Speed of sound in nitrogen as a function of temperature and pressure

Journal Item

Speed of sound in nitrogen as a function of temperature and pressure (L)

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(Received 26 May 2005; revised 15 June 2005; accepted 16 June 2005)

Speed of sound measurements in nitrogen by Younglove and McCarty [J. Chem. Thermodynam. **12**, 1121–1128 (1980)] are revisited and an empirical polynomial equation for the speed of sound is derived. The polynomial coefficients differ from those given by Wong and Wu [J. Acoust. Soc. Am. **102**, 650–651 (1997)] with the result that discrepancies between predicted and measured values at low temperatures are reduced. The maximal error over the complete temperature and pressure range from 80 to 350 K and 0.031 to 0.709 MPa is reduced from 5.38% to 0.78%. © *2005 Acoustical Society of America.* [DOI: 10.1121/1.2000748]

PACS number(s): 43.58.Dj [AJZ] Pages: 1272–1273

I. INTRODUCTION

Recently, there has been an increase in interest in the physical properties of nitrogen gas at low temperatures in the context of planetary science. Wong and Wu reported a polynomial expression for the speed of sound in nitrogen as a function of temperature and pressure.[1] They used the values from 150 to 330 K as measured by Younglove and McCarty[2] and obtained a remarkable accuracy of 380 ppm in that region. At some low temperatures and high pressures, however, the deviation between measured and calculated values can be as high as 5.38%. In our search for a reasonably accurate function specification applicable to a wider range of temperatures and pressures, we have calculated a new polynomial fit to the complete set of Younglove and McCarty's measurements.

II. TWO-DIMENSIONAL POLYNOMIAL APPROXIMATION

We assume that the speed of sound c in nitrogen gas as a function of temperature T and pressure p can be approximated by a polynomial expression of the form

$$c(p,T)=\sum_{i=0}^{m}\left(\sum_{j=0}^{n}a_{ij}T^j\right)p^i, \tag{1}$$

where the a_{ij} need to be found by matching the $c(p,T)$ function to experimental data. We assume throughout this paper that p is given in MPa and T in K.

Wong and Wu used a multicolumn coefficient curve matching procedure in the p, T domain, first defining coefficients A_i such that a least-squares fit is applied to the pressure-dependent measurements at temperatures T_k assuming

$$c(T_k)=\sum_{i=0}^{n}A_i p^i, \quad k=0,1,2,\ldots . \tag{2}$$

Then the a_{ij} can be found using

$$A_i=\sum_{j=0}^{n}a_{ij}T^j. \tag{3}$$

The function specification uses selected measurements by Younglove and McCarty at temperatures from 150 to 330 K, and is extremely accurate within this range, with a maximum deviation of 380 ppm. This is comparable with the experimental accuracy of 300 ppm. At lower and higher temperatures, however, this function specification is considerably less accurate, particularly with increasing pressure. We found deviations of more than 5% at 110 K and 1.44 MPa. An investigation revealed that the A_i column coefficients in the region from 150 to 330 K take values such that $|A_i| > |A_{i+1}|$, but this is not the case for very low and very high temperatures, explaining the higher error with increasing pressure.

We have used the full set of measurements by Younglove and McCarty. Temperatures were adjusted from the IPTS-68 to the ITS-90 system using an eighth-degree fit.[3] Given the magnitude of errors over the complete p,T range, however, we found this correction to be of minor importance. We then minimized the function of $(m+1)\times(n+1)$ variables

$$L=\sum_{\ell=1}^{237}(c_\ell - c(p_\ell,T_\ell,a_{11},a_{12},\ldots a_{mn}))^2, \tag{4}$$

where c_ℓ is the ℓth of the 237 measurements by Younglove and McCarty and $c(p_\ell,T_\ell,\ldots)$ is the speed of sound as calculated by Eq. (1). L being a quadratic form, $\partial L/\partial a_{ij}$ can be readily derived, and the solution can be found using conjugate gradients or variants of Newton's method. We used the latter, with Wong and Wu's values as a starting guess.

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TABLE I. Polynomial coefficients for our two-dimensional function specification for the speed of sound in nitrogen.

j	a_{1j}	a_{2j}	a_{3j}	a_{4j}
0	6.721 128E+01	−1.908 177E+02	5.908 631E+01	−6.511 627E+01
1	1.820 085E+00	2.971 323E+00	−1.063 665E+00	1.218 812E+00
2	−5.713 575E−03	−1.699 747E−02	6.173 142E−03	−7.949 396E−03
3	1.320 687E−05	4.219 848E−05	−1.390 550E−05	2.187 479E−05
4	−1.265 187E−08	−3.806 689E−08	9.852 826E−09	−2.165 669E−08

III. RESULTS

The polynomial coefficients a_{ij} for Eq. (1) with $n=4$, $m=3$ are given in Table I. The maximal deviation from Younglove and McCarty's measurements has been reduced to 0.78%. As the degree and order of our fit are only 4 and 3, respectively, some trade-offs had to be made in the midtemperature region from 150 to 330 K. Here the maximal deviation is 0.52%. In moderate temperature regimes, Wong and Wu's approximation remains more accurate. However, if we compare our fit to the measurements made by Ewing and Trusler,[4] the maximum deviation is reduced to only 0.22% in the region from 80 to 300 K. This value increases to 0.88% if we include their measurements at 373 K. In extreme conditions like those encountered in planetary science, our function specification maintains its accuracy over the whole range covered by measurements.

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