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Hydrodynamic Effects on Soiled Surfaces

- An Experimental Study and Theoretical Analysis -

PhD

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Chapter 5a

Step 1 – The Measurement and Determination of Shear Stress Magnitude
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\( d^{2} = \left[ \frac{1}{2} \left( \frac{1}{2} K_{t} \right) \frac{d^{2} \tau_{w}}{\rho u^{2}} \right]^{\frac{1}{2}} \) \quad Equation 5.4a .................................................................................................................. 124

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\[ x^* \leq 2.9 \quad y^* = 0.5x^* + 0.037 \]  
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\[ x^* \leq 5.6 \quad y^* = 0.8287 - 0.1381x^* + 0.1437x^* - 0.006x^* \]  
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\[ x^* \leq 2.9 \quad x^* = y^* + 2 \log(1.95y^* + 4.1) \]  
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\[ \Delta P = \frac{1}{h} \int \frac{\rho}{2} (u^2 + \sigma^2) dy = \frac{1}{h} \int \frac{\rho}{2} u^2 \left(1 + \frac{\sigma u^2}{u^2} \right) dy \]  
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\[ \Delta P = \frac{1}{h} \int \frac{\rho}{2} (u^2 + \sigma^2) dy = \frac{\rho}{2h} \int u^2 \left(1 + \frac{\sigma^2}{u^2} \right) dy \]  
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\[ C_f = 2 \left( \frac{u^*}{U} \right)^2 \]  
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\[ u^* = 2.6 \quad \text{per } 20 < y^* < 27 \]  
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\[ u^* = 2.3 \quad \text{per } 41 < y^* < 60 \]  
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**List of References**


Further Readings

- East L. F., Measurement of skin friction at low subsonic speed by the razor blade technique, Royal Technical Aircraft Establishment; R & M 3525, 1966.


Books


**Measuring Shear Stress Magnitude**

The main scope of step 1 was to develop and experiment a simple, yet reliable, method of measuring shear stress magnitude and eventually shear stress direction. As will be seen in steps 2 and 3 (see figure below and chapters that follow) the first step was fundamental for the correct interpretation and understanding of the hydrodynamic conditions in the water tunnel.

As is illustrated in the upper part of the next figure and described previously in chapter 1, step 1 involved experimenting (for the first time in water flows) two peculiar types of shear stress probe, namely small square and triangular shaped block probes [ref. 5.1a]. These probes were classified by their shape and ability to measure shear stress magnitude and direction.

---

**Figure 5.1a – Transition from Step 1 to Step 3**

The probes were placed across and along the tunnel and, in conjunction with a Preston tube, used to measure shear stress magnitude and characterise the tunnel. Of special interest was the variation of shear stress at right angles to the flow and in the area where the EMPA samples
were to be tested. Hence both shear stress wall profiles and shear stress magnitude were measured through a range of hydrodynamic conditions. Originally it was thought that a matrix or array of probes would be needed [ref. 5.2a] to quantify the shear stress where the EMPA samples would be tested. However, the experimentation revealed that this was not necessary because the shear stress was essentially constant and at least predictable for the hydrodynamic conditions in mind.

Before this could be established a total of six small block probes was installed and tested in the tunnel, these being:

- 3 square shaped probes equipped with one dynamic pressure tapping and one static pressure tapping. These probes were denominated RS1, RS4 and RS5, or RR1, RR4 and RR5, depending on the static pressure reference. One of these probes was mounted 50mm in front of the other two and aligned with the centre line (c.l.) of the tunnel.

- 3 triangular shaped probes equipped with three dynamic pressure tappings and one static pressure tapping. These probes were denominated DS2, DS3 and DS6, or DR2, DR4 and DR6, depending on the static pressure reference. These probes were placed across the tunnel and in-line with two of the square shaped probes.

All six probes were therefore calibrated for local and remote static pressure references that will be explained in greater detail further in this and the next chapter. The scope of the calibration was to compare the static pressure reference tapping of the probes to a remote tapping positioned nearby.

The six probes were installed as shown in the following two figures:
In order to ensure fully developed flow during measurements the probes were installed towards the rear end of the test section (see next figure). This also provided a sufficient length of tunnel (about 1.826m) to measure the pressure drop and use the Darcy equation to calculate the average shear stress [refs. 5.3a and 5.4a] in the test section.

The main part of the tunnel was therefore the test section of the tunnel, which is shown in the next three figures.
Figure 5.5a – Pictures of Test Section

The above three pictures show the test section of the tunnel: A) Assembled test section, B) Without tunnel roof, C) Inlet ramp.

Figure 5.6a – Ramped insert inside Test Section

A brief description of the shear stress measurement methods used in the tunnel experimentation will now be given.

**Direct and Indirect Methods of Measuring Wall Shear Stress**

The measurement of tangential wall shear stress is generally divided into two macro categories (shown in figure 5.7) i.e. Direct and Indirect, the latter category providing the majority of methods and devices.

Indirect shear stress measurements range from using simple coloured dyes to sophisticated velocity profile reconstruction using LASERs [ref. 5.5a]. The approach is always the same, that
is the measurements of one or more parameters are correlated to shear stress through analytical and/or empirical expressions. To date the methods available fall under one of seven categories, these being:

1. Visualisation and reconstruction of the boundary layer velocity profile using Optical methods [ref. 5.6a]
2. Image Velocimetry [refs. 5.7a and 5.11a]
3. Bubbles, Traces, Dyes, Smoke, Tufts, Clay etc. [ref. 5.8a]
4. Oil–Film Gauges [ref. 5.9a]
5. Heat Transfer probes [ref. 5.10a]
6. Mass Transfer probes [ref. 5.11a]
7. Pressure probes [ref. 5.12a]

The emphasis in this and following chapters will be on this last category and especially a peculiar type of pressure probe known as "small block" probe.

Perhaps the biggest drawback with indirect methods is that they all require some form of shear stress calibration. Ideally, this should be done with a direct measurement technique, which may not be feasible or rational e.g. measuring shear stress in artery blood flow. Henceforth an indirect method is often compared with another indirect method e.g. small block probes versus a Preston tube.

Direct methods also have their shortcomings, including, correct installation, force calibration, compensation and freedom of sensor movement so as to accurately detect the shear force.

Recent advances in nano–technology [ref. 5.13a] have seen attempts at bringing together indirect and direct methods, meaning that small chip–like sensors have been successfully experimented in real fluid flows.

An overall, and much more detailed, account of shear stress measurement is given by the author in [ref. 5.3a].
Figure 5.7a – Overall Picture of Direct and Indirect Shear Stress Measurements and Visualisation
Pressure Probes

A pressure probe is essentially a means for detecting the local flow conditions. This can range from a simple tapping in the wall to much more sophisticated devices (but nevertheless simple in concept) that sample the boundary layer.

Two good examples of the first kind of probe concern the measurement of flow conditions (e.g. flow rate) with the Thomson weir and the average shear stress with the tunnel pressure drop tappings. Generally these techniques only provide an overall picture of the flow conditions i.e. they ignore the local variations of velocity. Hence only average hydrodynamic conditions can be estimated and, in the case of pressure drop measurements, this entails a minimum stretch or length of duct\(^1\): in the water tunnel this was about 1826 mm.

To obtain the missing local information, especially that near-the-wall, a multitude of probes have been developed that measure the local pressures (static, stagnation and dynamic) in the and across the boundary layer. Through these measurements, it is possible to extrapolate the local velocities and hence derive shear conditions.

Consequently the approach of all these types of probe is to capture or sample the fluid, measure its pressure (without disturbing too much the rest of the flow) and then correlate it to shear stress via the wall velocity profile. Examples of these probes are:

1. Pitot tube
2. Prandtl tube
3. The Stanton tube
4. The Stanton gauge or razor blade
5. Sub-layer fence
6. The Preston Tube
7. Small block probes

\(^1\) In reality the pressure drop measurements over a shorter distance are only limited by the precision of the pressure transducer technology available today. However, nano-technology will change this scenario and eventually lead to smaller, more precise, transducers.
The development of the Stanton tube is really a derivation of the Pitot and Prandtl probes, whilst the Preston tube is subsequent modification of the Stanton tube.

Methods 3 to 7 have been extensively used to measure shear stress [ref. 5.14a]. In particular the Preston tube is considered a universal method for the measurement of shear stress and is now considered a reliable comparison for other indirect methods. It was therefore used in the water tunnel to calibrate the small block probes. The small block probes offer a cheap and reliable way of locally sampling the boundary layer with minimum flow disturbance.

Basically a block probe offers an opportunity to the flow to arrest itself and in doing so allows us to interrogate the local boundary layer conditions. There are several types of block probe ranging from "submerged step" to a square ridge.

A comparison of the pressure probe techniques is shown in the graph below [ref. 5.15a].

![Comparison of Pressure probe methods](image)

**Figure 5.8a – Comparison of Pressure probe methods**

The (triangular) block probe that has been designed and calibrated is similar to a "block cut-out" probe (see next figure) but has the added advantage of being able to measure shear stress direction as well as magnitude.
It also offers a wider range of applications because of its modular (layer) design. Essentially this means that the probe height can be adjusted to suit flow conditions. In figure 5.8a both the Preston tube and block probes were used and calibrated in the upper part of the curve i.e. \(20 \leq \Delta p/\tau \leq 700\).

Shear stress determination from pressure measurements is based on a peculiar law known as the "law of the wall" [ref. 5.16a] that assumes that the velocity profile is either linear or logarithmic. In practise the two profiles are merged by another profile which is neither linear nor logarithmic that describes what is known as the buffer area.

Moreover, it is common practise to extend the application of the linear profile higher in the boundary layer (e.g, \(y^+ = 20-25\)) and, provided pressure gradients are minimal, both linear and logarithmic profiles are rather lenient. (White, 1991). In fact for the logarithmic part of the curve deviation from logarithmic approximation only really starts well after \(y^+ = 100\), This is shown below:

Figure 5.9a – Pressure Gradients (Inner law)
In the tunnel measurements of steps 1 to 2 the block probes were designed to be located in the linear part (known as the laminar or sub-viscous layer) of the boundary layer (Vardy, 1990 and Brodkey 1995). Probe design was originally determined for $y^+ \sim 16$, hence the tunnel was designed and run with this constraint in mind.

A summary of the pressure based shear stress methods is provided in the next table.
### Table 5.1a – Summary of Shear Stress Determination methods based on Pressure Probe Measurements

<table>
<thead>
<tr>
<th>Method</th>
<th>Law</th>
<th>Scheme or picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanton Tube</td>
<td>[ \bar{u} = \frac{\tau_w}{\mu} ]</td>
<td><img src="image" alt="Stanton Tube" /></td>
</tr>
<tr>
<td></td>
<td>[ \tau_w \approx P^n_D ]</td>
<td></td>
</tr>
<tr>
<td>Stanton gauge</td>
<td>[ \frac{\Delta P h^2 \rho}{\mu^2} = \frac{\tau_w h^2 \rho}{\mu^2} ]</td>
<td><img src="image" alt="Stanton gauge" /></td>
</tr>
<tr>
<td>Pitot tube</td>
<td>[ \bar{u} = \sqrt{\frac{2(p_{stag} - p_{sat})}{\rho}} ]</td>
<td><img src="image" alt="Pitot tube" /></td>
</tr>
<tr>
<td>Sub-layer fence</td>
<td>[ \bar{u} = \frac{\tau_w}{\mu} ]</td>
<td><img src="image" alt="Sub-layer fence" /></td>
</tr>
<tr>
<td></td>
<td>[ \tau_w \approx \Delta P^n ]</td>
<td></td>
</tr>
<tr>
<td>Preston tube</td>
<td>[ \frac{\Delta P d^2}{\rho v^2} = f \left[ \frac{d_1 \tau_w}{\rho v^2} \right] ]</td>
<td><img src="image" alt="Preston tube" /></td>
</tr>
<tr>
<td>Small Block – (Dexter)</td>
<td>[ \frac{\Delta P d^2}{\rho v^2} = f \left[ \frac{d_1 \tau_w}{\rho v^2} \right] ]</td>
<td><img src="image" alt="Small Block - Dexter" /></td>
</tr>
<tr>
<td>Small Block – (Gaudet et al.)</td>
<td>[ \frac{\Delta P d^2}{\rho v^2} = f \left[ \frac{d_1 \tau_w}{\rho v^2} \right] ]</td>
<td><img src="image" alt="Small Block - Gaudet et al." /></td>
</tr>
<tr>
<td>Small block – triangular probe (Ward et al.)</td>
<td>[ \frac{\Delta P d^2}{\rho v^2} = f \left[ \frac{d_1 \tau_w}{\rho v^2} \right] ]</td>
<td><img src="image" alt="Small block - triangular probe (Ward et al.)" /></td>
</tr>
<tr>
<td>Small block – triangular probe (Ward et al.)</td>
<td>[ \frac{\Delta P d^2}{\rho v^2} = f \left[ \frac{d_1 \tau_w}{\rho v^2} \right] ]</td>
<td><img src="image" alt="Small block - triangular probe (Ward et al.)" /></td>
</tr>
</tbody>
</table>

A brief description and analytical explanation of the Preston tube and small block probes used will now be given.
The Preston Tube

The Preston tube is based on a small, thin walled, round section tube that is placed in contact with the wall where the shear stress needs to be evaluated. The construction of the tube used for the tunnel tests was very straightforward starting from a relatively large bore (ø8mm) pipe, to which the Druck pressure transducer was connected via tubing, terminating in a very thin, syringe-like, tube (ext. dia. 0.82mm circa). This final tube was sized so that it was well inside the boundary layer and to minimise flow disturbance. The Preston tube was shown in Fig. 3.5.

The Preston tube obeys the equation of the Pitot tube and therefore measures an impact pressure imparted by the fluid across the mouth of the tube. The required static pressure is measured by a wall tapping or port, which is placed in the vicinity where the Preston tube will be used to measure the dynamic pressure. In the tunnel tests the Preston tube was placed in front of the block probes.

The ratio of internal tube to outside tube diameter is known to be very lenient but provided it is >0.4, [refs. 5.16a and 5.17a], the Preston tube gives accurate results. As suggested by Preston ref. 5.18a the ratio for the tube diameters used for the water tunnel experiments was set to about 0.6. Preston assumed that the velocity gradient near a wall was a universal function dependent only on the shear stress and probe position within the boundary layer. That is to say:

\[ \overline{u} = f(\overline{\tau}_w, y) \quad \text{Equation 5.1a or} \]

\[ u = \left( \frac{\overline{\tau}_w}{\rho} \right)^{\frac{1}{2}} \left( \frac{yp(\overline{\tau}_w/\rho)}{\mu} \right)^{\frac{1}{2}} \quad \text{Equation 5.2a} \]

This relationship was confirmed by Preston for fully developed incompressible turbulent flows where he calculated the time-averaged shear stress through pressure drop measurements i.e.:

\[ \overline{\tau}_w = \frac{d}{4} \left| \frac{dP}{dx} \right| \quad \text{Equation 5.3a} \]
The attractiveness of the Preston tube lies in its simplicity and manoeuvrability within the boundary layer and across the surface of the wall. This means that local tunnel readings are made simply by moving the tube along the wall surface and reading off the imparted pressure. Another very useful aspect is that the tube can be placed in the measurement area and subsequently withdrawn to a remote area so that other measurements are not disturbed by the tube. Interestingly the Preston tube can be used for both compressible and incompressible flows provided it is within the laminar viscous sub-layer of the boundary layer (i.e. where \( \bar{U} = \bar{\tau}_w y/\mu \)).

Since the principle of operation is essentially that of the Pitot tube an effective centre is needed, this is defined as \( y = 1/2 K_i d_i \), where \( d_i \) is the outside diameter of the tube and \( K_i \) is a calibration coefficient. By assuming the tube lies within the boundary layer the Reynolds friction number can be defined as:

\[
d_i^* = \left[ \frac{1}{2} \left( 1 - \frac{1}{2} K_i \right) \frac{d_i^2 \bar{\tau}_w}{\rho u^2} \right]^{1/2}
\]

Equation 5.4a

Preston went on to find that for large values of \( d_i^* \) the difference between impact and static pressures could be related through:

\[
\frac{\Delta P}{\bar{\tau}_w} = \left[ \frac{1}{2} \left( 1 - \frac{1}{2} K_i \right) \frac{d_i^2 \bar{\tau}_w}{\rho u^2} \right]
\]

Equation 5.5a

Research by Young and Mass [ref. 5.19a] has shown that for large Re numbers the value of \( K_i \) is 1.30 provided that the ratio of inside to outside diameter is 0.6 for the Pitot tube.

Moreover, provided the flow can be considered 2D i.e. the flow is uniform spanwise, then we may write:

\[
\frac{\Delta P}{\bar{\tau}_w} = \frac{1}{2} \left( 1 - \frac{1}{2} K_i \right) d_i^2 \bar{\tau}_w \left\{ 1 + \left( \frac{r_w}{\bar{\tau}_w} \right)^2 \right\}
\]

Equation 5.6a
where \( \bar{\tau}_w \) is the time averaged wall stress and \( \tau_w^2 \) is the mean square of the fluctuations in the wall stress. Measurements carried out in a number of laboratories would indicate that \( \frac{\tau_w^2}{\bar{\tau}_w} \approx 0.15 \). For turbulent flow conditions [ref. 5.20a] the Preston tube cannot be made small enough to sample the region where \( \bar{U} = \frac{\tau_w y}{\mu} \). To get around this problem it is required that the Preston tube samples the region where the law of the wall is described and for which the following relationship is known to hold:

\[
\frac{\Delta P}{\bar{\tau}_w} = f \left[ \frac{d^2 \tau_w}{\rho \nu^2} \right]
\]

Equation 5.7a

Alternative forms developed by Preston are:

\[
\frac{\Delta P d_i^2}{4 \rho \nu^2} = f \left[ \frac{d^2 \tau_w}{4 \rho \nu^2} \right]
\]

Equation 5.8a

\[
y^* = \log_{10} \left( \frac{\tau_w d_i^2}{4 \rho \nu^2} \right) = -2.628 + \frac{7}{8} \log_{10} \left[ \frac{(P_o - P_w) d_i^2}{4 \rho \nu^2} \right]
\]

Equation 5.9a

where \( y^* \) is a dimensionless shear stress for incompressible, isothermal flow and \( x^* \) is dimensionless pressure difference for incompressible isothermal flow.

Extensive work has been carried out in fully developed turbulent pipe flows with different diameter Preston tubes [refs. 5.21 to 24]. Head and Ram developed a “universal calibration chart” (shown next) for the Preston tube for which its accuracy (±6%) is guaranteed for incompressible, two-dimensional, turbulent boundary layer with a moderate pressure gradient.
Table 5.2a – Preston tube Calibration table according to Head & Ram

In certain circumstances, the Preston tube is also applicable to boundary layers with small pressure gradients, Patel [ref. 5.25]. An example of this is the water tunnel where moderate positive (favourable) pressure gradients are applicable during shear stress measurements.

Patel (1964) published the results of tests with 14 different circular Pitot probes using three different pipe diameters. Patel obtained empirical equations for:

\[ y^* = \log_{10} \left( \frac{\tau_w}{4\rho u'^2} \right) = f \log_{10} \left( \frac{(P_o - P_w) d^2}{4\rho u'^2} \right) = \theta(x^*) \]  

Equation 5.10a

over three ranges of \( y^* \):

<table>
<thead>
<tr>
<th>( \frac{\Delta P}{d^2} \times 10^{-3} )</th>
<th>( \frac{\Delta P}{d^2} \times 10^{-3} )</th>
<th>( \frac{\Delta P}{d^2} \times 10^{-3} )</th>
<th>( \frac{\Delta P}{d^2} \times 10^{-3} )</th>
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<td>4.1 21.49</td>
</tr>
</tbody>
</table>

Page : 126
1. $3.5 < y^* < 5.3$
2. $1.5 < y^* < 3.5$
3. $y^* < 1.5$ (viscous sub-layer) with $y^* = 0.37 + 0.5x^*$

NB. ranges 1 and 2 ($1.5 < y^* < 5.3$) can be approximated with $y^* = 0.8287 - 0.1381x^* + 0.1437(x^*)^2 - 0.006(x^*)^3$.

Patel studied the affects of adverse and favourable pressure gradients and recommended the use of the Preston tube only for $-0.007 < \Delta < 0.015$ when $d_t^+ \leq 200$, where the parameter $\Delta$ is defined by:

$$\Delta = \frac{\sqrt{(dP/dx)}}{\rho u^+} \quad \text{Equation 5.11a}$$

In the case of the water tunnel the value of $\Delta$ is typically $<-0.007$ for $40000 \leq \text{Re} \leq 150000$, i.e. the range for which the shear stress was measured. Moreover the value of the Reynolds friction number as defined by eq. 5.18, is $d_t^+ \leq 40$, thus confirming the applicability of the Preston tube for the tunnel measurements.

**Small Block Probes**

The measurement of dynamic pressure near the wall using a Preston tube can be exploited using what are known as block probes. The scope of block probes is to place a small, thin, obstacle or block in contact with the wall and against the flow stream. In this way the fluid is made to "jump" the block on which the fluid impacts a dynamic force and hence a dynamic pressure defined as:

$$P_d = \frac{\rho u^2}{2} \quad \text{Equation 5.12a}$$

This "jump" effect is shown in the figure next.
The imparted force can be measured in terms of the dynamic pressure via a small tap or port placed at the "mouth of the block (see Fig. 5.11a). A pressure gradient is formed by the difference between the dynamic and static pressure across the probe. This ultimate reading can be obtained locally or at a remote area away from the probe.

It is observed in figure 5.8a that the block probe used in the water tunnel experiments is really a pseudo-Stanton tube because the wafer or sandwich construction means that the dynamic pressure has a rectangular mouth.

The pressure gradient can be correlated to the local shear stress since it is a function of the local friction velocity and the height of the probe, thus:

\[ P_d = f(u^*, h) \]

Equation 5.13a

where \( P_d \) is a function of the type (shape) of block used.

This relationship shows that the dynamic pressure is strongly correlated to the law of the wall, defined as follows:

\[ \frac{\ddot{u}}{u^*} = f\left(\frac{u^* h}{v}\right) \]

Equation 5.14a

where \( h \) is the height of the probe and the friction velocity is defined as:
Thus, there are four variables included in the use of block probes: shear stress, $\tau_w$, fluid density, $\rho$, fluid viscosity, $\mu$, the height of the block or "mouth", $h$, that is:

$$\Delta P = f(\tau, \rho, \mu, h)$$  \hspace{1cm} Equation 5.16a

An illustration of the small block concept is shown below:

![Very thin block](image)

**Figure 5.12a – Concept of Small Block Probe**

The small block technique has undergone several developments and now includes several local dynamic pressure measurements as well as a local static pressure measurement as well. An example of a small block probe fitted with one dynamic port and one local static port is shown next.

![Fluid trap Dynamic Static pressure port](image)

**Figure 5.13a – Recent small block probe design**

The advantage of measuring the local dynamic and static pressures near each other is that the local shear stress measurement is more "realistic" i.e. it is almost local. This is particularly true if the size of the probe is small and flow disturbance can be minimised.

Clearly it is virtually impossible to exclude flow disturbance and hence the static pressure measurement is somewhat effected. This can be compensated by measuring another static pressure

\[ u^* = \sqrt{\frac{\tau_w}{\rho}} \]  \hspace{1cm} Equation 5.15a

---

2 This implies $u^* = y^*$ (Brodkey, 1995, pp.245-246)
pressure near the probe, as discussed for the Preston tube, and comparing it to the local probe pressure.

The difficulty with just one dynamic pressure is that the direction of the shear force is unknown. Hence previously mentioned developments have led to probes with three dynamic pressure ports that provide three force vectors that when added give the resultant magnitude and direction of shear. This is vital when the flow is likely to be three-dimensional such as in cross flow conditions or the true flow direction is unknown.

Dexter [ref. 5.26a] was the first to develop a small triangular-shaped block probe with three dynamic pressure ports, one on each face of the equilateral triangle.

In this specific case the scope of Dexter’s work was to develop a yawmeter. He thus measured three dynamic pressures (P₁, P₂ and P₃) and one static pressure in the vicinity of the probe. An illustration of his probe with the three dynamic pressure ports was given in Fig. 3.10a.

Gaudet et al. [ref. 5.27a] continued this work and developed a similar triangular-shaped small block but with the dynamic ports retracted. This was achieved by milling slots from the face to the port. In this way the capture of dynamic pressure was more reliable. A further modification was the inclusion of a centrally positioned static pressure port or tap (P₄). A picture of this probe was shown in Fig. 3.10b.

The small block probe that was further developed during the research for the water tunnel tests and was based on both Dexter and Gaudet probe designs. A view of the underside of the triangular probe is given below:

![Figure 5.14a – Underside and Complete view of Triangular probe](image-url)
The overall design of the probes was based on work by Dexter, Winter (1977), Cassani and Stellati (1989), Gaudet et al (1994) [ref. 5.28a] and a series of manual calculations or simple assumptions. In particular the height of both probe designs was determined by imposing $y^*<16$ since this ensured that (most of) the probe operated in the viscous laminar sub-layer. The tunnel operating conditions were used to calculate a theoretical maximum pressure drop of about 800Pa (based on a simple pipe flow model) and shear stress of about 8Pa. The Druck pressure transducer was chosen to suit these conditions with a FSD of 1000Pa.

The triangular probe was designed with a side length to thickness ratio of 20 (as suggested by Gaudet). Also all tappings (for both probes) were 0.9-1.1mm in diameter.

The square probe was designed with one dynamic port and one static pressure and is mounted parallel to the flow direction. An example of the use and relative positioning of the triangular and square shaped probes in the water tunnel experiments was shown in figure 3.3.

In this figure the remote static pressure port was placed in-line with the probes and this pressure is compared with the local probe static pressure during the measurements.

**Validation of the Small Block Probes**

The most obvious way of obtaining the validation of the probes is to compare the pressures recorded during the Preston tube experiments. Alternatively, one could compare the small block performance with the theoretical pressures based on the law of the wall. The first approach can be achieved by examining the work of Preston, Patel or Head and Ram. For example, Patel used the following assumptions:

\[
F(x^*, y^*) = 0 \\
y^* = f(x^*)
\]

where:

\[x^* = \log_{10}\left(\frac{\Delta P d_i^2}{4 \rho \nu^2}\right)\] \hspace{1cm} \text{Equation 5.17a} \hspace{1cm} \[y^* = \log_{10}\left(\frac{\tau_w d_i^2}{4 \rho \nu^2}\right)\] \hspace{1cm} \text{Equation 5.18a}
Patel split the relationship between $x^*$ and $y^*$ into three functions depending on the value of $x^*$:

$$\begin{align*}
\text{if } x^* \leq 2.9 \quad & y^* = 0.5x^* + 0.037 \quad \text{Equation 5.19a} \\
\text{if } x^* \leq 5.6 \quad & y^* = 0.8287 - 0.1381x^* + 0.1437x^* - 0.006x^* \\
\text{if } x^* \leq 2.9 \quad & y^* = y^* + 2\log(1.95y^* + 4.1) \quad \text{Equation 5.21a}
\end{align*}$$

This last equation is solved by iteration and agreed accuracy for the three equations is 5–6%.

The water tunnel experiments were carried out with dynamic pressures below 1000Pa, which implies a max. shear stress of 8.1Pa circa and hence measurements refer to equation 5.20a ($x^* \leq 5.6$).

Another approach is to calculate the theoretical pressure drop using the Darcy equation or a more refined method based on the following integral:

$$\Delta P = \frac{1}{h} \int_{0}^{h} \frac{\rho}{2}(u^2 + \sigma_u^2) dy = \frac{1}{h} \int_{0}^{h} \frac{\rho}{2}u^2 \left(1 + \frac{\sigma_u^2}{u^2}\right) dy \quad \text{Equation 5.22a}$$

In this integral $\sigma_u^2$ is the fluctuating component of the local velocity, $u$. This integral is solved by knowing (or assuming) the relationship between $u$ and $y$ as well, i.e. the trend of $\sigma_u/u$, which also depends on the local friction properties of the fluid. Kutateladze [ref 5.29a] and Huser and Biringen [ref 5.30a] suggest that eq. 5.22a can be re–written using $u$ the average local velocity and $u'$ the fluctuating part of the same velocity component, that is: $u$ and $u'$ are a function of $Y^*$.

$$\Delta P = \frac{1}{h} \int_{0}^{h} \frac{\rho}{2}(u^2 + u'^2) dy = \frac{\rho}{2h} \int_{0}^{h} u'^2 \left(1 + \frac{u'^2}{u^2}\right) dy \quad \text{Equation 5.23a}$$

The function $Y^* = \sigma_u/u$ was studied by Sherman [ref. 5.31a], using Kutateladze's data, where the standard deviation of the velocity fluctuations very near a smooth wall in fully developed channel flow were plotted against $Y^*$ resulting in the following graph.
As can be seen from the above graph for values below $y^+\sim 12$ the plots can be considered virtually linear [ref. 5.32a]. In fact the streamwise plot results provide the following:

\[ \sigma_U / u^* = 0.28y^+ = 0.28 \frac{u}{u^*} \]

Equation 5.24a

If one considers that velocity fluctuations are essentially along one axis then use of equation 5.22a allows the part of the integral in brackets to be reduced to a fixed value of 1.0625 for
y^*<11.6 (and even as high\(^3\) as 16). The height of the dynamic port section of the small block probe has been calculated for y^*<16 for water.

With this information it is possible to calculate the theoretical differential pressure described with equation 5.23a. Should values of y^* be higher, which means we are entering the logarithmic part of the boundary layer, then the integral becomes more complex.

This integral can be solved by splitting it into two parts, a linear and a logarithmic part, described by the following equations:

\[
\frac{\bar{u}}{u^*} = f\left(\frac{u^* h}{v}\right) \quad \left(\frac{u^* h}{v}\right) < 11.6 \quad \text{Equation 5.25a}
\]

\[
\frac{\bar{u}}{u^*} = 5.75 \log_{10}\left(\frac{u^* h}{v}\right) + 5.5 \quad \left(\frac{u^* h}{v}\right) \geq 11.6 \quad \text{Equation 5.26a}
\]

The final integral looks like this:

\[
\overline{\Delta P}_{y^* \geq 11.6} = \frac{1}{h} \int \left[ A + B \right] = \frac{\rho}{2} \left( \frac{2 \sigma}{u^*} \right)^2 \left( \frac{h^3}{3} \right) + \frac{1}{h^2} \left( \frac{2 \sigma}{u^*} \right)^2 \left( \frac{2.5 \ln \left( \frac{u^* y^*}{v} \right) + 5.5}{(1.0625)} \right) \cdot dy
\]

Alternatively, equations 5.24a and 5.25a can be replaced by the Spalding equation but this is in implicit form and requires much more computation effort.

A once favoured graphical method, known as the Clauser method after its originator, consists of using a chart (the Clauser chart) in which u*/U and y/A) are linked via a coefficient C\(_f\) [ref. 5.34a]. The value of C\(_f\) is determined using:

\[
C_f = 2 \left( \frac{u^*}{U} \right)^2 \quad \text{Equation 5.27a}
\]

Where u* is the local (friction) velocity and U is the main stream velocity. The chart is used by superimposing a series of points that yields a certain C\(_f\) trace as shown below. In this way the

\(^3\) Bocchiola [ref. 5.32a] uses a slightly different relation: \(\frac{C_f}{u^*} = 0.25 y^* = 0.25 \frac{u^*}{u}\).

\(^4\) The upper limit \(y^*\) depends on the presence of local pressure gradients.
friction velocity and hence shear stress can be estimated, further details on this method can be in [ref. 5.3]. The Clauser chart, prepared by Winter (1977), is shown next:

Figure 5.17a – Clauser chart

Preston Tube Experimental Results

The tabulated results and graphs that follow are the outcome of calculations that follow the Patel calibration method and equations 5.13a thru 5.23a. In particular the calculations refer to two ranges of $y^*$ for smooth walls [ref. 5.35a], namely:

$$y^* < 11.6 \quad \frac{u}{u^*} = \frac{u^*}{h} = y^*$$

$$y^* \geq 11.6 \quad \frac{u}{u^*} = 2.5 \ln \left( \frac{u^* h}{v} \right) + 5.5$$

In order to solve $\Delta P = \frac{1}{y^*} \int \frac{P}{2} \int (y) u'^2 dy$ the pressure measurements have been averaged over 10 seconds at a sampling rate of 100 samples/second. Before doing this the pressure measurement transient was left to expire (Brodkey, 1995), hence:

$$\overline{\Delta P} = \frac{1}{h} \int \frac{1}{2} \left( \frac{u'^2}{u^2} + u'^2 \right) dy = \frac{1}{h} \int \frac{u'^2}{u^2} \left( 1 + \frac{u'^2}{u^2} \right) dy$$

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To solve this latter equation the ratio of average local (point) velocity ($u_{\text{bar}}$) and the relative fluctuating turbulent part is needed. For the range $11.6 < y^* < 20$ the following relationship is suggested:

$$\frac{u'}{u^*} = 2.7 \quad \text{Equation 5.28a}$$

For larger values of $y^*$ the following relationships are suggested by Huser and Biringen (1993):

- $u'/u^* = 2.6$ per $20 < y^* < 27$ \quad \text{Equation 5.29a}
- $u'/u^* = 2.5$ per $27 < y^* < 41$ \quad \text{Equation 5.30a}
- $u'/u^* = 2.3$ per $41 < y^* < 60$ \quad \text{Equation 5.31a}
Remarks

As will be seen in the following graphs the dimensionless shear stress profiles confirm that the shear stress is not constant across the tunnel.

The variation in shear stress is probably caused by secondary circulation across the tunnel [ref. 5.36]. This is illustrated below:

![Diagram](image.png)

*Figure 5.18a - Tunnel Secondary Circ. (Knight and Patel, 1985)*

Further, all the resulting profiles are very similar and, at least for the range of Reynolds numbers investigated, do not differ. This was considered important for the determination of the position of the soiled samples tested in step 3.
Table 5.3a – Preston Tube Experimental Results for Re=41397

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<th>Re=41397</th>
<th>Y</th>
<th>DP (Pa)</th>
<th>Cv</th>
<th>X*</th>
<th>Y*</th>
<th>□ (Pa) [Patel]</th>
<th>□*</th>
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Figure 5.19a – Plot of DP, τ and τ* versus Y (Re=41397)
Table 5.4a – Preston Tube Experimental Results for Re=51464

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<th>Y*</th>
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MIN. / MAX. / AVERAGE / ST. DEV:

- MIN. / MAX. / AVERAGE / ST. DEV:

![Figure 5.20a – Plot of DP, τ and τ* versus Y (Re=51464)](image)
Table 5.5a – Preston Tube Experimental Results for Re=60034

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<th>Y*</th>
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Figure 5.21a – Plot of DP, τ and τ* versus Y (Re=60034)
Table 5.6a – Preston Tube Experimental Results for Re=68591

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<th>Y*</th>
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Figure 5.22a – Plot of DP, \(\tau\) and \(\tau^*\) versus Y (Re=68591)
### Table 5.7a – Preston Tube Experimental Results for Re=80029

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![Figure 5.23a – Plot of DP, τ and τ* versus Y (Re=80029)](image-url)
Table 5.8a – Preston Tube Experimental Results for Re=87186

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<th>Y*</th>
<th>□ (Pa) [Patel]</th>
<th>□*</th>
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Figure 5.24a – Plot of DP, τ and τ* versus Y (Re=87186)
Table 5.9a – Preston Tube Experimental Results for Re=100320

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<th>Y*</th>
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Figure 5.25a – Plot of DP, τ and τ* versus Y (Re=100320)
Table 5.10a – Preston Tube Experimental Results for Re=119583

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<th>Y*</th>
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Figure 5.26a – Plot of DP, τ and τ* versus Y (Re=119583)
A very useful way of validating step 1 experimental results is to compare them with data provided by Knight and Rhodes [ref. 5.34a] for a duct of similar section ratio (6.25 vs. 6) i.e. duct width/height. In the following graph all Preston tube measurements are presented in dimensionless form (i.e. $\tau^*$) and compared to the average of these results and those of Knight and Rhodes.

Figure 5.27a – Comparison with Knight and Rhodes findings

As can be seen the comparison is favourable and demonstrates the validity of the experiments and tunnel measurements.
Block Probes Experimental Results

To assess the performance of the block probes the experimental results of the square and triangular block probes are presented next.

Table 5.11a – τ & DP data for Square Probes (Local & Remote Refs)

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<th>(\tau) (RS5)</th>
<th>(\tau) (RR1)</th>
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Figure 5.28a – Square Probes Behaviour (\(\tau\) for Local and Remote Pressure Refs)
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<td>147378</td>
<td>321.02</td>
</tr>
</tbody>
</table>

Table 5.12a – Tau & DP data for Tri.-Probes (Local & Remote Refs)

![Graph](image)

Figure 5.29a – Tri.-Probes Behaviour (τ for Local and Remote Pressure Refs)
Summary and Conclusions

The development of the square and triangular probes has provided a simple and yet effective way of determining the magnitude of wall shear stress. The determination of shear stress via pressure measurements is certainly feasible and applicable to other parts of this project e.g. steps 4 and 5.

Probe reliability is also confirmed and measurements with the Preston tube have substantially validated both types of probe. Having to choose between these two probes the final choice falls on the triangular probe because it offers a wider range of shear stress direction detection and has a slightly better accuracy in the tunnel. Although, to be fair, the square probe offers better immunity from pressure surge because the dynamic pressure tapping is more shielded and is easier to build.

The overall shear stress profile across the tunnel was essentially constant up to 130mm circa from the centre line of the tunnel (see Fig. 5.27a). All the Preston tube sampling and resulting profiles confirmed that the best area for the EMPA samples is in the region between 20≤Y≤120mm. In other words within this range the shear stress is substantially predictable.

Further measurements along the tunnel taken with the Preston tube and the forward square probe (see Fig. 5.3a) have shown that in an area of about 100mm x 100mm the shear stress is substantially constant and/or can be calculated with a good degree of accuracy.

Further discussion concerning the measurement area will be given in the next two chapters.
Chapter 5b

Step 2 - The Measurement and Determination of Shear Stress Direction
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List of Equations

\[ K = \frac{P_1 - P_2}{\frac{1}{2}(P_1 + P_2) - P_3} \]  
Equation 5.1b .................................................................................................................. 163

\[ \alpha = 60 - \theta \]  
Equation 5.2b .................................................................................................................. 163

\[ \beta = \left( \frac{P_1 - \frac{P_2 + P_3}{2}}{P_1 - \frac{P_2 + P_3}{2}} \right)_\theta = \frac{\Delta P_\theta}{\Delta P_0} \]  
Equation 5.3b .................................................................................................................. 166

\[ \beta = \frac{P_1 - P_{rf} - \left( \frac{P_2 + P_3 - 2P_{rf}}{2} \right)_\theta}{P_1 - P_{rf} - \left( \frac{P_2 + P_3 - 2P_{rf}}{2} \right)_0} \]  
Equation 5.4b .................................................................................................................. 167

\[ \beta^* = \frac{9P_1}{P_1} = \left( \frac{P_1 - P_{rf}}{P_1 - P_{rf}} \right)_\theta \]  
Equation 5.5b .................................................................................................................. 167

\[ \frac{\Delta P^0 \cdot h^2 \cdot \rho}{\mu^2} \]  
Equation 5.6b .................................................................................................................. 173
\[
\frac{\Delta p^0}{\tau} \quad \text{Equation 5.7b}
\]

List of References


Angular Measurements of Shear Stress

The main scope of step 2 was to determine the direction of shear stress since this could have had an effect on the interpretation of step 3 results. There are two ways of doing this with the triangular block probe:

- By placing a series of probes at different angles with respect to the flow and then taking pressure recordings for each angle and probe.
- By rotating (on a disk or rotatable platform) a series of probes at different angles with respect to the flow and then taking pressure recordings.

The first method is liable to differences in probe construction and at least 6–10 probes would be needed to obtain the necessary angular calibration curve. A further problem would be mounting them in a sufficiently small area without causing probe cross disturbance.

The second method still had the same probe tolerance liability but required a limited number of probes (1 to 5 probes). Consequently the latter method was chosen and one probe was placed at the centre of a manually, rotatable, circular disk. This provided accurate control of the probe's angular position and implied that a large number of angles could eventually be experimented.

The drawback was that this required the design and building of a second tunnel roof, rotating disk and, above all, a means of sealing disk and roof. Just as in step 1 the disk and probe were located in an area of fully developed flow with the Preston tube just in front of the disk.

The new roof layout is shown next.
In order to ensure an adequate water seal while maintaining easy disk rotation, a solid Lexan disk was made and an "L" shaped recess machined in the tunnel roof. The seal was achieved using silicone grease spread thickly between the roof and disk, this, together with the water pressure (see arrows in figure below), effectively provided both a seal and easy manual rotation of the disk during tests.

The installation of the rotating disk, seen from outside the water tunnel, is shown next along with the silicone tubing connected to the probe's needles:

Figure 5.1b – New Roof Layout

Figure 5.2b – Rotating disk and seal

Figure 5.3b – Installation of Rotating Disk
In order to position the probe at a specific angle small 5 degree divisions (increments) were machined into the roof, as shown below:

![Divisions](image)

**Figure 5.4b – Five degree divisions around Rotating Disk**

The probe's pressure tappings were realised with the same method used in step 1 i.e. small medical needles and silicone tubing, and positioned as follows.

![Diagram](image)

**Figure 5.5b – Disk and Probe Layout**

The above disk is shown at an arbitrary angle $\theta$ and two static pressure tappings (local and remote) are shown. During tests the disk was positioned at $10^\circ$ intervals between $10^\circ$ and $60^\circ$ and differential pressure measurements taken repeatedly for a given hydrodynamic condition.

Two sets of recordings were made, one with the probe's (i.e. local) static pressure tapping and one with the remote static pressure tapping indicated by the letter R in the test results summary.
The remote static pressure tapping was placed in line with the probe centreline, approx. 30mm from the wall, and on the roof of the tunnel. The dynamic pressure tappings are numbered according to the flow direction i.e. the angle of incidence, hence the highest possible dynamic pressure is indicated as P1, P2 etc.

Interpretation of the pressure recordings is as follows:

- P1, P2 and P3 are differential pressure recordings taken using the probes static pressure tapping and with the disk at ZERO incidence angle i.e. $\theta=0^\circ$ ("probe ref. Pressure" or simply "local").

- Similarly P1R, P2R and P3R all refer to the remote static pressure tapping (R meaning "ext. ref. or remote") again at $\theta=0^\circ$. This angle is referred to tapping 1 while angle $\theta$ is the probe's yaw angle.

- Recordings were taken every 2 rotational increments (i.e. every $10^\circ$) hence the recordings are marked; 10P1, 10P2, 10P3, 20P1, 20P2, 20P3 etc and 10P1R, 10P2R, 10P3R, 20P1R, 20P2R, 20P3R etc

The probe pressure measurements have been recorded in different hydrodynamic conditions i.e. from approx. Re=82000 to 143000 at 20000 intervals (see table 5.2b). For each Reynolds number 42 probe measurements were taken (6 for each $10^\circ$ angle increment) plus one Preston tube recording. Hence a total of 172 measurements were made.

Before angular measurements were attempted the probe was calibrated with the Preston tube. This was done by comparing the Preston tube data to P1 and P1R measurements for another 4 Reynolds numbers i.e. Re=51k, 71k, 113k and 130k. This data is given next.
Table 5.1b – Preston tube Recordings for Shear Stress Magnitude

<table>
<thead>
<tr>
<th>Re</th>
<th>$J_{med}$</th>
<th>$U$ (m/s)</th>
<th>$Q$ (m³/s)</th>
<th>$T°C$</th>
<th>$v$ (m²/s)</th>
<th>$\rho$ (kg/m³)</th>
<th>$\square$ (Nm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51280</td>
<td>0.005</td>
<td>0.689</td>
<td>0.0103</td>
<td>14.6</td>
<td>1.161E-3</td>
<td>999.19</td>
<td>1.16</td>
</tr>
<tr>
<td>71330</td>
<td>0.010</td>
<td>0.958</td>
<td>0.0143</td>
<td>14.5</td>
<td>1.155E-3</td>
<td>999.21</td>
<td>2.080</td>
</tr>
<tr>
<td>112810</td>
<td>0.024</td>
<td>1.528</td>
<td>0.0220</td>
<td>14.3</td>
<td>1.161E-3</td>
<td>993.24</td>
<td>4.500</td>
</tr>
<tr>
<td>130460</td>
<td>0.031</td>
<td>1.767</td>
<td>0.0260</td>
<td>14.3</td>
<td>1.161E-3</td>
<td>999.24</td>
<td>6.040</td>
</tr>
</tbody>
</table>

In tables 5.1b and 5.2b the average pressure drop ($J_{med}$) is calculated according to Darcy's equation (column 2) along with the average shear stress calculated using Patel's calibration [ref. 5.1b] and the DP for the Preston tube (last column).

Table 5.2b that follows refers to the four hydrodynamic conditions (identified by 4 Re numbers) used for the angular calibration of the probe.

Table 5.2b – Shear Stress Magnitude using Patel's calibration for the Preston tube

<table>
<thead>
<tr>
<th>Re</th>
<th>$J_{med}$</th>
<th>$U$ (m/s)</th>
<th>$Q$ (m³/s)</th>
<th>$T°C$</th>
<th>$v$ (m²/s)</th>
<th>$\rho$ (kg/m³)</th>
<th>$\square$ (Nm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>81968</td>
<td>0.0136</td>
<td>1.104</td>
<td>0.0165</td>
<td>14.5</td>
<td>1.155E-3</td>
<td>999.21</td>
<td>2.627</td>
</tr>
<tr>
<td>100595</td>
<td>0.0194</td>
<td>1.355</td>
<td>0.0203</td>
<td>14.5</td>
<td>1.155E-3</td>
<td>999.21</td>
<td>3.667</td>
</tr>
<tr>
<td>121107</td>
<td>0.0272</td>
<td>1.640</td>
<td>0.0246</td>
<td>14.0</td>
<td>1.171E-3</td>
<td>993.28</td>
<td>5.482</td>
</tr>
<tr>
<td>142629</td>
<td>0.0354</td>
<td>1.912</td>
<td>0.0287</td>
<td>14.7</td>
<td>1.149E-3</td>
<td>999.18</td>
<td>7.094</td>
</tr>
</tbody>
</table>

The tables that follow are the pressure recordings for each angular position of the disk and probe taken over a 10 second ensemble period. The pressure signal is expressed both in Pa and the original voltage provided by the Druck pressure transducer. A measure of the variance and coefficient of variance (CV) is also given. Of special importance is the static reference pressure, which assumes two values. The first pressure is measured locally (i.e. centre of the probe) and is reported "Probe pressure reference or local" hereon. The other reference is the remote static pressure taken in the vicinity of the probe and reported "ext. ref." or "remote" hereon.

$J_{med}$ is a dimensionless form of the pressure drop per meter.
<table>
<thead>
<tr>
<th>Pressure Recording reference</th>
<th>DP (Pa)</th>
<th>Transducer Signal (Volt)</th>
<th>Variance (Volt)</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP Preston tube</td>
<td>228.65</td>
<td>1.143</td>
<td>0.010034</td>
<td>0.00</td>
</tr>
<tr>
<td>P1</td>
<td>79.16</td>
<td>0.396</td>
<td>0.021</td>
<td>0.02</td>
</tr>
<tr>
<td>P2</td>
<td>6.27</td>
<td>0.031</td>
<td>0.008</td>
<td>0.01</td>
</tr>
<tr>
<td>P3</td>
<td>7.12</td>
<td>0.036</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>P1R</td>
<td>41.77</td>
<td>0.209</td>
<td>2.188</td>
<td>0.03</td>
</tr>
<tr>
<td>P2R</td>
<td>-32.01</td>
<td>-0.160</td>
<td>0.004</td>
<td>0.02</td>
</tr>
<tr>
<td>P3R</td>
<td>-30.00</td>
<td>-0.150</td>
<td>0.002</td>
<td>0.01</td>
</tr>
<tr>
<td>10P1</td>
<td>73.87</td>
<td>0.369</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>10P2</td>
<td>8.86</td>
<td>0.044</td>
<td>0.015</td>
<td>0.01</td>
</tr>
<tr>
<td>10P3</td>
<td>3.91</td>
<td>0.020</td>
<td>0.009</td>
<td>0.02</td>
</tr>
<tr>
<td>10P1R</td>
<td>40.21</td>
<td>0.201</td>
<td>0.003</td>
<td>0.01</td>
</tr>
<tr>
<td>10P2R</td>
<td>-26.21</td>
<td>-0.131</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>10P3R</td>
<td>-30.86</td>
<td>-0.154</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>20P1</td>
<td>69.29</td>
<td>0.346</td>
<td>0.011</td>
<td>0.01</td>
</tr>
<tr>
<td>20P2</td>
<td>10.72</td>
<td>0.054</td>
<td>0.004</td>
<td>0.01</td>
</tr>
<tr>
<td>20P3</td>
<td>0.46</td>
<td>0.002</td>
<td>0.008</td>
<td>0.02</td>
</tr>
<tr>
<td>20P1R</td>
<td>35.40</td>
<td>0.177</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>20P2R</td>
<td>-22.17</td>
<td>-0.111</td>
<td>0.002</td>
<td>0.02</td>
</tr>
<tr>
<td>20P3R</td>
<td>-33.03</td>
<td>-0.165</td>
<td>0.005</td>
<td>0.02</td>
</tr>
<tr>
<td>30P1</td>
<td>58.61</td>
<td>0.293</td>
<td>0.009</td>
<td>0.02</td>
</tr>
<tr>
<td>30P2</td>
<td>11.57</td>
<td>0.058</td>
<td>0.003</td>
<td>0.05</td>
</tr>
<tr>
<td>30P3</td>
<td>-4.08</td>
<td>-0.020</td>
<td>0.006</td>
<td>0.18</td>
</tr>
<tr>
<td>30P1R</td>
<td>26.77</td>
<td>0.134</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>30P2R</td>
<td>-20.24</td>
<td>-0.101</td>
<td>0.001</td>
<td>0.02</td>
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<tr>
<td>30P3R</td>
<td>-35.73</td>
<td>-0.179</td>
<td>0.002</td>
<td>0.01</td>
</tr>
<tr>
<td>40P1</td>
<td>47.85</td>
<td>0.239</td>
<td>0.006</td>
<td>0.02</td>
</tr>
<tr>
<td>40P2</td>
<td>12.30</td>
<td>0.061</td>
<td>0.004</td>
<td>0.05</td>
</tr>
<tr>
<td>40P3</td>
<td>-7.61</td>
<td>-0.038</td>
<td>0.010</td>
<td>0.13</td>
</tr>
<tr>
<td>40P1R</td>
<td>19.41</td>
<td>0.097</td>
<td>0.001</td>
<td>0.02</td>
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<tr>
<td>40P2R</td>
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<td>-0.180</td>
<td>0.003</td>
<td>0.01</td>
</tr>
<tr>
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<td>33.88</td>
<td>0.169</td>
<td>0.010</td>
<td>0.03</td>
</tr>
<tr>
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<td>17.27</td>
<td>0.086</td>
<td>0.011</td>
<td>0.06</td>
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<td>-11.41</td>
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<td>0.046</td>
<td>0.002</td>
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<td>-0.041</td>
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<td>50P3R</td>
<td>-35.72</td>
<td>-0.179</td>
<td>0.003</td>
<td>0.02</td>
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<tr>
<td>60P1</td>
<td>20.88</td>
<td>0.104</td>
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<td>60P2</td>
<td>24.77</td>
<td>0.124</td>
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<tr>
<td>60P3</td>
<td>-13.17</td>
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<td>60P1R</td>
<td>-1.69</td>
<td>-0.008</td>
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</tr>
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<td>60P2R</td>
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<td>0.001</td>
<td>0.02</td>
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<td>60P3R</td>
<td>-35.98</td>
<td>-0.180</td>
<td>0.003</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 5.3b – Experimental data for Re=81968
<table>
<thead>
<tr>
<th>Pressure Recording reference</th>
<th>DP (Pa)</th>
<th>Transducer Signal (Volt)</th>
<th>Variance (Volt)</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP Preston tube</td>
<td>345.78</td>
<td>1.729</td>
<td>0.055</td>
<td>0.001</td>
</tr>
<tr>
<td>P1</td>
<td>119.55</td>
<td>0.598</td>
<td>0.026</td>
<td>0.001</td>
</tr>
<tr>
<td>P2</td>
<td>12.61</td>
<td>0.063</td>
<td>0.013</td>
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<td>P3</td>
<td>11.25</td>
<td>0.056</td>
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<td>0.008</td>
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<td>P1R</td>
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<td>-40.51</td>
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<tr>
<td>P3R</td>
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<td>10P1</td>
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<td>10P2</td>
<td>15.00</td>
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<td>0.014</td>
<td>0.004</td>
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<tr>
<td>10P3</td>
<td>6.69</td>
<td>0.033</td>
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<td>0.006</td>
</tr>
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<td>62.41</td>
<td>0.312</td>
<td>0.009</td>
<td>0.001</td>
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<tr>
<td>10P2R</td>
<td>-37.82</td>
<td>-0.189</td>
<td>0.001</td>
<td>0.001</td>
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<tr>
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<td>-0.235</td>
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<td>0.001</td>
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<tr>
<td>20P1</td>
<td>104.42</td>
<td>0.522</td>
<td>0.010</td>
<td>0.001</td>
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<td>13.64</td>
<td>0.068</td>
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<td>0.006</td>
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<td>20P3</td>
<td>0.00</td>
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<td>56.16</td>
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Table 5.4b – Experimental data for Re=100595
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<th>CV</th>
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Table 5.5b – Experimental data for Re=120107
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Table 5.6b – Experimental data for Re=142629

Notes: In the event of processing error by the data acquisition system the variance is reported 0.000.
Experimental Data Analysis and Probe Angular Calibration

The following approach was originally derived by Gaudet, Savory and Toy [ref. 5.2b] for the calibration of a similar triangular block probe. However, a more direct method for establishing the direction of the shear stress is reported here [ref. 5.3b].

In essence a series of parameters (k, α, β, θ) are determined that provide the direction of the shear stress simply by knowing the probe's pressure data.

Phase I

First the K coefficient is determined:

\[ K = \frac{P_1 - P_2}{\frac{1}{2}(P_1 + P_2) - P_3} \]  

Equation 5.1b

This coefficient links the angle α to the three differential pressure measurements: α is defined as:

\[ \alpha = 60 - \theta \]  

Equation 5.2b

The experimental data for K (local and external static reference pressure) are provided below for the four Reynolds numbers tested, followed by the relative plots:

**Table 5.7b – K Data**

<table>
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<th>α = 60 - θ</th>
<th>Re=81968</th>
<th>Re=100595</th>
<th>Re=120107</th>
<th>Re=142629</th>
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<td>K (ext. ref.)</td>
<td>K (local)</td>
<td>K (ext. ref.)</td>
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<td>1.452</td>
<td>1.622</td>
<td>1.422</td>
</tr>
<tr>
<td>30</td>
<td>1.201</td>
<td>1.205</td>
<td>1.223</td>
<td>1.124</td>
</tr>
<tr>
<td>20</td>
<td>0.943</td>
<td>0.946</td>
<td>0.881</td>
<td>0.884</td>
</tr>
<tr>
<td>10</td>
<td>0.449</td>
<td>0.477</td>
<td>0.465</td>
<td>0.459</td>
</tr>
<tr>
<td>0</td>
<td>-0.108</td>
<td>-0.095</td>
<td>-0.137</td>
<td>-0.116</td>
</tr>
</tbody>
</table>
Figure 5.6b – K Plot for Re=81968

Figure 5.7b – K Plot for Re=100595
Figure 5.8b – $K$ Plot for Re=120107

Figure 5.9b – $K$ Plot for Re=142629
The following graph, in which the average values of K are plotted, shows that K is not dependent on the static reference pressure. This is confirmed also by equation 5.1b and the previous plots for K.

Figure 5.10b – Average values of K for 82000<Re<143000

Clearly once the α–K graph is complete it is possible to determine the value of α for any probe angle between 0 and 60°. Consequently to determine the direction of the probe shear stress all that is needed are the recordings of the three probe pressures (local or remote referenced i.e. P1, P1R, P2 or P2R etc.) and the above graph.

**Phase 2**

Next the β coefficient is determined:

$$\beta = \frac{(P_1-\frac{P_2+P_3}{2})}{\frac{P_2+P_3}{2}} = \frac{\Delta P_0}{\Delta P_0}$$

Equation 5.3b
This coefficient represents a dimensionless ratio that relates the zero angle pressure differential to another at a different angle (θ) and avoids the complexity of discriminating between local and remote static pressures.

\[
\beta = \frac{\left( p_1 - p_{\text{rf}} \right) - \left( \frac{p_2 + p_3 - 2p_{\text{rf}}}{2} \right)_0}{\left( p_1 - p_{\text{rf}} \right) - \left( \frac{p_2 + p_3 - 2p_{\text{rf}}}{2} \right)_0} \quad \text{Equation 5.4b}
\]

*NB Pressure differences are indicated with a capital letter 'P' while simple pressures are denoted with a small letter 'p'. Also 'θ' indicates a zero angle of incidence while \( \Theta \) is any other angle up to 60°.*

This latter equation shows how the static reference pressure and dynamic pressures are related and why the reference pressure disappears. It also shows that \( \beta \) is simply a ratio in which the difference between the dynamic pressure on tapping 1 (the one facing the flow) is compared to the average of the two other dynamic pressure tappings (2 and 3). This is also a convenient way of understanding what is happening in terms of flow conditions at the rear of the probe.

Of course this is not absolutely essential since only the differential pressure could be used. For example, for tapping 1 one may write:

\[
\beta^* = \frac{\Phi p_1}{p_1} = \frac{\left( p_1 - p_{\text{rf}} \right)_\theta}{\left( p_1 - p_{\text{rf}} \right)_0} \quad \text{Equation 5.5b}
\]

Here the reference pressure becomes important and the same relationship could be obtained for tapping 2 or 3, although it seems more logical to use tapping 1 since this was facing the flow and has a positive value between 0≤θ≤60.

Moreover this approach could be used for any probe, including the square shaped probe, although behaviour (and calibration) would change from probe to probe especially because of position and size tolerances.
Both the values of $\beta$ and $\beta^*$ vary between 0 and 1 and the major difference lies in the dependency on the reference pressure: for $\beta$ it is null while $\beta^*$ changes significantly (as can be seen in eq. 5.5b). This can be observed in the following graphs.

**Figure 5.11b – $\beta$ vs. $\theta$ for Re=81968**

**Figure 5.12b – $\beta$ vs. $\theta$ for Re=100595**
Similarly for $\beta$ averaged for all four Reynolds numbers:

Figure 5.15b – Average $\beta$ for 82000<Re<143000 circa
On the other hand $\beta^*$ depends on the reference pressures, although in the next two graphs (again for $82000<\text{Re}<143000$) $\beta^*$ behaviour can be seen to be essentially uniform.

![Graph 1](image1.png)

**Figure 5.16b – Average values for $\beta^*$ vs. $\theta$ (Probe ref.)**

![Graph 2](image2.png)

**Figure 5.17b – Average values for $\beta^*$ vs. $\theta$ (Ext. ref.)**

The dependency of $\beta^*$ on the reference pressure can be illustrated also along side data by Gaudet et al.
Phase 3

Once the direction of the shear stress is determined the next step is to determine its magnitude. This is done by associating it with the Preston tube measurements and the relative Patel calibration curve originally ideated for the Preston tube with a zero incidence angle.
In essence the Preston tube and Patel's calibration are used to determine the shear stress from which an estimation is obtained for the probe.

Thus the Preston tube measurements are used as the reference shear stress for probe calibration which implies that probe pressure (differential) should first be measured for an incidence angle of zero degrees. As a comparison the resulting difference between the Preston tube and probe measurements (P1 or P1R) can be seen in the previous four tables (first two rows).

Hence both the local and remote differential pressure recordings were recorded first for P1 followed by pressures P2 and P3 (and P2R and P3R) for all subsequent angles.

With these recordings in hand equation 5.4b (or 5.5b) can be re-arranged to provide the unknown pressure differential at angle \( \theta \), namely:

\[
\Delta P_1 = \beta^* P_1
\]

or

\[
\left[ p_1 - p_{\text{ref}} - \left( \frac{p_2 + p_3 - 2p_{\text{ref}}}{2} \right) \right]_0 = \beta \left[ p_1 - p_{\text{ref}} - \left( \frac{p_2 + p_3 - 2p_{\text{ref}}}{2} \right) \right]_0
\]

Effectively once the left-hand terms, i.e. reference differential pressure (\( \Delta P \)) and \( \beta^* \) (or \( \beta \)), are obtained the unknown differential pressure (right-hand term) can be estimated. Clearly \( \beta^* \) requires less pressure recordings than \( \beta \) but is dictated by where the reference pressure is taken.

**Phase 4**

With the differential pressure for angle \( \theta \) (e.g. \( \Delta P_1 \)) known the next step is to associate this to the shear stress, \( \tau \). Phase 4 was originally suggested by Gaudet, Savory and Toy and consists of slightly modifying Patel's calibration.

Basically the idea is to plot two dimensionless ratios in which both contain the required differential pressure (indicated \( \Delta P^o \) by Gaudet et al.) but only one has the unknown shear stress magnitude (\( \tau \):

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Clearly phase 3 will lead to \( 9P1 = \Delta P^o \), and \( h \) shown above in equation 5.6b is the probe (mouth) thickness i.e. 0.20mm.

The missing shear stress magnitude (\( \tau \)) is provided by the Preston tube measurements. Clearly the assumption here is that the Preston tube precision of 6% is acceptable [ref. 5.4].

As there were two reference pressures the end result involves two plots, \( \Delta P^o/\tau \) vs. \( \rho h^2 \Delta P^o/\nu^2 \), one for the local static pressure reference and one for the remote static pressure reference. The plots are shown below for low (1000<\(Re<26000\)) Reynolds numbers emulated specifically in the water tunnel so that a direct comparison can be made later with work by Gaudet et al.

\[
\frac{\Delta P^0 \cdot h^2 \cdot \rho}{\mu^2} \quad \text{Equation 5.6b}
\]

\[
\frac{\Delta P^0}{\tau} \quad \text{Equation 5.7b}
\]

The two curves are evidently different. This is caused by the wake at the back of the probe and flow over the probe's static pressure tapping. That is to say the probe provokes a disturbance in the flow on top of the probe such that local static pressure recordings are negatively (by 20/30\%) effected. This was also predicted in the CFD work.
As a result the better curve is that referred to the remote static pressure reference. A similar curve can be plotted using the β calibration curve as shown below:

![Graph showing local and remote calibration plots](image)

**Figure 5.20b – Probe calibration plots for β**

What can now be shown is a direct comparison with work by Gaudet et al. both for β (β*) and K. The following curves were averaged over the Reynolds range mentioned for the previous two curves:

![Graph showing local and remote plots](image)

**Figure 5.21b – β=f(θ)**
Summary and Proposed Method

The previously described four phases seem a complicated and lengthy process to obtain shear stress magnitude and direction from the probe’s pressure data. So it will be condensed, presented in a flow chart and summarised here:

The more direct process is that for $\beta^*$ ($\beta$ is simply just longer) hence this will be illustrated:

- Calculate $K$ and obtain the value of $\theta$.
- Calculate $\beta^*$ for the reference pressure desired i.e. local or remote.
- Calculate $\Delta P^\circ$, interpolate from graph (see figure 5.24b) and obtain $\tau$ (clearly $\tau$ will refer either to the local or remote reference pressure). Alternatively the equations provided in figure 5.24b can be used.

To make things even easier $K$ and $\beta$ graphs can be plotted together as follows:
Figure 5.23b – Combined K and β* Graphs

Note that the approach here is to interpolate α so that θ can be found before moving onto β (or β*). In alternative to this an equation can be used that interprets the data for K directly. In fact taking simply the average data in table 5.7b for $K_{\text{local}}$ and using Table Curve 2D we obtain:

$$\alpha = -0.1532704 + 0.1393937K^{0.667198}$$

for $K_{\text{local}}$ and $K_{\text{remote}}$

The next and final graph is used to determine $\tau$ once the $\Delta P^o$ is known.
Figure 5.24b – Graph for $\tau$ interpolation
Figure 5.25b – Flow Chart for Shear Magnitude and Direction Determination
Chapter 5c

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Further Bibliographic references

- Sedov L. I., Similarity and Dimensional Methods in Mechanics, MIR Publishers Moscow, 1982
Preface and Scope of Step 3

The scope of step 3 was to determine the effects of hydrodynamic shear stress on the removal of soil from a plain weave cotton textile.

In this chapter a link will be created between steps 1–2 and the investigation of step 3. That is to say step 3 bridges the tunnel experimentation with washing machine conditions and tests. Three activities were involved in this investigation, these being:

1. A SEM investigation to quantify and classify the tested soils.
2. Exposure of soiled samples for a specific test duration (approx. 40mins) to range of shear stress ($0.14 \leq \tau_w \leq 7.74$ Pa).
3. Exposure of soiled samples for a set shear stress ($\tau_w \approx 2.3$ Pa) for an increasing test duration (from 1 to 150mins).

The investigation was based on the IEC 60456 standard [ref. 5.1c] used for domestic washing machine appliance performance testing. Experimentation in step 3 was therefore based on the use of 4 EMPA 105 [ref. 5.2c] reference soils (carbon black and mineral oil, pigs blood, chocolate and milk, and red wine). These soils are deposited on a plain 1/1 weave made of cotton fibre and are commonly known as EMPA stripes.

Correlating Tunnel and Washing Machine Reynolds Numbers

The hydrodynamic conditions described in steps 1 and 2 were established on the basis of what happens in the washing machine. In other words the tunnel hydrodynamic conditions have been equated and correlated to washing machine conditions.

This was not an easy task because no model was available, hence models [refs. 5.3 and 5.4c] have been developed to aid this correlation and task. There was also doubts about the scalability and similarity [ref. 5.5c] of some of this project. In fact originally the idea was to test a scaled-up model of a cotton weave in the tunnel.
As can be imagined there is no straightforward answer to the question of correlating tunnel and washing machine conditions because of the sheer complexity of the wash load motion. Henceforth it was concluded that all models would have some assumptions and/or simplifications. Hydrodynamically speaking the most convenient way to compare the tunnel and washing machine is to arrive at a Reynolds number i.e. compare the classical Reynolds number for the tunnel to that defined by Schmidt for mixing vessels [ref. 5.6c].

The Schmidt number is a dimensionless number that considers the drum to be a vessel in which two substances are mixed i.e. the wash load and water, and states a ratio between inertial and viscous effects, this is why it is considered a particular form of the original Reynolds number. The Schmidt number considers the inertial effects to be described by the drum speed and diameter while the viscous effects are handled by the kinematic viscosity of the wash water. Hence the Schmidt interpretation of the Reynolds number is written:

\[
\text{Re}_s = \frac{ND^2}{v} \quad \text{Equation 5.1c}
\]

Using this interpretation we may state that the tunnel Reynolds number test range (15k≤Re≤155k at 15°C) was equivalent to a typical European (where D=0.5m) washing machine drum speed of 4≤N≤43 RPM circa. This result is very similar to the typical drum speed of the washing machine used in the FMT and S experimentation described in step 4 (see also Fig. 5.2c).

However, although the Schmidt approach is substantially correct it does include several imperfections, these being:

- In equation 5.1c there is no indication of the effects of drum depth. Hence any drum with the same diameter is satisfactory which can’t be the case because the eventual mixing effects of the wash load during drum rotation is dependent on the space available.
The definition of the drum diameter is not an easy task because of the ribs inside the drum. This inevitably leads to discrepancies in calculations i.e. with or without ribs.

Washing machine drums are perforated hence the amount of water in the drum will depend on the number and size of the perforations and drum size.

The wash load size, volume, mass etc are not considered.

With these considerations in mind the Schmidt number could be modified as follows:

$$Re^*_w = \frac{ND^2}{v} \frac{D}{L}$$  \hspace{1cm} \text{Equation 5.2c}

where $L$ represents the drum depth and $D$, the drum diameter (in meters), is defined as the average diameter of the inner ring made-up of the "dry" wash load. To establish this diameter the wash load is subjected to the standard washing cycle and after adequate tumbling, the wash load is spun so as to measure the inner ring of "dry" wash load shown in the next figure.

![Diagram of Drum Diameter based on Wash Load]

\textit{Figure 5.1c – Definition of Drum Diameter based on Wash load}

The author therefore suggests that the Schmidt number be further modified as follows:

$$Re^*_w = \frac{ND_{av}^2}{v} \frac{D_{av}}{L}$$  \hspace{1cm} \text{Equation 5.3c}
By introducing these modifications the equivalent drum speed for Re experimented in the water tunnel rises to $10 \leq \text{N} \leq 70$ RPM circa, but still remains accountable in the tunnel tests and covers the drum speeds shown in figure 5.2c.

![SPEED](image)

Figure 5.2c – Typical Drum Rotation Cycle (RPM vs. t in secs)

The inclusion of a second dimensionless group or ratio (D/L) satisfies several if not all 4 of the previously mentioned imperfections. For example, for a given ratio it is impossible to overload the washing machine, in fact the Bauknecht machine used for the tests had a maximum volume equivalent to 5kg of wash load. Tests were also carried out between 2 to 5kg i.e. 40 to 100% of the maximum load volume.

Yet another way of comparing tunnel and washing machine conditions is to use similarity. For example thus can be achieved between two different washing machines by concurrently accomplishing geometric, kinematic and dynamic similarity [ref. 5.7c] through parametric modelling.

In the case of washing machine the measures of length, velocity and acceleration are used.
Henceforth if we consider the first parametric measure to be that of length and this, for example, is defined as the ratio of fabric plug centre radius to that of the radius of drum. Hence $R_{pm}$ is described by:

$$R_{pm} = \frac{R_p}{R_D} \quad \text{Equation 5.4c}$$

A ratio of radii is chosen because it is a common denominator for length, velocity and acceleration.

Clearly $R_{pm}$ can take any value between 0 to 1, although both limits are physically impossible, i.e. $0 < R_{pm} < 1$. Another interesting comment is that we are looking for a relationship between length, speed and acceleration hence any point in a fabric may be considered for this type of analysis.

Moving on we may also state that the velocity parameter is:

$$v_{pm} = \frac{v_p}{v_D} = \frac{\omega R_p}{\omega R_D} = R_{pm} \quad \text{Equation 5.5c}$$

This is a very interesting result because it implies that two machines, which have the same $R_{pm}$ value and run at the same speed will be dimensionally equivalent in geometric and kinematic terms.

In dynamic terms the centrifugal force that pushes the fabric plug outwards towards $R_D$ involves the centripetal acceleration that may be written in parametric form:

$$a_{pm} = \frac{v_{pm}^2}{R_{pm}} = \frac{R_{pm}^2}{R_{pm}} = R_{pm} \quad \text{Equation 5.6c}$$

This simple approach does not take into full consideration the true motion of the fabric plug since it assumes 2D and not 3D motion. This could be tackled by estimating or measuring the motion of the fabric, although this implies knowing the position vectors in all 3 dimensions at
any instant in time, which is clearly not an easy task. However, it can be done in two dimensions, as will be shown in chapters 5d and 5e.

To emphasise the complexity of perfect similarity Bear [ref. 5.8c] states (for perfect similarity in porous media) the contribution of each component that makes up the total position, velocity or acceleration vector is needed (versus time).

Further discussion on modelling will be given in chapters 5d and 5e.

**Soil and Weave Characterisation**

Before the testing of the EMPA stripes in the water tunnel there was a need to quantify and qualify the physical characteristics of the soil [ref. 5.9c] so as to be able to relate them to the boundary layer and weave surface. A specific SEM (Scanning Electronic Microscope) investigation was therefore carried out.

The findings were that the four standard EMPA 105 soils have specific shapes, sizes and locations on the textile fibre surface. It was also possible to reveal the dimensions of the weave, thread and fibres as well as where the soils are typical resident. The SEM analysis findings are summarised in chapter 6 but can be viewed in the following table and figures:

Table 5.1c – EMPA Soil Characterisation based on SEM investigation BEFORE tests i.e. new soils

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Average Soil Size in μm</th>
<th>Soil shape</th>
<th>Soil Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon black</td>
<td>5–20</td>
<td>Cluster of spheres</td>
<td>On fibre and sometimes lodged between fibres</td>
</tr>
<tr>
<td>(and Mineral oil)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>5–20</td>
<td>Cluster of spheres or elliptical</td>
<td>On fibre and sometimes lodged between fibres</td>
</tr>
<tr>
<td>Chocolate (and Milk)</td>
<td>20–50 or higher</td>
<td>Foamy or strand-like</td>
<td>Across fibres and pores.</td>
</tr>
<tr>
<td>Wine</td>
<td>1–5</td>
<td>Speckle or small sphere</td>
<td>On fibre, generally too small to bridge fibre gap</td>
</tr>
</tbody>
</table>
Table 5.2c – EMPA Soil Characterisation based on SEM investigation AFTER tunnel tests

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Average Soil Size in μm (1E-6m)</th>
<th>Soil shape</th>
<th>Soil Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon black</td>
<td>5–20</td>
<td>Cluster of spheres</td>
<td>No change</td>
</tr>
<tr>
<td>(and Mineral oil)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>&lt;10</td>
<td>Cluster of flakes</td>
<td>On fibre. Fibres are squashed.</td>
</tr>
<tr>
<td>Chocolate (and Milk)</td>
<td>20–50 or higher</td>
<td>Foamy or strand-like</td>
<td>No change</td>
</tr>
<tr>
<td>Wine</td>
<td>1–25</td>
<td>Speckles or clusters</td>
<td>On fibre, Fibres are slightly squashed</td>
</tr>
</tbody>
</table>

Figure 5.3c – SEM pictures of EMPA soils (as new & unwashed): mag. 1000-4000X
Figure 5.4c – SEM pictures of Pigs blood and Red wine soils AFTER tunnel tests; magn. 1000-4000X

A. Close-up of EMPA stripe, B. Close-up of thread section, C. Plan view of weave, D. Close-up of weave and threads, E. Close-up of pore, F. Close-up of fibres with soil, G. Close-up of fibre and soil
The above figure illustrates the extent the EMPA stripe contours can vary and that the soil can locate itself in the most difficult weave locations, thus making detachment particularly difficult. Also the soil cannot be modelled strictly by a simple sphere shape although the soil does tend to cluster and form in bunches or balls. In some cases, such as chocolate and milk soil, the soil splinters and has a spread like form across the fibres. In theory abrasion would help here although this has the drawback that some of the soil could also be rubbed into the fibre surface. Further, although the pores are relatively "regular" they offer a high flow resistance because of shape and size. This can be appreciated by examining the figure below in which the change in pore shape and size across 16 levels (i.e. layers of constant thickness) of a plain and twill weave is shown:

![Figure 5.6c - Pore shape and size according to weave type](image)

This last figure exemplifies why flow through weave pores is slowed down due to pore distortion and why the fluid prefers to flow across the weave surface rather than through it [ref. 5.10c].

A further table can be used to summarise the observations linked to the physical characteristics of the (dry) weave illustrated in figure 5.5c. Further information about the waviness of the weave surface is given in chapter 5d.
Table 5.3c – Weave characteristics based on SEM investigation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Estimated weave roughness (surface waviness) is 100–300 μm.</td>
</tr>
<tr>
<td>b</td>
<td>Weave height is approx. 700–900 μm.</td>
</tr>
<tr>
<td>c</td>
<td>Weave is relatively regular, although rarely perfectly regular.</td>
</tr>
<tr>
<td>d</td>
<td>Weave regularity is bi-directional and valid for both sides of stripe.</td>
</tr>
<tr>
<td>e</td>
<td>Weave thread width is approx. 400–450μm.</td>
</tr>
<tr>
<td>f</td>
<td>Initial pore mouth size is typically about 3–5 fibre diameters (50–120 μm).</td>
</tr>
<tr>
<td>g</td>
<td>Pore mouth is fluted and has a complex cross-section.</td>
</tr>
<tr>
<td>h</td>
<td>Loose fibres are common and are strung across the weave.</td>
</tr>
<tr>
<td>i</td>
<td>Weave is typically porous and sponge-like due not only to the hollowness of the cotton fibre but also to the weave pattern.</td>
</tr>
</tbody>
</table>

This latter list of observations shows that the EMPA samples have a waviness more or less in-line with height of the block probes, thus confirming the integrity of steps 1–3. The drawing below illustrates:

![Probe and needle assembly and comparison with EMPA sample X-section](image)

Figure 5.7c – Probe and needle assembly and comparison with EMPA sample X-section

Also since the no-slip condition of the boundary layer starts at the (fibre) wall surface the surface waviness can be compared favourably with the roughness of a flat (plate) surface. This roughness tends to be lower in practise because the water tunnel (static) pressure compresses.
the weave thus making it flatter [ref. 5.11c] and essentially smooth. Also loose weave fibres assume the direction of flow and therefore act just as tufts\(^1\) do in wind tunnel measurements.

Another consideration is that the weave for the EMPA samples is a standard 1/1 plain weave meaning it is geometrically symmetric. Hence there is no preferred sample mounting direction (e.g. weave pattern direction) needed for step 3 experimentation in the water tunnel\(^2\).

Soil particle height was found to be generally lower than 50\(\mu\)m, which implies that there is a dimensional scale factor of at least 1:10 between soil and weave thread and even more if the longitudinal dimension (both along weft and warp direction) is considered. Hence even if the soil was perfectly located on the crest of the weave profile it could be considered more as (thread) surface roughness than a distinct obstacle to the flow. This means that the soil sits comfortably in the initial part of the boundary layer (both for tunnel and washing machine) and is virtually hidden in the weave (see parts A and D of figure 5.5c). This concept of boundary layer and soil location–interaction will now be explained.

### The Boundary Layer and Tunnel Test Conditions

The water tunnel experimentation in steps 1 to 3 was conducted in turbulent flow conditions, as is testified by the Reynolds number test range (approx. 15k\(\leq\)Re\(\leq\)155k).

In fluid flow (turbulent or non) the boundary layer is a region of the fluid flow attached to the wall (e.g. textile surface, tunnel wall, machine door, drum wall etc) where the fluid is stratified and slowed down. The flow will eventually come to rest at the wall surface, termed the no-slip condition, and therefore the flow inside the boundary layer will have a specific velocity contour (see upper part of figure 5.8c) and hence a shear stress profile.

---

\(^1\) Another example are algae on a reef

\(^2\) This was confirmed by experimentation in step 3. This was done by changing the orientation of the EMPA samples in step 3 tests and comparing soil removal efficiency.
It is important to realise that the no-slip condition is valid for any condition where a fluid meets a wall e.g. the block probes, EMPA samples, washing machine door surface, sphere etc., irrespective of whether the surface is at rest or in motion.

The boundary layer height, which decreases as flow velocity increases, is important for the soil removal process because the soil will be subjected to hydrodynamic shear stress (and force) caused by the friction effects of the fluid starting from the wall. Henceforth determination of the b. l. height or what constitutes it provides an indication of what the soil actually "feels" e.g. the magnitude of shear that it is subjected to.

For turbulent flows the boundary layer is essentially split into three layers [refs. 5.12c and 5.13c]; a). laminar or viscous sub-layer, b). buffer layer and c). turbulent eddy layer (see figure 5.6c) while in laminar flows only one laminar layer exists.

The effects of the above three layers (in turbulent flow) on the overall behaviour of the boundary layer changes as the water tunnel hydrodynamic conditions become more turbulent. That is to say as the turbulence increases the boundary layer will become more 'squashed' against the wall surface and the outer layer becomes more dominant with respect to the two layers. The boundary layer thickness, displacement thickness and the momentum thickness are also all dependent on the flow velocity in the tunnel.

There is also a further complication due to the transition from when the flow starts to when the flow, and thus b. l., may be considered fully developed [ref. 5.14c].

This is shown in the top r.h. corner of the next figure.
When trying to envisage what is happening to the soil within the boundary layer the immediate issue is know or assume the flow velocity. In this way it is possible to emulate what is happening in the washing machine directly in the water tunnel. Unfortunately there are at least two distinct flow domains in the washing machine, namely, inside the wash load and outside, and both have peculiar flow conditions.

On the outside of the wash load, e.g. near the washing m/c door, flow velocities of meters per second are quite common [ref 5.15c] and the flow is very turbulent also due to the reversing motion of the drum. Step 4 tackles this domain in greater detail.

It is a very different story inside the wash load and there is some contradiction about what actually happens in terms of flow: step 5 and chapter 5e, specifically tackle this issue.

In the meantime an assumption can be from mass transfer measurements [ref 5.16c]. Here the flow velocities are in the order of mms$^{-1}$ (which implies laminar flow) i.e. there is a factor of at least 1000 between the inside and outside of the wash load. However, the higher the flow velocity the higher the shear stress and turbulence.
So it is logical to conclude that the better cleaning (hydrodynamic) forces occur in turbulent flows and this explains why these conditions have been emulated in the tunnel tests. What is also important to realise is that the soil is suspended in a viscous layer of fluid that is virtually at rest [ref. 5.17c] whether it be in laminar or turbulent flow.

Further, because the intent of the laminar viscous sub-layer (where the soil sits) is to dampen any attempt at mixing with the upper layers it is logical to assume that the best way to remove soil from the fibre is to squash, break-down and/or attack the laminar viscous sub-layer. Clearly, the water cannot be removed\(^3\) since it is needed to aid heat and mass transfer\(^4\) e.g. convey the detergent.

One of the effects of drum rotation and wash load motion is to subject the fluid and soil to inertial effects, almost as if one is shaking and attacking the fluid thus subjecting the soil to gravitational and inertial effects. Thus the circular motion of the drum also improves detergent mixture and heat-mass transfer.

Another way of achieving the same goal is to subject the b. l. to cavitation or push gaseous mixtures through the wash load so as to modify the boundary layer characteristics [ref. 5.18c]. However, attempts at doing this with air [ref. 5.19c] by Daewoo in their "Bubble Washer" have not shown much progress.

This leaves the option of further "squashing" the b. l. by increasing the flow rate (i.e. higher Re). Not only will this reduce the b. l. thickness but it will also change all three sub-layers [ref. 5.20c]. Hence the laminar viscous sub-layer will become increasingly “squashed” against the wall and the formation of other, more chaotic, layers on top of it. This provides an opportunity for the soil to be exposed to more turbulence than before and was contemplated while designing the water tunnel.

\(^3\) Clearly the water would be replaced by air, i.e. another (gaseous) fluid.

\(^4\) This raises the question of what is the best fluid to achieve this goal.
In general the hydrodynamic conditions emulated for step 3 (15k ≤ Re ≤ 155k) provided laminar viscous sub-layer between 0.1mm–0.35mm and the boundary layer height up to 0.8mm circa. Thus all four soils were certainly in the linear part of the boundary layer velocity profile.

More important is that it is possible to scale the shear stress magnitude measured by the block probes (or Preston tube) to obtain an indication of what force the soil was actually subjected to in the tunnel.

Alternatively Kovich [ref. 5.21c] proposes a model based on drag, which can be applied by initially assuming that the soil particle is subjected to a drag force defined by:

$$F_D = \frac{1}{2} C_D \rho \bar{U}^2 R^2$$

Equation 5.7c

The immediate issue with this model is that an indication of the average (local) flow velocity ($U_{bar}$) is needed while the magnitude of the drag coefficient is determined by assuming laminar flow [ref. 5.22c] e.g. using:

$$C_D = 18 \text{Re}^{-0.6}$$

Equation 5.8c

Kovich used Van de Brekel's flow data to calculate the drag force and plot this against the particle size.

However, this data refers to mass transfer measurements that are averaged over the wash load and do not consider what happens in the boundary layer i.e. locally. In spite of this Kovich provided a useful graph for understanding just why the hydrodynamic force alone cannot overcome the Van de Walls force (the principal adhesion force) for very small particles [ref. 5.23c].

In this graph (see next figure) both forces are plotted and compared for a series of particle sizes.
From this graph it is evident that particles <0.25 microns (1 micron = 1 μm) are virtually impossible to remove (hydrodynamically). This has been suggested by Cutler and practise shows that particles below 0.1 microns cannot be removed [ref. 5.24c].

As mentioned previously it is possible to obtain an indication of the local velocity from the probe measurements thus removing one of the difficulties with the Kovitch method. This is achieved by translating the dynamic pressure of the probe into a velocity and, then using the drag coefficient, to estimate the drag force.

However the drag coefficient given in eq. 5.8c needs to be related to the local conditions as well. Thus to calculate the local velocity the assumption is that the dynamic pressure in the laminar viscous sub-layer is linear and therefore can be simply scaled (down) to provide a resultant local velocity. To do this one assumes a fixed particle size and shape, say 20 microns in diameter and spherical in shape and relate this to the total laminar viscous sub-layer height.
Thus the particle is more or less one tenth the height of the probes dynamic tapping and one tenth of the measured dynamic pressure.

Using this scale factor the following results are obtained:

**Table 5.4c – Soil Dynamic Velocity Stress and Shear Force**

<table>
<thead>
<tr>
<th>Re (Main stream)</th>
<th>Main stream Velocity (m/s)</th>
<th>Probe Pressure (Pa)</th>
<th>Estimated Probe Dynamic Vel. (m/s)</th>
<th>Estimated Dynamic Vel. on Soil (m/s)</th>
<th>Soil Shear Force (20μm dia.) in N</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
<td>0.2</td>
<td>20</td>
<td>0.2</td>
<td>0.02</td>
<td>2e-9</td>
</tr>
<tr>
<td>80000</td>
<td>1.1</td>
<td>79</td>
<td>0.4</td>
<td>0.04</td>
<td>5.2e-9</td>
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<tr>
<td>142000</td>
<td>1.9</td>
<td>300</td>
<td>0.77</td>
<td>0.077</td>
<td>12.9e-9</td>
</tr>
</tbody>
</table>

The next proposal is to use the shear stress measured by the probe and scale this down (by a ratio of front areas) to obtain a local shears stress (thus force) against the soil particle. A more precise and theoretical value can be obtained with equation 5.22a in step 1.

With a value of shear stress in hand and the soil's front surface area (that exposed to the flow) the shear force can be determined. In this way the following table is obtained:

**Table 5.5c – Soil Shear Stress and Shear Force**

<table>
<thead>
<tr>
<th>Re (Main Stream)</th>
<th>Main stream Velocity (m/s)</th>
<th>Probe square (Pa)</th>
<th>Estimated Probe mouth force (N)</th>
<th>Estimated Soil Force (20μm diameter) in N</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
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<td>64E-9</td>
<td>0.06E-9</td>
</tr>
<tr>
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<td>800E-9</td>
<td>0.8E-9</td>
</tr>
<tr>
<td>142000</td>
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<td>7.5</td>
<td>2400E-9</td>
<td>2.4E-9</td>
</tr>
</tbody>
</table>
This last shear force estimated for Re=142000 is shown in figure 5.9c and demonstrates that the hydrodynamic conditions were not enough to remove the soil. In fact for the two cases presented the Re number would probably have to be in excess of 250000 to stand any chance of overcoming the adhesion forces.

**Experimental Test Set-up for Step 3**

In the water tunnel experimentation the EMPA test samples were fitted to a flanged fabric holder plug which was fitted into the “roof” and mounted virtually flush to lower face of tunnels upper wall. The plug was essentially a square flange to which a sample of soiled textile could be attached (see also figure 3.8).

Since fluid flow was found to be symmetric and reasonably constant across the wall face the exact position of the plug was not critical and could arbitrarily positioned in either one of the two halves of the roof surface.

Figure 5.10c – EMPA sample position and $\tau$ profile measurements

However, the shear stress profile did vary as depicted in the next illustration.
This last figure shows the importance of the shear stress profile and where the EMPA samples needed to be positioned. In the end the samples were positioned between $Y=20$ and $Y=120$ mm, where the shear stress profiles were consistently constant (see steps 1–2).

The water temperature was that of the mains water line and hence varied between 15–20°C. The water supply could not be regulated and, hence heated, because of the very high flow rates and, above all, the absence of water heating plant.
The plug was designed so that it could be easily locked accurately in place and removed via a handle arrangement to aid assembly and test sample inspection. The test sample was fixed (pressed not glued) to the fabric holder plug by means of a brass frame that was locked in place by means of small grub screws. The fabric holder plug was designed and accurately machined so that the test sample was mounted almost flush to the roof surface but just protruding (≈0.2mm) into the boundary layer of the fluid flow.

**Figure 5.13c – Fabric holder plug details**

Two test modes were used:

**Mode 1** Each test sample was tested in steady state conditions for 40 minutes, excluding the time (initially several minutes) to fill and regulate the water tunnel.

**Mode 2** Test samples were exposed for a varying amount of time (from 1 to 150mins) to understand time dependency.

In mode 2 in order to reduce the filling time the tunnel pressure and flow rate was lowered so that the plug was fitted and tested within only 1 minute. This was particularly successful for tests lasting less than 5 – 8 minutes.
To monitor test conditions the tunnel was also equipped with all the relevant pressure transducers to determine the local average pressure drop across the “roof” or upper internal wall face of the tunnel. The flow rate was also monitored using the Thomson weir. In practise therefore the same measurement and data acquisition system used in steps 1 and 2 was also used for step 3. Further details are provided in chapter 3.

**Experimental Results**

The cleaning action, if any, in the water tunnel is that due to the application of hydrodynamic forces within the boundary layer. This is best defined by the measure of tangential shear stress that occurs within the boundary layer, that is strictly related to Reynolds number and the main stream velocity. This topic will be discussed again in chapter 6 but using a separate DOE involving the washing machine and soaking tests.

In the water tunnel tests two types of information have been be revealed, 1. the effects of shear (Reynolds number) and 2. the effects of shear vs. time on the cleaning efficiency.

**Cleaning Efficiency versus Reynolds Number**

In the first case the outcome was that soil removal was virtually unaffected by shear stress (see next figure). This is very interesting results because if one considers that as Re increases the sub-laminar layer thickness decreases and the shear stress increases the chances of removing soil should improve. This occurred only for Pigs blood soil while all other soils were either unaffected or actually responded negatively (Wine soil). Even taking into account optical (reflectance) measurement error (<1%) the above trends cannot be explained away.
Figure 5.14c – Tangential Fluid Shear Stress Effects

This confirms that the hydrodynamic forces tested in the tunnel were incapable of overcoming the adhesion forces, even at very high flow velocities (up to 2m s\(^{-1}\)).

To provide an idea of what this means at \(Re \approx 155k\) the laminar sub-layer thickness is approx. 50–60E–6 m (\(\tau = 7.7 \text{ Nm}^{-2}\)) while at \(Re \approx 20k\) the thickness rises to approx. 330–360E–6m (\(\tau = 0.2 \text{ Nm}^{-2}\)). The effects on the boundary layer and shear stress magnitude as \(Re\) increases is illustrated below:
Re vs. Tau vs. Sub-laminar layer thickness
(Pigs blood results)

A possible explanation could be that the soil is fundamentally rubbed or squashed into the fibre and/or the physical properties change. In the first instance no evidence of soil being pushed into the fibre was found in the SEM photographs that were taken after the tunnel tests although the soil shape was different, almost pressed. Hence it is more likely that the soils rheological properties change.

Time Dependency of Soil Removal

As there was some doubt about the change in rheological properties of the soils and consequent effects on their removal the water tunnel experiments were repeated and samples exposed for different, increasing, times. As mentioned previously no significant difference in soil removal was noticed as the Reynolds number increased from 15k to 155k, hence the time dependency test condition was set at an arbitrary value between Re=70k to Re=85k, i.e. approx. half way between 15k and 155k.

The samples were exposed to test intervals from 1 to 150 minutes with the longest time referred to as the “infinite” exposure time.
Not surprisingly the time dependency of soil removal was different for all four soils, in particular, chocolate was substantially zero. However, pigs blood and red wine did have a clear distinct time dependency. Last but not least the carbon black soil had a (very) slight time dependency.

In all three cases the time dependency assumes a typical initial exponential signature, which is typical of first order systems, for the first 10 to 20mins and thereafter becomes practically linear. This is best observed if the rate of change of soil removal efficiency is plotted against time as shown below:

![% Cleaning rate vs. Time for all Four Soils](attachment:figure_5.16c.png)

Figure 5.16c – Time Dependency of Soil Removal (up to ≈50mins)

The fact that we observe a first order response in soil removal implies three things:

1. The system (soil and fibre) has storage (or dissipative) properties. That is, both fibre and soil absorb the water and this could change the rheological properties of the soil.

2. The system undergoes a step response, which is logical since the water tunnel literally fills in seconds and virtually wets the test sample instantaneously.
3. There is obviously mass transfer occurring and this is known [ref. 5.25c] to have an exponential response for washing machine operation. It therefore seems likely that the action of the fluid on the soil is principally tied to the modification of the soil’s mechanical, physical and chemical characteristics. Henceforth the function of the fluid could therefore be to change the rheology of the soil [refs. 5.26c and 5.27c] and fibre, so that this effects both the mechanical properties and adhesive forces [ref. 5.28c]. The hydrodynamic conditions are obviously beneficial both to dislodge the soil from the fibre surface and to ensure that it never returns to the original site.

**Associated Research**

It is convenient to compare Step 3 findings with work by other researchers although almost all of it was conducted in ideal (solid spherical particles not soil) or peculiar conditions that are not easily comparable with the EMPA sample tests described for step 3.

This work will now be briefly mentioned, further references can be found in the literature reviews.

Gim, Lesniewski, and Middleman (1995) conducted experiments to understand the effects of impinging jets on solid spheres and found that the spheres tended to roll rather slide. No other mechanisms were discussed.

Das et al (1995) completed studies on the hydrodynamic detachment of particles from solid surfaces. They explain that to detach a particle from a surface one must have a hydrodynamic force greater than the force of adhesion. This confirms that the findings mentioned previously in this chapter and also work by Kovich.

They also state that the detachment of the particle is dependent on several factors such as, particle diameter, surface roughness, and physical and electrochemical properties of the fluid.
They further concluded that in simple shear flow spherical particles release due to rolling as compared to sliding or lifting.

Soltani and Ahmadi (1993) examined particle removal in turbulent flow. The authors also wanted to determine if removal occurred via rolling, sliding, or lifting in turbulent flow. They concluded that rolling of the particle was the dominant mechanism for removal and that hydrodynamic torque acting on the particle was significant.

Musselman and Yarbrough (1987) employed high velocity spray washing to determine parameters influencing particle removal. They state that cleaning techniques on smaller particles is less effective because of the lack of ability to impart a force on the particle. The authors indicate that the drag force produced by the shear stress depends on the frontal area. Increasing the frontal area increases the drag force beyond the point that exceeds the adhesion force. They concluded that sufficient drag force could be generated to remove particles as small as 0.1µm.

Donovan, Yamamoto, and Periasamy (1993) summarise several cleaning methods for removing particles from a wafer surface. They state that scrubbing uses hydrodynamic drag and can remove particles effectively for diameters larger than 1µm.

**Conclusions**

The water tunnel tests have confirmed that under closely monitored tangential fluid shear stress conditions sustained in the purpose-built water tunnel (15k≤Re≤155k, 0.2≤U≤1.97ms⁻¹ and 0.14≤τw≤7.74N/m²) the removal of all four standard soils (carbon black, chocolate, pigs blood and wine) was totally unsatisfactory. Further tests have also been carried out at considerably lower flow speeds and soil removal continues to be extremely poor. It therefore seems highly probable that soil removal by shear stress alone, and as found in today's washing machine, is inadequate to remove these types of soil. Henceforth other factors such as detergency, temperature, abrasion etc. are more likely to play a more fundamental role.
Equally important is the unearthing of the time dependency of soil removal. Both pigs blood and wine soils have a marked first order time dependency and this dependency expires within the first 10–20 minutes. Carbon black also shows similar behaviour but to a much lower extent. Chocolate on the other hand showed no appreciable time dependency. Hence the removal of these two latter soils must certainly be accomplished and accompanied by other means (abrasion, detergency etc.).

The role of the boundary layer is to ensure that the soil is submerged in a viscous fluid of varying thickness and turbulence that also conveniently facilitates the transfer of heat and mass. It is interesting to note that by simply increasing the water temperature from 15 to 60°C the kinematic viscosity falls by 60% that also leads to a reduction in the thickness of the laminar sub-layer where the soil hides. This is good news for soil removal because it implies that the soil is exposed and subjected not only to more turbulence and shear, but also aids the change in rheological properties.

Figures 5.4c and 5.5c show that pigs blood and red wine soils undergo some change (together with the fibre) and this can only occur if the mechanical properties of the soil are effected. Henceforth there is evidence that the soil changes from being a solid to at least a plastic, if not a viscoelastic material. This is attributed to (at least) a combination of mechanical, thermal and hydrodynamic effects during the tunnel tests. In this context the author hypothesises that the rheological properties of the soil play a role in soil removal in the washing machine.
## Test Results (Summary)

### Table 5.6c - Overall Cleaning results versus Re and \( \tau \)

<table>
<thead>
<tr>
<th>Re</th>
<th>Carbon Black</th>
<th>Pigs Blood</th>
<th>Chocolate</th>
<th>Wine</th>
<th>( \tau_{\text{mean}} ) (N/m(^2))</th>
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### Table 5.7c - Overall Cleaning results versus Time (Re= 80k)

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<th>Time</th>
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<th>Pigs Blood</th>
<th>Chocolate</th>
<th>Carbon Black</th>
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### Table 5.8c - Cleaning results versus Time (Re= 75 - 85k) for Pigs blood and Wine

#### Wine

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#### Pigs Blood

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Page: 212
### Chocolate

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Table 5.9c - Cleaning results versus Time (Re= 8.5 - 9.0k) for Chocolate and Carbon Black
Chapter 5d

Step 4 – The Measurement of External Wash Load Hydrodynamic Conditions
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List of Equations

\[ r = xi + yj \]  
Equation 5.1d ............................................................................................................... 225

\[ \Delta r = r_f + r_i \]  
Equation 5.2d ............................................................................................................... 225

\[ \Delta r = \Delta xi - \Delta yj \]  
Equation 5.3d .............................................................................................................. 226

\[ v = \sqrt{v_x^2 + v_y^2} \]  
Equation 5.4d ............................................................................................................... 226

\[ \tan \theta = \frac{v_y}{v_x} \]  
Equation 5.5d ................................................................................................................... 226

\[ \frac{\Delta v_x}{\Delta t} i + \frac{\Delta v_y}{\Delta t} j = a_x i + a_y j \]  
Equation 5.6d ............................................................................................................... 227

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Step 4 – The Measurement of External Wash Load Flow Velocity

In steps 1 to 3 the hydrodynamic conditions were established according to the theoretical flow conditions in the washing machine, in particular they were based on the flow through the washing machine drum perforations and presumambly through and across the wash load.

In the next two steps the aim will be to verify these conditions and, in particular, justify the choice of the hydrodynamic conditions in steps 1 to 3.

In steps 4 and 5 two, new, flow domains were therefore explicitly proposed, these being:

1. Flow through the wash load.
2. Flow on the outside of the wash load.

In this chapter the focus will be on the second domain.

An attempt at estimating the flow conditions on the outside of the wash load has already been made by Van Den Brekel [ref. 5.1d] who assumed that the flow from the inner drum perforations (the 'sump' area) to the outer drum is representative of the flow conditions on all the external surface of the wash load. This is a reasonable postulation but has several major drawbacks due to the following assumptions of this model:

1. The pressure head above the perforations is constant.
2. The perforations lie on a plane i.e. drum curvature is ignored.
3. Inner drum is stationary.
4. Water temperature is constant and uniform across all perforations.
5. Flow conditions are independent on the wash load.
6. The lower part of the drum represents the rest of the drum as well.

These assumptions were considered too restrictive by the author and therefore the objective of step 4 was to establish an alternative method(s) and compare experimental results.
In other words the goal was to bridge any gap between the tunnel measurements and the washing machine conditions.

**Experimental Methods**

Perhaps the easiest and most reliable way to understand the hydrodynamic conditions on the outside of the wash load is to measure the flow velocity on the inside surface of the washing machine door. This was done during the initial part of the wash cycle, with and without wash load.

This approach provided the following advantages:

- The conditions could be measured on a large surface area (the door) that is always in contact with the wash load both in static and dynamic conditions.
- Pressure head variation could be measured and eventually accounted for.
- New information regarding flow damping and oscillation could be retrieved with relative.
- Water temperature could be monitored accurately
- Flow conditions could be monitored on a flat surface.

The disadvantages essentially concerned:

- The assumption that the door represents all the external surface of the wash load
- A minimum amount of water was needed to wet the lower part of the door. In generally this was about 5 to 20 litres more than in normal washing conditions.

In spite of these two limitations three new methods have been developed and experimented, these being:

1. Block probes positioned on the inside surface or face of the washing machine door (shown in next figure). The probe pressure was recorded by the same data acquisition system [ref. 5.2d] used in step 5 except that the pressure transducer signal was wired directly to the DAQCARD instead of being transmitted by radio.
2. High speed filming of the entrained air bubbles in the water near the door and estimate an average water velocity.

By analysing air bubble motion i.e. the bubble dynamics, the average value of water flow velocity on the outside of the wash load could be estimated and later compared to the average dynamic pressure measured by the shear stress probe fitted on the inside surface of the door.

3. High speed filming of the water line.

The aim here was to determine the frequency and variation in pressure head of the wash water during drum movement.

The high speed filming involved analysing the acquired digital images with specific hardware and motion analysis software [ref. 5.3d]. This entailed acquiring velocity measurements from the tracking of the motion of entrained air bubbles and water line. In this way no colouring or tracer agents were needed and multiple objects (bubbles or water line) were simultaneously measured.
Experimental Results of Block Probes on Door

The first noticeable result was that the signature (behaviour) of the flow was typically oscillatory with a variable frequency between 2 to 7Hz and average of 4Hz. This frequency does not coincide with the natural drum (washing) frequency (1 to 1.5Hz\(^1\)). Further, although the dynamic pressure did vary it was sufficiently constant to determine average flow conditions. In particular equation 5.12a was used to calculate the velocity from the dynamic pressure measurements i.e. the flow velocity on the door and wash load.

This information was retrieved by recording the variation in dynamic pressure during and after drum rotation as shown next.

![Figure 5.2d - Typical Dynamic Pressure Data recording](image)

\(^1\) Van den Brekel also examined wash load motion and obtained graphs similar to Fig. 5.2d but with much lower frequencies (<0.1Hz).
In order to provide a statistically significant number of test data the dynamic pressure acquisition was repeated 10 times for each test. Each test had a recording time of 3 to 5 seconds with at least 2000 samples per second. Tests were realised for two load conditions:

a. Without wash load
b. With wash load (3kg dry load)

The results for both conditions have been summarised below:

**Table 5.1d - Dynamic Pressure Data for Block Probes**

<table>
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<th>Parameter</th>
<th>Magnitude</th>
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<td>Average dynamic pressure</td>
<td>250-500Pa</td>
</tr>
<tr>
<td>Velocity range</td>
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<td>Average velocity</td>
<td>0.72ms$^{-1}$</td>
</tr>
<tr>
<td>Frequency range</td>
<td>2 to 7Hz</td>
</tr>
<tr>
<td>Average frequency</td>
<td>4Hz</td>
</tr>
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</table>
Remarks

According to Van den Brekel the flow velocity through the washing drum perforations was found to be between 0.5 and 1.5 ms\(^{-1}\), i.e. on average higher than the block probe recordings. Further, Van Den Brekel also found that this depended on the wash load and measured values of 0.1 to 0.3 ms\(^{-1}\) when the machine was loaded. Also the vertical displacement of the wash load was found to be between 60 and 110 mm but with peaks as high as 210 mm.

The same block probe recordings were also realised in differential mode. That is the pressure transducer was connected to the block probes static and dynamic pressure tappings. The test set-up in this case is shown below:

![Figure 5.4d – Differential Pressure Data recording set-up](image)

The next graph shows the initial rest period followed by the motion of the drum and consequent effects on water flow near probe.
This graph shows the same oscillatory behaviour as before and reasonably constant differential pressure. This indicates that the shear stress is substantially constant.

The results of the differential pressure measurements are summarised below:

**Table 5.2d – Differential Pressure Data for Block Probes**

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<tr>
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</table>

* Based on work by Knight and Rhodes [ref. 5.4d] and step 1 calculations.

**High Speed Filming**

**Test Set-up and Test Equipment**

The high speed filming was carried out with a standard B/W CCD camera connected to a Weinberger AG (Speedcam) high speed digital image acquisition system. This system was connected to a Pentium II microprocessor based PC where all high filming was recorded in (512Mbyte) RAM before being transferred to CD-ROM. The camera was positioned in front or
slightly to one side of the washing machine (Bauknecht mod. WA3774S Excellence) depending on the illumination requirements and type of image acquisition. This set-up is shown below:

![Test set-up diagram](image)

Figure 5.6d – Test set-up (HS Digital Image Acq. System, Camera, Lighting and Washing M/C)

The frame rate of all filming was determined experimentally and set at 500FPS. To record the reference length or dimension a small 500lire coin (Ø25.8mm) was used for the bubble and water line filming. This allowed the conversion of the recorded image pixel size to millimeters. The graphs shown later are expressed in pixels but can be treated as mm since 1 pixel equals very nearly 1 mm.

For demonstration purposes only, a series of high-speed films were also carried out with a Kodak SR500C camera [ref. 5.6d] and the same quality of image was achieved. The only differences were found in ease of use (in favour of Kodak).

For each acquisition the system was run for a total of 4 seconds, i.e. the maximum recording length of the system, which is typical for HS digital filming. After each run the images were played back and edited on-line (directly on screen) before finally recording them on video-tape or CD-ROM.

Tests were carried with variable wash water levels 10 to 30litres of water and no wash load. These conditions facilitated filming and were helped further by the lack of reflectance from the target and filming close-up to the target, consequently the picture contrast was outstandingly good.
**Dynamics and Particle Motion Model**

In order to assess the dynamics of the air bubbles or water line (as well as sphere) it was necessary to first outline the method used to calculate the displacement, velocity etc.

For simplicity the method it is applied here to the sphere trajectory during free fall i.e. after release from the drum agitator [ref. 5.7d]. However, the method can be applied to any particle motion.

One first assumes that the sphere (or bubble, water line pixel etc) is represented by a particle [ref. 5.8d] whose general trajectory is shown next:

![Figure 5.7d - Particle (e.g. Bubble, Sphere etc) Trajectory Analysis](image)

In the above figure the position vector \( t \) locates the centre of the sphere relative to the origin of a reference frame. In two dimensions we write:

\[
r = xi + yj
\]

Equation 5.1d

where \( x \) and \( y \) are the particle (sphere) coordinates. These coordinates are obtained by converting the \( x \) and \( y \) pixels to mm.

The displacement of the sphere i.e. from its initial position \( i \) to its final position \( f \) (end of free fall or impact), is defined by:

\[
\Delta r = r_f + r_i
\]

Equation 5.2d

or from Eq. 5.1d:
\[ \Delta r = r_f + r_i = (x_f i + y_f j) - (x_i + y_i j) \]

\[ \Delta r = (x_f - x_i) i + (y_f - y_i) j \]

Letting \( \Delta x = x_f - x_i \) and \( \Delta y = y_f - y_i \), we have:

\[ \Delta r = \Delta x i - \Delta y j \]  \hspace{1cm} \text{Equation 5.3d} \]

The average velocity of the particle (sphere) \( \bar{v}_s \) for a given time interval \( \Delta t \) is:

\[ \bar{v}_s = \frac{\Delta r}{\Delta t} \]

we may also write:

\[ \bar{v}_s = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j = v_x i + v_y j \]

and

\[ v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \]

that is the velocity is the limiting value of the average velocity.

Further \( v_x = \frac{dx}{dt} \) and \( v_y = \frac{dy}{dt} \).

The magnitude of the velocity is the speed \( v \), that is:

\[ v = \sqrt{v_x^2 + v_y^2} \]  \hspace{1cm} \text{Equation 5.4d} \]

and the direction of the particle (sphere) can be expressed in terms of an angle \( \theta \) between the velocity vector and the \( x \) axis:

\[ \tan \theta = \frac{v_y}{v_x} \]  \hspace{1cm} \text{Equation 5.5d} \]

This angle is not used to calculate the critical angle i.e. the release angle after which the free fall starts. The critical angle is determined simply by using the \( x \) and \( y \) coordinates at the start of the free fall and was found to be approx. 45°.
Another useful part of the dynamics analysis is the acceleration of the sphere, which is defined as follows:

\[
\mathbf{\bar{a}} = \frac{\Delta \mathbf{v}}{\Delta t} \quad \text{or} \quad \mathbf{\bar{a}} = \frac{\Delta v_x}{\Delta t} \mathbf{i} + \frac{\Delta v_y}{\Delta t} \mathbf{j} = \mathbf{a}_x \mathbf{i} + \mathbf{a}_y \mathbf{j}
\]

where \( \mathbf{a}_x = \frac{dv_x}{dt} \) and \( \mathbf{a}_y = \frac{dv_y}{dt} \).

Through this calculation it is possible to calculate the force of the sphere due to its acceleration i.e. \( F = ma \).

**Experimental Results of Bubble and Water-line Dynamics**

Bubble tracking allows the monitoring of bubble trajectory and definition of bubble dynamics. This technique is applicable to bubble tracking for a relative short period of time (<0.5s) because after this the bubbles either split, disappear or are absorbed by other bubbles [ref. 5.9d]. However, bubble tracking for the washing machine is relatively easy to obtain and more than one bubble can be monitored at a time.

Since bubble dynamics were recorded at 500 FPS, the minimum time event was 2mseconds. The maximum event duration was 4 seconds.

The tracking of a bubble provided X–Y coordinates for each recorded frame. The X–Y coordinates were used to calculate a resultant position vector, which was then used to estimate the velocity.

This estimation was done every 2 msecs or averaged over a particular period e.g. 20 to 30 msecs, depending on the event, bubble size and bubble growth. In the data given next data has been averaged (with 0.002 second increments) over the an arbitrary trajectory which, on average, lasted \( \approx 0.12 \)s.
### Remarks - Bubble Dynamics

As can be seen from figures 5.8d and 5.9d typical bubble movement was quite linear, although it would have been better to monitor bubble movement over a longer period.

The observed motion was either towards the water line surface, towards the drum or along the same direction of the drum rotation. This is an important result because it means that the square shaped small block probe was probably inaccurate because it was not always perfectly aligned with the flow. In fact dynamic pressure measurements indicated lower velocities by a factor of at least 2. Hence the conclusion was that a triangular shaped probe is more suitable for these measurements.

Another equally important observation was while it is known that bubble dynamics tend to be affected by the friction between the bubble and wall (the door in this case) and bubble-bubble, the velocities measured were still undoubtedly high. Hence tunnel conditions were considered appropriate and approved. The knowledge gained from the analysis of water line motion and bubble confirmed that the flow velocity on the outside of the wash load is certainly measurable, high (meters per second) and is turbulent [refs. 5.10d and 5.11d].

---

2 An excellent account of bubble formation is given by Brodkey [ref. 5.12d].
Figure 5.8d – Typical Bubble Trajectory during drum Rotation

Figure 5.9d – Typical Bubble Trajectory during Drum rest
Water level tracking

<table>
<thead>
<tr>
<th>Post-processed Test Data</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical event duration</td>
<td>1.0–3.0s</td>
</tr>
<tr>
<td>Typical head variation</td>
<td>20mm</td>
</tr>
<tr>
<td>Typical head pressure</td>
<td>200Pa</td>
</tr>
<tr>
<td>Typical head rate</td>
<td>1000Pas⁻¹</td>
</tr>
<tr>
<td>Average bubble speed</td>
<td>0.17ms⁻¹</td>
</tr>
<tr>
<td>Maximum bubble speed</td>
<td>0.91ms⁻¹</td>
</tr>
<tr>
<td>Typical damped frequency</td>
<td>2.7Hz</td>
</tr>
<tr>
<td>Typical damping factor</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 5.4d – Water line (bubble) Dynamics Results Table

Remarks - Water-line Dynamics

The velocity of the water line was high with or without wash load. Again this result confirmed the conditions in the tunnel investigation. Equally interesting was the variation in water line that subjects the wash water to a pulsating pressure head. Hence it was concluded that the wash load was probably subjected to a pulsating flow rather than constant pressure head.
Chapter 5e

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\[ K_{rf} = \frac{\sqrt{1 - \left( \cos^{-1}(1 - 2K_f) - \frac{\sin 2\cos^{-1}(1 - 2K_f)}{2\pi} \right)}}{2} \quad \text{Equation 5.22e} \]
\[ D = 3.19 \frac{\gamma \mu m_{sl}}{K_e (1 - \sqrt{1 - K_f})} \quad \text{Equation 5.23e} \]
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\[ \bar{u} = u_t = \omega_D R_D = \frac{\pi N L_D}{30} \quad \text{Equation 5.25e} \]
\[ \Delta P_h = \rho g h_w \quad \text{Equation 5.26e} \]
\[ \bar{u}_h = k \frac{2 \Delta P_h}{\rho} \quad \text{Equation 5.27e} \]
\[ k = \frac{f \cdot T_D}{D} \quad \text{Equation 5.28e} \]
\[ f = \frac{64}{Re} \quad \text{Equation 5.29e} \]
\[ f = 0.0055 \left[ 1 + \left( \frac{20000 \cdot \frac{e}{D} + 10^4}{\frac{Re}{10^3}} \right)^{\frac{1}{3}} \right] \quad \text{Equation 5.30e} \]
\[ \Delta P_k = \frac{\gamma}{2} \rho v_{imp}^2 \quad \text{Equation 5.31e} \]
\[ \omega_D = \frac{\pi N}{30} \quad \text{Equation 5.32e} \]
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**List of References**


Other recommended References

- Heertjes P. M., Studies in Filtration: Blocking Filtration, Vol. 6, pp. 190 to 203, Laboratory of Chemical Engineering of the Technical University of Delft, 1956.
Preface to Step 5

The successful transfer or deployment of knowledge acquired in the water tunnel experiments is possible only if it can be used for comparison with washing machine performance (see also chapter 6). As has been illustrated in step 4 this implies bridging the gap between the water tunnel experimentation and the washing machine. In step 5 this work was continued and the notion was to attempt to determine the hydrodynamic conditions within the wash load. It was also the closing part of the Mass Transfer project.

Background to the Wash load measurements and Scope of Step 5

Flow measurements within the wash load have so far been carried out by a combination of permeability measurements [ref. 5.1e] of the textile fabric in static conditions and using reactor modelling, i.e. mass transfer.

Research by Van den Brekel [ref. 5.2e] revealed that the flow velocity measured in this latter way was very small (1E-3 ms⁻¹ or lower) and hence typically laminar or creep-like. Apart from this very little was known about wash load motion and flow. Moreover, conditions were considered typically diffusive i.e. slow [ref. 5.2e] and nothing was truly known about what really happens in the wash load. Therefore the fundamental question was how could (flow) conditions be monitored, especially locally, inside the wash load?.

The scope of step 5 was precisely this, to (remotely) monitor internal wash load conditions especially in terms of hydrodynamics. Designing a system to do this was difficult because no reliable data was available and thus flow conditions needed to be estimated. In particular this meant determining some range of pressure for the pressure transducer. Also the system to be developed required the following characteristics:

- No connecting leads or wiring i.e. remote and wireless.
- The device that "probes" the wash load must not disturb the natural behaviour of the wash load [ref. 5.3e].
Compatibility with different sensors including pressure, temperature, humidity, force etc.

Cheap, precise and reliable.

The approach was simplified by assuming that flow velocity was similar to that found in step 4 and basing system design on the flow conditions found in step 4. The reasoning behind this approach was elementary, for should step 5 provide no readable pressure data then the flow inside the wash load was obviously very slow thus proving Van den Brekel's results. On the other hand should step 5 reveal higher flow velocities then this would be interpreted accordingly and contrast the laminar flow results.

Step 5: H.S. Filming, Modelling and Remote Data Acquisition

To quantify the flow conditions within the wash load two techniques have been developed, these being:

a. High-speed image acquisition analysis to quantify the motion of wash load. In support of the high-speed image analysis a mathematical model has been developed [ref. 5.4e] that is based on a concept of concentrated wash load mass termed 'fabric plug'. The analysis of the motion of the wash load will be discussed in the next section.

b. Dynamic pressure measurement during wash load motion so as quantify the dynamic pressure of the wash water in the wash load that affords an indirect measurement of the average water speed on the surface of the wash load.

Figure 5.1e – Step 5 Activities
In order to obtain the dynamic pressure data a peculiar and unique remote data acquisition system [European patent pending] has been developed [ref. 5.5e].

The system is based on a piezo-resistive pressure transducer and a matched pair of battery operated hybrid transmitters and receivers with a working frequency of 433MHz. The transducer and transmitter were housed within a small plastic sphere (see chapter 3) designed to follow the wash load during its movement in the washing machine drum. A triangular block probe, as discussed for step 2, was fitted to the surface of the sphere to pick up the surface dynamic pressure of the sphere and relate this to the local surface velocity of the wash load. The receiver was mounted on the outside of the washing machine where it picked up the modulated signal from the transmitter, processed it and then stored it through a dedicated data acquisition system. The sphere was placed within the wash load during washing and is based on a fabric plug concept [ref. 5.3e] described in chapter 6 and in the model hereon.

The system was designed to offer the maximum flexibility and is therefore applicable to many other remote sensing circumstances including those outside domestic appliances.

**Overview of Modelling of a Horizontal-axis Domestic Washing Machine**

In this section two topics regarding the modelling of a horizontal-axis domestic washing machine are tackled and solved. They can be treated as complementary activities for the research.

In the first instance the design of the horizontal axis washing machine by means of parametric modelling is shown. The end product is a series of dimensionless factors that characterise the horizontal axis washing machine. These parameters can be interlaced and correlated to machine performance, including soil removal efficiency and effectiveness. In this way it is possible to optimise machine configuration such as drum diameter, angular speed, depth etc.

In the second instance the motion of the wash load has been modelled by means of an imaginary fabric plug and by dividing the rotation of the wash load into 3 distinct steps (A, B
and C). The resulting cycle has provided a means of calculating the theoretical fabric plug pressure drop, which is known to drive local hydrodynamic conditions including fluid flow.

**Parametric Modelling of a Horizontal Axis Washing Machine**

Up until now washing machine design has been dictated by market and manufacturing needs. Hence the correct “mixture” of chemistry, mechanics, physics and thermodynamics etc. that composes the washing process has been left to the experience of the design engineer. Here a method based on similarity criteria and dimensionless coefficients make it possible to express the basic machine design criteria in parametric form.

The main advantage of parametric modelling is that it simplifies machine performance comparison and design. It also provides direct design parameter selection, such as drum diameter, and directly relates other parameters, e.g. wash load volume to available drum volume.

The powerfulness of this approach also lies in the fact that machine performance (soil removal efficiency and effectiveness) may be included, indeed anything related to machine performance e.g. energy, water or detergent consumption.

In this section the optimisation of the drum size is considered to lead to the best washing performance for a given set of machine dimensional constraints.

Therefore the underlying objective is to set-up and relate a series of dimensionless factors that describe the machine both in terms of configuration and performance.

The most obvious starting point is perhaps the most understood design constraint of all, that is the overall dimensions of the machine. This is strictly correlated to size of the drum, i.e. D, the drum diameter D and L, drum depth.

These two fundamental geometric parameters instinctively lead to the definition of another three parameters i.e., drum circumference; drum cross-sectional area and drum volume.
The description of the machine continues with drum speed, fabric mass (wash load), fabric density, wash water volume etc. There are of course many others but in this chapter 7 dimensionless factors based on these parameters will be elaborated. If one looks at the laundry or wash load it may be said that it will either be in the "dry" or wet state depending on the stage of the washing process.

In the "dry" state, the laundry (wash load) takes up the following volume in the drum (see Fig. 5.1e):

\[ V_{wl} = \gamma_{wl} m_{wl} = A_h L \]  
Equation 5.1e

where \( \gamma_{wl} \) is the specific volume of the laundry (for modern fabrics, \( \gamma_l \approx 7.7 \pm 1 \times 10^{-4} \text{m}^3/\text{kg} \)); \( A_h \) is the cross-sectional area corresponding to the transverse filling of the drum, (m²), \( L \) is the textile thickness in meters and \( m_{wl} \) is the laundry or wash load in kg. In the wet state the volume will depend on the fibre origin, weave type and tightness (porosity) and whether or not the fibre is natural or synthetic (see tables 5.1e and 5.2e).

One first introduces a dimensionless coefficient to characterise the degree of "dry" laundry filling \(^1\) (K\(_f\)) of the drum cross section in static conditions:

\[ K_f = \frac{A_h}{A_D} \]  
Equation 5.2e

where \( A_D \) is the cross-sectional area of the drum in m\(^2\).

An alternative to \( K_f \) is to consider the total surface area of the laundry (\( A_h^* \)) with respect to the surface area of the drum (\( A_D^* \)). This is a useful parameter because washing machine performance regulations impose a certain amount of laundry specified in terms of surface area and mass (see table 5.3e). Furthermore, since surface area is used, this ratio would help correlate abrasion effects in soil removal.

Hence equation a.5.2e may be rewritten as:
From equations 5.1e and 5.2e one obtains:

$$A_h = K_f \pi R^2$$  \hspace{1cm} \text{Equation 5.4e}

One can also characterise the ratio of drum length to radius in the same conditions which produces the second parameter:

$$K_L = \frac{L}{2R}$$  \hspace{1cm} \text{Equation 5.5e}

The value of $K_L$ depends on the size and manufacturer of the machine, for example for Western European shallow washers $K_L \approx 0.3$ while for more standard depth washers $K_L \approx 0.8$. Similarly $K_h$, the ratio of the drum (water) filling height $h$ to its radius $R$, is:

$$K_h = \frac{h}{2R}$$  \hspace{1cm} \text{Equation 5.6e}

Its value is typically $0.1 \leq K_h \leq 0.8$ and may be calculated from the water filling height of the drum shown below.

![Figure 5.2e – Drum parameters](image)

The drum–filling ratio can also be calculated as a ratio of volumes. That is the ratio of washing water to drum volume:

$$K_f = \frac{F_h}{A_D}$$

\hspace{1cm} \text{Equation 5.3e}

1 There is clearly no reason why the wet area cannot be used to define $K_f$.
\[ K_h = \frac{V_h}{V_D} \quad \text{Equation 5.7e} \]

Clearly from a design point of view the ratio of volumes is possibly more convenient and can also be extended to the ratio of rinse water to drum volume or wash load mass.

Returning to figure 5.2e we can use standard formulae to determine the area \( A_h \) and relative total perimeter \( P' \):

\[
A_h = \frac{\pi R^2}{360} \left( \alpha - \frac{R^2}{2} \sin \alpha \right) \quad \text{Equation 5.8e}
\]

\[
P' = \frac{\pi R}{180} \left( \alpha + 2R \sin \frac{\alpha}{2} \right) \quad \text{Equation 5.9e}
\]

Where \((80 \leq \alpha \leq 180)\)

\[
\alpha = 2 \left( \cos^{-1} \left(1 - \frac{h}{R}\right) \right) \quad \text{Equation 5.10e}
\]

Referring to the shaded area in figure 5.2e the total perimeter \( P' \), (dashed line) of this area can be defined as the sum of the wetted drum perimeter, \( p \), and wash load water level (or an imaginary wash load) boundary length, \( W_L \). That is:

\[
P' = p + W_L \quad \text{Equation 5.11e}
\]

Referring to Fig. 5.2e this leads to:

\[
P' = p + 2 \sqrt{R^2 - (R - h)^2} = p + 2\sqrt{2Rh - h^2}
\]

hence:

\[
p = P' - 2\sqrt{2Rh - h^2} \quad \text{Equation 5.12e}
\]

The wetted perimeter, \( p \), can also be expressed in terms of \( R \) and \( h \) as follows:

\[
p = \frac{\pi R}{180} \left( \alpha + 2R \sin \frac{\alpha}{2} - 2\sqrt{2Rh - h^2} \right)
\]

\[
p = \frac{\pi R}{90} \cos^{-1} \left( \frac{1 - \frac{h}{R}}{R} \right) + 2R \sin \left[ \cos^{-1} \left( \frac{1 - \frac{h}{R}}{R} \right) \right] - 2\sqrt{2Rh - h^2} \quad \text{Equation 5.13e}
\]
This allows us to introduce another parameter, $K_p^2$, that defines the ratio of drum perforations covered by the wash load or water, that is:

$$K_p = \frac{\alpha + \frac{1}{\pi} \sin \alpha}{\frac{\sqrt{2Rh-h^2}}{\pi R}}$$

Equation 5.14e

Hence for any given value of $p$ we can calculate the number of perforations likely to be involved in water flow between the inner drum (illustrated in figure 5.2e) and the outer drum. Its value is typically $0.2 \leq K_p \leq 0.6$

This is shown below for the opened-out inner drum:

![Ratio of Perforations](image)

Figure 5.3e – Ratio of Perforations

Hence the number of perforations per unit area of drum is:

$$N_p = \frac{N}{\pi DL}$$

Equation 5.15e

or

$$n_p = N_p \cdot pL = N_p \cdot K_p \pi DL = N \cdot K_p$$

Equation 5.16e

Returning to $K_f$, its value may be calculated from the known relative filling height of the cylinder using:

$$K_f = \frac{A_k}{A_D} = \frac{\frac{\pi R^2}{360} \alpha - \frac{R^2}{2} \sin \alpha}{\pi R^2} = \frac{\alpha}{\frac{360}{2\pi}} \sin \alpha$$

2 It is also possible to use $K_{ph} = \frac{A_k}{A_D}$, which is the ratio of the area of perforated holes respect to the inner drum area
from equations 5.6e and 5.10e we may write:

\[ \alpha = 2\cos^{-1}\left(1 - \frac{2K_h R}{R}\right) \]

thus:

\[ K_f = \frac{\cos^{-1}\left(1 - 2K_h\right) - \sin\left(2\cos^{-1}\left(1 - 2K_h\right)\right)}{180} \]

Equation 5.17e

Typically 0.1 ≤ K_f ≤ 0.9.

In order to take into account the dynamic conditions of the laundry it is necessary to assume that the fabric radially fills the drum while running. In this way we can characterise the radial filling by introducing a radial-filling coefficient K_f. Where K_f is defined as follows:

\[ K_f = \frac{R_h}{R} \]

Equation 5.18e

If one assumes that the wash load occupies an area defined by a ring whose outside diameter is that of the drum, R and inside radius R_h we can formulate an expression for R_h and K_f.

\[ A_h = \pi\left(R^2 - R_h^2\right) \]

\[ K_f = 1 - \frac{R_h^2}{R^2} \]

Thus:

\[ R_h = R\sqrt{1 - K_f} \]

Equation 5.19e

From eq. 18:

\[ R_h = \frac{1}{2}\sqrt{1 - K_f} \]

Equation 5.20e

and the radial thickness of the wash load is:

\[ K_f = \frac{\sqrt{1 - K_f}}{4} \]
\[ \Delta R_h = R - R_h \]
\[ = \frac{D}{2} - \left( \frac{D}{2} \sqrt{1 - K_f} \right) \]

\[ \Delta R_f = \frac{R}{4} \left( 1 - \sqrt{1 - K_f} \right) \]  

Equation 5.21e

\[ K_{rf} = \frac{1 - \cos\left(1 - 2K_h\right) - \sin 2\cos^{-1}(1 - 2K_h)}{2} \]  

Equation 5.22e

Typically \(0.1 \leq K_{rf} \leq 0.4\)

If \(K_f\) and \(K_{rf}\) are plotted against \(K_h\) the following graphs are obtained:

![Figure 5.4e - Nomogram for values of Dimensionless Drum Coefficients](image)

One can now combine equations 5.1e to 5.22e and obtain:

\[ D = 3.19 \sqrt{\frac{\gamma_l \gamma_{ml}}{K_f \left( 1 - \sqrt{1 - K_f} \right)}} \]  

Equation 5.23e

In this equation there are four fundamental parameters:

- \(K_{rf}\), a washing machine design parameter
- \(K_f\), a combination of washing machine design and wash load size
- \(\gamma_l\), a characteristic of the type of textile and fibre origin
- \(\gamma_{ml}\), a characteristic of the type of textile and fibre origin
m_{wl}, a combination of washing machine operating conditions, type of textile and weave

If this data was known for the size of wash load, type of textile and above all, soil removal performance, it would be possible to reduce equation 5.23e into a closed form where

\[ D = f(m_{wl}) \]

only. This approach is also valid for K_F and K_L, since both can be related to D.

**Remarks**

All of the previously mentioned dimensionless parameters have been introduced to provide a description of the washing machine. However, there may be many other parameters and those proposed may or may not have an impact on washing performance. It is for this reason that the author suggests that experimentation be carried out and each parameter evaluated so as to gain insight into the dominant design features of horizontal axis washing machines.

For example, for European washing machines the value of K_L varies between approximately 0.3 to 0.8 depending on whether the machine is 'normal' or shallow in depth.

If one considers the textile to be cotton or polyester the following table of data is considered valid:

<table>
<thead>
<tr>
<th>Textile</th>
<th>Density, ( \rho ), (kg/m(^3))</th>
<th>Specific volume, ( \gamma ), (m(^3)/kg)</th>
<th>Typical Porosity for Plain 1/1 weave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cotton</td>
<td>Min 1300</td>
<td>Min 7.7E-4</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Max 1500</td>
<td>Max 6.6E-4</td>
<td></td>
</tr>
<tr>
<td>Polyester</td>
<td>Min 1200</td>
<td>Min 8.3E-4</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Max 1400</td>
<td>Max 7.1E-4</td>
<td></td>
</tr>
</tbody>
</table>

However because of the multitude of natural and synthetic fibres available the following list of densities for the most important fibres is also given:
Table 5.2e – Average Dry Density - Specific Volume values for typical Fibres [ref. 5.6e]

<table>
<thead>
<tr>
<th>Textile</th>
<th>Density, $\rho$, (kg/m³)</th>
<th>Specific volume, $\gamma$, (m³/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyester</td>
<td>1380</td>
<td>7.25E-4</td>
</tr>
<tr>
<td>Nylon</td>
<td>1140</td>
<td>8.77E-4</td>
</tr>
<tr>
<td>Acrylic</td>
<td>1170</td>
<td>8.55E-4</td>
</tr>
<tr>
<td>Spandex</td>
<td>1200</td>
<td>8.33E-4</td>
</tr>
<tr>
<td>Cotton</td>
<td>1450</td>
<td>6.89E-4</td>
</tr>
<tr>
<td>Silk</td>
<td>1350</td>
<td>7.41E-4</td>
</tr>
<tr>
<td>Wool</td>
<td>1310</td>
<td>7.63E-4</td>
</tr>
</tbody>
</table>

It will be noticed that there is some discrepancy between tables 5.1e and 5.2e but the overall average density remains 1300kg/m³ or 7.7E–4m³/kg. If one considers this average value then for a wash load that varies between 0.5 and 5kg the respective volume of textile is between 3.85E–4 to 38.5E–4m³.

Hence by conducting a series of experiments, say for a cotton wash load, and varying $K_f$, $K_l$, wash load mass versus soil removal performance one obtains an optimum value of $K_f$ and hence diameter of drum, D.

Research by Panfilov et al. [ref. 5.7e] based on a Russian household washing machine tested with reference to Russian standard GOST 8051–83 revealed the following optimum parameters for a cotton wash load: $K_f$=0.65 and $K_l$=0.6. Similarly the standard reference laboratory washing machine, known as the Wascator mod. FOM71MP LAB manufactured by Electrolux [ref. 5.8e], provides $K_l$=0.66 and 0.2≤$K_h$≤0.8.

In making reference to washing or soil removal performance standard IEC 60456 [ref. 5.8e] is used extensively in Western Europe. This standard defines the wash load or laundry as follows:

**Sheets and Pillowcases**

Textile type: Bleached cotton, finished fabric

Weave type: Plain 1/1
Mass per unit area : 185g/m² (±5%)
Sheet size : 1.5m x 2.6m (3.9m², 0.7215kg)
Pillowcase size : 1.6m x 0.8m (1.28m², 0.2368kg)

Hand-towels
Textile type : Bleached cotton, finished fabric
Weave type : Huckaback
Mass per unit area : 230g/m² (±5%)
Sheet size : 1m x 0.46m (0.46m², 0.1058kg)

The wash load for the performance tests is defined in table 5.3e below:

<table>
<thead>
<tr>
<th>Approximate Load (kg)</th>
<th>Number of sheets (kg)</th>
<th>Number of pillowcases (kg)</th>
<th>No. of Hand-towels (kg)</th>
<th>Approx. Total Surface Area (one side only) in m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 (0.7215)</td>
<td>2 (0.4736)</td>
<td>8 (0.8464)</td>
<td>10.1</td>
</tr>
<tr>
<td>2.5</td>
<td>1 (0.7215)</td>
<td>2 (0.4736)</td>
<td>12 (1.2696)</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1 (0.7215)</td>
<td>2 (0.4736)</td>
<td>17 (1.7986)</td>
<td>14.3</td>
</tr>
<tr>
<td>3.5</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>15 (1.587)</td>
<td>17.3</td>
</tr>
<tr>
<td>4</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>20 (2.116)</td>
<td>19.6</td>
</tr>
<tr>
<td>4.5</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>24 (2.5392)</td>
<td>21.4</td>
</tr>
<tr>
<td>5</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>29 (3.0682)</td>
<td>23.7</td>
</tr>
<tr>
<td>5.5</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>34 (3.5972)</td>
<td>26.0</td>
</tr>
<tr>
<td>6</td>
<td>2 (1.443)</td>
<td>2 (0.4736)</td>
<td>39 (4.1262)</td>
<td>28.3</td>
</tr>
</tbody>
</table>

The last column represents the surface area of one side of the cotton sheets hence the surface area would, in theory, be double the value indicated in table 5.3e.

Another interesting point is that if the diameter of the drum is 0.5m then the maximum value of $K'f$ becomes:

$$K'f = \frac{A^*h}{A^*_D} = \frac{A^*_h}{\pi R^2} \approx 144$$
Similarly for $A_h^* = 8.9 \text{m}^2$, $K'_{r} \approx 45$ and hence $45 \leq K'_{r} \leq 144$ for $8.9 \leq A_h^* \leq 28.3$, or double this if the values for both sides of the test fabric are calculated. Similarly $K_p$ can be expressed in terms of $K_h$:

**Fabric Plug Motion Analysis**

The scope here was to analyse fabric plug motion so as to address dynamic similarity and hence understand the viscous and inertial forces acting on fabric plug.

Closer observation of the washing process of a horizontal axis washing machine revealed that the clothes (wash load) undergo three distinct steps that are repeated for every drum rotation. In the following analysis the clothes are considered as being a single (concentrated) mass or "plug" of fabric i.e. fabric plug. The three steps can be defined as follows:

**Step A – Fabric Pulled through Water**

This generates a pressure drop across the fabric plug and an exchange in kinetic energy for the water, which flows through the plug [ref 5.10e].

**Step B – Fabric is Lifted out of Water**

As the plug emerges from the water it will rise and this creates a pressure head due to the position of the plug.

**Step C – Fabric Impacts Water**

When the plug reaches its maximum height for which it remains attached to the drum wall it will be projected out and back to the lower part of the drum. In doing so the plug acts as a mass which is launched at a certain angle and initial velocity. Clearly each step imposes a different flow condition, varying pressure drop and inertial forces.

---

3 this point is also defined by the critical angle  
4 the resulting trajectory will be shown to be parabolic
Figure 5.5e – The 3 Steps of the Washing Process

As can be seen in figure 5.5e it is assumed that each step imparts a pressure drop on the plug and that this determines a specific flow condition. A brief discussion of each step will now follow.

**Step A – Fabric Plug pulled through Water**

In this case a pressure drop is created across the fabric plug due to the exchange of kinetic energy produced by a difference in (average) velocity, \( \bar{u} \), between the fabric and water (of density \( \rho \)). The pressure drop also involves a correction factor, the resistance coefficient \( (C_r) \), that represents projected area to the fluid flow e.g. the spherical fabric plug. The pressure drop is therefore:

\[
\Delta P_k = C_r \frac{1}{2} \rho \bar{u}^2
\]

Equation 5.24e

If we assume the water in the washing machine to be at rest then the velocity \( \bar{u} \), can be that of the fabric motion during the rotation of washing machine drum, i.e. the tangential velocity component, \( u_t \). Hence we may write:
During the washing process the drum speed is typically between 30 to 60RPM and the average radius of the drum is between 0.2 and 0.25m. These values provide the following table of results based on equation 5.25e:

<table>
<thead>
<tr>
<th>RPM = 30</th>
<th>RPM = 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum radius = 0.2m</td>
<td>$u_i=0.628\text{ms}^{-1}$</td>
</tr>
<tr>
<td>Drum radius = 0.25m</td>
<td>$u_i=0.785\text{ms}^{-1}$</td>
</tr>
</tbody>
</table>

Table 5.4e – Tangential Velocities of Fabric

Hence the average velocity is approx. 1.1m/s which produces, according to eq. 5.24e, a pressure drop of approx. 300 to 600Pa (for $0.5 \leq C_r \leq 1.0$).

If we consider the Reynolds number in these conditions we need to establish a suitable geometric or characteristic length e.g. plug diameter, weave length/diameter/thickness, fibre diameter etc. Suppose the fabric plug diameter is 0.05m, ≈0.1kg of dry cotton) then the average Reynolds number for water at ambient temperature will be $Re \approx 53000$.

<table>
<thead>
<tr>
<th>X(m)</th>
<th>Re (water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabric plug diameter</td>
<td>0.05</td>
</tr>
<tr>
<td>Thread diameter</td>
<td>450E−6</td>
</tr>
<tr>
<td>Fibre diameter</td>
<td>18E−6</td>
</tr>
</tbody>
</table>

Table 5.5e – Reynolds Numbers for Dynamic Similarity

Note that if we consider larger diameter plugs, for example 0.1m (≈0.7kg of dry cotton), then the Reynolds number is simply multiplied by a factor of two. This leads to remark that within the fabric the flow can be extremely turbulent or laminar depending on the scale length,
definition of Reynolds number, and where we are considering the flow (as shown in the bottom two entries of previous table).

**Step B – Fabric Plug Lifted out of Water**

As soon as the fabric plug moves vertically (upwards or downwards) it is subjected to a variation in pressure because of a change in potential energy. Hence step B requires quantifying the pressure head. In a washing machine the clothes are placed in the inner drum, which is rotated during the washing process, and water is forced through the fabric plug and out through to the outer drum. This force is provided by the pressure head of the water in the drum.

The inner drum is perforated with a series of holes (measuring about $\approx 0.05\text{m}^2$ for the whole drum and $\approx 1/5$ of this value for the wetted part) that offer resistance to any seeping water hence some of the pressure head is lost because of the resistance of these drum holes to fluid flow. The water that seeps through the drum under the influence of a pressure head that is shown in figure 5.6e below:

![Diagram showing inner and outer drums with water level and fabric plug](image)

This pressure head can be calculated by:

$$\Delta P_h = \rho g h_w$$

Equation 5.26e
Thus assuming the water head is $h=0.1\text{m}$ and the fluid is water at ambient temperature this results in a maximum pressure head of $\approx980\text{Pa}$ and a range of $0\text{Pa}$ to $\approx980\text{Pa}$ during step B. Moreover, the centre of gravity of the plug will be at some distance (e.g. half the radius of the fabric plug) from the wall and so in practice the pressure head will be lower (by 10 to 20%) and vary across the plug.

The pressure head exerts a force on the water which seeps through the plug and out across each hole of the perforated inner drum and each hole can be considered a small orifice (see figure 5.3e) to which the Bernoulli equation can be applied. This equation allows us to quantify the outlet velocity of the water through the perforated drum holes that is assumed to be the flow velocity over the fabric plug. The water velocity is calculated using:

$$u_h = k \sqrt{\frac{2\Delta P_k}{\rho}}$$

Equation 5.27e

The factor $k$ is known as the pressure loss factor for the drum perforation or hole and may be calculated from:

$$k = \frac{f \cdot t_D}{D}$$

Equation 5.28e

where $f$ is the friction factor of the hole, $t_D$ and $D$ are, respectively, the length (drum thickness) and diameter of the hole. The friction factor can be determined for laminar flow and transient-turbulent flow conditions using:

**Laminar [ref. 5.9e]**

$$f = \frac{64}{\text{Re}}$$

Equation 5.29e

**Turbulent [ref. 5.10e]**

$$f = 0.0055 \left( 1 + \left( 20000 \cdot \frac{e}{D} + \frac{10^8}{\text{Re}} \right)^{1/3} \right)$$

Equation 5.30e

---

5 The sump pump flow velocity is typically 0.5 to $1\text{ms}^{-1}$. 

Page : 256
If one assumes a drum thickness of $6\times10^{-4}$m and a drum hole diameter of $5\times10^{-3}$m (which is typical for drawn metal plate) and consider $Re=10000-100000$ then the friction factor is approx. $f=0.02$: for laminar flow (with $Re=100-1000$) one obtains $f=0.64$ to 0.064, respectively. Hence $f$ ranges from 0.01 to 1 approx. which leads to a pressure loss factor from 0.0012 to 0.1.

Using equation 5.27e, and the pressure value of 980Pa, we obtain $u_h=0.002$ to 0.15ms$^{-1}$ approximately.

Remarks

It is interesting to note that if one was considering flow through the fabric cloth the pressure loss factor would be much higher [ref. 5.2e]. This doesn’t necessarily mean higher flow velocities because the pressure ($\Delta P_h$) may not be enough to move the fluid through the fabric anyway. In fact this is one way of justifying fluid flow over the fabric surface rather than through it.

Besides when the plug starts to rise the pressure head of the fabric plug will increase and only decrease once it crosses the critical slip angle$^6$. Also the fabric plug will be exposed to two different fluids (water and air) as it moves from step A to step B. This will also effect the fluid dynamics in and around the plug as it moves.

Step C – Fabric Plug Impact velocity

Once the fabric plug leaves the drum wall it will follow a parabolic trajectory [ref. 5.3e] until it hits or impacts the water. At impact a pressure is exerted on the plug which is proportional to the impact velocity i.e.:

$$\Delta P_h = \tfrac{1}{2} \rho v_{imp}^2$$

Equation 5.31e

The pressure due to impact may be considered instantaneous and is almost immediately dampened by the wash water. Through step C, i.e. during fabric plug fall, the pressure head
will vary due to position and both impact velocity and fabric plug position may be determined by considering it to be a solid body in motion and by solving the following model:

\[ V_t = \pi \cdot R_p \]

(Figure 5.7e - Model of fabric plug movement during step 3)

The preferred solution procedure is to consider the fabric plug as a body on which a system of forces acts and then use Newton's laws of motion to find a solution. Before doing this the angular rotational velocity of this body is defined by:

\[ \omega_\phi = \frac{\pi N}{30} \quad \text{Equation 5.32e} \]

One assumes that for domestic washing machines the average plug rotation radius (\(R_p\)) is 0.23m and the max. drum rotation speed is typically 50RPM. This provides the plug’s tangential velocity (\(V_t\)):

\[ V_t = w_D R_p = 1.204 \text{m} \cdot \text{s}^{-1} \]

The system of forces is shown next:

\[ \text{As was found during high speed filming} \]
The equilibrium condition i.e. the moment before which the fabric plug starts its trajectory is defined equating the forces on the plug, i.e.:

\[ m_p g \sin \phi = m_p \omega^2 R_p \]
\[ m_p g \sin \phi = m_p \frac{v_i^2}{R_p} \]

this leads to the critical plug slip angle:

\[ \phi_{crit} = \sin^{-1} \left( \frac{v_i^2}{R_p \cdot g} \right) \]  

Equation 5.33e

Hence if the drum rotation is 50RPM we have \( \phi_{crit} = 40^\circ \). We can use the same expression to determine the maximum drum velocity for which the plug does NOT detach itself from the drum. This is done by assuming \( \phi_{crit} = 90^\circ \), i.e. the plug is at the top of the drum and remains there. Hence we obtain:

\[ \sin \phi_{crit} = 1 = \frac{v_i^2}{R_p \cdot g} \]

Rearranging:

\[ v_{max} = \sqrt{R_p g} \]  

Equation 5.34e

that is \( v_{max} = 1.502 \text{ms}^{-1} \).
This maximum tangential drum speed for no plug slip is equivalent to approx. 62RPM, hence drum speeds are usually kept below this speed so as to improve wash load mixing and clothes friction. These effects are further amplified by alternating the rotation of the drum. We now move onto the calculation of $h_o$ which is defined as:

$$h_o = R_p \sin \phi + (R_D - h_w)$$

which maybe rewritten in terms of the tangential velocity:

$$h_o = \frac{v_t^2}{g} + (R_D - h_w) \quad \text{Equation 5.35e}$$

Assuming $(R_d-h_w)=0.1\text{m}$ and $N=50\text{RPM}$ we obtain $h_o=0.248\text{m}$.

\[F_{g} = ma\]

\[h_{x}\]

\[h_{o}\]

\[F = F_{g} = ma = m \ddot{r} = m \cdot g(-j) \quad \text{Equation 5.36e}\]

\[\begin{align*}
r(0) &= h_o \\
\ddot{r}(t) &= x \ddot{t} + y \ddot{j} \\
\ddot{r}(0) &= v_{x}(\cos \phi \ddot{t} + \sin \phi \ddot{j})
\end{align*}\]

Figure 5.9e – Calculation of $h_o$
this leads to 2 equations:

\[ x(t) = v_i t \cos \phi \]

\[ y(t) = v_i t \sin \phi - \frac{1}{2} gt^2 \]

The fabric plug will follow a parabolic path described by:

\[ y(t) = x \tan \phi - \frac{g x^2}{2 v_i^2 \sec^2 \phi} \]  \hspace{1cm} \text{Equation 5.37e}

The time for the fabric plug to reach its greatest height is:

\[ t = \frac{v_i^2 \sin \phi}{2g} \]  \hspace{1cm} \text{Equation 5.38e}

Hence for the conditions discussed we obtain: \( t = 0.043 \text{s} \), \( x = 0.034 \text{m} \) and \( y_p = 0.031 \text{m} \). The total distance travelled after release is established by:

\[ h_y = h_o + y_p \]  \hspace{1cm} \text{Equation 5.39e}

Hence \( h_y = h_o + y_p = 0.031 + 0.248 = 0.279 \text{m} \)

The horizontal distance travelled after release is:

\[ h_x = \frac{v_i^2 \cos \phi}{g} \]  \hspace{1cm} \text{Equation 5.40e}

Thus \( h_x = 0.146 \text{m} \). We may summarise the results as shown in the figure below:

\[ \text{Figure 5.10e – Summary of Fabric Plug Trajectory after Release} \]
We can move onto the calculation of the impact velocity by assuming it to be equal to the terminal velocity of the plug when it hits the water. This is done by first determining the time of travel:

$$t_I = \frac{2v_y \sin \phi}{g}$$  \hspace{1cm} \text{Equation 5.41e}

Hence \(t_I = 0.188\) s (for \(y_p = 0\)). The terminal velocity is reached when \(h_y = 0\) which for the given conditions is approx. 0.338 s. Thus the total time to reach the water level is 0.526 s and this can be used to determine \(v_{\text{term}}\) by differentiating the two expressions obtained from eq. 43 i.e.:

$$v_{\text{term}} = \sqrt{(v_y \cos \phi)^2 + (v_y \sin \phi - gt)^2}.$$

Hence the terminal velocity is 4.31 m/s at \(h_c = 0.262\) m.

Remarks

This ultimate result ignores the effects of resistance to air so to understand the true terminal velocity it would be necessary to establish the true geometry of the textile that offers resistance to air-flow. A typical pressure graph versus time reflecting pressure variation during one revolution of the drum (steps A to C) is shown below:

Figure 5.11e – Plug Pressure Variation for Steps A to C

The textile or fabric plug is obviously a simplification of what actually happens in a real domestic washing machine. Direct observation of the wash load shows that the motion of the
textiles will vary depending on the volume available for rotation\(^7\), which is dependent on the machine (drum size), the wash load size and on the wash cycle e.g. wash water.

In fact as the wash load increases so the motion of the plug in the wash load gradually becomes restricted since the space available for it to move is reduced.

This effect leads to a preferred concentration of the wash load i.e. the centroid of the wash load will take-up a preferred area within the drum. Radiotracer studies [ref. 5.2e] show that the preferred position of the centroid lies at a mid-radius 7–8 o’clock position when facing the drum, although localised fabric lumps occur elsewhere as well. Also as free drum volume decreases so the vertical lifting height of the plug tends to increase because of the support effects of the fabric behind it during rotation. The subsequent effect (step C) is the rolling of the fabric, which implies that the fabric folds and unfolds generating a churning effect. Not surprisingly if the vertical distance of the fabric is plotted against time the resulting profile is a series of spikes showing that the fabric motion is erratic.

The overall churning effect also conditions the diffusion and convective regions in the fabric plug resulting in an unsteady mass-transfer problem with variable geometry. Ganguli and van Eendenburg [ref. 5.13e] sustain that wash load agitation forces the wash water to penetrate the load only up to a certain distance while the rest of the region is stagnant leading to the so called effective diffusion length.

\(^7\) In fact the high speed filming returned a maximum height of approx. 110mm with about 3kg of wash load.
**High Speed Filming of the Wash load and Sphere**

In the last chapter the analysis of the bubble trajectory provided an indication of what was happening (e.g. the flow velocity) on the inside door face. In particular the bubble dynamics were first modelled and assessed with high speed filming techniques.

The same approach was used in step 5 except that the bubble analysis now becomes the sphere motion analysis and the high speed filming was taken directly in front of the machine door of both the sphere and wash load. Consequently the same instrumentation and methodology discussed for step 4 were also valid for step 5.

In this section only the impact (free fall) part of the sphere motion is discussed since the plug essentially follows the drum before and after this. Thus the format of the summary of results is provided as per the bubble dynamics analysis i.e. with the following graph and tables.

![Figure 5.12e – Typical Sphere Trajectory during CW Free fall](image)
Post-processed Test Data

<table>
<thead>
<tr>
<th>Post-processed Test Data</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical event duration</td>
<td>0.1–0.25s</td>
</tr>
<tr>
<td>Typical Y displacement</td>
<td>50–110mm</td>
</tr>
<tr>
<td>Critical Angle</td>
<td>45°</td>
</tr>
<tr>
<td>Maximum acceleration</td>
<td>0.502ms⁻²</td>
</tr>
<tr>
<td>Maximum force</td>
<td>0.1N</td>
</tr>
<tr>
<td>Impact velocity</td>
<td>1.60ms⁻¹</td>
</tr>
<tr>
<td>Maximum sphere speed</td>
<td>1.60ms⁻¹</td>
</tr>
<tr>
<td>Average sphere speed</td>
<td>0.87ms⁻¹</td>
</tr>
<tr>
<td>Minimum sphere speed</td>
<td>0.50ms⁻¹</td>
</tr>
</tbody>
</table>

Table 5.6e – Sphere Dynamics (2.5kg wash load) Results Table

A finer illustration of what happens to the sphere during the final 10mseconds of free fall is given below:

Table 5.7e – Sphere Dynamics over final 10msecs of free fall

<table>
<thead>
<tr>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>t (secs)</th>
<th>\(\dot{x}) (ms⁻¹)</th>
<th>\(\dot{y}) (ms⁻¹)</th>
<th>v (ms⁻¹)</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>174.716</td>
<td>94.387</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>175.721</td>
<td>92.379</td>
<td>0.002</td>
<td>0.502</td>
<td>1.004</td>
<td>1.123</td>
<td>63.4</td>
</tr>
<tr>
<td>175.721</td>
<td>90.371</td>
<td>0.004</td>
<td>0</td>
<td>1.004</td>
<td>1.004</td>
<td>/</td>
</tr>
<tr>
<td>176.725</td>
<td>89.366</td>
<td>0.006</td>
<td>0.502</td>
<td>0.502</td>
<td>0.710</td>
<td>45</td>
</tr>
<tr>
<td>177.725</td>
<td>86.354</td>
<td>0.008</td>
<td>0.502</td>
<td>1.506</td>
<td>1.588</td>
<td>71.56</td>
</tr>
<tr>
<td>177.729</td>
<td>85.350</td>
<td>0.01</td>
<td>0</td>
<td>0.502</td>
<td>0.502</td>
<td>/</td>
</tr>
</tbody>
</table>

Over (the final) 10ms we may conclude:

\[\Delta x = 177.729 - 174.716 = 3.012E^{-3}\text{m}\]
\[ \Delta y = 94.387 - 85.350 = 9.037 \times 10^{-3} \text{m} \]

\[ x = 0.301 \text{m/s} \quad y = 0.904 \text{m/s} \quad \text{hence} \quad v = 0.953 \text{m/s} \]

Hence we conclude that over the final 10ms of the free fall of the sphere the maximum speed is approx. 1.6m/s and the average speed is 0.95m/s. Another useful part of the dynamics analysis is the acceleration of the sphere, which is defined as follows:

\[ \bar{a} = \frac{\Delta v}{\Delta t} \quad \text{or} \]

\[ \bar{a} = \frac{\Delta v_x}{\Delta t} i + \frac{\Delta v_y}{\Delta t} j = a_x i + a_y j \]

Equation 5.42e

where \( a_x = \frac{dv_x}{dt} \) and \( a_y = \frac{dv_y}{dt} \).

Through this calculation it is possible to calculate the force of the sphere due to its acceleration i.e. \( F = ma \). Typical values are \( a_{\text{max}} = 0.5 \text{m/s}^2 \) and \( F = 0.1 \text{N} \) for the sphere during free fall.
The Sphere and the Remote Data Acquisition System

The remote data acquisition system was divided into two parts:

1. The first or transmitter part consisted of a hollow plastic sphere which houses the needles, silicone tubing, pressure transducer, battery and transmitter with integrated antenna.

2. The second or receiver part was external to the drum and consists of the receiver section of the system.

The sphere, shown below open before and after wiring, had the transmitter and transducer fitted in one half and the other half occupied by the battery and external connections. The size of the sphere was approximately 96mm in diameter and weighed less than 200gr fully assembled.

The receiver section included a hybrid receiver (Aurel mod. RF290A-300), relative power supply, a sample and hold board (mod. SC2040), a PCMCIA data acquisition card (mod. DAQcard AI-16E-4), both by National Instruments, and the data acquisition program developed using LABVIEW version 5.1.

Figure 5.13e – Inside the sphere

(upper right: battery, lower right: pressure transducer; lower left transmitter and V–F converter)
The data acquisition card and SW was installed on a IBM 600 Thinkpad laptop where the experimental data was both acquired and processed.

**System Operation**

System operation was as follows:

The pressure transducer detected the dynamic pressure on a small triangular type block probe as described for step 2. Only one pressure tapping was connected and the static pressure was referred to as the pressure inside the sphere.

The probe therefore acted as a small flat Pitot tube and converted the pressure into a voltage signal, after which it was conditioned and then sent to a voltage to frequency converter. The output frequency was then modulated and transmitted outside the washing drum via a miniature battery operated radio transmitter (Aurel mod. TX–SAW – I.A.) at 433MHz (see Fig. 5.14e a).

The matched radio receiver (see Fig. 5.14e b) is located within a short distance (<1m)\(^8\) on the outside of the washing machine and its output signal is sent to the sample and hold board and then to the data acquisition card and laptop where the experimental data is stored and processed.

![Transmitter and Receiver Circuits](image)

**Figure 5.14e – a) Transmitter circuit, b) Receiver circuit**

To start the acquisition the sphere is placed in the washing drum with the wash load and the data acquisition started. The washing cycle is activated and the pressure data recorded by the

---

\(^8\) The system was tested up to about 10m.
pressure transducer was transferred to the laptop mass memory where it was subsequently retrieved, processed and stored. The complete system is shown below:

![Complete System Diagram]

**System Calibration**

Before system experimentation two calibration or characterisation procedures were involved, these being:

1. Flow characterisation of the sphere and probe.
2. Electrical characterisation of the transmitter–receiver and data acquisition section

In the first case the sphere was calibrated by subjecting the probe (and sphere) to a series of specific water flow conditions. A Yokogawa MT120 digital pressure manometer recorded the dynamic pressure so it was possible to relate flow rate, local flow velocity and dynamic pressure. Typical calibration conditions were \( Q = 2 \times 10^{-3} \text{s}^{-1} \), \( P_d = 100-500 \text{Pa} \), \( u = 0.2-1 \text{ms}^{-1} \) (outlet). The test rig is pictured shown next:
The sphere was calibrated both with and without the remote acquisition system, in this way the procedure was identical for both cases. The calibration method consisted of applying a fixed flow rate while monitoring the water temperature. The flowing water was directed towards the probe so as to ensure that it was fully immersed. Close-up filming was also carried out to ensure that this was so and also establish what was happening near the probe. The flow rate was held constant by using a constant pressure head establish by the overflow which was positioned 580mm above the outlet.
The flow rate was measured by diverting the flow (through the metering valve) from the sphere to a measuring cylinder. The time taken to reach a given quantity of water (usually 30-100cc) in the cylinder was used to determine the average flow rate from which the average flow velocity was calculated. To reduce experimental error the measurements were repeated at least 10 to 20 times for each flow rate and for each of the three probe pressure tappings (P1, P2 and P3).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>T (°C)</th>
<th>Average Flow (ml/s)</th>
<th>Stdev</th>
<th>Average Pressure (KPa)</th>
<th>Stdev</th>
<th>% flow</th>
<th>Density (Kg/m³)</th>
<th>No. of Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.38</td>
<td>3.983</td>
<td>0.1362</td>
<td>0.250</td>
<td>0.03120</td>
<td>93.9387</td>
<td>997.936</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>21.60</td>
<td>3.665</td>
<td>0.059212</td>
<td>0.220</td>
<td>0.03476</td>
<td>86.4387</td>
<td>997.887</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>19.70</td>
<td>4.2400</td>
<td>0.03192</td>
<td>0.358</td>
<td>0.02925</td>
<td>100.0000</td>
<td>998.295</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>18.46</td>
<td>3.3360</td>
<td>0.05166</td>
<td>0.150</td>
<td>0.02012</td>
<td>78.6792</td>
<td>998.541</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>19.20</td>
<td>1.9580</td>
<td>0.01435</td>
<td>0.229</td>
<td>0.02778</td>
<td>46.1792</td>
<td>998.396</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>19.70</td>
<td>3.6580</td>
<td>0.05621</td>
<td>0.225</td>
<td>0.01242</td>
<td>86.2736</td>
<td>998.295</td>
<td>24</td>
</tr>
<tr>
<td>7</td>
<td>20.01</td>
<td>1.4320</td>
<td>0.01410</td>
<td>0.153</td>
<td>0.01186</td>
<td>33.7736</td>
<td>998.231</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>20.41</td>
<td>3.3810</td>
<td>0.04106</td>
<td>0.172</td>
<td>0.01440</td>
<td>79.7406</td>
<td>998.147</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>20.80</td>
<td>2.5950</td>
<td>0.02186</td>
<td>0.177</td>
<td>0.01632</td>
<td>61.2028</td>
<td>998.063</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>20.90</td>
<td>1.9190</td>
<td>0.02676</td>
<td>0.155</td>
<td>0.02417</td>
<td>45.2594</td>
<td>998.041</td>
<td>10</td>
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<tr>
<td>11</td>
<td>23.91</td>
<td>3.7420</td>
<td>0.05596</td>
<td>0.410</td>
<td>0.03601</td>
<td>88.2547</td>
<td>997.342</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>23.90</td>
<td>3.0215</td>
<td>0.03540</td>
<td>0.367</td>
<td>0.04151</td>
<td>71.2623</td>
<td>997.344</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>19.80</td>
<td>3.7359</td>
<td>0.04031</td>
<td>0.346</td>
<td>0.02159</td>
<td>88.1116</td>
<td>998.274</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>19.66</td>
<td>3.4450</td>
<td>0.05783</td>
<td>0.344</td>
<td>0.05308</td>
<td>81.2500</td>
<td>998.541</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>19.45</td>
<td>3.7330</td>
<td>0.04521</td>
<td>0.367</td>
<td>0.03280</td>
<td>88.0425</td>
<td>998.346</td>
<td>15</td>
</tr>
</tbody>
</table>

The measurements were first carried out with the probe incidence angle equal to zero and a total of over 700 measurements were collected. The standard deviation for the flow rate measurements was typically <2%, thus demonstrating the validity of the test rig flow regulation. However, the recorded pressures showed a much higher standard deviation (between 5 to 15%) which was attributed essentially to the accuracy of the sphere position (and hence probe) with respect to the flow.

Following these measurements a series of probe angles (10 to 60° at 10° increments) were also tried so as to establish the angular sensitivity of the measurement. Although the calibration procedure was rather rudimentary it sufficed to determine a calibration curve for the probe and
sphere. As is shown below before any form of correction or filtering was introduced the experimental data did show a typical quadratic behaviour showing that the flow velocity on the surface of the sphere could be predicted simply by recording the dynamic pressure (see eq. 5.12a).

![Figure 5.18e – P1, P2 and P3 Dynamic Pressure (Raw) Data](image)

To determine the (average) flow velocity at the probe the dynamic pressure was used and obtained from the general Bernoulli equation:

\[ P_{\text{tot}} = \rho g H + P_{\text{atm}} + \frac{1}{2} \rho u^2 \]  

Equation 5.43e

The first two terms were accounted for through the recording of the baseline for each test i.e. zero calibration of the MT120 instrument or washing machine. That is to say the baseline was recorded and then used to nullify this affect in eq. 5.43e. In this way the pressure recordings could be solely attributed to the flow at the probe.
Typical pressure and flow rate data errors were below 10% and 3% respectively (see table 5.8e). After adequate filtering and compensation the typical correlation coefficient was $0.95 \leq R^2 \leq 0.97$.

An example of how the data spread changes after filtering and correction (e.g. temperature correction of water density) is shown next for the same data and that after for P1 pressure tapping only.

![Graph of pressure vs flow rate](image)

**Figure 5.19e – Preliminary Filtering and Correction of all pressure data**

The above graph was obtained after using the Lowess filtering technique which is a locally weighted regression smoothing algorithm that performs a full least-squares computation for each data point.
Interestingly the probe generally behaved better at low flow rates and this was accredited to better water flow–to–probe orientation and thus wetting of the probe during measurements. As mentioned previously a close–up video of the calibration sequence was also made. From this video it was possible to observe the exact flow in and around the probe and therefore obtain further flow (visual) information. In particular it was possible to observe that the flow over the probe reacted in way very similar to a block (broad–crested) weir [ref. 5.11e]

Once the probe was hydrodynamically characterised the transmitter and receiver connection was calibrated by supplying several different waveforms (see figure 5.21e) with a waveform generator to the transmitter input (i.e. after the V–F converter) and recording the received signal. This test was repeated for a series of signal conditions in order to be able to fully characterise the system and quantify parameters such as the gain, noise, signal–to–noise etc. The gain of the system was found to be between 4 and 6 with a low–pass (<150Hz) Butterworth filter.
The test set-up is illustrated in the figure below followed by a series of traces obtained during this procedure:

```
Function Generator ----> T ----> R ----> SC2040 Sample-Hold
```

**Figure 5.21e – Calibration of the transmitter–receiver section**

**Figure 5.22e – Examples of Transmitter and Received signals**

With the electronics calibrated the sphere was further sealed with a silicone sealant so as to avoid any ingress of water during tests. The sphere was then calibrated again but this time with the remote signal, a typical recording is shown below for a specific flow rate (circa 3.5ml/s):

**Figure 5.23e – Typical Transmitted Pressure Signal from Test Rig**
To acquire test data the sphere was placed in the washing machine and the following types of base line recordings made:

1. Base line with sphere at rest and at the bottom of the drum.
2. Base line with sphere at rest and the top of the drum
3. Baseline with wash load, no water and with the drum in motion.

With this done the average value (approx. 20–30 ±3Pa) for the three baselines was used as the offset for all future recordings with wash load, sphere and wash water.

Essentially the baseline average value was subtracted from the recordings made with the sphere during drum motion. An example of baseline type 3 is shown below:

![Figure 5.24e - Example of Washing machine baseline](image)

Sphere Test Results

Rather than show all of the test results three typical recordings for the wash load and wash water conditions will be shown and discussed. The results, presented in the form of graphs and tables, concern three conditions, these being:

1. Standard wash load conditions i.e. 5kg of dry wash load as per IEC 60456 and the nominal wash water for the machine used i.e. 12.7kg at 20°C.
2. As per condition 1 but with an extra 10kg of water at 20°C.

9 pressure variation for condition 3 was approx. 3Pa.
3. As per condition 2 but with an extra 20kg of water at 20°C.

The received signal in volts is plotted against time for all three conditions. All three graphs have been filtered (FFT filtering and/or Lowess filter prior to plotting). This was essential because of the noise (in the form of spikes) introduced during transmission.

The tabled data provides an indication of the maximum peak-to-peak reached (both in volts and Pa) and also an indication of the fastest rate of change of pressure (Pas⁻¹).

Table 5.9e – Example of Std. 5kg Wash load (12.7kg of water)

<table>
<thead>
<tr>
<th>Test Data</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (average)</td>
<td>0.45 to 2.5Hz</td>
</tr>
<tr>
<td>Peak-to-peak signal</td>
<td>2.45V</td>
</tr>
<tr>
<td>Peak-to-peak pressure</td>
<td>615Pa</td>
</tr>
<tr>
<td>Max. Pressure change rate</td>
<td>850Pas⁻¹</td>
</tr>
</tbody>
</table>

Figure 5.25e – Std. 5kg Wash load (example)
Test Data

<table>
<thead>
<tr>
<th></th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (average)</td>
<td>1 to 4 Hz</td>
</tr>
<tr>
<td>Peak-to-peak signal</td>
<td>1.45V</td>
</tr>
<tr>
<td>Peak-to-peak pressure</td>
<td>360Pa</td>
</tr>
<tr>
<td>Max. Pressure change rate</td>
<td>650Pas$^{-1}$</td>
</tr>
</tbody>
</table>

Table 5.10e – Std. 5kg Wash load + 10kg of extra water

Std. 5kg Wash load with 10kg of extra water

![Graph showing signal in volts vs elapsed time in seconds]

Figure 5.26e – Std. 5kg Wash load + 10kg of extra water (example)

Test Data

<table>
<thead>
<tr>
<th></th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (average)</td>
<td>0.5 to 3.5 Hz</td>
</tr>
<tr>
<td>Peak-to-peak signal</td>
<td>1.75V</td>
</tr>
<tr>
<td>Peak-to-peak pressure</td>
<td>430Pa</td>
</tr>
<tr>
<td>Max. Pressure change rate</td>
<td>530Pas$^{-1}$</td>
</tr>
</tbody>
</table>

Table 5.11e – Std. 5kg Wash load + 10kg of extra water
Figure 5.27e – Std. 5kg Wash load + 20kg of extra water (example)

Summary

The three most significant observations can be summarised as follows:

1. The dynamic pressure is sufficiently high to state that the flow velocity over the sphere was well in excess of $\text{mms}^{-1}$. On the basis of the recordings made the flow velocity range was approx. 0.5 to 1.2$\text{ms}^{-1}$, although if the extreme values are also accounted for the true range reaches as far as 1.5$\text{ms}^{-1}$. This result is also in-line with what was discussed in step A (300–600Pa, $u=1.1\text{ms}^{-1}$).

Also Van den Brekel states that the (theoretical) plug pressures are: pull=300Pa, lift=490Pa, Impact=4124Pa and an average of 500Pa during washing.

2. The variation in pressure does follow a curve very similar to that predicted by the fabric plug model. In fact there is a definite cyclic signature and thus the wash load is subjected to this type of flow, pressure and motion.

3. The rate of change of dynamic pressure is high although the frequency for the above mentioned cyclic behaviour is low.
In view of these experimental results it can be concluded that the flow velocity inside the wash load can reach values in the order of meters per second. Further discussion of these results will be given in the final part of the thesis, "Round-up of Results, Findings and Next Steps".
Chapter 6 – Further Research Steps and Additional Washing Machine

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<tr>
<td>Remarks</td>
<td>329</td>
</tr>
<tr>
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<td>330</td>
</tr>
<tr>
<td>Remarks</td>
<td>331</td>
</tr>
<tr>
<td>Detergent Effects</td>
<td>332</td>
</tr>
</tbody>
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\[ H = D \left( 1 - \frac{D}{2a} \right) \]  
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Further Recommended Readings

Computational Fluid Dynamics and the Research

Foreword

Part of the development of the tunnel and probes in step 1 was achieved with the use of CFD [ref. 6.1], in particular with FLUENT/UNS version 5, a renowned CFD package. Almost all the CFD work tackled tunnel design although some work was carried out to experiment the integration of CAD, rapid prototyping and FLUENT for the weave model.

This section therefore focuses on the findings of a series of numerical simulations of the tunnel aimed at assessing the effects of hydrodynamic flow conditions on tunnel and probe behaviour [ref. 6.2]. Hence the main scope of the simulation was to predict the hydrodynamic behaviour of the tunnel in terms of wall shear stress, velocity contours, pressure drop etc. in specific sections of the test section.

A total of four simulation runs were made and each run was based on FLUENT's standard K- Epsilon model and usage of the standard wall functions [ref. 6.3].

The simulation essentially focused on the central part or test section of the tunnel, where the experimentation was carried out.

The meshing of the test section was done with GAMBIT, FLUENT's latest meshing tool, and divided in two domains. The first domain had a rough mesh of approximately 23000 cells situated before and after the measurement zone of the tunnels test section while the second domain, where the probes were mounted and tested, had approximately 204000 cells.

Scope of the Simulation

The overall scope of the simulation activity was to establish:

- Static and total pressure mapping of the tunnel, tunnel walls and probes.
- Velocity vector mapping near the probe and along the tunnel at specific sections.
- Determination of tunnel pressure drop
Comparison of simulated results with and without the probe.

Evaluation and visualisation of wall shear stress in and around the area of shear stress measurement.

Comparison between simulated and experimental results.

FLUENT was chosen because of its renowned robustness in industrial applications and solution models available. Moreover it numerically solves the N–S equations by a marching solution through space and/or time using finite volumes. The end result is a numerical description, using structured or unstructured grids, of the complete flow field of interest, i.e. tunnel with or without the shear stress probe.

In essence FLUENT is a simulation tool that makes 2D or 3D fluid flow investigation possible on a computer without the need to build and test a physical model first. The simulation discussed here refers to an unstructured approach of the flow domains and has been used in two specific circumstances, these being:

1. To simulate the tunnel with and without the triangular shear stress probe placed in-line and opposing the main flow stream of the tunnel.

2. To simulate the behaviour of the shear stress probes at different hydrodynamic conditions.

The hydrodynamic conditions were simulated in four specific cases identified by the relative Reynolds number, i.e. Re= 51464, 60035, 82000 and 119583.

This range corresponded to the central part of the Reynolds number test range for the soil removal investigation in step 3, as well steps 1 and 2.
Effects of Block probes and Preston Tube

Both the square and triangular block probes interfere with the flow since they are invasive methods of measuring shear stress.

One way of understanding these effects is to simulate the probes in the tunnel in real flow conditions. This was done for the triangular shaped probe and as shown below:

![Velocity Vectors around Triangular Probe]

Figure 6.1 – Velocity Vectors around Triangular Probe

As can be seen above the probe's pressure tappings have a certain amount of re-circulation and the static tapping also has an uneven pressure across its face.

As expected the flow around the tips and edges of the probe also had turbulence so after some attempts at simulating it was decided to emphasise this by increasing the fineness of the mesh both for and near the probe as shown next and in figure 6.4b.
In general the probes only marginally affected the shear stress measurements since probe, Preston tube and CFD all provided similar results. This implies that the CFD simulation was satisfactorily validated and could in fact, be repeated with a slightly inferior mesh.

**Solution Model**

Since the water tunnel was run in turbulent flow conditions the relative numerical solutions were affected by the presence of the tunnel walls.

In particular the need to satisfy the no-slip condition at the wall brings about the requirement to use what are called near-wall models, which significantly impact the fidelity of the numerical solution. This happens because the solution variables have large gradients near the wall, which leads to obvious hesitation about the size of the mesh to be used from the wall outwards.

In summary, accurate representation of the flow near the wall e.g. the probe, tunnel roof etc., determines successful predictions of the wall-bounded turbulent flows which are dependent both on the model and mesh.
Surprisingly even though the mesh was rough near the wall the predicted and experimental results still only differed by less than 10%.

A further difficulty with turbulent flows is that they involve time-varying fluctuations of velocity and scalar quantities. FLUENT tackles such situations by solving for the ensemble-average [ref. 6.4] of the turbulence, which leads to a resulting N-S equation set that includes the Reynolds stresses, \( \rho \bar{u}_i \bar{u}_j \). The solution of this equation set depends on using the appropriate closure model for the problem. FLUENT incorporates three models: the standard k-\( \varepsilon \) model, the Renormalisation Group k-\( \varepsilon \) (RNG) model and the Reynolds stress model (RSM). For the tunnel simulation the std. k-\( \varepsilon \) model was used although FLUENT suggests this model only for Re numbers greater than 50000.

The std. k-\( \varepsilon \) model is based on the Boussinesq hypothesis that states that the Reynolds stresses are proportional to the mean velocity gradients that include a turbulence kinetic energy term (k) and the rate of dissipation of turbulent kinetic energy term (\( \varepsilon \)). The main strengths and weaknesses of the std. k-\( \varepsilon \) model are:

**Table 6.1 – Strengths and Weaknesses of the std. k-\( \varepsilon \) model**

<table>
<thead>
<tr>
<th><strong>Strengths</strong></th>
<th><strong>Weaknesses</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Model is robust.</td>
<td>Relies on a &quot;semi&quot; empirical basis (valid) for Re&gt;50000 in simple steady state flows.</td>
</tr>
<tr>
<td>It is cost efficient in use and effective for std. Industrial applications.</td>
<td>The eddy viscosity is isotropic which implies the Reynolds stresses are all of the same order and there is only one length and velocity scale for all directions.</td>
</tr>
<tr>
<td></td>
<td>All three previously mentioned models cannot be applied near the wall without a satisfactory wall approach.</td>
</tr>
</tbody>
</table>
FLUENT can tackle the near-wall region in one of two ways:

- Through special "Wall-functions"
- Using a 'Near-wall' model

The simulation work discussed here is based on the first approach. This first approach undertakes the problem by using semi-empirical formulae that essentially bridge the viscosity affected region between the wall and fully-turbulent or core region. These functions essentially assist, without modifying, FLUENT's turbulence models such as the k-ε model. The main advantages of the wall functions, which is favoured in industry, are:

- Saving of computational resources.
- The solution is cost effective, robust and reasonably accurate for most cases.

The second approach is where the near-wall region is resolved all the way down to the wall. The wall-functions are basically a collection of formulae based on experimental observation and include:

- Laws-of-the-wall for mean velocity (and other scalars)
- Formulae for near-wall turbulent quantities

FLUENT also allows the choice of two approaches using:

a. Standard wall functions.

b. Non-equilibrium wall functions.

The simulation discussed refers to the standard wall functions since these have been used also for the manual calculations [refs. 6.5, 6.6 and 6.7]

**The Tunnel and Geometric Model**

In order to optimise the mesh and computation time the system included only the essential parts i.e. test section of tunnel and probe. Thus the geometric model was constructed so that the resulting two-domain model was made as simple. This effectively reduced the tunnel length from

---

1 It can also use a two-layer zonal model but this is not discussed here.
>1m to 4m circa, with special focus on the test section and the area near the probe or measurement reference planes.

The final geometric model therefore consisted of parts 4 to 6 (see table 3.1 and Fig. 3.1) but with only half the solid, wetted tunnel boundaries being modelled (since the tunnel was effectively symmetric it was split along the centreline) and simulated as shown below:

Figure 6.3 – Complete Geometric Model of Tunnel

The test section of the tunnel included the ramped insert and the final model was generated with GAMBIT, FLUENT's pre-processor, which also provides the meshing tool, thus rendering the pre-processing phase particularly efficient.

**Mesh and Initial Conditions**

Since the model essentially involved solving symmetric flow in steady state conditions the tunnel was split exactly at its centreline, which considerably simplified the mesh and reduced the number of cells for computation by approximately 50%.

The meshing of the model was hybrid i.e. it was divided into two distinct domains (see Fig. 6.4a): Domain a). the probe and probe vicinity, Domain b). the approach or entrance and converging exit of the tunnel, including ramps. The mating of the two domains, i.e. tunnel with the probe, was automatic and made by combining Hex and Tetra type grids, which is another interesting feature of FLUENT.
The mesh was accomplished using GAMBIT, FLUENTs, all-in-one, pre-processor that provides excellent flexibility also in terms of domain and cell type generation. FLUENT allows the use of quad cells with an unstructured approach using PAVE cells (a type of quad cell) based on Cooper's scheme.

![Mesh Details](image)

**Figure 6.4 – Mesh Details**

a) Two domains  

b) Close-up of probe domain

**Mesh Characteristics**

- Large macro grid (tunnel inlet and exit) : 23100 cells
- Concentrated grid (probe and probe vicinity) : 204609 cells
- Number of nodes : 84465
- Number of faces : 473035
- Skewness : 0.79
- Aspect ratio : 4.2

**Solver Settings – Convergence Results**

- Steady state turbulent flow
- K–Epsilon model and standard wall functions
- Relaxation parameters : K=0.8, Epsilon=0.8
- Model used for numerical solution near wall: standard wall functions
No. of iterations to complete convergence: <500
Residuals continuity: <1E-4
K and Epsilon: typ. <1E-4
X, Y and Z velocity components: <1E-6
Computation time: min. 31380s (Re=51464) max. 34680s (Re=119583)
Computer: SUN Ultra 10 Workstation – 500Mbyte RAM

Initial conditions
The dynamic viscosity of the water was determined using:
\[ \mu^{-1} = 555.38579 + 19.93T + 0.1052T^2 \]
The density of the water was determined using:
\[ \rho = 999.6143 + 0.2091T - 0.0622T^{3/2} \]
Reference pressure = 101325Pa.
Yaw angle = 0°, probe position: “in-line” for all four cases.

Table 6.2 – Overall Hydrodynamic Test Conditions

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Re</th>
<th>Water T°C</th>
<th>Average Flow Velocity (ms⁻¹)</th>
<th>Dynamic Viscosity (Nsm⁻²)</th>
<th>Density (kgm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51464</td>
<td>13.2</td>
<td>0.718</td>
<td>1.195E-3</td>
<td>999.391</td>
</tr>
<tr>
<td>2</td>
<td>60035</td>
<td>14.4</td>
<td>0.811</td>
<td>1.157E-3</td>
<td>999.226</td>
</tr>
<tr>
<td>3</td>
<td>82000</td>
<td>14.5</td>
<td>1.105</td>
<td>1.154E-3</td>
<td>999.212</td>
</tr>
<tr>
<td>4</td>
<td>119583</td>
<td>13.6</td>
<td>1.65</td>
<td>1.182E-3</td>
<td>999.339</td>
</tr>
</tbody>
</table>

Table 6.3 – Shear Stress Stations

<table>
<thead>
<tr>
<th>No.</th>
<th>Inlet</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (m)</td>
<td>0.0</td>
<td>3.22</td>
<td>3.25</td>
<td>3.28</td>
<td>3.35</td>
<td>3.4</td>
<td>3.45</td>
<td>3.55</td>
<td>3.6</td>
<td>3.65</td>
<td>3.7</td>
<td>3.75</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Table 6.4 – Velocity Stations

<table>
<thead>
<tr>
<th>Station Ref.</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
<th>No. 6</th>
<th>No. 7</th>
<th>No. 8</th>
<th>No. 9</th>
<th>No. 10</th>
<th>No. 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(m)</td>
<td>3.5</td>
<td>3.6</td>
<td>3.7</td>
<td>3.8</td>
<td>3.9</td>
<td>4.0</td>
<td>4.1</td>
<td>4.2</td>
<td>4.3</td>
<td>4.4</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Figure 6.5 – Upper Wall Plots

**Re=51464**

**Experimental Data**

![Experimental Data Graph](image1)

**CFD Simulation**

![CFD Simulation Graph](image2)

Wall Shear Stress
Re=51464, x=3.35m
FLUENT 5.1 (3D, segregated, k-w)

**Re=60035**

**Experimental Data**

![Experimental Data Graph](image3)

**CFD Simulation**

![CFD Simulation Graph](image4)

Wall Shear Stress
Re=60035, x=3.35m
FLUENT 5.1 (3D, segregated, k-w)

Page : 297
Overall Experimental Results versus CFD predictions

Table 6.5 – Average Shear Stress (τ = Pa) at Centre line of Tunnel

<table>
<thead>
<tr>
<th>Re</th>
<th>Experimental Data</th>
<th>CFD Simulation</th>
<th>%Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>51464</td>
<td>1.26</td>
<td>1.36</td>
<td>+7.9</td>
</tr>
<tr>
<td>60035</td>
<td>1.74</td>
<td>1.7</td>
<td>-2.2</td>
</tr>
<tr>
<td>82000</td>
<td>2.68</td>
<td>2.8</td>
<td>+4.5</td>
</tr>
<tr>
<td>119583</td>
<td>5.84</td>
<td>5.94</td>
<td>+1.7</td>
</tr>
</tbody>
</table>
**Summary and Conclusions**

The main outcome of the simulation was two fold:

a.) The confirmation of the shear stress magnitude measured during experimentation with the Preston tube and shear stress probe, with a discrepancy of less than 10% between simulated and experimental measurements.

b.) Reliable prediction of shear stress probe and tunnel behaviour.

The four cases evaluated with FLUENT confirm that this package performs effortlessly and provides more than adequate results. Of particular interest is the high performance of FLUENT’s solver and standard wall functions, which in spite of the rather crude mesh near the wall, and especially across the height of the probe, proved to be accurate within 10% of the experimental results.

Interestingly as the Re number decreased the discrepancy between experimental and CFD results actually increase. This seems to go against common knowledge in CFD (e.g. more turbulence adds more uncertainty) but FLUENT indicate a lower limit of about Re=50000 for reliable data using the wall functions.

Equally comforting was the comparison of shear stress profiles and the effects of the side wall between 120 to 150mm (z-direction). Moreover the computation for each case was typically completed within 8–9 hours against the equivalent experimentation lasting at least 2 twice as long and obviously susceptible to experimental error.

The next logical step would be to simulate the angular calibration of the probe.
**Roughness and Waviness Measurements**

In step 3 the position of the EMPA samples was controlled by ensuring that the fabric plug was suitably sized and the mounting procedure was respected for each test. However, nothing was known of the surface characteristics of the EMPA samples hence it was considered useful to measure the roughness and waviness of the sample surface.

All these measurements were realised using a OPTIMET Conoscan 2000 non-contact dimensional measurement instrument [ref. 6.8]. This instrument is based on a low power LASER light source, which scans the surface and then optically translates the reflected light into a vertical (Z-axis) holographic measurement. Thus the instrument is capable of 3D non-contact dimensional measurement at very high scan rates i.e. it can measure profiles of moving surfaces without actually touching the surface. This non-contact feature is essential for weave profiles because the cotton fibres and thread of the EMPA sample obviously bend very easily under pressure thus rendering traditional contact roughness measurements inappropriate. An alternative method will also be discussed.

The samples were mounted on a thick (10mm), flat, Perspex substrate using the same twin-sided tape used to bond the probes. The samples were pressed onto the tape and then left to settle (3 days) so that the natural shape of the weave could return. The samples tested were as follows:

<table>
<thead>
<tr>
<th>EMPA Sample</th>
<th>EMPA sample serial No. 105.</th>
</tr>
</thead>
<tbody>
<tr>
<td>White reference (no soil)</td>
<td>Art. 221</td>
</tr>
<tr>
<td>Chocolate (and milk)</td>
<td>Art. 112</td>
</tr>
<tr>
<td>Carbon black (and mineral oil)</td>
<td>Art. 101</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>Art. 111</td>
</tr>
<tr>
<td>Red Wine</td>
<td>Art. 114</td>
</tr>
</tbody>
</table>
For each acquisition the instrument was run at least six times for each sample in both scanning directions over a distance of 5mm. This data was then averaged and the results are reported in table 6.10 and 6.11.

The samples were obtained directly from EMPA stripes [ref. 6.9] as used for IEC 60456 tests [ref. 6.10]. The size of each sample was 75mmx75mm and a total of 5 samples were tested i.e. 1 white (no soil) reference sample and 4 soiled samples in accordance with table 6.6. The samples were then mounted on the base or platform of the Conoscan 2000 and a series of preliminary test runs were realised. The scope of these runs was to establish the best instrument settings, e.g. scanning rate etc before proceeding with the measurements.

The greatest difficulty encountered in the measurements was with the wet samples since the retained water tended to reflect the LASER source (see table 6.8). However, after several attempts the resulting procedure was to wet the samples so as to avoid excessive soaking and then tests were carried out immediately to avoid evaporation of the retained water.

Another important factor was the scanning rate, which was eventually set at 5000 points per mm hence each scan took approximately 7–8 seconds to complete. All measurements were taken along the weave pattern line (vertically or horizontally) but measurements were also taken diagonally without observing significant measurement variation.

All measurements were realised at an ambient temperature between 20–25°C and relative humidity 60–70%. Once the measurements had been recorded and concluded the experimental data was processed with the VIEWER program provided with the CONOSCAN 2000.

This software provided all the necessary information to assess roughness, including standard engineering units (e.g. Ra). A series of filtering techniques were also available and used where the profile was most erratic.
The final settings for the OPTIMET Conoscan 2000 are reported in the table below:

Table 6.7 – Instrument Settings and Measurement Conditions

<table>
<thead>
<tr>
<th>Instrument Condition–Parameter Setting</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMPA sample size</td>
<td>75mm x 75mm</td>
</tr>
<tr>
<td>Scan length</td>
<td>5mm</td>
</tr>
<tr>
<td>Scanning points</td>
<td>5000</td>
</tr>
<tr>
<td>Frequency (points or acquisitions per second)</td>
<td>700Hz</td>
</tr>
<tr>
<td>Absolute accuracy</td>
<td>2microns</td>
</tr>
<tr>
<td>Repeatability</td>
<td>0.1microns</td>
</tr>
<tr>
<td>Minimum working range</td>
<td>0.5mm</td>
</tr>
<tr>
<td>Linearity of working range</td>
<td>0.5%</td>
</tr>
<tr>
<td>LASER spot size</td>
<td>8microns</td>
</tr>
<tr>
<td>Resolution</td>
<td>5microns</td>
</tr>
<tr>
<td>LASER power setting</td>
<td>17</td>
</tr>
<tr>
<td>Lens size</td>
<td>16mm</td>
</tr>
</tbody>
</table>

To guarantee data reliability the VIEWER statistical functions were extensively used, especially the ratio of “good to bad points” and the signal–to–noise (SNR) ratio. In normal conditions the instrument manufacturer advises a SNR value of 30% or higher, the outcome of the analysis is shown below:

Table 6.8 – SNR values for EMPA samples

<table>
<thead>
<tr>
<th>EMPA Sample</th>
<th>SNR % (Dry)</th>
<th>SNR % (Wet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White reference (no soil)</td>
<td>48</td>
<td>37</td>
</tr>
<tr>
<td>Chocolate (and milk)</td>
<td>44</td>
<td>32</td>
</tr>
<tr>
<td>Carbon black (and mineral oil)</td>
<td>58</td>
<td>47</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>47</td>
<td>34</td>
</tr>
<tr>
<td>Red Wine</td>
<td>37</td>
<td>38</td>
</tr>
</tbody>
</table>
Another useful parameter to judge the validity of experimental data is the number of “good” and “bad” points i.e. the number of acquisitions the instrument classifies are valid. The ratio of bad-to-total points for each sample is shown below:

<table>
<thead>
<tr>
<th>EMPA Sample</th>
<th>Bad–to–Total points % (Dry)</th>
<th>Bad–to–Total points % (Wet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White reference (no soil)</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>Chocolate (and milk)</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td>Carbon black (and mineral oil)</td>
<td>&lt;1</td>
<td>13</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>14</td>
<td>~60</td>
</tr>
<tr>
<td>Red Wine</td>
<td>27</td>
<td>32</td>
</tr>
</tbody>
</table>

This latter table shows that the wet samples are effectively the most difficult to analyse although it should be realised that even in the worst case (wet pigs blood) there are over 2000 points that are considered “good” e.g. over 5mm there is one good acquisition every 0.0025mm.

Moreover, all of our dry measurements were below 30% even before filtering was attempted.

To facilitate the understanding the experimental data tables 6.10 and 6.11 are expressed directly in mm. Examples of profiles are also given in the (next) test results section.

**Test Results**

The test results have been summarised under two headings, these being:

1. **DRY**, these measurements were taken when the sample was dry i.e. as the EMPA samples are supplied

2. **WET**, these measurements were taken by wetting the EMPA samples but without soaking them excessively.
For each measurement the average value peak-to-peak was recorded as shown below. This measurement is a rough measurement based on the profile of the sample.

Table 6.10 – Peak-to-peak measurements in mm

<table>
<thead>
<tr>
<th>EMPA sample</th>
<th>DRY</th>
<th>WET</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>0.9–1.2</td>
<td>0.7</td>
<td>Irregular when wet</td>
</tr>
<tr>
<td>Chocolate</td>
<td>0.7–0.8</td>
<td>1.1</td>
<td>More aperture (weave spread) when wet</td>
</tr>
<tr>
<td>Carbon Black</td>
<td>1.2</td>
<td>1.1</td>
<td>More aperture (weave spread) when dry</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>1.2</td>
<td>1.1</td>
<td>More aperture (weave spread) when wet</td>
</tr>
<tr>
<td>Red Wine</td>
<td>1.2</td>
<td>1.1</td>
<td>More aperture (weave spread) when wet</td>
</tr>
</tbody>
</table>

A more accurate result is obtained if the VIEWER SW is used automatically to calculate the average Z measurement. This measurement consists of calculating the average value (height) along the Z-axis starting from the average value of all the data. That is to say it is approximately half the peak-to-peak value: this data is given below:

Table 6.11 – Average Z measurements in mm

<table>
<thead>
<tr>
<th>EMPA sample</th>
<th>DRY</th>
<th>WET</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>0.535</td>
<td>0.581</td>
<td>8</td>
</tr>
<tr>
<td>Chocolate</td>
<td>0.526</td>
<td>0.603</td>
<td>13</td>
</tr>
<tr>
<td>Carbon Black</td>
<td>0.546</td>
<td>0.501</td>
<td>8</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>0.451</td>
<td>0.495</td>
<td>9</td>
</tr>
<tr>
<td>Red Wine</td>
<td>0.474</td>
<td>0.596</td>
<td>21</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.506</strong></td>
<td><strong>0.555</strong></td>
<td><strong>12</strong></td>
</tr>
</tbody>
</table>
One can see that the overall apparent roughness of the samples lies between 1.012 and 1.110mm. However, this apparent roughness is not really true for the water tunnel experiments because the water pressure of the tunnel tends to squash the sample fibres. This very important conclusion was reached after analysing the water tunnel samples using the SEM. The SEM analysis showed that the samples were compressed to such an extent that the true roughness was considerably less than shown in tables 6.10 and 6.11. In order to estimate the true roughness the SEM pictures were examined and an estimation of the amount of compression determined. This was done by dimensionally comparing the squashed (real) to un-squashed (ideal) fibres i.e. the apparent fibre diameter to the nominal fibre diameter and using $H$ to estimate the true roughness, this is depicted below:

![Diagram of fibre diameters comparison](image)

**Figure 6.6 – Ideal and Real fibre diameters comparison**

We assumed that the roughness measurements refer to the nominal diameter of the fibre hence once $H$ was determined the true water tunnel experimented EMPA roughness could be extrapolated.

The magnitude of $H$ was established by assuming that the fibre cross-sectional area remained constant and the new shape was elliptical in shape. By simple geometry it was possible to relate the magnitude of $H$ to the nominal fibre diameter and the major axis value ($a$) as follows:

$$H = D \left(1 - \frac{D}{2a}\right)$$  \hspace{1cm} \text{Equation 6.1}

It was found that the compression effect of the water tunnel pressure amounted to about 10–20% of reduced weave height although this percentage contains some relaxation due to the
drying of the EMPA samples prior to the SEM analysis. We concluded that during water tunnel experimentation the EMPA sample roughness was below 1mm and most probably between 0.7 and 0.8mm i.e. the diameter of the Preston tube. Two examples of EMPA sample profiles are given below.

**Examples of Typical EMPA Roughness Profiles (not to scale)**

![Wet EMPA sample profile](image1)

![Dry EMPA sample profile](image2)

Figure 6.7 – Carbon Black (wet and dry)

![Wet EMPA sample profile](image3)

![Dry EMPA sample profile](image4)

Figure 6.8 – White Reference sample

**Alternative Method**

Apart from carrying out the same measurements using a contact method it is also possible to measure roughness by encapsulating the samples in a plastic transparent resin as used in metallurgy and measuring the samples under a microscope. The measurement can be done in loco i.e. under the microscope or by analysing a scanned image of the sample with suitable SW.
This ultimate method requires that the image of the sample be imported to an image analysis SW package such as Sigma Scan [ref. 6.11] using a simple flat bed scanner or by taking a still picture in digital format.

Once the image has been captured the SW can analyse the profile of the weave and statistical data obtained. This data can include cross-sectional area, peak-to-peak etc.

The foremost criticism towards this method is that the samples are effectively suspended in a resin, which is then heated, compressed and cooled. This excludes any possibility of preparing and analysing wet samples and also means that the suspension can be tricky and lead to twisting.

The validity of end result is therefore very debatable although it does mean that samples can be stored indefinitely. As a simple matter of comparison between non-contact and the resin methods the peak-to-peak measurement for white reference sample was found to be 0.47–0.58mm versus 0.9–1.2mm given in table 5 i.e. approximately least 50% less.

Clearly the harder the surface the less likely the method will damage or affect the sample surface, thus the method is ideal for solid samples (e.g. metallic samples).

An example of various images obtained with the encapsulating method is shown below:

Figure 6.9 – Alternative analysis of scanned weave profiles

Starting from the left the EMPA samples are encapsulated in small resin blocks, which are then scanned on a flat-bed scanner. The scanned images can then be compared to a reference object.
(in the figure above it is solid circular rod) with known dimensions and the final EMPA sample roughness calculated directly from the cross-section of the weave.

**SEM Investigation – Soil Size, Shape and Location**

The scope of the SEM investigation [ref. 6.12] was to understand the location of the soil and determine both size and shape. The investigation is also complementary both to the roughness measurements and step 3.

The SEM investigation used the EMPA samples as the soil sources hence all the standard specifications for these samples are valid. A summary is provided below

**EMPA sample Specifications**

**Base fabric characteristics**

Reflectance : $90 \pm 1\% \ (89.2 \ measured \ for \ EMPA \ 127)$

Pre-treatment : Bleached but without optical brightner

Fluidity index : 4 to 5 poises

Weave details :

- Warp $34 \pm 2$ double thread of 30tex/cm
- Weft $20 \pm 2$ threads of 50tex/cm
- Mass per unit area – $200 \pm 10 \ g/m^2$

EMPA Batch Numbers : EMPA 127 art. 105

**Carbon black characteristics**

Average grain size : 0.0295 microns

Carbon content : 96%

Average surface area : $94m^2/g$

Mineral oil : Ondina oil 33 (Shell)

Reflectance at 460nm : 29.8 (mean) stdev=0.2
Pigs Blood characteristics

Pigs blood and 10g/l ammonium citrate added

Chocolate and Milk characteristics

Reflectance at 460nm: 25.0 (mean) stdev=0.1

Red Wine characteristics

Alicante red wine

Reflectance at 460nm: 41.2 (mean) stdev=0.2

EMPA Measuring conditions

Instrument: Spectraflash 500
Illuminant: D65/10°
Wavelength range: 420 to 750nm
UV filter: at 420nm
Measuring diameter: 28mm
Gloss: excluded

SEM photography was taken at 4 magnifications, 32X, 480X, 1000X and 4000X for all four soils (carbon black, pigs blood, chocolate–milk and wine). The first magnification showed the weave surface status, while the remaining magnifications were used to quantify the soil size, shape and location. It was found that before washing, three of the four soils (carbon black, pigs blood and wine) were substantially spherical, chip-like or elliptical in shape while chocolate–milk was strand/foam-like and tended to impregnate the weave. This ultimate soil is therefore widespread across the fibres while the other 3 soils are sporadic and the soil is distributed in a stand-alone manner. Soil location was either on the fibre, in between the fibres and/or in the weave pores. The attachment of the soils was generally extensive and was typically along the equivalent length of one diameter of the soil particle. Thus pure point contact (e.g. sphere on
curved surface) was very rare and the soils showed a tendency to develop along one or two axes. Thus adhesion is more rooted and spread in and across the surface of the fibre.

Soil particle size varied but was generally between several microns to tens of microns. The chocolate soil was the most difficult to classify because of the impregnation effects but was similar to the size of the fibre.

The white reference sample provided a white cotton fibre approximately 10–20 microns in diameter with thread size of 0.4–0.5 mm. Pore size was typically 1 to 5 microns at the maximum diagonal. It was therefore concluded that all soils sit well inside the boundary layer and within the laminar sub-layer [ref. 6.26]. Furthermore the size of the soil was at least between 1/10 to 1/30th of the size of the EMPA sample. Using the SEM photographs the best classification of average soil size results in the following table:

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Average Soil Size</th>
<th>Soil shape</th>
<th>Soil Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon black (and Mineral oil)</td>
<td>5–20</td>
<td>Cluster of spheres</td>
<td>On fibre and sometimes lodged between fibres</td>
</tr>
<tr>
<td>Pigs Blood</td>
<td>5–20</td>
<td>Cluster of spheres or elliptical</td>
<td>On fibre and sometimes lodged between fibres</td>
</tr>
<tr>
<td>Chocolate (and Milk)</td>
<td>20–50 or higher</td>
<td>Foamy or strand-like</td>
<td>Across fibres and pores.</td>
</tr>
<tr>
<td>Wine</td>
<td>1–5</td>
<td>Speckle or small sphere</td>
<td>On fibre, generally to small to bridge fibre gap</td>
</tr>
</tbody>
</table>

Table 6.12 – EMPA Soil Characterisation based on SEM investigation (before washing i.e. new soils)
On washed samples of EMPA soiled cloth it was found that the soil size and/or shape tended to change i.e. the change was visible. However, it was impossible to determine the extent of the change because this could not be done on exactly the same unwashed soil particles.

On a closing note it is worth noting data suggested by Cutler [ref. 6.13] for some general soil data found on garments:

Table 6.13 – General Soil Data suggested by Cutler

<table>
<thead>
<tr>
<th>Soil Category</th>
<th>Soil Type</th>
<th>Average Soil range or size in microns</th>
<th>Soil shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Siliceous</td>
<td>Sand</td>
<td>50-2000</td>
<td>Particle</td>
</tr>
<tr>
<td></td>
<td>Silt</td>
<td>2-50</td>
<td>Particle</td>
</tr>
<tr>
<td></td>
<td>Clay</td>
<td>0.02-2</td>
<td>Particle</td>
</tr>
<tr>
<td>Skin</td>
<td>Epithelial</td>
<td>25-30</td>
<td>Leaf</td>
</tr>
<tr>
<td>Sweat</td>
<td>15-50</td>
<td></td>
<td>Leaf</td>
</tr>
<tr>
<td>Sebum</td>
<td>N.A.</td>
<td></td>
<td>Blanket</td>
</tr>
</tbody>
</table>
It is also interesting to note that the average human body weight loss of skin soil is approximately 3 to 15 grams per day [ref. 6.14-6.15].

**Soil Removal Investigation in the Washing Machine**

The standard washing machine performance tests based on IEC 60456 are aimed at assessing soil removal efficiency so as to be able to classify and compare overall washing machine performance in standard conditions. Thus they are not intended to assess single factors such as detergency, warping effects etc. The scope of the soil removal investigation was to assess single factors or parameters. Six parameters were chosen on the assumed importance in the washing process, these being:

Detergency, Textile Abrasion, Water Temperature, Textile Bending (e.g. Twisting, Churning, Warping), tangential fluid flow shear stress and soaking at Very Low Flow Speeds.

The investigation therefore set-off to:

- Quantify the importance of known machine parameters including: Detergency, Textile Abrasion, Water Temperature, Textile Bending, Twisting, Churning, Warping.
- Compare tangential fluid flow shear stress and soaking at Very Low Flow Speeds with the previous four parameters.

**Designing the Experiments**

The experimental work was based on two techniques:

1. **OFAT** - One Factor At a Time

2. **DOE** - Design of Experiments and in particular the full factorial or $2^k$ model [refs. 6.16 and 6.17] of DOE.

The six parameters and two levels were considered:
Table 6.14 – DOE Factors

<table>
<thead>
<tr>
<th>Parameter or Factor</th>
<th>Low level</th>
<th>High level</th>
<th>Where</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detergency</td>
<td>Without</td>
<td>With</td>
<td>W. Machine</td>
</tr>
<tr>
<td>Textile abrasion</td>
<td>NO</td>
<td>YES</td>
<td>W. Machine</td>
</tr>
<tr>
<td>Water temperature</td>
<td>15°C</td>
<td>60°C</td>
<td>W. Machine</td>
</tr>
<tr>
<td>Textile bending etc</td>
<td>NO</td>
<td>YES</td>
<td>W. Machine</td>
</tr>
<tr>
<td>Shear stress</td>
<td>Low Re</td>
<td>High Re</td>
<td>Tunnel</td>
</tr>
<tr>
<td>Soaking</td>
<td>Low T°C or</td>
<td>High T°C or Long T°C or Long Time</td>
<td>Tub</td>
</tr>
</tbody>
</table>

The major difficulty in assessing these parameters is that there were 3 different prototypes involved: washing machine, water tunnel and tub. This implies that a maximum of (the first) four parameters could be assessed simultaneously, this was done using MINITAB SW [refs. 6.16, 17 and 6.25]. The remaining two factors were therefore assessed singularly and then compared against the OFAT parameters. This entailed a specific experiments (OFAT) matrix that will be explained first and then the DOE results with the first four parameters will be examined.

**Comparison of Washing Machine, Tunnel and Tub Test Results**

One of the advantages of comparing all six parameters is that more than two conditions (levels) can be investigated albeit needing more experimentation. For example, to consolidate the study of the transient effects of pure shear the water tunnel measurements have been conducted from 1 to 150mins, even though only the first 20minutes were found to be relevant. Similarly several water temperatures were tested in the washing machine tests.

All experimentation, including the 4 parameter DOE, involved several simplifications and/or standardised procedures, these being:
✔ Detergency: Only standard washing machine performance test detergent was used and this was in accordance with IEC 60456.

✔ Abrasion: Two degrees of abrasion were evaluated. In the first instance the abrasion effects were simultaneously contemplated between textile cloths and textile cloths against rigid surfaces (e.g. wash load against drum or door). In the DOE the abrasion was avoided by inserting small studs in the sphere.

✔ Temperature: The water temperature was between 15 to 60°C, depending on the test condition.

✔ There was no discrimination between bending, twisting, warping or churning i.e. they were all classified as one, single, effect.

✔ Pure fluid shear stress: This was essentially what happened in water tunnel and therefore referred to high speed, parallel water flow.

✔ Pure fluid soak: This referred to soaking or steeping of the textile with water flow at very low flow speeds (<0.015ms⁻¹).

✔ Only one standard Whirlpool washing machine was used for the FMT and sphere tests.

The overall matrix of essential experimentation is shown below:

Table 6.15 – OFAT Experiments Matrix

<table>
<thead>
<tr>
<th></th>
<th>S and FMT (15°C)</th>
<th>Sphere (15°C)</th>
<th>Soak (60°C)</th>
<th>S&amp;T (15°C)</th>
<th>Tunnel (15°C)</th>
<th>Soak (15°C vs. time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detergent</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abrasion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Temperature</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warping</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Pure Shear</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Pure Soak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
In addition to the above tests several experiments were made more robust and strengthened by cross relating the experimental results. For example, in order to confirm the temperature effects the soak tests were compared at 60 and 15°C.

To consolidate the soil removal measurements two optical methods were used and a third digital image technique experimented. The former methods referred to the standard photocolorimetric reflectance measurements in use in the washing and textile industries with particular reference to IEC 60456. A third method, involving a standard flat-bed scanner, is also feasible but is not discussed further [ref. 6.18]. This technique is based on a high-resolution flat-bed scanner and also provides measurements of hue, intensity and luminance.

The preparation and employment of the 4 standard soiled samples was based entirely on the IEC 60456 standard which states that the scope of each of the 4 soils is as follows:

- Carbon black and mineral oil (abbr. to Carbon Black or CB) is to assess the mechanical and thermal action towards scouring
- Pigs blood (abbr. PB) to assess the removal of protein pigments
- Chocolate and milk (abbr. Chocolate or Ch) to assess the removal of organic pigments.
- Red wine (abbr. Wine or W) to assess bleaching effects

Test Apparatus and Method

Three test set-ups have been used, these being:

Set-up 1 – Washing machine set-up

This is where the Free Moving Textile (FMT) and Sphere (S) samples (see next figure) were simultaneously subjected to a standard washing efficiency cycle at different temperatures. Two washing machines were used: for low temperature testing (+15°C) a Bauknecht type WA 2587 was used while all other tests involved a type WA 3774S Excellence machine. Both machines
are in fact identical except for a minor SW change, e.g. to allow lower washing temperature settings for WA2587.

![Bauknecht brand washing machine]

Figure 6.11 – Washing m/c Set-up

The free moving textile (FMT) samples were simply the original EMPA samples, 4 soiled and 1 white (clean) stripes attached (clipped) to towels that make-up the remaining part of the wash load as prescribed by standard IEC 60456. Hence these tests were identical to the official tests except for varying test temperatures. The sphere (S) tests were conducted in the same wash loads as the FMT samples.

The scope of the sphere samples was to evaluate the difference between free moving and rigidly fixed textile samples due to warping.

The (polystyrene) spheres were fitted with small (50% surface area of the above FMT stripes) triangular shaped pieces of soiled textile cut directly from the same EMPA batch of stripes as used for the FMT samples. The pieces of soiled textile were fixed to the surface of polystyrene spheres by simply inserting them into pre-cut grooves on the surface of spheres. Hence no adhesive was used.

The surface of the sphere was divided into two halves with four pieces on the upper half and four on the bottom. Hence a total of 8 pieces of soiled textile were tested for each sphere, with
at least two spheres for each test. No white (clean) textile was fixed to the surface of the sphere, as this was part of the FMT stripes tested simultaneously with the spheres.

The total dry weight of the sphere with the 8 soiled samples was 10 grams and the only weight increase during washing was that absorbed by the soiled samples.

Indeed the polystyrene density was chosen so as to minimise the water absorption and in fact was so small that it can be considered negligible with respect to the total wet sphere weight. The position of the test pieces for the sphere are shown below:

Figure 6.12 – Position of soiled samples on sphere surface

Set-up 2 – Water Tunnel (T) Tests Set-up:

As discussed in step 3 (chapter 5c).

Set-up 3 – Very Low Speed or Soak Test Set-up

This experimental set-up was simply a small reservoir (tub) in which the test samples were immersed i.e. soaked or steeped, at various temperatures for different exposure times without detergent. The water, which is heated to a precise (±1°C) fixed temperature, flows through and out of the reservoir in a controlled manner with the test samples at rest. The average flow speed was set to 0.01–0.015 m s⁻¹ and theoretically represents the flow speed within the fabric during washing [ref. 6.19]. The scope of this test was to measure the effects of very low speeds in combination with thermal effects (16, 30, 40, 50 and 60°C).
To investigate the importance of warping the same test rig was used and a special test, termed "Soak and Twist" (S&T), was developed. In this test set-up textile samples were twisted (back and forth) while immersed in the tank. The test set-up is shown below:

![Diagram of test set-up](image)

**Figure 6.13 - Very Low Flow Speed Tests**

**Sphere Size Definition**

The size of the sphere was determined through specific experimentation and was aimed essentially at ensuring that the size was representative of real wash load conditions in a horizontal axis machine.

Four sizes of sphere were used in a fake (no detergent) standard washing test (at 15°C, 40°C and 60°C) using the same density of polystyrene ($\rho = 25\text{kg/m}^3$) for each sphere. The spheres were inspected for surface damage after each cycle both with and without EMPA samples. In both circumstances the ideal sphere diameter was found to be between 80 to 120mm although there was a tendency for more surface damage for the larger diameter as water temperature increased.

The spheres that were tested with real EMPA samples inserted in the surface of the sphere provided similar washing efficiency performance to those of FMT samples (see appendix 3),
especially as the temperature increased. Tests with these covered spheres have confirmed that the ideal diameter was between 80–120mm.

Another discriminating factor for sphere size was the increase in weight during the washing cycle. Although this was dependent on the density and homogeneity of the polystyrene the best results were obtained with 90–100mm diameter spheres and $\rho = 25\,\text{kg/m}^3$.

Interestingly the 90–120mm sphere diameter range also provided the most suitable surface area to carry out the colorimeter measurements. Summarising, the final selected sphere diameter was chosen to be 90mm and this was based on the following test results:

- Least surface damage of the polystyrene sphere
- Least weight increase of the sphere
- Nearest washing efficiency results to standard FMT results
- Minimum and most suitable surface area for colorimeter measurements
## Overall Test Data Summary

Table 6.16 – Average Soil Removal Efficiency Measurements based on EMPA stripes

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>FMT</th>
<th>Sphere</th>
<th>Tunnel</th>
<th>Soak</th>
<th>S &amp; T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wh</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>NO</td>
<td>15 to 20°C</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>CB</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15 to 20°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
</tr>
<tr>
<td>PB</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15 to 20°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
</tr>
<tr>
<td>Ch</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15 to 20°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
</tr>
<tr>
<td>W</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15, 30,40,50 &amp; 60°C</td>
<td>15 to 20°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
<td>15, 25,40,50 &amp; 60°C</td>
</tr>
<tr>
<td>Transient</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Detergent</td>
<td>With and Without</td>
<td>With and Without</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Total number of Measurements</td>
<td>500</td>
<td>400</td>
<td>450</td>
<td>320</td>
<td>60</td>
</tr>
<tr>
<td>Total number of Samples</td>
<td>100</td>
<td>80</td>
<td>90</td>
<td>64</td>
<td>12</td>
</tr>
<tr>
<td>Total No. of Tests</td>
<td>10</td>
<td>10</td>
<td>90</td>
<td>64</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes

The optical test results were obtained with two photolorimeters defined as: 1. “Old” - refers to a Zeiss colorimeter (see tab.5.6c), 2. “New” refers to a Datacolor SF600 Plus CT colorimeter (see tabs.5.7-8c). Wh – White; CB – Carbon black (and mineral oil); PB – Pigs Blood; Ch – Chocolate (and milk); W – Wine. All measurements refer to average value (based on at least 6 readings) taken from minimum area of 1500mm² of textile, i.e. at least twice the minimum nominal area for the correct use of both colorimeters.
**Water Temperature Effects**

In accordance with table 6.15 the reference test condition was the soak tests at +15°C without detergent. Hence in this condition all of the other five factors (detergency, abrasion etc.) can be excluded (see Fig. 6.13). The assessment of temperature effects can be made by comparing two different soak temperatures e.g. 15 and 60°C for a fixed test duration.

The table below shows test results after 20min$^2$ of soaking.

<table>
<thead>
<tr>
<th>Soak T°C</th>
<th>Wh</th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°C</td>
<td>88.6</td>
<td>29.3</td>
<td>40.6</td>
<td>29.3</td>
<td>58.5</td>
<td>157.7</td>
</tr>
<tr>
<td>60°C</td>
<td>89.7</td>
<td>31.9</td>
<td>40.1</td>
<td>22.2</td>
<td>62.6</td>
<td>156.8</td>
</tr>
<tr>
<td>% Change (15 vs 60°C)</td>
<td>1.2</td>
<td>+8.9</td>
<td>-1.2</td>
<td>-24.2</td>
<td>+7.0</td>
<td>-0.6</td>
</tr>
<tr>
<td>Unwashed</td>
<td>92.2</td>
<td>31.3</td>
<td>41.8</td>
<td>20.4</td>
<td>51.3</td>
<td>146.6</td>
</tr>
<tr>
<td>% change (60°C vs. unwashed)</td>
<td>-2.8</td>
<td>+1.9</td>
<td>-1.7</td>
<td>+8.8</td>
<td>+22.0</td>
<td>+6.9</td>
</tr>
</tbody>
</table>

This table had some conflicting yet interesting results. For example, the overall difference in soil removal is virtually nil (-0.6%) and it would therefore seem logical to conclude that temperature does not play any role at all, except possibly between 25-40°C. To illustrate this point the overall soil removal results for soak tests from 15 to 60°C are shown below:

![Water Temperature Effects on Overall Soil Removal](image)

Figure 6.14 – Water Temperature Effects (Soak tests)

---

2 This time was based on results from the soak tests.
However, a closer look at figure 6.15 reveals that specific soils undergo change and this accounts for the conflicting results in the above table.

For example only the chocolate soil remains substantially the same while carbon black and wine improve (9% and 7% respectively) and pigs blood actually decreases by 24%. Even if one takes into account experimental error etc. this latter result seems nonsensical. We will reconsider this result further in this section but the conclusion for the time being is that temperature does effect at least 3 of the 4 tested soils.

A more logical result is obtained when the 60°C test results are compared to the unwashed samples. Here the overall difference increases by 6.9% thus indicating an improvement in performance as temperature increases. Furthermore the pigs blood soil has improved thus reversing the previous statement (22.2% versus 20.4%). In fact it is actually better at 15°C than at 60°C. This could indicate that the temperature effect is combined with a soaking effect i.e. the soil shape, size, location and spread change to such an extent that the soil is “different”.

This introduces the doubt that the soil changes physically i.e. rheologically [ref. 6.20] and that time will probably be a factor. This will be dealt with in the next section dedicated to soak effects.

Equally interesting is the change in soil removal versus temperature using the same soak methodology, the results are shown in the next graph.
Figure 6.15 – Water Temperature Effects for single soils

**Remarks**

From this graph it can be seen that while wine soil removal improves pigs blood soil tends to worsen. The other two soils seem unresponsive and only show signs of conditioning above 40°C.

**Soak Effects**

The consumer knows the usefulness of soaking or steeping clothes although it involves time. The perception that the soil is “softened and loosened” by soaking implies some modification in the mechanical properties of the soil. Hence exposing the soiled samples with no detergent was considered a logical approach to verify this thought. This was done in the soak tests where the soiled samples were soaked for 1-150mins at 15°C (see next figure).
This graph illustrates that 20 minutes is a sufficient exposure time except possibly for pigs blood which stabilises after 30–35 minutes. Thus the test time (duration=20mins) for the water temperature tests is validated with this graph.

**Remarks for Soak Temperature and Time**

The fact that the overall soil removal performance seems only marginally affected by the water temperature does not condemn the use of heat to remove soil rather one should conclude that temperature alone is not enough. Moreover, heat will facilitate fibre swelling [ref. 6.21] and boundary layer thinning [ref. 6.22] and both aid abrasion effects.

The soil will also swell (thus changing in size) and, rheologically speaking, the soil will tend to become either plastic or viscoelastic. This could explain why other mechanisms, such as abrasion when combined with temperature create a cross–mechanism dependency that enhance the likelihood of soil removal.

---

3 Although soaking overnight was not verified i.e. for an extended period of time.
**Abrasion Effects**

Abrasion is defined (here) as the physical mating of surfaces in relative motion with an intermediate fluid (water). This could be classified as a case of hydrodynamic lubrication [ref. 6.23] or permanent deformation of the soil. The idea of using a sphere and mounting test samples firmly on its surface was developed to observe the effects of keeping at least one mating textile surface rigid during washing so that abrasion caused by the sliding or wiping of the compliant textile surfaces was disrupted i.e. was less likely.

In accordance with table 6.1 the reference test condition was taken to be the sphere test at 15°C without detergency.

In this condition we can exclude the effects of detergency, pure fluid shear stress, soaking and warping (bending, twisting, churning etc.) and also temperature, providing the tests are conducted at a low temperature e.g. 15°C.

Henceforth in order to provide a baseline the sphere test was conducted at a low temperature e.g. 15°C and can be compared to the water tunnel tests, as shown below:

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (15°C)</td>
<td>38.4</td>
<td>46.5</td>
<td>28.1</td>
<td>60</td>
<td>173.0</td>
</tr>
<tr>
<td>Tunnel (15°C)</td>
<td>30.3</td>
<td>39.4</td>
<td>29.3</td>
<td>58.2</td>
<td>157.2</td>
</tr>
<tr>
<td>% Change (S vs. T)</td>
<td>+26.6</td>
<td>+18</td>
<td>-4.1</td>
<td>+3.1</td>
<td>+8.9</td>
</tr>
<tr>
<td>Unwashed</td>
<td>31.3</td>
<td>41.8</td>
<td>20.4</td>
<td>51.3</td>
<td>146.6</td>
</tr>
<tr>
<td>% Change</td>
<td>+22.7</td>
<td>+11.2</td>
<td>+37.7</td>
<td>+17</td>
<td>+15.3</td>
</tr>
</tbody>
</table>

If we compare tunnel to sphere results with no detergent (at 15°C) we obtain an increase in overall cleaning performance equal to approximately 9%. To ensure that this result is due to abrasion only, we need to subtract the soak effect which was seen previously to be about 6.9%
(157.7 versus 146.6), i.e. very similar to the shear stress effects. Hence, in these conditions, it was concluded that abrasion accounts for about 2%.

Interestingly the water tunnel produced a similar result and abrasion seems to have little effect on wine soil and actually worsens for pigs blood.

Clearly the test samples that were firmly attached to the surface of the sphere did not bend or twist during washing, hence the contact was between a compliant surface (free moving textile or FMT samples) and sphere surface samples, that are considered rigid and non-compliant in this discussion. This indicates that the worsening effect for pigs blood, as previously hypothesised, could be caused by a spreading or flattening of the soil, which implies warping can be excluded.

**Warping Effects (incl. Bending–Twisting–Churning)**

Warping is essentially classified here as textile cloth motion and therefore attributed to the FMT tests. Hence the FMT tests with no detergent can be compared to the other extreme of this condition that is represented by the sphere and water tunnel tests. All three conditions are summarised below:

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMT (15°C)</td>
<td>37.8</td>
<td>30.9</td>
<td>47.8</td>
<td>62.4</td>
<td>178.8</td>
</tr>
<tr>
<td>S (15°C)</td>
<td>38.4</td>
<td>28.1</td>
<td>46.5</td>
<td>60</td>
<td>173.0</td>
</tr>
<tr>
<td>Tunnel (15°C)</td>
<td>30.3</td>
<td>29.3</td>
<td>39.4</td>
<td>58.2</td>
<td>157.2</td>
</tr>
<tr>
<td>% Change (FMT vs. S)</td>
<td>–1.6</td>
<td>9.1</td>
<td>2.7</td>
<td>3.8</td>
<td>3.2</td>
</tr>
<tr>
<td>% Change (FMT vs. T)</td>
<td>19.8</td>
<td>5.2</td>
<td>17.6</td>
<td>6.7</td>
<td>12.1</td>
</tr>
<tr>
<td>% Change (S vs. T)</td>
<td>+21.1</td>
<td>–4.3</td>
<td>15.3</td>
<td>3</td>
<td>9.1</td>
</tr>
<tr>
<td>Unwashed</td>
<td>31.3</td>
<td>41.8</td>
<td>20.4</td>
<td>51.3</td>
<td>146.6</td>
</tr>
<tr>
<td>% Change</td>
<td>+22.7</td>
<td>+11.2</td>
<td>+37.7</td>
<td>+17</td>
<td>+15.3</td>
</tr>
</tbody>
</table>
In this table the comparison between FMT and S shows that in the absence of warping for the sphere there is a difference of only 3% (at 15°C). It is reasonable to attribute this to the abrasion effects.

When the FMT results are compared to the tunnel tests (where there is clearly no abrasion and warping) the difference rises to 12%. Hence we conclude that warping (at 15°C) accounts for approximately 9% of the overall cleaning. This occurs especially for the carbon black (as anticipated in the IEC60456 standard) and pigs blood soils.

To confirm this conclusion we may examine the difference between the sphere and tunnel tests (after the shear stress effects have been accounted for – see table 6.7) where only abrasion is substantially the difference, again we see a value of approximately 2–3%. This result summons the need for cross comparison. For example what happens to warping when water temperature increases? This can be examined by comparing the sphere and soak tests at 60°C.

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (60°C)</td>
<td>38.6</td>
<td>26.3</td>
<td>44.8</td>
<td>61.3</td>
<td>171</td>
</tr>
<tr>
<td>Soak (60°C)</td>
<td>31.9</td>
<td>20.4</td>
<td>40.1</td>
<td>62.6</td>
<td>155</td>
</tr>
<tr>
<td>% Change (S vs. Soak)</td>
<td>17.4</td>
<td>22.4</td>
<td>10.5</td>
<td>–2.1</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Hence the increase of temperature in the absence of warping seems to increase the likelihood of soil removal for CB, PB and Ch, while wine remains substantially the same. The difference is high (approximately 9%) but this could be correlated also to fibre and soil swelling effects as well as a change in soil rheological properties.

In fact in the previous conclusions regarding the effects of water temperature the difference between unwashed and soaked samples at 60°C was approximately 7%. Hence as temperature increases the warping effects change, but only marginally (2% circa).
Tangential Fluid Shear Stress Effects

As discussed in step 3 in the water tunnel tests two types of information can be revealed, i.e. the effects of shear and the effects of shear versus time. The water tunnel also excludes the presence of detergent, warping, abrasion and temperature. The only other variable or effect soaking, but this can be excluded by examining the data in table 6.21.

Table 6.21 – Tangential Fluid Shear Stress Effects

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnel (15°C)</td>
<td>30.3</td>
<td>39.4</td>
<td>29.3</td>
<td>58.2</td>
<td>157.2</td>
</tr>
<tr>
<td>S (15°C)</td>
<td>38.4</td>
<td>28.1</td>
<td>46.5</td>
<td>60</td>
<td>173.0</td>
</tr>
<tr>
<td>Soak (15°C)</td>
<td>29.3</td>
<td>40.6</td>
<td>25.2</td>
<td>58.5</td>
<td>153.5</td>
</tr>
<tr>
<td>% Change (T vs. Soak)</td>
<td>+3.3</td>
<td>-3</td>
<td>+14</td>
<td>-0.5</td>
<td>+2.3</td>
</tr>
<tr>
<td>Unwashed</td>
<td>31.3</td>
<td>41.8</td>
<td>20.4</td>
<td>51.3</td>
<td>146.6</td>
</tr>
<tr>
<td>% Change (T vs. unwashed)</td>
<td>-3.2</td>
<td>-5.7</td>
<td>+43.6</td>
<td>+13.5</td>
<td>+7.2</td>
</tr>
</tbody>
</table>

The difference between pure soaking and the shear stress is only 2.3% which is mainly caused by the removal of pigs blood soil. When the water tunnel conditions are compared to the unwashed samples the amount of soil removed rises to 7.2% hence if the soaking effect is subtracted the shear stress accounts for approximately 5% of the soil removed. This seems a reasonable conclusion until the results are examined for a range of Re as shown next.
This figure shows that soil removal is virtually unaffected by shear stress especially if one considers that as Re increases the sub-laminar layer thickness of the boundary layer decreases and the shear stress increases. There seems to be some effect on pigs blood (positive) and wine (negative) but not enough to merit further investigation (for this range of Reynolds number).

**Remarks**

The amount of soil removed exclusively by shear stress is very small even for very high Reynolds numbers (e.g.155k). In these conditions the laminar boundary layer thickness approximately 60E-6m i.e. 2–3 times larger than the dry soil. Even taking into account swelling effects the soil still sits inside the laminar sub-layer of the boundary layer.

Moreover, for all four soils the soil removal is virtually constant between Re=15k to Re=155k and very similar results have been noticed in the soak tests i.e. where the flow speeds are smaller by a factor of 15 or more.

This leads one to conclude that for Reynolds numbers up to 155k the fluid shear stress alone DOES NOT adequately remove soil. Another equally important conclusion is that the overall
cleaning performance is very similar to the soak tests where the fluid flow velocity was very low. Thus on the basis of this investigation, whether or not flow velocity is low (tens of mm/s) i.e. laminar or high (up to 2m/s) i.e. turbulent, seems irrelevant.

**Time Dependency of Soil Removal**

As there is some doubt about the change in rheological properties of the soils and consequent effects on their removal the water tunnel experiments were repeated and samples exposed for different, increasing, times. As mentioned previously no significant difference in soil removal was noticed as the Reynolds number increased from 15k to 155k, hence the time dependency test condition was set at an arbitrary value of Re=70k to Re=80k, i.e. approximately half way between 15k–155k.

The samples were exposed to test intervals from 1 to 150 minutes with the longest time referred to as the “infinite” exposure time.

Not surprisingly the time dependency of soil removal was different for all four soils, in particular, chocolate was substantially zero. However, pigs blood and red wine did have a clear distinct time dependency. Last but not least the carbon black soil had a very slight time dependency but this result could be justified (partially) by experimental error.

In all three cases the time dependency assumes a typical initial exponential signature, which is typical of first order systems, for the first 10 to 20mins and thereafter becomes practically linear. This is best observed if the rate of change of soil removal efficiency is plotted against time as shown next.
The fact that we observe a first order response [ref. 6.24] in soil removal implies two things:

1. The system (soil and fibre) has storage (or dissipative) properties. That is, both fibre and soil absorb the water.

2. The system undergoes a step response, which is logical since the water tunnel literally fills in seconds and wets the test sample instantaneously.

It therefore seems plausible that the action of the fluid on the soil is principally tied to the modification of the soil’s mechanical properties. Henceforth the idea of a change in rheological properties does not seem too far-fetched and the properties of the fluid also assume a new role and function.

Remarks

If the rheological properties do play an important role then the soil turns from being a solid to a plastic and possibly a viscoelastic material.

Thus from the temporal behaviour of pigs blood and wine shown in figure 6.18 these soils should be investigated first. Moreover the fact that carbon black and chocolate seem unaffected could demonstrate that these two soils need to be broken down on more than one front i.e. with the help of detergency, mechanical action etc.
To demonstrate this hypothesis one idea could be attempt to "film" the removal of the soil especially during the first 10 minutes of exposure to say soaking or shear stress conditions. This could be done in a specifically built test apparatus using high-speed digital image processing equipment, a high magnitude (x400 to x1000) microscope and small scaled down version of the water tunnel using heated water. This would also allow us to investigate the conditioning effects of temperature on soil rheology as well as shear stress.

**Detergent Effects**

The test results discussed here refer to the test detergent prescribed by IEC 60456. Detergent effects were evaluated by comparing test conditions where detergent can be added or removed easily. Thus water tunnel tests cannot be used for this purpose. However, the FMT and sphere tests are a good way of seeing the effects of detergent especially if the water temperature is kept low e.g. +15°C. For example for the sphere tests we observe:

![Cleaning Efficiency With and Without detergent for Sphere at 15°C](image)

*Figure 6.19 – Detergent Effects (Sphere tests)*

Figure 6.9 includes abrasion effects (approximately 2%) but direct comparison as shown in the next table leads us to conclude that detergent effects are high (approximately 28%) and this rises to 39% with respect to unwashed (FMT) soiled samples.
Table 6.22 – Detergent effects (S 15°C)

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (15°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with detergent</td>
<td>41.2</td>
<td>52</td>
<td>70.4</td>
<td>76.5</td>
<td>240.1</td>
</tr>
<tr>
<td>S (15°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without detergent</td>
<td>38.4</td>
<td>28.1</td>
<td>46.5</td>
<td>60</td>
<td>173.0</td>
</tr>
<tr>
<td>% Change</td>
<td>6.8</td>
<td>46</td>
<td>33.9</td>
<td>21.6</td>
<td>27.9</td>
</tr>
</tbody>
</table>

Similarly for the FMT tests conducted at 15°C we observe:

Table 6.23 – Detergent effects (FMT 15°C)

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>Ch</th>
<th>PB</th>
<th>W</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMT (15°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with detergent</td>
<td>45.4</td>
<td>62.3</td>
<td>83.1</td>
<td>83.1</td>
<td>273.9</td>
</tr>
<tr>
<td>FMT (15°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without detergent</td>
<td>37.8</td>
<td>47.8</td>
<td>31</td>
<td>62.4</td>
<td>178.8</td>
</tr>
<tr>
<td>% Change</td>
<td>16.7</td>
<td>23.3</td>
<td>62.7</td>
<td>24.9</td>
<td>34.7</td>
</tr>
</tbody>
</table>

In these results there is obviously the effects of warping and abrasion, nonetheless detergency boosts results by approximately 35% in FMT conditions i.e. domestic wash load conditions. Moreover, since abrasion seems to account for 2% (at 15°C) it seems logical to conclude that detergency effects for FMT conditions include a proportion of warping effects. If we subtract the FMT and S test results i.e. 27.9 and 34.7%, the difference is approximately 7%. From this result we further subtract 2% for abrasion and conclude that warping at 15°C accounts for...
approximately 5% of the soil removal. This is similar to the 9% mentioned previously for the warping results. The FMT results are plotted below:

![Figure 6.20 – Detergent Effects (FMT tests)](image)

![Figure 6.21 – Overall Test Results With and Without Detergent](image)

**Overall Cleaning Efficiency Without Detergency**
Overall Cleaning Efficiency with Detergency

![Bar chart showing overall cleaning efficiency with detergency at various temperatures.](chart.png)
**DOE and the Washing Machine**

Designed experiments provide a logical framework for comparing factors (e.g. detergency, temperature etc.) in terms of some response of interest, in this case the amount of soil removed [ref. 6.25]. DOE is based on the analysis of variance (abbrev. ANOVA) and in the case to be discussed the design was entirely randomised with two levels for each of four factors [ref. 6.17] namely; detergent, water temperature, abrasion and warping.

Of particular importance was the choice of response since this parameter is usually needed to be optimised and/or is under scrutiny. In the case of this investigation it was natural to choose the amount of soil removed i.e. % clean. Other responses could have been the degree of fibre damage, cleaning time, amount of detergent, detergent effectiveness etc.

The main objective of the DOE was to statistically "weigh" which of the four $2^k$ DOE factors were the more significant for the removal of soil. To do this required the following data or conditions:

- A continuous output: in this case the quantitative variable was provided by the optical measurements that establish how clean the EMPA samples are after treatment. This is expressed as the % clean for all four EMPA soils.
- One single prototype i.e. one washing machine in which all the tests were conducted.
- Four factors that could be assessed on two distinct levels.
- All other factors are classified as noise.
- Statistical analysis tool: in this case provided by MINITAB.
- Establish a confidence level for the analysis: In this case it was set to 99%.

These circumstances can be summarised as follows:
The noise or error (Z) accounts for any parameter that has not been considered as a factor, for example, the shear stress or soaking effects. Clearly if Z is high in magnitude the experimentation has left out an important factor (or factors) and this adds uncertainty to the analysis. Also since there were 4 factors the full factorial design ($2^4$) of experiments was 16 experiments (or runs), which may be reduced to 8 experiments for half-factorial design ($2^{4-1}$).

This is shown below and taken directly from MINITAB:

Since most of the data was already available from the previous tests it was decided to process 16 runs.
Next the levels were set for the parameters:

- **Without**
  - (+1) Detergency
  - (+1) Water temperature

- **With**
  - (-1) Detergency
  - (-1) Water temperature
  - (+15°C)
  - (+60°C)
  - NO (+1)
  - YES (-1)

- **Abrasion**
  - YES (-1)

- **Warping**
  - NO (+1)
  - YES (-1)

- **% Clean**

**Figure 6.24 – Parameter levels**

The DOE was then set-up using MINITAB and a randomised experimental design generated as follows:

**Figure 6.25 – DOE based on 4 factors and 16 runs**
Following this a series of test runs were carried out according to the order assigned by MINITAB (see column C2 in figure above) and the experimental data collected and processed.

In order to verify the effects of abrasion and warping two new test methodologies were created. In the first case the sphere tests included the insertion of small protruding pins in the surface of the spheres. In this way EMPA sample movement was impeded and abrasion with mating textiles avoided. The pin heads measured about 2mm in diameter and 100 pins were equi-spaced over the whole of each sphere surface.

The second methodology required the absence of abrasion but with warping, hence the spheres could not be used for this purpose. This goal was achieved by covering the FMT samples with a thin but very open spaced net. This allowed both adequate EMPA sample movement and flexibility while ensuring that mating textile surfaces were avoided or limited. Both methods are shown next.

![Sphere with Pins](image1)

![FMT samples with net](image2)

Figure 6.26 – a) Sphere with Pins b) FMT samples with net

The resulting statistical analysis provided the following results:
Pareto Chart of the Effects
(response is % Clean, Alpha = .05)

Figure 6.27 – Test results before Filtering

Interaction Plot - Data Means for % Clean

Figure 6.28 – Interaction between factors
Pareto Chart of the Standardized Effects
(response is % Clean, Alpha = .05)

Figure 6.29 – Interaction between factors

Figure 6.30 – Final (weighted) results of factors

Remarks

The previous four charts show that the principle factor is the detergent, which when combined with the heating of the water, practically covers 89-92% of the amount of soil removed.
Abrasion and warping account only for about 7% of the removal of soil (as for the previous OFAT tests) while 1% error implies that the four factors chosen are the main factors for the washing machine tested. Clearly the experiment is not conclusive because:

a) More responses are needed so as to provide an overall picture, for example with different detergents.

b) Tests need to be extended and confirmed by carrying out the same on more washing machines.
Chapter 7 - Round-up of Results, Findings and Next Steps
Research Round-Up and Findings

Originally the scope of the research was to quantify the height of the boundary layer since it was thought that only this information was worthy of investigation for this part of the Mass Transfer project.

However, it was soon found that the boundary layer contains much more information and from this reasoning a whole series of parameters ranging from the laminar sub-layer thickness to the shear stress were considered worthy of investigation.

Of the many parameters meriting attention the effects of hydrodynamics (especially shear) on soil removal was judged to be an excellent platform to understand the interaction between boundary layer and the soil during washing. Consequently the hydrodynamic shear stress has taken the central part of this research but other parts, such as the measurement of shear stress, flow conditions, wash load motion etc. have been equally significant.

To establish the importance of shear stress on soil removal the first obstacle has been the development of a reliable measurement technique. The chosen technique was based on pressure recordings and a particular branch of indirect shear stress measurement termed "small block probe". In consequence two types of probes have been successfully developed not only to measure shear stress magnitude but also direction in water flows. A simple method of achieving both shear stress magnitude and direction was developed from straightforward pressure measurements. Furthermore, to the best of the author's knowledge, all previous attempts involved using these probes in, more amicable, air-flows.

This achievement implied the design and development of a purpose-built water tunnel, which was also used to correlate hydrodynamic effects and soil removal from soiled cotton samples. The probes, in conjunction with a Preston tube, have also been used to examine the shear stress profiles in the water tunnel, which were in-line both with other research findings (Knight and Patel, 1985), and the CFD activity promoted by the author.
Once the tunnel was fully characterised and knowledge was gained about the working conditions of the tunnel special EMPA soiled samples were subjected to pure tangential shear stress. The scope was to understand just how much soil could be removed in ideal shear conditions. This experimentation has led to the confirmation that shear stress alone up to 7.7 Pa circa tunnel (15k ≤ Re ≤ 155k, 0.2 ≤ U ≤ 1.97 ms⁻¹ and 0.14 ≤ τ ≤ 7.74 Nm⁻²) is inadequate for soil removal. These conditions were later correlated to the washing machine (10 ≤ RPM ≤ 43) using a modified version of the Reynolds number, which is based on mixing vessel theory developed originally by Schmidt. Moreover the only soil removed occurred during the first 10 to 20 mins of exposure and had a typical exponential decay signature. Interestingly the estimated minimum flow condition to expose the soil to the turbulent layer of the boundary layer in order to increase the likelihood of shear removing soil particles of the largest diameter is in excess of Re=400k (100RPM) at ambient temperature. Unfortunately this speed is unadvised in the washing drum because the wash load would not perform its usual elliptical orbit and would stick to the drum during rotation, hence reducing mixing, abrasion and warping (churning) effects. Moreover work by Kovich also seems to offset this theory because velocities in excess of 8 ms⁻¹ have proved futile. In defence of higher flow velocities it is worth emphasising that the role of the laminar sub-layer is to ensure the soil is engulfed in virtually motionless fluid and the only way to make it thinner is to change the fluid properties, especially viscosity, for example by increasing temperature. Indeed the increase in water temperature in the washing cycle is likely also to improve matters for this very reason.

In order to compare the tunnel conditions to real washing conditions the research has also involved the modelling and characterisation of the wash load motion during washing. It has been demonstrated both mathematically and practically (through high speed filming) that the wash load performs an elliptical orbit during the initial part of the washing process. Hence detergent and wash load mixing are improved by reversing the drum rotation and thus wash load orbit.
This also helps keep the flow as chaotic as possible and expose the soil to as much turbulence, abrasion and warping as possible. It was also possible to observe that the motion of the wash load can also be 3D depending on load i.e. it tends to move between the rear of the drum and door. Even more important is that the research has revealed at least two distinct flow domains in the washing machine, namely inside and outside the wash load, as well as other domains including flow across the wash load and across the drum. The resultant flow velocities are high, even up to several meters per second have been both predicted and measured. This shows that the soil is indeed subjected to turbulent and high-speed flow conditions and (this alone) justifies the previously mentioned tunnel conditions and validates the hydrodynamics effects discussed previously.

On the basis of these findings one can also conclude that soil removal in the washing machine by water flow alone, through or across, the weave is insufficient. This leads one to declare that the principle scope of water in today's machine is to convey heat and mass, which becomes increasingly more difficult the tighter the weave and wash load gets. Hence to promote better flow conditions within the wash load the best idea seems to reduce flow resistance by, for example, swelling the weave (to increase porosity) and agitating the wash load better. Having stated this, attempts at pushing water through the wash load by air entrainment (Daewoo) has so far proved unacceptable but clearly this seems a logical step and worth further investigation. The results imply that other mechanisms such as detergency, heat transfer, abrasion, warping etc. are certainly required to remove soil. In fact in a separate investigation shear stress, soaking, abrasion, warping were investigated and these amounted to less than 15% of all soil removed thus leaving detergency and water temperature as the prime factors in domestic washing. From this result one could conclude that washing research should focus on these latter two factors but one must not forget that fibre care is also as equally important as soil removal.
The author also concludes that the rheological properties of the soil are prone to change mainly because of the combined soaking and heating effects of the water, surfactants in the detergent and motion of wash load and soil. This change means that the soil cannot necessarily be treated as being a solid for all the washing process and indeed could well be considered a plastic or viscoelastic material. This has definite implications for the adhesion phenomena (since the mechanical properties are certainly not constant during constant) and the author suggests that adhesion models should account for this.

The main outcome of the research may be summarised as follows:

- Hydrodynamic shear stress up to approx. 8Pa ($15k \leq Re \leq 155k$, $0.2 \leq U \leq 1.97ms^{-1}$ and $0.14 \leq \tau \leq 7.74Nm^{-2}$) is inadequate to remove soil from the standard EMPA textile surfaces.
- There are at least two different flow domains within a horizontal-axis washing machine, one on the inside of the wash load and one on the outside.
- Flow conditions on both the inside and outside of the wash load are turbulent and velocities up to several meters per second have been recorded. It is therefore very likely that, given the porosity of the EMPA sample, the surface flow velocity is high, and much higher than that of the pore.
- Only 10-15% of the soil removed in the washing machines can be attributed to abrasion, warping (due to churning) and hydrodynamic shear stress, the rest is attributed to detergency and heat transfer.
- Soiled EMPA samples placed on a polystyrene sphere provide a lower cleaning efficiency (approx. 5%) than the standard stripes but have a lower data spread (typically ~0.5% versus ~3-5%). Consequently the sphere concept is better indicated for performance measurements and machine comparison.
The system used to acquire the flow data is also applicable to other parameters including temperature, humidity etc. For this reason Whirlpool has requested a patent for the sphere concept.

Remote data acquisition could also be a useful and better way for monitoring and releasing detergent in a controlled fashion.

Typical (laminar) flow conditions near fibre wall and pore imply that the soil is submerged in the viscous part of the boundary layer that varies in thickness during washing.

Increasing the water temperature from, say, 15 to 60°C makes the kinematic viscosity fall by 60% and likewise reduces the thickness of the laminar viscous sub-layer where the soil hides. This is good news for soil removal because it implies that the soil is exposed and subjected not only to more turbulence but also increased shear.

Flow in the washing machine is oscillatory and unsteady. Typical dynamic pressures are between 200 to 600Pa and the variation in water level line shows that the wash load is subjected to a pulsating rather than constant pressure head.

Typical frequencies (of water flow and wash load) are below 10Hz.

Only pigs blood and wine soil removal was found to be time dependent. This time dependency was of the first order type (which is typical of rheological and diffusion phenomena) and expired within the first 10–20 minutes of exposure, irrespective of flow conditions (up to Re=155k).

Carbon black provided a similar behaviour but to a much lower extent while chocolate soil showed no appreciable time dependency. Hence the removal of these two latter soils must certainly be accomplished and accompanied by other means (abrasion, detergency etc.).

From the SEM investigation there was evidence that during washing the soils undergo physical change, to the extent that the soil seems to change from being a solid to at least a plastic, if not a visco-elastic material. This is most likely caused by a combination of
mechanical, thermal and hydrodynamic effects during washing. This implies that the rheological properties of the soil changes.

So where does one go from here?

The final comment concerning rheology suggests that the most beneficial next step in this research would be the observation of the change in soil properties as it would link-up very conveniently with the adhesion studies and provide a better adhesion model. Thus a natural extension of the tunnel research would involve a state-of-the-art high(er) speed water tunnel equipped with high speed and high quality filming apparatus. This could prove conclusively that the soil undergoes rheological change during washing. The new tunnel could also provide means of understanding the link between hydrodynamic shear stress, water temperature and detergent.

Another area of research deemed promising would be to combine the effects of shear stress and frequency. This is because the tunnel tests were carried out in steady state conditions while measurements in the washing machine show transient or frequency-dependent conditions. The author believes that shock waves, pulsating flow conditions and/or vortices for example could be a valid starting point for future research.

There is of course the whole question of flow and wash load motion that has only just been tapped and certainly merits further attention. One activity in mind would be to measure wash load motion for different water temperatures, fills and loads so that a more complete model could be assembled. Although motion is predominantly 2D a more complete 3D model would also be advantageous for wash load dynamics modelling, in this way tumbling and spinning models could be combined.

It also seems reasonable to continue wash factor testing based on the DOE approach, testing not only more factors but also different levels e.g. more detergents, different water hardness values, machines etc. This work could be combined with the suggested parametric modelling so that the
design engine has not only an idea of what counts in today's machine design but builds a library of ideal test conditions for machine comparison.

On an equal academic standing, but one with no (or little) connection to future soil removal research, is the further development of the small block probes and application of the Preston tube. Smaller probes (50% size reduction) could be built with relatively little effort and a better understanding of shear stress over an extensive area could be obtained. Also the CFD activity could be extended not only to the angular calibration of the probe but to a wider range of simulated hydrodynamic conditions.

The sphere activity explained in step 5 (chapter 5c) has provided an ideal way of investigating what is happening to the flow in the washing machine and could be extended to more test conditions as well as different sensors. For example, this activity needs to include more flow conditions so that the sphere is tested on a wider scale e.g. in different machines and different wash loads. It therefore combines neatly with the application of the parametric modelling discussed previously.

Following the patent application for the sphere discussion has led to a need to investigate the application of nano sensors and not only for future shear stress research. In fact a second patent application is under evaluation that basically uses radio wave activated chips on board garment labels. The scope is to equip clothing with these special labels not only used to help the machine understand what is in the drum, and thus set the right washing cycle, but also as a means of locating clothes within the households.

On a closing note whatever research activity follows the scope must be to evaluate and find better techniques that break down the bond between soil and fibre. The author believes that the path to better soil removal is to either weaken the bond between soil and fibre, change the soils chemical-physical properties or both.
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Presentations, Reports and Publications

This final chapter provides an overview of research documentation in the form of presentations, publications and principal project reports prepared by the author during the Mass Transfer project. In addition two undergraduate dissertations concerning steps 1 and 2 will also be briefly discussed.

The names of the authors are provided in order of involvement i.e. participation, and the research documentation given in chronological order.

Abstracts of the publications will be given first followed by the scope and executive summaries of the reports issued within Whirlpool Corporation since March 1997.

Microelectronic Sensor for the Measurement of Shear Stress within a Domestic Washing Machine

David Ward, M. Lazzaroni, G. Menduni, M. Viterbo

Paper presented at IMAPS'99, Annual Italian conference for Microelectronics Applications, July 1999, University of Milan, Italy

Abstract

A wireless monitoring system is proposed that allows remote dynamic pressure sensing and data acquisition for the first time inside a domestic washing machine.

The scope of the system is to measure the dynamic pressure inside the wash load so as to relate it to the average flow velocity. The system is based on matching, single channel, 433Mhz transmitter and receiver units that transmit and receive pressure data supplied by a piezo-resistive pressure transducer connected to special small block probes developed by the authors.

The system is suitable for other single channel monitoring applications including temperature, humidity, conductivity etc.
Numerical Simulation of a Rectangular-section Water Duct equipped with Small Block Shear Stress Probes

David Ward, M. Viterbo, G. Menduni, D. Bocchiola

Paper presented at CAPI'99, Annual Italian conference for Advanced Numerical Simulation, Nov. 1999, University of Milan, Italy

Abstract

This paper presents the findings of a series of numerical simulations of a rectangular-section water duct that has been purpose-built to investigate the effects of hydrodynamic shear stress on the removal of soil from textile surfaces.

The scope of the simulation was to predict the hydrodynamic behaviour of the duct in terms of wall shear stress, velocity contours, pressure drop etc. in the test section of the duct.

The simulation and investigation included the development of miniature shear stress probes in the form of small triangular and square shaped thin (7-9x0.35mm) blocks attached to the inner face of the upper wall of the duct. These probes were used to measure local duct wall shear stress both in magnitude and direction. The measurement is based on the “law of the wall” and is compared with Preston tube measurements.

The simulation has been carried out using version 5 of FLUENT/UNS and a total of four runs were made. Each run was based on FLUENT's standard K-Epsilon model and usage of the standard wall functions. The simulation essentially focussed on the central part or test section of the duct, which measures 3.5m in length (of a total length of 11.05m), where the experimentation and simulation was affected. The meshing of the test section was done with GAMBIT, FLUENT's latest meshing tool, and divided in two domains. The first domain had a
rough mesh of approx. 23000 cells situated before and after the measurement zone while the second domain, where the probes were mounted and tested, had approx. 204000 cells.

The main outcome of the simulation was two fold:

a.) The confirmation of the shear stress magnitude measured during experimentation with the Preston tube and shear stress probe, with a discrepancy of less than 10% between simulated and experimental measurements.

b.) Reliable prediction of shear stress probe and tunnel behaviour.

The paper also includes a brief discussion on indirect shear stress measurement techniques with particular focus on pressure measurement, the Preston tube and small block probes.

Modelling of a Horizontal-Axis Domestic Washing Machine

David Ward

Journal of the Textile Institute

Vol. 91, Part 1, No. 2, 2000

Abstract

This paper examines the design of the horizontal axis washing machine by means of parametric modelling and the description of clothes motion during the washing process.

A series of dimensionless factors have been determined that characterise the horizontal-axis washing machine. These parameters can be linked and correlated to machine performance, including soil removal efficiency and effectiveness. In this way it is possible to optimise machine parameters such as drum diameter, angular speed, depth etc.

The motion of the wash load has also been modelled by means of an imaginary fabric plug and by dividing the rotation of the wash load into 3 distinct steps. The resulting cycle has provided a means of calculating the theoretical fabric plug pressure drop, which is known to drive local hydrodynamic conditions including fluid flow.
Two methods of determining fabric plug impact velocity are also proposed including one based on the “added” mass concept. The question of machine comparison and scalability is also tackled by means of geometric, dynamic and kinematic similarity.

La Misura dell'Attrito di Parete tramite Sonde di Pressione (una Rassegna ed una Proposta)

"Measuring Skin Friction with Pressure Probes (An Overview and Proposal)"

ACQUA – Italian Journal of Hydraulics, June 2000

Daniele Bocchiola, Giovanni Menduni, Marco Viterbo, David Ward

Abstract

The paper first provides a brief classification of the methods available for the measurement of wall shear stress. The authors then focus on methods, namely block probes that are based on the measurement of differential pressure readings. The theoretical background to these methods is then explained and the most favoured versions reported further. The paper is concluded with the description and discussion of an experiment in which block probes, appropriately developed by the authors, were tested to measure both shear stress magnitude and direction in water flows.

Remote Measurement and Monitoring of Critical Washing Process Data directly inside the Washing m/c drum

IEEE Instrumentation and Measurement Technology Conference

Baltimore Hilton and Towers, Baltimore, Maryland, USA May 1-4, 1999

M. Lazzaroni, D. Ward, E. Pezzotta, G. Menduni, D. Bocchiola

Abstract

The system proposed allows the remote sensing of pressure inside the wash load of a domestic washing machine via a wireless data acquisition system.

The system is based on a piezoresistive pressure transducer and a matched pair of battery
operated hybrid transmitters and receivers with a working frequency of 433MHz. The transducer and transmitter are housed within a small plastic sphere designed to follow the wash load during its movement in the washing machine drum. A small block probe is fitted to the surface of the sphere to pick up the surface dynamic pressure of the sphere and relate this to the local surface velocity. The receiver is mounted on the outside of the washing machine where it picks up the modulated signal from the transmitter and stores it through a dedicated data acquisition system.

The system has been designed to offer the maximum flexibility and is therefore applicable to many other remote sensing circumstances including those outside domestic appliances. The system is capable of measuring pressure within the washing machine and providing an estimation of the flow conditions within the drum during the rotation of the wash load. Perhaps the most appealing part of this work is that the system certainly lends it self to a whole series of transducer and monitoring applications within the washing machine e.g. temperature, humidity etc.

Also the cost of the whole system is not prohibitive since it is based on off-the-shelf industrial solutions thus providing both reliability and cost effectiveness.

Clearly the advent of smaller transducers and hybrid wireless monitoring systems will push this type of solution even further, even to the stage where the garments themselves will have not only identification but also monitoring features.

**Literature Review - Textiles**


**Scope and Executive Summary**

The scope of the literature review was to acquire a basic understanding of textiles and relative fabrication so as to relate this knowledge to the fluid dynamics inside the washing machine.
Ample documentation was found concerning textiles ranging from weaving to dyeing however very little was found concerning fluid mechanics. What was found involved mainly the filtration and separation industries where textiles are used as the porous media.

Henceforth the major outcome of the literature review is that the most urgent goal is to acquire accurate information of the local fluid flow velocity and domain in general. For the time being we may state that past research has identified typical flow velocities below 1ms⁻¹ and Reynolds numbers below 1000. However, much higher magnitudes may be useful to verify the adhesion study or, for example, to create flow conditions to demonstrate the influence of vorticity in soil removal. In fact perhaps the measurement of flow characteristics within the weave is the most demanding task at this stage of the project, especially if one considers that to do this will require the development of purpose-built tools, including micro-sensors.

**Literature Review – Shear Stress Measurement**


**Abstract**

This literature review, which is also found in chapter 4 or issues 1 and 2, of the Boundary Layer report, concerns the measurement of hydrodynamic shear stress.

As discussed in the previous chapters a fluid flowing past a wall exerts both a normal and tangential stress on it. The measurement of normal stress is relatively straightforward and generally requires only a simple tap in the wall to which a pressure gauge e.g. a manometer or pressure transducer is connected. Difficulties in this instance may arise due to damping of the piping, sensor pressure range, sensor frequency response etc. However, the major drawback with normal stress measurement is that it generally provides limited information about the fluid flow.

On the other hand the measurement of tangential or shear stress is considerably more complex
but potentially yields considerably more information about the fluid flow and is therefore more rewarding than normal stress measurements.

The measurement of wall shear stress is generally divided into two macro categories (shown in figure 5.7a) i.e. Direct and Indirect, the latter category providing the majority of methods and devices. Perhaps the biggest drawback with indirect methods is that they all require some form of calibration. Ideally, this should be done with a direct measurement technique, which may not be feasible or rational. Henceforth an indirect method is often compared with another indirect method e.g. small block probes versus a Preston tube and theoretical value of shear stress.

Direct methods are also not without shortcomings, since they need adequate installation, calibration, compensation and a certain degree of freedom in order for the sensor to accurately detect the shear force.

Ideally, the measurement of shear stress should be supported by the visualisation of the flow and possibly CFD simulation using programs such as FLUENT, ANSYS etc. In this way it is possible to observe and/or predict turbulence, secondary flows, pressure gradients and other flow phenomena.

Thus, this chapter addresses both shear stress measurement and flow visualisation concurrently.

CFD simulation is briefly discussed in chapter 5.

Similarity and General Considerations for the Weave Model

D. Ward and M. Viterbo, Whirlpool Internal Report, Issue 1, June 1998

Scope and Executive Summary

The scope of this report was to address the issues involving the design of the similarity model necessary for the understanding of fluid flow within the textile or fabric. The similarity issue concerns the problem of scaling the weave and relative fluid flow and, more specifically, tackling the geometric, dynamic and kinematic similarities.
This document is therefore focused on addressing how the microscopic level hydrodynamic flow conditions can be scaled to a more convenient macroscopic level, such as that found in the water tunnel. The need for scaling up the weave and flow is to simplify the analysis of the boundary layer and thus provide an acceptable answer to fundamental questions such as: what is hydrodynamic shear force near the fabric, weave, thread or fibre surfaces?, what is the pressure drop that drives the velocity field through a weave? etc. The objective was to design and build a scaled model of the fabric which can be tested in a tailor made water tunnel.

To achieve perfect similarity it is necessary to include geometric (size), dynamic (force) and kinematic (motion) similarity in the dimensioning of the model. This is true whether the model is scaled-up or scaled-down.

There is ample evidence of achieving geometric similarity, and to a lesser extent dynamic similarity, while kinematic similarity is particularly difficult. This is especially true if one intends modelling motion such as machine and fabric motion.

To this end the author suggests that a compromise be accepted and the model be based on geometric and dynamic similarities with the inclusion of simplifying assumptions to accommodate kinematic effects.

The most significant assumption is to consider the fabric as a porous media, hence we consider relative motion only i.e. whether the fabric or fluid moves is deemed irrelevant. To implement this approach suitable model adjustments are made by exploiting the work of past researchers in the fields of filtration and textile development.

Next to the similarity issue is the question of shear force measurement. The primary aim of this work is to establish the shear force via the measurement of dynamic and static pressures within the similarity model’s boundary layer. With this data in hand a comparison will be made with the findings of the adhesion studies at Purdue University.
Boundary Layer Theory, Shear Stress Measurement and the Washing machine-Tunnel

Investigation

D. Ward, Whirlpool Internal Report, issue 2, Dec. 1999

(formerly the Boundary Layer report, issue 1 Dec. 1998)

Scope and Executive Summary

The Mass Transfer project extends from understanding the adhesive forces at a nanomicroscopic scale level to the macroscopic scale level of the boundary layer of the fluid flow at the weave-thread surfaces and possibly further.

This report tackles this latter aspect and focuses on shear stress theory and measurement in turbulent flow, with particular reference made to the water tunnel or duct tests.

This focus is by far from casual since shear, in general, is considered a fundamental indicator of soil removal by hydrodynamic forces and turbulent flow is known to provide the highest surface shear stresses.

Thus, it is hoped that by determining the turbulent shear stress near the surface of soil or weave it is possible to evaluate its importance in terms of washing performance. This report is also a broad-based report covering many practical aspects of boundary layer theory, and explaining how tunnel and washing machine conditions are correlated. Moreover, because of the broad background of potential readers the report is aimed at providing a detailed overview without going into unnecessary depths, for example the Navier-Stokes equations are only briefly introduced.

Summarising, the scope of this document is to provide an overall background to boundary layer theory with particular emphasis on hydrodynamic shear stress, shear stress measurement methodologies and generally discuss the washing machine-tunnel investigation carried out so
far. The intent is therefore to explain the characteristics of the boundary layer, how they are measured and/or calculated and how they relate to soil removal from textile surfaces.

High Speed Filming of the Wash load in a Horizontal-axis Washing m/c

D. Ward, Whirlpool internal report, Issue 1, Dec. 1999

Scope and Executive Summary
The scope of this report was to verify the theoretical model described in the paper titled “Modelling of a Horizontal axis Washing Machine” and determine a first approximation of the impact velocity of the wash load and wash water velocity near the door.
The investigation has demonstrated the validity of mathematical model although more from a qualitative than a quantitative perspective. The parabolic shape of the free fall has been confirmed and the critical (release) angle found to be 45° (against 40° predicted by the model).
The impact velocity shows the largest difference between model (4-4.5ms⁻¹) and experimental measurements (1.6ms⁻¹). However, the high speed filming has shown why this discrepancy exists, in particular the sphere is subject to resistance caused by the air and rolling effects caused by the adjacent fabric. Other differences were found in the total distance travelled in the x and y directions, in both cases reductions of 20 to 60% were found with respect to the model predictions. However, when the real conditions are put in the model the impact velocity discrepancy is reduced to just 0.1-0.2ms⁻¹, a more than acceptable result.
Clearly to upgrade the model implies investigating more load conditions and acquiring more high speed imaging data.
Analysing bubble movement demonstrated that the flow velocity on the outside of the wash load is indeed considerably higher than inside it. This demonstrates that there are at least two flow domains (1. Inside 2. Outside) and the ratio between the two is at least 100. Outside velocity is far from insignificant with an average velocity of approximately 2.2ms⁻¹, although this value will
decrease as wash load increases. My personal prediction is that the velocity will drop to $1\text{ms}^{-1}$ when the machine is fully loaded (5kg) since the wash load tends to dampen movement because of compactness and rolling effects.

Finally the water line motion was also analysed. The most significant outcome was that the pressure head rate (variation of pressure per second) was remarkably high being typically $1000\text{Pas}^{-1}$. This change in pressure provides an insight into just how the water may be “pumped” through the wash load hence the diffusion through the wash load is characterised by a pumping effect and not a constant pressure head.

**Roughness and Waviness measurements of EMPA soiled samples**

D. Ward, Whirlpool internal report, Issue 1, Dec. 1999

**Scope and Executive Summary**

The scope of this research activity was to determine the surface profile characteristics of EMPA samples used in the water tunnel investigation and establish the impact this may have on the removal of soil in the water tunnel investigation.

The investigation has demonstrated the validity of a non-contact surface roughness measurement method using the Conoscan 2000 surface profiler and relative Viewer SW. Hence we now have a reliable method for determining the roughness of flexible surfaces including those of the EMPA samples used in washing machine performance tests and the water tunnel experimentation.

The overall outcome of the investigation is that the typical roughness of the EMPA samples is lower than $1\text{mm}$ and therefore the boundary layer (in both the washing machine and water tunnel) on the EMPA sample surface amply contains both soil and weave profile.

The water tunnel experimentation therefore remains valid and the measurements reported here confirm that surface roughness and profile do change when the weave is wet. No further work is
foreseen for the measurement of weave roughness.

**Una Metodologia per la Misura Puntuale Dell'attritto di Parete in Correnti Cilindriche (A Method for Measuring Local Skin Friction in Cylindrical flows)** Politecnico di Milano, 1998

**Author**: Daniel Bocchiola  
**Supervisor**: Prof. Giovanni MENDUNI  
**Co-supervisor**: Ing. David WARD

**Abstract**

In this thesis an indirect method of measuring localised skin friction in closed channel water flows is discussed. The method is based on two types of small block probes in the form of a thin square and triangular shaped obstacles, equipped with static and dynamic pressure tappings. Both probes exploit the 'law of the wall' and local pressure recordings to provide the magnitude of local wall shear stress.

The thesis discusses an experiment carried out in a tailor-made water tunnel, which shows how the probes can be calibrated to reveal wall shear stress magnitude within 40k<Re<160k. The calibration technique used is the Preston tube, a development of the Stanton tube, which has been used to characterise also the shear stress contours of the upper tunnel wall. These contours were found to be in accordance with work carried out by Knight and Rhodes.

The major outcome of the research was that the small block probes can be used to measure wall shear stress up to 8Pa in closed channel water flows.
Abstract

In this thesis an indirect method, known as ‘small block probes’, for the measurement of shear stress conditions in closed channel water flows is discussed. These probes, in the form of a thin triangular shaped obstacles, are equipped with static and dynamic pressure tappings. The exploitation of the differential pressure recordings and the ‘law of the wall’ provide a direct indication of wall shear stress.

The thesis discusses an experiment carried out in a tailor-made water tunnel, which shows how the probes can be calibrated to simultaneously reveal both wall shear stress magnitude and direction.

One of the major difficulties in the indirect measurement of wall shear stress magnitude and direction is the calibration and comparison with a more reliable, and preferably direct, method. However, reliable direct methods are hard to come by and hence the approach preferred by many researchers is another indirect method known as the ‘Preston tube’, a development of the Stanton tube.

The research therefore discusses the findings of both the Preston tube and small block probes experimentation and links them to the theory behind the ‘law of the wall’.

The major outcome of the research was that the small block probes can be used to measure wall
shear stress up to 8Pa in closed channel water flows as well as predict the direction of the shear simply measuring differential pressures (static and dynamic) across the probes.

**Virtual Reality Presentation**

**Whirlpool 1999-2000, IMAPS'99 and CAPI'99**

Presenter and Author : David Ward
Co-author : Marco Viterbo
Support : TTM group (Virtual Lab.)

**Scope and Executive Summary**

Project communication was one of the toughest challenges in the Mass Transfer research project because of its globalness.

It was therefore decided to use virtual reality to convey the project approach and illustrate some of the results using advanced audio-visual technology. This was the first time VR was used in R & D. A total of three presentations were prepared with each one being an upgrade of the previous one both in terms of quality and quantity of information.

Presentations were given not only during project meeting updates but also at IMAPS’99 and CAPI’99 using virtual reality presentations by the TTM (Time-to-Market) group.

The VR presentations were basically a series of animated or real video scenes that demonstrated one or more of the 5 project steps. The VR presentations were aimed at a general audience with a technical background and stored on CD-ROM so that they could be projected using standard information technology.

All presentations were in colour with a maximum duration of no longer than 5 minutes.
Weave Model Construction using Stereolithography ... An example of Rapid-Prototyping and the advance use of Pro-E and Fluent

Whirlpool Internal Seminar held by David Ward and Marco Maritan
March 1998

Scope and Executive Summary

Stereolithography is an example of rapid prototyping which allows the design engineer to first design solid models with advanced CAD systems like Pro-E and then build the models using a special machine based on stereolithography technology.

The machine works by solidifying photosensitive wax-like resin using a UV or LASER light source exactly where the model needs to be solid. In practice the machine can be visualised as a 3D printer, which creates layers of solidified special resin according to the 3D model downloaded into the machine.

The model therefore consists of a series of layers, one on top of each other, with each layer having an area of solidified resin and an area of semi-solid resin-like wax. At the end of the process all this resin-like wax is simply washed away leaving the solid model behind.

The stereolithography technology was presented within the R & D group of Whirlpool in March 1998, since then this technology has become a standard tool for prototype building.

The scope of the presentation was not only to illustrate applications of stereolithography but also to the development of the weave threads and weave model. This included showing how the threads and model were developed with Pro-E SW and the use of the same solid model transferred to FLUENT where a CFD simulation was successfully carried out. The model was subsequently used to in conjunction with small, triangular shaped, block stress to measure flow conditions on the models surface. These blocks were connected externally via pipe tubing routed and integrated in the model itself so that pressure measurements could be recorded.
Abstract

In this thesis the design, development and testing of a remote data acquisition system is discussed. The system, based on the sphere concept and used in step 5 discussed previously, uses a matched pair of hybrid radio transmitter and receivers. The scope of the system was to establish the flow velocity within the wash load during the initial washing phase of the fabric care process. The system used a small, triangular shaped, block probe connected to a miniature pressure transducer to detect the dynamic pressure, which was then converted into an average flow velocity.

The research activity also involved the development of a purpose-built LABVIEW program.
Remote Measurement and Monitoring of Critical Washing Parameters Inside a Domestic Washing Machine

D. Ward, E. Pezzotta, M. Lazzaroni, M. Viterbo, G. Menduni

Paper presented at NIDays2000, National Instruments, Annual Italian conference, 28th Nov. 2000, Milan, Italy

Abstract

This concise paper discusses the measurement system based on National Instruments SW (LABVIEW) and HW (SC2040 and DAQCard). The scope was to describe the measurement system from a SW and HW point of view. The paper therefore emphasises the systems side of the washing machine measurements and briefly discusses the integration of HW and SW.

The paper was presented at the NiDays2000 conference organised by National Instruments Italy.

Wash Load Motion: A Theoretical Approach And Experimental Analysis

D. Ward and D. Conrad

Accepted for the 52nd International Appliance Technical Conference, Ohio State University, Ohio USA, March, 2001

Abstract

This paper is essentially an extended version of the Journal of the Textile Institute paper regarding the modelling of the wash load motion. The paper includes the experimental part of the fabric plug model and hence provides a description of the high speed filming technique used for this purpose.
European Patent Application

Title: System for the remote measurement and monitoring of critical washing parameters inside a domestic washing machine

Inventors: David Ward and Ivano Frattini, CTED, Whirlpool Europe

No. MI2000A002122

Dear Reader,

The CD-ROM attached to this thesis includes a virtual reality simulation illustrating the five steps of my research. There is also one example of a high speed motion film of the fabric plug which requires running the motion picture SW provided. All relative thesis word files and project powerpoint files are provided. These have been created using Win95 and Office 97.

It is advised to run all files attached on a PC with at least 64Mbyte of RAM, 100Mbyte disk space and a P2 processor.

Further info. can be found on http://www.geocities.com/dawardit/index.htm

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