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“Stokes of Pembroke S.W. & a very good one” – The mathematical education of George Gabriel Stokes

By June Barrow-Green

My title comes from the diary of Joseph Romilly, Registry of the University of Cambridge (1: p.45). Romilly, who was writing on 22 January 1841, was remarking on the fact that Stokes had not only triumphed in the gruelling Cambridge Mathematical Tripos examination, winning the coveted title of Senior Wrangler, but that he had excelled in doing so in a year when the papers were notoriously difficult.\(^1\) It was a notable achievement but it was a prize hard won after several years of preparation, and not only years spent at Cambridge. When Stokes arrived at Pembroke College, he had spent the previous two years at Bristol College, a school which prided itself on its success in preparing its students for Oxford and Cambridge.

In this chapter I shall follow Stokes on his path to the senior wranglership, tracing his mathematical journey from its beginnings in Ireland to its close at the end of his final year of undergraduate study.

Early education

Stokes began his education at home, being tutored in Latin by his father and learning arithmetic from the local Parish Clerk, George Coulter.\(^2\) Even at a young age Stokes showed evidence of mathematical talent. Coulter, who taught using Voster’s *Arithmetic* \(^4\), a popular text specifically ‘adapted to the Commerce of Ireland’, was delighted by the fact that the young Stokes quickly worked out for himself ‘new ways of doing sums’ \(^3\) far better than those given by Voster. Coulter also reported that ‘clever people’ were astonished by Stokes’s facility with the rule of false position.\(^3\)

In 1832, aged 13, Stokes was sent to Dublin where he lived with his uncle, John Stokes, and attended the famous school of Dr Richard Henry Wall. Dr Wall’s school, officially known as the Seminary for General Education, was located on Hume Street (the building is now part of a hospital). Among other pupils who at some time attended the school were the engineer William Thomas Mulvaney and the mathematician George Johnston Allman. The course of instruction included mathematics, arithmetic and book-keeping, as well as weekly lectures on mechanics \(^5\). Stokes’ mathematical ability once again attracted attention, this time it was his solutions to geometrical problems which caught a master’s eye \(^3\).

Stokes’s father died in 1834 and soon after the decision was made to send Stokes to school in England. The school chosen was Bristol College, the choice being made on the recommendation of

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\(^1\) Wranglers were students who had gained first-class honours in the Mathematical Tripos. Romilly, who was writing after attending the meeting where the order of merit had been decided, had been 4\(^{th}\) wrangler in 1813 and he ‘long remembered his sufferings when he sat for the examination’ (which was partly oral in his time) so was well-qualified to judge \(^2\): p.152).

\(^2\) Most of what is known about Stokes’s early education comes from the memoir written by his daughter, Mrs Laurence Humphry, written after Stokes’s death \(^3\): p. 3–5).

\(^3\) Elias Voster set up a school in Cork for teaching arithmetic and book-keeping. His text was first published in 1768 and ran into multiple editions during the 18\(^{th}\) and 19\(^{th}\) centuries. It is not known which edition was used by Stokes.
Stokes’ second brother, William Haughton, who was a friend of the Irish-born principal of the College, Joseph Henry Jerrard. Both of them had studied at Trinity College Dublin before going up to Caius College, Cambridge in 1824. William graduated as 16th wrangler, coming rather higher on the Mathematical Tripos list of 1828 than Jerrard who was 27th senior optime that year, although Jerrard was placed in the first class in the Classical Tripos. Humphry described Jerrard as ‘a mathematician of some note’ (3: p.6) but evidence for her claim has proved hard to find. Graduating as 27th senior optime in the Mathematical Tripos would not warrant such an epithet. It is possible that Humphry confused Joseph with his elder brother, George Birch Jerrard, who was educated at Trinity College Dublin and who was well-known in his day for his publications on the theory of equations. It seems less likely that she confused him with his younger brother, Frederick William Hill Jerrard, who was 8th wrangler at Cambridge in 1833 and two months later came to the College as the first professor of mathematics and natural philosophy (7: p.3), since the youngest Jerrard displayed no interest in mathematics after leaving the College, making his career in the church.

Bristol College

In 1829 a group of the ‘more intelligent citizens’ of Bristol, conscious of the lack of an institution of higher education in their city, proposed ‘the erection of a college, with an efficient staff of masters and lectures, theological instruction according to the doctrines of the Church of England being also provided for such pupils as might desire to avail themselves of it’ (8: p.140–142). An Outline of the Plan of Education to be pursued in the Bristol College (9), published by the Council of the College in 1830, provides information about the structure of the College, professors to be appointed etc. The prospectus also makes clear that the teaching of mathematics was considered essential—something that was not the case in all schools at the time, Eton and Harrow providing conspicuous examples (1: p.90–91)—as it was part of a ‘liberal education’, the notion so strongly advocated by William Whewell (10). And the course of study of mathematics at Trinity College, Cambridge, was to provide the model. As the Outline stated (9: p6,9-10):

‘Experience has proved, that a close application to the exact sciences is the best discipline for the mind, and the most suitable preparation for its advancement in the schools of philosophy. The Mathematics are therefore justly held to be an essential part of every liberal education. ... The Mathematics will be taught in separate classes. ... It is intended however to adopt, with no more alteration than can be avoided, the plan at present pursued in Trinity College, Cambridge. It is expected, that the student will have been grounded in the elements of Geometry and Algebra, while in the junior classes. He will then proceed to Plane Trigonometry, to the higher parts of Algebra, and having become acquainted with the Differential and Integral Calculus, to the theory of Curves, and successively to Statics and Dynamics, Conic Sections, and the first three sections of Newton’s Principia. Thus far he may advance in the first and second years : in the third, he will be occupied

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4 Senior optimes were students who had gained second-class honours in the Mathematical Tripos. The Classical Tripos, established in 1822, was a voluntary examination open only to students who had already gained honours in the Mathematical Tripos. The Classical Tripos became independent of the Mathematical Tripos in 1850.

5 Today George Birch Jerrard is best remembered for his mistaken belief that he had found a formula for solving equations of the fifth degree. In 1824, Niels Henrik Abel had proved that such a formula (based on only fundamental algebraic operations) could not exist but Jerrard refused to accept it, publishing erroneous results and persisting in his belief to the end of his life (6).

6 The founding of the college is described in detail by John Latimer, former editor of the liberal leaning Bristol Mercury, and author of the highly regarded four-volume Annals of Bristol (8).
with the principles of Hydrostatics and Optics, and with the remainder of the first book of the
*Principia*, as well as with Spherical Trigonometry and Physical Astronomy.  

When the College opened its doors on 17 January 1831 it had about thirty pupils, considerably fewer
than originally envisaged. Due to the constitution of the College with respect to religion—
theprofessional instruction was voluntary—the Bishop of Bristol had been openly hostile to the plan
which meant that the founders had been unable to raise as much capital as they had hoped.
Feelings had run high with a heated discussion having taken place within the pages of *The Bristol
Mercury*, and comparisons being made with ‘that moral pest-house—the London University’ (11: p.3). Nevertheless, despite the opposition, the College quickly achieved success with many of its
pupils later attesting to its value.  

But despite its academic success, the College was to be short-lived. The problem once again lay in its
religious tolerance. Sons of Nonconformists were allowed to attend the school without having to
participate in religious instruction and this did not sit well with a section of the local clergy. By 1840 the
latter had garnered enough support, including the Bishop of Gloucester and Bristol, to open a rival
school. Bristol College was unable to withstand the competition and closed at Christmas 1841. As one
commentator put it: ‘its promoters were a generation before their contemporaries, and the institution
was of too liberal a character for the age.’ (8: p.141)

The first mathematics tutor of the College, who was also the Vice-Principal, was George Butterton
who replaced the original appointee Charles Smith who had been taken ill. Both Butterton and
Smith were 8th wranglers, the former in 1827 and the latter in 1828. By March 1833, the number of
students had increased to the extent that it was found necessary to establish a professorship of
mathematics and natural philosophy in the senior department and it was then that Frederick William
Hill Jerrard was elected to the post.

Thus by the time Stokes entered Bristol College in 1835, there were two members of staff who were
high-ranking wranglers, with one of them, the youngest Jerrard, having left Cambridge only two
years before, both well placed to implement the course of Cambridge-style mathematics detailed in
the prospectus. There is no record of how Stokes fared under their instruction but one teacher at
the College whom Stokes did recall was the writer Francis Newman, younger brother of John Henry
(later Cardinal) Newman, and ‘a man of charming character’ (3: p.6). Newman, who had
distinguished himself at Oxford, gaining a double first in classics and mathematics in 1826, joined the
College in 1834 as classical tutor to lecture for Joseph Henry Jerrard who was unwell. However, it
was his mathematics that made his mark with Stokes. Humphry (3) recalled that Stokes considered
he ‘owed much’ to Newman’s teaching, and he ‘subsequently corresponded with him on
mathematical subjects when both had become famous’ (p.6). Although today Newman is
remembered as a social and religious reformer, his first publication was a pamphlet on Taylor series,
and he went on to write numerous books and articles on mathematics, including a book on
elementary geometry which was published 1841, the year after he left the College (12; 13).

Stokes’s ability for mathematics was readily apparent to those around him at the College. According
to Lord Rayleigh, there was ‘a tradition that he did many propositions of Euclid, as problems, without
looking at the book’ (14: p.200). At the end of his first year Stokes was awarded a ‘Grand Prize’,

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7 *Bristol Mercury*, Letter to the Editor, 1 December 1829.
8 One famous pupil was the writer Walter Bagehot who subsequently studied mathematics at University
College London with Augustus De Morgan.
9 In 1851 Newman taught elementary geometry at the Ladies’ College (later Bedford College) in succession to
Augustus De Morgan; among those who attended his classes was the novelist George Eliot (18: p.77).
which included a prize in mathematics and a certificate of honour in English Prose Composition (15: p.3). At the end of his second and final year, in which he was examined in, elementary dynamics, mechanics, the differential and integral calculus, and trigonometry, with oral testing on the last three, he did not win the first mathematics prize but was awarded a prize for ‘Extraordinary Progress in Mathematics’, which possibly indicates that when he arrived at the College he was not as well prepared in mathematics as some of his fellow students (16: p.4).

His final examinations over, the question now was where should Stokes go to next. His father and all three brothers had studied at Trinity College Dublin (TCD), but they had not been to a school which explicitly prepared students for Oxford or Cambridge. In fact Stokes’s brother William had entered TCD at the age of 17 before going up to Cambridge five years later. But such a pattern of study was not unusual. The level of mathematics taught at TCD and other universities, such as Edinburgh University, Glasgow University or University College London, was much lower than at Cambridge, and in the 19th century several mathematics students at Cambridge, including high-ranking wranglers such as William Thomson and James Clerk Maxwell, had already studied for a degree elsewhere.

On the final day of Stokes’s examination, Joseph Jerrard wrote to Stokes: ‘I have strongly advised your brother [William] to enter you at Trinity, as I feel convinced that you will in all human probability succeed in obtaining a Fellowship at that College’ (3: p.6). William, who in 1836 had become Dean of Caius College, counselled Stokes to take care when choosing his college (17: p.300). However, when Stokes went up to Cambridge he did not go to Trinity but to Pembroke.

Why Stokes went to Pembroke and not to Trinity as advised by Jerrard is not known. Stokes himself seems never to have given a reason. In 1901, when in his early eighties he was asked to recall his student days, he gave no clue, only writing: ‘I entered Pembroke College, Cambridge in 1837. In those days boys coming to the University had not in general read so far in mathematics as is the custom at present; and I had not begun the differential calculus when I entered the College, and had only recently read analytical sections.’ (3: p.7) It appears, however, that Stokes’s memory must have been playing tricks on him since, as noted above, the subjects in his final examination at Bristol College included the differential and integral calculus.

Cambridge

At the beginning of the nineteenth century, the normal route to a Bachelor of Arts degree at Cambridge was through the Senate-House Examination, popularly known as the Tripos. The examination was primarily in mathematics but included other subjects, such as logic, philosophy, and theology. It was not until 1824, when the Classical Tripos was examined for the first time that it began to be referred to as the Mathematical Tripos, although students could enter the Classical Tripos only if they had already obtained honours in the Mathematical Tripos, a situation which pertained until 1850. As the century progressed the examination took on an ever-increasing significance. There was a shift from oral to written examinations, with success in the final examination being paramount, and a concomitant rise in private tutoring without which such success was virtually impossible. A high place in the order of merit garnered national recognition and was a passport to the career of one’s choice.

Mathematics was the core of study at Cambridge not because it was preparation for a career as a mathematician because it provided a fundamental part of a liberal education. The reason for studying Euclid’s Elements was not simply to learn geometry. It was a training of the mind. That being said, knowledge of Euclid provided (at least some) access to the single most important text a Cambridge mathematics student had to study: Isaac Newton’s notoriously difficult Principia. Written primarily in the language of geometry, the Principia provided the most certain demonstration of
man’s knowledge of the natural world. It is no wonder that it played a dominating role in the university’s undergraduate course.

Undergraduates generally began their studies under the direction of a college lecturer whose duties were to guide the reading of the students and to prepare them for the rigours of the college and Senate-House examinations. When Stokes arrived at Pembroke, the college mathematical lecturer was John Mills (5th Wrangler 1831) so presumably it was Mills who acted as Stokes’s first point of call. Little is known about the particular nature of Stokes’s studies in his first year except for the fact that he came second in the college examinations, pipped by a certain John Sykes. Sykes’ father came from Cambridge so perhaps Sykes had been even better prepared in Cambridge-style mathematics than Stokes. The following year Stokes came out on top with Sykes coming only third. In his second and third years, Stokes was privately tutored by the famous mathematical coach, William Hopkins (of whom more below). Sykes it seems was not tutored by Hopkins although he would have certainly received private tuition.

As well as the tuition provided by the college, there were also lectures delivered by the professors. Not all students attended the lectures of the mathematics professors, and not all of the mathematics professors lectured. While Stokes was an undergraduate neither of the Lucasian professors—Charles Babbage, who held the chair from 1828 to 1839, and Joshua King, who held the chair from 1839 to 1849 (when he was succeeded by Stokes)—lectured, although both of them examined for the Smith’s Prize (19), King having previously examined several times for the Tripos. George Peacock, Lowndean professor of astronomy and geometry, advertised lectures on ‘Science of astronomy and practical methods of observation; use of Instruments. Geometry, and general principles of Mathematical Reasoning’ and it is likely that Stokes attended them. Stokes certainly attended the lectures of the Plumian professor of astronomy and experimental philosophy, James Challis, today best remembered for failing to identify the planet Neptune. Challis offered ‘Laws of Hydrodynamics, Pneumatics, and Optics with special reference to the Mathematical Theories of Light and Sound. Explanations exhibited experimentally; explanations given of Principles employed in Mathematical Reasoning’ (20).

As well as Euclid’s Elements and Newton’s Principia, there were several mathematical textbooks which Cambridge students were expected to study, many of which had been written by former Cambridge wranglers and were designed specifically for students of the university. Amongst the most prolific and influential of writers who produced books in this category was John Hymers (2nd Wrangler 1826), a fellow of St John’s, who successfully combined his college career with private tutoring, and who examined for the Tripos in 1833 and 1834. With a reputation for being ‘profoundly versed in mathematics,’ Hymers had ‘a vast acquaintance with the mathematics of the continent’ (23) which was evident in many of his books. For example, the second edition of his Integral Equations (1835) introduced English students to the newly discovered topic of elliptic functions, while his Treatise on Conic Sections and the Application of Algebra to Geometry (1837) became the standard textbook on analytic geometry.

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10 John Sykes ended up as third wrangler to Stokes. He later examined for the Mathematical Tripos several times. He went on to have a long career in education, including examining for The Society of Arts.
11 The term ‘coach’ for a private tutor is thought to have originated in Oxford in around 1830 and to have come into use in Cambridge in the early 1830s (21: p.89).
12 Alex Craik in his comprehensive study of Hopkins has found no evidence of Sykes being coached by Hopkins (1: p.4).
13 Robert Leslie Ellis who was the senior wrangler a year ahead of Stokes attended Peacock’s lectures (22: p.xv), so it is likely that Stokes did also.
The 1816 translation of Lacroix’s introductory textbook on the differential and integral calculus by Babbage, Herschel and Peacock, which was an important stimulus for the introduction of analytical methods into Cambridge, was followed by a number of new books which treated their subjects from an analytical perspective. Among these were the textbooks of William Whewell on mechanics (1819) and dynamics (1823). Whewell’s *Treatise on Dynamics* is also notable for being one of the first books to treat the subject from a deliberately pedagogical perspective, as well as for promoting a problem-solving approach (21: p.146-147).

George Biddell Airy’s *Mathematical Tracts*, was another staple of an undergraduate’s diet. Originally published in 1826 while Airy was Lucasian professor, the second edition of 1831, which would have been studied by Stokes, included a new section on the wave theory of light. It provided an analytical approach to problems of physical astronomy, the shape of the Earth, and to its precession and nutation. And Airy did not shy from explaining why he considered other texts unsuitable: the standard text on Lagrange’s calculus of variations by Lacroix was ‘singularly confused and unintelligible’ (a judgement with which later authors concurred). A more advanced text espousing a similar analytical approach which appeared a few years later was John Pratt’s *The Mathematical Principles of Mechanical Philosophy and their Application to the Theory of Universal Gravitation* (1836). Pratt (3rd Wrangler 1833), a former student of Hopkins, provided a particularly clear account of the shape of the earth. Another book Stokes could have read is Mary Somerville’s *Mechanism of the Heavens* (1831), Somerville’s interpretation of Laplace’s *Mécanique Céleste*. It was promoted by both Whewell and Peacock, the latter writing to the author in 1832 that he ‘had little doubt that it will immediately become an essential work to those of our students who aspire to the highest places in our examinations’. (24: p.172)

**William Hopkins – ‘the senior-wrangler maker’**

Stokes’s coach, William Hopkins went up to Cambridge in 1822 as an undergraduate at the age of twenty-nine, a change in family circumstances allowing him to abandon his inherited occupation of gentleman farmer. Graduating as 7th wrangler in 1827—a strong year in which Augustus De Morgan (1806-1871) came 4th—he would have been a candidate for a college fellowship but as a married man he was ineligible. The same year he was appointed to a lectureship in Peterhouse and Esquire Bedell to the University (a partly administrative and partly ceremonial post). In 1839 he was one of only two candidates for the Lucasian Professorship, the chair having become vacant on the resignation of Charles Babbage. But it was a contest he was doomed to lose. The other candidate was Joshua King, senior wrangler in 1819, President of Queens’ College and one of the eight electors. King won the vote by a majority of seven to one.

Hopkins was the first of the Cambridge coaches to make a permanent living from private tutoring. He rapidly developed a reputation as an outstanding teacher and his results were remarkable. Between 1828 and 1849, he ‘personally trained almost 50% of the top ten wranglers, 67% of the top three, and 77% of senior wranglers’, which amounted to 108 in the top ten, 44 in the first three, and 17 senior wranglers, and earned him the sobriquet ‘senior wrangler maker’. (21: p.84–85).

As Hopkins’ reputation grew, he was able to pick and choose his students. By taking students in their second, or occasionally their third year, he had time to assess their abilities and select the most promising before taking them into his tutelage. Stokes was typical in this respect. He began
studying with Hopkins in his fourth term and stayed with him until his final examinations. Hopkins taught in small classes, putting students of equal ability together. As Warwick (21) has described ‘this meant that the class could move ahead at the fastest possible pace, the students learning from and competing against each other’ (p.84). Or as one of his obituarists wrote:

‘The secret of his success as a teacher was the happy faculty he had of drawing out the thoughts of his pupils and make them instruct each other, while he took care that the subjects under discussion were treated in a philosophical manner so that mere preparation for the senate-house examination was subordinate to sound scientific training.’\(^\text{15}\)

A first-hand account of how Hopkins ran his classes was given by William Thomson (later Lord Kelvin). Thomson, who started with Hopkins in a class of about five students in the year after Stokes graduated, wrote to his father James Thomson (professor of mathematics at the University of Glasgow):

‘What we have had already approximates very much to the plan which you pursue with your class. He [Hopkins] asked us all questions on various points in the differential calculus, in the order of his manuscript, which he has given us to transcribe, and gave us exercises on the different subjects discussed, which we are to bring with us tomorrow. He says he can never be quite satisfied that a man has got correct ideas on any mathematical subject till he has questioned him viva voce. I can judge very little yet of the other men whom I meet with him, but I hope they are not extremely formidable.’ (26: p.76)

Although the number in the class was relatively small, the competitive nature of the Senate-House examination was clearly in evidence.

Of the atmosphere of the class, Francis Galton, a student with Hopkins from 1841, painted a rather convivial picture:

‘Hopkins to use a Cantab expression is a regular brick; tells funny stories connected with different problems and is no way Donnish; he rattles us on at a splendid pace and makes mathematics anything but a dry subject by entering thoroughly into its metaphysics. I never enjoyed anything so much before. ... [He charges] only £72 per annum instead of £100 as currently reported: this will make a jolly difference to my finances.’\(^\text{16}\)

But this letter was written when Galton was in only his first year of study with Hopkins and as time progressed the unremitting strain of competition began to tell on him. In the end his health broke down completely and he never graduated (19: p.183). The brilliant Robert Leslie Ellis, senior wrangler the year before Stokes, unusually studied with Hopkins only in his final year and only so that ‘his reading should be arranged and put in a form suitable for the Cambridge examinations’. Although Ellis detested the system which he described as ‘the crushing down of the mind and body for a worthless end’,\(^\text{17}\) he knew that if he was to have any chance of being a high wrangler he had to go to Hopkins.

\(^{15}\) The Gentleman’s Magazine, 1866, p. 706, quoted in (25: p.40).

\(^{16}\) Letter from Francis Galton to his father, 11 November 1841, quoted in (21: p.85).

\(^{17}\) Diary of Robert Leslie Ellis, 3 December 1838. Trinity College Cambridge, Add Ms a.219.1. For further comments about Ellis’ dislike of the Mathematical Tripos, see (21: p.184-5).
In contrast to Ellis, the system appears to have suited Stokes rather well. He kept the notes he made while reading with Hopkins, although they are rather difficult to read.\textsuperscript{18} There are sets of notes for several different courses including differential and integral calculus, mechanics, dynamics, optics, hydrostatics, hydrodynamics, sound and light, and calculus of variations.\textsuperscript{19} From these it is possible to get a good idea of Hopkins’ style of teaching. The standard theory is given, examples are worked through and others are left for the student to complete. Hopkins was also particularly well-placed to teach on topics covered in Airy’s \textit{Tracts} since he would have attended Airy’s lectures while he was an undergraduate.

None of Hopkins’ own notes appear to have survived although he did publish a textbook, \textit{Elements of Trigonometry} (1833). A feature of this text is his use of history of mathematics both to elucidate and to generate interest in the mathematics discussed. (Another textbook, on the differential calculus, was promised but never materialised.\textsuperscript{20}) An insight into Hopkins’ views on mathematics and what informed his teaching can be gleaned from a pamphlet he wrote in 1841 in response to proposals to exclude from the Tripos applications of mathematics to astronomy, optics and hydrodynamics, which would have rendered the retention of partial differential equations and higher geometry pointless:

‘It is only when the student approaches the great theories, as Physical Astronomy and Physical Optics, that he can fully appreciate the real importance and value of pure mathematical science, as the only instrument of investigation by which man could possibly have attained a knowledge of so much of what is perfect and beautiful in the structure of the material universe, and in the laws which govern it. It is then that he can form an adequate conception of the genius that has been developed in the framing of those theories, and can feel himself under those salutary influences which must be ever exercised on the mind of youth by the contemplation of the workings of lofty genius, in whatever department of science or literature it may have been called into action.’ (29: p.10)

Thus for Hopkins mathematics was important because it was the means by which the secrets of the universe could be revealed. In short, Hopkins ‘regarded the Newtonian system of the world and the wave theory of light as the crowning achievements of the mathematical investigation of nature.’ (17: p.302) The proposals would also have meant that there was little point in retaining partial differential equations and higher geometry. Little wonder Hopkins was opposed.

Hopkins’ pamphlet also provided a response to a recent attack by George Peacock (30) on private tutoring. Peacock had made no bones about his views:

‘The rapid growth of the private tuition in late years, which is due to various causes, is an evil of the most alarming magnitude, not merely as a great and ruinous increase of the expenses of academical education, but as threatening to supersede the system of public instruction, both in the colleges and in the university.’ (p.153)

\textsuperscript{18} Stokes, G.G. (1838-1840). MSS Stokes Collection PA 2-24, Manuscripts and University Archives, Cambridge University Library. Wood notes that Stokes’s ‘writing was so bad that he eventually became one of the first people in Britain to make regular use of a typewriter’ (27: p.53).

\textsuperscript{19} The other courses in Stokes’s notes are: Example of tracing a curve and differential equations, constrained motion of a point, impulsive force and impact, motion of two or three bodies attracting one another and lunar inequalities, astronomical instruments.

\textsuperscript{20} A generation later, Isaac Todhunter would choose the opposite path, taking up textbook writing in preference to teaching, and thereby amassed a small fortune (28).
He considered even the best coaching to be a system ‘of forced culture’ which although it ‘may accelerate the maturity of the fruit’ it was ‘inconsistent with the healthy and permanent productiveness of the tree’. His solution was to ban private tutoring altogether, although he did concede it would cause financial hardship to some talented men, and, without explicitly mentioning him by name, he did acknowledge Hopkins’ ‘great skill and pre-eminent success as an instructor of youth.’ (30: p.156).

Since a student aspiring to high honours had to engage a private tutor, an important factor in the debate, aside from the threat to the university, was the cost of private tutoring. In 1839 Ellis paid Hopkins £42 for a year’s worth of coaching, a fee which indicates that he was being coached six days a week (21: p. 87). The fact that Galton was charged £72 p.a. indicates that he was having additional coaching in the long vacation (29: p.153). In 1852 Hopkins himself estimated that to a good student the cost of private tutoring for three years was approximately £150 (1: p.100). Stokes was fortunate that Hopkins’ fees were not an issue.

Hopkins had a lasting influence on Stokes, and not only a pedagogical one. While Stokes was still an undergraduate, it was Hopkins who advised him to study hydrodynamics (3: p.9), the subject in which he began his research.

Examinations

During the early decades of the nineteenth century, the senate-house examination underwent a number of reforms. At the beginning of the century, it lasted for three days but it gradually got extended so that by the time Stokes came to sit it in 1841 it was six days, having been extended from five in 1839, with the papers getting progressively more difficult. The questions were of two types: bookwork and problems. The former required students to reproduce standard definitions, theorems and proofs, while the latter tested students’ ability to apply what they had learnt to increasingly technical and challenging problems. These were not problems to be found in the back of textbooks but problems constructed specifically for the examination, and it was not unusual for the examiners to base questions on their own research (21: p.156). Importantly, it was the problems that effectively determined the order of merit.

There were two examinations every day apart from Sunday, 2½ hours in the morning and 3 hours in the afternoon, making a total of 33 hours examination altogether. For the first two papers, students were not allowed to use the differential calculus. Every undergraduate had to take the first four papers and a failure to pass resulted in the student being ‘plucked’, i.e. not allowed to continue his studies.\textsuperscript{22}

The papers were set by two Moderators and two Examiners. They undertook essentially the same tasks, the only difference being that the Moderators were responsible for the ‘papers of original problems’, i.e. for the more difficult ones (22: p.xix). Usually at least one of the Moderators continued as either a Moderator or an Examiner in the following year, thereby ensuring continuity. In Stokes’s year, the Moderators were Alexander Thurtell (4\textsuperscript{th} Wrangler 1829) and Edwin Steventon (3\textsuperscript{rd} Wrangler 1830), and the Examiners were Henry Wilkinson Cookson (7\textsuperscript{th} Wrangler 1832), Edward...

\textsuperscript{21} Hopkins was thus earning something in region of £750 a year through his coaching, a substantial amount and a sum much higher than the stipend of most of the university professors.

\textsuperscript{22} Until 1850 students who had not obtained honours in the Mathematical Tripos were not allowed to sit the Classical Tripos. It was therefore possible for high-achieving classics students to fail to get a degree.
Brumell (3rd Wrangler 1837), none of whom made a distinguished career in mathematics. Stokes himself would be a Moderator in 1846–1848 and an Examiner in 1849.

The preparation for the examination was a punishing experience and it is little wonder that the health of students was sometimes compromised. The American Charles Bristed, who studied for the Tripos between 1841 and 1844 and wrote a book chronicling his experience, declared:

‘Indeed a man must be healthy as well as strong—“in condition” altogether to stand the work. For in the eight hours a-day which form the ordinary amount of a reading man’s study, he gets through as much work as a German does in twelve; and nothing that our students go through can compare with the fatigue of a Cambridge examination. If a man’s health is seriously affected, he gives up honors at once, unless he be a genius like my friend E[lli], who “can’t help being first.”’ (31: p.331)

Ellis suffered from poor health throughout his time at Cambridge and indeed for most of his life. Stokes was more robust. He told his daughter that ‘he never read more than eight hours a day, even before an examination’ and that ‘he had never been reduced to binding his head up with a wet towel.’ (3: p.7)

The examination produced its own casualties. In 1842 C.T. Simpson, who was second wrangler to Cayley, ‘almost broke down from over exertion ... and found himself actually obliged to carry a supply of ether and other stimulants into the examination in case of accidents’ (31: p.126). While some students were unable to stay the course altogether:

‘A singular case of funk occurred at this examination [1843]. The man who would have been second (also a Johnian) took fright when four of the six days were over, and fairly ran away—not only from the examination, but out of Cambridge, and was not discovered by his friends or family till some time after.’ (31: p.163)

The ‘man’ in question was Thomas Minchin Goodeve who had been expected to be second but had ended up as ninth, his days of absence from the Tripos thus proving not too calamitous. Indeed, as Bristed himself observed, the papers of the last two days affected the places of only the best ten or fifteen students.

When it came to Stokes’s turn, he had to tackle 175 questions over the course of the 12 papers. The papers were designated pure mathematics, natural philosophy or problems. See the Appendix for the ‘Plan of the Examination’ and copies of the first and the last papers. All the papers, together with the order of merit, are reproduced in (32: p.189–208). The solutions to the three designated ‘Problems’ papers are given in (33).

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23 Robert Leslie Ellis, the senior wrangler of 1840, said of Bristed: “I rather like him though he is vain and a little American: decidedly clever.” Diary of Robert Leslie Ellis, 25 December 1843. Trinity College Cambridge, Add Ms a.218.41.
24 Perhaps surprisingly and unlike many Tripos students, Goodeve was not put off mathematics by the experience. He went on to have a solid career teaching mathematics at a number of institutions including the Royal Military College Woolwich, Kings College London, and the Royal College of Science.
25 The papers were designated pure mathematics, natural philosophy or problems. See the Appendix for the ‘Plan of the Examination’ and copies of the first and the last papers. All the papers, together with the order of merit, are reproduced in (32: p.189–208). The solutions to the three designated ‘Problems’ papers are given in (33).
The spaces which a body describes when urged by any finite force, whether that force is determinate and immutable, or is continually increased or continually decreased, are at the very beginning of the motion in the squared ratio of the times.

In other words, the position varies as the square of the time, providing the initial acceleration is non-zero.  

Students were thus not only expected to be able to prove lemmas from the *Principia*, they were also expected to remember how they were numbered! The majority of the other questions were on algebra, the calculus, mechanics, dynamics, astronomy, hydrostatics and optics, with only a few on heat, electricity and magnetism.

The results were announced in the Senate-House on the 22nd January, ten days after the final examination. The following day the degrees were conferred by the Vice-Chancellor at a ceremony in the Senate House (see Figure 2). Although no description exists of Stokes's ceremony, Ellis recorded his experience in the previous year:  

‘When all was ready [William Hopkins] and the other Esquire Bedell made a line with their maces, and Burcham led me up [the Senate House]. Instantly my good friends of Trinity and elsewhere, two or three hundred men, began cheering most vehemently, and I reached the Vice [Chancellor's] chair surrounded by waving handkerchiefs and most head rending shouts. Burcham nervous. I felt his hand tremble as he pronounced the customary words “vobis presento hunc juvenem.” Then I took the oaths of allegiance and supremacy and I knelt before the Vice [Chancellor] who pattered over the “Auctoritate mihi &c” and then shaking hands wished me joy. I walked slowly and stiffly down the Senate House – more cheering.’

By the end of the proceedings, Ellis had turned so pale that he was made to sit down and a young woman from the crowd offered him some smelling salts! Stokes, one assumes, was made of sterner stuff.

As was common practice, the results of the examination were widely reported, both in the national press – The Times of London regularly printed a full list of the successful students – and in the local press in which often a ‘local hero’ was celebrated. It so happened that in 1841 the results caused a particular stir due to the high number of failures among classics students. The *Hull Packet* (34) took up the story:

‘The number of names on the printed list, published previously to the examination, contains 145 names; that on the return list is 117. It must not, however, be taken for granted, that all whose names were on the first list went into the Senate House, and that, consequently, 28 had the misfortune of being “plucked”. I believe that 25 is the exact number of these unhappy ones, - amongst them are some of the best classics in the University, who are debarred from the privilege of going in for the classical tripos and the medal. For the latter honour, even a junior optime cannot go in. Trinity [College] is like a little town in a roar this morning, on the disagreeable subject of a pair of their best classical scholars (of the house too) being plucked for plus and minus.’

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26 Newton, when investigating orbital motion in the *Principia*, proved propositions involving force by considering accelerations (34: p.291).

27 Diary of Robert Leslie Ellis, 15 January 1840. Trinity College Cambridge, Add Ms a.218.41. Thomas Burcham was the Father (today Praelector) of Trinity College. For further details of the ceremony, see (21: p.207-208).
The failures were the subject of correspondence in *The Times*, and Bristed (31) too remarked on them, noting that ‘the mathematical examination was very difficult and made great havoc amongst the classics.’ (p.85) Later that same year, whether or not prompted by the ‘havoc’, Peacock (30) voiced his concerns:

‘The problems which are proposed in the senate-house are very generally of too high an order of difficulty, and are not such as naturally present themselves as direct exemplifications of principles and methods and require for their solution a peculiar tact and skill, which the best instructed and most accomplished student will not always be able to bring to bear upon them. It is not unusual to see a paper of questions proposed for solution in the space of three hours, which the best mathematician in Europe would hesitate to complete in a day.’ (p.153)

Given that the statutes explicitly stated that the papers should not contain more questions than well prepared students could answer within the time allowed, the difficulty of the papers was clearly a legitimate cause for concern.29

Putting Stokes’s triumph in the context of the uproar over the stiffness of the examination gives Romilly’s remark about Stokes being ‘a very good’ senior wrangler extra resonance, especially since Romilly was a Trinity man. But Stokes’s trials were not over with his victory in the Senate-House examination.

Shortly after the results had been declared, the top wranglers knuckled down again to compete for the Smith’s Prize.30 This took the form of further examination papers, each one of which was sat over the course of a day and was set by a different examiner. Unlike the Tripos, the questions were usually geared towards evincing an original or creative approach.31 In general only the most distinguished wranglers sat the examination, so the numbers entering were usually small, and it was not unknown for the number of candidates to be the same as the number of prizes.

In 1841 the Smith’s Prize examiners were the three mathematics professors: Peacock, Challis and King. Each paper consisted of around 25 questions ranging over a variety of subjects from pure mathematics to the construction of astronomical instruments, and often included a discursive element. For example, one question on Peacock’s paper asked for a solution to the functional equation:

\[(\varphi x)^2 = \varphi(2x) + 2,\]

another asked for an explanation of the construction of the ‘Huyghenian eye-piece’, while Peacock’s final question asked for short dissertations on half a dozen different topics including the theory of parallels and the theory of the rainbow. Detailed knowledge of classic texts was assumed. One of King’s questions gave a description of a problem in Newton’s *Principia* and then referred to different solutions by Newton in different editions of the *Principia* and to three further solutions by Lagrange in his *Calcul des Fonctions*, none of which were given but all of which were expected to be known.

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28 *The Times*: 25 January 1841, p.5, and 29 January 1841, p.5; the former supporting the status quo as ‘judicious and right’, and the latter vehemently opposing it.
29 The relevant statute stated: ‘That there be not contained in any paper more Questions than Students well prepared have generally been found able to answer within the time allowed for that Paper.’ The problems with the examination—both with its form and its content—generated much discussion which culminated in major reforms in 1848. Not only was the examination revised but a formal syllabus of mathematical studies was also introduced. For details see (21: 101-103).
30 For details of the history of the Smith’s Prize together with a list of winners, see (19).
31 Famously, in 1854 Stokes himself set a Smith’s Prize question which included the first appearance in print of what would later become known as Stokes’s theorem.
Perhaps predictably, Challis’ paper focused more on applications and physical astronomy, including several questions on subjects such as the wave theory of light and the lunar theory.

Once again, Stokes won the day.\textsuperscript{32} Although the prize was worth £25, its real value was in the academic prestige attached to winning. The competition was a much sterner test than the Tripos and although to the outside world a prizeman did not carry the cachet of a senior wrangler, within the confines of the Cambridge mathematical community the honour was recognised as the ultimate achievement.\textsuperscript{33}

On the 23\textsuperscript{rd} January, with all the results announced, Pembroke celebrated Stokes’s remarkable success by holding a grand dinner in his honour and making him a Fellow of the College. The world was now his oyster. As one relative wrote to him,\textsuperscript{34} he now had only to determine whether he would be ‘Prime Minister of England, the Lord Chancellor or Archbishop of Canterbury’.

**Appendix: The plan of the Senate-House examination together with the first and the last question papers.**

The plan is taken from the Cambridge University Calendar for 1841 (38), while the two Tripos papers are taken from the Cambridge University Magazine (39).

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\textsuperscript{32} The second prize was won by the second wrangler, Henry Cadman Jones of Trinity, who afterwards made a career at the Bar. Cadman was present at the Stokes’s Jubilee in Cambridge in 1899 and the fact that he was able to come and offer his congratulations was considered one of the ‘most pleasing features’ of the occasion (37: p.25).

\textsuperscript{33} When William Thomson won the first Smith’s Prize having come a disappointing second in the Senate-House, William Hopkins wrote to Thomson’s father ‘The examination, as you are probably aware, is altogether of a higher character than that of the Senate-House, being, in fact, intended to furnish a higher test of the merits of the first men.’ (19: p.280)

\textsuperscript{34} Letter from Nicholas Fenwick to Stokes, 1 February 1841. Stokes Manuscripts. Add. MS 7656. F69.
SENATE-HOUSE EXAMINATION.

WEDNESDAY, Jan. 6, 1841.....9 to 11½.

[N.B. The Differential Calculus is not to be employed.]

1. The angles which one straight line makes with another upon the one side of it are either two right angles, or are together equal to two right angles.

2. Describe an isosceles triangle, having each of the angles at the base double of the third angle. Shew that if the points of intersection of the circles, in Euclid's figure, be joined with the vertex of the triangle and with each other, another triangle will be formed similar and equal to the former.

3. State the difference between interest and discount, and find the discount on £397, 6s. 8d. due 9 months hence, at 4 per cent. per ann.

4. Find the number of feet, inches and parts, in the side of a square whose area is 14 feet 11 inches.

5. Define the least common multiple of two quantities, and prove that it measures every other common multiple of them. Find the least common multiple of $3x^3 - 5x + 2$ and $4x^3 - 4x^2 - x + 1$.

6. Solve the following equations:

\[
\begin{align*}
(1) \quad & \frac{1}{x-2} - \frac{2}{x+2} = \frac{3}{5}, \\
(2) \quad & x - y = 12 \quad \text{and} \quad x^2 + y^2 = 74
\end{align*}
\]

and shew that a quadratic equation cannot have more than two roots.

7. Shew that if any one term of the series for $(1 + x)^\pm n$, where $x$ is a proper fraction, be numerically less than the one which precedes it, the same is true of all the following terms.

Find the coefficient of $x^r$ in the expansion of \[(x - \frac{1}{x})^{3n-1}.

8. Prove that $\tan A = \pm \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}$, and shew which of the two signs ought to be used in particular cases.

9. Explain the use of subsidiary angles in adapting algebraical formulæ to numerical calculation.

Ex. (1) $a \sin x + b \cos x = c$, \hspace{1cm} (2) $x = \sqrt{a - b} + \sqrt{a + b}$.

10. Find the equation to a straight line passing through a given point in the axis of $x$, and making an angle of $45^\circ$ with the straight line whose equation is \[\frac{x}{a} - \frac{y}{b} = 1.

11. In the parabola shew that $SY^2 = AS \cdot SP$, and prove that there is only one point where the focal distance is perpendicular to the tangent.

12. Find the equation to the ellipse, referred to the centre as origin of co-ordinates, and a system of conjugate diameters as axes. Find
the angle between the axes when the equation is reduced to the form

\[ x^2 + y^2 = c^2. \]

13. Shew how to transform an equation into one which shall want its second term. Take away the second term from the equation

\[ x^3 - 6x^2 + 12x + 19 = 0, \]

and find the three roots of the resulting equation.

14. Prove that in a spherical triangle,

(1) Any one side is greater than the difference between the two others.

(2) The sum of any two angles is greater or less than 180° according as the sum of the two opposite sides is greater or less than 180°.

15. Find an expression for the side of a spherical triangle in terms of another side and the angles which are adjacent to that side.

16. How are different systems of logarithms distinguished? Having given the logarithm of a number in one system, shew how to find it in another. Assuming the series for \( \log (1 + x) \) prove that

\[
\log_a (x + 2a) = 2 \log_a (x + a) - \log_a x - \left\{ \frac{a^2}{(x + a)^3} + \frac{a^4}{3(x + a)^4} + \frac{a^6}{5(x + a)^5} + \text{&c.} \right\}
\]

17. Prove that

\[
\frac{\sin \theta}{\theta} = \left(1 - \frac{\theta^2}{2}\right) \left(1 - \frac{\theta^3}{2^2 \cdot 2}\right) \left(1 - \frac{\theta^4}{3^2 \cdot 3}\right) + \text{&c.}
\]

Explain fully the advantages of a set of tables which give the numerical values of \( \frac{\sin \theta}{\theta} \), when \( \theta \) is small.
TUESDAY, Jan. 12, 1841......1 to 4.

1. Explain how Newton's method of approximation may be adjusted so as to furnish the real roots of an equation to any required degree of accuracy.

2. Explain how the position of the invariable plane of a system of material particles is determined. Extend the method to the case of a system of bodies of finite magnitude. Upon what suppositions may the position of the invariable plane of the solar system be found, and how far are they correct?

3. A given number of concentric spherical shells, the thickness of each being uniform, are separated by non-conducting media; find the effect of their mutual influence when they are electrized.

4. Explain the nature of developable and twisted surfaces, and shew that the hyperboloid of one sheet is a twisted surface.

5. State the results which have been obtained in the theoretical investigation of the earth's figure independently of any assumed law of the density, and calculate the length of a degree of latitude at any place on the earth's surface in terms of the length of a degree at the equator.

What reason is there for supposing that the variations of pressure in the interior of the earth are proportional to the squares of the densities?

6. Find the variation of \( \int_x^y F(x, y, z, \frac{dy}{dz}, \frac{dx}{dz}, \&c.) \) subject to the condition that \( f(x, y, z, \frac{dy}{dz}, \frac{dx}{dz}, \&c.) = 0 \), and apply the result to determine the curve of quickest descent in a medium resisting as any function of the velocity.

7. Explain the effect of eccentrical refraction through a lens in producing distortion. A ray of light is incident upon a glass lens in a direction parallel to the axis; find the tangent of the angle which the emergent ray makes with the axis, to a second approximation. Find also the ratio between the radii when the coefficient of the small term, for a given focal length, is a minimum.

8. Assuming the properties of the axes of elasticity, find the velocity of transmission of a wave of light after refraction at the surface of a uniaxal crystal. If the crystal be bounded by planes perpendicular to the axis investigate the difference of retardations of the ordinary and extraordinary rays. State briefly how it is shown by experiment that plane and circularly polarized light differ from each other.

9. Enunciate the propositions by which it is proved (1) that the eccentricities of the planetary orbits, (2) that their inclinations to the ecliptic, are always small. How are they affected by the circumstance that all the planets revolve in the same direction about the sun? Shew that the variations of the inclination and longitude of the node are given by the equations

\[
\frac{d\left(\tan i_i\right)}{dt} = \frac{n_i a_i}{\mu \tan i_i \sqrt{1-e_i^2}} \frac{dR}{d\Omega_i},
\]

\[
\frac{d\Omega_i}{dt} = -\frac{n_i a_i}{\mu \tan i_i \sqrt{1-e_i^2}} \frac{dR}{dt}.
\]
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(38) Cambridge University Calendar. Published by Benjamin Flower, Cambridge, 1841, p16.


**Figure Captions**

Figure 3.1 Stokes as senior wrangler in 1841 (Courtesy of the Master and Fellows of Trinity College, Cambridge).

Figure 3.2 Arthur Cayley, senior wrangler 1842, being presented to the Vice-Chancellor, William Whewell, at the Senate House to have his degree conferred. Andrew Warwick considers it probable that the figure on the left with his right hand on the table is William Hopkins (21: p.207). Taken from Huber, V.A., *The English Universities* (tr. F. Newman), Vol. 1, William Pickering London, 1843 frontispiece.