Large Families of Ternary Sequences with Aperiodic Zero Correlation Zone Sequences for a Multi-Carrier DS-CDMA System

Conference or Workshop Item

LARGE FAMILIES OF TERNARY SEQUENCES WITH APERIODIC ZERO CORRELATION ZONES FOR A MC-DS-CDMA SYSTEM

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Abstract - A new method for generating families of ternary spreading sequences is presented. The sequences have aperiodic zero correlation zones and large families are created for a specific sequence length. The sequences are proposed as spreading sequences to provide high capacity and cancel multipath and multiple access interference (MAI) in a single carrier (SC) or multi-carrier (MC) direct-spread code division multiple access (DS-CDMA) system. A Multi-carrier DS-CDMA system is simulated that employs the new sequences as spreading sequences in a multipath channel. Bit error rates (BER) and frame error rates (FER) for a range of E_b/N_o values are presented and it is demonstrated that the proposed sequences improve the BER and FER performance when used in place of masked Walsh Codes for the frequency selective fading channel evaluated, when a single correlator receiver is used on each sub-carrier.

Keywords - Ternary sequences, Multicarrier DS-CDMA, zero correlation zones.

I. INTRODUCTION

CDMA is the underlying technology of many of today's communication systems, including emerging third generation (3G) cellular systems. The properties of the spreading sequences employed in these systems impinge on the basic bit error ratio and it is therefore constructive to design and develop these sequences in parallel with the systems themselves, with system performance, capacity and complexity in mind.

cdma2000 3x or 3x-MC (multi-carrier) is a 3G MC-DS-CDMA system, where narrowband DS-CDMA signals are transmitted using different sub-carriers. Three carriers each of 1.25MHz bandwidth are used which means its total bandwidth is similar to that of the single carrier wideband CDMA system also being used for 3G. A RAKE receiver is used on each carrier to capture the power from the delayed paths caused by the multipath channel. The main driving force behind a multi-carrier option in the 3G mobile cellular standard is its backward compatibility to the CDMA based 2G system, IS-95, already in use and it is seen as a natural path of evolution for operators with existing IS-95 networks.

A. Zero Correlation Zone Sequences

Zero correlation zone (ZCZ) sequences have recently been proposed and investigated as a means of reducing multiple access interference (MAI) in CDMA systems. These are sequences that have a window around the in-phase correlation position where the correlation sidelobes are zero. This means that the interference between users separated by delays that are within this window or interference due to delayed replicas of a users signal due to the multipath channel will be eliminated. Much of the research has been based upon binary sequences [1] and lower bounds for the ZCZ of binary sequences were presented in [2]. One of the most significant problems of these codes is generating a sufficient number of sequences with a useful ZCZ. To deal with this problem, Cha [3] introduced a class of ternary sequences, which have elements from the alphabet {0, +1, -1}, with larger ZCZs then the comparative binary sequences and are constructed by chip shifting a ternary preferred pair (TPP). The above sequences have a zero correlation window for the periodic correlation functions. In a DS-CDMA system both periodic and odd correlation values are of equal importance and therefore it is desirable to find sequences that minimise both these functions. The sequences proposed by Li [4] have excellent correlation parameters in that they have aperiodic zero correlation zones and also the correlation values outside this window are restricted to +/-1. These codes are generated by the insertion of long zero strings of varying lengths between elements of an orthogonal code group. The codes proposed in this paper, are generated by the insertion of equal length zero padding vectors between elements of multiple orthogonal seed sets and utilizing the ZCZ to create orthogonality between the subsets created from each seed. Large family sizes with aperiodic correlation zones are generated

The sequences are proposed as spreading sequences for a single carrier (SC) or multi-carrier (MC) quasi-synchronous DS-CDMA system or synchronous DS-CDMA system operating in a multipath channel. By implementing a MC-DS-CDMA system, within the same overall system bandwidth, the number of resolvable paths per sub-carrier is reduced due to the reduction in chip rate, making it easier to find large enough sets of sequences with the required zero correlation zones. In this paper a downlink MC-DS-CDMA system is simulated that employs the new ternary spreading sequences to improve the error performance in a frequency selective channel.

II. SEQUENCE CONSTRUCTION

A. Generation Method

The construction method starts with the generation of multiple orthogonal ZCZ ternary subsets of sequences. Each ternary subset is created from a different binary seed set, where M is the number of seed sets and subsequently ZCZ subsets. Equal length zero padding vectors are inserted between elements of each sequence of every seed set. The seed sets are of size $L_0 \times L_0$ and although sequences within each seed are orthogonal, no orthogonality between subsets is required. Therefore, depending on the overall number and properties of the sequences required any number of sequences may be used. Seed sequences may even be re-used, providing the size of the correlation sidelobes outside the ZCZ is not important. The construction method can also be extended to orthogonal quadriphase seed sets, of which greater numbers exist, that generate 5-valued ZCZ sets of sequences.

The length of the zero padding Z_0 inserted between elements of the sequences determines firstly, the overall length of the sequences generated and secondly, their ZCZ value. Each seed set is transformed into a subset of sequences of length $L = L_0(Z_0+1)$ with an aperiodic ZCZ of $Z_{cz}^{(0)} = 2Z_0 + 1$. The initial ZCZ of these individual subsets can then be utilized to provide orthogonality between sequences of all subsets and provided that the original ZCZ is large enough, also provide ZCZ properties across all subsets. This is achieved by chip shifting each subset a different number of chips, τ_m for $m = 1,2...M$, where M is the number of seed sets. Each sequence created from the same seed set is shifted the same number of chips. s_m^l represents the *l*-th sequence, where $l = 1,2...L_0$, created from zero padding the *m*-th seed set and S_m is the subset of sequences. The overall set of ML_0 sequences is given by P_M and their overall ZCZ is given in (2).

$$P_M = \begin{bmatrix} S_1(\tau_1) \\ S_2(\tau_2) \\ \vdots \\ S_M(\tau_M) \end{bmatrix} \qquad (1)$$

$$Z_{cz}^{(M)} = 2Z_M + 1 \qquad (2)$$

Where Z_M is the minimum number of zeros between elements of all sequences.

$$Z_M = \left\lfloor \frac{Z_0+1}{M} \right\rfloor - 1 \qquad (3)$$

When a specific ZCZ is required, Z_M can be used to calculate the chip-shift applied to each set, $\tau_m = (Z_M+1)(m-1)$ and to ensure the complete set of sequences have ZCZ properties, i.e. $Z_M \geq 1$ the initial design condition given in (4) must be met.

$$\left\lfloor \frac{Z_0+1}{M} \right\rfloor \geq 2 \qquad (4)$$

It is worth noting that although the cross-correlation ZCZ reduces as M increases and more sets are combined, the autocorrelation ZCZ remains the same for each sequence and is independent of the value of M. In a DS-CDMA application, this means the self-user multipath rejection capability remains unchanged, regardless of the number of sequences/users. An example set of sequences is illustrated below for *M*=2, $L_0 = 4$ and $Z_0 = 3$.

$$s_1 \Rightarrow \begin{bmatrix} +++- \\ ++-+ \\ +-++ \\ +--- \end{bmatrix} \Rightarrow S_1(\tau_1) \Rightarrow \qquad s_2 \Rightarrow \begin{bmatrix} ++++ \\ +-+- \\ ++-- \\ +--+ \end{bmatrix} \Rightarrow S_2(\tau 2) \Rightarrow$$

$$P_2 = \left\{ \begin{matrix} \begin{bmatrix} 000+000+000+000- \\ 000+000+000-000+ \\ 000+000-000+000+ \\ 000+000-000-000- \end{bmatrix} \\ \begin{bmatrix} 0+000+000+000+00 \\ 0+000-000+000-00 \\ 0+000+000-000-00 \\ 0+000-000-000+00 \end{bmatrix} \end{matrix} \right\}$$

Fig.1. Example sequence set

B. Sequence Properties

Obviously, the size of the ZCZ depends on the length of the initial zero vector and the number of seed sets used and these in turn affect the length and the number of sequences respectively. Also influencing the size of the ZCZ is the size of the orthogonal seed sets used. Different sizes and numbers of seeds can be used to generate sequences of the same length with varying properties. The larger the seed set, the less seeds required to generate the same number of sequences, but also the shorter the zero padding can be for the sequence length to remain constant. This affects firstly, the maximum ZCZ that the sequences will exhibit, in that smaller seed sizes give rise to higher achievable ZCZ values. For a given sequence length, the maximum ZCZ for any seed set size is when M=1. The maximum attainable ZCZ in terms of the overall sequence length and the seed set size is given in (5).

$$Z_{cz}^{(1)} = \frac{2L}{L_0} - 1 \qquad (5)$$

As an example, the family sizes for a final sequence length of 64 constructed from seed sets of sizes 4, 8 and 16 for different ZCZ sizes are illustrated in Table 1. As well as the ZCZ values being based upon the aperiodic correlation functions as opposed to just the periodic such as the sequences based on ternary preferred pairs (TPP), the new method produces larger family sizes, although the maximum ZCZ given in [3] by (0.75L+1) is not always obtainable.

Table 1
Family sizes for various ZCZs for length 64 sequences

N	**ZCZ**							
ZCZ	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**
Sequence types:								
TPP	24	16	12	8	8	6	6	6
seed size 4x4	32	20	16	12	8	8	8	4
seed size 8x8	32	16	16	8	8	8	8	-
seed size 16x16	32	16	16	-	-	-	-	-
ZCZ	**19**	**21**	**23**	**25**	**27**	**29**	**31**	**-**
Sequence types:								
TPP	4	4	4	2	2	2	2	-
seed size 4x4	4	4	4	4	4	4	4	-
seed size 8x8	-	-	-	-	-	-	-	-
seed size 16x16	-	-	-	-	-	-	-	-

The second effect of using differently sized seed sets is the change in the deficiency of the sequences generated. The deficiency $\delta(S)$ of a ternary sequence s_m^l is defined in [5] to be the number of zero elements in s_m^l. This determines the energy efficiency of the sequences, η.

$$\eta = \frac{E}{L \max\left\{\left|s_m^l\right|^2\right\}} \tag{6}$$

Where E is the sequence energy.

For binary sequences (or complex sequences with a constant magnitude of unity) then, $E=L$, and η is maximised and equal to 1. For ternary sequences the numerator of (6) is reduced by $\delta(S)$ and so the energy efficiency is also reduced.

Unlike the sequences proposed in [4] the correlation values of the new sequences outside the ZCZ window are not restricted to +/-1 and are particularly high when seed sequences are re-used, but the ratio of numbers of sequences to sequence lengths is comparatively high. Therefore, the sequences are proposed for applications that require a large family of sequences where the relative delay between users or paths is strictly limited within the ZCZ.

III. SIMULATION

A MC-DS-CDMA system has been simulated to evaluate the performance of the new ZCZ sequences. As it is proposed for a downlink application the system is synchronous. A users data is generated in frames and each frame is firstly encoded using a rate 1/2 convolutional code with constraint length 9 and generator polynomial [561;753]. The resulting codeword is then serial-to-parallel converted into 3 lower rate bit streams; each sub-stream represents data that modulates a different carrier. Each data sub-stream is spread by the user's allocated orthogonal spreading code. For the purposes of evaluating the effect of the ZCZ sequences on the Inter-symbol Interference (ISI) introduced due to multipath on each sub-carrier and with the aim of optimising the ISI cancellation, the carriers are assumed not to overlap, as in [7], and therefore the adjacent carrier interference has been assumed to be zero. In practical systems, sub-carriers may be chosen to be orthogonal and therefore without multipath fading do not interfere and yet have good bandwidth efficiency. However, once multipath occurs, interference between carriers may be introduced and as a function of the separation of the carriers [8].

The channel is modelled as a tapped delay line consisting of a number of fixed delay resolvable Rayleigh faded paths. The complex low-pass impulse response of the channel for carrier m and user k is given below.

$$h_{k,m}(t) = \sum_{p=1}^{P} \beta_{m,p} e^{j\gamma_{m,p}} \delta\left(t - \tau_p\right) \tag{7}$$

Where P is the number of resolvable paths and $\beta_{m,p}$, $\gamma_{m,p}$ and τ_p are the path gain, phase and time delay of the m-th carrier and p-th path. $\beta_{m,p} e^{j\gamma_{m,p}}$ is a complex Gaussian random variable with zero mean and variance σ_p. Due to the system representing the downlink, all users experience the same channel. Therefore β and γ are only independent for different carriers and paths and τ is assumed to be only independent for each path.

Simulations have been performed for a 3-path channel (P=3), each path separated by one chip and with equal powers. The received signals are despread on each sub-carrier using a single correlator receiver and the 3 output bit streams parallel-to-serial converted back into a single data stream and soft-decision decoded.

A spreading factor of 64 is used in all the simulations and the performance of a system employing the ZCZ ternary sequences is compared to the performance when using the channelisation and scrambling codes as used in cdma2000. Two types of ZCZ ternary sequences, created using differently sized seed sets and zero padding, are evaluated. The first is created from a Hadamard matrix seed with L_0=16 and a zero padding vector length Z_0=3. The second type has L_0=4, M=4 and Z_0=15. Both have a ZCZ larger than or equal to the delay spread.

In the masked Walsh Code simulations, the spreading factor of the Walsh Codes is also 64 and length 2^{15} chip PN codes, are used to spread the data in quadrature at the same chip rate as the Walsh Code spread data. Results are illustrated for various numbers of simultaneous users.

IV. RESULTS

The results illustrated in Fig.2 and Fig.3 show the BER and FER performances for Walsh codes masked by PN scrambling codes (mWC) and the ternary ZCZ sequences (ternZCZ) with a 16x16 seed set. The results are illustrated for 1,4 and 8 simultaneous users of the system.

Significant improvements in BER and FER performances can be seen, for the channel evaluated, when the new ZCZ Sequences are used in place of masked Walsh codes. The improvement is achieved without the use of RAKE receivers so the complexity of the system remains low.

As the number of users increases, the improvement over the masked Walsh Codes increases due to little or no additional multiple access interference being introduced as the number of users rises.

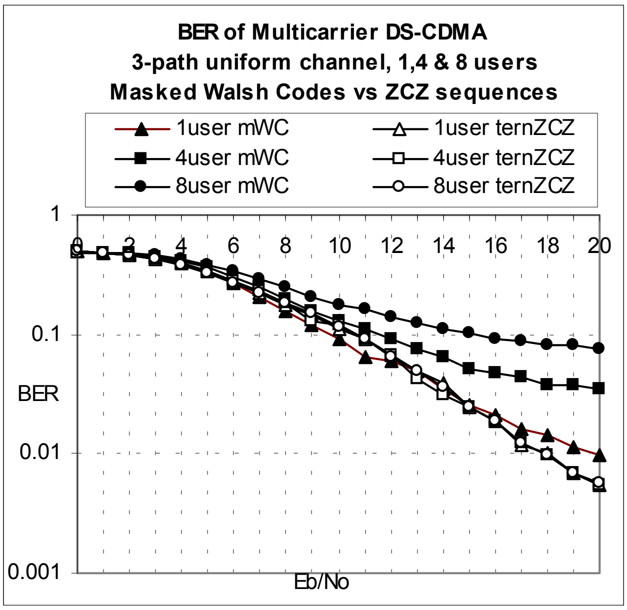

Fig. 2. BER Performances in a multipath Channel

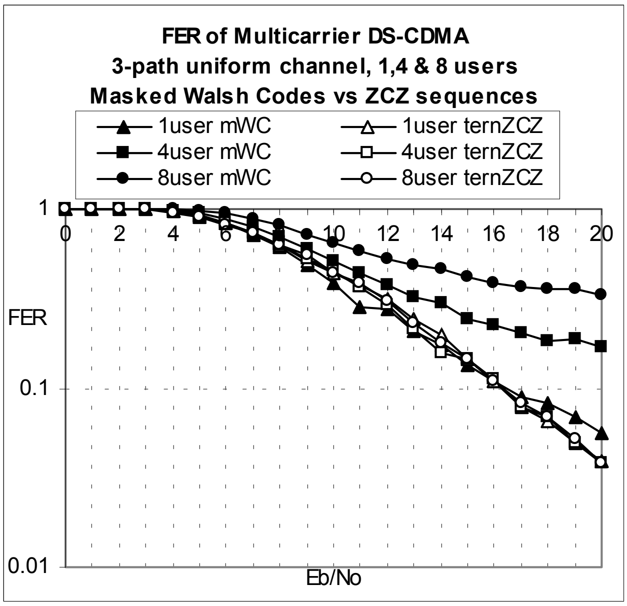

Fig. 3. FER Performances in a multipath Channel

Fig. 4 and Fig. 5 compare the BER and FER performances of the 2 types of ternary ZCZ sequences, created from the different sized seed sets, as described previously. Even though the capacity of the ternary simulations has been doubled up to 16 users, both ternary simulations are seen to have a lower BER and FER than the Walsh Code result for 8 users. The Performances of the 2 ternary type sequences is very similar, but the sequences created from the 16x16 seed set, that have a lower deficiency, have a slightly improved performance over those created from multiple 4x4 seed sets.

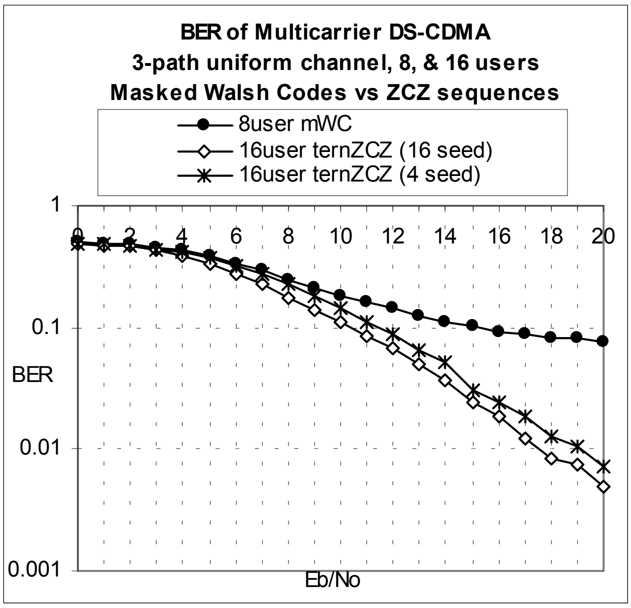

Fig. 4. BER comparison for different ternary sequences

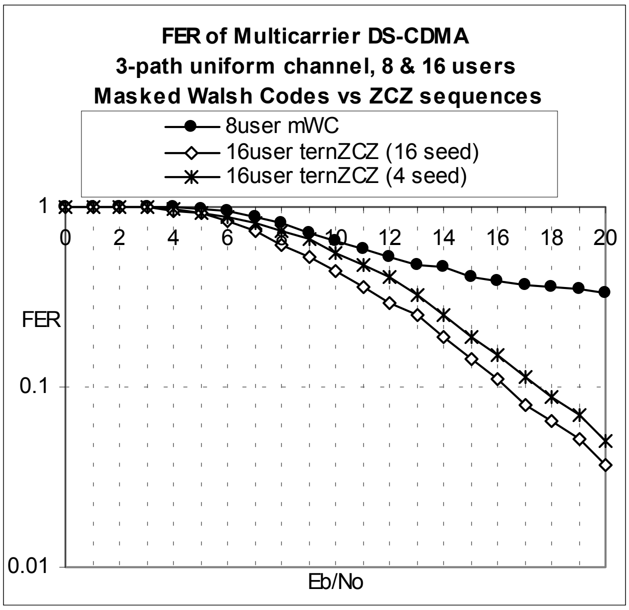

Fig. 5. FER comparison for different ternary sequences

V. CONCLUSIONS

In summary, a novel method for generating sets of ternary orthogonal sequences with zero correlation zone properties has been described. The beneficial properties of the sequences include aperiodic zero correlation properties for the elimination of multipath interference, large family sizes for high capacity and flexible sequence lengths and corresponding zero correlation values.

The application of the ternary ZCZ sequences to a synchronous MC-DS-CDMA system has been presented. A MC-DS-CDMA system based upon the cdma2000 3x downlink has been simulated. The simulated results show improvement in BER and FER performance, for the 3-path channel evaluated, when the new sequences are used in place of masked Walsh Codes. The improvement in performance increases as the number of users increases.

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