialogue between different perspectives of activity theory could be established.

Activity Theory is well established as a tradition which has been applied in education, linguistics, anthropology, cultural research and more recently in computer science and can be used to provide a broad conceptual framework which can be used to describe the structure, development and context of T.D. tasks.

Kuutti (1990) defines an activity as the fundamental context within which human actions take meaning. Activities have motive, are collective phenomena, have a subject - an individual or collective, exist in a material environment and transform it, are historically developing phenomena, develop in response to contradiction, and are realised through the conscious and purposeful action(s) of the participant(s). The emphases are on the transformation of activities, on their cultural mediation in specific settings. Hasan et al (1997) describes how the theory is applied to the analysis of dynamic problems which are highly context dependent and involve the use of tools to support high level human activity such as strategic decision making tasks. Scribner describes how skilled practical thinking is goal-directed and varies adaptively with the changing properties of the problems and the changing conditions in the task environment (1984, p.39).

According to activity theory the unit of analysis is an activity composed of subject, object, actions and operation. Actions are goal-
directed processes which must be undertaken to complete a drawn solution to a drawing problem. These actions are conscious since they have a goal which must be fulfilled. The actions engaged in can be referred to as the tasks which are associated with a particular solution to a problem. The activity has both internalised and externalised elements and their division is artificial, since both are necessary to complete the activity. The activity is mediated by the drawing instruments which were created to control the behaviour associated with the completion of the activity. Further, activity theory requires that our interaction with reality should be analysed in the context of development. Thus the activity involves the student and the drawing instruments and is both internal and external at the same time. The salient point is that the internalised and externalised activity is fused or unified - one can not be separated from the other.

Thus, the activity of producing a drawn solution is undertaken by the T.D. student (the subject) who is motivated towards the solution of that problem (the object) which is mediated by tools (in this case the drawing instruments) in collaboration with others (the T.D. community - teachers, fellow students and those who set examination questions). The structure of the activity is constrained by cultural factors or conventions (rules) and social strata (division of labour) within the context of the activity. The conditions in which T.D. activity takes place for the post-primary student differ in many respects from those experienced by the modern drawing office practitioner. These differences were clearly illustrated by the experiences of a professional draughtsman during the initial part of this study (described later in Chapter 3). He had successfully completed post-primary T.D. and a further programme of study at an Institute of Technology before being employed (for more than a decade) by a number of civil engineering companies. He is highly knowledgeable and skilled in the T.D. used in industry. However, his competence in the T.D. knowledge used in school classrooms and in the solution of examination problems was less comprehensive. The structure of the activity involved in the solution of T.D. problems is (obviously) different for the classroom and the drawing office. This would appear to support a view of cognition which is situated in the context in which the activity is being carried out.
Situated cognition theory builds on activity theory. It is a theory of knowledge acquisition. Lave (1988) describes how learning, as it normally occurs, is a function of the activity, context and culture in which it occurs, i.e. it is situated. She argues that this is in contrast with most of the learning activities which takes place in classrooms where abstract knowledge taken out of context is used. She argues further that social interaction is a crucial component of situated learning with learners gradually becoming involved in a community of practice which embodies certain beliefs and behaviours to be acquired. Beginners or novices move from the periphery of this community to its centre gradually, becoming more active and engaged with the culture as they do so, eventually assuming the role of expert. Lave and Wenger (1991) described this process as legitimate peripheral participation. Situated learning is also described as being usually unintentional, rather than deliberate with the acquisition of knowledge and skills taking place gradually as the novice learns from the expert in the context of everyday activities. Thus, the two major principles of situated cognition are: firstly, that knowledge needs to be presented in authentic contexts, i.e. in settings and applications that would normally involve such knowledge; and secondly, that learning requires social interaction and collaboration.

Interestingly, situated cognition theory has been applied in the context of technology-based learning activities for schools that focus on problem solving skills (Cognition and Technology Group at Vanderbilt, 1993) which may be regarded as being broadly similar to the subject (which is technology related) and focus (problem solving) of the investigation reported here. It has also been applied in the context of artificial intelligence (Suchman, 1988).

This situated view has been developed further into the notion of cognitive apprenticeship by Brown et al (1989). Learners, according to this view, are supported in the domain in which the learning takes place, as they acquire, develop and use the cognitive tools in authentic domain activity. This is achieved, both in and out of school, through collaborative social interaction and the social construction of knowledge.
These alternative perspectives emphasise that the actions associated with drawing a solution can not be divorced from the associated cognition in the way the received perspective separates them.

Actively using the drawing instruments leads to an implicit understanding of the contexts in which they are used. This understanding continually changes as a result of the interaction of the instruments and the contexts in which they are used. Learning and acting become indistinct and a life-long process of learning results from acting in (different) situations (Seely Brown et al, 1989, p33).

This explanation of learning in (different) T.D. contexts, coupled with a view of the student engaging in individual participation in socially organised practices discussed further by Hennessy (1993, p.2) falls within the relatively new theoretical framework characterised as 'situated' cognition.

A theory of T.D. which emphasises Lave's view of the critical nature of the social and physical contexts in which activities that produce solutions to drawing problems are undertaken may be postulated. The solution of such problems is embedded in the context of the activities associated with the act of drawing solutions with instruments in pure and applied geometrical contexts. The situation in which the solution is prepared structures cognition (Miller & Gildea, 1987). Lave has described everyday cognition (1977; 1988) where problem solving is active and flexible and where the solution of problems often poses further dilemmas to which entirely satisfactory solutions may not be possible in a given set of circumstances. Thus, problem solving is structured into and by ongoing activity with a variety of methods being used, depending on the circumstances encountered. This differs considerably from the problem solving required to solve Spatial Ability tests, which require correct, experimenter-determined answers. It is also different from the type of problem solving which T.D. students must carry out when solving T.D. problems in examinations.

Since perception forms a major part of the graphicacy perspective it is interesting to note that Gaughran did not include any reference to the work of Gibson (1966; 1977; 1979) on perception, affordances and the ecological approach to visual perception. One of the major theses of Gibson is that perception depends entirely upon information in the stimulus array, rather than on sensations influenced by cognition. He
further proposed that perception is a direct consequence of the properties of the environment and does not require any sensory processing of the type Gaughran describes in support of his view of perception. Gibson's theories were most well developed in respect of the visual system (although they were intended as general theories of perception) and recommended that instruction should emphasise the stimulus characteristics of the environment that provide perceptual cues. This view of perception could be adapted, it would appear, to address issues of perception in T.D.

What is Problem Solving?

The received view of T.D. outlined in syllabus documentation appears to assume that a linear form of problem solving, related to developing spatial ability, can be taught and applied across contexts - i.e. problem solving directly related to the space factors which can be used to solve pure and applied geometrical problems. Scribner (1984, p.38) describes how the problem solving process is restructured by the knowledge and strategy repertoire available to the problem solver. Hennessy (1993, p.25) argues that an approach which ignores the context of the problem is inherently unproductive. She argues further, that direct teaching for transfer across contexts is necessary and that this should include training in task-specific skills, self-regulation and awareness such as that described by Browne & Campione (1984). Interestingly, she also argues that direct training for transfer can be successful in facilitating conceptual development.

The notion of internalising the actions associated with progression through the S.F.'s is consistent with Piaget's notion of formal operations. However the constitution of visualisation, another form of mental activity, is not so readily agreed. It (visualisation) has been variously described as 'the process of mentally understanding visual information' (Bertilone et al, 1997, p.16) 'the ability to think in three dimensions' (Giesecke et al, 1980, p.164), or in Gaughran's case the manipulation of mental images of perceived or imagined configurations - an amalgam of all the S.F.'s. There is a subtle difference between Gaughran's definition and some of the views of visualisation discussed during the earlier S.A. debate. For example, French (1951) discusses visualisation in terms of mental movement about real or imagined configurations. This equates with Gaughran's S.F. 3, mid-way through
the developmental hierarchy. Gaughran would describe this form of mental activity (when isolated from the hierarchy of S.F.'s - if that is possible) as 'spatialising', a far less complex mental activity to the abilities associated with S.F.'s 4 and 5, where the ability to mentally rotate or alter configurations has developed. Spatialising, would appear to be related to concrete operational thinking, where the student is essentially still thinking in terms of actual movement about a configuration. She/he is therefore, not engaged in formal operational thought. The differences between the definitions alluded to above are subtle but significant. There is agreement (e.g. Bertilone et al, 1997; Giesecke et al, 1980) that it is very important for students to be able to visualise, in the absence of a clear definition of what visualisation actually is. In this context, it is worth noting that as recently as 1996 Gimmestad Baartmans and Sorby published a book on 3-D spatial visualisation for undergraduate engineering students in which the nature of visualisation is defined only in terms of its usefulness in various engineering and architectural endeavours. To define it in terms which relate solely to the development of S.A. can therefore, be seen as a convenience from the received perspective of T.D., and or as a major deficiency in terms of the study of the nature of T.D. where its significance is universally accepted.

What of T.D. Knowledge?

By emphasising those aspects of graphicacy associated with S.A. development the received view of T.D. effectively de-emphasises the nature and use of knowledge associated with the subject. It is worth noting that the T.D. syllabus makes several references to knowledge and indeed that the whole programme is organised around the knowledge associated with the topics dealt with while learning, development and problem solving are discussed only in relation to the development of graphicacy. Theoretical perspectives of other post-primary subjects based on views of knowledge have already been developed and the basis for a view of T.D. from a knowledge perspective is outlined next.

Technical Drawing: A Knowledge Perspective

Although other theoretical perspectives on learning, development and problem solving have gained widespread currency, the 'graphicacy-
Piagetian cognitive-constructivist perspective' advanced by the Department of Education and Science is the one which is currently used to inform practice in Irish post-primary classrooms. Alternative theoretical perspectives of learning and development, most notably those of Bruner (1966 and 1968 for example) and Vygotsky (1962 and 1978 for example), which can address the same issues have been generally ignored by those who have been charged with the development of subject syllabi in Ireland since the implementation of the new T.G. syllabus in 1990. These alternative views, for example activity theory and situated cognition described earlier, tend to emphasise the importance of the social and cultural aspects of the interaction between the mature and immature (the teacher and the student in the classroom context) above the mental activities associated with development from a constructivist perspective. The received view of T.D. and other alternatives however, are not of necessity, mutually exclusive. Cobb (1994) for example, contends that 'self-organisation and enculturation into established practices' (p.13) are complementary perspectives and that we should not be forced into making a choice between cognitive constructivist and sociocultural perspectives. In this context, in my view, the singular approach to the syllabus adopted officially may be viewed as inadequate since there are other complementary perspectives which are either overlooked or ignored. Selecting and using one theoretical perspective to address issues which may equally be addressed by others (or in conjunction with others) is less than satisfactory when the effectiveness of the alternative views has not been explored. Therefore, in order to understand the T.D. domain more fully, I would argue, it would be advantageous to examine other alternative explanations of how learning, development and problem solving may occur.

As in Gaughran's case, my own views on

- the importance of social interaction in teaching and learning,
- literature relating to problem solving in terms of knowledge use,
- the debate on the ability of T.D. to effect S.A. development, and
- the difficulties associated with making assumptions based on observed behaviours in the received view that I was encountering on a daily basis in the T.D. classroom
have all come together to prompt the research reported in the following chapters. Knowledge is regarded as a major factor to be considered in issues related to learning and instruction (deJong and Fergussen-Hessler, 1996, p.105) and therefore, a view of T.D. formulated around its definition and use, would be very beneficial in helping us understand aspects of T.D. (for example learning and problem solving) that, in my view, the received view does not adequately describe or explain. Glaser's views of the importance of domain knowledge in problem solving give some insights into how such a view for T.D. might be formulated (1984, p.97).

The knowledge associated with T.D. is referred to in the syllabus document. The course is viewed as one which

provides the students with a body of knowledge [...] in topics which are appropriate and meaningful in a technological world. (Rialacha & Clar, 1994, p. 192)

Part 1 of the programme (Plane & Solid Geometry) contains 'topics requiring conceptual knowledge of a purely theoretical form' while Part 2 (Building Applications) 'emphasises the role of T.D. as the language of technology and contains topics which involve varied and interesting applications of the type of problem encountered in modern technology' (Rialacha & Clar, 1994, p. 192).

Although the nature of knowledge associated with T.D. is not specifically defined in the T.D. Syllabus, there are references to knowledge which may be regarded as procedural where the aims of the programme to 'develop problem-solving as well as the physical skills of manipulation and dexterity' (Rialacha & Clar, 1994, p.192) are clearly identified. The introductory section also identifies 'conceptual knowledge of a purely theoretical nature' (p.192) in respect of Part 1 of the programme, where Plane & Solid Geometry is described in detail. Thus, the types, levels and qualities of knowledge discussed by McCormick (1996), deJong & Fergussen-Hessler (1996), Stevenson (1994), Gott (1988), and Schoenfeld (1985), among others, is outlined in the official documentation which details the received view of T.D.. This knowledge is not however, defined in explicit terms. One reason for this, I contend, is that T.D. in Ireland is now thought of only in...
terms of the received view. The received view has become, in metaphorical terms, the vehicle through which any alternative tenor is explained (Sfard, 1997, p. 341). Thus, any alternative view of T.D. which is proposed is thought of in terms of the current received views.

**Procedural Knowledge**

A simplistic manner of discriminating procedural knowledge in T.D. (as outlined in the syllabus) may, for example, contend that much of the drawing that is done, in the physical sense, is procedural in nature. Thus, when what is required to complete a drawn solution is known, the construction of the drawing on the board in incremental stages is procedural - how to do it knowledge (McCormick, 1996, p.2).

However, all procedures are not the same - there are different levels of complexity. These levels are discussed by Stevenson (1994, pp. 13-14) who identifies three orders - the first is automatic and addresses known goals. For example, when students are presented with a table of co-ordinates from which the projections of intersecting planes or skew lines must be drawn. The second allows the achievement of unfamiliar goals using specific procedures. For example, when students draw an auxiliary view. The third order allows cognition to be switched between the former two. For example, when students draw the projections of solids in contact before introducing another solid which must be placed in a specific position. deJong & Fergusson-Hessler (1996, p.107) discuss these levels in terms of deep and surface knowledge with the implication that deep knowledge is better than surface knowledge. McCormick (1996, p.5) emphasises that recognising that such levels exist, is more important than precisely identifying the levels themselves. From the T.D. students perspective, access to procedures at all levels may (will) be required in the preparation of graphical solutions to problems at Leaving Certificate level and students therefore, must learn (and be taught) to use simple and/or complex procedures to complete adequate drawn solutions to address the problems they will encounter as part of their T.D. programme. This may be achieved, to a certain degree, in the absence of student understanding of the concepts associated with the problem or its solution. The use of simple specific procedures, for example when constructing a drawing of two intersecting planes from co-ordinates, can be learned very early in a T.D. programme with students.
producing appropriate orthographic projections of the planes without
the concepts associated with this specific procedure and resulting
drawing, being accessible to those operating on a procedural level
only. The same may be true for procedures at the different levels
described above.

From a knowledge perspective, problem solving in T.D. may be
regarded as a particular form of procedural knowledge which is
employed in particular sets of circumstances, i.e. particular procedures
which lead to the production of appropriate drawn solutions when
T.D. problems on specific topics are posed in particular ways.
Problems can be classified as either 'well-structured' or 'ill-structured'
(Biehler and Snowman, 1997, p.374). Well-structured problems are clearly
stated and can be solved by the application of specific procedures, or
algorithms. These types of well-structured problems also fit Barnes'
(1989, p.78) description of tasks, as activities where one knows exactly
what to do in a given situation to get a required result. The question
of whether such problems are problems at all then must be raised.
Ill-structured problems are more complex, provide fewer cues to the
procedures required to complete solutions and have less definite
criteria to determine whether the solution is adequate. Only small
portions of Higher Level of T.D. questions could be described in this
way. In such cases students do not know (immediately) the course of
action which is needed to complete the solution (Andre, 1989, p. 61).
Problem solving in T.D. at the Higher Level may then be described
as the application of knowledge that results in the attainment of a
satisfactory solution to ill-structured problems - deJong &
However, if similar ill-structured problems appear on examination
papers in successive years teachers will have devised procedures that
will allow the students to develop appropriate solutions, turning what
was an ill-structured problem when it first appeared into a task, not a
well structured problem. Previous experience of solving well- or
ill-structured problems is a major discriminating factor in determining
how students bring their knowledge to bear on a particular situation
which must be addressed ,(Hennessy, 1993, pp.1-41). As I will show
later, conceptual knowledge is crucial in this discrimination.

Thus, in T.D. examinations students are presented with a problem,
which requires the execution of particular procedures at various levels
to produce a drawn solution which is regarded as appropriate. Each question, on each topic, has a number of procedural approachs which lead to the production of an appropriate solution. There is no general problem solving strategy required. There may be similarities between the procedures employed in the different topic areas but success in the solution of questions within a topic area depends on the student developing a level of expertise in the use of procedures associated with the particular contexts - i.e. there is no set of procedures which can be used in each of the fourteen geometric areas of the T.D. examination, there is no general problem-solving strategy (see McCormick, 1996, pp.5-6 for a discussion of such strategies). There are however, procedures which may be employed when something unfamiliar appears in an examination question which may prove useful in the formulation of a solution without guaranteeing success.

Students therefore, need to be 'familiar with a broad range of general problem-solving techniques' called 'heuristics' by Schoenfeld (1985, p.12). These are more generally defined as *rules of thumb*, which prompt strategic action in particular circumstances. *Rules of thumb* help the student to identify instances where a particular construction is appropriate. Students often however, experience difficulty in the transfer or application of these rules. Prawat (1989, p.4) discusses how the pervasion of 'access failure' is cited in studies of problem solving as the major cause of poor performance. In T.D., where the application of geometric principles forms a major part of the programme, this transfer problem where it occurs, may be regarded as a major deficit in student knowledge. Others, for example Greeno and Riley (1987, pp. 289-313) would argue that it is the transfer metaphor which is deficient and not student knowledge. It is a problem which may persist (McCormick, 1996, p. 14), possibly because particular constructions are associated with the original situations where they were encountered (Meadows, 1993, p.81). For example, the 'planes method' used when finding the horizontal distance between two skew lines is often not recognised by students as the same construction that is encountered in mining application problems. This may also be viewed as student knowledge being less well structured (deJong and Fergusson-Hessler, 1996, p.108) than expert knowledge. Thus, for students to gain maximum benefit from knowledge of *rules of thumb* the situations where they may be appropriately used must be clearly outlined (Schoenfeld, 1985) for them.
Declarative Knowledge

The notion of a body of knowledge independent of those involved (Hoyles, 1989, p.121) in teaching or learning T.D. is, I believe implicit, in the T.D. syllabus. The texts commonly associated with T.D. outline principles and methodologies associated with the subject in minute detail and are primarily concerned with providing the students with geometric resources in the sense described by Schoenfeld (1985) covering all relevant principles, rules, constructions and conventions for the programmes they are designed to serve. This type of knowledge may be viewed in terms of facts concerned with T.D. and as such could be categorised as declarative knowledge as described by Anderson (1990). These facts relate to particular geometric topics and as such may be regarded as unrelated, except insofar as they are all geometric facts.

This selection of texts is extensive, and numerous volumes have been published in Ireland in recent years. These include works where topics related to the Junior Certificate and Leaving Certificate Syllabi are covered by Hogg (1985), O’Broin (1986 and 1991), O’Callaghan (1991 and 1996), O’Connor (1991 and 1997), Curtin & Curran (1993), and Gaughran (1992 and 1996). When used in the classroom (many T.D. teachers use these texts for personal reference only) these can contribute to the development of students' geometric resources and/or act as reference material. Many volumes are also available for those who wish to follow more advanced programmes in Plane & Solid Geometry. For example, the work of Abbott (1929) has been used as standard text by T.D. teachers in Ireland for generations while a similar text by Giesecke et al (1980) has enjoyed similar popularity in the United States. More modern texts such as that by Bertilone et al (1997) which contain chapters dealing with modern architectural and engineering drawing office practice are becoming more widely used by those more experienced in the domain.

These texts deal with topics from Plane and Descriptive Geometry (required by the T.G. Syllabus, 1990, p.7) in a graded format with the governing principle usually outlined at the beginning, followed by constructions which progress from simple to more difficult. These constructions are dealt with individually and it is only when they are

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appropriately linked by the student that they develop as conceptual knowledge. This may be viewed as gradually introducing the student to a store of geometric 'resources' (Schoenfeld, 1985, p. 12) to utilised in the context of solving problems related to topics on the syllabus. However, in the absence of specific instructions concerning the relationships between the items of knowledge or appropriate procedures to be employed in particular circumstances, this form of knowledge on its own is less than useful when geometric problems are encountered. Indeed, the sheer volume of constructions and principles may impact negatively on the students when faced with the circumstances of a particular problem. These texts may be viewed as containing a form of geometric knowledge which is inert and is therefore, of little benefit to the T.D. student. Knowing these facts without being able to identify the relationships between them across topic areas, or operationalise them into appropriate procedures is of no benefit to the problem solving described above in terms of the use of procedures.

The presentation of these texts would benefit greatly therefore, from alterations which emphasise the relationships between related constructions and identify appropriate specific and strategic procedures together with rules of thumb which would 'facilitate both knowledge acquisition and utilisation' (Prawat, 1989, p.3).

Conceptual Knowledge

Conceptual knowledge is viewed as being on the higher end of a continuum from declarative knowledge. The meanings attributed to a drawing by the one who makes it or by the one who reads it is however, of a conceptual nature since the relationships which link items of knowledge may be understood only in respect of the concepts represented (McCormick, 1996, p.2).

The meaning which an expert in the domain can immediately attribute to a drawing of the planes mentioned above for example, is not immediately accessible to a novice who, having learned a simple procedure can produce an appropriate drawing, but because she/he lacks 'conceptual understanding' (McCormick, 1996, p. 3) does not understand its significance. Just as Hiebert (1986) has distinguished between procedural and conceptual knowledge and the role each plays
in learning in Mathematics, similar roles for procedural and conceptual knowledge can be identified for T.D.. Whereas earlier studies of the nature and use of knowledge has separated procedural and conceptual knowledge it should be noted that more recently (for over a decade) the focus has been 'on the relationship between the two' (E819 Study Guide, p.67) rather than on the differences between them. This recent trend of examining the relationships between procedural and conceptual can usefully be pursued in studies into the knowledge associated with T.D.. This is the case because it is very difficult to isolate procedures used in T.D. which are solely procedural and without a conceptual content. Similarly, in order to produce an adequate drawn solution to a T.D. problem the concepts associated with it can only be represented in a drawn solution following the execution of the procedures in which the associated concepts are embedded. Concepts and procedures do not appear in isolation in the drawn solutions which the students produce.

Seely Brown et al (1989, p. 33) have written about concepts in terms of being 'both situated and progressively developed through activity' and not therefore, 'abstract, self-contained entities'. I think it is useful in the case of T.D. to think of concepts in this way since they are developed, I would argue, through student engagement in the activity associated with the preparation of drawings where procedures and concepts may not be easily, or usefully, separated.

Thus, when students are faced with questions from Ordinary Level T.D. papers for example, questions whose content varies little from year to year, the tendency is to view the preparation of the solutions as procedural only because the questions have been framed to allow the use of procedures which have been practised many times in classroom tasks (see Barnes, 1989, p.78 for a definition). The completion of such procedures in the examination is sufficient to satisfy the problem solving requirements for assessment purposes, whether the concepts associated with the completion of procedures is understood or not. Higher Level T.D. questions also tend to have a strong procedural emphasis in which there is an embedded conceptual content, but completion of a full solution is usually not possible unless the student can display a knowledge of the concepts associated with the topic. These concepts then have a procedural element. The procedures needed to do this are not necessarily complex, but the
level of understanding required to guide their use requires more than the procedural content, i.e. if the student can not recognise the relationships between the items of knowledge needed to address the problem in the question then appropriate procedures may not be executed.

In order to facilitate conceptual understanding of Plane & Solid Geometry teachers often use solid or pictorial models of the two- and three-dimensional configurations and mechanisms such as hatching and shading to recover some of the three-dimensional qualities lost in the two-dimensional drawing on the drawing board (see Gregory et al, 1995, p.149 for a discussion). This helps students to 'accurately perceive the three-dimensional nature' (Coren and Ward, 1989, p.276) of the configuration in its absence and thus, the two-dimensional representation of an object or configuration acts as an aid to visualisation of the real or imagined object itself. Vernon (1970) argues that shading and hatching of drawings restores some of their three-dimensionality and makes their interpretation easier. This is because three-dimensional drawings are more primitive and easier to understand than their two-dimensional counterparts. That two-dimensional drawings act as an aid to understanding real or imagined two- or three-dimensional configurations is a view of visualisation which is different to the received one, and one which has not been sufficiently explored.

Metacognition

The notion of metacognition includes both procedural and conceptual knowledge and self-regulatory mechanisms (Duell, 1986 cited in McCormick, 1996). Meadows describes this as 'reflecting upon one's own knowledge (or lack of knowledge) in respect of what may be required to solve the problem' (1993, pp.78-79). Self-regulation involves planning, checking outcomes of strategies, evaluating and revising strategies. Self-regulation strategies, if they are to be learned by the student, require specific treatment by the teacher, since only by teaching when and how to employ specific procedures can strategic knowledge be developed in the student. This is particularly so when strategic knowledge is to be transferred to new situations (McCormick,1996, p.6). Schoenfeld (1985) describes how when teaching mathematical problem solving, he encouraged his students to
ask themselves questions continually about what they were doing, what they were trying to achieve and what they would do next until the questions were internalised and led to improved student performance.

Visualisation From a Knowledge Perspective

From a knowledge perspective, visualisation, may be viewed as a form of knowledge with which the student uses her/his procedural and conceptual knowledge of the topic being dealt with, together with the drawings of the solution as they are constructed, stage by stage, in order to suggest further action. It is in effect, a way of knowing by looking at the drawings already completed, what procedural and conceptual knowledge needs to be used to construct an appropriate solution. In this way visualisation could be compared to a metacognitive or control function which encompasses self-regulation.

However, what distinguishes visualisation is its association with the drawing under construction. It is not solely knowledge about cognitive resources (procedural and/or conceptual) or about when it is appropriate to switch between them. It is knowledge inextricably linked to the action of constructing the drawn solution.

Indeed students often use their partially constructed drawings to dictate their approach toward completing a solution where knowledge relating to the topic is deficient in some way. Thus, rather than being solely in the head, visualisation is rather, inextricably linked to the actions of making the drawings on the board. It can therefore, be dealt with specifically by the T.D. teacher and learned by the T.D. student in the classroom.

These comparisons can be made across all the topics in which students are examined in the Leaving Certificate Examination in T.D..

Conclusion: The Need to Explore a Knowledge Perspective

In the literature review I have outlined the received view of learning, development and problem solving within the T.D. domain based on the
received graphicacy - Piagetian rationale for the subject from official literature. This received view emphasises the development of the mental abilities associated with the development of spatial ability described by Gaughran and a staged view of human development as outlined in Piaget's theory. Thus the Department of Education and Science views T.D. from a cognitive constructivist perspective where student learning and development is seen in terms of development through an hierarchy of space factors which leads the student from concrete to formal operational thought and to the development of spatial ability. Development is associated with the notion of readiness and progression is via an hierarchical structure. Problem solving is viewed as the ability to mentally visualise and manipulate images of configurations within a formal operational structure.

I have also outlined what I perceive as difficulties with this received view, namely its reliance on particular theoretical and methodological perspectives, in an educational context where other theoretical perspectives also exist, and which could serve to enhance our understanding of the domain. These difficulties serve to prompt research into alternative explanatory perspectives.

An alternative knowledge perspective is also outlined. Although a knowledge approach has been adopted in the study of other post-primary subjects there has not been any research of this nature undertaken in the T.D. classroom. Research into student competence in the T.D. domain (of which there has been very little at post-primary level in Ireland or anywhere else) has centred on addressing the relationship between developing S.A. and problem solving competencies. The relationship of students use of procedural, declarative and conceptual knowledge to their effectiveness in terms of solving two- and three-dimensional T.D. problems has received no attention and consequently little is known of students use of knowledge in T.D. problem solving. This is an important deficit in our understanding of T.D. since we know that changes in knowledge of domain-specific content may underlie changes attributed to the growth of capabilities (Siegler and Richards, 1982) in a domain.  

It is my view that T.D. teachers emphasise many aspects of the alternative theories (described earlier) which could be applied to T.D.
in apparent contradiction of the graphicacy perspective. For example, they emphasise

- learning in social contexts,
- the socio-cultural and social-constructivist aspects of classroom interaction,
- access to domain resources for use in appropriate contexts, and
- the strategic use of procedural and conceptual knowledge to address well- and ill-structured T.D. problems presented in written or graphical form to agreed standards.

The received description of how students become, and act as problem solvers in terms of the received view is inadequate since it makes assumptions based on observed behaviours. A view of T.D. focused on knowledge seems an equally valid approach in explanatory terms, but one which required some investigation.

Thus, the literature review undertaken here highlights the prevailing views on T.D. while outlining an alternative approach which needs to be investigated in order to provide a more complete view of all aspects of the domain. The process of addressing the issues of student development in the T.D. domain, detailed analysis of the types and levels of knowledge associated with the subject, learning grounded in the activity associated with T.D. and its social context, and the nature of problem solving are addressed by the study detailed in the following chapters. The emphasis of the study reported in the following chapters is on the relationships between the various types of knowledge. Unlike the work in the graphicacy area reviewed earlier, some of which was not related to T.D. in (or out of) the classroom context, the work reported in the following chapters involved students in naturalistic classroom situations, working on everyday tasks with the minimum disruption of day-to-day classroom routine.

**Defining the Focus of the Research**

The research that has influenced T.D. has been focused on views of general development from a cognitive constructivist perspective. It also incorporates views of spatial ability development. It is somewhat surprising that students use of different types of knowledge, the relationships between their use, and students effectiveness in problem
Solving using knowledge in T.D. has received no attention from researchers (although it has received attention in other subject areas). As a consequence of this lack of research, little is known about how knowledge use relates to how students solve T.D. problems. Indeed, procedural and conceptual knowledge are not specifically defined in any official documentation relevant to the Technical Drawing programme, although elements of the programme which may be regarded as procedural and conceptual are alluded to in the Syllabus.

Thus, the research reported in the following chapters was designed to address these deficits.

The introduction to the T.D. syllabus describes how the course sets out to develop problem solving in two- and three-dimensions. Problem solving is viewed as important, possibly, because of T.D.'s association with other technological subjects and hence, 'the technological method' (Murphy and McCormick, 1997, p.1).

Though the syllabus envisages two distinct types of 'problem', those of a purely theoretical nature relating to plane figures and geometric solids, and those involving the use of geometric principles to address technological problems, I have already noted that the 'problems' which students must solve in the T.D. examination may only be tasks.

Thus, the development of students' problem solving, as I have already noted, is central to the aims of the T.G. and T.D. programmes. Currently, problem solving is located in the graphicacy tradition and a cognitive constructivist perspective. As a possible alternative to the received view of T.D. the knowledge perspective allows students' problem solving to be approached from (at least) three directions -

1. how do students operate procedurally,
2. how do they link items of knowledge - how they conceptualise, and
3. how do they inter-relate procedural and conceptual knowledge when drawing solutions?

Exactly how students become good problem solvers is not clear from the received rationale for the subject. From research into other (somewhat related areas of the curriculum) it appears that the inter-
relationship of students' use of procedural and conceptual knowledge plays a significant role in the development of problem solving skills within any domain. Whatever form the development of students problem solving abilities takes, in the received wisdom, its importance lies in its facilitation of spatial thinking. Whether this takes the form of internal graphical discussion and reflection on the concept of space and all it holds from the cognitive constructivist perspective - similar to Vygotsky's 'speech for oneself' (Britton, 1989, p.211) in the language domain or to Piaget's 'formal operations' (Donaldson, 1987, p.139) - or through the inter-related use of knowledge and the action of drawing is a question which needs to be addressed. Similarly, a distinction must be made between knowledge which is constructed through 'self-directed problem solving' (Wood, 1988, p.5) - i.e. problem solving to learn from the constructivist standpoint and through social interaction in the classroom - where students are taught and learn how to problem solve in a given subject area.

At this stage I had identified a large number of questions related to the nature of procedural and conceptual knowledge and their inter-relationship. The existing views on learning and problem solving were grounded in a constructivist view, one in which the role of the teacher is limited to facilitating learning by providing an appropriate learning environment for the student. This viewpoint appears to limit the importance of instruction, contingent control of learning and the perfection of self-regulation during social and instructional interactions discussed by Wood (1988, pp. 9-10) and others. I felt these were specific areas worthy of study related to a knowledge perspective of problem solving.

Thus, the title for my research study was formulated -

'The Inter-Relationship of Procedural and Conceptual Knowledge in Two- and Three-Dimensional Spatial Problem Solving of Technical Drawing Students'.

Thus, the study reported here was designed specifically to examine the following questions -

1. how are procedural and conceptual knowledge defined for T. D.?
2. what is known about how T.D. students use procedural and conceptual knowledge?

3. how is spatial problem solving defined and how does it develop in students?

4. is there a connection between T.D. students use of procedural and conceptual knowledge and their effectiveness as two- and three-dimensional spatial problem solvers?
Chapter 3
Methodology and Methods

Introduction

As outlined in Chapter 2., The Department of Education and Science in Ireland proposes a cognitive constructivist perspective of learning, development and problem solving in T.D. for students in junior and senior Cycle in Irish post-primary schools. Official literature emphasises internalised mental activity associated with Spatial Ability and problem solving is equated with the development of the space factors which constitute it. The research which led to the development of current view of the subject can be located methodologically within the ‘positivist paradigm’ (Denzin and Lincoln, 1994, p.13). It relies heavily on experimental models of generating data in contrived situations and on statistical analysis.

The developing views on the nature of spatial ability were then used in conjunction with the theory of development espoused by Piaget (e.g. 1952; 1957) to define a coherent rationale for T.D.. Piaget’s theory was used, I believe, because it offered explanations of staged development, where student behaviour could be retrospectively linked to learning which was assumed to have taken place and because the methodologies employed by Piaget in the generation and analysis of data had some congruence with the spatial ability research.

Whatever personal views other teachers hold in respect of the official cognitive constructivist philosophies their operation in classrooms (based on my experience) emphasises the importance of the social interactions, the context in which drawn solutions to problems are made, and T.D.-specific knowledge.

T.D. knowledge has traditionally been thought of in terms of the constructions associated with plane and descriptive geometry. However, in my view, this underestimates the role of such knowledge in students' learning, development and problem solving. Unlike the received view of knowledge, I am proposing a view of its development on an interpersonal social level first, in transactions
between the teacher and the student (Rogoff and Lave, 1984, p.4) prior to its appearance on an intrapsychological internal level (Vygotsky, 1978, p.57 and Wertsch, 1985), all the while being mediated through the use of drawing instruments at the drawing board. This mediation through the use of instruments in the process of making drawn solutions is similar to what Vygotsky described as 'activity theory' (1978, p.40). My experience of T.D. classrooms suggests that there is a dichotomy between the theory used to guide the subject and the practice of teachers in the classroom. If this practice is effective, in terms of facilitating student learning, questions must be asked about whether the received view can adequately explain the processes of learning, development and problem solving which it advocates.

In order to inform practice we must first understand it more fully, before trying to change it. When the senior and junior post-primary syllabi were last changed, in the mid-1980's and 1990 respectively, no attempt was made to critique the vocational philosophy which was being discarded or to assess or justify the necessity for change. Therefore, the possible classroom implications of the philosophical view that was introduced was not assessed. There was a dearth of research into technological subject areas at the time which could inform such an assessment. It is not surprising that the research that would inform the new T.D. syllabus was sourced outside educational settings where spatial ability theories were already well established. One of the main issues arising from the introduction of a rationale for T.D. which placed such a great emphasis on spatial ability was that the problem solving involved in tests to assess spatial ability development is considerably different to that encountered in the T.D. classroom.

The general Aptitude Tests used in Ireland today (e.g. those developed by the Psychological Corporation, 1990) have sections devoted to Abstract (pp.17-22) and Mechanical Reasoning (pp. 28-39), and Space Relations (pp. 40-51) similar in many respects to the tests used early in the debate. Subjects are presented with problem and answer figures. Each problem figure is accompanied by a series of answer figures from which the subject must select an answer. The correct selection may be made by chance (e.g. if four answer figures are given to a problem figure there is a 25% chance of randomly selecting the correct answer figure) so there is an element of
inaccuracy present which must be considered when assessing test results. Gaughran's tasks are very similar but they can be directly related to his definition of the Space Factors which constitute spatial ability. He describes how a variety of tests can be applied to each S.F. and how it is regarded as advisable to use a selection of these when assessing their development. Solid and pictorial (abstract) models of the figures or configurations are also used during the testing process. These are regarded as aids to Perception and Visual Perception. The complexity of the tests reflects the hierarchical nature of the spatial ability sub-factors.

When tasks are successfully completed the subject is assumed to be thinking in a particular way - i.e. inferences about the state of development of spatial ability are based on the nature of the solutions the student identifies. Similarly, failure to identify appropriate solutions is regarded as indicative of deficiencies in the development of particular S.F.'s.

Efforts are made to apply this type of reasoning to tasks which students undertake in the T.D. classroom and examinations although few similarities appear to exist between tasks undertaken in the different contexts. For example, the tasks used to assess S.F.'s are very well defined in terms of their format and solution - i.e. they are presented in a particular way and have a particular correct solution, one which the student must identify, not construct. Students are aware that the solution to the problem lies in the answer figures that are supplied - it is a matter of selecting the correct one. These tests are completed by individual students in test situations, i.e. they are not carried out as 'normal' classroom tasks might be. No effort is made to render the activities associated with the tests personally meaningful or purposeful in the manner authentic activity is described by Murphy & McCormick (1997, p.2) where knowing, doing and participating in the ordinary practices of the culture are emphasised. The received view of T.D., based largely on such tests, may therefore, be regarded as deficient in many ways.

While the school T.D. community which students are learning to become participants in is relatively well defined the broader T.D. community encompassing those involved in Engineering, Building and other technologies is less clear. Schools have difficulties in respect of
separation from this broader community of T.D. practitioners but these are compounded by the perspective of problem solving advocated by the received graphicacy perspective, where problem solving is disassociated from activity. Although there are major differences between the practice of T.D. in classroom and industrial contexts a knowledge perspective of the types of activities which take place in 'normal' T.D. classrooms constitute, I would argue, a more appropriate basis for a theory of T.D. which can be related to both T.D. communities.

The research which is used to guide T.D. as a subject should be carried out in naturalistic classroom settings, since this would be much more likely to generate a theory of T.D. which can be reconciled with classroom practice. This is the case because T.D. teachers are in a position to design authentic classroom tasks which will progressively lead to student enculturation into the school T.D. community (Lave, 1992, p. 85). This is achieved by explicit teaching of problem solving procedures through interaction and discussion with students (Murphy and McCormick, 1997, p.5). A theory grounded in classroom activity is much more likely to advance our existing knowledge and understanding of the subject in ways which the received philosophy generated using positivist methodologies could not. The study reported here seeks to determine what the meaning of T.D. is for one of the main group of actors - the students in the classroom. Student activity was observed and their explanations for these activities recorded. This is important since the subject may not be fully understood if viewed exclusively from any particular perspective - for example, the T.D. teacher or policy maker. Thus, the study undertaken here seeks to begin the process of formulating a possible alternative theory by addressing two of the issues identified above - the context in which students prepare drawn solutions to problems and their use of knowledge in doing so; i.e. problem solving for T.D. at the drawing board in the T.D. classroom. The study also includes detailed analyses of the types and levels of T.D. knowledge and the nature of problem solving used when constructing solutions to T.D. examination questions in naturalistic classroom settings.

The social interaction involved in teaching and learning in a T.D. classroom is an issue which warrants separate study with a singular
focus. As such, it was too large an issue to be addressed in a study of the scale described here.

**My Methodological Approach**

All researchers are philosophers in that ‘universal sense in which all human beings [...] are guided by highly abstract principals’ (Bateson, 1972, p. 320). These principles combine beliefs about ontology (what is the nature of reality?), epistemology (what is the relationship between the inquirer and the known?) and methodology (how do we know the world or gain knowledge of it?). These beliefs shape how the researcher sees the world and acts in it. The ontological, epistemological and methodological beliefs of a researcher may therefore, be regarded as an individual paradigm or interpretative framework, a ‘basic set of beliefs that guides action’ (Guba, 1990, p.17). So it is with my work. Denzin and Lincoln (1994, pp.13-14) describe four interpretative paradigms which structure qualitative research: positivist and post-positivist; constructivist-interpretive; critical; and feminist-poststructural. They also describe how these four general paradigms become more complicated at the level of concrete specific interpretative communities. The final three paradigms, alluded to above, work against and alongside (and some within) the positivist and post-positivist models. They all work within relativist ontologies (with multiple constructed realities), interpretative epistemologies (with the knower and the known interacting and shaping one another), and interpretative, naturalistic methods. Each also has its own set of assumptions, criteria for evaluation and typical theoretical statements which are identifiable as being consistent with the particular paradigm. So it is with my study.

The study described in the following chapters could broadly be described as falling within the constructivist-interpretive paradigm. A relativist ontology is assumed (there are multiple realities); as is a subjectivist epistemology (where the knower and the subjects create understanding); and a naturalistic (in the natural world) set of methodological procedures are adopted. The findings are presented in terms of the criteria of a grounded theory (Strauss and Corbin, 1990) and terms such as credibility, transferability, dependability and confirmability replace the usual positivist criteria of internal and external validity, reliability and objectivity.
Although the methodological philosophy which guides the study can be defined in terms of the constructivist-interpretive paradigm mentioned above with its origins in the 1920’s, it may also be regarded as practitioner research, a relatively new recruit to the many traditions of educational research. Practitioner research has its origins in the teacher researcher movement of the 1970’s that focused on curriculum research and development and the critical appraisal of classroom practice using action research (Stenhouse, 1975; Open University, 1976). This type of research, undertaken in educational institutions, often by the teacher in her/his own classroom, is focused on teaching and learning, and its main purpose is to improve practice. This form of research is also intended to develop the professional judgement and expertise of the teacher engaging in the research process. In order for this to take place the teacher researcher must be committed to developing her/his self-awareness viz a viz values, preconceptions and tacit pedagogic theories. There must also be a commitment to reflect honestly on her/his behaviour and to share these reflections with sympathetic colleagues. This is not an easy undertaking but the benefits derived from developing habits of critical self-reflection makes an enormous contribution to the teachers self-confidence and professional expertise. There have been many trends in action research (e.g. Elliot, 1981; Kemmis and McTaggart, 1981) but I do not wish to be associated too closely with any one of them - mine is a more eclectic approach. My research was not strictly action research, since I was not studying the effect of making changes. However, like Bassey (1990, p.35), I view the purpose of carrying out my research as an attempt to make some claim to knowledge - to show something which was not known before, following a systematic, critical and self-critical investigation in my own classroom.

This kind of research was necessary for a number of reasons. Firstly, the research on which the rationale for the T.D. syllabus is based was conducted under a positivist paradigm that had particular views of learning, development and problem solving. As I have outlined in Chapter 2, the current rationale may be regarded as incomplete in its description and explanation of the activities associated with T.D.. Alternative theories, developed for other subjects, may offer a fuller description and explanation of the activities associated with T.D.. The research reported here is an effort to develop some of these.
alternative theories into a perspective of T.D.. Secondly, the research which is currently used to guide T.D. was conducted (almost) exclusively outside the classrooms which it has come to influence so profoundly. Experimental tasks (which are very dissimilar to the everyday T.D. classroom tasks) were administered in artificial settings (which are very dissimilar to classroom settings) and the results of these tests were statistically analysed. Therefore, it was necessary to conduct research in the T.D. classroom for the T.D. classroom, where students problem solving activities could be assessed in a naturalistic setting. Thirdly, as outlined in Chapter 2, a number of researchers have questioned whether teaching T.D. can influence student development in the way described by the advocates of these views. This question arises because learning and development are not regarded as synonymous by the current guiding theories. The combination of all three factors have led to the research reported here being conducted in the manner described in later sections.

Like the constructivist paradigm model described above practitioner research may be viewed as involving the studied use and collection of a variety of empirical materials. These describe routine and problematic moments and meanings in the lives of T.D. students in an effort to 'get a better fix on the subject matter at hand' (Denzin and Lincoln, 1994, p. 2), i.e. how T.D. students use knowledge in problem solving.

My reading on learning and knowledge has helped to formulate the focus of my research. My efforts to investigate how students use knowledge in solving T.D. problems has however, been governed by the desire to be open. I did not have any preconceived notions about students' knowledge use before this study began. I had some thoughts about what might be happening but these were based on studies which had been carried out in other areas of the curriculum - no empirical work of the type I proposed to undertake had previously been carried out in T.D. classrooms. I used soft data generated from observing, interviewing and audio-recording students in conversation. I emphasised the process students worked through in generating solutions and did not concentrate on the final solutions which they produced to problems. I tried to be interpretative. I used a small number of students (because my classes were relatively small) in naturalistic classroom settings and developed an analysis of the process grounded in the data. This approach reflects my desire to secure an
in-depth understanding of how students used knowledge to solve T.D. problems by using a strategy which was rigorous in the collection, analysis and interpretation of data; had breadth, in the variety of data generating activities; and depth, in the detail of the data generation, analysis and interpretation. Since objective reality can not be captured this strategy offers an ‘alternative to validation’ (Flick, 1992, p.194).

Within this paradigm I began the study by clearly defining what would be its focus - what concrete problem would I investigate? Once this focus had been determined a specific research strategy of inquiry was devised to investigate it. This strategy was divided into three main activities: the collection of empirical materials (data); the analysis of these empirical materials; and their interpretation. Thus, the study described here was designed to investigate a general curriculum and assessment issue associated with knowledge and the curriculum (Ed.D. Research Proposal Guidance Notes, p.2). It may be viewed as descriptive and explanatory (E835 Study Guide, p.36) in respect of the nature of the use of procedural and conceptual knowledge related to problem solving in two- and three-dimensions carried out by T.D. students. I undertook research into this curriculum issue using a qualitative model similar to that described in detail in the E835 Study Guide (p.66) and in the tradition described in Atkinson et al (1993, pp.16-31). Naturalism (E835 Study Guide, p. 91) is advocated as a key concept where normal classroom activities could proceed (relatively) unaffected in any substantive way by the activities associated with the research process.

In summary, the methodology described above was necessary for a number of reasons. The previous research used to inform T.D. was in the cognitive constructivist tradition and therefore, positivistic. It relied on data generated in experimental settings and statistical analysis. The Literature Review reported in Chapter 2 also questions this type of research and its methods. There are several aspects to my approach which are detailed in the following section. It is constructivist-interpretive, with interpretation of students descriptions and explanations of the activities associated with problem solving forming the basis of the theory of knowledge and its use in T.D.. The research was conducted in the T.D. classroom with data generating techniques designed to be as unobtrusive as practicable. Further, as the teacher-researcher I had access to and knowledge of the setting
not accessible to an outside researcher and while my dual role during the course of the study was different to the singular role I would normally adopt, I argue that the natural dynamics of the classroom were not markedly disturbed.

**My Specific Approach**

The research was conducted in my normal T.D. classroom in the consecutive years of the study. Thus, I researched naturally occurring cases and used continuing analysis of data from year to year to inform my work as it progressed (Kemmis, 1993, pp.177-190). This is similar to action research but I was not adapting practice as a result of my analysis during the study. Relatively small numbers of students were involved as the study progressed and the selection of those who participated may be regarded as an 'opportunity sample' (E835 Study Guide, p. 92) since I made a conscious decision to involve only students to whom I had direct, easy access. I did not wish to disrupt the work of my colleague T.D. teachers. My sampling did not therefore, conform to the optimal 'intentional, systematic and theoretically guided' variety described by Hammersley (1984, p.51) but the group of students I worked with during the Main Study was typical of T.D. classes in my school, which in turn is a typical Voluntary Secondary School. I was conscious that there are issues relating to generalisability which may be associated with using small numbers (Schofield, 1993, pp. 91-113) but was satisfied that the procedures adopted could be readily replicated by other teacher researchers in other classrooms in other institutions where larger numbers take T.D.. In this way I believed it to be credible, transferable, confirmable and dependable in terms of data collection, analysis and interpretation.

Since I intended to use qualitative methods, carrying out an Initial Study, in 1997, allowed me to explore and assess the collection and analysis of data from a number of sources. It also allowed me to sharpen the interviewing and observation techniques which were an essential part of the overall study design. From a practical perspective, it also allowed me to assess my ability to continue to operate as the classroom teacher while engaging in the research process.
Collecting Empirical Materials

The Initial Study was carried out in 1997. My own work and that of three of my colleague T.D. teachers, the work of four of my students, and an architectural draughtsman were reported.

The methods used to collect data were designed to enable data which informed the research questions to be collected in naturalistic settings with minimal disruption of everyday classroom functioning. Data were generated from five sources -

a) an analysis of the knowledge content of the various tasks associated with completing appropriate solutions to Leaving Certificate T.D. questions from two areas of the syllabus. This analysis was based on recording how I, as an expert teacher, completed a 'Planes' question and informed a novice of the tasks that needed to be completed,

b) observations of students followed by interviews. Two students were observed preparing solutions to a 'Planes' question. Two others were observed preparing solutions to a 'Mining' question. General notes were made on the moves they made in preparing their drawn solutions. Following these observations described the students were interviewed. These interviews were audio-recorded and focused on how the solutions were prepared. They were relatively informal. They took place shortly (5-10 minutes) after students had completed the tasks and were conducted during regular T.D. classes. Other students worked on task during the interviews,

c) two students were audio-recorded as they discussed the solution of the 'Mining' problem prior to beginning work on their drawn solutions,

d) three of my colleague T.D. teachers were interviewed informally. The interviews centred on their views of what students found problematic when dealing with the
'Planes' and 'Mining' questions. These brief discussions also focused on examination preparation in these topics and the teachers views of what the examination was assessing, and

e) a drawing office practitioner worked on Leaving Certificate T.D. questions and was subsequently interviewed. This interview focused on the difficulties he encountered in preparing solutions to the 'Planes' and 'Mining' questions addressed by the students.

It was envisaged that further insights into the types, levels and qualities of knowledge discussed by McCormick (1996), deJong & Fergussen-Hessler (1996), Stevenson (1994), Gott (1988), and Schoenfeld (1985) and identified in T.D. related literature would result from data generated by the methods described above. Specifically, (a) was designed to examine expert knowledge associated with the two topics 'Planes' and 'Mining'. This required not only examination of the knowledge associated with the topics but also of how the knowledge identified might be taught. Observing students at work (b) allowed students use of procedures to be recorded. It was felt that interview data and audio-recording of pairs (c) would provide some insights into students' conceptual knowledge and possibly to their metacognition. I felt that this was important since this type of cognition could not be observed directly. It was viewed as a means of gaining access to students cognition in case my questions during interviews might not elicit the types of responses which would indicate these. I felt that these insights might emerge while students were engaged in naturalistic conversations about the nature of solutions to problems. Interviews with teachers (d) were designed to highlight their guiding philosophies and teaching strategies. Examining how a draughtsman (e) would approach questions on 'Planes' and 'Mining' was expected to offer insights into how an outside (school) expert in the domain would approach unfamiliar problems as there is evidence that the type of knowledge used in T.D. in schools (for example when addressing examination questions) is radically different from the knowledge used and valued by experts outside them (Barnes, 1989, p. 75).

Despite the brevity of the Initial Study and the small number of students involved, it generated data which addressed, in some measure,
the research questions as I had intended. In order to address the issues in greater detail I proposed more in-depth interviews with students, recording students as they discussed problems, working with students across a range of abilities on all Leaving Certificate topics over a prolonged period while continuing to interrogate the literature in all relevant areas.

For the Main Study the number of elements involved in the generation of data were reduced with a view to focusing more directly on the areas of procedural, declarative and conceptual knowledge associated with the students problem solving in the T.D. domain. Data were to be generated from -

1. analysis of the remaining twelve topics which are examined in the Leaving Certificate T.D. examination. This analysis of knowledge involved the categorisation of the specific and strategic procedures required to complete appropriate drawn solutions in these topic areas. These procedures formed the basis for analytical categories based on Primary (P.P.), Complementary (C.P.) and Incremental Procedures (I.P.) and Rules of Thumb (R.o.T.). These were formulated with reference to the literature on types of knowledge and are explained in detail at the beginning of Chapter 4.

2. observation of students working on the solution of Leaving Certificate questions from these twelve areas of the syllabus. These observations considered students use of these specific and strategic procedures - i.e. P.P.'s, C.P.'s, and I.P.'s. (some examples are included in Appendix 15.

3. interviews with students following the observations at (2) above. These interviews provided data on students' use of R.o.T.'s, declarative and conceptual knowledge of topics, and their self-regulation and metacognition. It would not have been possible to observe these. Appendix 16 show extracts from examples of these, and

4. audio-recordings of pairs of students discussing the solution of a Leaving Certificate question in one of the twelve topic areas
of the syllabus mentioned at (1) above (Appendix 17). These audio-recordings were intended to offer further insights into students conceptual knowledge, self-regulatory procedures and metacognitive processes since it was intended that these might be verbalised during discussion of the problems.

Analysis of Topics

Analysis of the areas of the syllabus at (1) above allowed definition and explanation of the nature and levels of procedural, declarative and conceptual knowledge associated with tasks students would be required to carry out to be defined and explained. It was intended that analysis of the remaining twelve topics would be informed by earlier experience of analysing topics for the Initial Study. This analysis could be viewed in terms of compiling definitions of knowledge which would allow data generated from observations, interviews, and audio-recordings of pairs of students at work to be categorised in terms of the knowledge content of their solutions. This analysis of unstructured data was designed to be undertaken after a block of observations, interviews and audio-recordings over a four week period were completed.

Observation of Students

Each Monday and Tuesday of the collection period I worked with the class group on the scheduled topics during the first of the double period. During the second period all students worked on the solution of a related Leaving Certificate problem. The organisation of the classroom allowed two students to be observed from a position in the aisle between rows. As they worked on task, I observed two students closely (No.2 above). Data from these observations were recorded on prepared schedules. These schedules were designed so that students behaviour was not pre-categorised rather, they were intended to record the sequence of drawn solutions. They also allowed for brief notes on questions arising from the observations and any general comments on the lesson to be recorded.

This format was a familiar one for my students. During the course of everyday T.D. lessons I regularly adopt it, with most of my time during the second class period being devoted to observing student
...progress on tasks and assisting individuals when required. At the end of these double periods I normally collected the drawn solutions and marked them before the next scheduled T.D. class. I did not use the results as part of the analysis of the solutions. I continued to work in this way throughout the data collection period. Thus, double class periods were not changed radically or disrupted to facilitate data collection.

Student Interviews

The interviews described at (3) above were intended to provide further insights into students thought processes during the construction of their individual solutions. These interviews were intended to be brief (approximately 5 minutes in each case) and had to be completed before the end of the double period to minimise disruption of ensuing classes. During these interviews I questioned the students on their approach to completing their individual solution or problem solving strategy because I was interested in how these could be related to the received and/or knowledge perspectives outlined in Chapter 2.

I was also interested in what students found problematic, because I wanted to see how this corresponded to how the received and knowledge perspectives define problems. I also addressed questions about their solutions which I had noted earlier on the observation schedule. These interviews were audio-recorded. I envisaged that the data produced here would allow insights into how students conceptualise T.D. problems.

Audio-Recording of Pairs of Students

Each Thursday during the study two students were selected on a rotating basis and presented with a Leaving Certificate question to which a drawn solution had to be produced. The combinations of students were altered such that each student worked with all the others during the course of the data gathering process. During the discussion and construction of the solution their conversations were audio-recorded in an attempt to gain insights into the nature of their problem solving, particularly their use of R.o.T.'s, their declarative and conceptual knowledge, the self-regulatory process they employed and their metacognitive functioning. The type of cognition involved was
not observable but I felt that working in pairs would result in the students having to externalise their thinking in order to communicate their impressions of the problem to the other student. I tried to encourage this further by assigning one of the pair the responsibility of drawing the solution while the other was to instruct how this should be done.

Procedures

Two students were to be observed and interviewed during the double class period each Monday and Tuesday during each collection period. Thus, sixteen individual students were observed and interviewed during each four week data collection block.

Each Thursday during the data collection period I selected two students to discuss the solution of a problem. These discussions were audio-recorded (4 above). I did not propose to discuss the contents of the tapes with the students afterwards because of time constraints. Selection of students for this element of the data collection was again to be based on seating arrangements in the T.D. room. I involved all the students in the data collection process so I used a rotating system for observations, interviews and audio-recording of pairs.

In addition to highlighting students' use of procedures these observations, interviews and audio-recordings of pairs at work also afforded an insight into students' declarative knowledge. I also envisaged that examination of the strategies employed in the solution of problems as students work in pairs would provide insights into their metacognition or self-regulation processes because students would be encouraged to externalise these to one another. The data produced here did indeed allow some insights into students self-regulation, as they monitored their progress in terms of identified strategies that produced desired outcomes. This involved students planning, checking, evaluating and revising their strategies.

Analysing and Interpreting the Data

During preliminary analysis carried out during the Initial Study data were analysed in respect of their effectiveness in addressing the research questions. The initial observations allowed the students
construction of solutions to be recorded. They also allowed for improvements to the instrument to be developed in order to effectively record the nuances of constructions during the main study which was to follow. Thus, for example, the observation instrument used in the main study showed an expert solution of the problem on which the sequence and nuances of the student's could be noted.

Similarly, the interviews undertaken during the Initial Study allowed me to sharpen my interviewing technique in order to optimise the quality of data which interviews during the main study would provide. For example, the initial study allowed me to develop open-ended questions which allowed students to offer descriptions and explanations of what they were doing and why they were doing it, rather than some of the closed questions I had asked during the Initial Study.

Analysis of data from the Initial Study also allowed me to make a judgement as to the effectiveness of the different sources and how they might (or might not) be used in the main study.

Specifically, the observation data were to be examined in respect of students use of procedural, declarative and conceptual knowledge. In order to accomplish this I had to develop categories of these procedures (referred to earlier and described in detail in Chapter 4).

Interviews and audio-recordings of students working in pairs were transcribed. The observation schedules and transcripts were then annotated in an effort to identify possible problem solving strategies, and relationships between types of knowledge. This initial coding of the data was revised as the research process progressed (E835 Study Guide, p.171) and as different analytical categories emerged, for example when different levels of procedures were identified across the range of topics examined during the study.

Following these processes I was faced with the task of interpreting an ever increasing mountain of empirical materials. I proceeded to construct my interpretation of the data by searching for themes related to the types of procedural and conceptual knowledge which were emerging in data from the different sources. This interpretation was also an ongoing process, and was specifically designed to be such, with each block of data collection followed by processing and analysis.
the precursor to the construction of my interpretation of that batch of empirical materials. It consisted of my ongoing attempts to make sense of what I had learned during that particular block of data collection, processing and analysis - with the large amounts of data finally being organised into themes which were mutually related (E835 Study Guide, p.171). The final cumulative interpretation of the data was written up as a grounded theory of problem solving in T.D. which is based on students' use of knowledge in constructing solutions to T.D. problems.

Role of Researcher

Stenhouse (1975, pp.142-165) has described the teacher researcher as an extended professional possessing all the attributes of a good teacher but having additional skills, perspectives and involvement's. Among the many attributes he describes the desire to question and test theory using skills which have been developed to interrogate one's own teaching. This is achieved through systematic self-study, the study of the work of other teachers and through the testing of ideas by classroom research procedures (E835 Offprints Reader, p.32). My focus during this study was not on my own performance but on testing ideas using classroom research.

Hammersley would argue against such a role for the teacher for two main reasons; firstly, he argues that the collective teacher mind has little time to reflect on practice, never mind engage in systematic research; and secondly, government policy on the role of the teacher is at odds with the concept of teacher researcher (E812 Study Guide, p.38). He also argues that the general and specific arguments against conventional educational research (viz a viz its irrelevance, invalidity and undemocratic nature) and in favour of teacher research are not convincing and that the teacher researcher role is undesirable both from the teaching and research viewpoints (Hammersley, 1993, pp. 246-264). While recognising the importance of Hammersley's views in this regard, I would argue that a T.D. teacher researcher possessing the characteristics of the extended professional described by Stenhouse (1975), as a major actor in the classroom situation, is better equipped to conduct research in the T.D. classroom despite the difficulties associated with carrying out the dual role adequately.
The research reported here will have to satisfy the demands for systematics, rigour and explicitness (Hammersley, 1986, p. 255) demanded by both T.D. teachers and the research community. If it succeeds in satisfying both groups, who are routinely sceptical for their own reasons, it can be argued that it will have avoided the construction of a false consciousness which might otherwise have developed as a result of inadequacies of the teacher researcher who conducted it.

**Ethical Issues**

Before the research could begin I had to negotiate access to the students with the school authorities at the beginning of the Initial Study and again at the beginning of the 1998/1999 school year for the Main Study. This negotiation process was also undertaken September for the 1999/2000 school year. The school Principal at the beginning of the research process was very supportive and permission to carry out the study in the school was readily given. I also had to negotiate with the students themselves (and their parents) since I felt it was not sufficient to secure institutional consent for the research to go ahead. Therefore, I entered negotiations with the students and their parents. I explained the nature and general aims of the proposed study to both groups and I emphasised the distinction between data I proposed to collect for research purposes and work that would normally be corrected in preparation for T.D. examination purposes. I felt that this aspect of the negotiation was extremely important since the experience of Irish Senior Cycle students centres around their preparation for the Leaving Certificate Examination. The students who took part in the main study sat this examination in June, 1999 and for that reason I felt that the classroom work required to prepare them for their main state examination should not be disrupted by the procedures I had devised to collect data, for my purposes. Consequently (as described earlier) the lesson format used relates closely to that employed during the normal course of T.D. classes at this stage of the Leaving Certificate programme. However, my observations did mean that I was not freely available to offer immediate assistance to students working on tasks who may have required it - something which concerned me throughout the data collection process. I tried to compensate for this deficiency by asking students to note difficulties they experienced on their drawings (which
I collected after each lesson). I noted in my journal any difficulties students described in this way and addressed them during the Thursday class each week of the study.

During my negotiations with students and their parents at the outset of the main study I also received authorisation to use quotations from transcripts of interviews and audio-recordings in the compilation of my various reports. I also agreed to make the various drafts of the final report of the main study available to any student or parent who wished to view it but I retained the right to include my own interpretation of the data produced during the study (E621, p.10).

At the end of the data generation and analysis process I also negotiated permission to access and use the Leaving Certificate Examination results of the students’ who were part of the study in order to assess the significance of the differences between groupings of students I had formulated during the data analysis process.

Much of the data generated in this study came from observations of students at work and interviews with them. Therefore, I felt a great responsibility to develop my ability to observe carefully and listen attentively to what the data are saying. I believe this was one of the main challenges I faced in order to adequately perform my main roles as the researcher in the classroom. I had been developing these skills over time (during the M.A. modules) and was conscious that they were constantly in need of fine-tuning (E835 Study Guide, p.90.) The longevity of the study, together with the frequency of observations and interviews helped greatly in this regard.

Summary

It was difficult to categorise the research undertaken here since it did not fit easily into any one qualitative tradition - such as those described by Atkinson et al (1993, pp.16-31). I regarded it as falling within the enlightenment model, since it was designed to provide new insights into problem solving in T.D. from a student perspective. My interpretation of the data may serve to influence practitioners in T.D. classrooms and to shape their orientation to practice by suggesting alternative descriptions and theoretical frameworks within which to
locate their everyday encounters with students (E835 Study Guide, p27).

Every effort was made during the study to provide as objective an account (Phillips, 1993, p.66) as was possible. The veracity of the descriptions of how data were collected, analysed and interpreted is therefore, significant. There may be no consensus as to how, precisely, educational research should be judged but the extent to which my account represents the phenomena to which it addresses itself may address matters of validity. My interpretation of the data is based on my presuppositions, and my account of the research should therefore, be read with these in mind. My methodological stance suggests that there is no absolute proof of my interpretations. However, the reader can assess the validity of the study using the criteria of plausibility and credibility outlined in the E835 Study Guide (p.34). The topic researched contributes to existing knowledge by examining the nature, use and inter-relationship of knowledge in T.D. problem solving - something not previously done. It may therefore, be regarded as relevant since it was intended to begin to develop a theory of problem solving for T.D. based on naturalistic classroom activities, thus, making a significant contribution to our understanding of T.D. as a post-primary subject. Once a knowledge view of problem solving was developed a comparison could be made with the current view. Thus, possible prescriptions for teaching problem solving (based on empirical research) and not a 'fitted' theory could be advanced. The comparison of the two theories could also raise the issue of how problem solving could or should be assessed for accreditation purposes. Thus, the criteria for relevance were addressed in the Main Study.
Chapter 4

Findings & Discussion

Introduction

The findings reported here are based on the application of my analysis of the knowledge required to solve typical Leaving Certificate T.D. examination problems to how the students in the study solved these problems. All the topics which appear on typical Leaving Certificate T.D. examination papers (and their constituent parts) were analysed in these terms. This involved relating the literature on problem solving and knowledge to the tasks students are typically required to complete and to data generated from observations, interviews, and audio-recordings (detailed in Chapter 3) of students preparing drawn solutions. These solutions could then be described and explained in terms of knowledge rather than graphicacy.

In the following sections I will discuss problem solving in terms of the procedural and conceptual knowledge used in the construction of solutions for the topics discussed. This discussion provides an alternative perspective on problem solving to the received graphicacy view - one based on the interpretation of empirical data generated in a normal T.D. classroom. As part of this overall discussion I discuss

* how the problems students encounter may be related to knowledge, rather than to aspects of graphicacy,

* how I grouped students into groups because of the common characteristics they displayed in terms of their knowledge use while problem solving,

* self-regulation, visualisation and differences between individuals from the groups, and

* a discussion of the findings of this study in terms of how problem solving is related to the graphicacy and knowledge perspectives.
Problem Solving

As a T.D. teacher and expert problem solver in the T.D. classroom I bring my considerable domain-specific knowledge to bear on every new T.D. problem I encounter. My approach to solving problems is not a general one - rather it is topic-specific. However, there is a certain amount of overlap, since topics in the Building Applications section of the programme are just that - applications of Plane & Solid Geometry. Because of my position as a T.D. teacher I also have a professional interest in T.D. problem solving (in and out of the classroom) which makes it meaningful and relevant for me. My T.D. teacher colleagues share this interest and if (when) faced with a particular problematic situation we are in a position to work collaboratively to arrive at an appropriate solution. The post-primary T.D. students who took part in this study did not approach problem solving in the same way that I (or my colleagues) do. One of the main reasons for this was that their T.D.-specific knowledge was much more limited. This was chiefly due to their relative lack of experience. Thus, unlike the expert T.D. problem solver, the majority of students in this study brought only a limited knowledge of the topics to the situation in which they were trying to solve problems. Glaser's thesis (1984, p.98) that being able to distinguish the characteristics and organisation of their (limited) knowledge will lead to a better understanding how they become more expert problem solvers is appropriate here. A two-year programme in Leaving Certificate T.D. lasts for approximately 72 weeks. Students may spend as little as 5 weeks studying each of the 14 topics which will be examined. Understandably, their knowledge of each topic is limited. However, like the expert T.D. problem solver, the students in the study also appeared to make extensive use of their knowledge in each topic area, even though their knowledge was limited. One major difference was that despite the intended (and real) application of Plane & Solid Geometry in Building Applications students did not appear to recognise the applied Geometry but rather, treated Building topics as separate problems.

Although students are goal-directed the literature would suggest that students' goals are slightly different from the (T.D.) expert. For example, students interest in solving T.D. problems is primarily directed at achieving a pass grade in the Leaving Certificate
Examination. They may not (necessarily), have the same interest in problem solving per se as a T.D. professional, in or out of school, would have.

The problems which students choose to address in the Leaving Certificate T.D. examination from a given selection indicates both the topics which students prefer to engage with and topics they find problematic - because they avoid them. The most popular topic areas are used for illustration here. These tell us a lot about what type of problem solving students like (choose) to engage in but there are aspects of these preferred topics that students often find problematic. The preferred problems tend to be well-structured, vary little from year to year, have strong procedural elements with only sub-sections which require the use of conceptual knowledge. Alternatively, the problems students avoid (especially in the Leaving Certificate Examination, the results of which may shape their future lives in many respects) tend to be less well-structured, are subject to change from year to year, and have relatively large sections which require the use of conceptual knowledge prior to executing relatively straightforward procedures.

Selection of Topics Used For Illustration

Students are examined in 14 topics at Leaving Certificate level (seven Plane & Solid Geometry and seven Building Applications). All 14 were analysed in the terms described above during this research project but only four from each area of the syllabus are reported in this chapter. These are detailed in Table 4.1. below.

Table 4.1 - Leaving Certificate Questions Used For Illustration

<table>
<thead>
<tr>
<th>Paper 1 - Plane &amp; Solid Geometry</th>
<th>Paper 2(b) - Building Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intersecting Planes &amp; Skew Lines</td>
<td>Perspective Projection</td>
</tr>
<tr>
<td>Solids in Contact</td>
<td>Hyperbolic Paraboloid Roofs</td>
</tr>
<tr>
<td>Interpenetrations</td>
<td>Mining Applications</td>
</tr>
<tr>
<td>Conic Sections</td>
<td>Roadworks Geometry</td>
</tr>
</tbody>
</table>

I have chosen to present my findings in this way for two main reasons. Firstly, according to the most recently published Chief T.D.
Examiner's Report (1999, pp. 1 - 26.) questions related to these topics (with the exception of Conic Sections) are the most frequently and successfully answered in the Leaving Certificate T.D. examination. As such, they afford examples of both popular and successful (since a relatively small percentage fail the examination) problem solving. This popularity and success was also mirrored in the choices made by the students during this study, especially in their Trial Leaving Certificate Examination held in March, 1999. In this trial examination, Conic Sections proved to be the fourth most popular question from Paper 1, hence its inclusion here instead of Special Curves, the fourth most popular question from Paper 1 according to the Chief Examiner's Report (1999, p.4). Secondly, the questions chosen reflect both the proportion of two-dimensional to three-dimensional topics examined and attempted by students. Three of the seven questions on Paper 1 relate to two-dimensional topics - Areas of Figures, Special Curves and Conic Sections (although Conic Sections may also be viewed as a three-dimensional topic). Areas of Figures almost invariably proves to be the least popular question from Paper 1 while the popularity of Special Curves and Conic Sections is as described above.

Relating Problems to Knowledge

Leaving Certificate questions are generally sub-divided into tasks, similar to those described by Barnes (1989) and discussed in Chapter 2, which are outlined in the following sections. The types of procedures which the students must complete when addressing these tasks are also discussed and may be regarded as similar to the first two orders described by Stevenson (1994), where both familiar and unfamiliar goals can be achieved by the application of specific procedures.

Different topics require the completion of primary, complementary and incremental procedures of different levels of complexity. These are also described in the following sections where the levels of P.P., C.P. and I.P.'s associated with the topics which are used for illustration are detailed.

A description of the tasks associated with the topics used for illustration is included in Table 4.2. below.
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Intersecting Planes &amp; Skew Lines</strong></td>
<td><em>Perspective Projection</em></td>
</tr>
<tr>
<td><strong>Task 1:</strong> Draw projections of planes from co-ordinates.</td>
<td><strong>a)</strong></td>
</tr>
<tr>
<td><strong>Task 2:</strong> Determine the line of intersection.</td>
<td><strong>Task 1:</strong> Reproduce given plan.</td>
</tr>
<tr>
<td><strong>b)</strong></td>
<td><strong>Task 2:</strong> Draw the set-up for the perspective view.</td>
</tr>
<tr>
<td><strong>Task 3:</strong> Determine the dihedral angle.</td>
<td><strong>Task 3:</strong> Locate the auxiliary vanishing points.</td>
</tr>
<tr>
<td><strong>c)</strong></td>
<td><strong>Task 4:</strong> Locate starting position for perspective view.</td>
</tr>
<tr>
<td><strong>Task 4:</strong> Draw projections of required line.</td>
<td><strong>Task 5:</strong> Complete perspective view.</td>
</tr>
<tr>
<td><strong>d)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Task 5:</strong> Draw skew lines from co-ordinates.</td>
<td></td>
</tr>
<tr>
<td><strong>Task 6:</strong> Find the shortest distance.</td>
<td></td>
</tr>
<tr>
<td><strong>Solids in Contact</strong></td>
<td><strong>Hyperbolic Paraboloid Roofs</strong></td>
</tr>
<tr>
<td><strong>a)</strong></td>
<td><strong>a)</strong></td>
</tr>
<tr>
<td><strong>Task 1:</strong> Reproduce given diagram and complete the projections.</td>
<td><strong>Task 1:</strong> Draw the given directrices.</td>
</tr>
<tr>
<td><strong>Task 2:</strong> Draw given sphere in secondary position before moving into required position</td>
<td><strong>Task 2:</strong> Draw the projections of required number of elements.</td>
</tr>
<tr>
<td><strong>b)</strong></td>
<td><strong>Task 3:</strong> Determine the plane director.</td>
</tr>
<tr>
<td><strong>Task 3:</strong> Determine the radius of second sphere before moving into position.</td>
<td><strong>Task 4:</strong> Draw the traces of the plane director in the required position.</td>
</tr>
<tr>
<td><strong>c)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Task 4:</strong> Draw the projections of an enveloping cone.</td>
<td></td>
</tr>
<tr>
<td><strong>Task 5:</strong> Draw the traces of the required tangent plane.</td>
<td></td>
</tr>
<tr>
<td><strong>Interpenetrations</strong></td>
<td><strong>Mining Applications</strong></td>
</tr>
<tr>
<td><strong>a)</strong></td>
<td><strong>a)</strong></td>
</tr>
<tr>
<td><strong>Task 1:</strong> Draw the projections of the given solid.</td>
<td><strong>Task 1:</strong> Draw the boreholes from specifications.</td>
</tr>
<tr>
<td><strong>Task 2:</strong> Draw the set-up for the interpenetrating prism.</td>
<td><strong>Task 2:</strong> Determine the top and bottom of the stratum.</td>
</tr>
<tr>
<td><strong>Task 3:</strong> Complete the projections of the interpenetrating prism.</td>
<td><strong>Task 3:</strong> Draw triangular plane.</td>
</tr>
<tr>
<td><strong>Task 4:</strong> Draw the required auxiliary plan.</td>
<td><strong>Task 4:</strong> Determine strike, dip and thickness.</td>
</tr>
<tr>
<td><strong>Task 5:</strong> Determine entry/exit points of prisms edges and positions where prism surfaces cross edges of second solid.</td>
<td><strong>b)</strong></td>
</tr>
<tr>
<td><strong>Task 6:</strong> Complete projections of lines of interpenetration.</td>
<td><strong>Task 5:</strong> Draw the initial projections for extra borehole.</td>
</tr>
<tr>
<td><strong>Conic Sections</strong></td>
<td><strong>Task 6:</strong> Address the requirements for this borehole.</td>
</tr>
<tr>
<td><strong>a)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Task 1:</strong> Produce initial diagram.</td>
<td><strong>Roadworks Geometry</strong></td>
</tr>
<tr>
<td><strong>Task 2:</strong> Draw the required curve (usually an Ellipse).</td>
<td><strong>a)</strong></td>
</tr>
<tr>
<td><strong>b)</strong></td>
<td><strong>Task 1:</strong> Determine the works required for level road.</td>
</tr>
<tr>
<td><strong>Task 3:</strong> Produce the initial diagram.</td>
<td><strong>Task 2:</strong> Determine fill for sloping road.</td>
</tr>
<tr>
<td></td>
<td><strong>Task 3:</strong> Determine cut for sloping road.</td>
</tr>
<tr>
<td></td>
<td><strong>Task 4:</strong> Determine works for parking area or other extra works.</td>
</tr>
</tbody>
</table>
A discussion of the types of knowledge required to address problems in the topics referred to are described in the following section. This may be regarded as a commentary on the creation of analytical categories.

**Procedural Knowledge**

Typical drawn solutions to T.D. problems are completed in a number of stages which require the identification and completion of a number of procedures guided by the application of rules of thumb. The number of procedural elements and rules of thumb are topic-specific. Apart from its role in allowing the drawing of a solution to be constructed procedural knowledge also serves to link declarative and conceptual knowledge. A brief description of the types of procedures identified is given in Table 4.3. below. Procedural knowledge is then discussed in detail in the following sections.

Table 4.3. - Types of Procedures

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>The completion of the initial drawing from a given diagram or written specifications. In most instances it involves the completion of the projections (elevation and plan) of a 3-D configuration. When dealing with 2-D topics, for example Areas of Figures, it involves the completion of the rudiments of the drawing required for an appropriate solution.</td>
</tr>
<tr>
<td>Complementary</td>
<td>These are required to advance the construction of an appropriate solution and are completed following the completion of a P.P.. However, they may also be used following the completion of an I.P. (below) in order to address an aspect of a solution not previously addressed by a P.P. or I.P..</td>
</tr>
<tr>
<td>Incremental</td>
<td>Usually involves the construction of successive auxiliary views or staged constructions. May be drawn directly after a P.P. or following a C.P.. I.P.'s may also include working backwards from 2nd auxiliary views to the original projections.</td>
</tr>
</tbody>
</table>
With the exception of Roadworks Geometry where a contour map to a given scale is provided, all solutions must be drawn from scratch. Drawn solutions are begun by completing an initial diagram that provides the basis from which the overall solution can be constructed. I have called the completion of this part of the solution a Primary Procedure (P.P.). The complexity of this initial diagram varies with the topic and therefore there are different levels of P.P.. These range, for example, from the transformation of written specifications into diagrammatic form for questions on Conic Sections (P.P. - Level 1) shown in Figure 4.1. to drawing the projections (elevation and plan) of a geometric solid (P.P. - Level 4) for questions on Interpenetrations shown in Figure 4.2. below.

Thus, P.P.'s range from the completion of simple diagrams involving the construction of simple plane figures to the completion of the projections of solids. However, all are consistent with descriptions in the literature of procedures which allow for the completion of specific aspects of tasks associated with a required solution. Solutions in all topics require the completion of at least one P.P. while a number of P.P.'s may be required (for example the question on Intersecting Planes & Skew Lines requires separate P.P.'s for the planes and skew lines elements. Once completed, the solution can only be advanced by completing further more complex procedures. These I have called Complementary (C.P.) and Incremental Procedures (I.P.).

Similar to P.P.'s, every topic examined requires the completion of at least one Complementary Procedure (C.P.). These constitute the next procedural stage in the completion of a solution, usually following on directly from the P.P.'s. In effect, this means that each C.P. (and again there are several levels identified) can be directly related to an initial procedure, a P.P., which precedes it. In cases where three-dimensional topics are being addressed C.P.'s usually involve the construction of a first auxiliary view, i.e. a view other than one which is perpendicular to either the Horizontal Plane (a plan) or the Vertical Plane (an elevation). When the line of intersection between two Intersecting Planes is not given, for example, an auxiliary elevation (or plan) is usually drawn in order to locate it. Alternatively, two horizontal lines may be used (both procedures are C.P.'s - Level 2). This allows the line of intersection to be identified (see Figure 4.3 below). Thus, the view (C.P.) can follow on directly from a P.P.
Figure 4.1. - Primary Procedure (P.P. - Level 1) Conic Sections

Draw a triangle $FPF_1$ where $FP = 130\text{mm}$, $PF_1 = 70\text{mm}$ and $FF_1 = 100\text{mm}$. $F$ and $F_1$ are the foci of a double hyperbola and $P$ is a point on the curve.

(i) Draw a portion of the double hyperbola.

(ii) Determine the asymptotes to the curve.

Primary Procedure (P.P. - Level 1).
A regular pentagonal right pyramid has a side of base 70 mm and an altitude of 120 mm and rests with one of its triangular faces on the horizontal plane as shown. Also shown are the projections of a square prism of 40 mm side which penetrates the pyramid.

Draw the projections of the solids showing all lines of interpenetration.

Used to construct an initial diagram. However, C.P.'s are also used during the latter stages of the construction of a solution in order to address an aspect of a solution that have not previously been dealt with. For example, Task 5 in questions on Conic Sections usually requires the completion of a tangent and the location of the centre of curvature (C.P. Level 3). This is the final part of the solution (shown in Figure 4.4 below) and requires the use of a construction which
To determine the dihedral angle between the planes ABC and ADE.

Complementary Procedure (C.P. - Level 2).

relates back to previous procedures, but only indirectly to the initial P.P.. Although C.P.'s contribute significantly to the completion of solutions they are not sufficiently complex to address all aspects of a solution. Incremental Procedures (I.P.) are required to do this.
To determine the tangent and centre of curvature at P.

Incremental Procedures (I.P.'s) of different levels must be used to complete all questions. In all the three-dimensional topic areas except Perspective Projection, where solutions are drawn in Orthographic and Perspective Projection, these I.P.'s usually involve the construction of successive auxiliary views. That is, the I.P. is completed only after two successive steps - a first auxiliary view followed by a second projected from the first. Determining the dihedral angle (Task 3), between the Intersecting Planes, is an example of such an I.P..
To determine the projections of a cone at B such that it is in contact with the cone A and the sphere C.

Alternatively, in the cases of Solids in Contact and Projection of Solids (Figure 4.5. above) for example, I.P.'s may require the positioning and movement of 3-D geometric solids instead of the consecutive 2-D auxiliary views described above. The solutions of...
Task 3 in Solids in Contact questions require the completion of such an I.P. (Level 3). A sphere must be drawn (using a locus to determine its radius) in a known position before being moved into a required position. I.P.’s used in Perspective Projection involve the complex inter-relating of the Orthographic Projection of the three-dimensional object being drawn with the set-up for the Perspective Projection. This results in the staged construction of the overall drawn solution (see Figure 4.6. above).
Table 4.4. - Levels of Procedures Associated with Tasks.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary</strong></td>
<td>Tasks 1 &amp; 5 Level 1</td>
<td>Task 1 Level 2/3</td>
<td>Task 1 Level 3/4</td>
<td>Task 1 &amp; 3 Level 1</td>
</tr>
<tr>
<td><strong>Complementary</strong></td>
<td>Task 2 Level 2 Task 4 Level 2</td>
<td>Task 4 Level 3 Task 5 Level 4</td>
<td>Task 2 Level 2 Task 5 Level 3</td>
<td></td>
</tr>
<tr>
<td><strong>Incremental</strong></td>
<td>Task 3 Level 1 Task 6 Level 2</td>
<td>Task 2 Level 2 Task 3 Level 3</td>
<td>Task 6 Level 3</td>
<td>Task 4 Level 3</td>
</tr>
<tr>
<td><strong>Building Applications</strong></td>
<td>Perspective Projection Hyperbolic Mining Applications</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Primary</strong></td>
<td>Task 1 Level 2/3</td>
<td>Task 1 Level 2</td>
<td>Task 1 Level 3</td>
<td>Task 1 Level 1</td>
</tr>
<tr>
<td><strong>Complementary</strong></td>
<td>Task 2 Level 3 Task 3 Level 2 Task 4 Level 4</td>
<td>Task 2 Level 3 Task 4 Level 4</td>
<td>Task 2 Level 3 Task 5 Level 4</td>
<td></td>
</tr>
<tr>
<td><strong>Incremental</strong></td>
<td>Task 5 Level 4</td>
<td>Task 3 Level 3</td>
<td>Task 6 Level 3</td>
<td>Task 2 Level 3 Task 3 Level 3 Task 4 Level 3</td>
</tr>
</tbody>
</table>

The I.P.'s used in the completion of solutions to two-dimensional topics may also involve movement and the overall solutions are invariably constructed in stages. Thus, complex movements are completed in the cases of questions relating to Special Curves or complex constructions are completed in stages for Areas of Figures questions. In the two-dimensional questions discussed here (Conic Sections) I.P.'s usually involve the completion a portion of a curve (almost invariably an Hyperbola) following the completion of an initial diagram from written specifications (a P.P.).

A brief summary of all levels of procedures required to complete the tasks in each topic area is given in Table 4.4. above.

William P. Bolger - M7137754
Table 4.5. - Rules of Thumb Associated with Topics.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rules of Thumb</strong></td>
<td><strong>Over-up-Down to draw planes/lines.</strong>&lt;br&gt;Use an edge view of one plane + a level line when line of intersection must be determined. 2 auxiliaries needed for dihedral angle - 1st to show true length, 2nd to show point view of line of intersection. Use the “Planes Method” when “horizontal” distance between skew lines is required.</td>
<td><strong>Draw projections of one given solid.</strong>&lt;br&gt;Use ‘secondary positions’ to place given, sphere. Use loci to determine radius of 2nd sphere. Use enveloping cones to determine H.T. of tangent planes. Use the projections of a known point to determine the V.T.</td>
<td><strong>Questions are based on eccentricity and other major properties.</strong>&lt;br&gt;<strong>Ellipse</strong> - Focal distances = major axis and eccentricity, less than 1. <strong>Parabola</strong> - Locus of a point equidistant from a line and a point. Ecc. = 1. <strong>Hyperbola</strong> - Difference of focal distances = major axis. Ecc. greater than 1.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rules of Thumb</strong></td>
<td><strong>Start with the spectator,</strong>&lt;br&gt;<strong>Draw the given plan,</strong>&lt;br&gt;<strong>Draw the elevation only if necessary.</strong>&lt;br&gt;<strong>Complete the set-up,</strong>&lt;br&gt;<strong>Including auxiliary vanishing points.</strong>&lt;br&gt;<strong>Start the perspective where the picture plane is in contact with the building/object being drawn.</strong></td>
<td><strong>I.L.P. roofs are quadrilaterals.</strong>&lt;br&gt;Pairs of directrices are opposite each other. Elements are drawn between pairs of directrices. Elements are equally spaced along the directrix. For plane director, construct a triangular plane attached to one outer element. Use lines parallel in I.L.P. and V.P. to determine direction of traces.</td>
<td><strong>Determine top and bottom line on stratum.</strong>&lt;br&gt;<strong>Construct a triangular plane attached to top/bottom line.</strong>&lt;br&gt;<strong>Use a level line on the triangular plane to determine direction of auxiliary which determines dip and thickness.</strong>&lt;br&gt;<strong>For extra borehole work back from the auxiliary.</strong></td>
<td><strong>Draw works for level roads first.</strong>&lt;br&gt;<strong>Use a run which will give a difference in level equal to the vertical interval on the map.</strong>&lt;br&gt;<strong>For fill, start at the bottom of the gradient.</strong>&lt;br&gt;<strong>For cut, start at the top of the gradient.</strong>&lt;br&gt;<strong>Use different coloured pencils for cut and fill.</strong></td>
</tr>
</tbody>
</table>

All questions also require a knowledge of Rules of Thumb (R.o.T.’s). These are simple guiding rules which indicate which procedures are appropriate in given circumstances - they are topic-specific. R.o.T.’s for the topics covered in the text are outlined in Table 4.5. above. In the case of Hyperbolic Paraboloid Roofs for example, students must remember that roof outlines are quadrilateral - something which may not be immediately obvious from a given diagram where adjacent directrices may appear to be in line. Similarly, pairs of directrices (of which each roof has two) are always opposite each other. In order to be opposites they may not be adjacent. This is important because the elements run between opposite directrices.
Conceptual Knowledge

Declarative Knowledge (including R.o.T.'s) in the form of facts, constructions and rules usually (though not always) topic-specific is also required to complete appropriate solutions. That plane figures have area and not volume is an example of a geometric fact; examples of R.o.T.'s are given above; and the bisection of a line is an example of a construction. I have called this knowledge the students Geometric Database (G.D.). These resources may be viewed as being on the lower end of a continuum with Conceptual Knowledge at the upper end. As discussed in Chapter 2, Conceptual knowledge requires the relationships between the items of knowledge necessary for the completion of an appropriate solution to be identified. For example, the completion of a solution of a Conic Sections question requires the linking of various items of knowledge in the construction of an appropriate solution.

Question

Draw a triangle FPF₁, where F, P and F₁ are the vertices of the triangle, and the sides FP, PF₁ and FF₁ are 130 mm, 70 mm and 110 mm long respectively. F and F₁ are the foci of a double Hyperbola and P is a point on the curve.

Draw a portion of the double Hyperbola.

In order to translate the written information into a diagram (a P.P. Level 1) the student must know how to draw a triangle given the lengths of its sides, a construction from their Geometric Database. A Rule of Thumb relating to the major properties of the Hyperbola (the difference between the focal distances is equal to the major axis) indicates that the length of the major axis of the conic is 60 mm, since the distance from one focus to a point on the curve subtracted from the distance from the same point to the second focus (FP - PF₁) equals 60 mm. This rule extends to the difference between focal distances for any point on either branch of the curve.

Further facts about the Hyperbola must also be known before the solution can be advanced. For example, students must know that the major axis is measured (half way) on either side of the mid-point of the line FF₁ (between the focal points) and that the curve is
symmetrical before the solution can be advanced. The I.P. used to complete the two branches of the curve links the items of knowledge described above with a procedure which enables points to be located that are positioned in relation to the focal points, such that the differences between the focal distances is constant (the length of the major axis), while constructing both branches (both sides of the axis) simultaneously, emphasising the symmetrical nature of the double curve. Thus facts, rules and constructions (declarative knowledge) are linked with procedural knowledge in the formulation of a solution.

Although each of the types of knowledge (procedural, declarative and conceptual) described above have been written about separately in the literature on knowledge this study appears to support the Hiebert and Lefevre's (1986) thesis that the types are linked. Their separation here would be a false one, since procedural aspects of solutions have embedded declarative and conceptual knowledge while conceptual elements encompass declarative and procedural elements. The types of knowledge are linked in that they are different aspects of the same thing.

When student data relating to the use of the types of knowledge described above were analysed I found that similarities between some students work allowed for the grouping together of several individuals. These groups are described in the next section.

Groups Derived From Analysis Of Observation, Interview And Audio-Recorded Data.

Following analysis of the data generated from observations, interviews and audio-recordings in the manner described above the eighteen students who took part in the Main Study were divided into four groups because of the similarities encountered in the data relating to these individuals.

The similarities in the data related to students' use of procedural, declarative and conceptual knowledge. Table 4.6. below lists the students allocated to each group while the sections which follow include summaries of data for individuals within groups. It should also be noted that there is a degree of overlap here and that there are
similarities as well as differences between the students who I have positioned within particular groups.

Table 4.6. - Groups Derived From Analysis of Data.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carl</td>
<td>Pat</td>
<td>Micheál</td>
<td>Kenneth</td>
</tr>
<tr>
<td>Gary</td>
<td>David</td>
<td>Stephen</td>
<td>Cathal</td>
</tr>
<tr>
<td>Páraic</td>
<td>Michael</td>
<td>Martin</td>
<td>Enda</td>
</tr>
<tr>
<td>Oliver</td>
<td>Justin</td>
<td>Denis</td>
<td>Trevor</td>
</tr>
<tr>
<td></td>
<td>Derek</td>
<td>Brian</td>
<td></td>
</tr>
</tbody>
</table>

**Group 1**

This group was composed of students who consistently completed all tasks associated with all questions from Papers 1 & 2(b). They demonstrated their knowledge of primary, complementary and incremental procedures and all rules of thumb necessary to guide the completion of appropriate solutions. They also completed all levels of procedures consistently and appropriately. Their comprehensive knowledge of the concepts associated with all topics was evidenced in both their drawn solutions, verbal explanations and audio-recordings, where the appropriate linkages between procedures and knowledge from their extensive geometric databases were evidenced. Their draughting skills were also very well developed, allowing the completion of drawings to acceptable conventional standards within acceptable time limits (it is worth noting that Paraic was the best draughtsman from Group 1). Thus, these students displayed comprehensive knowledge of all topics and possessed the necessary draughting skills which allowed this knowledge to be translated into appropriate drawings in the time allocated. Consequently, they scored very highly in T.D. examinations during the study and also in their Leaving Certificate T.D. examination (Carl A1, Gary A1, Paraic B2, and Oliver B2).

The data generated during this study reflects slight variations in the extent of Paraic and Oliver's knowledge relative to Carl and Gary's. Whereas Carl and Gary, during observations, always completed solutions with apparent ease and were able to explain why and how these solutions were completed during interviews, Paraic and Oliver...
sometimes found that tasks which required the use of conceptual knowledge sometimes proved problematic (summaries of data for this group are included at the end of this section). Paraic, for example, sometimes had difficulty competing Task 4 (I.P. Level 4) of Conic Sections questions. Similarly, Oliver sometimes encountered difficulties with Task 6 (I.P. Level 3) in Mining Applications questions where items of knowledge need to be linked to determine an appropriate solution. Carl described the difficulties encountered when facing the final part of Mining Application problems -

Finding the strike, dip and thickness is not a problem...
It's the last part of the question that can be difficult...
You have to do something with another borehole...
It's not really what has to be done that's difficult -
it's how to do it on the drawing....

Carl is referring to conceptual knowledge, the links which must be made between items of declarative and procedural knowledge relating to the stratum (based on the lines on its top and bottom, how altitudes are measured from a common datum, how the stratum and altitudes can be related to another skew borehole, and how all these may be related to each other on the drawing) in order to address the particular problem posed in the question.

Similarly, questions on Intersecting Planes always include a task which requires that students use conceptual knowledge. This part of the problem is invariably addressed last, to allow time to consider possible solutions while other aspects of the solution of the problem are completed. Gary describes this process -

I do the dihedral angle first, once I have the line of intersection. Then I do the skew lines part...
While I'm doing these I can look at the drawings and think about what's needed to solve the extra bit...
It usually doesn't take long to draw the extra bit once you know what needs to be done....

All higher level T.D. problems include such a task. Its successful completion requires that students link items of their declarative and procedural knowledge into an appropriate conception of the problem,
leading to relevant procedures which will allow a solution to be drawn. During this study Carl and Gary were very successful in making these links in all topic areas while Paraic and Oiver sometimes ran into difficulties in a number of topic areas. These included Solids in Contact, Conic Sections and Mining Applications. Paraic describes what is generally required and the parts of problems he has difficulty with -

You know with Higher-Level questions that there'll be a certain amount that you can do, because they always ask you to do the same kind of things. But to do well in the exams you have to be able to finish the other parts of the questions as well. You never know with these extra parts what you'll be asked to do, so you need to be well up on everything to do with the question......Sometimes I can't do these bits...

In order to complete all the tasks students must also demonstrate very high levels of draughtsmanship to complete solutions within given time limits (approximately 35 minutes per question). Completing solutions within the time allocated is difficult and students complain about the amount of drawing needed for certain questions and how this militates against them in examination conditions, since much of the drawing is mechanical and lower level. Carl and Oliver describe these difficulties.

Carl I like perspective questions - the solutions are usually very straightforward - but getting finished inside 35 minutes is often a bit of a struggle.

Oliver The problem with the question on planes and skew lines is that there's so much to do. You have to draw the views, find the dihedral angle (another two auxiliaries), draw the skew lines, find the shortest distance between them, and do something else, all within 35 minutes. It's much easier to do the interpenetration (even though I don't like it so much) because you only have three views to draw.

This has important implications for choice of questions in examinations in terms of what is being tested. Summaries of the data from observations and interviews for this group are detailed in Tables 4.7. and 4.8. below.
Table 4.7. - Summary of Observation Data for Group 1.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carl</td>
<td>Consistently completed all procedures. Task 4 (C.P.) completed last.</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gary</td>
<td>Consistently completed all procedures. Task 4 (C.P.) completed last.</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Perspective Projection</th>
<th>Hypothetical Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carl</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gary</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pávlic</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td>Occasionally had difficulty completing Task 6 (I.P.).</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
</tr>
<tr>
<td>Oliver</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td>Occasionally had difficulty completing Task 6 (I.P.).</td>
<td>Consistently completed all procedures. Tasks always completed in order.</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.8 - Summary of Interview Data for Group 1.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Carl</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gary</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Piaraic</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 3 (I.P.) related to setting up loci of possible centres of new sphere (conceptual).</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 4 related to identifying properties of conic to which question related.</td>
</tr>
<tr>
<td><strong>Oliver</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 3 (I.P.) related to setting up loci of possible centres of new sphere (conceptual).</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 4 related to identifying properties of conic to which question related.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Carl</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gary</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Piaraic</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Items of knowledge required to complete Task 6 (I.P.) not linked.</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
</tr>
<tr>
<td><strong>Oliver</strong></td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Items of knowledge required to complete Task 6 (I.P.) not linked.</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
</tr>
</tbody>
</table>
Group 2

This group is composed of students similar in most respects to those outlined above in terms of their use of procedural knowledge, rules of thumb and geometric databases (Pat, David, Michael, Justin, Derek). They consistently completed all levels of procedures appropriately. However, they did not consistently complete (or always attempt) the solution of parts of the questions which required their knowledge of the concepts associated with the question to be used to construct an appropriate solution. That is, apparently they did not always identify and or act on the relationships between items of knowledge as the students from Group 1 did. Their draughting skills were also of a very high standard (particularly David's - who was easily the best draughtsman in all four groups), and these students completed solutions (although sometimes incomplete) to very high standards well within imposed time limits. Thus, students from this group consistently completed the majority of tasks across the range of topics examined during this study. However, they were usually unsuccessful when faced with a problem for which they were required to link items of knowledge not previously linked in a known procedure.

Thus, from a knowledge perspective these students were similar in many respects to the students from Group 1 described above, in terms of knowledge use. However, their failure to complete the final tasks in each of the topics suggests that their declarative and conceptual knowledge of the topics was deficient. Somewhat paradoxically, these students regularly displayed a knowledge of quite complex incremental procedures in all topic areas but were simultaneously unable to complete relatively simple complementary procedures in many of the same topic areas. This appears to suggest that their declarative and conceptual knowledge was much less comprehensive than the students in Group 1 despite their procedural sophistication. Despite the high standard of draughting skills displayed by students from this group, the speed at which solutions were competed was often slower than for students from Group 1. Thus, it took them longer to complete fewer tasks. The most significant difference is that fewer tasks were completed. Speed in drawing solutions becomes significant in examination settings where the number of tasks students from Group 1 completed in the allotted time had a significant impact on the grade which they could achieve, relative to students from Group 2.

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Together with the difficulty of completing solutions within a 35 minute time-frame students from Group 2 frequently found drawing complete solutions problematic, i.e. they often failed to complete all the tasks associated with a problem. For example, these students were usually successful in completing the Primary Procedure required to complete the diagram of boreholes from the written specifications on the Mining Applications question. The two separate Complementary Procedures required to determine the dip, strike and thickness of a stratum did not prove problematic. However, when they were required to use their conceptual knowledge to complete another task (usually involving another complementary procedure) they frequently failed to do so. Justin described this deficiency during interview:

Justin: I'm able to do the strike, dip and thickness OK and I can do the set up for the last bit of the question sometimes as well, but I never seem to get all parts of the mining question completed.

WPB: Why do you think you never completely finish this question?

Justin: Well, I know how to do part (a), it doesn't matter what specifications are given. But most of the time I don't know exactly what they want for part (b), although sometimes I do try to do something, just to get a few marks.

Such problems require the linking of topic-specific procedural and declarative knowledge and the formation of a concept of the problem. An inability to complete such problems may be regarded as failure to conceptualise the problem.

Summaries of the data from observations and interviews for this group are detailed in Tables 4.9. and 4.10. below.

Group 3

This group was composed of students similar to those in Group 2 above but who were not consistent in their completion of primary, complementary or incremental procedures. They sometimes also made errors in their application of rules of thumb which led to the construction of inappropriate solutions. Their Geometric Databases often proved inadequate in terms of knowledge of facts and...
Table 4.9 - Summary of Observation Data for Group 2.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids In Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
</table>

Building Applications | Perspective Projection | Hyperbolic Paraboloid Roofs | Mining Applications | Roadworks Geometry |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>Occasionally had difficulty completing Task 5 (I.P.).</td>
<td>Consistently completed all Tasks successfully.</td>
<td>Occasionally had difficulty completing Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Task 4</td>
</tr>
<tr>
<td>David</td>
<td>Occasionally had difficulty completing Task 5 (I.P.).</td>
<td>Consistently completed all Tasks successfully.</td>
<td>Occasionally had difficulty completing Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Task 4</td>
</tr>
<tr>
<td>Justin</td>
<td>Occasionally had difficulty completing Tasks 4 &amp; 5 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Occasionally had difficulty completing Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Task 4</td>
</tr>
<tr>
<td>Derek</td>
<td>Occasionally had difficulty completing Tasks 4 &amp; 5 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Occasionally had difficulty completing Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Occasionally had difficulty completing Task 4</td>
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</tbody>
</table>
Table 4.10. - Summary of Interview Data for Group 2.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>Items of knowledge required to complete Task 4 (C.P.) not being linked.</td>
<td>Linking of items of knowledge required to complete Task 3 (I.P.) not being made.</td>
<td>Items of knowledge required to complete Task 6 (I.P.) not being linked.</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 5 (I.P. &amp; C.P.) not being linked.</td>
</tr>
<tr>
<td>David</td>
<td>Items of knowledge required to complete Task 4 (C.P.) not being linked.</td>
<td>Linking of items of knowledge required to complete Task 3 (I.P.) not being made.</td>
<td>Items of knowledge required to complete Task 6 (I.P.) not being linked.</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 5 (I.P. &amp; C.P.) not being linked.</td>
</tr>
<tr>
<td>Michael</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Linking of items of knowledge required to complete Tasks 3, 4 &amp; 5 (I.P. &amp; C.P.s) not being made.</td>
<td>Items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 5 (I.P. &amp; C.P.) not being linked.</td>
</tr>
<tr>
<td>Justin</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Linking of items of knowledge required to complete Tasks 3, 4 &amp; 5 (I.P. &amp; C.P.s) not being made.</td>
<td>Items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 5 (I.P. &amp; C.P.) not being linked.</td>
</tr>
<tr>
<td>Derek</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Linking of items of knowledge required to complete Tasks 3, 4 &amp; 5 (I.P. &amp; C.P.s) not being made.</td>
<td>Items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being linked.</td>
<td>Items of knowledge required to complete Tasks 4 &amp; 5 (I.P. &amp; C.P.) not being linked.</td>
</tr>
</tbody>
</table>

Building Applications

<table>
<thead>
<tr>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>Occasional difficulties with the I.P. necessary to complete Task 5.</td>
<td>All knowledge required to complete Tasks evidenced.</td>
<td>Links between items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being made.</td>
</tr>
<tr>
<td>David</td>
<td>Occasional difficulties with the I.P. necessary to complete Task 5.</td>
<td>All knowledge required to complete Tasks evidenced.</td>
<td>Links between items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being made.</td>
</tr>
<tr>
<td>Michael</td>
<td>Difficulties with Task 4 (C.P.) resulted in Task 5 (I.P.) also not being completed.</td>
<td>Difficulties with the I.P. required to complete Task 3 also resulted in Task 5 (C.P.) not being completed.</td>
<td>Links between items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being made.</td>
</tr>
<tr>
<td>Justin</td>
<td>Difficulties with Task 4 (C.P.) resulted in Task 5 (I.P.) also not being completed.</td>
<td>Difficulties with the I.P. required to complete Task 3 also resulted in Task 5 (C.P.) not being completed.</td>
<td>Links between items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being made.</td>
</tr>
<tr>
<td>Derek</td>
<td>Difficulties with Task 4 (C.P.) resulted in Task 5 (I.P.) also not being completed.</td>
<td>Difficulties with the I.P. required to complete Task 3 also resulted in Task 5 (C.P.) not being completed.</td>
<td>Links between items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.) not being made.</td>
</tr>
</tbody>
</table>
constructions associated with the completion of some procedures. These students encountered difficulties in all the topics encountered. They were most successful with solutions to Intersecting Planes & Skew Lines questions where difficulties were confined to Tasks 4 & 6 (C.P. Level 2 and I.P. level 2 respectively) where conceptual knowledge and working in reverse order from the second auxiliary is required. In all other topic areas complementary and incremental procedures caused difficulties where items of knowledge had to be linked in order to complete a solution. These students often displayed quite good draughting skills but, because they often failed to complete a sufficient numbers of tasks across the required number of topics in T.D. examinations, they did not score highly in examinations (Martin and Micheál achieved C1 grades - with scores in the 65-69% range; Denis achieved a C2 with a score in the 60-64% range; while Stephen and Brian both achieved D1 grades with scores in the 50-54% range).

The students from this group usually completed the earlier tasks in all topic areas. That is, they usually completed only the projections that were given in the diagram accompanying the question together with some parts of tasks which required the completion of first auxiliary views (C.P.'s). Final tasks, which required the completion of consecutive auxiliary views, were seldom attempted. If attempted, the nature of the solutions were sometimes difficult to categorise and were invariably incorrect.

Thus, students in Group 3 were not as successful in completing many of the tasks which students from Group 2 included in their solutions. They encountered some difficulties across the range of topics examined. However, they did frequently complete the majority of tasks associated with some questions. In Intersecting Planes, for example, they often completed the projections of the planes (a Primary Procedure) and found the dihedral angle (an Incremental Procedure) without difficulty. In their solutions to Skew Lines problems (part of the same question) they frequently completed the projections of the lines (another Primary Procedure) but failed to find the shortest distance between the lines (another Incremental Procedure). This failure is sometimes due to the fact that they choose the wrong procedures to address the problem. I.e., the rule of thumb which guides the use
of an appropriate procedure is incorrectly applied. Denis describes how this sometimes happens -

I know there are two methods, one for the shortest perpendicular line and one for the shortest horizontal line. I know we call one the planes method, but what confuses me is that you have to use two auxiliary views whichever method you choose.

The rule of thumb advises that the planes method should be used if the shortest horizontal line is required while the auxiliary views method is required to find the shortest (perpendicular) distance. As Denis correctly states, it is necessary to use consecutive auxiliary views (an incremental procedure) in both instances - hence, his confusion. He is typical of a student who gets the procedure associated with a rule of thumb confused. He often went on to complete the shortest distance, only to discover that it was not what was required in the solution. When this happened, the time within which the problem had to be solved had elapsed and he was unable to rectify his error, even when he recognised he had made one -

I left the Skew Lines 'til last, went through the whole construction, and only realised at the end that I used the wrong method to find the shortest line - there wasn't enough time left then to go back and start over...

Other students from this group sometimes partially completed the shortest distance between skew lines, having identified the correct approach using the rule of thumb. However, having completed the Incremental Procedure, which allows the position of the shortest line to be located, they fail to bring the shortest line back through the views to the original projections of the Skew Lines - they fail to complete the process in reverse. This is a conceptual deficiency - they are not linking their knowledge of the projection system to the procedure, which allows the shortest line (found on the second auxiliary view) to be brought back to the original projections of the Skew Lines.

Summaries of the data from observations, interviews and audio recordings for this group are detailed in Tables 4.11. and 4.12. below.
Table 4.11. - Summary of Observation Data for Group 3.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids In Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Michaél</strong></td>
<td>Difficulty completing Task 6 (I.P.). Rarely (usually unsuccessfully) attempted Task 4 (C.P.).</td>
<td>Difficulty completing Task 3 (I.P.) and Tasks 4 &amp; 5 (C.P.s)</td>
<td>Difficulty completing Tasks 4, 5 &amp; 6 (C.P., C.P. &amp; I.P.)</td>
<td>Difficulty completing Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Stephen</strong></td>
<td>Difficulty completing Task 6 (I.P.). Rarely (usually unsuccessfully) attempted Task 4 (C.P.).</td>
<td>Difficulty completing Task 3 (I.P.) and Tasks 4 &amp; 5 (C.P.s)</td>
<td>Difficulty completing Tasks 4, 5 &amp; 6 (C.P., C.P. &amp; I.P.)</td>
<td>Difficulty completing Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Martin</strong></td>
<td>Difficulty completing Task 6 (I.P.). Rarely (usually unsuccessfully) attempted Task 4 (C.P.).</td>
<td>Difficulty completing Task 3 (I.P.) and Tasks 4 &amp; 5 (C.P.s)</td>
<td>Difficulty completing Tasks 4, 5 &amp; 6 (C.P., C.P. &amp; I.P.)</td>
<td>Difficulty completing Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Denis</strong></td>
<td>Difficulty completing Task 6 (I.P.). Rarely (usually unsuccessfully) attempted Task 4 (C.P.).</td>
<td>Difficulty completing Task 3 (I.P.) and Tasks 4 &amp; 5 (C.P.s)</td>
<td>Difficulty completing Tasks 4, 5 &amp; 6 (C.P., C.P. &amp; I.P.)</td>
<td>Difficulty completing Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Brian</strong></td>
<td>Difficulty completing Task 6 (I.P.). Rarely (usually unsuccessfully) attempted Task 4 (C.P.).</td>
<td>Difficulty completing Task 3 (I.P.) and Tasks 4 &amp; 5 (C.P.s)</td>
<td>Difficulty completing Tasks 4, 5 &amp; 6 (C.P., C.P. &amp; I.P.)</td>
<td>Difficulty completing Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Reefs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
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</thead>
</table>
Table 4.12. - Summary of Interview Data for Group 3.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micheal</td>
<td>Difficulties with C.P.s and I.P. required to complete Tasks 3, 4 &amp; 5. Not linking items of knowledge.</td>
<td>Not linking items of knowledge required to complete Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td>Martin</td>
<td>Difficulties with C.P.s and I.P. required to complete Tasks 3, 4 &amp; 5. Not linking items of knowledge.</td>
<td>Not linking items of knowledge required to complete Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td>Denis</td>
<td>Difficulties with C.P.s and I.P. required to complete Tasks 3, 4 &amp; 5. Not linking items of knowledge.</td>
<td>Not linking items of knowledge required to complete Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td>Brian</td>
<td>Difficulties with C.P.s and I.P. required to complete Tasks 3, 4 &amp; 5. Not linking items of knowledge.</td>
<td>Not linking items of knowledge required to complete Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 5 &amp; 6 (C.P. &amp; I.P.).</td>
<td>Not linking items of knowledge required to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
</tbody>
</table>
Many of the tasks these students compete in their solutions are correct - but they frequently leave many tasks incomplete or unattempted.

Group 4

Students in this group successfully completed only the less complex initial and complementary procedures and often had difficulty with higher level procedures from these classifications. Thus, (usually) only those tasks which involved the construction of the initial diagram (P.P.'s) in each topic area were made without any further attempt to address the major tasks involved in the completion of solutions. Their knowledge of rules of thumb was also seriously deficient and their solutions were often inappropriate as a result. These students rarely attempted incremental procedures but when they did they invariably made serious errors, often related to the deficiencies in their knowledge of rules of thumb alluded to above. Although some of these students displayed quite high levels of draughtsmanship they tended not to score well in T.D. examinations because they frequently attempted only a very limited number of the tasks in each topic area. In the Leaving Certificate T.D. examination Kenneth, Cathal, Enda, and Trevor all achieve D3 grades - scoring in the 40-44% range.

The students from Group 4 frequently found (even) drawing the projections of all but the simplest solids or configurations (Primary Procedures) problematic. Many questions require that some work must be carried out in order to reproduce the diagram given in the question. This must be completed before the tasks which address the problem can be addressed. Students from this group often draw inappropriate views or ignore the specifications outlined in the question. Enda describes how he drew the projections of a square pyramid which had to be rotated into a position where its base was inclined at 30° to the Horizontal Plane.

WPB Enda, can you describe what you have done here?
Enda I just drew the elevation and plan that was given in the question.
WPB Are they the same - your projections and the given diagram?
Enda Yes, I think so.
Enda's concept of what was portrayed in the question was flawed, and he continued with his solution based on his original misconception.

Students from this group often completed first auxiliary projections (C.P.'s) but found it difficult to complete successive auxiliary views (I.P.'s). The rules of thumb which guide work on first and second auxiliary views were often not rigorously applied - i.e. the students appeared to know the rules but failed to apply them rigorously. For example Trevor describes drawing an auxiliary plan in an Interpenetrations problem -

Trevor I projected from the elevation and measured from the plan.
WPB And is your auxiliary plan correct?
Trevor I know I've done it the right way...

He had competed an auxiliary plan. However, he did not apply the correct measurements along the appropriate projection lines. So although he followed the rule of thumb for auxiliary plans his application of the rule was not accurate enough (because of his poor draughting skills) and a misshapen auxiliary view resulted. In turn, this made it impossible to complete the appropriate incremental procedure required to complete the solution.

Summaries of the data from observations and interviews for this group are detailed in 4.13. and 4.14. below.

Table 4.15. below gives the Leaving Certificate T.D. Examination results for the students who are discussed above in terms of the groups into which I placed them. It is worth noting that some of the students from Group 3 performed better in the examination than their counterparts in Group 2.
### Table 4.13 - Summary of Observation Data for Group 4.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Solids in Interpenetrations</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kenneth</strong></td>
<td>Consistently failed to complete Tasks 1, 2 &amp; 3 (P.P.s). Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4, 5 &amp; 6 (C.P.s &amp; I.P.s). Sometimes had difficulty with Task 1 (P.P.).</td>
<td>Completed Tasks 1 &amp; 3 (P.P.s). Occasionally failed to complete Task 2 (C.P.). Invariably had difficulty completing Tasks 4 &amp; 5 (I.P. &amp; C.P.).</td>
</tr>
<tr>
<td><strong>Cathal</strong></td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4, 5 &amp; 6 (C.P.s &amp; I.P.). Sometimes had difficulty with Task 1 (P.P.).</td>
<td>Completed Tasks 1 &amp; 3 (P.P.s). Occasionally failed to complete Task 2 (C.P.). Invariably had difficulty completing Tasks 4 &amp; 5 (I.P. &amp; C.P.).</td>
</tr>
<tr>
<td><strong>Enda</strong></td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4, 5 &amp; 6 (C.P.s &amp; I.P.). Sometimes had difficulty with Task 1 (P.P.).</td>
<td>Completed Tasks 1 &amp; 3 (P.P.s). Occasionally failed to complete Task 2 (C.P.). Invariably had difficulty completing Tasks 4 &amp; 5 (I.P. &amp; C.P.).</td>
</tr>
<tr>
<td><strong>Trevor</strong></td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 &amp; 6 (C.P.s &amp; I.P.s).</td>
<td>Consistently failed to complete Tasks 2, 3, 4, 5 &amp; 6 (C.P.s &amp; I.P.). Sometimes had difficulty with Task 1 (P.P.).</td>
<td>Completed Tasks 1 &amp; 3 (P.P.s). Occasionally failed to complete Task 2 (C.P.). Invariably had difficulty completing Tasks 4 &amp; 5 (I.P. &amp; C.P.).</td>
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</tbody>
</table>

#### Building Applications

<table>
<thead>
<tr>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kenneth</strong></td>
<td>Consistently failed to complete Tasks 3, 4 &amp; 5 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Cathal</strong></td>
<td>Consistently failed to complete Tasks 3, 4 &amp; 5 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Enda</strong></td>
<td>Consistently failed to complete Tasks 3, 4 &amp; 5 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
<tr>
<td><strong>Trevor</strong></td>
<td>Consistently failed to complete Tasks 3, 4 &amp; 5 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3, 4 (C.P.s &amp; I.P.).</td>
<td>Consistently failed to complete Tasks 2, 3 &amp; 4 (I.P.s).</td>
</tr>
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</table>
Table 4.14 - Summary of Interview Data for Group 4.

<table>
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<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Intersecting Planes &amp; Skew Lines</th>
<th>Solids in Contact</th>
<th>Interpenetrations</th>
<th>Conic Sections</th>
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</thead>
</table>

Building Applications

<table>
<thead>
<tr>
<th>Perspective Projection</th>
<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenneth</td>
<td>Procedural difficulties with Task 2 (C.P.). Not linking items of knowledge to complete Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Procedural difficulties with Tasks 3, 4 &amp; 5 (C.P.s). Not linking items of knowledge to complete Task 6 (I.P.).</td>
<td>Not linking items of knowledge which would allow Tasks 2, 3 &amp; 4 (I.P.s) to be completed.</td>
</tr>
<tr>
<td>Cathal</td>
<td>Procedural difficulties with Task 2 (C.P.). Not linking items of knowledge for Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Procedural difficulties with Tasks 3, 4 &amp; 5 (C.P.s). Not linking items of knowledge to complete Task 6 (I.P.).</td>
<td>Not linking items of knowledge which would allow Tasks 2, 3 &amp; 4 (I.P.s) to be completed.</td>
</tr>
<tr>
<td>Ends</td>
<td>Procedural difficulties with Task 2 (C.P.). Not linking items of knowledge for Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Procedural difficulties with Tasks 3, 4 &amp; 5 (C.P.s). Not linking items of knowledge to complete Task 6 (I.P.).</td>
<td>Not linking items of knowledge which would allow Tasks 2, 3 &amp; 4 (I.P.s) to be completed.</td>
</tr>
<tr>
<td>Trevor</td>
<td>Procedural difficulties with Task 2 (C.P.). Not linking items of knowledge for Tasks 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Procedural difficulties with Tasks 3, 4 &amp; 5 (C.P.s). Not linking items of knowledge to complete Task 6 (I.P.).</td>
<td>Not linking items of knowledge which would allow Tasks 2, 3 &amp; 4 (I.P.s) to be completed.</td>
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Table 4.15. - Leaving Certificate T.D. Examination Results

<table>
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<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carl A1</td>
<td>Pat B3</td>
<td>Micheál D1</td>
<td>Kenneth D3</td>
</tr>
<tr>
<td>Gary A1</td>
<td>David B3</td>
<td>Stephen D1</td>
<td>Cathal D3</td>
</tr>
<tr>
<td>Páraic B2</td>
<td>Michael C1</td>
<td>Martin C1</td>
<td>Enda D3</td>
</tr>
<tr>
<td>Oliver B2</td>
<td>Justin D1</td>
<td>Denis C2</td>
<td>Trevor D3</td>
</tr>
<tr>
<td>Derek C1</td>
<td>Brian D1</td>
<td></td>
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</tr>
</tbody>
</table>

A1 = 100%-90%; A2 = 89%-85%; B1 = 84%-80%; B2 = 79%-75%; B3 = 74%-70%; C1 = 69%-65%; C2 = 64%-60%; C3 = 59%-55%; D1 = 54%-50%; D2 = 49%-45%; D3 = 44%-40%.

Self-Regulation

According to the literature, self-regulation involves planning, checking outcomes of strategies, evaluating and revising strategies. Examples of these control functions were evidenced in the data from interviews and audio-recordings from many of the topics examined. A summary of the data are included in Table 4.16. However, I encountered evidence of self-regulation or metacognitive function only in students from Groups 1 and 2. Students from the other groups may also have used self-regulation but only students from Groups 1 and 2 were observed to begin a solution, discard the work they had begun and approach the work using different procedures, though not necessarily different types of procedure. Students from these groups explained their changes in approach in the interviews which followed. For example, during an audio-recording of Pat from Group 2 and Denis from Group 3 discussing and working on the solution of Task 4 of an Intersecting Planes & Skew Lines problem Pat obviously decided that the procedure they were executing was not going to be successful - they were trying to find the projections of a line perpendicular to one of the planes they had drawn earlier -

Pat Draw a horizontal line across the elevation passing through point C.
Denis I've done that - now what comes next?
Pat O.K. - draw that line on the plan.
Denis By projecting the ends that pass through AD and BD?
Pat Yes, that's right - then draw a line from C perpendicular to that on the plan - no wait a minute! Go back to the
start again. We have to get an edge view of ABD first...
Draw a level line on ABD...OK now draw an auxiliary
to show the level line as a point...

Denis  You mean along the true length?
Pat   Yes...now draw C on this auxiliary.

Denis  OK I've done that...the line we want is at right angles to
the edge view of the plane...right?
Pat   That's it on the auxiliary, but we still have to get it back to
plan and elevation.

The original procedure Pat was following was a strategic C.P. used to
find the shortest horizontal line from a given point to a plane. As the
construction progresses Pat appeared to realise (possibly as a result of
visualisation as described in Chapter 2) that the resultant line was not
going to be perpendicular to the plane. He then reverted to another
strategic procedure (another C.P.) which did indeed yield the desired
result. He appears to have been evaluating the outcomes of the earlier
procedure as Denis drew.

WPB  Why did you draw those three first auxiliary elevations?
Gary I wasn't sure exactly where to take the auxiliary to find the
distance that was asked for. I thought I knew where to draw
the view, so I drew the first auxiliary, but the line I wanted
wasn't right when I'd finished it. So I drew another view, there
wasn't much involved really, just the projection of six points,
so it didn't take too long.

WPB  But you drew another auxiliary?
Gary Yes. When I drew the second one I knew where I'd been
going wrong. So the third auxiliary gave me the correct
solution, I think?

This is an example of the interaction of the drawing he was making
and his thinking about possible solutions. He did not have a
procedure which he could automatically use here. So he used
complementary procedures which he had used in other problem solving
situations. The construction of these different C.P.'s to advance his
thinking on the problem is an example of a third order procedure or
control function such as that described by Stevenson (1994, pp.
13-14) where Gary was switching between specific and strategic
procedures to determine a solution - which his third auxiliary view enabled him to do.

Another example sees Carl attempt a number of times to determine the traces of a tangent plane to touch two solids in a 'Solids in Contact' question.

He was observed drawing a cone enveloping a sphere and noting the position of the horizontal trace of this cone on the Horizontal Plane. He then drew traces of a plane and marked them VTH (the designation of a plane; Vertical Trace and Horizontal Trace. These are combined into VTH). However, he almost immediately reversed the orientation of the enveloping sphere and drew another set of traces related to this second cone - the traces of the required plane. Afterwards he explained his actions.

Carl I knew that I'd need to draw a cone around the sphere to get the angle of the plane. But when I had it drawn it didn't seem to me to be sloping in the direction I thought it should. So I inverted the cone around the sphere and drew the traces again. Once I saw the traces drawn I knew it was tilted in the right direction.

When Justin (Group 2) was working on the solution of a skew lines question he was observed drawing an auxiliary elevation at right angles to the plan of one of the skew lines. This auxiliary view gave him the true length of one of the skew lines and constitutes one part of an Incremental Procedure. However, it was not an appropriate first view to draw in the Incremental Procedure which was required to complete the task he was working on. Once this view was completed however, he began work on a Complementary Procedure to construct a triangular plane, seeming to abandon his earlier work. Once the Complementary Procedure was completed he went on to complete an appropriate incremental procedure which allowed him to complete the task correctly.

I asked Justin during the interview which followed why he had abandoned his original work -
Justin When I had the auxiliary elevation drawn (the first one I drew) I realised that I had picked the wrong method. I wouldn't have been able to find the shortest horizontal distance between the lines if I carried on and drew the second auxiliary. So I had to go back and start over. But I didn't have time to draw the whole lot again so I just left it as it was and started off by drawing my triangle. When I was ready to draw the auxiliaries there wasn't much room and I ended up with them superimposed on the first elevation I drew - but that didn't effect the solution, it was correct the second time round.

In the example above Justin checks the outcomes of his strategies and evaluates them. This demonstrates elements of self-regulation.

Similarly Pat (Group 2) appeared to change his approach to drawing an auxiliary plan when drawing the solution of an Interpenetration question. This question involved a cone and a triangular prism and it was the approach to drawing the cone on the auxiliary plan that he changed - demonstrating an aspect of self-regulation.

Then he was observed dividing the projections of the cone (elevation and plan) to show twelve generators equally positioned. However, when it came to drawing the auxiliary plan he did not use these generators at all, drawing the elliptical base of the cone on this auxiliary view using the concentric circles method (a very quick method of drawing an ellipse). When I asked why he had done this in the interview which followed he explained -

Pat I started to draw the elevation and plan of the given cone and I automatically began to divide it to show generators, like we always do. I knew I'd be able to draw the auxiliary plan using these generators but then I thought... I'm going to have to draw lines on the surface of the cone to find the entry and exit points, and if I already have generators drawn I'm going to be totally confused. So I decided to leave the generators out of the auxiliary plan and just draw the shape of the base using concentric circles.

Whereas the work of students from Groups 1 and 2 evidenced at least some aspects of the self-regulatory process students from Groups
## Table 4.16 - Summary of Audio-Recording of Pairs Data Across Groups.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
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<th>Conic Sections</th>
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<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 3</strong></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 4</strong></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
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<th>Hyperbolic Paraboloid Roofs</th>
<th>Mining Applications</th>
<th>Roadworks Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective.</td>
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<td></td>
<td></td>
</tr>
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<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
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<tr>
<td><strong>Group 4</strong></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
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3 and 4 seldom made any attempt to check the outcomes of their strategies or evaluate their solutions in respect of their attainment of desired goals. For example, when Cathal attempted Perspective problems he seldom varied his approach. He always began by attempting to draw the projections of the building or monument which was the subject of the problem. It is not necessary (usually) to draw an elevation and plan - a R.o.T. advises this. However, Cathal did consistently draw both elevation and plan, apparently unaware of the first three rules of thumb for these problems which advise that

* the drawn solution should begin with the spectator,

* that the plan should be completed next, and

* that the elevation of the building or monument should only be drawn when absolutely necessary.

Cathal usually continued with the set-up for the perspective and positioned the picture plane touching the corner nearest the spectator, irrespective of the required position. He seldom attempted any part of the perspective view - rather he treated it as an auxiliary elevation, and even here he did not use proper Orthographic or Perspective Projection. He continually used the wrong procedures. In his work Cathal did not appear to display any aspect of self-regulation.

From the examples cited above some aspects of the types and levels of knowledge discussed in the literature can be identified.

**Visualisation**

Viewed from a graphicacy perspective, visualisation involves the mental manipulation of two- or three-dimensional configurations. Great importance is attached to this notion and it is regarded as one of the two most important aspects of spatial ability (the other being internalisation) and an essential attribute in solving two- and three-dimensional problems.

I believe there is evidence to support a view of visualisation which is not internalised mental activity as discussed in Chapter 2, but rather a form of knowledge which allows students to predict what drawn
solutions are likely to resemble. This allows some (though not all) students to see the development of the solution before it is drawn on the paper. This form of knowledge would be used in conjunction with procedural, declarative conceptual knowledge to build possible solutions. The source of this form of knowledge is the students experience of preparing solutions to problems.

For example, students from Groups 1 and 2 usually position their initial drawings (P.P.'s) on the paper such that subsequent work may be adequately accommodated. In Interpretation questions for example, these students know that the given projections (plan and elevation) of the intersecting solids have to be drawn, followed by an auxiliary plan which will allow the exact nature of the interpenetration to be determined. Auxiliary plans are usually (though not always) projected downwards from the given elevation so space to accommodate this view must be left on the drawing paper. Experience of doing Interpenetration questions (and other questions) allows students to know, by developing their own R.o.T.'s, that when the projections of solids are positioned in particular ways in the question that the overall shape of the solution (encompassing all views) will require particular spacing and orientation on the drawing paper. Students often describe this as 'planning out their drawing'. Here they are not referring to the content of the drawings which will constitute a solution, rather they are speaking of the arrangement of the drawings. Thus students visualise how the drawings which make up the solution will look when drawn out on the paper. They are using their knowledge of what drawings are likely (in their experience) to comprise an appropriate solution. In this way visualisation is a type of knowledge separate from the procedural, declarative or conceptual knowledge of the topics being dealt with.

Students are not necessarily (though they may be) using their procedural or conceptual knowledge of what the content of the solution may be when visualising what the solution is likely to be.

Carl's (from Group 1) visualisation appeared to be particularly well developed. This was evidenced in the layout of the drawings which comprised his solutions across the range of topics examined. His visualisation of the solution usually allowed him to position all the drawings required for particular solutions without ever superimposing
(drawing one over another) them. In one interview which followed his completion of the solution to an Interpenetrations problem Carl explained -

I look at the first bit of the drawing I've made... or sometimes at the diagram that's given on the question paper...they're all much the same, just different solids, and I can see the direction I have to draw the auxiliary plan in...I know then by the space on the drawing sheet where I have to put it...

He was using his experience of drawing interpenetrating solids to suggest how the arrangement of drawings would appear when complete. He approached drawing all his solutions in a similar fashion. Students were in a position to visualise the arrangement of drawings for solutions only after the projections had been drawn. This highlights the link between the activity involved in the drawing of the solution as opposed to the notion of visualisation as mental activity alone advanced by the graphicacy perspective. This is usual in my experience and is evidenced in some of the interview data. For example, I asked David if he had an image of a pair of Intersecting Planes from reading the co-ordinates -

No...obviously I know the positions of the corners of the panes from the co-ordinates...but until I index them in elevation and plan and join them in the given order I don't know what they look like...even then it's hard sometimes to picture them...you know, especially on the plan...

Students from Groups 3 and 4 did not appear to visualise their finished solutions. Despite using A2 (relatively large) drawing paper throughout the study, these students almost invariably run out of space for some parts of their solutions - especially when there is pressure of time during examinations. At the very least they could be described as less successful in visualising. For example, when drawing his solution to a Mining question Kenneth was observed to draw an X-Y line very close to the left hand side of the drawing paper. He subsequently ran out of space when he tried to draw the auxiliary views (C.P.'s) necessary to complete the projections of the boreholes.
When questioned afterwards about his positioning of the solution he appeared not to have been able to visualise what the completed solution would entail -

Kenneth I was just trying to draw the boreholes. I marked in the altitudes of the points and their positions on the plan, but when I drew the directions on plan I didn't have any room for the auxiliaries.

WPB Did you think about the solution before you started?
Kenneth Yes, I knew I'd have to draw two auxiliaries, one for each of the boreholes, but I just ran out of space to draw them.

WPB But you had a lot of space to the right of your drawing?
Kenneth Yes, but I couldn't draw the views over there!

Although Kenneth appeared to be aware of the procedures required to complete the question he appeared simultaneously to be unable to visualise how the overall solution might emerge from completing these procedures.

Visualisation may be viewed as a form of knowledge where the student uses the drawings which result from using procedural (and conceptual) knowledge, stage by stage during construction, to suggest further action. What distinguishes it from other types of knowledge described here (and elsewhere) is its association with the drawing under construction. It is not solely knowledge about cognitive resources (procedural and/or conceptual) or about when it is appropriate to switch between them. It is knowledge inextricably linked to the action of constructing the drawn solution. It is in effect, a way of knowing by looking at the drawings already completed, what procedural (and conceptual) knowledge needs to be used to construct an appropriate solution. In this way visualisation could be compared to a metacognitive function which encompasses self-regulation.

Indeed students often use their partially constructed drawings to dictate their approach to completing a solution where knowledge relating to the topic is deficient in some way. Derek from Group 2 for example often used the projections of boreholes drawn in the
earlier tasks in Mining Application questions when attempting Tasks 5 & 6 the extra borehole. He was attempting to relate the work he had already completed to the solution of this final part of the problem.

Thus, rather than being solely in the head, visualisation is rather, inextricably linked to the actions of making the drawings on the board. It is also inextricably linked to the procedural (and conceptual) knowledge and can therefore, be dealt with specifically by the T.D. teacher and learned by the T.D. student in the classroom.

These comparisons can be made across all the topics in which students are examined in the Leaving Certificate Examination in T.D. Papers 1 & 2(b).

Individual Differences & Student Profiles

For the purpose of illustrating the differences between the individual students I have summarised the main differences between four students, one from each of the groups, in respect of the topics reported in the text. This summary is shown in Table 4.17 below.

Carl from Group 1

This students consistently completed all the tasks required to draw appropriate solutions to questions across the full range of topics examined at Leaving Certificate level. That includes both topics detailed in the text and in appendices.

From a knowledge perspective therefore, he could be said to have comprehensively demonstrated that his knowledge of the topics is extensive. This success is also reflected in the grade A1 (90 - 100%) he received in his Leaving Certificate T.D. Examination. He knew how to complete all simple (Primary) and strategic (Complementary and Incremental) procedures. He also demonstrated the declarative knowledge (constructions, facts and rules of thumb) necessary to compete tasks in conjunction with appropriate procedures. He also displayed a level of control and self-regulation which allowed him to use his declarative and conceptual knowledge with appropriate procedures to appropriately address the question.
Table 4.17. - Differences Between Selected Individuals Across Groups

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Carl Group 1</td>
<td>Comprehensive knowledge of all concepts, procedures, and rules of thumb.</td>
<td>Difficulty in linking knowledge of solids to address Task 3 (I.P.). Therefore, difficulties with Tasks 4 &amp; 5.</td>
<td>Difficulty with in/out and cross-over points in Task 5 (C.P.). Therefore, Task 6 (I.P.) frequently not complete.</td>
<td>Task 4 (I.P.) often not complete. Link of properties of conics to question often not made.</td>
</tr>
<tr>
<td>Michael Group 2</td>
<td>Knowledge of procedures and rules of thumb. Difficulty linking knowledge to appropriately address Task 4 (C.P.).</td>
<td>Difficulties with Task 3 (I.P.). Often unable to link items of knowledge to determine locus of centres of possible spheres and therefore, radius of required sphere.</td>
<td>Often had difficulty with Task 4 (C.P.). Therefore, Task 5 (C.P.) and Task 6 (I.P.) were often not completed.</td>
<td>Tasks 4 &amp; 5 (I.P. &amp; C.P.) were frequently not completed. Unable to link properties of conics to question to be addressed.</td>
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</tbody>
</table>

Building Applications | Perspective Projection | Hyperbolic Paraboloid Roofs | Mining Applications | Roadworks Geometry |
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<tbody>
<tr>
<td>Carl Group 1</td>
<td>Comprehensive knowledge of all concepts, procedures, and rules of thumb.</td>
<td>Difficulties with Task 6 (I.P.), linking knowledge which would allow the construction of a solution.</td>
<td>Usually displayed a comprehensive knowledge of concepts, procedures, and rules of thumb.</td>
<td></td>
</tr>
<tr>
<td>Michael Group 2</td>
<td>Knowledge of procedures and rules of thumb. Difficulty in completing Task 5 (I.P.).</td>
<td>Sometimes encountered difficulties in addressing Task 3 (I.P.).</td>
<td>Difficulties with Task 6 (I.P.), linking knowledge which would allow the completion of Task 6 (I.P.) and the construction of a solution.</td>
<td></td>
</tr>
<tr>
<td>Martin Group 3</td>
<td>Often encountered difficulties with Tasks 3 &amp; 4 (C.P.s) and Task 5 (I.P.). Knowledge of rules of thumb.</td>
<td>Frequently encountered difficulties in addressing Tasks 3 &amp; 4 (I.P. &amp; C.P.). Not linking items of knowledge.</td>
<td>Sometimes had difficulties with Tasks 4 (C.P.) and in linking knowledge which would allow the completion of Task 6 (I.P.) and the construction of a solution.</td>
<td>Invariably had difficulty in completing Task 4.</td>
</tr>
<tr>
<td>Kenneth Group 4</td>
<td>Frequently encountered difficulties with Tasks 3 &amp; 4 (C.P.s) and Task 5 (I.P.).</td>
<td>Often had difficulty with Task 2 (C.P.). Frequently encountered difficulties in addressing Task 3 &amp; 4 (I.P. &amp; C.P.).</td>
<td>Frequently had difficulties with Tasks 4 (C.P.) and in linking knowledge which would allow the completion of Task 6 (I.P.).</td>
<td>Invariably had difficulty in completing Tasks 2, 3 &amp; 4.</td>
</tr>
</tbody>
</table>
The graphicacy perspective would attribute his success in completing solutions to the development of his spatial ability. Thus, in the question on Interpenetrations for example, when he completed the projections of the intersecting solids (Tasks 1-3) the drawn solution is regarded as the final (communicative) stage of graphicacy - i.e. the completion of these projections is regarded as evidence that he has developed a three-dimensional mental image of the solids based on the two-dimensional image which is presented to him in the question. This assumes a certain type of cognition based on evidence provided in the drawn solution. A knowledge view of what has been done, on the other hand, would say that he has successfully completed procedures - that is the only assumption that is made - he knew how to complete the procedures (P.P. and C.P.'s).

Drawing the auxiliary plan (Task 4 C.P. Level 3), is evidence that he is aware of the strategic nature of this auxiliary view *viz a viz* the completion of an adequate solution. When he drew his $X^1-Y^1$ line perpendicular to the angle of the interpenetrating prism he was using a strategic procedure and rule of thumb which would yield the desired result. That he drew this is not evidence, as the graphicacy perspective would argue, that he can imagine a rotating three-dimensional configuration or imagine how the configuration would appear if viewed from a position other than that given in the diagram accompanying the question. It indicates rather, in knowledge terms, that he is aware of the procedural knowledge involved in constructing consecutive views which will address the questions asked.

Similarly, when completing the exact points (C.P. Level 4) and lines of interpenetration (I.P. Level 3) he is using strategic procedures in conjunction with rules of thumb to construct an appropriate solution. It is a control or self-regulatory function governing all his knowledge which allowed him to select an appropriate strategy from those available to him.

**Michael from Group 2**

This student consistently completed the tasks associated with the range of topics as discussed above for all students in Group 2. When his performance is examined it reveals that he was less successful in completing specific and strategic procedures than Carl from Group 1.
This is reflected in the Cl grade (65-69%) he received in the Leaving Certificate T.D. examination. He was however, reasonably successful in completing tasks during observations and both interview and audio-recorded data indicated that his declarative and procedural knowledge was reasonably comprehensive. However, on the occasions where it was necessary to link items of knowledge to complete aspects of a solution he frequently failed to do so. This failure to complete final tasks across the range of topics suggests that his knowledge of the concepts associated with the topics and solutions was deficient despite his declarative and procedural knowledge. His draughting skills however, were also of a very high standard and his solutions (although often incomplete) reflected this. Since he often did not complete all required tasks he usually completed his solutions well within the limits of time imposed on him.

Thus, for example, when drawing the solution of a Mining Applications question Michael readily completed Tasks 1 - 4 (P.P. and C.P.'s) with relative ease allowing him to score relatively highly. He was also aware of the necessity to work back from the auxiliary view when addressing Tasks 5 - 6 but was often unable to apply this rule of thumb in constructing his solutions. Thus, although he demonstrated a high level of procedural knowledge in completing Tasks 1 - 4 and possessed the declarative knowledge which was necessary to address Tasks 5 - 6 he was in practice, unable to link the items of knowledge in order to complete the solution.

Martin from Group 3

This student successfully completed only earlier tasks across the range of topics. Later tasks involving the drawing of first and second auxiliary views (C.P.'s and I.P.'s) were usually only partially, and often incorrectly, completed. Despite his relatively limited success in completing solutions during the study he performed relatively well in the Leaving Certificate T.D. examination where he achieved a Cl (65 - 69%) grade.

In terms of his knowledge and its use, Martin was similar to students from Group 2 above, but he was not as consistent in the completing the procedures required to complete all tasks. He also made errors in
the application of rules of thumb and his declarative and conceptual knowledge also proved inadequate.

For example, although he frequently completed Task 3 (C.P. Level 4) of the Mining Applications question - including drawing the projections of a level line on the triangular plane - he frequently failed to complete the procedure required to address Task 4 (C.P. Level 5) despite (apparently) being aware of the necessity to draw an auxiliary elevation related to the level line. Thus his failure to complete earlier procedures resulted in an inability to complete later ones.

When dealing with Roadworks Geometry he frequently completed all necessary I.P.'s required to deal with the works required for sloping roads. However, when addressing Task 4 (a further I.P. Level 3) he usually failed to link the knowledge used earlier for the work associated with the road to the work required for the parking area.

Despite his procedural, declarative and conceptual shortcomings Martin displayed quite high levels of draughtsmanship in the work he completed. However, since he regularly failed to complete a number of tasks, he did not do very well in T.D. examinations held during the study. However, as mentioned earlier, he did quite well in the Leaving Certificate T.D. examination (presumably because the deficiencies highlighted during the study became the focus of his preparation for the State examination).

**Kenneth from Group 4**

This student completed the earlier tasks across the range of topics successfully. He usually completed P.P.'s for example Tasks 1 and 3 of Conic Sections questions, but frequently had difficulty with ensuing C.P.'s or I.P.'s. Thus, he sometimes completed first auxiliary elevations (C.P.'s) but tasks which required the completion of second auxiliary views (I.P.'s) were consistently left unattempted. This was the case across the complete range of topics.

From a knowledge perspective, he may be regarded as being similar to but much less successful than Martin from Group 3 above. His knowledge of rules of thumb was also deficient and his attempted solution (especially) to final tasks were usually inappropriate as a
result. When answering Skew Lines questions (Task 6) for example, he frequently employed an inappropriate procedure (I.P.) when trying to determine the shortest distance between them. He appeared to be confusing, or unable to distinguish between the procedures necessary to determine the shortest horizontal or perpendicular distances. Thus, although he did (sometimes) attempt I.P.s he invariably made serious errors related to his knowledge of rules of thumb.

He did however, display quite high draughting skills - but since his solutions did not address all the tasks required for a complete solution he did not score well in T.D. examinations held during the study or in the Leaving Certificate T.D. examination where he achieved a D3 (40 - 44%) grade.

Discussion of Findings

The analysis of the data generated during my study, I believe, gives a different, more comprehensive and convincing account of problem solving and visualisation from a knowledge perspective than that provided by the received view. The following discussion compares the received and knowledge perspectives and outlines how the knowledge perspective offers a more convincing explanation of the nature of problem solving and visualisation. I will show how every aspect of the students construction of a drawn solution to a T.D. problem can be described and explained in detail. In contrast to the received view, inferences about how students are thinking are not made based on the nature of the final solution presented, rather every step in the production of a solution is described and explained in terms of the knowledge associated with it.

Problem Solving From A Graphicacy Perspective

The received view of problem solving in T.D. is centred on the notion that in order to be a good problem solver a student must have a highly developed spatial ability. From this perspective for example, in general terms Carl from Group 1 could be described as having demonstrated abilities (during the course of this study) which are consistent with a fully developed graphicacy. Based on his performance when faced with T.D. problems, i.e. following exposure to an external stimulus (the diagram or written specifications which accompanied the!
questions) he would be regarded (presumably) as having the ability to generate mental images of the three-dimensional (in most cases) configurations (Perception) which were then interpreted and to which he assigned significance (Visual Perception) when these images were compared to his existing mental schemata. Again, based on the solutions he produced, he would be regarded having displayed the ability to work through the full compliment of hierarchical Space Factors (S.F.’s) which comprise Spatial Ability as it is described by Gaughran. Similarly, when he completed all the drawings associated with a complete solution to T.D. problems to conventional standards he displayed his ability to Communicate Graphically.

The act of drawing the solutions is the final one in the whole problem solving process from this perspective. It is completed purely for the purpose of communicating with others. That is, making the drawings is not part of the problem solving process per se. Before the solution is drawn on the paper the student is viewed as having formed and categorised the necessary mental images and as having manipulated these images in order to address the problem posed.

In specific terms, when for example, he completed the projections of intersecting solids from the diagram which accompanied the question (completing Tasks 1 & 2) he demonstrated the ability to hold and compare images (S.F.1). That is, he would be regarded as having the ability to see in three-dimensional mental images what is represented in the diagram in two-dimensions while also being able to compare and check for correspondence the geometric images in the given diagram with those images he has drawn as part of his solution. When he completed the projections of the solids he was demonstrating the ability to rotate solids in a single plane (S.F. 2). In this way the interpenetrating solid is assumed to have been imagined firstly, in a horizontal position parallel to the Vertical Plane, before being rotated about the Horizontal Plane into the required position.

When he completed Task 3 (drawing an auxiliary plan which shows the true shape of the interpenetrating prism) successfully he demonstrated the ability to view three-dimensional objects from different spectator positions (S.F.3), or to have rotated the objects in his imagination (S.F.4), or to have done both. That is, he would be viewed as having either the ability to imagine how the intersecting solids will appear when viewed from positions other than that given in
the diagram (perpendicular to the Planes of Projection) or, to imagined
the total configuration rotating about two axes (consecutively or
simultaneously) until his viewing position is in line with the
interpenetrating prism. When he completed the projections of the
intersection of the two solids he could be viewed as having
demonstrated the ability to manipulate the elements within the
configuration in his imagination, suggesting a developed S.F. 5. Lines
can be imagined for example, on the surface of one solid and moved
in the imagination on the surface to an advantageous position where
the exact points of interpenetration between the edges of the
interpenetrating prism and the other solid are located.

Because he correctly completed a drawing of an appropriate solution,
it would be assumed (because he can communicate his ideas
graphically) that Carl possesses the mental abilities of Perception,
Visual Perception, Spatial Ability - constituting a very high level of
overall Graphicacy. Since the capacity to problem solve in T.D. from
a Graphicacy perspective is equated to the ability to work through the
hierarchy of Space Factors which constitute Spatial Ability, Carl
would also be viewed as an extremely competent T.D. problem solver.

The overall graphicacy of many of the other students in the study,
who were unable to produce adequate drawn solutions to problems
across the range of topics, would be regarded as being flawed. The
evidence provided by their drawn solutions would be regarded as
indicating, in Michael's (Group 2) case for example, that his Spatial
Ability was only partially developed. This is the case since he
demonstrated abilities consistent with S.F.'s 1 - 4 only in the
construction of many of his solutions. Since he often failed to
complete the final tasks in completing solutions (many of which
require the abilities associated with S.F. 5 to complete them) it would
be assumed that S.F. 5 is not fully developed in his case. Michael
would not be regarded as a very competent problem solver since his
Spatial Ability is not fully developed.

Therefore, it is the mental manipulations which constitute the problem
solving. The construction of the drawing is subordinate to this mental
activity and is completed in order to communicate these mental
manipulations - to the classroom teacher for regular assessment of
student progress or to a Leaving Certificate examiner for certification purposes.

Examinations in T.D. are supposed to assess the students state of development and ability to problem solve within the domain. However, as stated above, problem solving is carried out as a mental activity which can not be observed directly or, I believe, inferred following the production of a drawn solution, appropriate or otherwise. When students complete appropriate solutions to problems posed in a question or task assumptions are made about the type of cognition which takes place prior to the completion of the drawing. That is, the student is assumed to have arrived at the drawn solution following progression through the stages of mental activity outlined above.

To summarise, the image this Graphicacy theory conjures up is one of a student being presented with T.D. problem, sitting in front of a drawing board with a blank drawing sheet, apparently not doing anything to complete a solution. After a period of apparent inactivity (during which the student was however, extremely mentally active) she/he quickly puts pencil to paper and completes a drawn solution to the problem. The degree of success of the solution completed is then attributed to the level the student has reached on the spatial ability hierarchy of sub-factors.

This view of problem solving, revolving about spatial ability, appears to suggest that problem solvers may have this ability innately. If this is the case, there are serious questions to be asked about whether a T.D. programme constructed around the space factors which are regarded as constituting spatial ability can (at all) influence the development of problem solving in T.D. students at post-primary level. It is my view that student development under the current philosophy is subject to influence by the classroom T.D. teacher only if spatial ability is not an innate ability.

However, this image is not borne out by my experiences in T.D. classrooms or by the nature of the student activities which were examined during this research study.
The study reported here describes a model of problem solving which emphasises the students use of different types of knowledge associated with T.D. when formulating drawn solutions. The different types and levels of knowledge discussed in the literature can readily be discriminated and related to the procedural nature of completing drawn solutions while the guiding concepts associated with the topics can be examined in detail. A diagrammatic model of a Knowledge Perspective for T.D. is shown in Figure 4.7. at the end of this section.

During this research project I have analysed all the tasks which must be completed when preparing solutions to problems posed in typical examination questions for their knowledge content and have used this analysis to develop a view of problem solving in T.D. which provides an alternative explanation to the received views. I have also carried out extensive data gathering in my T.D. classroom and believe that analysis of this data clearly supports my alternative views on problem solving. The importance of this view lies not only in its alternative nature but also in its development following research carried out in a T.D. classroom situation. Furthermore, each stage of the processes involved when students formulate and construct a solution can be identified and explained more readily with reference to a knowledge perspective. I do not make the type inferences about students cognition which are made by those who support the graphicacy perspective. I am rather, advancing the thesis that students drawn solutions to T.D. problems are better explained in terms of their use of procedural and conceptual knowledge and that this leads to better prescriptions for T.D. teachers than those which are suggested by the received view with its inferences of mental activity from drawn solutions. I believe there is also evidence that the separation of procedural and conceptual knowledge in the literature, together with the distinctions which are made between both types, may not be as clear as previously thought, at least not in T.D.. I also believe that the findings of this study indicate that visualisation, regarded as a mental ability from a graphicacy perspective, may in fact be a further form of knowledge that links procedural and conceptual knowledge and that is used by T.D. students in the problem solving process.
From a knowledge perspective the T.D. student reads the question and begins work immediately on the initial diagram required to construct a drawn solution. The production of (given) diagrams from given information given can be viewed in terms of the student completing Primary Procedures - the set-up from which further construction of the solution to a problem may proceed. These may be thought of as the specific procedures referred to in the literature. Once this initial work has been completed the student progresses through the solution using a series of single or consecutive steps which I have termed Complementary and Incremental Procedures respectively. The distinction between procedural and conceptual knowledge is less distinct here, I feel, than is described in the literature since there are concepts embedded in the procedures used and procedural elements in the students concepts of what is to be done to complete an appropriate solution. Quite simple concepts often require complex procedures to translate them into drawn solutions. Conversely, quite complex concepts can often be translated into drawings using quite simple procedures. But the procedures and concepts are inextricably linked in both cases, though it may be argued that this does not make them less distinct. I think it is also important not to assume, as Hiebert and Lefevre (1986) have suggested, that students are consciously aware of the complete conceptual implications of procedures they know how to complete.

When examined from a knowledge perspective the example of Carl completing a solution to an Interpenetrations problem (used above to illustrate a graphicacy perspective) is equally valid, and does not make any specific inferences about the type of cognition in which he is engaged. When he completed Task 1 - the projections of one of the given solids Carl completed a Primary Procedure. He demonstrated that he knew how to complete this simple procedure. That is all he did. Its completion was (almost) automatic. This initial drawing was then supplemented by the addition of the set-up for the interpenetrating prism (Task 2) and the projections of the interpenetrating prism (Task 3) and an auxiliary plan (Task 4). Thus, this knowledge of these strategic procedures was demonstrated. Similarly, operating with specific procedures to achieve unfamiliar goals, he completed (Tasks 5 and 6) the projections of the solids including the lines of interpenetration.
Continuing on the procedural path, students choices of strategies are governed by a self-regulatory or control function which governs all the students knowledge, procedural, declarative and conceptual. Under this model the drawings created at every stage of the construction of a solution are used to assist in the completion of the final solution. That is, instead of having mental images which may be manipulated, the student has a drawing in progress from the time the Primary Procedures are undertaken to the completion of strategic Complementary and Incremental Procedures which address the particular problem. Students may not be seeing mental images, I would argue, before they commit their solutions to paper. Rather, they see a developing image of the configuration on the paper in front of them which indicates further steps that (may) need to be taken to complete the solution. Students think when they draw and also use the drawings as they are constructed in order to advance their thinking about a specific problem. This link between thinking and acting was discussed in Chapter 2.

Failure to complete a total solution therefore, should not be regarded as a deficiency in an ability (innate or otherwise) which cannot be compensated for. It is rather a lack of, or deficiency in, appropriate knowledge. There is scope for the deficit of knowledge to be learned, and therefore by implication, taught. Problem solving from this perspective relates to the strategic use of procedural knowledge and is, as such, something which can be learned and taught with specific reference to a variety of knowledge types.

Moreover, the production of drawings is more significant from a knowledge perspective. The emphasis on internalisation and visualisation in the Graphicacy Perspective places making drawings in a subordinate role to mental activity. In effect, the solution is regarded as having been completed mentally before it is committed to paper with the drawing of solutions being completed (mainly) for communicative purposes. The intention is to communicate the nature of cognition to another - to the examiner in an examination setting. Theoretically, it is the necessity to display the nature of student cognition and ability for assessment purposes which makes the production of drawings necessary in the T.D. classroom and examination. They are not (strictly) necessary to satisfactorily solve the problem posed. The Knowledge Perspective also acknowledges the
communicative significance of the drawings produced while the processes involved in their production are also viewed as extremely important. When preparing solutions to problems students are not only communicating their competence to the examiner but they are also using the drawings, incrementally made, to assist in the formulation of an appropriate solution. Therefore, through the drawings, as they are given and or produced, the students come to see the nature of the problem (not visualisation as mental activity). The drawings also suggest where the next step in the solution will come from. It is in constructing the drawings that the student mediates her/his knowledge of the problem under consideration. Therefore, it is in constructing a drawing that the student solves the problem.
Chapter 5

Conclusions

The study described in the previous chapters was designed to address a number of questions related to the nature of the problem solving engaged in by T.D. students. This is currently described and explained in relation to the notion of graphicacy as defined by Gaughran. The study outlined in the previous chapters has endeavoured to describe and explain the process of problem solving from a knowledge perspective, by trying to adapt the definitions of types and levels of knowledge from the literature to the T.D. subject area, where it has not been used before. This attempt to adapt has led to the development of types and levels of knowledge which specifically relate to the activities students engage in when constructing drawn solutions to T.D. problems. In doing so, it has offered descriptions and explanations of problem solving and visualisation which relate the use of conceptual and procedural knowledge to the activities involved in making drawings of solutions to problems.

In the following sections I will discuss the significance of these new definitions of T.D. knowledge and the way in which students use knowledge to solve T.D. problems. Following this discussion are sections on how the conclusions drawn from this study might impact on teaching and learning problem solving in T.D. classrooms together with suggestions about how this study might have been improved and suggestions for further study in the area.

Types of Knowledge?

It is clear from the discussion of the literature reported in Chapter 2 that the knowledge associated with T.D. is not defined specifically in any of the official documentation associated with the subject. Knowledge is mentioned in the syllabus rationale. However, the definitions of procedural, declarative and conceptual knowledge in literature related to other subjects (notably in Mathematics and Science) are not defined for T.D.. These definitions from other subject
areas have indeed proved useful in addressing T.D. from a knowledge perspective, but it was not possible to directly translate them because of the major differences in the types of activities students engage in when drawing the solutions to T.D. problems. For example, there are major differences between the nature of activities associated with solving Mathematical and T.D. problems. The Theorem of Pythagoras, for example, can be proven relative simply using either a simple equation or diagram. In T.D. the theorem is proven by drawing the actual squares on the sides of the right angled triangle whereas the mathematical proof of the theorem is achieved using squared numbers, the same result achieved by different approaches. Early discussions in the research literature described the differences between types of knowledge but there has been a growing recognition that these distinctions are not as pronounced as was thought. However, the identification of the nature of the relationships between knowledge types is still relatively unclear. In the study reported here, it was very difficult to separate the types of knowledge into the distinct categories described in the research literature. Indeed, in many respects, the separation of knowledge into distinct types when addressing the nature of problem solving in T.D. was possible, because of the overlap of characteristics attributed to the different knowledge types. In the study reported, for example, the knowledge referred to in the literature as procedural was found to have declarative and conceptual attributes. This is the case despite their definition as distinct types. Similarly, conceptual knowledge, i.e. the relating of items of knowledge was found to have both declarative and procedural elements. Furthermore, the study appears to suggest that visualisation, traditionally thought of in terms of the internal imagery associated with developed spatial ability, is rather, a form of knowledge which can not be defined in terms of the categories of knowledge described for T.D. or other areas of activity. Thus, visualisation is not a sudden 'seeing' but a development as the activities involved in drawing a solution take place. In this way, the study reported here may have made a contribution to the discussion of the importance of the relationships between types of knowledge which is gaining prominence in the research literature from other areas of activity.

How Technical Drawing Students Use Knowledge

The study reported here has examined the definition of knowledge in a
T. D. context (as described above) and how T. D. students use this knowledge to solve T. D. problems. Prior to the study reported here the procedural, declarative and conceptual knowledge defined for other subject areas had not been examined in a T. D. context nor had T. D. students use of knowledge in their preparation of drawn solutions been studied or reported in any research. Once types of knowledge identified and defined in other areas of activity were examined in a T. D. context however, the distinctions originally made between them is now being questioned following investigations into their inter-relationships. This was the case in the study reported here, and although problem solving in T. D. (especially as it relates to preparing solutions to examination questions) may be viewed in terms of procedural knowledge it would not be wise to view it solely in procedural terms. For example, each Primary, Complementary and Incremental procedure identified here was found to have embedded in it, declarative and conceptual elements. Similarly, the conceptual knowledge associated with understanding individual topic areas, was found to have both declarative and procedural elements. So, for example, when a student used a specific Primary Procedure to draw the projections of two Intersecting Planes appropriately, s/he may or may not have been operating on a procedural level. The student may also have been aware of the declarative and conceptual implications of the procedure executed. The declarative elements relate to knowledge about the nature of planes (both the intersecting planes which are the subject of the procedure and the planes of projection used to position them) and rules of thumb which guide the execution of the procedure. The concepts involved in an appropriate solution demanded the recognition of relationships between items of declarative and procedural knowledge necessary to complete the drawing. A student with conceptual knowledge could relate all the items of knowledge required for the completion of the procedure and recognise the significance of the relationships. It should also be noted however, that students in the study were often in a position to complete specific procedures without necessarily demonstrating an understanding of the full conceptual implications of the procedure they were completing. I.e. they may have known how to advance the solution procedurally without necessarily making the connections between the types of knowledge associated with conceptualisation. Furthermore, to fulfil the requirements of such a task in the examination, it is only necessary to know how to complete the
procedure - conceptual understanding is not a requirement of completing such procedures in an examination situation. Similarly, with strategic procedures, which also have declarative and conceptual elements, students are required to demonstrate only a knowledge of how to complete the procedure to adequately fulfil the examination requirements. Therefore, problem solving in T.D. examinations may be viewed in terms of students completing procedures, i.e. procedural knowledge.

It should be noted, that the definition of strategic problem solving was different for each of the groups identified in the study and described in Chapter 4. However, problem solving had common characteristics across the four groups identified. It involved the use of different levels of the Primary, Complementary and Incremental procedures and rules of thumb identified to complete the tasks necessary to complete a drawn solution to a T.D. problem. Students from Groups 1 & 2 completed these specific and strategic procedures without difficulty and also displayed a high level of conceptual understanding of their significance. Students from Groups 3 & 4 however, often had difficulty completing all but the Primary and Complementary procedures. They also had difficulty with the higher levels of these, with most Incremental procedures, and their declarative knowledge was also deficient. They often failed to complete the correct procedures or used procedures which were inappropriate to the task being addressed, or a combination of both. The completion of tasks for which no known procedure was available involved a self-regulatory function and only students from Group 1 were regularly successful in completing the solutions to such tasks. These students were successful because their T.D. knowledge, of all types, was much more comprehensive than that of students in the other three groups.

The knowledge of constructions, rules, and facts about all the topics covered by the complete T.D. programme may be regarded as Declarative Knowledge. However, declarative knowledge should be viewed in terms of how it is used interactively with procedural and conceptual knowledge and not as a separate, inert body of knowledge. Rather, it is useful only when the student can readily identify its significance in relation to the solution of a problem. Many T.D. textbooks are organised around the declarative knowledge associated with individual T.D. topics. These may prove to be of only limited use, because students need specific instructions regarding the significance and
particular uses of items of such knowledge. This type of knowledge may be regarded as being at the lower end of a continuum at which conceptual knowledge is at the upper end. Conceptual knowledge allows the student to identify the relationships between such items of knowledge so that the solution of a particular drawing problem may be appropriately addressed. For example, a simple Conic Sections may be used to indicate how a number of items of knowledge must be linked in order to determine an appropriate solution.

Draw a straight line DVF, where DV is 45mm long and VF is 28mm long. F is the focus of an ellipse, V is the vertex and D is a point on the directrix. Draw a portion of the curve.

Students must complete the simple Primary Procedure necessary to show the given information in diagrammatic form. In order to advance the solution they must know what the terms given in the question refer to in terms of the Ellipse specifically, since the three Conic Sections on the syllabus share common characteristics. They must know that the Ellipse has two major properties and also the rule of thumb which suggests which of these properties to use to complete the drawn solution. A Complementary Procedure which enables the eccentricity graph to be drawn must then be used in conjunction with a construction to determine points on the curve (common to all three conics) in order to advance the solution. Thus, items of knowledge are linked in the development of the solution of this problem, as in others from other topic areas. Procedural, declarative and conceptual knowledge are very closely inter-related in the drawing of the solution.

The development of T.D. concepts, as is the case in other subject areas also, is not achieved by transmission. The students must be able to modify their existing conceptual models to incorporate those required in to solve drawing problems. When viewed from a constructivist perspective, problem solving is regarded as a very important part of the process of knowledge construction in the individual student. From the knowledge perspective it is also an essential element, but one which can be taught as a form of procedural knowledge. This is not achieved in the process of individual discovery which is implied by the received view of T.D. but through a process of social interaction and enculturation into the T.D. world, beginning in the classroom.
The types of knowledge elaborated in the study reported here are T.D. specific and although the conclusion drawn may be used to inform views of knowledge in other subject areas, it is not my claim that they should be applied directly in other domains.

**Problem Solving**

In the Leaving Certificate T.D. examination students are required to answer four of seven questions on each of the two papers. These questions are regarded as two- or three-dimensional spatial problems since they relate to Plane and Descriptive (Solid) Geometry and its application in the Building Technology. Each problem involves the completion of a number of tasks. The level of difficulty of these tasks is traditionally thought to relate to the level of development of students spatial ability (incorporating visualisation) from the received graphically perspective, but it may also be related to the types of knowledge required to prepare the solution.

According to the received graphically view of T.D. problem solving is inextricably linked with spatial ability. That is, good problem solvers are automatically regarded as having a highly developed spatial ability. Thus, when a student prepares an appropriate solution to a T.D. problem inferences are automatically made about the state of development of her/his spatial ability. The examination assesses the extent of the students development in the domain by assessing their ability to solve given problems. But since problem solving occurs as mental actions, the level of development can only be inferred from the drawn solutions produced. Thus, based on these drawn solutions, an assessment of the type of cognition involved in their compilation is made. This relation of particular demonstrated behaviours to cognition (which is assumed to have taken place) is tenuous at best and may not accurately reflect the development of spatial ability, even in situations where correct solutions to problems have been produced.

Only students from Group 1 in the study reported here for example, consistently completed the solution of all tasks in questions they addressed, would be regarded as having fully developed graphically. That is, because they completed all tasks satisfactorily, it is assumed that all the components of graphically are completely developed and
Students in the three other groups, who did not consistently complete appropriate solutions, would be assumed not to have a fully developed graphicacy, because their spatial ability would be assumed to be deficient or incomplete. However, assumptions made on the basis of these solutions may be inaccurate in respect of graphicacy development. Students from Groups 2, 3 and 4 were in a position to complete major sections of the solutions required to address problems they were presented with. Analysis of some of the tasks they completed from a graphicacy perspective, would indicate that these students often appeared to have the ability to successfully operate at the higher space factors while failing to complete tasks which required them to operate at lower space factors in the hierarchy. For example, students were often unable to complete given drawings successfully. This required that they hold and compare images - Space Factor 1. Simply, they could not reproduce what was given in a diagram and compare their reproduction to the original for correspondence. Although unable to complete given diagrams successfully these students frequently completed auxiliary elevations correctly. The completion of such views can be categorised as pertaining to Space Factor 3 - orientation, the ability to imagine objects viewed from alternative positions. Thus, students demonstrated the ability to operate at the higher space factor while the ability to operate at the lower level was not demonstrated. This occurred across the range of topics examined during the course of the study and represents a serious inconsistency in the graphicacy perspective. Thus, based on the evidence provided by drawn solutions, it would be unwise to make assumptions about the nature of the development of students' spatial abilities, especially since the space factors which constitute spatial ability are regarded as hierarchical. Despite these inconsistencies in respect of their spatial ability development, these students, would still have (and did) scored reasonably well in the Leaving Certificate examination. Indeed all the student from these three groups succeeded in passing the Higher Level examination - despite (what would be regarded as) their undeveloped spatial ability and visualisation according to the received graphicacy perspective. Indeed, it could be argued that these students succeeded in passing the examination because of the (limited) extent of their knowledge. Thus, from a knowledge perspective the tasks which a solution requires, may be completed as the Initial, Complementary and Incremental procedures identified in the study reported here - without reference to spatial ability or other aspects of the graphicacy perspective. Thus, students may complete appropriate
solutions without having the mental abilities associated with perception, visual perception or spatial ability. They apparently do not have to form or manipulate mental images of the configurations they are dealing with, or complete the solution of the problem as mental actions before committing it to paper. The graphicacy perspective may therefore, be seen as an alternative, though less convincing, view of problem solving in T.D. than a knowledge perspective.

Examination questions also include (usually) one task where the student is required to demonstrate fully developed visualisation and spatial ability from the received perspective or conceptual knowledge from a knowledge perspective in order to develop a suitable solution. Students with deficient visualisation and spatial ability seldom address such tasks successfully. From a knowledge perspective however, such tasks generally require the completion (only) of a relatively simple procedure which appropriately addresses the question. Thus, it is the identification of the relationships between the items of knowledge detailed in the question which allows the student to identify appropriate procedures to construct the solution.

How is Spatial Problem Solving Defined?

I believe that the graphicacy perspective is advancing a general view of problem solving, one which is applicable across the complete range of topics around which examination questions are formulated. The examination questions seek therefore, to embody tasks which require students to exercise the abilities associated with graphicacy in their solution. Thus, an examination question from any section the course, will require students to exercise their graphicacy. The student must work through this mental process, from the formulation of images through to the construction of an appropriate drawing of the solution. It is assumed that this process is applicable to questions from all sections of the course since it is assumed to be a general transferable technique. This is one major difficulty with the received view of problem solving in T.D., and one which was not evidenced in students work on examination questions during this study. Students - especially from Groups 2, 3 and 4 - were sometimes successful in completing solutions in some topic areas while failing to do so in others. For example, students were sometimes quite successful in drawing the intersection between solids for Interpenetration questions (regarded as
requiring the development of space factors from the upper end of the hierarchy) while sometimes failing to complete earlier stages of the question appropriately (involving space factors lower on the hierarchy). This evidence, I believe, disputes the received view of problem solving as a general transferable technique for T.D..

Thus, following examination of drawn solutions, the assumptions about the nature and extent of the students' abilities which are made, may be based on a perception of problem solving as a general technique which could be open to question. It may therefore, be unwise to infer that a student who can complete a comprehensive drawn solution appropriately in one area, can do so in others.

Further, the student is assumed to have developed these abilities during the programme of study in T.D.. Thus, assumptions, not only about the nature and extent of the abilities the student possesses, but also about what has been learned during study of the subject are also made. If a student completes the solution of a particular problem requiring particular space factors in any topic area she/he is assumed to be able to solve all similar problems in that area. Further, since the ability to problem solve (to a particular space factor level) in one area has been demonstrated it should also be possible for the student to problem solve at the same space factor level in other topic areas.

When a student makes a drawing of an incomplete solution, therefore, not only is an incomplete state of development of graphicacy including problem solving indicated for that particular topic, but it is also indicated for all other areas also. This is possibly the main inconsistency with the received view of problem solving in T.D. - if problem solving is not developed how is it that students can solve some problems? Or, if problem solving is developed why is it that some problems can not be solved? It appears that the received definition of problem solving is unsatisfactory!

During this study students described problems solely in terms of how the tasks associated with the completion of a satisfactory solution in a particular topic area could be completed. They described the identification and completion of procedures necessary to complete tasks in individual areas as solving problems. These descriptions are at odds with that of the received view in a number of ways.
Firstly, students look upon the solution of problems from each topic area differently. They do not (necessarily) associate the construction of a solution in one topic area with that in another, even when this is intended by the programme. From the graphicacy perspective this may not prove problematic, since problem solving is viewed as a general ability. In the study however, the students did not always make the connections between associated questions from both areas of the syllabus. So for example, when students draw the solution of a Skew Lines question from the Plane & Solid Geometry paper, they do not associate it with the Mining question from the Building Applications paper, despite the programme intention of showing the use of pure geometry in applied settings. The students view the two problems as separate - even though the geometric and procedural content are not.

Secondly, it indicates that students view problem solving as the identification and completion of procedures which lead to the construction of an appropriate drawn solution. Thus, Primary, Complementarly and Incremental procedures must be identified and related to topic specific resources which will allow an appropriate solution to be constructed. It is the completion of procedures following their identification as appropriate and not mental activity which was viewed as problem solving. During the course of this study, the students described in detail the procedures which were necessary to complete specific tasks associated with the full range of topics covered.

Thirdly, occasionally students referred to trying to picture the solution of the problem. However, they were not speaking about mental images of the configurations such as those described in the received graphicacy perspective. Rather, they were referring to visualisation from the knowledge perspective described in Chapter 4. They were trying to picture the ordered collection of procedures which lead to the development of the overall drawn solution. At every stage of its construction, the drawing served to suggest how the student should proceed. Thus, visualisation may be viewed in terms of its association with the drawn solution under construction - not as the manipulation of mental images.

Thus, students regarded problem solving as procedurally completing the tasks which allow for the construction of appropriate drawn solutions.
in individual topic areas. These procedures are general purpose and can be used across the complete range of topics. However, the need to use particular procedures can be identified only after the student has recognised the relationship between all the items of knowledge for a particular question. This will also require the use of declarative knowledge. However, it is the identification of the relationship between these items of knowledge - conceptual knowledge - which additionally facilitates the construction of an appropriate solution. This can then be completed following the correct application of the procedure.

Learning To Problem Solve

The main function of problem solving from the received perspective is in the development of students' knowledge - students problem solve to learn. Thus, from a graphicacy perspective, students think and learn about the T.D. domain by exercising their problem solving abilities in real and/or hypothetical situations. This may be viewed as similar to how Hiebert et al (1996, p.12) describe how curriculum and instruction should be based on allowing students to problematize the subject. Their thinking is progressed by the solution of these real or imagined problems. Like many other aspects of the received view of T.D., it is assumed that this function of problem solving can be used in other real or imagined problematic situations the student will encounter. The T.D. teacher, according to the received view, helps to progress this thinking and learning by designing a programme of study which relates specifically to the space factors which constitute spatial ability. Thus, programmes are graded so that exercises regarded as relating to Space Factor 1 are dealt with first. Only after the student has demonstrated that she/he can address problems related to this space factor should exercises progress to those related to Space Factor 2, and so on. Thus the teacher facilitates the progression of thinking and development related to spatial ability. Thus, according to the general and accepted constructivist view, the T.D. teacher can not transmit new concepts to the students in the classroom. These can only be learned when the student is able to modify existing conceptual models, for example when progressing from Space Factor 3 to Space Factor 4. Thus, the role of the teacher may be viewed as one of providing the environment which can best facilitate this change in student thinking. The ability of a classroom teacher to influence the development of problem solving from the received perspective therefore, may be viewed as constrained by
student readiness and the ability to advance her/his thinking and development of knowledge through problem solving.

Relevance to Professional Practice

Teaching and Learning

Using the received view of T.D. it is very difficult to make definitive statements about the (exact) nature of students' development, learning or problem solving since the nature of cognition is being linked to the evidence provided by drawn solutions.

The official documentation does not recommend a specific pedagogy which will influence the development of the processes which comprise graphiacy except to say that 'it is important that the students be involved in a variety of activities which are both interesting and relevant' (T.G. Teacher Guidelines, 1990, p. 3.), and that a variety of teaching aids (building blocks, charts, iso or squared geo-boards, two- and three-dimensional puzzles, colour and computers) should be employed in the classroom. Specific details concerning the development of learning problem solving are however, not referred to except to say that strategies developed by the teacher should be staged to address the hierarchy of space factors which lead to a developed spatial ability. Problem solving is therefore, directly linked to the space factors which constitute spatial ability, an internalised form of mental activity.

When solutions are related to a knowledge perspective however, exact relationships between the solution which has been completed and the nature of the knowledge used by the student can be identified with much more clarity and certainty. The teacher in the classroom is in a much stronger position to influence the learning, development and problem solving of the student from this perspective. The procedural, declarative and conceptual knowledge which is required to address problems as they are currently presented can be treated specifically for each of the topics examined. This specific treatment of the knowledge used in the construction of solutions will result in teaching and learning which will be much more effective in terms of addressing the problems posed. Thus, a comprehensive pedagogy for T.D. can be based on a theory of the subject which is knowledge-based. The procedures which are required to complete appropriate drawn solutions to the tasks.
involved in T.D. problems can be given specific treatment by the T.D. teacher and their use practised both in and out of the classroom. Thus, specific Primary and strategic Complementary Incremental Procedures may be taught and learned in the classroom. Similarly, control and self-regulation as form of procedural knowledge may be given specific treatment with students being taught how to plan a course of action, implement the plan by executing the procedures before evaluating the outcomes against desired goals. Thus, problem solving as a form of procedural knowledge can be taught to T.D. students.

Students effectiveness as problem solvers may then be enhanced by specific treatment in the classroom of the domain resources (constructions) and rules of thumb (heuristics). Students Geometric Databases (declarative knowledge) will then be comprised of constructions composed of particular procedures and rules to guide action in executing them for all the topics which the syllabus deals with. Particular attention can then be paid to indicating instances where it is appropriate to use this form of knowledge.

Similarly, the links between various items of knowledge (conceptual knowledge) within a particular topic area (or between topic areas) can be specifically addressed by the teacher in the classroom.

The importance of the activities associated with making drawings can also receive particular attention from the teacher. Attention to draughtsmanship can be used to heighten students awareness of the procedural, declarative and conceptual knowledge of a topic. This increased awareness can be the result of indistinct learning from drawing (acting) in different topic areas (situations), or from specific treatment of the knowledge associated with drawing activities by the teacher.

Since visualisation (linked to the cognition associated with spatial ability in the received view of T.D.) is inextricably linked to the drawings which constitute solutions to problems from a knowledge perspective, it is very important that students are taught how to look at drawings under construction in ways which will suggest further action. At every stage of the construction of a solution the partially completed drawing will suggest further action which should be taken. This way of
knowing what to do next by looking at drawings during construction must receive specific treatment by the teacher if students are to learn how the procedures necessary to complete an appropriate solution may be identified from a drawing in progress.

Thus, all the types of knowledge identified in the literature can be applied to the T.D. classroom situation, where they may receive specific treatment by the teacher.

The solutions to T.D. problems which students prepare can therefore, be described and explained in knowledge terms in a much more comprehensive manner than the descriptions and explanations advanced by the received view of T.D..

Assessment

Under the current regime, T.D. is assessed using a norm-referenced terminal examination. This reflects a tendency to 'generalise from limited evidence' (Nuttall, 1989, p. 265) development and achievement within the T.D. domain. The ability of such an assessment strategy to serve the learner in a constructive way is questionable and I believe that a criterion-referenced model of assessment similar to that described in the Hargreaves Report (in Broadsfoot et al, 1989, p. 298) would prove more advantageous for students, teachers, parents and the wider T.D. community. Such a strategy, reflecting current best practice in assessment, would encompass assessment through examinations, application of knowledge, evidence of personal and social skills, positive attitudes, motivation and commitment and may be regarded as the most comprehensive way of assessing the students knowledge, competence as problem solvers and experience across the range of activities associated with T.D.. Thus an assessment strategy which employs an aggregation (Task Group Report: 1987 - Preface) of the results of classroom tasks, problems, formal tests, group projects and classroom dialogue is proposed.

Classroom tasks would be similar to the tasks which students must complete in the current examination. However, emphasis would be placed on collaboration between students, and between students and the T.D. teacher. These tasks would require the completion of first order specific (Primary) procedures directed toward known goals and/or
second order procedures directed at unfamiliar goals but operating on specific strategic (Complementary and Incremental) procedures and would provide the opportunities for student learning in the classroom. Further, tasks would also contain problems which would require the exercise of a control function where cognition would have to be switched between first and second order procedures in order to complete them. Analysis of the tasks associated with current examination question suggests that many of the procedures which students are required to complete are lower order. In order to (accurately) assess the extent of (all) T.D.-specific knowledge examination questions must reflect the need to use all types and levels of knowledge in formulating solutions. This would appear not to be the case at present, where large portions of questions can be addressed by applying specific and strategic procedures.

Formal tests would allow for comparisons of results in respect of use of all levels of procedures and drawing conventions.

Group projects would address procedural, declarative and conceptual knowledge and would also provide opportunities for peer group teaching and learning. Such projects would be open-ended and extensive in scope and would have to be completed within a negotiated time-frame. These could also fulfil homework requirements over the period of time allocated to them while engaging students in personally and culturally meaningful activities.

Students involvement in classroom lessons and their demeanour - i.e. their cognitive, social, personal, psychomotor skills, and attitudes to learning described by Broadfoot et al (1989, p. 296) - needs to be recorded. Thus, a method of recording classroom dialogue and student activity which evidences the quality and quantity of student involvement in classroom activity and dialogue over the assessment period must be developed.

This assessment procedure is designed to inform the T.D. teacher of the development of each students knowledge for diagnostic as well as assessment purposes and to inform students of the nature of development of their knowledge as they progress through the T.D. programme. This would require that formal records of student performance across the range of assessment instruments be kept by the
classroom teacher, and is directly opposed to the views on formal
assessment for accreditation advanced by the teacher unions (including
the union of which I am a member).

Students would have to be aware that their overall assessment in T.D.
will be derived from an aggregation of all these assessment procedures.

I believe that assessment strategies similar to that described above are
currently operated informally by many T.D. teachers with an aggregated
form of assessment based on classroom performance, homework, test
results and general attitude being used to assess student development
and performance - this however does not extend to assessment for
accreditation purposes. One major advantage of the proposed assessment
strategy would be the development of a perception of the pupil's 'actual
intelligence across a range of activities' (Sternberg: 1989, p.241), and
not solely based on T.D. examination results. Another advantage is its
ability to distinguish qualitative differences between individual students
(and between the sexes) based on evidence from a number of sources.

What of C.A.D.?

As discussed earlier in this dissertation, there is a great difference
between the activities engaged in by T.D. students in post-primary
schools and the activities engaged in by T.D. professionals working in
technologically based industries. All the drawings prepared in the
classroom are drawn on the board with drawing instruments. The use
of Computer Aided Design (C.A.D.) is now pervasive in industry and
it is unlikely that the T.D. students who took part in the study
reported here will spend much time at the drawing board should they
take up employment as technicians or professionals in industry. I do
not necessarily believe that this is an argument for bringing C.A.D. into
the classroom, and currently there is no requirement to do so at Senior
Cycle level (although there is a section on computer graphics in Junior
Cycle T.G.). I believe that it is important to develop students
geometric knowledge and problem solving before they have access to a
computer (which will undoubtedly produce the drawing more quickly
and accurately). This can be achieved satisfactorily using a knowledge
perspective and traditional means (board and instruments). Much of the
drawing done in industry is process and product specific and although
general C.A.D. training is necessary for out of school T.D. it is
perhaps more appropriate that these industry-specific C.A.D. skills should be taught and learned in the context in which they will be used since they may have little relevance for technicians or professionals in other areas of activity.

If computer drawing is to feature more prominently in future T.D. schools programmes then substantial investment in the necessary computer hardware and software is essential - perhaps it is the need for this investment which has resulted in computer drawing not featuring in the current Senior Cycle syllabus.

Disseminating the Findings of this Study

To my Colleague T.D. Teachers

In order that the maximum number of students may benefit from approaching T.D. from a knowledge perspective it will be necessary to inform those involved in teaching the subject at post-primary level in the first instance. Informing teachers of the benefits of this approach will be undertaken in a number of ways. I intend to prepare a series of articles based on this study for publication in the quarterly journal published by the Association of Materials Technology and Graphics Teachers and the equivalent journal of the Association of Engineering Teachers. These articles would focus on describing and explaining T.D. activities from a knowledge perspective. The problem solving involved in individual topics will then be examined from a knowledge perspective, highlighting how this provides a more comprehensive explanation of the activities associated with drawing solutions to T.D. problems. These articles will allow T.D. teachers who are members of these associations to have direct access to the ideas raised in this study.

To Future T.D. Students

I also intend to prepare a textbook which will define the tasks associated with examination type questions in terms of the knowledge discussed in this study, one that can be used in T.D. classrooms. This would be an important development since the format of current textbooks tend to ensure that they are not used in the classroom context. Thus, they tend not to be useful from either the teaching or
learning perspective. A review of the whole Junior and Senior Cycle Curricula is currently underway and the National Council for Curriculum and Assessment will shortly be seeking submissions from subject teachers. This will be an ideal opportunity to influence national policy on T.D. and I will take the opportunity to make a detailed submission based on this study.

To Others

The study reported here will be of interest to other educationalists not directly concerned with T.D. or other related technological subjects. The characteristics of the traditional types of knowledge which this study has raised will be of interest to a wider audience and therefore, articles will also be submitted to education journals whose readership is interested in the definition of types of knowledge and problem solving. Indeed, one of the most important contributions made by this study was in identifying visualisation as a form of knowledge in the solution of T.D. problems. This highlights the danger of extrapolating to other situations knowledge types identified in others subject areas and serves to highlight the need to conduct research in all subjects on the curriculum. Therefore, this is not significant only in T.D - other subject specific forms of knowledge may be identified in subjects which have not been the focus of research interest in Ireland or elsewhere.

Things I Would Do Differently

Official Comment on Study

Despite several attempts over the course of this research project I was not able to get any reaction from The Department of Education & Science in respect of the official views of learning, development and problem solving in T.D.. I wrote on several occasions to the Chief Inspector with responsibility for T.D. asking him to outline the nature of official views, because they were not very clear from official documentation. I never received any acknowledgement from him. He has since retired, and a new inspector with sole responsibility for T.D. has yet to be appointed. I think that it is unlikely that any comment will be forthcoming from official quarters until this appointment has been made (I have written to the Department of Education & Science in the interim period, again without receiving any acknowledgement).
During the course of my work I also made several attempts to contact Mr. Gaughran at the University of Limerick. The first of these was by e-mail about the time my Initial Study was completed. I received an acknowledgement approximately one year later. Although I outlined the nature of my research focus to him at the time, no worthwhile correspondence resulted from the initial contact. I feel that comment from the T.D. Inspector and from Mr. Gaughran, during the course of my study would have been extremely helpful as I tried to formulate my views of the received perspective of the subject. As it was, I had to interpret the official perspective and formulate my own views of the subject in relation to this interpretation. I was left with no option but to do this since I did not have access to official comment and no research, other than that conducted by Gaughran and reported in the Chapter 2, has been carried out into T.D.. It is interesting to note that in recent papers Gaughran (1996a; 1996b; 1997a; 1997b) continues to espouse the importance of spatial ability and spatial cognition and I believe that, had a dialogue between his perspective and that reported here taken place during this study, it would have proved mutually beneficial in informing both perspectives.

Social Interactions in the Technical Drawing Classroom

The teacher in a T.D. classroom is in a better position to influence the development of student problem solving in the classroom from a knowledge perspective. Since problem solving centres around the identification and use of procedures which can be used to address the solution of the tasks associated with particular topics it can be the subject of specific treatment by the teacher. Thus, the Initial. Complementary and Incremental procedures which are appropriate for each topic can be taught specifically by the classroom teacher. The declarative knowledge for specific topics can also be addressed in order that the students have access to knowledge of facts, rules and principles in particular topic areas. Further, the relationships between items of knowledge which forms conceptual knowledge can also be the subject of specific treatment by the teacher. Thus, all the elements which are required to complete a drawn solution to drawing problems can be given specific treatment by the teacher in the classroom. Therefore, the interaction of teacher and students takes on a greater
significance from a knowledge perspective. This is recognised by the philosophical perspectives outlined in Chapter 2 and I believe the nature of the interactions in the T.D. classroom should become the focus of further studies.

**Further Examination of Technical Drawing Knowledge**

The topics studied as part of the T.D. programme at post-primary level in Ireland, although comprehensive, represent only a selection of geometry and its applications and post-primary T.D. would benefit from further in-depth studies of types of knowledge. Although the knowledge associated with fourteen geometrical topics was examined during this study the seven topics associated with Part 2(a) of the programme - Engineering Applications - was not examined. This should be undertaken as soon as possible, preferably by a teacher with expertise in this area of the syllabus, before other programmes in T.D. are examined in the particular contexts in which they are undertaken.

**The Context in Which Drawings Are Made**

The importance of the physical context in which the solutions to T.D. problems are made also needs to be investigated further. For example, (I have done this to some extent) a rationale for the subject could be developed which relates to Vygotsky's activity theory, with its emphasis on the structure of activity, object-orientedness, internalisation and externalisation, mediation and development. Similarly, an extension of the present study which would examine in more detail how the knowledge identified is situated in the context of the activity and the culture in which it is developed and used (Seeley Browne et al, 1989) could also benefit future T.D. students.

**Work With More Students**

Twenty two students in total were associated with this study, four in the Initial Study and eighteen in the Main Study. I did not regard these relatively small numbers as problematic and it did not affect the generation or analysis of data from my methodological perspective. However, others may regard the number of participating students as small for a study which is intend to influence the provision of T.D. for about 20,000 students annually. Although my methodology was
qualitative, I attempted to have the significance of the differences within and between the groups I identified statistically analysed so that the results the students achieved in their Leaving Certificate examination could be compared to these results. So far I have not succeeded in achieving this. This is primarily the result of dealing with such small numbers, but can also be related to difficulties encountered because a specific score is not given. Such an analysis is not an essential feature of my overall analysis, but may have been used to support the interpretative analysis which I was using. In this way, larger numbers of students might benefit future studies.

The method of selection of students to participate was opportunistic. Future studies might benefit from samples which are more representative of the students taking the subject at post-primary level. For example, the number of girls who take T.D. is slowly increasing in my school and in my Year 5 class this year there are four girls, about 20% of the class group. Whereas, Sara who took part in my study, represented less than 5% of the student involvement.

**More Research**

Very little research has been conducted in technological subjects in Ireland even though these subjects are among the oldest on the curriculum. I believe that research into these subjects would benefit teachers and students alike.
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Technical Drawing Marking Scheme (1997), Department of Education & Science Examinations Branch, Athlone, Co. Westmeath, Ireland.


Thurstone, L.L. (1938) 'Primary Mental Abilities' Psychometric Monograph, 1.


Appendices

William Patrick Bolger

M7137754

The Inter-Relationship of Procedural and Conceptual Knowledge in Two- and Three-Dimensional Spatial Problem Solving of Technical Drawing Students

DOCTOR OF EDUCATION (EdD)

2001
Appendix 1 - Other Topics Addressed in the Leaving Certificate T.D. Examination.

<table>
<thead>
<tr>
<th>Paper 1: Plane &amp; Solid Geometry</th>
<th>Paper 2: Building Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Areas of Figures</td>
<td>Roof Geometry</td>
</tr>
<tr>
<td>Special Curves</td>
<td>Projection of Shadows</td>
</tr>
<tr>
<td>Oblique Planes</td>
<td>Parabolic Shell</td>
</tr>
<tr>
<td></td>
<td>Structures</td>
</tr>
</tbody>
</table>
### Appendix 2 - Descriptions of Tasks and Levels of Procedures Associated with Other Topics Detailed in Appendix 1.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Building Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Areas of Figures</strong></td>
<td><strong>Roof Geometry</strong></td>
</tr>
<tr>
<td>a) Task 1: Draw the initial figure (P.P. - Level 1).</td>
<td>a) Task 1: Draw the initial figure, usually portion of the plan (P.P. - Level 2).</td>
</tr>
<tr>
<td>Task 2: Complete the given figure. (C.P. - Level 4).</td>
<td>Task 2: Complete the projections of the roof (C.P. - Level 3/4). This may also involve an I.P. - Level 3/4.</td>
</tr>
<tr>
<td>Task 4: Divide the area as required. (I.P. - Level 3).</td>
<td></td>
</tr>
<tr>
<td><strong>Special Curves</strong></td>
<td><strong>Projection of Shadows</strong></td>
</tr>
<tr>
<td>Task 2: Draw the locus of the given point for the movement required, usually a cycloid (I.P. - Level 3).</td>
<td>b) Task 2: Determine the true angle of light (C.P. - Level 2).</td>
</tr>
<tr>
<td>Task 3: Draw the locus for the second movement, usually an involute (I.P. - Level 3).</td>
<td>Task 3: Determine the shadow cast by the main part of the building (C.P. - Level 3/4).</td>
</tr>
<tr>
<td><strong>Oblique Planes</strong></td>
<td>Task 4: Determine the shadow cast by the sphere (I.P. - Level 4).</td>
</tr>
<tr>
<td>a) Task 1: Draw the initial diagram, usually the partial plan of a solid cut by an oblique plane (P.P. - Level 2/3).</td>
<td></td>
</tr>
<tr>
<td>Task 2: Determine the traces of the plane VTH (C.P. - Level 2/3).</td>
<td></td>
</tr>
<tr>
<td>Task 3: Complete the projections of the solid (C.P. - Level 2/3).</td>
<td></td>
</tr>
<tr>
<td>b) Task 4: Complete the projections of a given solid, often a tetrahedron) when the corners of the base are in given positions relative to H.P. (I.P. - Level 4).</td>
<td></td>
</tr>
<tr>
<td><strong>Parabolic Shell Structures</strong></td>
<td></td>
</tr>
<tr>
<td>a) Task 1: Draw the initial diagram (P.P. - Level 2).</td>
<td></td>
</tr>
<tr>
<td>b) Task 2: Complete the parabolas in elevation and end view (C.P. - Level 3).</td>
<td></td>
</tr>
<tr>
<td>c) Task 3: Complete the plan (I.P. - Level 2).</td>
<td></td>
</tr>
<tr>
<td>Task 4: Complete any sections that are required (C.P. - Level 3/4).</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix 3 - Rules of Thumb Associated with Topics Detailed in Appendix 1.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rules of Thumb</strong></td>
<td>Mean proportion usually involved. Many questions based on Circle Theorems. Conversions from polygons to triangles to rectangles to squares. Proportional lines also useful for areas in given ratios.</td>
<td>Curves are usually cycloidal. Plot locus for each stage of movement sequentially - tracing paper with moving circle is often very useful here. Ditto for involutes - sequential plotting and tracing paper.</td>
<td>Determine traces by extending given parts of section to H.P. &amp; V.P. Draw an auxiliary elevation to determine remaining cut surface. A secondary figure is required to position base relative to H.P. or V.P.. Use true lengths of sides here.</td>
</tr>
<tr>
<td><strong>Building Applications</strong></td>
<td><strong>Roof Geometry</strong></td>
<td><strong>Projection of Shadows</strong></td>
<td><strong>Parabolic Shell Structures</strong></td>
</tr>
<tr>
<td><strong>Rules of Thumb</strong></td>
<td>Use auxiliary views to complete projections showing surfaces at given inclinations. Use traditional construction for dihedral angle in reverse where dihedral angle is given. Use two auxiliaries (or traditional construction) to determine dihedral angle.</td>
<td>Use an auxiliary elevation at right angles to plan direction of light to determine true angle. Draw elliptical shade and shadow cast by sphere by determining the axes of the ellipses.</td>
<td>Identify positions of vertices in elevation and end view before drawing parabolas. Use translated parabola on end view to determine widths of vertical sections for plan. Note where vertical sections are cut by section planes to complete views.</td>
</tr>
</tbody>
</table>
Appendix 4 - Summary of Observation Data Group 1.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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<tr>
<td>Carl</td>
<td>Consistently completed all procedures. Tasks completed in order.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gary</td>
<td>Consistently completed all procedures. Tasks completed in order.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paral</td>
<td>Occasionally had difficulty with Tasks 2, 3 &amp; 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
</tr>
<tr>
<td>Oliver</td>
<td>Occasionally had difficulty with Tasks 2, 3 &amp; 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Roof Geometry</th>
<th>Projection of Shadows</th>
<th>Parabolic Shell Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carl</td>
<td>Consistently completed all procedures. Tasks completed in order.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gary</td>
<td>Consistently completed all procedures. Tasks completed in order.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paral</td>
<td>Occasionally had difficulty with Task 3.</td>
<td>Occasionally had difficulty with Task 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
</tr>
<tr>
<td>Oliver</td>
<td>Occasionally had difficulty with Task 3.</td>
<td>Occasionally had difficulty with Task 4.</td>
<td>Occasionally had difficulty with Task 4.</td>
</tr>
</tbody>
</table>
### Appendix 5 - Summary of Interview Data Group 1.

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<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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</thead>
<tbody>
<tr>
<td>Carl</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gary</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parale</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 3 concerning identification of locus of point through construction lines</td>
<td>Difficulty with Task 4 concerned how to link the items of knowledge to suggest the I.P. required.</td>
</tr>
<tr>
<td>Oliver</td>
<td>Declarative (rules of thumb and constructions), procedural and conceptual knowledge evidenced.</td>
<td>Difficulty with Task 3 concerning identification of locus of point through construction lines</td>
<td>Difficulty with Task 4 concerned how to link the items of knowledge to suggest the I.P. required.</td>
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### Appendix 6 - Summary of Observation Data Group 2.

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<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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</thead>
<tbody>
<tr>
<td><strong>Pat</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3-&lt;br&gt;LP.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2, 3 &amp; 4-&lt;br&gt;C.P.'s and I.P..</td>
</tr>
<tr>
<td><strong>David</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3-&lt;br&gt;LP.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2, 3 &amp; 4-&lt;br&gt;C.P.'s and I.P..</td>
</tr>
<tr>
<td><strong>Michael</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3-&lt;br&gt;LP.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2, 3 &amp; 4-&lt;br&gt;C.P.'s and I.P..</td>
</tr>
<tr>
<td><strong>Justin</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3-&lt;br&gt;LP.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2, 3 &amp; 4-&lt;br&gt;C.P.'s and I.P..</td>
</tr>
<tr>
<td><strong>Derek</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3-&lt;br&gt;LP.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2, 3 &amp; 4-&lt;br&gt;C.P.'s and I.P..</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building Applications</th>
<th>Roof Geometry</th>
<th>Projection of Shadows</th>
<th>Parabolic Shell Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pat</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P. &amp; I.P..</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P..</td>
</tr>
<tr>
<td><strong>David</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P. &amp; I.P..</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P..</td>
</tr>
<tr>
<td><strong>Michael</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P. &amp; I.P..</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P..</td>
</tr>
<tr>
<td><strong>Justin</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P. &amp; I.P..</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P..</td>
</tr>
<tr>
<td><strong>Derek</strong></td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 2 &amp; 3&lt;br&gt;C.P. &amp; I.P.'s.</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P. &amp; I.P..</td>
<td><strong>Difficulty with</strong>&lt;br&gt;Tasks 3 &amp; 4-&lt;br&gt;C.P..</td>
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### Appendix 7 - Summary of Interview Data Group 2

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>Items of knowledge required to complete Tasks 2, 3 &amp; 4 not being linked.</td>
<td>Confusion with procedures required to complete Tasks 2 &amp; 3.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
</tr>
<tr>
<td>David</td>
<td>Items of knowledge required to complete Tasks 2, 3 &amp; 4 not being linked.</td>
<td>Confusion with procedures required to complete Tasks 2 &amp; 3.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
</tr>
<tr>
<td>Michael</td>
<td>Items of knowledge required to complete Tasks 2, 3 &amp; 4 not being linked.</td>
<td>Confusion with procedures required to complete Tasks 2 &amp; 3.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
</tr>
<tr>
<td>Justin</td>
<td>Items of knowledge required to complete Tasks 2, 3 &amp; 4 not being linked.</td>
<td>Confusion with procedures required to complete Tasks 2 &amp; 3.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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<tr>
<td>Derek</td>
<td>Items of knowledge required to complete Tasks 2, 3 &amp; 4 not being linked.</td>
<td>Confusion with procedures required to complete Tasks 2 &amp; 3.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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<table>
<thead>
<tr>
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<th>Parabolic Shell Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat</td>
<td>Items of knowledge required to complete Tasks 2 &amp; 3 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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<tr>
<td>David</td>
<td>Items of knowledge required to complete Tasks 2 &amp; 3 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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<tr>
<td>Michael</td>
<td>Items of knowledge required to complete Tasks 2 &amp; 3 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
</tr>
<tr>
<td>Justin</td>
<td>Items of knowledge required to complete Tasks 2 &amp; 3 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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<td>Derek</td>
<td>Items of knowledge required to complete Tasks 2 &amp; 3 not being linked.</td>
<td>Items of knowledge required to complete Tasks 3 &amp; 4 not being linked.</td>
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Appendix 8 - Summary of Observation Data Group 3.

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<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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<tbody>
<tr>
<td>Michelle</td>
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<tr>
<td>Stephen</td>
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<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<tr>
<td>Martin</td>
<td>Occasionally had difficulty with Tasks 2, 3 &amp; 4.</td>
<td>Occasionally had difficulty with Tasks 2 &amp; 3.</td>
<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<tr>
<td>Denis</td>
<td>Occasionally had difficulty with Tasks 2, 3 &amp; 4.</td>
<td>Occasionally had difficulty with Tasks 2 &amp; 3.</td>
<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<tr>
<td>Brian</td>
<td>Occasionally had difficulty with Tasks 2, 3 &amp; 4.</td>
<td>Occasionally had difficulty with Tasks 2 &amp; 3.</td>
<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<tr>
<th>Building Applications</th>
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<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<tr>
<td>Stephen</td>
<td>Occasionally had difficulty with Tasks 2 &amp; 3.</td>
<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<td>Martin</td>
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<td>Occasionally had difficulty with Tasks 3 &amp; 4.</td>
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<td>Denis</td>
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<td>Brian</td>
<td>Occasionally had difficulty with Tasks 2 &amp; 3.</td>
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### Appendix 9 - Summary of Interview Data Group 3.

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<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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<tbody>
<tr>
<td>Micheál</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
<td></td>
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<tr>
<td>Stephen</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
<td></td>
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<tr>
<td>Martin</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Denis</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Brian</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Micheál</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Stephen</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Martin</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Denis</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Brian</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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## Appendix 10 - Summary of Observation Data Group 4

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<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
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</tr>
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### Appendix 11 - Summary of Interview Data Group 4.

<table>
<thead>
<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
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<th>Oblique Planes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenneth</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Cathal</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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</tr>
<tr>
<td>Enda</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trevor</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
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<th>Parabolic Shell Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kenneth</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Cathal</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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<tr>
<td>Enda</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
<td></td>
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</tr>
<tr>
<td>Trevor</td>
<td>Procedural difficulties with all but Task 1. Some rules of thumb not known, others not followed. Not linking items of knowledge to complete C.P.'s or I.P.'s.</td>
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</table>
### Appendix 12 - Summary of Audio-Recording of Pairs Data Across Groups

<table>
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<tr>
<th>Plane &amp; Solid Geometry</th>
<th>Areas of Figures</th>
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<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective</td>
<td></td>
<td></td>
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<tr>
<td><strong>Group 3</strong></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
<td></td>
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<tr>
<td><strong>Group 4</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
<td></td>
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<table>
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<tr>
<th>Building Applications</th>
<th>Roof Geometry</th>
<th>Projection of Shadows</th>
<th>Parabolic Shell Structures</th>
</tr>
</thead>
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<tr>
<td><strong>Group 1</strong></td>
<td></td>
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<td></td>
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<td></td>
<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective</td>
<td></td>
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<tr>
<td><strong>Group 2</strong></td>
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<td>Evidence of planning, checking, evaluating and revising strategies. Evidence also of visualisation from a knowledge perspective</td>
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<tr>
<td><strong>Group 3</strong></td>
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<tr>
<td></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
<td></td>
<td></td>
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<tr>
<td><strong>Group 4</strong></td>
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<tr>
<td></td>
<td>Discussed the completion of P.P.'s, C.P.'s and I.P.'s. Seldom completed other than P.P.'s. C.P.'s and I.P.'s Discussed only in terms of how to complete procedures. No evidence of self-regulation or visualisation from a knowledge perspective.</td>
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## Appendix 13 - Differences Between Selected Individuals Across Groups

<table>
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<th>Plant &amp; Solid Geometry</th>
<th>Areas of Figures</th>
<th>Special Curves</th>
<th>Oblique Planes</th>
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</thead>
<tbody>
<tr>
<td><strong>Gary Group 1</strong></td>
<td>Comprehensive knowledge of all procedures, concepts and rules of thumb.</td>
<td>Knowledge of procedures and rules of thumb. Sometimes had difficulty with Tasks 3 &amp; 4.</td>
<td>Knowledge of procedures and rules of thumb. Sometimes had difficulty with Task 4.</td>
</tr>
<tr>
<td><strong>Stephen Group 3</strong></td>
<td>Often had difficulties completing Task 2. Usually completed Tasks 1 &amp; 2 but seldom completed Task 3.</td>
<td>Usually completed Tasks 3 &amp; 4.</td>
<td>Seldom completed Tasks 2, 3 &amp; 4.</td>
</tr>
<tr>
<td><strong>Trevor Group 4</strong></td>
<td>Frequently had difficulties completing Task 1 &amp; 2. Usually completed Tasks 1 &amp; 2 but seldom completed Task 3.</td>
<td>Usually completed Tasks 1 &amp; 2. Sometimes attempted Task 2 but seldom completed it or Task 3.</td>
<td>Seldom completed Tasks 2, 3 &amp; 4. No rules of thumb.</td>
</tr>
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</table>

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<thead>
<tr>
<th>Building Applications</th>
<th>Roof Geometry</th>
<th>Projection of Shadows</th>
<th>Parabolic Shell Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>David Group 2</strong></td>
<td>Knowledge of procedures and rules of thumb. Sometimes had difficulties with Tasks 2 &amp; 3.</td>
<td>Knowledge of procedures and rules of thumb. Sometimes had difficulty with Task 4.</td>
<td>Tasks 1 &amp; 2 usually completed. Tasks 3 &amp; 4 often attempted but usually not completed.</td>
</tr>
<tr>
<td><strong>Stephen Group 3</strong></td>
<td>Seldom completed Task 2. Often unsuccessful in completing Task 3.</td>
<td>Tasks 1 &amp; 2 usually completed. Tasks 3 often completed but Task 4 usually not completed.</td>
<td>Tasks 1 &amp; 2 usually completed. Tasks 3 &amp; 4 often attempted but usually not completed.</td>
</tr>
<tr>
<td><strong>Trevor Group 4</strong></td>
<td>Seldom completed Task 2 successfully. Usually unsuccessful in completing Task 3.</td>
<td>Tasks 1 usually completed. Task 2 often unsuccessfully attempted. Tasks 3 &amp; 4 sometimes attempted but usually not completed.</td>
<td>Tasks 1 completed. Task 2 occasionally attempted. Tasks 3 &amp; 4 sometimes attempted but usually not completed.</td>
</tr>
</tbody>
</table>

William P. Bolger - M7137754 162
1. Given the horizontal and vertical projections of two planes ABC and DEF.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<td>55</td>
<td>.15</td>
<td>95</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) Determine the line of intersection between the planes.
(b) Determine the dihedral angle between the planes.
(c) Determine the horizontal trace and the vertical trace of a plane which passes through the points B, C and E.
(d) On a separate diagram, draw the projections of the skew lines AC and DE and show the projections of the shortest horizontal distance between them.
Fig. 1 shows a circle and an irregular pentagon ABCDE. The area of the triangle CDE is half that of the quadrilateral ABCE.

(a) Draw the given figure showing all constructions clearly.

(b) On a separate diagram, inscribe an equilateral triangle in the pentagon ABCDE having one vertex at C and the other two vertices on the sides AB and DE, respectively.

Fig. 2 shows the elevation of a right cone A and a sphere B which are in contact with each other.

(a) Draw the elevation and plan of the solids.

(b) Draw the projections of another sphere in the position C so that it rests on the horizontal plane, is in contact with the cone and touches the sphere B at a point 40mm above the horizontal plane.

(c) Draw the traces of a tangent plane to the cone and sphere B.
4. Fig. 3 shows the projections of an equilateral triangular prism of 95mm side lying on the horizontal plane.

Also shown are the projections of a solid whose cross-section is a quadrant of a circle of 40mm radius which penetrates the prism.

Draw the projections of the solids showing all lines of interpenetration.

![Fig. 3](image)

5. Fig. 4 shows the projections of a cylinder which lies on the horizontal plane. The diameter of the cylinder is 100 mm and its length is 140 mm. Also shown is a straight line PQ on the surface of the cylinder.

The cylinder rolls clockwise along the horizontal plane for one complete revolution. During the rolling of the cylinder the point P moves along the straight line to Q.

Draw the locus of the point P in plan and elevation for the combined movement.

![Fig. 4](image)
(a) Draw a straight line DVF, where DV is 45mm long and VF is 28mm long. F is a focus of an ellipse, V is the vertex and D is a point on the directrix.

Draw a portion of the curve.

(b) Draw a triangle AFP where AF = 35 mm, FP = 70 mm and AP = 85 mm. F is one of the focal points of a double hyperbola, A is a point on the transverse axis, AP is a tangent to the curve and P is the point of contact.

Draw a portion of the double curve.

Fig. 5 shows the plan and elevation of a regular tetrahedron of 90mm side. The tetrahedron has been cut by an oblique plane as shown.

The true length of the edge DF is 57mm and the cut surface DEF is inclined at 40° to the horizontal plane.

(a) Draw the given plan and elevation.

(b) On a separate diagram, draw a plan and elevation of the complete tetrahedron when the corner C rests on the horizontal plane and the corners A and B are 20mm and 60mm, respectively, above the horizontal plane.
INSTRUCTIONS

(a) Answer four questions.
(b) All questions carry equal marks.
(c) Construction lines must be shown on all solutions.
(d) Write the number of the question, distinctly, on the answer paper.
(e) First or third angle projection may be used.
(f) All measurements are given in metres.

1. Draw a perspective view of the structure shown in Fig. 1. The picture plane passes through the corner A, the spectator S is 10 m from the corner A and the horizon line is 9 m above the ground line.

Use auxiliary vanishing points where appropriate.

Scale 1 : 100

\[ \text{Fig. 1} \]
Fig. 2 shows the outline plan and elevation of a roof. Surface B has a pitch of 50°, surface C has a pitch of 45° and surface E has a pitch of 40°. The dihedral angle between the surfaces A and B is 130°.

Draw the given plan and elevation and determine the dihedral angle between the surfaces D and E. Scale 1 : 100

Fig. 2

Fig. 3 shows the outline plan and elevation of a building.

Draw the given views and determine the shadows and shade in plan and elevation when the direction of the light is as shown in the figure. Scale 1 : 200

Fig. 3
Fig. 4 shows the outline plan and elevation of a building. It is in the form of a hyperboloid of revolution. The outline of an entrance which projects from the main building is also shown.

Draw the plan and elevation of the building.

Scale 1 : 200

On a contour map A and B are two points whose altitudes are 125 m and 130 m, respectively. On the map B is located 125 m east of A. A skew bore-hole at A is drilled in a north-easterly direction in plan and has an actual inclination of 65° to the horizontal plane. It reveals the top and bottom surfaces of a stratum at altitudes of 105 m and 40 m, respectively.

A skew bore-hole at B is drilled in a south-westerly direction in plan and has an actual inclination of 50° to the horizontal plane. It reveals the top and bottom surfaces of the stratum at altitudes of 85 m and 70 m, respectively.

(a) Determine the dip, strike and thickness of the stratum.

(b) Another skew bore-hole at B is drilled in a south-easterly direction in plan. The length of the hole as it goes through the stratum is 30 m. Determine the inclination of this bore-hole to the horizontal plane and find the distance from B to the top surface of the stratum along the bore-hole.

Scale 1 : 1000
Fig. 5 shows the outline plan of a roof which contains five hyperbolic paraboloid roof surfaces. The roof perimeter is square in plan. The corners A, B, C and D are at ground level, corners G and E are 12 m above ground level and corners F and H are 16 m above ground level.

(a) Draw the given plan of the hyperbolic paraboloid surfaces ABEF and EFGH and project an elevation.

(b) Show the curvature of the roof along a line joining G to B.

(c) Determine the traces of the plane director for the edges AF and BE of the surface ABEF and having its horizontal trace passing through B.

The accompanying drawing shows ground contours at five-metre vertical intervals. AB is the line of a proposed roadway and CDEFG is a proposed parking area.

The roadway AB has the following specifications:

(i) formation width 15 m;
(ii) formation level at A is 80 m;
(iii) gradient A to B is 1 in 20 rising;
(iv) side slopes for cuttings 1 in 1.5;
(v) side slopes for embankments 1 in 2.

In the parking area the sides DE and GF are level and the gradient from C to D is 1 in 20 falling. The side slopes for cuttings and embankments are the same as for the roadway.

On the drawing supplied show the earthworks necessary to accommodate the roadway and parking area.
Appendix 15 - Examples of Observation Data.

Carl - Observation
Tuesday 11th May, 1999.

Perspective Projection

1. Draw a perspective view of the structure shown in Fig. 1. The picture plane passes through the corner A, the spectator S is 10 m from the corner A and the horizon line is 9 m above the ground line. Use auxiliary vanishing points where appropriate.

Scale 1 : 100

Task 1. Began by positioning spectator below plane in relation to it.

Task 2. No difficulty working sequential front. Didn't draw elevation.

Task 3. Set up correct viewing point, but marked heights on line and showed using colouring pencils.

Ground line & horizon line - (but there on side away from spectator not shown here)

2/3/4 Comments

Task 1. Started drawing perspective view at edge A seen to the right PP.

He went on to build up the perspective as an angular sided front.

Task 2. Showed sequential front first.

Task 3. This was constructed using lines to show as shown in the solution.

1/6/99.
3. Fig. 3 shows the outline plan and elevation of a building.

Draw the given views and determine the shadows and shade in plan and elevation when the direction of the light is as shown in the figure.

![Outline plan and elevation of a building.](image)

Scale 1:200

---

**Gary - Observation**

**Tuesday 21st September, 1999**

**Projection of Shadows**

Began with plan, then went on to draw elevation.

Looked at this intersection of the cylinder & inclined surface for a while before completing it. Used "Derogaphic" paper to trace shadows.

Just drew an aux. of the spherical dome. Drew one light & divided it to face a light to determine shade/light.

Again — spent some time looking at completed aux. & plan.

Drew cast shadow on HP first for main building. Shaded on vertical surface.

Then used project lines of angle of light on plan to determine shadow cast by dome after it'd drawn in the shading on the dome. Be used a dia. & projection & plan from aux. to do this.
1. Given the horizontal and vertical projections of two planes ABC and DEF.

<table>
<thead>
<tr>
<th>Plane</th>
<th>H</th>
<th>V</th>
<th>W</th>
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<tbody>
<tr>
<td>A</td>
<td>45</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>215</td>
<td>25</td>
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<tr>
<td>C</td>
<td>45</td>
<td>150</td>
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<td>D</td>
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<td>200</td>
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<tr>
<td>E</td>
<td>45</td>
<td>250</td>
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<tr>
<td>F</td>
<td>45</td>
<td>140</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) Determine the line of intersection between the planes.
(b) Determine the dihedral angle between the planes.
(c) Determine the horizontal trace and the vertical trace of a plane which passes through the points B, C, and E.
(d) On a separate diagram, draw the projections of the skew lines AC and DE and show the projections of the shortest horizontal distance between them.

Task 1: Completed

Used edge view of a plane (ABC) to determine it → Level line on surface → True length on plane → Plan to show a point view of the line located the line by using where DE F intersect line (edge view) of ABC, → projections to plan to elevation.

Task 2: Completed

Found dihedral 105°
Para. 1 — Observation
Tuesday 12th January, 1999.

Interpenetrations

Fig. 3 shows the projections of an equilateral triangular prism of 95 mm side lying on the horizontal plane. Also shown are the projections of a point whose cross section is a quadrant of a circle drawn on the plane of the prism.

Table 1

<table>
<thead>
<tr>
<th>Paragraph</th>
<th>Difficulty</th>
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<tbody>
<tr>
<td>3</td>
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<td>5</td>
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<td>9</td>
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<td>10</td>
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</tr>
</tbody>
</table>

Note: The surface under Fig. 4 shows the plane of projection of the prism. The angles of the prism are drawn to scale.
Appendix 16 - Examples of Interview Data

Extract from Interview with Justin Cummins on Intersecting Planes & Skew Lines

Justin completed a very satisfactory solution to the question, answering all parts to a very high standard measured by examination standards. He complete his solution in 30 minutes, a good five minutes under the time allocation for each question in the examination. I was interested in the order in which he prepared his solution and if there was any portion of the question which he found problematic.

WPD When you were drawing your solution you didn’t answer the question in the order in which it was given on the question paper - why was that?

Justin I drew the planes from the co-ordinates the way you showed us.

WPD What do you mean?

Justin The measurements in the first column were very big so I reduced them by taking the lowest measurement given from the others.

WPD What about parts a, b and c?

Justin I did the dihedral angle second. The line of intersection was given so I was able to start drawing the auxiliaries straight away.

WPD How did you know where to start with the auxiliary views?

Justin I drew the first auxiliary at right angles to the line of intersection and then I had the true length of it. The second auxiliary I drew was along the true length to give a point view of it.

WPD What did that do for you?

Justin It gave me the planes as lines and I was able to see the dihedral angle between them then.

WPD You then moved onto part c. Why was that?

Justin Well I knew that I just had to redraw the skew lines and draw two auxiliary views to find the shortest horizontal line between them.

WPD Was the solution to this part straightforward so?

Justin No. There are two ways of doing this. If you want the shortest line between the skew lines you have to use the auxiliary views method. And if you want the shortest horizontal line between them you use the
planes method. I knew I needed to use the planes method because the word horizontal is in the question.

WPB You left part b of the question to last, why was that?

Justin I wasn't able to figure out how to do it straight away. I couldn't figure out what to do even though I had the elevation and plan of the planes drawn. So I decided to leave it for a while and then to come back to it after I had the Skew lines finished.

WPB So how did you figure out how to finish part b?

Justin I went back and drew a horizontal line through C in elevation. I could see that this line cut through the sides of the other quadrilateral plane, showing a level line in elevation on this plane. So I drew this line on the plan and the shortest line from C to it was at right angles to it.

WPB Was there any part of the question you had any problem with?

Justin Yes. Until I drew the level line passing through C I couldn't figure out how to finish part b. But when I saw the line on ABDE I was able to finish it.
Interview with Ken following observation of him completing a Perspective Projection question (Q.1 1996 Paper 2(b) Higher level) on Monday, December 7th, 1998.

WPB Ken, you drew the plan and elevation first but you left out the curve?

Ken Yes, at the start but I came back to it.

WPB Then what did you do?

Ken I picked a centre on the centre-line and drew a curve I thought looked right.

WPB And then?

Ken I used the same distance on the compass to draw the curve on the elevation. I drew up from the centre on the plan and from one of the points I had I drew an arc to intersect it and then I drew the arc again.

WPB Did that work out?

Ken No - I got a curve that didn't go through the two points that I had.

WPB Why was that do you think?

Ken I don't know, I thought I had it done right.

WPB Had you ever done anything like that before?

Ken Yes, I think we drew something like that before in class.

WPB What about Junior Cert.?

Ken I didn't do drawing before fifth year at all.

WPB Anyway, you moved on to set up for the perspective?

Ken Yes, I drew all the points. The angle of vision first, the central vision ray, the plan of the picture plane and the vanishing parallels to give me the vanishing points.

WPB Any problems there?
Ken  No, I don’t think so. I drew the ground line and the horizon line after that and marked in the positions of the vanishing points.

WPB  What about the auxiliary vanishing points mentioned in the question?

KenI  I didn’t get around to that. I don’t know if I’d have been able to find it.

WPB  You didn’t try to draw the perspective view itself?

Ken  I was trying to figure out where to start but the plan of the picture plane wasn’t touching the nearest corner of the plan like it usually does - it was passing through the plan - I know we did some like this, but I couldn’t remember how to get started.

WPB  The picture plane was passing through the plan but you didn’t know where to start the perspective is that it - how many places was the plan cut by the picture plane?

Ken  It passed through two edges, but that was no use to me - I just couldn’t think of how to find a starting height.

WPB  What about the heights of the edges you mentioned, did you know those?

Ken  Yes, I suppose so.

WPB  What did you think of your overall solution?

Ken  Well it wasn’t so bad - I had the plan done properly. The set-up for the perspective was alright, even though I hadn’t found the auxiliary vanishing point.

WPB  What about the perspective view you were asked to draw?

Ken  I suppose that wasn’t very good.
Paraic Kennedy
Leaving Certificate - Higher Level 1991 - Question 5 - Interpenetrations

WPB I noticed that you began your solution by drawing the pyramid in a secondary position?

PK Yes. I drew it when it was on the horizontal plane.

WPB Why was it necessary to do that?

PK Well, I couldn't draw the elevation that was given in the question right away.

WPB Why was that?

PK Because I didn't know the height of the pyramid when the base was tilted up.

WPB So how did you go about finding the height?

PK I drew the pyramid when its base was on the ground. Then I rotated it so that one of its sides was perpendicular to the ground.

WPB How exactly did you do that?

PK Once I had the pyramid rotated into the correct position I was able to draw the edge of the base that was left on the ground on the plan and elevation. I knew the apex of the pyramid was straight above the centre of the base. So I finished the elevation and plan of that side.

WPB What about the rest of the solid?

PK Well, I knew the height the other side of the base...the one that was off the ground...from the original elevation I drew. So I just drew two lines from the base of the vertical side at 30° up to that height to give me the other side of the base. Then I finished the base in the elevation and plan and drew the edges of the pyramid in.

---

1 This assumes that the sloping lines of the base are parallel to the edges of intersecting prism - which is not the case here. The angle of the sloping edges is determined by the inclination of the pyramid into a position where one of its triangular edges is standing perpendicular to the horizontal plane.
WPB You took it that the sloping lines of the base of the pyramid were at 30° in the elevation - is that right?

PK Yes - is that not right?

WPB Well let me ask you this! Was there any other way you could have drawn the elevation and plan of the pyramid with the base inclined as its shown in the question?

PK I thought you'd have to draw it in position on the ground first before you could rotate it!

WPB So you didn't think about any other way of drawing the pyramid in the required position?

PK No.2

WPB Once you had the pyramid drawn you went on to draw the prism in position?

PK Yes. I started with the bottom edge in elevation. Then I drew the section as it was given so that I could put in all the lines in elevation.

WPB What about the plan then?

PK I started this from the given edge. Then I took the measurements from the distances I'd marked at right angles to the angle of the prism in the elevation. I took the order of the edges from left to right.3

WPB What then?

PK Well I had the elevation and plan of the two solids complete then. So I started on the auxiliary plan.

WPB What did you do there?

PK I drew the XI-Y line at right angles to the angle of the prism in elevation. Then I projected the five corners that made up the pyramid down from the elevation. I took the measurements from X-Y back to the plan. I did the prism the same way.4

2 There are other ways of drawing the pyramid in the required position. Probably the easiest one would be to draw an auxiliary elevation along the edge that rests on the Horizontal Plane on plan. Once this view would be constructed the plan could be projected directly from it - then the elevation.

3 This is a complementary procedure he has just described.

4 Another complementary procedure - the drawing of an auxiliary plan - is described here.
WPB Once you had this finished what did you do?

PK Well, I could see\(^5\) from the auxiliary plan that all the exit points for the prism were on the vertical side of the pyramid. So I finished this side off first.

WPB How did you do that?

PK I just projected each point that struck the vertical side on the plan up to the elevation.

WPB You didn't use the auxiliary view?

PK I didn't have to for that side.\(^6\)

WPB What about the other side?

PK Well, I used lines on the surface there.\(^7\)

WPB Did you have any problems with this?

PK No, not really. Two of the edges were at the back and two at the front. So I couldn't join them up straight away. I had to find the cross-over points.\(^8\)

WPB How many of those were there, and how did you find them?

PK There were just two on this problem. I found them on the auxiliary view first, projected them to the edges in elevation and then to the plan.\(^9\)

WPB How did you know there were only two?

---

\(^5\) Visualising the solids intersecting?

\(^6\) When a surface is vertical the intersection of any line (edge) with it can be pinpointed immediately. He seems not to be just following the procedures associated with Orthographic Projection here.

\(^7\) He knows what procedure to follow here to locate the intersection of the edges on these sloping surfaces.

\(^8\) Again, he knows what procedure to use to deal with an intersection that is not straightforward.

\(^9\) It is not always easy to determine exactly how many cross-over points there are on a problem like this. Again, he seems to have a good picture of what is happening where the solids intersect.
PK When I followed the edges around on the auxiliary view I had to cross the edge of the pyramid, once on the top and once on the bottom.  

WPB Was that it then?

PK Yes.

WPB What did you find most difficult about this question?

PK The hardest part was trying to figure out how to draw the pyramid in the position that it was given in.

WPB What was difficult about this?

PK Well I drew the pyramid on the ground first, then rotated it so that a side was vertical. But I wasn't sure how to connect these views with the elevation that had to be drawn.

WPB How did you do this in the end?

PK I drew the sloping edges of the base at 30°.

WPB Was that correct do you think?

PK I think so.

---

10 He's describing another simple procedure which will allow the number of cross-over points to be determined.
11 Trying to devise procedures which will allow for the construction of a solution.
12 He was able to come up with a procedure which allowed for the position of the apex of the pyramid to be determined in elevation. However, his completion of the elevation and plan of the pyramid highlighted the disjointed nature of the construction he devised. He assumed the sloping lines were at 30° and continued from there. His pyramid was incorrect. However, all the procedures he followed thereafter would have led to a correct solution being drawn had this complementary procedure been carried out satisfactorily.
Appendix 17 - Examples of Audio Recorded Pairs Data.

Gary Raftery & Carl Newell:

Gary  Right...we know the areas are 5:4:3. So that means that ABC is obviously 5, ACD is obviously 4, and ADE is obviously 3. So what you do there is draw a line from E perpendicular to AC and divide the line...we'll say from B to a point on AC...we'll call it G...

Carl  You don't have to call it anything - you can just draw it in position...

Gary  Right...then divide that line into five parts. Then you take four of the parts and measure it off from G up the other side, you'll get the height of the triangle. You'll get the height of the triangle and then all you have to do is...ACD is a 70° angle so...

Carl  We don't know what that is but we'll find it out though when we draw the 25° line...it's just that's the ratio for the other part of the line.

Gary  O.K. that's it yeah! That's what you do. Where the line AD cuts that line you'll just get that triangle and then it's the exact same thing for the next one. All you have to do is draw another perpendicular from C to AD and divide that into 4. Then take 3 of those parts from the line AD and you'll knock that off perpendicular to AD. Then you go at 25° to the line AC.

Carl  all right Gary I get the point

Gary  Do you?

Carl  I do - but listening to the tape I don't think he'll have a clue what's going on.

1 They've realised that the given base AB should not be used when dealing with these adjoining triangles. Instead of the obvious, they are taking AC as the base, drawing a perpendicular to it, and dividing it into the ratio required.  
2 This is correct. They have now determined the perpendicular height of the triangle ACD.  
3 Gary is assuming that the three triangles are similar.  
4 Carl has spotted that the triangles are not similar and that all that is required to locate D is to draw a line parallel to AC at the appropriate height such that it intersects the second 25° line - to give D.  
5 Gary has now seen Carl's point and describes how the final triangle ADE is to be constructed using a similar construction.  
6 Actually, they have described the construction very clearly - so I was able to follow it without any difficulty.
Gary Did you do something wrong Carl?

Carl I got mixed up with your instructions Gary!

Gary You missed taking a height there...definitely! We're trying to construct the third triangle here.

Carl Carl would have it constructed ages ago if he wasn't getting so many instructions!

Gary You should be getting $75^3$ there to the horizontal line AB.

Carl I have that already done Gary. $^7$

David Gleeson joined in at this point.

David You sound like you're a bit behind there lads (pause - playing with recorder. It was turned on and off several times).

Gary From A draw two straight lines which will divide the figure into three parts so that their areas are in the ratio of 3:4:5. Their areas are in the ratio of 3:4:5...sorry! O.K. - join E to B. Yeah! That makes a quadrilateral out of the five sided figure. And we still have the same base AB and a triangle ABE. Is the base still AB? $^8$

Carl Yes.

Gary The base is still AB but it's a continuation of AE $^9$ - so now we have...from B...right, that makes sense Carl...3:4:5...now you're going to divide the opposite side into the ratio of 3:4:5 are you? $^10$

$^7$ They're just messing about here. They have completed the construction of the irregular pentagon - part (a) of the question.

$^8$ Gary has realised that he will not be able to complete part (b) of the question while the figure is an irregular polygon. He has decided therefore, to sub-divide it into a triangle ABE and quadrilateral BCDE as a first step in completing part (b).

$^9$ This refers to changing the area of the quadrilateral BCDE. This is done in two stages. Firstly BCDE is divided into two triangles BCE and CDE. Then into a single triangle by moving the vertex D, such that it falls on the line AE extended. This gives an overall triangle equal in area to the figure ABCDE.

$^{10}$ They've managed to convert the given irregular pentagon into a triangle of equal area. Now, Gary is inviting Carl to divide the side of the triangle opposite B into the required proportions. Since the bases of these triangles are in the correct proportions and they all have the same perpendicular height the resulting triangles have areas in the required proportions.
Carl Yes, 12.
Gary 3:4:5 is twelve parts...that's 3+4+5=12.
Carl Yes, 'cause 3 and 5 is 8, and 8 and 4 is 12!
Gary O.K. - 3 and 4 is 7 and 5 is 12!
Carl That's an equally valid point Gary.
Gary Now...it's actually a very easy question this! Now, bring them back...no, back up to this point here Carl!
Carl That's what I'm doing, if you'd move your elbow so I can move the 'tee square' Gary.
Gary You're connecting them to the wrong point there Carl?
Carl No - I'm connecting them to the right point! You must still be thinking about your Irish Oral!
Gary Now...we're back on the right road again...3:4:5. Does that mean any specific order? It could be 3:4:5 in the other order could it?
Carl No...it would be up here then, it wouldn't be bisecting the five-sided figure at all.
Gary Yeah! I was just testing you there to see.
Carl Is that it Gary? Here I was thinking that you just didn't know.

---

11 This is a reference to the Oral Examination for the Leaving Certificate which these two students were due to take the following morning.
12 Gary is wondering if the extended line AE should be divided in the required ratio from A or from the other end of the line?
13 Carl is just pointing out that the proportion must begin at A, otherwise the two lines dividing the triangle will not fall within the original irregular pentagon ABCDE.
14 This is just a bit of banter between the two students. In fact, it can be very difficult to determine where such a proportional division of a line should begin in order to achieve the desired result. That Carl can say definitely which end of the line he should start from, without trying it out, is quite unusual. He obviously has a good 'picture' of the overall solution.
Gary So now...that's that finished. Today is the 22nd of April...that's it now...we just lost it up there a bit with the indices...  

Carl No...I took the...I brought the...when I was dividing it into five I brought it back onto the line, draw five sections on the line...  

Gary But you can just index it like the original.  

Carl Oh! I know what you mean. A, B and this is C, isn't it?  

Gary That's that now, that's it all finished!  

Carl That's us finished!  

15 This is just the system of allocation letters or numbers to the important points on the drawing. The original polygon was indexed ABCDE - various other important points were located during the construction. Gary makes reference to calling important points by an index early on the tape. Carl rightly points out that it isn't strictly necessary to call a point anything. I try to impress on the students that there is a need for indices only if the drawing becomes confusing - if one can follow the construction without indexing every important point that's located, then indices may be omitted completely.  

16 Carl seems to be referring to the point G that Gary was talking about at the beginning of the construction, when he was dividing a perpendicular from B to AC into 5 equal parts-calling the point where the perpendicular fell on AC, G?  

17 Yes - they have just completed correct solutions to parts (a) and (b) of the question.
Audio-recording of Enda and Ken at work on an Hyperbolic Paraboloid Shell problem on Thursday 13/5/1999. Enda directs Ken as they seek to complete the projections of the roof shell including elements, before finding the curvature along a roof diagonal.

Enda Right, draw the outline of the plan as it's given.

Ken O.K., that's straightforward enough - what about these scale measurements? What's 12m to a scale of 1:200?

Enda The short sides are 60mm long and the longer ones are twice that.

Ken Right - I've that done now, where do I go from here?

Enda Divide up the opposite sides into the same number of parts - there are seven elements so divide it into six parts.

Ken Do I have to do both sides like that?

Enda No - it's symmetrical so you can just project them from side to side. Then up to the elevation.

That's it - now finish off the elevation.

Ken Should I index these, I can never remember how they're supposed to be joined.

Enda Do if you like, but you don't need to really - there's only five elements in the middle when you take out the two outside ones and we should be able to keep track of them.

Ken O.K. so - have I got it done right?

Enda Yes I think so! That's that finished now we have to do the curve from A to D.

Ken Do I have to draw an end view or what?

Enda I don't think you can do it on an end view - the line is at an angle on the plan - try an

Actually it wasn't correct - elements weren't plan - incomplete.

William P. Bolger - M7137754
Complementing Auc.(3)

auxiliary looking at right angles to the line on the plan.

Ken Is this it - I'll have to mark some points for the heights won't I?

Enda Take the positions from where the elements cut the line on the plan. Project them onto the auxiliary and measure the heights from the elevation.

Ken But I haven't got the heights marked in the elevation?

Enda Have you not? Just project them from the plan up - be careful it has to be accurate, otherwise you'll get bumps on the curve.

That's it - now take the heights of those points and mark them on the auxiliary. Yes that's it - it's a bit bumpy, but I think it's O.K.
Audio-recording of Gary and Olive discussing the solution of a Roof Geometry Question on Tuesday, 18/5/1999. The extract below refers to a dihedral angle in reverse construction - i.e. the dihedral angle between two roof surfaces is given and the students are trying to find the outline plan of the roof. Oliver draws as Gary directs him.

Gary: O.K. you can draw the bottom part of the wall plate on the plan and mark the position of the hip rafter - yes that's right. Now draw the elevation of the hip as well.

Oliver: What about the next hip, between the next two surfaces?

Gary: It's all right for the moment, we're only dealing with the two we have the dihedral angle for.

We have to use the traditional construction here - well maybe you can use auxiliary views as well - but we'll stick to the traditional method.

Draw the line at right angles to the plan of the hip. O.K. now find its true length.

Oliver: Will I do it on the plan or draw an auxiliary view?

Gary: Draw it on the plan, we're using the traditional construction. That's it - take the height and rebat it. O.K. now mark the point on the wall plate we have.

Oliver: Back where the line at right angles to the hip cuts it is it?

Gary: Yes. All right - mark a line from where that cuts the hip at right angles to the true length of the hip and rebat it down. Now you can draw on side of the dihedral angle from the point on the hip back to the point on the wall plate.

Oliver: I've that done now how do I get the other half?

Gary: Just get your protractor and mark off the angle with the point on the hip as the vertex - where that line cuts the line at right angles to the hip is the point on the other wall plate we need.