Families of Complementary Distributions

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Families of complementary distributions

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SUPPLEMENTARY MATERIAL: proofs of results in Section 3

Using

\[ E_H(T^2) = 1 - 2 \int_0^1 tH(t) \, dt \quad \text{and} \quad \int_0^1 tH^{-1}(t) \, dt = E_H\{TH(T)\}, \]

gives rise to the following proof:

Proof of Theorem 3.5

\[ E_{G_p}(X^2) = 1 - 2 \int_0^1 xG_p(x) \, dx \]

\[ = 1 - 2 \left\{ \frac{(1 - p) / 3 + (2p - 1)}{p} \int_0^1 xM_p^{-1}(x) \, dx \right\} \]

\[ = 1 - 2 \left\{ \frac{(1 - p) / 3 + (2p - 1)E_{M_p}\{SM_p(S)\}}{p} \right\} \]

\[ = 1 - 2 \left\{ \frac{(1 - p) / 3 + (2p - 1)}{p} \times \right\} \]

\[ \left[ p^2 E_{F\{VF(V)\}} + p(1 - p)(1 + E_{F\{V^2\}})/2 + (1 - p)^2/3 \right] / p \]

\[ = p(2p + 1)/3 - 2p(2p - 1)E_{F\{VF(V)\}} - (2p - 1)(1 - p)E_{F\{V^2\}}. \]

The theorem results by elementary methods.

Using

\[ E_H(T^3) = 1 - 3 \int_0^1 t^2H(t) \, dt \quad \text{and} \quad \int_0^1 t^2H^{-1}(t) \, dt = E_H\{TH^2(T)\} \]

the proof of Theorem 3.6 ensues:

Proof of Theorem 3.6

\[ E_{G_p}(X^3) = 1 - 3 \int_0^1 x^2G_p(x) \, dx \]

\[ = 1 - 3 \left\{ \frac{(1 - p) / 4 + (2p - 1)}{p} \int_0^1 x^2M_p^{-1}(x) \, dx \right\} \]

\[ = 1 - 3 \left\{ \frac{(1 - p) / 4 + (2p - 1)E_{M_p}\{SM_p^2(S)\}}{p} \right\} \]

\[ = 1 - 3 \left\{ \frac{(1 - p) / 4 + (2p - 1)}{p} \times \right\} \]

\[ \left[ (1 - p)(p^2 + 2p + 3)/12 + p^3 E_{F\{VF^2(V)\}} + p^2(1 - p)E_{F\{V^2F(V)\}} \right. \]

\[ + \left. p(1 - p)^2 E_{F\{V^3\}}/3 \right] / p \]

\[ = p(1 + p + 2p^2)/4 - 2p - 1 \times \left[ 3p^2 E_{F\{VF^2(V)\}} \right. \]

\[ + \left. 3p(1 - p)E_{F\{V^2F(V)\}} + (1 - p)^2 E_{F\{V^3\}} \right]. \]
Therefore,

\[ S_{G_p}(X) = p(1 + p + 2p^2)/4 - (2p - 1) \times [3p^2E_F\{VF^2(V)\} + 3p(1 - p)\{VF^2(V)\} + (1 - p)^2E_F(V^3)] \]

which can be rearranged to give the result of the theorem.

**Proof of Theorem 3.7**

Further manipulation completes the proof.

**Proof of Theorem 3.8**

Further manipulation completes the proof.
\[(1 - p)(4p - 4p^2 - 1)/2 + 6p(2p - 1)(1 - p)\left[p^2 E_F\{VF(V)\}\right.
+ p(1 - p)\left\{1 + E_F(V^2)\right\}/2 + (1 - p)^2/3] - 6(1 - p)^2(2p - 1)\times
\left[(1 - p)(p^2 + 2p + 3)/12 + p^3 E_F\{VF^2(V)\} + p^2(1 - p)E_F\{V^2 F(V)\}\right.
+ p(1 - p)^2 E_F(V^3)/3] - 6(1 - p)(2p - 1)^2 \left[p^2 E_F\{V^2 F(V)\}\right.
+ p(1 - p)\left\{1 + 2E_F(V^3)\right\}/3 + (1 - p)^2/4]
+ 3p(2p - 1)^2 \left\{(1 - p)/3 + pE_F(V^2)\right\}
- 2(2p - 1)^3 \left\{(1 - p)/4 + pE_F(V^3)\right\}
- p^2(2p - 1) \left\{(1 - p)/2 + pE_F(V)\right\}/p^3
\]

and the result follows almost immediately.